Chapter 8
Indefinite Noun Phrases
Any
Expanded Account of Quantifiers

A. Indefinite Noun Phrases

1. Indefinite Articles

In English, and many other languages, a common-noun-phrase (CNP) may be prefixed by an indefinite article, the resulting phrase being what may be called an *indefinite noun phrase* (INP). The following are example sentences from English, in which 'a' serves as an indefinite article.

that is **a** dog
Jay owns **a** dog
a dog is in the yard
there is **a** dog in the yard
every man who owns **a** dog feeds it
a dog is happy if it is well-fed
a dog is **a** mammal
a dog can hear sounds **a** human can't
Jay is looking for **a** dog

Note in particular that, if we delete the word 'a', we obtain phrases that standard English regards as syntactically ill-formed.[1] On the other hand, there are many languages that lack indefinite articles, the biggest of which are Latin, Russian, and Mandarin, which freely admit sentences like 'that is dog'.

Furthermore, even English eschews indefinite articles when the common-nouns are plural-nouns or mass-nouns,[2] as in the following examples.

those are dogs	that is milk
Jay owns dogs	Jay has milk
dogs are in the yard	milk is in the refrigerator
there are dogs in the yard	there is milk in the refrigerator
every man who owns dogs feeds them	every man who has milk drinks it
dogs are happy if they are well-fed	milk stays fresh if it is refrigerated
dogs are mammals	milk is food
dogs can hear sounds humans can't	milk can be made into cheese
Jay is looking for dogs	Jay is looking for milk

Note also that *colloquial spoken* English often employs unstressed 'some' ["səm"] as an indefinite article, which can prefix many of the nouns in the list.[3]

Given the strong structural similarities among these examples, and given the absence of indefinite articles in a large number of languages, we propose to use the term *indefinite noun phrase* to refer to all such phrases. More specifically, we propose to use this term to refer to any common-noun-phrase that plays an NP-role (subject, object, …), whether prefixed by an overt indefinite article or not.

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[1] Supposing we reject the reading according to which 'dog' is a proper-name, and the reading according to which 'dog' is a mass-noun [referring presumably to dog-matter].

[2] Plural-nouns, which are marked in English by the suffix 's', are a species of count-noun. A count-noun refers to one or more discrete entities. A mass-noun has singular-number usually, but does not refer to a discrete entity, but rather to indefinitely-divisible "matter". See Chapter 9 [Number Words] for further discussion.

[3] Also note that Spanish has plural indefinite articles, 'unos' and 'unas', and French has a plural indefinite article 'des' and a mass indefinite article 'de'. For example, if a French waiter asks you "d'eau?", he is asking whether you would like **səm** water.

2. Initial Hypothesis – INPs are QPs

An indefinite-noun-phrase (INP) is a common-noun-phrase (CNP) that plays an NP-role. By way of accounting for this behavior, the following hypothesis seems fairly natural.

(IH) Indefinite-noun-phrases are quantifier-phrases; in particular:

(1) ‘a’ is a variant of ‘some’,
which attaches to singular-nouns,
and which may not be deleted in the final form (pronunciation).

(2) ‘səm’ is a variant of ‘some’,
which attaches to plural-nouns and mass-nouns,
and which may be deleted in the final form (pronunciation).

IH accounts for the following examples.

1. Jay owns a dog

Jay+1	owns	a	dog	+2
		$\lambda P_0 \bigvee x P x$	$\mathbf{D}_0$	
		$\bigvee x \mathbf{D} x$		$\lambda x.x_2$
	$\lambda x_2 \lambda y_1 \mathbf{O} y x$	$\bigvee\{ x_2 \mid \mathbf{D} x \}$		
J_1	$\bigvee\{ \lambda y_1 \mathbf{O} y x \mid \mathbf{D} x \}$			
$\bigvee\{ \mathbf{O} J x \mid \mathbf{D} x \}$ $\exists x\{\mathbf{D} x \,\&\, \mathbf{O} J x\}$				
there is an x such that: x is a **dog-individual** and Jay owns x				

2. Jay owns dogs

Jay+1	owns	[səm]	dogs	+2
		t	$\mathbf{D}_0$	
		$\bigvee x \mathbf{D} x$		$\lambda x.x_2$
	$\lambda x_2 \lambda y_1 \mathbf{O} y x$	$\bigvee\{ x_2 \mid \mathbf{D} x \}$		
J_1	$\bigvee\{ \lambda y_1 \mathbf{O} y x \mid \mathbf{D} x \}$			
$\bigvee\{ \mathbf{O} J x \mid \mathbf{D} x \}$ $\exists x\{\mathbf{D} x \,\&\, \mathbf{O} J x\}$				
there is an x such that: x is a **dog-plurality** and Jay owns x				

3. Jay owns land

Jay+1	owns	[səm]	land	+2
		$\lambda P_0 \bigvee x P x$	$\mathbf{L}_0$	
		$\bigvee x \mathbf{L} x$		$\lambda x.x_2$
	$\lambda x_2 \lambda y_1 \mathbf{O} y x$	$\bigvee\{ x_2 \mid \mathbf{L} x \}$		
J_1	$\bigvee\{ \lambda y_1 \mathbf{O} y x \mid \mathbf{L} x \}$			
$\bigvee\{ \mathbf{O} J x \mid \mathbf{L} x \}$ $\exists x\{\mathbf{L} x \,\&\, \mathbf{O} J x\}$				
there is an x such that: x is a **land-mass** and Jay owns x				

IH also accounts for the following example, by treating ‘is’ as identity.

4. Rex is a dog

Rex +1	is [ID]	a	dog	+2
		$\lambda P_0 \bigvee y P y$	$\mathbf{D}_0$	
		$\bigvee y \mathbf{D} y$		$\lambda y.y_2$
	$\lambda y_2 \lambda x_1 [x=y]$	$\bigvee\{ y_2 \mid \mathbf{D} y \}$		
R_1	$\bigvee\{ \lambda x_1 [x=y] \mid \mathbf{D} y \}$			
$\bigvee\{ R=y \mid \mathbf{D} y \}$ $\exists y\{ \mathbf{D} y \,\&\, R=y \}$ $\mathbf{D} R$				

3. Problems with the Initial Hypothesis

Although IH accounts for some sentences, it has trouble accounting for other sentences, including the following.

- **a** dog is **a** mammal
- Jay is looking for **a** dog
- **a** dog is happy if it is well-fed

Let's see what happens when we apply IH to these examples.

5. a dog is a mammal

a	dog	+1	is [ID]	a	mammal	+2
$\lambda P_0 \vee x P x$	$\mathbf{D}_0$			$\lambda P_0 \vee y P y$	$\mathbf{M}_0$	
$\vee x \mathbf{D} x$		$\lambda x.x_1$		$\vee y \mathbf{M} y$		$\lambda y.y_2$
			$\lambda y_2 \lambda x_1[x=y]$	$\vee\{ y_2 \mid \mathbf{M} y \}$		
$\vee\{ x_1 \mid \mathbf{D} x \}$			$\vee\{ \lambda x_1[x=y] \mid \mathbf{M} y \}$			
$\vee\{ x=y \mid \mathbf{D} x \;\&\; \mathbf{M} y \}$ $\exists x \exists y\{ \mathbf{D} x \;\&\; \mathbf{M} y \;\&\; x=y\}$ $\exists x\{\mathbf{D} x \;\&\; \mathbf{M} y\}$						

Compare this example with the following very similar sentence.

6. a dog is barking

a dog +1	is barking
$\vee\{ x_1 \mid \mathbf{D} x \}$	$\lambda x_1 \mathbf{B} x$
$\vee\{ \mathbf{B} x \} \mid \mathbf{D} x \}$ $\exists x\{\mathbf{D} x \;\&\; \mathbf{B} x\}$	

The difference between these examples seems to be that, whereas the latter is about a dog-*particular*, the former is naturally read as about dog-*kind*. The particular/kind distinction also figures in the following sentence.

7. Jay is looking for a dog

Jay+1	is-looking-for	a dog +2
	$\lambda y_2 \lambda x_1 \mathbf{L} x y$	$\vee\{ y_2 \mid \mathbf{D} y \}$
J_1	$\vee\{ \lambda x_1 \mathbf{L} x y \mid \mathbf{D} y \}$	
$\vee\{ \mathbf{L} \mathrm{J} y \mid \mathbf{D} y \}$ $\exists y\{\mathbf{D} y \;\&\; \mathbf{L} \mathrm{J} y\}$		
there is an x such that: x is a dog and Jay is looking for x		

According to this reading, there is at least one particular dog that Jay is looking for. Although this is an admissible reading, there is another reading according to which Jay is not looking for a particular dog. Rather, 'a dog' indicates the *kind* of thing Jay is looking for. The Initial Hypothesis cannot produce this reading.

The following produces a similar ambiguity.

8. a dog is happy if it is well-fed

a dog	+1 –1	is happy	if	(–1) it +1	is well-fed
$\vee x \mathbf{D} x$	$\lambda x\{x_1 \times x_{-1}\}$			$\lambda x_{-1}{:}x_1$	$\lambda x_1 \mathbf{W} x$
$\vee\{ x_1 \times x_{-1} \mid \mathbf{D} x \}$		$\lambda x_1 \mathbf{H} x$	$\lambda \mathrm{X} \lambda \mathrm{Y}(\mathrm{X} \rightarrow \mathrm{Y})$	$\lambda x_{-1}\, \mathbf{W} x$	
$\vee\{ \mathbf{H} x \times x_{-1} \mid \mathbf{D} x \}$			$\lambda x_{-1}\, \lambda \mathrm{Y}\, (\mathbf{W} x \rightarrow \mathrm{Y})$		
$\vee\{ \mathbf{W} x \rightarrow \mathbf{H} x \mid \mathbf{D} x \}$ $\exists x\{ \mathbf{D} x \;\&\; (\mathbf{W} x \rightarrow \mathbf{H} x) \}$					

This reading says there is a particular dog who is happy if well-fed.[4] A much more natural reading treats 'a dog' as a general/generic noun, which IH does not account for.

[4] We can also treat 'if' as conditional-assertion, which is left as an exercise. It does not produce a better reading.

4. New Proposal

By way of accounting for indefinite noun phrases, we propose the following.

(1) Indefinite noun phrases have type C.
(2) Type C is equivalent to type ΣD,
where Σ is a special new junction (sum).
(3) Σ-phrases are sometimes promoted to Π-phrases,
where Π is a special new junction (product).
(4) Although the article 'a' is syntactically a determiner,
it is semantically an adjective.[5]

These ideas are formally presented in the following sections.

5. Sum (Σ)

We originally proposed that common-noun-phrases have type C [$=_{df}$ D_0→S], which means that they are a special kind of predicate. We now propose that we can *equally well* treat CNPs as mereological-sums of entities, based on the following schematic example.[6]

'dog'	denotes	dog_1	*and*	dog_2	*and*	…	*and*	dog_k
		list of all the dogs						

This is not *logical-and*, but rather *mereological-and*, also called *mereological-sum*, for which we propose the following notation.[7]

'dog'	denotes	dog_1	+	dog_2	+	…	+	dog_k
		$\Sigma\{ dog_1, dog_2, \ldots, dog_k \}$						

By way of incorporating this into our formal language, we propose yet another junction – Σ (sum) – characterized as follows.

if	A	is a type	then	ΣA	is a type

if	α	is an expression of type	A
and	Φ	is a formula	
then	Σ{α | Φ}	is an expression of type	ΣA
	reads:	the **sum** of all α such that Φ	

ΣνΦ	$=_{df}$	Σ{ν | Φ}	ν is a variable of any type; Φ is any formula
α + β	$=_{df}$	Σ{ν | ν=α ∨ ν=β}	ν not free in α or β

ΣD = D	a sum of entities is itself an entity
ΣS = S	a sum of sentences is itself a sentence

As noted earlier, we also propose that CNPs have interchangeable types – C and ΣD – which we call **CNP-Duality**, which is formally rendered by the following bi-directional inference principle.

[5] Indeed, this is precisely our earlier proposal, in Chapters 4 and 5, according to which 'a' has type C→C, which is also consistent with our later treatment of number words [Chapter 9], where we propose that number-words are fundamentally adjectives, and 'a' is synonymous with 'one'.
[6] This very similar to treating common noun as denoting *sets* of entities. In set theory, sets are primitive, and functions are derivative, but every subset A of a set S has an associated function – namely, its *characteristic function* χ_A from S into {T,F}. In particular, an item α is a member of set A iff $\chi_A(\alpha)$=T.
[7] There are mereological subtleties in distinguishing 'dog' from 'dogs'. See Chapter 9 [Number Words].

$\lambda\nu_0\Phi$	⊣⊢	$\Sigma\nu\Phi$
ν is a variable of type D		

Converting $\lambda\nu_0\Phi$ to $\Sigma\nu\Phi$ is employed when a CNP is asked to play a functional-role (subject, object, etc.), or alpha-role (for binding alpha-pronouns), since $\Sigma\nu\Phi$ admits case-marking, but $\lambda\nu_0\Phi$ does not.

6. Indefinite Articles

We propose that ‘a’ is fundamentally a number-word,[8] which is a modifier-adjective, categorially rendered as follows.

a	C→C	(D_0→S)→(D_0→S)	$\lambda P_0\, \lambda x_0\, \mathbf{1}(P)[x]$

Here, ‘**1**(P)’ is understood as follows.

$$\mathbf{1}(P)[\alpha] \quad =_{df} \quad \alpha \text{ is a "unit P"}$$

Here, what counts as a "unit P" depends upon P.[9] Usually, a unit-P is simply a P. Indeed, if we disregard collective-nouns, plural-nouns, measure-nouns, and mass-nouns, as is customary in elementary logic, then the domain consists exclusively of singular-particulars (individuals), which are all unital, in which case ‘a’ is semantically redundant.

7. Existential Readings of INPs

In this section, we show how our new proposal reproduces the examples that IH gets right – namely, those examples in which INPs behave like existential-quantifier phrases. This is based on the following composition principles for Σ.[10]

Σ-Composition	
α	α, β, γ are any expressions
$\Sigma\{\, \beta \mid \Phi \,\}$	Φ is any formula
α ; $\beta \hookrightarrow \gamma$	any sub-derivation of γ from $\{\alpha,\beta\}$
$\Sigma\{\, \gamma \mid \Phi \,\}$	Σ admits **all** α
Σ-Simplification	
$\Sigma\{\, \Psi \mid \Phi \,\}$	Φ, Ψ are formulas
$\exists \underline{\nu}\{\Phi \,\&\, \Psi\}$	$\underline{\nu}$ are all the variables C-free in Φ,Ψ

Notice that these look very much like the rules for ∨. The difference is that Σ admits all phrases, whereas ∨ does not. A more important difference, however, is that Σ corresponds to no specific morpheme; rather it arises precisely when CNPs are converted to entity-sums via CNP-duality, which is illustrated in some of the following derivations.[11]

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[8] This seems plausible in consideration of the fact that many languages use the same word-form to translate both ‘a’ and ‘one’; for example – German *eine*, French *une*, Spanish *una* .
[9] See Chapter 9 [Number Words].
[10] This is not the whole story of how Σ behaves, since there is also Σ-promotion; see later.
[11] In these examples, the nouns are interpreted according to their appropriate number (singular, plural, mass), and ‘a’ is treated as semantically redundant.

9. Rex is a dog

[treating 'is' as copular]

Rex +1	is [COP]	a	dog
		$\varnothing$	$\lambda x_0\mathbf{D}x$
	$\lambda P_0{:}P_1$	$\lambda x_0\mathbf{D}x$	
R_1	$\lambda x_1\mathbf{D}x$		
$\mathbf{D}R$			

[treating 'is' as identity]

Rex +1	is [ID]	a	dog	+2
		$\varnothing$	$\lambda x_0\mathbf{D}x$	
		$\lambda x_0\mathbf{D}x$ $\Sigma y\mathbf{D}y$		$\lambda y.y_2$
	$\lambda y_2\lambda x_1[x{=}y]$	$\Sigma\{\, y_2 \mid \mathbf{D}y \,\}$		
R_1	$\Sigma\{\, \lambda x_1[x{=}y] \mid \mathbf{D}y \,\}$			
$\Sigma\{\, R{=}y \mid \mathbf{D}y \,\}$ $\exists y\{\, \mathbf{D}y\ \&\ R{=}y \,\}$ $\mathbf{D}R$				

10. a dog is barking

a	dog	+1	is-barking
$\varnothing$	$\lambda x_0\mathbf{D}x$ $\Sigma x\mathbf{D}x$		
$\Sigma y\mathbf{D}y$		$\lambda x.x_1$	
$\Sigma\{\, x_1 \mid \mathbf{D}x \,\}$			$\lambda x_1\mathbf{B}x$
$\Sigma\{\, \mathbf{B}x \mid \mathbf{D}x \,\}$ $\exists x\{\mathbf{D}x\ \&\ \mathbf{B}x\}$			
here			

11. Jay owns a dog

Jay +1	owns	a	dog	+2
		$\varnothing$	$\lambda y_0\mathbf{D}y$ $\Sigma y\mathbf{D}y$	
		$\Sigma y\mathbf{D}y$		$\lambda y.y_2$
	$\lambda y_2\lambda x_1\mathbf{O}xy$	$\Sigma\{\, y_2 \mid \mathbf{D}y \,\}$		
J_1	$\Sigma\{\, \lambda x_1\mathbf{O}xy \mid \mathbf{D}y \,\}$			
$\Sigma\{\, \mathbf{O}Jy \mid \mathbf{D}y \,\}$				
$\exists y\{\, \mathbf{D}y\ \&\ \mathbf{O}Jy \,\}$				

12. Jay owns dogs

Jay +1	owns	dogs	+2
		$\lambda y_0\mathbf{D}y$ $\Sigma y\mathbf{D}y$	$\lambda y.y_2$
	$\lambda y_2\lambda x_1\mathbf{O}xy$	$\Sigma\{\, y_2 \mid \mathbf{D}y \,\}$	
J_1	$\Sigma\{\, \lambda x_1\mathbf{O}xy \mid \mathbf{D}y \,\}$		
$\Sigma\{\, \mathbf{O}Jy \mid \mathbf{D}y \,\}$ $\exists y\{\, \mathbf{D}y\ \&\ \mathbf{O}Jy \,\}$			

13. Jay owns land

Jay +1	owns	land	+2
		$\lambda y_0\mathbf{L}y$ $\Sigma y\mathbf{L}y$	$\lambda y.y_2$
	$\lambda y_2\lambda x_1\mathbf{O}xy$	$\Sigma\{\, y_2 \mid \mathbf{L}y \,\}$	
J_1	$\Sigma\{\, \lambda x_1\mathbf{O}xy \mid \mathbf{L}y \,\}$		
$\Sigma\{\, \mathbf{O}Jy \mid \mathbf{L}y \,\}$ $\exists y\{\, \mathbf{L}y\ \&\ \mathbf{O}Jy \,\}$			

8. Generic Readings of INP's; Entity-Sums as Entities

The examples in the previous section show how INPs can simulate existential-quantifier-phrases. We still need to show how INPs behave in the problematic examples we mentioned. For example, we still need to account for the reading of

Jay is looking for a dog

according to which 'a dog' does not indicate a *particular* thing Jay is looking for, but rather indicates the *kind* of thing Jay is looking for.

To account for the *generic* reading of 'a dog', we take advantage of the following type-identity.

$$\Sigma D = D$$

In other words, any sum of entities is itself an entity. We add to this a derivative principle for marked entities.

$$[\Sigma D]_k = \Sigma[D_k]$$

$$[\Sigma\{x \mid \Phi\}]_k = \Sigma\{x_k \mid \Phi\}$$

Our current example provides an opportunity to invoke compound-entities.[12] In particular, we can

[12] Various types of compound-entities appear in the philosophical literature, including *mereological-sums* and *pluralities*. These "logical" compounds should be distinguished from natural-organic compounds like molecules and polymers. See Chapter 19 [Formal Appendices].

treat *looking-for* as a relation between cognitive-agents and entities, the latter of which may be simple or compound.[13] Since ‘a dog’ can (in effect) be a QP or a DNP, ‘looking for a dog’ is correspondingly ambiguous, as seen in the following two derivations.[14]

<table>
<tr><td>Jay +1</td><td>is-looking-for</td><td>a dog</td><td>+2</td></tr>
<tr><td></td><td></td><td>$\Sigma x\mathbf{D}x$</td><td>$\lambda x.x_2$</td></tr>
<tr><td></td><td>$\lambda x_2\lambda y_1\mathbf{L}yx$</td><td colspan="2">$\Sigma\{ x_2 \mid \mathbf{D}x \}$</td></tr>
<tr><td>j_1</td><td colspan="3">$\Sigma\{ \lambda y_1\mathbf{L}yx \mid \mathbf{D}x \}$</td></tr>
<tr><td colspan="4">$\Sigma\{ \mathbf{L}\mathrm{j}x \mid \mathbf{D}x \}$
$\exists x \{ \mathbf{D}x \ \& \ \mathbf{L}\mathrm{j}x \}$</td></tr>
</table>

<table>
<tr><td>Jay +1</td><td>is-looking-for</td><td>a dog</td><td>+2</td></tr>
<tr><td></td><td></td><td>$\Sigma x\mathbf{D}x$</td><td>$\lambda x.x_2$</td></tr>
<tr><td></td><td>$\lambda x_2\lambda y_1\mathbf{L}yx$</td><td colspan="2">$\Sigma\{ x_2 \mid \mathbf{D}x \}$
$[\Sigma x\mathbf{D}x]_2$</td></tr>
<tr><td>j_1</td><td colspan="3">$\lambda y_1\mathbf{L}[y, \Sigma x\mathbf{D}x]$</td></tr>
<tr><td colspan="4">$\mathbf{L}[\mathrm{j}, \Sigma x\mathbf{D}x]$</td></tr>
</table>

In the first derivation, we use Σ-composition in the usual manner, so ‘a dog’ behaves like a QP. But in the second derivation, we treat ‘a dog’ as a type-D phrase, which denotes a compound-entity, and which serves as the argument for ‘is looking for’. According to the first reading, there is a dog that Jay is looking for; he stands in relation **L** to a particular dog. According to the second reading, Jay stands in relation **L** to a compound-entity – namely, the sum of all dogs – what we might describe as *dogs-as-a-whole*.

To see that this is not as exotic as it might sound at first, consider what it means to be looking for a spatially-complex entity, such as India.[15] One stands in relation **L**, not to any particular part of India, but to India-as-a-whole. We propose that looking for a dog, or dogs, can be similarly "holistic".

In the above example, ‘looking for’ is given a purely extensional interpretation; the relation **L** stands between *actual* entities. Oftentimes, however, ‘looking for’ is *intensional* in nature. For example, looking for a unicorn is different from looking for a dragon, although the extensions of ‘unicorn’ and ‘dragon’ are identical, both being empty.

This suggests that what we seek is not so much an entity, simple or complex, but a more abstract item – a state of affairs. For example, seeking a unicorn might be understood as seeking *to-behold-a-unicorn*.[16] Then looking for a unicorn is seeking a state-of-affairs in which one beholds a unicorn. But notice that one does not stand in the seek-relation to ***a* particular** state of affairs; rather, one stands in the seek-relation to a *sum* of states-of-affairs.[17]

Entity-sums are also useful is in explaining *generic readings* of common nouns such as in the following examples

14. children like dogs
15. fruit-flies like a banana [18]

If one construes indefinite-noun-phrases as entity-sums, then one can interpret these as asserting a relation between children-as-a-whole and dogs-as-a-whole, and between fruit-flies-as-a-whole and bananas-as-a-whole.

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[13] Treating ‘looking for’ as an *extensional* predicate may seem implausible on the face of it. See later examples.

[14] In *The Empire Strikes Back*, Luke Skywalker says “I am looking for someone”, to which Yoda replies “found someone, you have, I would say, hmmm?” Note that ‘someone’ often replaces the indefinite ‘a person’; see Section 16 [Other Forms that Act Like INPs].

[15] For example, Columbus was ostensibly looking for India, but instead found America, which he *thought* was India, which resulted in lexical chaos that lingers today.

[16] More generally, one seeks to stand in some tacitly understood relation to a unicorn – for example, *owning*. Also, if I am seeking a spouse, I may be seeking to be related to someone who is a spouse (of mine, or of someone else), or I may be seeking to be spousally-related to someone.

[17] This is also true for other words like ‘want’. Wanting (say) a pony is wanting *to have a pony*, and wanting a cheeseburger is wanting *to have* a cheeseburger. Presumably, having a pony is different from having a cheeseburger. Can you have your cheeseburger and eat it too? What we want is a state of affairs, even if it involves a concrete particular (a particular pony or cheeseburger). Speaking of cheeseburgers, my dog often begs for table scraps. I explain to her that she wants table scraps, and I want world peace, but we both have to wait!

[18] This comes from Groucho Marx, which is a follow-up to ‘time flies like an arrow’. A variant joke might be for Groucho to pull out a very crooked arrow, and say, “… but not this one”.

children +1	like	dogs +2
	$\lambda y_2 \lambda x_1 \mathbf{L}xy$	$[\Sigma x \mathbf{D}x]_2$
$[\Sigma x \mathbf{C}x]_1$	$\lambda x_1 \mathbf{L}[x, \Sigma x \mathbf{D}x]$	
$\mathbf{L}[\ \Sigma x \mathbf{C}x, \Sigma y \mathbf{D}y\ ]$		

fruit-flies +1	like	a banana +2
	$\lambda y_2 \lambda x_1 \mathbf{L}xy$	$[\Sigma x \mathbf{B}x]_2$
$[\Sigma x \mathbf{F}x]_1$	$\lambda x_1 \mathbf{L}[x, \Sigma y \mathbf{B}y]$	
$\mathbf{L}[\ \Sigma x \mathbf{F}x, \Sigma y \mathbf{B}y\ ]$		

[19]

Bear in mind that the generic reading is not forced by *compositional*-semantics. Both the existential and the generic readings are semantically admissible. Other (lexical, pragmatic) criteria must be invoked in order to decide which reading is appropriate. Consider the following pair.

- Jay likes dogs
- Jay owns dogs

One seems generic; the other seems existential. See Section 10 for further discussion.

Next, consider the following examples

16. a dog is a mammal
17. dogs are mammals

which we are inclined to understand as generic and perhaps nomic (i.e., law-like).

Suppose all the indefinite noun phrases above are understood to be generic, so they denote compound entities. We then have the following semantic trees.

a dog +1	is	a mammal +2
	?	$[\Sigma x \mathbf{D}x]_2$
$[\Sigma x \mathbf{D}x]_1$	**?**	
?		

dogs +1	are	mammals +2
	?	$[\Sigma x \mathbf{M}x]_2$
$[\Sigma x \mathbf{D}x]_1$	**?**	
?		

So how do we interpret 'is/are'? The usual suspects are *existence*, *predication*, and *identity*.[20] Since the arguments are entities, the connector must be a transitive verb, but the only transitive form of 'be' is identity. But the relation expressed in these two sentences is not symmetric, so it is not identity. So the usual suspects do not work.

Rather, it seems we have yet another variety of 'be'. For these particular examples, the most natural semantic account is that 'be' denotes the *species-genus* relation, which is categorially rendered as follows.

is[G]	$D_2 \rightarrow (D_1 \rightarrow S)$	$\lambda y_2 \lambda x_1 [x \sqsubseteq y]$	[21]

The above semantic derivations can then be completed as follows.[22]

a dog +1	is	a mammal +2
	$\lambda y_2 \lambda x_1 [x \sqsubseteq y]$	$[\Sigma x \mathbf{D}x]_2$
$[\Sigma x \mathbf{D}x]_1$	$\lambda x_1\ [x \sqsubseteq \Sigma x \mathbf{M}x]$	
$\Sigma x \mathbf{D}x \sqsubseteq \Sigma x \mathbf{M}x$		

dogs +1	are	mammals +2
	$\lambda y_2 \lambda x_1 [x \sqsubseteq y]$	$[\Sigma x \mathbf{M}x]_2$
$[\Sigma x \mathbf{D}x]_1$	$\lambda x_1\ [x \sqsubseteq \Sigma x \mathbf{M}x]$	
$\Sigma x \mathbf{D}x \sqsubseteq \Sigma x \mathbf{M}x$		

[19] In the following, we reduce generic-*plurals* to generic-*singulars*, which may not be completely proper.

[20] For logicians at least!

[21] Set A is included in set B [A⊆B] if and only if every member of A to also be a member of B. Note, however, that the species-genus relation [⊑] is modal/nomic in character, so the symbol is more "boxy", standing between *possible* pluralities.

[22] We finesse the difference between the singular 'a dog'/'a mammal' and the plural 'dogs'/'mammals'. See Chapter 9 [Number Words].

9. General Readings of INPs; Σ-Promotion

We have discussed *existential readings*, and *generic readings*, of INPs, but there are also *general readings*, according to which INPs mimic universal quantifier phrases.[23]

Examples in mathematics abound, including the following examples due to Pythagoras.[24]

- **a** number is rational if and only if it is the quotient of two integers
- the square of the hypotenuse of **a** right triangle is equal to the sum of the squares of its other two sides.

Physics also provides examples, including the following due to Newton.[25]

- **a** body at rest will remain at rest unless acted upon by an external force
- **a** body in motion will remain in motion unless acted upon by an external force

There are also poetic uses of ‘a body’, as in the following verse.[26]

- if **a** body meet **a** body coming through the rye;
  if **a** body kiss **a** body, need **a** body cry.

Finally, there are more mundane examples such as the following from earlier.

- **a** dog is happy if it is well-fed

Elementary logic students are taught that sentences like these are *best* translated as formulas whose overall forms are:

$\forall x\{\mathbf{N}x \rightarrow \ldots\}$	every number…
$\forall x\{\mathbf{R}x \rightarrow \ldots\}$	every right triangle…
$\forall x\{\mathbf{B}x \rightarrow \ldots\}$	every body…
$\forall x\{\mathbf{D}x \rightarrow \ldots\}$	every dog…

The trick for the logic student is to fill in the “…”. The trick for the formal semanticist is to provide a theoretical account of how INPs (with or without indefinite articles) combine with other phrases so that ultimately they get interpreted as universal-quantifiers.

Our proposal is that:

(0) INPs are CNPs called upon to serve as NPs;
(1) INPs are fundamentally entity-sums (ΣD),
(2) Entity-sums may sometimes be *promoted* to entity-products (ΠD),[27]
which have their own special compositional properties,
and which ultimately get simplified via universal-quantification.

By way of implementing this proposal, we first formally introduce a new junction – Π (product) – as follows.

if	A	is a type	then	ΠA	is a type

if	α	is an expression of type	A
and	Φ	is a formula	
then	$\Pi\{\alpha \mid \Phi\}$	is an expression of type	ΠA
	reads:	the **product** of all α such that Φ	

[23] Indeed, one might re-interpret the genus-species relation discussed in the previous section so as to involve a disguised universal quantifier.
[24] Circa 570 to circa 490 BCE.
[25] Isaac Newton, *Philosophiæ Naturalis Principia Mathematica* (1687).
[26] Robert Burns, “Comin thro the Rye” (1782).
[27] Of course, the tricky part then is to specify precisely when/how promotion takes place. See later.

$\Pi\nu\Phi$	$=_{df}$	$\Pi\{\nu \mid \Phi\}$	ν is a variable of any type; Φ is any formula
$\alpha \otimes \beta$ [28]	$=_{df}$	$\Pi\{\nu \mid \nu{=}\alpha \vee \nu{=}\beta\}$	ν not free in α or β

ΠS = S	ΠD ≠ D

The following are the associated composition-rules.

Π-Composition	
α	α, β, γ are any expressions
$\Pi\{\ \beta \mid \Phi\ \}$	Φ is any formula
$\alpha\ ;\ \beta \hookrightarrow \gamma$	any sub-derivation of γ from $\{\alpha,\beta\}$
$\Pi\{\ \gamma \mid \Phi\ \}$	Σ admits **all** α
Π-Simplification	
$\Pi\{\ \Psi \mid \Phi\ \}$	Φ, Ψ are formulas
$\forall\underline{\nu}\{\Phi\rightarrow\Psi\}$	$\underline{\nu}$ are all the variables free in Φ,Ψ
OBLIG ATORY	no variables are externally-bound

Π is very similar to $\wedge$; in particular, they both simplify to a universal formula. The difference pertains to scope. First, Π admits all phrases. Second, $\Pi\{\Psi|\Phi\}$ only simplifies if Π binds all the variables in Φ,Ψ. This is described by saying that Π is a *maximal-scope* quantifier.

10. Σ-Promotion – The Simple Hypothesis

The remaining question then is:

What are the restrictions on Σ-promotion;
how/when does an entity-sum (ΣD) get promoted to an entity-product (ΠD)?

The simplest semantic hypothesis is:

(SH) an entity-sum (ΣD) may be freely promoted to an entity-product (ΠD).

In other words, there are no *formal semantic* restrictions on Σ-promotion. So every INP *officially* admits three readings, illustrated as follows.

'a dog'	translates as	$\Sigma x\mathbf{D}x$	existential reading
			generic reading
		$\Pi x\mathbf{D}x$	universal reading

Furthermore, whether a given reading is plausible/sensible/felicitous is not a matter of *formal* (compositional) semantics, but is rather a matter of lexical semantics and pragmatics.

For example, the following two sentences are *formally* on a par.

(1) Jay owns dogs
(2) Jay loves dogs

So, according to the simple hypothesis, these both admit three readings.

[28] Unfortunately, Π is not quite the infinitary-counterpart of $\times$. First, $\times$ can combine expressions of different types, but Π can only combine expressions of the same type. More importantly perhaps, Π is contractive [$\alpha\otimes\alpha{=}\alpha$, but $\times$ is anti-contractive [$\alpha\times\alpha\neq\alpha$].

existential	(1a) Jay owns some dogs	(2a) Jay loves some dogs
generic	(1b) Jay owns dogs-as-a-whole	(2b) Jay loves dogs-as-a-whole
universal	(1c) Jay owns all dogs	(2c) Jay loves all dogs

Some of these readings are more plausible than others. For example, (1a) is a plausible reading of (1), whereas (1b) and (1c) seem a bit wacky. These may be eliminated by lexical and pragmatic considerations.[29] On the other hand, (2b) is the most plausible of (2), while (2a) and (2c) are less plausible.

What if the INP is in subject position, as in the following examples?

(3) dogs are barking
a dog is barking

(4) dogs bark
a dog barks

The generic-reading and universal-reading of (3) are implausible. Conversely, the existential-reading of (4) is less plausible.[30] These discrepancies may be explained by reference to the lexical entries for 'bark'; according to one entry, it denotes an *event* or *state*, which encourages an existential reading; according to another entry, it denotes a *trait*, which encourages a generic or universal reading.[31]

Perhaps the above sentences can be understood as universally quantified. It is a much bigger stretch to read

(5) Rex is a dog
(6) Rex and Lassie are dogs

as saying:

Rex is every dog
Rex and Lassie are all (the) dogs

Perhaps, these can be dismissed by insisting that *copula-be* is the default reading of 'be'.

OK, but what about the following?

(7) Jay owns a dog

Surely, this does not plausibly say that

Jay owns *every* dog.

Can this reading be dismissed by appeal to the lexical entry for 'owns'? Maybe not!

The following example is also very troubling for the Simple Hypothesis.

(8) there is a dog in the yard
there are dogs in the yard

Surely, these do not – even remotely plausibly – say that

every dog is in the yard.

11. Revised Hypothesis – Restrictions on Σ-Promotion

In light of the numerous problematic sentences in the previous section, we reject the Simple Hypothesis, according to which Σ may be freely promoted to Π, replacing it with an account according to which Σ may be promoted to Π under special circumstances. What circumstances? We propose the following Σ-promotion rules.

---

[29] In particular, the lexical entry for 'own' would include the following clause,

$\mathbf{O}[\alpha,\Sigma\nu\Phi] = \Sigma\{\mathbf{O}[\alpha,\nu] \mid \Phi\}$

which reduces the generic-reading to the existential-reading. The universal-reading is pragmatically eliminated by noting that (1) seems to have neither nomic or modal force.

[30] Poetic/antique usage allows eventive readings. Consider the following line from a 13th Century nursery rhyme.

hark, hark, the dogs do bark
the beggars are coming to town…

[31] See later chapter for further discussion of *events* versus *states* versus *traits*.

Σ	*may** be promoted to Π by: 1. every 2. no 3. not 4. if-clauses…[32] 5. nomic contexts [33]
	* Promotion is *not obligatory*.

12. Examples

To see how this works, let's do a few examples.

18. Jay does **not** own dogs
    Jay does **not** own a dog

We concentrate on the second one. According to the revised hypothesis, 'a dog' is translated as $\Sigma x\mathbf{D}x$. Lexical considerations pertaining to 'own' obviates the generic-treatment of Σ . This leaves the quantifier-treatment of Σ. So the question is whether Σ can be promoted. It can (but need not be) promoted, as seen in the following derivations.

read INP as wide-Σ; plus promotion

Jay +1	doesn't	own	a dog +2
		$\lambda y_2\lambda x_1\mathbf{O}xy$	$\Sigma\{ y_2 \mid \mathbf{D}y \}$
	$\lambda X{:}{\sim}X$	$\Sigma\{ \lambda x_1\mathbf{O}xy \mid \mathbf{D}y \}$	
J_1	ⓟ $\Pi\{ \lambda x_1{\sim}\mathbf{O}xy \mid \mathbf{D}y \}$		
$\Pi\{ {\sim}\mathbf{O}Jy \mid \mathbf{D}y \}$ $\forall y(\mathbf{D}y \rightarrow {\sim}\mathbf{O}Jy)$			

ⓟ 'doesn't' promotes Σ to Π.

read INP as narrow-Σ

Jay+1	doesn't	own	a dog +2
		$\lambda y_2\lambda x_1\mathbf{O}xy$	$\Sigma\{ y_2 \mid \mathbf{D}y \}$
	$\lambda X{:}{\sim}X$	$\Sigma\{ \lambda x_1\mathbf{O}xy \mid \mathbf{D}y \}$ $\lambda x_1\,\Sigma\{ \mathbf{O}xy \mid \mathbf{D}y \}$ $\lambda x_1\,\exists y(\mathbf{D}y\ \&\ \mathbf{O}xy)$	
J_1	$\lambda x_1\ {\sim}\exists y(\mathbf{D}y\ \&\ \mathbf{O}xy)$		
${\sim}\exists y(\mathbf{D}y\ \&\ \mathbf{O}Jy)$			

Notice that the resulting formulas are logically equivalent. Also notice that, unlike ∧, Σ admits 'not', which promotes it to Π. But promotion is optional, so we also have the following derivation.

read INP as wide-Σ; no promotion

Jay +1	doesn't	own	a dog +2
		$\lambda y_2\lambda x_1\mathbf{O}xy$	$\Sigma\{ y_2 \mid \mathbf{D}y \}$
	$\lambda X{:}{\sim}X$	$\Sigma\{ \lambda x_1\mathbf{O}xy \mid \mathbf{D}y \}$	
J_1	$\Sigma\{ \lambda x_1{\sim}\mathbf{O}xy \mid \mathbf{D}y \}$		
$\Sigma\{ {\sim}\mathbf{O}Jy \mid \mathbf{D}y \}$ $\exists y(\mathbf{D}y\ \&\ {\sim}\mathbf{O}Jy)$			

This does not seem so plausible. The oddness of this sort of reading seems even more obvious in the following example.

19. Jay is **not** a dog

Consider the following derivation in which we treat 'is' as identity, and accordingly treat 'a dog' as an accusative-marked INP.

[32] Including 'if' itself. Also 'if and only if', since the latter is constructed from 'if'. See Chapter 11 [Definite Descriptions; *Only*].

[33] Unfortunately, these contexts are seldom overtly pronounced – for example, using the prefixed phrase 'it is a law of physics [mathematics, dog theory] that…'. rather, they must be intuited by the addressee.

read INP as wide-Π

<table>
<tr><td>Jay+1</td><td>is</td><td>not</td><td>a dog +2</td></tr>
<tr><td></td><td>$\lambda y_2\lambda x_1[x{=}y]$</td><td>$\lambda X{:}{\sim}X$</td><td></td></tr>
<tr><td></td><td colspan="2">$\lambda y_2\lambda x_1[x{\neq}y]$</td><td>$\Sigma\{\ y_2 \mid \mathbf{D}y\ \}$</td></tr>
<tr><td>J_1</td><td colspan="3">ⓟ $\Pi\{\ \lambda x_1[x{\neq}y] \mid \mathbf{D}y\ \}$</td></tr>
<tr><td colspan="4">$\Pi\{\ J{\neq}y \mid \mathbf{D}y\ \}$
$\forall y\{\mathbf{D}y \rightarrow J{\neq}y\}$
$\sim\mathbf{D}J$</td></tr>
</table>

ⓟ 'is not' promotes Σ to Π.

read INP as wide-Σ

<table>
<tr><td>Jay+1</td><td>is</td><td>not</td><td>a dog +2</td></tr>
<tr><td></td><td>$\lambda y_2\lambda x_1[x{=}y]$</td><td>$\lambda X{:}{\sim}X$</td><td></td></tr>
<tr><td></td><td colspan="2">$\lambda y_2\lambda x_1[x{\neq}y]$</td><td>$\Sigma\{\ y_2 \mid \mathbf{D}y\ \}$</td></tr>
<tr><td>J_1</td><td colspan="3">$\Sigma\{\ \lambda x_1[x{\neq}y] \mid \mathbf{D}y\ \}$</td></tr>
<tr><td colspan="4">$\Sigma\{\ J{\neq}y \mid \mathbf{D}y\ \}$
$\exists y\{\mathbf{D}y\ \&\ J{\neq}y\}$</td></tr>
</table>

The first reading agrees with our natural intuitions, whereas the second reading seems wacky. But notice that the wacky readings of our last two examples seem much better if we append a remark as follows.

<table>
<tr><td>20. Jay does not own a dog…</td><td rowspan="2">(namely) this/that one (pointing at a particular dog)</td></tr>
<tr><td>21. Jay is not a dog…</td></tr>
</table>

I call this the *Dangerfield Adjustment*, because Rodney Dangerfield once quipped:[34]

22. I own a suit for every occasion; unfortunately, this is it!

This line is funny because we originally hear the scopes reversed from how they end up. Indeed, the pronoun-binding restraints introduced by the coda make a narrow-scope reading of 'a suit' impossible. We come back to this example later.[35]

How do INPs interact with relative pronouns? Consider the following example.

23. every man who owns a dog is happy

Disregarding the generic-reading, which is obviated by the verb 'owns', we have three readings, according to how we accord scope.

1. wide-Σ; no promotion

<table>
<tr><td>every</td><td>man</td><td>who +1</td><td>owns</td><td>a dog +2</td><td>+1</td><td>is happy</td></tr>
<tr><td></td><td></td><td></td><td>$\lambda y_2\lambda x_1\mathbf{O}xy$</td><td>$\Sigma\{\ y_2 \mid \mathbf{D}y\ \}$</td><td></td><td></td></tr>
<tr><td></td><td></td><td>$\lambda x_0{:}x_1$</td><td colspan="2">$\Sigma\{\ \lambda x_1\mathbf{O}xy \mid \mathbf{D}y\ \}$</td><td></td><td></td></tr>
<tr><td></td><td>$\lambda x_0\mathbf{M}x$</td><td colspan="3">$\Sigma\{\ \lambda x_0\mathbf{O}xy \mid \mathbf{D}y\ \}$</td><td></td><td></td></tr>
<tr><td>$\lambda P_0\wedge xPx$</td><td colspan="4">$\Sigma\{\ \lambda x_0(\mathbf{M}x\ \&\ \mathbf{O}xy) \mid \mathbf{D}y\ \}$</td><td></td><td></td></tr>
<tr><td colspan="5">$\Sigma\{\ \wedge x(\mathbf{M}x\ \&\ \mathbf{O}xy) \mid \mathbf{D}y\ \}$</td><td>$\lambda x.x_1$</td><td></td></tr>
<tr><td colspan="6">$\Sigma\{\ \wedge\{\ x_1 \mid\ \mathbf{M}x\ \&\ \mathbf{O}xy\ \} \mid \mathbf{D}y\ \}$</td><td>$\lambda x_1\ \mathbf{H}x$</td></tr>
<tr><td colspan="7">$\Sigma\{\ \wedge\{\ \mathbf{H}x \mid\ \mathbf{M}x\ \&\ \mathbf{O}xy\ \} \mid \mathbf{D}y\ \}$
$\exists y\{\ \mathbf{D}y\ \&\ \forall x\{\ \mathbf{M}x\ \&\ \mathbf{O}xy\ .\!\rightarrow \mathbf{H}x\ \}\ \}$</td></tr>
</table>

This does not seem so plausible, but is considerably improved by the Dangerfield Adjustment.

The following two readings are more plausible, and are indeed logically equivalent.

---

[34] The joke appears, much earlier, in Beatrice Burton's 1925 novel *The Flapper Wife*, p 156.

[35] See Chapter 13 [Pronoun Binding Revisited 1].

2. wide-Σ; with promotion

<table>
<tr><th>every</th><th>man</th><th>who +1</th><th>owns</th><th>a dog +2</th><th>+1</th><th>is happy</th></tr>
<tr><td></td><td></td><td></td><td>$\lambda y_2 \lambda x_1 \mathbf{O}xy$</td><td>$\Sigma\{\ y_2 \mid \mathbf{D}y\ \}$</td><td></td><td></td></tr>
<tr><td></td><td></td><td>$\lambda x_0{:}x_1$</td><td colspan="2">$\Sigma\{\ \lambda x_1 \mathbf{O}xy \mid \mathbf{D}y\ \}$</td><td></td><td></td></tr>
<tr><td></td><td>$\lambda x_0 \mathbf{M}x$</td><td colspan="3">$\Sigma\{\ \lambda x_0 \mathbf{O}xy \mid \mathbf{D}y\ \}$</td><td></td><td></td></tr>
<tr><td>$\lambda P_0 \wedge x P x$</td><td colspan="4">$\Sigma\{\ \lambda x_0(\mathbf{M}x\ \&\ \mathbf{O}xy) \mid \mathbf{D}y\ \}$</td><td></td><td></td></tr>
<tr><td colspan="5">ⓟ $\Pi\{\ \wedge x(\mathbf{M}x\ \&\ \mathbf{O}xy) \mid \mathbf{D}y\ \}$</td><td>$\lambda x.x_1$</td><td></td></tr>
<tr><td colspan="6">$\Pi\{\ \wedge\{\ x_1 \mid \mathbf{M}x\ \&\ \mathbf{O}xy\ \} \mid \mathbf{D}y\ \}$</td><td>$\lambda x_1\ \mathbf{H}x$</td></tr>
<tr><td colspan="7">$\Pi\{\ \wedge\{\ \mathbf{H}x \mid \mathbf{M}x\ \&\ \mathbf{O}xy\ \} \mid \mathbf{D}y\ \}$
$\forall y\{\ \mathbf{D}y \rightarrow \forall x\{\ \mathbf{M}x\ \&\ \mathbf{O}xy\ .\!\!\rightarrow \mathbf{H}x\ \}\ \}$</td></tr>
</table>

ⓟ ‘every’ promotes Σ to Π.

3. narrow-Σ

<table>
<tr><th>every</th><th>man</th><th>who +1</th><th>owns</th><th>a dog +2</th><th>+1</th><th>is happy</th></tr>
<tr><td></td><td></td><td></td><td>$\lambda y_2 \lambda x_1 \mathbf{O}xy$</td><td>$\Sigma\{\ y_2 \mid \mathbf{D}y\ \}$</td><td></td><td></td></tr>
<tr><td></td><td></td><td>$\lambda x_0{:}x_1$</td><td colspan="2">$\Sigma\{\ \lambda x_1 \mathbf{O}xy \mid \mathbf{D}y\ \}$
$\lambda x_1\ \Sigma\{\ \mathbf{O}xy \mid \mathbf{D}y\ \}$
$\lambda x_1\ \exists y(\mathbf{D}y\ \&\ \mathbf{O}xy)$</td><td></td><td></td></tr>
<tr><td></td><td>$\lambda x_0 \mathbf{M}x$</td><td colspan="3">$\lambda x_0\ \exists y(\mathbf{D}y\ \&\ \mathbf{O}xy)$</td><td></td><td></td></tr>
<tr><td>$\lambda P_0 \wedge x P x$</td><td colspan="4">$\lambda x_0\{\mathbf{M}x\ \&\ \exists y(\mathbf{D}y\ \&\ \mathbf{O}xy)\}$</td><td></td><td></td></tr>
<tr><td colspan="5">$\wedge\{\ x \mid \mathbf{M}x\ \&\ \exists y(\mathbf{D}y\ \&\ \mathbf{O}xy)\ \}$</td><td>$\lambda x.x_1$</td><td></td></tr>
<tr><td colspan="6">$\wedge\{\ x_1 \mid \mathbf{M}x\ \&\ \exists y(\mathbf{D}y\ \&\ \mathbf{O}xy)\ \}$</td><td>$\lambda x_1\ \mathbf{H}x$</td></tr>
<tr><td colspan="7">$\wedge\{\ \mathbf{H}x \mid \mathbf{M}x\ \&\ \exists y(\mathbf{D}y\ \&\ \mathbf{O}xy)\ \}$
$\forall x\{\ \mathbf{M}x\ \&\ \exists y(\mathbf{D}y\ \&\ \mathbf{O}xy)\ .\!\!\rightarrow \mathbf{H}x\ \}$</td></tr>
</table>

13. Pronoun-Binding by INPs (Donkey Sentences)

In the previous example,

every man who owns a dog is happy

‘a dog’ can be narrow-existential or wide-universal, the resulting formulas being logically equivalent. Sometimes, however, the narrow-existential reading is not feasible, which in particular arises in sentences in which an INP binds a pronoun. Such sentences are often called "donkey sentences" because the earliest examples concerned donkeys, such as the following.[36]

24. every man who owns a donkey beats it

We prefer kinder and gentler examples, and dogs, so we offer the following substitute.

25. every man who owns a dog feeds it

First, the generic-reading is obviated by the verb ‘owns’. That leaves the QP-reading(s). The wide-∃ reading goes as follows.

[36] Geach, P. T. (1962), p. 143. The original wording employs ‘any’ rather than ‘every’.

<table>
<tr><td>every</td><td>man</td><td>who +1</td><td>owns</td><td>a dog</td><td>+2 −1</td><td>+1</td><td>feeds</td><td>(−1) it +2</td></tr>
<tr><td></td><td></td><td></td><td></td><td>$\Sigma y\mathbf{D}y$</td><td>$\lambda y(y_2 \times y_{-1})$</td><td></td><td>$\lambda y_2\lambda x_1\mathbf{F}xy$</td><td>$\lambda y_{-1}{:}y_2$</td></tr>
<tr><td></td><td></td><td></td><td>$\lambda y_2\lambda x_1\mathbf{O}xy$</td><td colspan="2">$\Sigma\{\, y_2 \times y_{-1} \mid \mathbf{D}y \,\}$</td><td></td><td colspan="2"></td></tr>
<tr><td></td><td></td><td>$\lambda x_0.x_1$</td><td colspan="3">$\Sigma\{\, \lambda x_1\mathbf{O}xy \times y_{-1} \mid \mathbf{D}y \,\}$</td><td></td><td colspan="2"></td></tr>
<tr><td></td><td>$\lambda x_0\mathbf{M}x$</td><td colspan="4">$\Sigma\{\, \lambda x_0\mathbf{O}xy \times y_{-1} \mid \mathbf{D}y \,\}$</td><td></td><td colspan="2"></td></tr>
<tr><td>$\lambda \mathrm{P}_0 \wedge x\mathrm{P}x$</td><td colspan="5">$\Sigma\{\, \lambda x_0(\mathbf{M}x \,\&\, \mathbf{O}xy) \times y_{-1} \mid \mathbf{D}y \,\}$</td><td></td><td colspan="2"></td></tr>
<tr><td colspan="6">$\Sigma\{\, \wedge x(\mathbf{M}x \,\&\, \mathbf{O}xy) \times y_{-1} \mid \mathbf{D}y \,\}$</td><td>$\lambda x.x_1$</td><td colspan="2"></td></tr>
<tr><td colspan="7">$\Sigma\{\, \wedge\{x_1 \mid \mathbf{M}x \,\&\, \mathbf{O}xy\} \times y_{-1} \mid \mathbf{D}y \,\}$</td><td colspan="2">$\lambda y_{-1}\, \lambda x_1\, \mathbf{F}xy$</td></tr>
<tr><td colspan="9">$\Sigma\{\, \wedge\{x_1 \mid \mathbf{M}x \,\&\, \mathbf{O}xy\} \times \lambda x_1\mathbf{F}xy \mid \mathbf{D}y \,\}$
$\Sigma\{\, \wedge\{\mathbf{F}xy \mid \mathbf{M}x \,\&\, \mathbf{O}xy\} \mid \mathbf{D}y \,\}$
$\exists y\{\mathbf{D}y \,\&\, \forall x\{(\mathbf{M}x \,\&\, \mathbf{O}xy) \rightarrow \mathbf{F}xy\}\,\}$</td></tr>
</table>

Notice once again that Σ admits every phrase, in virtue of which it gains wide scope. This reading is accordingly formally admissible, although it is not so plausible. However, as before, its plausibility is considerably improved by the Dangerfield Adjustment.

More plausible is the following derivation, in which Σ is promoted to Π.

<table>
<tr><td>every</td><td>man</td><td>who +1</td><td>owns</td><td>a dog</td><td>+2 −1</td><td>+1</td><td>feeds</td><td>(−1) it +2</td></tr>
<tr><td></td><td></td><td></td><td></td><td>$\Sigma y\mathbf{D}y$</td><td>$\lambda y(y_2 \times y_{-1})$</td><td></td><td>$\lambda y_2\lambda x_1\mathbf{F}xy$</td><td>$\lambda y_{-1}{:}y_2$</td></tr>
<tr><td></td><td></td><td></td><td>$\lambda y_2\lambda x_1\mathbf{O}xy$</td><td colspan="2">$\Sigma\{\, y_2 \times y_{-1} \mid \mathbf{D}y \,\}$</td><td></td><td colspan="2"></td></tr>
<tr><td></td><td></td><td>$\lambda x_0.x_1$</td><td colspan="3">$\Sigma\{\, \lambda x_1\mathbf{O}xy \times y_{-1} \mid \mathbf{D}y \,\}$</td><td></td><td colspan="2"></td></tr>
<tr><td></td><td>$\lambda x_0\mathbf{M}x$</td><td colspan="4">$\Sigma\{\, \lambda x_0\mathbf{O}xy \times y_{-1} \mid \mathbf{D}y \,\}$</td><td></td><td colspan="2"></td></tr>
<tr><td>$\lambda \mathrm{P}_0 \wedge x\mathrm{P}x$</td><td colspan="5">$\Sigma\{\, \lambda x_0(\mathbf{M}x \,\&\, \mathbf{O}xy) \times y_{-1} \mid \mathbf{D}y \,\}$</td><td></td><td colspan="2"></td></tr>
<tr><td colspan="6">ⓟ $\Pi\{\, \wedge x(\mathbf{M}x \,\&\, \mathbf{O}xy) \times y_{-1} \mid \mathbf{D}y \,\}$</td><td>$\lambda x.x_1$</td><td colspan="2"></td></tr>
<tr><td colspan="7">$\Pi\{\, \wedge\{x_1 \mid \mathbf{M}x \,\&\, \mathbf{O}xy\} \times y_{-1} \mid \mathbf{D}y \,\}$</td><td colspan="2">$\lambda y_{-1}\, \lambda x_1\, \mathbf{F}xy$</td></tr>
<tr><td colspan="9">$\Pi\{\, \wedge\{x_1 \mid \mathbf{M}x \,\&\, \mathbf{O}xy\} \times \lambda x_1\mathbf{F}xy \mid \mathbf{D}y \,\}$
$\Pi\{\, \wedge\{\mathbf{F}xy \mid \mathbf{M}x \,\&\, \mathbf{O}xy\} \mid \mathbf{D}y \,\}$
$\forall y\{\mathbf{D}y \rightarrow \forall x\{(\mathbf{M}x \,\&\, \mathbf{O}xy) \rightarrow \mathbf{F}xy\}\,\}$</td></tr>
</table>

ⓟ 'every' promotes Σ to Π.

What about the narrow-∃ reading?

<table>
<tr><td>every</td><td>man</td><td>who +1</td><td>owns</td><td>a dog</td><td>+2 −1</td><td>+1</td><td>feeds</td><td>(−1) it +2</td></tr>
<tr><td></td><td></td><td></td><td></td><td>$\Sigma y\mathbf{D}y$</td><td>$\lambda y(y_2 \times y_{-1})$</td><td></td><td>$\lambda y_2\lambda x_1\mathbf{F}xy$</td><td>$\lambda y_{-1}{:}y_2$</td></tr>
<tr><td></td><td></td><td></td><td>$\lambda y_2\lambda x_1\mathbf{O}xy$</td><td colspan="2">$\Sigma\{\, y_2 \times y_{-1} \mid \mathbf{D}y \,\}$</td><td></td><td colspan="2"></td></tr>
<tr><td></td><td></td><td>$\lambda x_0.x_1$</td><td colspan="3">$\Sigma\{\, \lambda x_1\mathbf{O}xy \times y_{-1} \mid \mathbf{D}y \,\}$</td><td></td><td colspan="2"></td></tr>
<tr><td></td><td>$\lambda x_0\mathbf{M}x$</td><td colspan="4">$\Sigma\{\, \lambda x_0\mathbf{O}xy \times y_{-1} \mid \mathbf{D}y \,\}$
$\lambda x_0\, \Sigma\{\, \mathbf{O}xy \times y_{-1} \mid \mathbf{D}y \,\}$</td><td></td><td colspan="2"></td></tr>
<tr><td>$\lambda \mathrm{P}_0 \wedge x\mathrm{P}x$</td><td colspan="5">※</td><td>$\lambda x.x_1$</td><td colspan="2">$\lambda y_{-1}\, \lambda x_1\, \mathbf{F}xy$</td></tr>
<tr><td colspan="9">※</td></tr>
</table>

This derivation fails because Σ-simplification is not permitted at any node, because the sum is not a sum of sentences.[37]

[37] This issue is taken up again in Part C [Expanded Account of Quantifiers].

In order to apply Σ-simplification, we can remove its alpha-marker [–1], but then ‘a dog’ does not bind ‘it’, as seen in the following derivation.

<table>
<tr><td>every</td><td>man</td><td>who +1</td><td>owns</td><td>a dog</td><td>+2</td><td>+1</td><td>feeds</td><td>(–1) it +2</td></tr>
<tr><td></td><td></td><td></td><td></td><td>$\Sigma y \mathbf{D}y$</td><td>$\lambda y.y_2$</td><td></td><td>$\lambda y_2 \lambda x_1 \mathbf{F}xy$</td><td>$\lambda y_{-1}{:}y_2$</td></tr>
<tr><td></td><td></td><td></td><td>$\lambda y_2 \lambda x_1 \mathbf{O}xy$</td><td colspan="2">$\Sigma\{\ y_2 \mid \mathbf{D}y\ \}$</td><td></td><td colspan="2"></td></tr>
<tr><td></td><td></td><td>$\lambda x_0.x_1$</td><td colspan="3">$\Sigma\{\ \lambda x_1 \mathbf{O}xy \mid \mathbf{D}y\ \}$
$\lambda x_1 \exists y\{\mathbf{D}y\ \&\ \mathbf{O}xy\}$</td><td></td><td colspan="2"></td></tr>
<tr><td></td><td>$\lambda x_0 \mathbf{M}x$</td><td colspan="4">$\lambda x_0 \exists y\{\mathbf{D}y\ \&\ \mathbf{O}xy\}$</td><td></td><td colspan="2"></td></tr>
<tr><td>$\lambda P_0 \bigwedge x P x$</td><td colspan="5">$\lambda x_0\{\mathbf{M}x\ \&\ \exists y\{\mathbf{D}y\ \&\ \mathbf{O}xy\}\}$</td><td></td><td colspan="2"></td></tr>
<tr><td colspan="6">$\bigwedge x\{\mathbf{M}x\ \&\ \exists y\{\mathbf{D}y\ \&\ \mathbf{O}xy\}\}$</td><td>$\lambda x.x_1$</td><td colspan="2"></td></tr>
<tr><td colspan="7">$\bigwedge\{\ x_1 \mid \mathbf{M}x\ \&\ \exists y\{\mathbf{D}y\ \&\ \mathbf{O}xy\}\ \}$</td><td colspan="2">$\lambda y_{-1}\ \lambda x_1\ \mathbf{F}xy$</td></tr>
<tr><td colspan="9">$\lambda y_{-1} \bigwedge\{\ \mathbf{F}xy \mid \mathbf{M}x\ \&\ \exists y\{\mathbf{D}y\ \&\ \mathbf{O}xy\}\ \}$
$\lambda y_{-1}\ \forall x\{\ \mathbf{M}x\ \&\ \exists y\{\mathbf{D}y\ \&\ \mathbf{O}xy\}\ .\!\!\rightarrow \mathbf{F}xy\ \}$</td></tr>
</table>

Although this is a permissible reading, it leaves the anaphoric-pronoun ‘it’ dangling (open). On the other hand, it can be bound by embedding the sentence in a wider sentence – for example, as follows.

> if a stray dog [–1] appears, then
> every man who owns a dog feeds (–1) it

Here, ‘it’ is anaphoric to ‘a stray dog’, not ‘a dog’.

Recall our earlier examples from mathematics, physics, poetics, and dog-theory.

- **a** number is rational if and only if it is the quotient of two integers;
- the square of the hypotenuse of **a** right triangle is equal to
  the sum of the squares of its other two sides.
- **a** body at rest will remain at rest unless acted upon by an external force;
- **a** body in motion will remain in motion unless acted upon by an external force.
- if **a** body meet **a** body coming through the rye;
  if **a** body kiss **a** body, need **a** body cry.
- **a** dog is happy if it is well-fed.

Most of these are donkey-sentences, by our definition, since they contain anaphoric pronouns bound by indefinite noun phrases. And even the ones without overt anaphoric pronouns can be rewritten (less elegantly perhaps) so they contain anaphoric pronouns.

Let's concentrate on the last one, which is the simplest. The INP ‘a dog’ is naturally understood as a maximal-scope universal-quantifier that binds ‘it’.

26. a dog is happy if it is well-fed

<table>
<tr><td>a dog +1 –1</td><td>is happy</td><td>if</td><td>(–1) it is well-fed</td></tr>
<tr><td>$\Sigma\{\ x_1 \times x_{-1} \mid \mathbf{D}x\ \}$</td><td>$\lambda x_1 \mathbf{H}x$</td><td>$\lambda X \lambda Y[Y/X]$</td><td>$\lambda x_{-1}\ \mathbf{W}x$</td></tr>
<tr><td colspan="2">$\Sigma\{\ \mathbf{H}x \times x_{-1} \mid \mathbf{D}x\ \}$</td><td colspan="2">$\lambda x_{-1}\ \lambda Y\ [Y/\mathbf{W}x]$</td></tr>
<tr><td colspan="4">Ⓟ $\Pi\{\ \mathbf{H}x \times \lambda Y[Y/\mathbf{W}x] \mid \mathbf{D}x\ \}$
$\Pi\{\ \mathbf{H}x/\mathbf{W}x \mid \mathbf{D}x\ \}$
$\Pi\{\ \mathbf{H}x \mid \mathbf{D}x\ \&\ \mathbf{W}x\ \}$
$\forall x\{\ \mathbf{D}x\ \&\ \mathbf{W}x\ .\!\!\rightarrow \mathbf{H}x\ \}$</td></tr>
</table>

Ⓟ ‘if it is well-fed’ promotes Σ to Π.

The following is an equivalent formulation of this principle.

27. if a dog is well-fed, then it is happy

<table>
<tr><td>if</td><td>a dog +1 –1</td><td>is well-fed</td><td>then (–1) it is happy</td></tr>
<tr><td rowspan="2">$\lambda X \lambda Y[Y/X]$</td><td>$\Sigma\{ x_1 \times x_{-1} \mid \mathbf{D}x \}$</td><td>$\lambda x_1 \mathbf{W}x$</td><td rowspan="3">$\lambda x_{-1}\, \mathbf{H}x$</td></tr>
<tr><td colspan="2">$\Sigma\{ \mathbf{W}x \times x_{-1} \mid \mathbf{D}x \}$</td></tr>
<tr><td colspan="3">ⓟ $\Pi\{ \lambda Y[Y/\mathbf{W}x] \times x_{-1} \mid \mathbf{D}x \}$</td></tr>
<tr><td colspan="4">$\Pi\{ \lambda Y[Y/\mathbf{W}x] \times \mathbf{H}x \mid \mathbf{D}x \}$
$\Pi\{ \mathbf{H}x/\mathbf{W}x \mid \mathbf{D}x \}$
$\Pi\{ \mathbf{H}x \mid \mathbf{D}x \,\&\, \mathbf{W}x \}$
$\forall x\{ \mathbf{D}x \,\&\, \mathbf{W}x \mathbin{.\!\!\rightarrow} \mathbf{H}x \}$</td></tr>
</table>

ⓟ 'if' promotes Σ to Π.

Continuing with examples from dog-theory, we consider the following, which has two INPs with corresponding pronouns.

28. if a man owns a dog, then he feeds it

<table>
<tr><td>if</td><td>a man +1 –1</td><td>owns</td><td>a dog +2 –2</td><td>then (–1) he feeds (–2) it</td></tr>
<tr><td rowspan="3">$\lambda X \lambda Y[Y/X]$</td><td rowspan="2">$\Sigma\{ x_1 \times x_{-1} \mid \mathbf{M}x \}$</td><td>$\lambda y_2 \lambda x_1 \mathbf{O}xy$</td><td>$\Sigma\{ y_2 \times y_{-2} \mid \mathbf{D}y \}$</td><td rowspan="4">$\lambda y_{-2}\, \lambda x_{-1}\, \mathbf{F}xy$</td></tr>
<tr><td colspan="2">$\Sigma\{ \lambda x_1 \mathbf{O}xy \times y_{-2} \mid \mathbf{D}y \}$</td></tr>
<tr><td colspan="3">$\asymp \Sigma\{ \mathbf{O}xy \times x_{-1} \times y_{-2} \mid \mathbf{M}x \,\&\, \mathbf{D}y \}$</td></tr>
<tr><td colspan="4">ⓟ $\Pi\{ \lambda Y[Y/\mathbf{O}xy] \times x_{-1} \times y_{-2} \mid \mathbf{M}x \,\&\, \mathbf{D}y \}$</td></tr>
<tr><td colspan="5">$\Pi\{ \lambda Y[Y/\mathbf{O}xy] \times \mathbf{F}xy \mid \mathbf{M}x \,\&\, \mathbf{D}y \}$
$\Pi\{ \mathbf{F}xy/\mathbf{O}xy \mid \mathbf{M}x \,\&\, \mathbf{D}y \}$
$\Pi\{ \mathbf{F}xy \mid \mathbf{M}x \,\&\, \mathbf{D}y \,\&\, \mathbf{O}xy \}$
$\forall x \forall y\{\mathbf{M}x \,\&\, \mathbf{D}y \,\&\, \mathbf{O}xy \mathbin{.\!\!\rightarrow} \mathbf{F}xy\}$</td></tr>
<tr><td colspan="5">for any x,y : if x is a man, and y is a dog, and x owns y, then x feeds y</td></tr>
</table>

ⓟ 'if' promotes Σ to Π.

In the above derivation, '≍' indicates that the two sums combine via parallel-composition into a big sum. This also happens in the following equivalent formulation.

29. a man feeds a dog if he owns it

<table>
<tr><td>a man +1 –1</td><td>feeds a dog –2</td><td>if</td><td>(–1) he +1 owns (–2) it +2</td></tr>
<tr><td>$\Sigma\{ x_1 \times x_{-1} \mid \mathbf{M}x \}$</td><td>$\Sigma\{ \lambda x_1 \mathbf{F}xy \times y_{-2} \mid \mathbf{D}y \}$</td><td>$\lambda X \lambda Y[Y/X]$</td><td>$\lambda y_{-2}\, \lambda x_{-1}\, \mathbf{O}xy$</td></tr>
<tr><td colspan="2">$\asymp \Sigma\{ \mathbf{F}xy \times x_{-1} \times x_{-1} \mid \mathbf{M}x \,\&\, \mathbf{D}y \}$</td><td colspan="2">$\lambda y_{-2}\, \lambda x_{-1}\, \lambda Y[Y/\mathbf{O}xy]$</td></tr>
<tr><td colspan="4">ⓟ $\Pi\{ \mathbf{F}xy \times \lambda Y[Y/\mathbf{O}xy \mid \mathbf{M}x \,\&\, \mathbf{D}y \}$
$\Pi\{ \mathbf{F}xy/\mathbf{O}xy \mid \mathbf{M}x \,\&\, \mathbf{D}y \}$
$\Pi\{ \mathbf{F}xy \mid \mathbf{M}x \,\&\, \mathbf{D}y \,\&\, \mathbf{O}xy \}$
$\forall x \forall y\{\mathbf{M}x \,\&\, \mathbf{D}y \,\&\, \mathbf{O}xy \mathbin{.\!\!\rightarrow} \mathbf{F}xy\}$</td></tr>
</table>

We have read all the above examples as wide-scope universal formulas. What about the existential and generic readings of the INPs. The existential readings are not very plausible, but their plausibility is improved by appending the phrase 'namely, this one'.[38]

Generic readings of donkey-sentences compute rather oddly, as the following derivation illustrates.

[38] The wide-existential reading is complicated by the conditional, which does not interact so well with existentials.

men	who +1	love	dogs	+2 –1	+1	feed	(–1) them +2
			$\Sigma y\mathbf{D}y$	$\lambda y(y_2 \times y_{-1})$		$\lambda y_2\lambda x_1\mathbf{F}xy$	$\lambda y_{-1}{:}y_2$
		$\lambda y_2\lambda x_1\mathbf{L}xy$	$[\Sigma y\mathbf{D}y]_2 \times [\Sigma y\mathbf{D}y]_{-1}$				
	$\lambda x_0.x_1$	$\lambda x_1\mathbf{L}[x,\Sigma y\mathbf{D}y] \times [\Sigma y\mathbf{D}y]_{-1}$					
$\lambda x_0\mathbf{M}x$	$\lambda x_0\mathbf{L}[x,\Sigma y\mathbf{D}y] \times [\Sigma y\mathbf{D}y]_{-1}$						
$\lambda x_0\{\mathbf{M}x \,\&\, \mathbf{L}[x,\Sigma y\mathbf{D}y]\} \times [\Sigma y\mathbf{D}y]_{-1}$ $\Sigma x\{\mathbf{M}x \,\&\, \mathbf{L}[x,\Sigma y\mathbf{D}y]\} \times [\Sigma y\mathbf{D}y]_{-1}$					$\lambda x.x_1$		
$[\Sigma x(\mathbf{M}x \,\&\, \mathbf{L}[x,\Sigma y\mathbf{D}y])]_1 \times [\Sigma y\mathbf{D}y]_{-1}$						$\lambda y_{-1} : \lambda x_1\mathbf{F}xy$	
$\mathbf{F}[\ \Sigma x(\mathbf{M}x \,\&\, \mathbf{L}[x,\Sigma y\mathbf{D}y]),\ \Sigma y\mathbf{D}y\ ]$							
(men who love dogs-as-a-whole)-as-whole feed dogs-as-a-whole							

14. More on Scope and Binding

Recall that, according to our account, scope-restrictions for quantifiers are implemented via admissibility-restrictions on the associated junctions. Furthermore, quantifier scope-ambiguity results, not from structural-ambiguity, but from compositional-ambiguity, which arises from the symmetry of the junction-composition rules. Specifically, in computing

Ж$_1\{\alpha \mid \Phi\}$	Ж$_2\{\beta \mid \Psi\}$
??	

one can equally legitimately apply:

Ж$_1$ -composition	Ж$_1$ has wide-scope
Ж$_2$ -composition	Ж$_2$ has wide-scope
parallel-composition	Ж$_1$ and Ж$_2$ have equal scope

Recall the following example.

30. Jay doesn't own a dog

Ignoring the generic-reading and the wide-Σ reading, we have the following admissible derivations, which produce equivalent formulas.

wide-Π

Jay +1	doesn't	own	a dog +2
		$\lambda y_2\lambda x_1\mathbf{O}xy$	$\Sigma\{\ y_2 \mid \mathbf{D}y\ \}$
	$\lambda X{:}{\sim}X$	$\Sigma\{\ \lambda x_1\mathbf{O}xy \mid \mathbf{D}y\ \}$	
J1	ⓟ $\Pi\{\ \lambda x_1{\sim}\mathbf{O}xy \mid \mathbf{D}y\ \}$		
$\Pi\{\ {\sim}\mathbf{O}\mathrm{J}y \mid \mathbf{D}y\ \}$ $\forall y(\mathbf{D}y \rightarrow {\sim}\mathbf{O}\mathrm{J}y)$			

narrow-Σ

Jay+1	doesn't	own	a dog +2
		$\lambda y_2\lambda x_1\mathbf{O}xy$	$\Sigma\{\ y_2 \mid \mathbf{D}y\ \}$
		$\Sigma\{\ \lambda x_1\mathbf{O}xy \mid \mathbf{D}y\ \}$	
	$\lambda X{:}{\sim}X$	$\lambda x_1\ \exists y(\mathbf{D}y \,\&\, \mathbf{O}xy)$	
J1	$\lambda x_1\ {\sim}\exists y(\mathbf{D}y \,\&\, \mathbf{O}xy)$		
${\sim}\exists y(\mathbf{D}y \,\&\, \mathbf{O}\mathrm{J}y)$			

Notice that these read the sentence as equivalent to:

31. Jay owns no dog

One might accordingly be inclined to say that 30 and 31 are *synonymous*. But this would mean that the following are also synonymous.

32. if Jay doesn't own a dog, then Jay doesn't feed it
33. if Jay owns no dog, then Jay doesn't feed it

The difference between the latter concerns pronoun-binding; whereas 'a dog' can bind 'it', 'no dog' cannot bind 'it', as shown in the following derivations.

34. if Jay doesn't own a dog, then Jay doesn't feed it

if	Jay +1	doesn't	own	a dog +2 –1	then Jay doesn't feed (–1) it
			$\lambda y_2 \lambda x_1 \mathbf{O}xy$	$\Sigma\{ y_2 \times y_{-1} \mid \mathbf{D}y \}$	
		$\lambda X{:}{\sim}X$	$\Sigma\{ \lambda x_1 \mathbf{O}xy \times y_{-1} \mid \mathbf{D}y \}$		
	J1	$\Pi\{ \lambda x_1{\sim}\mathbf{O}xy \times y_{-1} \mid \mathbf{D}y \}$			
$\lambda X \lambda Y[Y/X]$	$\Pi\{ {\sim}\mathbf{O}\text{J}y \times y_{-1} \mid \mathbf{D}y \}$				
$\Pi\{ \lambda Y[Y/{\sim}\mathbf{O}\text{J}y] \times y_{-1} \mid \mathbf{D}y \}$					$\lambda z_{-1} {\sim}\mathbf{F}\text{J}z$
$\Pi\{ {\sim}\mathbf{F}\text{J}y/{\sim}\mathbf{O}\text{J}y \mid \mathbf{D}y \}$ $\Pi\{ {\sim}\mathbf{O}\text{J}y \mid \mathbf{D}y \;\&\; {\sim}\mathbf{F}\text{J}y \}$ $\forall x \{ \mathbf{D}x \;\&\; {\sim}\mathbf{O}\text{J}x \;.\!\!\rightarrow {\sim}\mathbf{F}\text{J}x \}$					

35. if Jay owns no dog, then Jay doesn't feed it

if	Jay +1	owns	no dog +2 –1	then Jay doesn't feed (–1) it
		$\lambda y_2 \lambda x_1 \mathbf{O}xy$	$\bigcirc\{ y_2 \times y_{-1} \mid \mathbf{D}y \}$	
	J1	$\bigcirc\{ \lambda x_1 \mathbf{O}xy \times y_{-1} \mid \mathbf{D}y \}$		
$\lambda X \lambda Y[Y/X]$	$\bigcirc\{ \mathbf{O}\text{J}y \times y_{-1} \mid \mathbf{D}y \}$			
⨯				$\lambda z_{-1} {\sim}\mathbf{F}\text{J}z$
⨯				

Note that Π admits 'if', but ⋀ and ○ do not. Compare the latter derivation with the following, which also does not complete.

36. if Jay owns every dog, then Jay feeds it

if	Jay +1	owns	every dog +2 –1	then Jay feeds (–1) it
		$\lambda y_2 \lambda x_1 \mathbf{O}xy$	$\bigwedge\{ y_2 \times y_{-1} \mid \mathbf{D}y \}$	
	J1	$\bigwedge\{ \lambda x_1 \mathbf{O}xy \times y_{-1} \mid \mathbf{D}y \}$		
$\lambda X \lambda Y[Y/X]$	$\bigwedge\{ \mathbf{O}\text{J}y \times y_{-1} \mid \mathbf{D}y \}$			
⨯				$\lambda z_{-1} \mathbf{F}\text{J}z$
⨯				

As in earlier examples, we can complete these derivations if we remove the alpha-marker from the INP. For example, the 'no dog' example computes as follows.

if	Jay +1	owns	no dog +2	then Jay doesn't feed (–1) it
		$\lambda y_2 \lambda x_1 \mathbf{O}xy$	$\bigcirc\{ y_2 \mid \mathbf{D}y \}$	
	J1	$\bigcirc\{ \lambda x_1 \mathbf{O}xy \mid \mathbf{D}y \}$		
$\lambda X \lambda Y[Y/X]$	$\bigcirc\{ \mathbf{O}\text{J}y \mid \mathbf{D}y \}$ ${\sim}\exists y\{\mathbf{D}y \;\&\; \mathbf{O}xy\}$			
$\lambda Y[Y/{\sim}\exists y\{\mathbf{D}y \;\&\; \mathbf{O}xy\}]$				$\lambda z_{-1} {\sim}\mathbf{F}\text{J}z$
$\lambda z_{-1} \; [{\sim}\mathbf{F}\text{J}z \,/\, {\sim}\exists y\{\mathbf{D}y \;\&\; \mathbf{O}xy\}]$				

Notice that the anaphoric pronoun 'it' is left dangling (open). This can be bound in a wider sentence, as in the following sentence.

if a stray dog [–1] appears, then
if Jays owns no dog, then Jay doesn't feed (–1) it

Here, 'it' is anaphoric to 'a stray dog'.

Next, we reconsider how INPs scopally interact with ordinary quantifiers. For example, consider the following.

37. every man owns a dog

every man +1	owns	a dog +2
	$\lambda y_2\lambda x_1\mathbf{O}xy$	$\Sigma\{y_2 \mid \mathbf{D}y\}$
$\bigwedge\{x_1 \mid \mathbf{M}x\}$	$\Sigma\{\lambda x_1\mathbf{O}xy \mid \mathbf{D}y\}$	
$\bigwedge\{\Sigma\{\mathbf{O}xy \mid \mathbf{D}y\} \mid \mathbf{M}x\}$ $\forall x\{\mathbf{M}x \rightarrow \exists y(\mathbf{D}y \,\&\, \mathbf{O}xy)\}$		

38. no man owns a dog

no man +1	owns	a dog +2
	$\lambda y_2\lambda x_1\mathbf{O}xy$	$\Sigma\{y_2 \mid \mathbf{D}y\}$
$\bigcirc\{x_1 \mid \mathbf{M}x\}$	$\Sigma\{\lambda x_1\mathbf{O}xy \mid \mathbf{D}y\}$	
$\bigcirc\{\Sigma\{\mathbf{O}xy \mid \mathbf{D}y\} \mid \mathbf{M}x\}$ $\sim\exists x\{\mathbf{M}x \,\&\, \exists y(\mathbf{D}y \,\&\, \mathbf{O}xy)\}$		

These derivations ascribe narrow-scope to 'a dog'. Can we ascribe wide-scope to 'a dog'; are the following readings admissible?

every man +1	owns	a dog +2
	$\lambda y_2\lambda x_1\mathbf{O}xy$	$\Sigma\{y_2 \mid \mathbf{D}y\}$
$\bigwedge\{x_1 \mid \mathbf{M}x\}$	$\Sigma\{\lambda x_1\mathbf{O}xy \mid \mathbf{D}y\}$	
$\Sigma\{\bigwedge\{\mathbf{O}xy \mid \mathbf{M}x\} \mid \mathbf{D}y\}$ $\exists y\{\mathbf{D}y \,\&\, \forall x(\mathbf{M}x \rightarrow \mathbf{O}xy)\}$		

no man +1	owns	a dog +2
	$\lambda y_2\lambda x_1\mathbf{O}xy$	$\Sigma\{y_2 \mid \mathbf{D}y\}$
$\bigcirc\{x_1 \mid \mathbf{M}x\}$	$\Sigma\{\lambda x_1\mathbf{O}xy \mid \mathbf{D}y\}$	
$\Sigma\{\bigcirc\{\mathbf{O}xy \mid \mathbf{M}x\} \mid \mathbf{D}y\}$ $\exists y\{\mathbf{D}y \,\&\, \sim\exists x(\mathbf{M}x \,\&\, \mathbf{O}xy)\}$		

These seem odd, but they are admissible, considering other sentences with similar forms sound OK if a bit surprising. Recall the Dangerfield Adjustment.

39. every man owns a dog… namely, this one
40. no man owns a dog… namely, this one

Recall that the coda makes a wide-existential reading plausible; indeed it makes a narrow-existential reading impossible.

Another advantage of allowing these combinations is that it enables us to properly render the following two examples, which offer a dog-theoretic version of the logical notions of *freedom* and *bondage*.

41. if no man owns a dog, then it is free

if	no man +1	owns	a dog +2 −1	then (−1) it is free
		$\lambda y_2\lambda x_1\mathbf{O}xy$	$\Sigma\{y_2 \times y_{-1} \mid \mathbf{D}y\}$	
	$\bigcirc\{x_1 \mid \mathbf{M}x\}$	$\Sigma\{\lambda x_1\mathbf{O}xy \times y_{-1} \mid \mathbf{D}y\}$		
$\lambda X\lambda Y[Y/X]$	$\Sigma\{\bigcirc\{\mathbf{O}xy \mid \mathbf{M}x\} \times y_{-1} \mid \mathbf{D}y\}$ $\Sigma\{\sim\exists x(\mathbf{M}x\&\mathbf{O}xy) \times y_{-1} \mid \mathbf{D}y\}$			
ⓟ $\Pi\{\lambda Y[Y/\sim\exists x(\mathbf{M}x\&\mathbf{O}xy)] \times y_{-1} \mid \mathbf{D}y\}$				$\lambda y_{-1}\,\mathbf{F}y$
$\Pi\{\lambda Y[Y/\sim\exists x(\mathbf{M}x\&\mathbf{O}xy)] \times \mathbf{F}y \mid \mathbf{D}y\}$ $\Pi\{\mathbf{F}y/\sim\exists x(\mathbf{M}x\&\mathbf{O}xy) \mid \mathbf{D}y\}$ $\forall y\{\mathbf{D}y \,\&\sim\exists x(\mathbf{M}x\&\mathbf{O}xy) .\!\rightarrow \mathbf{F}y\}$				

42. if every man owns a dog, then it is bound

if	every man +1	owns	a dog +2 −1	then (−1) it is bound
		$\lambda y_2\lambda x_1\mathbf{O}xy$	$\Sigma\{y_2 \times y_{-1} \mid \mathbf{D}y\}$	
	$\bigwedge\{x_1 \mid \mathbf{M}x\}$	$\Sigma\{\lambda x_1\mathbf{O}xy \times y_{-1} \mid \mathbf{D}y\}$		
$\lambda X\lambda Y[Y/X]$	$\Sigma\{\bigwedge\{\mathbf{O}xy \mid \mathbf{M}x\} \times y_{-1} \mid \mathbf{D}y\}$ $\Sigma\{\forall x(\mathbf{M}x\rightarrow\mathbf{O}xy) \times y_{-1} \mid \mathbf{D}y\}$			
ⓟ $\Pi\{\lambda Y[Y/\forall x(\mathbf{M}x\rightarrow\mathbf{O}xy)] \times y_{-1} \mid \mathbf{D}y\}$				$\lambda y_{-1}\,\mathbf{B}y$
$\Pi\{\lambda Y[Y/\forall x(\mathbf{M}x\rightarrow\mathbf{O}xy)] \times \mathbf{B}y \mid \mathbf{D}y\}$ $\Pi\{\mathbf{B}y/\forall x(\mathbf{M}x\rightarrow\mathbf{O}xy) \mid \mathbf{D}y\}$ $\forall y\{\mathbf{D}y \,\&\, \forall x(\mathbf{M}x\rightarrow\mathbf{O}xy) .\!\rightarrow \mathbf{B}y\}$				

Compare these derivations with the following derivations.

43. if every man owns a dog, then he feeds it

‘a dog’ wide

<table>
<tr><td>if</td><td>every man +1 –1</td><td>owns a dog +2 –2</td><td>then (–1) he feeds (–2) it</td></tr>
<tr><td rowspan="2">$\lambda X\lambda Y[Y/X]$</td><td>$\wedge\{ x_1 \times x_{-1} \mid \mathbf{M}x \}$</td><td>$\Sigma\{ \lambda x_1\mathbf{O}xy \times y_{-1} \mid \mathbf{D}y \}$</td><td rowspan="3">$\lambda y_{-2} : \lambda x_{-1} : \mathbf{F}xy$</td></tr>
<tr><td colspan="2">$\Sigma\{ \wedge\{\mathbf{O}xy \times x_{-1} \mid \mathbf{M}x\} \times y_{-1} \mid \mathbf{D}y \}$</td></tr>
<tr><td colspan="3">※</td></tr>
<tr><td colspan="4">※</td></tr>
</table>

‘every man’ wide

<table>
<tr><td>if</td><td>every man +1 –1</td><td>owns a dog +2 –2</td><td>then (–1) he feeds (–2) it</td></tr>
<tr><td rowspan="2">$\lambda X\lambda Y[Y/X]$</td><td>$\wedge\{ x_1 \times x_{-1} \mid \mathbf{M}x \}$</td><td>$\Sigma\{ \lambda x_1\mathbf{O}xy \times y_{-2} \mid \mathbf{D}y \}$</td><td rowspan="3">$\lambda y_{-2} : \lambda x_{-1} : \mathbf{F}xy$</td></tr>
<tr><td colspan="2">$\wedge\{ \Sigma\{ \mathbf{O}xy \times y_{-2} \mid \mathbf{D}y \} \times x_{-1} \mid \mathbf{M}x \}$</td></tr>
<tr><td colspan="3">※</td></tr>
<tr><td colspan="4">※</td></tr>
</table>

These fail because ‘if’ does not combine with ‘every man owns a dog’ as configured. In particular, unlike Σ and Π, ∧ does not admit ‘if’. Also the ∧-phrase does not simplify, since it is not a conjunction of sentences.

One might wonder whether Σ gets promoted by ‘every man’ or ‘no man’; it does not, even though it is promoted by ‘every’ and ‘no’.[39] Otherwise, a legitimate reading of

every man owns a dog

is: every man owns *every* dog

On the other hand, we have (facetiously) suggested that we are discussing dog-*theory*, which means that the sentences carry modal/nomic force. In that case ‘a dog’ refers to dogs-in-general.

But we don't want Σ-promotion to occur when the sentence is clearly not nomic, as in the following example.

every man is walking a dog

15. Further Examples

By way of further illustrating our scheme, we consider a few somewhat complex examples. The following illustrate that INPs can bind pronouns in a variety of syntactic configurations. They also show how ‘if’ can be iterated.

44. if a man owns a donkey, he beats it, if it kicks him

<table>
<tr><td>if</td><td>a man [–1] owns a donkey [–2]</td><td>(–1) he beats (–2) it</td><td>if</td><td>(–2) it kicks (–1) him</td></tr>
<tr><td>$\lambda X\lambda Y[Y/X]$</td><td>$\Sigma\{ \mathbf{O}xy \times x_{-1} \times y_{-2} \mid \mathbf{M}x \,\&\, \mathbf{D}y \}$</td><td rowspan="2">$\lambda y_{-2}\, \lambda x_{-1}\, \mathbf{B}xy$</td><td>$\lambda X\lambda Y[Y/X]$</td><td>$\lambda x_{-1}\, \lambda y_{-2}\, \mathbf{K}yx$</td></tr>
<tr><td colspan="2">ⓟ $\Pi\{ \lambda Y[Y/\mathbf{O}xy] \times x_{-1} \times y_{-2} \mid \mathbf{M}x \,\&\, \mathbf{D}y \}$</td><td colspan="2" rowspan="2">$\lambda x_{-1}\, \lambda y_{-2}\, \lambda Y[Y/\mathbf{K}yx]$</td></tr>
<tr><td colspan="3">① $\Pi\{ \lambda Y[Y/\mathbf{O}xy] \times \mathbf{B}xy \times x_{-1} \times y_{-2} \mid \mathbf{M}x \,\&\, \mathbf{D}y \}$
$\Pi\{ [\mathbf{B}xy/\mathbf{O}xy] \times x_{-1} \times y_{-2} \mid \mathbf{M}x \,\&\, \mathbf{D}y \}$</td></tr>
<tr><td colspan="5">$\Pi\{ [\mathbf{B}xy/\mathbf{O}xy] \times \lambda Y[Y/\mathbf{K}yx] \mid \mathbf{M}x \,\&\, \mathbf{D}y \}$
$\Pi\{ [\mathbf{B}xy/\mathbf{O}xy]/\mathbf{K}yx \mid \mathbf{M}x \,\&\, \mathbf{D}y \}$
$\Pi\{ \mathbf{B}xy \mid \mathbf{M}x \,\&\, \mathbf{D}y \,\&\, \mathbf{K}yx \,\&\, \mathbf{O}xy \}$
$\forall x\forall y\{ \mathbf{M}x \,\&\, \mathbf{D}y \,\&\, \mathbf{K}yx \,\&\, \mathbf{O}xy \,.\!\rightarrow \mathbf{B}xy \}$</td></tr>
<tr><td colspan="5">for any x,y: if x is a man, and y is a donkey, and y kicks x, and x owns y, then x beats y</td></tr>
</table>

① This employs alpha-duplication, which allows an NP to bind indefinitely-many pronouns.[40]

[39] See Unit C [Expanded Account of Quantifiers], where we show that we can remove ‘every’ and ‘no’ from the list of Σ-promoting phrases.

[40] See Chapter 7 [Pronouns].

45. if **he** owns **it**, a man beats a donkey, if **it** kicks **him** [41]

<table>
<tr><td>if</td><td>(–1) he owns (–2) it</td><td>a man [–1] beats a donkey [–2]</td><td>if</td><td>(–2) it kicks (–1) him</td></tr>
<tr><td>$\lambda X \lambda Y[Y/X]$</td><td>$\lambda y_{-2}\, \lambda x_{-1}\, \mathbf{O}xy$</td><td rowspan="2">$\Sigma\{\, \mathbf{B}xy \times x_{-1} \times y_{-2} \mid \mathbf{M}x \;\&\; \mathbf{D}y \,\}$</td><td>$\lambda X \lambda Y[Y/X]$</td><td>$\lambda x_{-1}\, \lambda y_{-2}\, \mathbf{K}yx$</td></tr>
<tr><td colspan="2" rowspan="2">$\lambda y_{-2}\, \lambda x_{-1}\, \lambda Y[Y/\mathbf{O}xy]$</td><td colspan="2">$\lambda x_{-1}\, \lambda y_{-2}\, \lambda Y[Y/\mathbf{K}yx]$</td></tr>
<tr><td colspan="3">① ⓟ $\Pi\{\, [\mathbf{B}xy/\mathbf{K}yx] \times x_{-1} \times y_{-2} \mid \mathbf{M}x \;\&\; \mathbf{D}y \,\}$</td></tr>
<tr><td colspan="5">$\Pi\{\, [\mathbf{B}xy/\mathbf{K}yx] \times \lambda Y[Y/\mathbf{O}xy] \mid \mathbf{M}x \;\&\; \mathbf{D}y \,\}$
$\Pi\{\, [\mathbf{B}xy/\mathbf{K}yx]/\mathbf{O}xy \mid \mathbf{M}x \;\&\; \mathbf{D}y \,\}$
$\Pi\{\, \mathbf{B}xy \mid \mathbf{M}x \;\&\; \mathbf{D}y \;\&\; \mathbf{O}xy \;\&\; \mathbf{K}yx \,\}$</td></tr>
<tr><td colspan="5">for any x,y: if x is a man, and y is a donkey, and x owns y, and y kicks x, then x beats y</td></tr>
</table>

46. if he owns a donkey, a man beats it, if it kicks him

<table>
<tr><td>if</td><td>(–1) he +1</td><td>owns</td><td>a donkey +2 –2</td><td>a man –1 beats (–2) it</td><td>if (–2) it kicks (–1) him</td></tr>
<tr><td rowspan="3">$\lambda X \lambda Y[Y/X]$</td><td rowspan="2">$\lambda x_{-1} : x_1$</td><td>$\lambda y_2 \lambda x_1 \mathbf{O}xy$</td><td>$\Sigma\{y_2 \times y_{-2} \mid \mathbf{D}y\}$</td><td rowspan="4">$\lambda y_{-2}\, \Sigma\{\mathbf{B}xy \times x_{-1} \mid \mathbf{M}x\}$</td><td rowspan="5">$\lambda y_{-2}\, \lambda x_{-1}\, \lambda Y\, [Y/\mathbf{K}yx]$</td></tr>
<tr><td colspan="2">$\Sigma\{\, \lambda x_1 \mathbf{O}xy \times y_{-2} \mid \mathbf{D}y \,\}$</td></tr>
<tr><td colspan="3">$\lambda x_{-1} : \Sigma\{\, \mathbf{O}xy \times y_{-2} \mid \mathbf{D}y \,\}$</td></tr>
<tr><td colspan="4">ⓟ $\Pi\{\, \lambda x_{-1}{:}\lambda Y[Y/\mathbf{O}xy] \times y_{-2} \mid \mathbf{D}y \,\}$</td></tr>
<tr><td colspan="5">$\Pi\{\, \lambda x_{-1}{:}\lambda Y[Y/\mathbf{O}xy] \times \Sigma\{\mathbf{B}xy \times x_{-1} \mid \mathbf{M}x\} \times y_{-2} \mid \mathbf{D}y \,\}$
$\Pi\{\, \Pi\{\mathbf{B}xy/\mathbf{O}xy \times x_{-1} \mid \mathbf{M}x\} \times y_{-2} \mid \mathbf{D}y \,\}$
$\Pi\{\, \mathbf{B}xy/\mathbf{O}xy \times x_{-1} \times y_{-2} \mid \mathbf{D}y \;\&\; \mathbf{M}x \,\}$</td></tr>
<tr><td colspan="6">$\Pi\{\, \mathbf{B}xy/\mathbf{O}xy \times \lambda Y\, [Y/\mathbf{K}yx] \mid \mathbf{M}x \;\&\; \mathbf{D}y \,\}$
$\Pi\{\, (\mathbf{B}xy/\mathbf{O}xy)/\mathbf{K}yx \mid \mathbf{M}x \;\&\; \mathbf{D}y \,\}$
$\Pi\{\, \mathbf{B}xy \mid \mathbf{M}x \;\&\; \mathbf{D}y \;\&\; \mathbf{K}yx \;\&\; \mathbf{O}xy\}$
$\forall x \forall y\{\, \mathbf{M}x \;\&\; \mathbf{D}y \;\&\; \mathbf{O}xy \;\&\; \mathbf{K}yx \;.\!\!\rightarrow \mathbf{B}xy \,\}$</td></tr>
</table>

47. if a man owns it, he beats a donkey, if it kicks him +++FINISH+++

<table>
<tr><td>if</td><td>a man [–1] owns a donkey [–2]</td><td>(–1) he beats (–2) it</td><td>if</td><td>(–2) it kicks (–1) him</td></tr>
<tr><td>$\lambda X \lambda Y[Y/X]$</td><td>$\Sigma\{\, \mathbf{O}xy \times x_{-1} \times y_{-2} \mid \mathbf{M}x \;\&\; \mathbf{D}y \,\}$</td><td>$\lambda y_{-2}\, \lambda x_{-1}\, \mathbf{B}xy$</td><td>$\lambda X \lambda Y[Y/X]$</td><td>$\lambda x_{-1}\, \lambda y_{-2}\, \mathbf{K}yx$</td></tr>
<tr><td colspan="5">for any x,y: if x is a man, and y is a donkey, and y kicks x, and x owns y, then x beats y</td></tr>
</table>

48. if he owns it, he beats a donkey, if it kicks a man +++FINISH+++

<table>
<tr><td>if</td><td>a man [–1] owns a donkey [–2]</td><td>(–1) he beats (–2) it</td><td>if</td><td>(–2) it kicks (–1) him</td></tr>
<tr><td>$\lambda X \lambda Y[Y/X]$</td><td>$\Sigma\{\, \mathbf{O}xy \times x_{-1} \times y_{-2} \mid \mathbf{M}x \;\&\; \mathbf{D}y \,\}$</td><td>$\lambda y_{-2}\, \lambda x_{-1}\, \mathbf{B}xy$</td><td>$\lambda X \lambda Y[Y/X]$</td><td>$\lambda x_{-1}\, \lambda y_{-2}\, \mathbf{K}yx$</td></tr>
<tr><td colspan="5">for any x,y: if x is a man, and y is a donkey, and y kicks x, and x owns y, then x beats y</td></tr>
</table>

+++what are all the combinatorial possibilities+++

[41] Notice that the relative-scope of the two occurrences of 'if' does not affect the truth conditions. When I was a child, my mom once said the following humorous version of this form.

if we had ham, we could have ham and eggs, ... if we had eggs!

16. Other Forms that Act Like INPs

1. Personal INPs

When applied to special domains, quantifier-phrases occasionally take on special forms, including ‘always’, ‘never’, ‘everywhere’, and ‘somewhere’. When the special domain is persons, we have the following transformations.[42]

every person	⇒	every**one**
any person	⇒	any**one**
some person	⇒	some**one**
no person	⇒	no **one**
a person	⇒	some**one** [※ a **one**][43]

Notice that this means that ‘someone’ is ambiguous between the QP ‘some person’ and the INP ‘a person’, which must be carefully distinguished.

Also, there are exceptions such as the following.

49. a person is a moral agent

This philosophical claim is presumably nomic (law-like), and accordingly licenses counterfactual reasoning. But if we replace ‘a person’ by ‘someone’, we obtain

someone is a moral agent

which does not seem so clearly to be nomic. Similarly,

50. a person likes dogs

admits a generic reading, but if we replace ‘a person’ by ‘someone’, we obtain

someone likes dogs

which does not seem so clearly to be generic.

Nevertheless, there are uses of ‘someone’ that behave like an indefinite noun phrase.

51. if someone owns a dog, s/he feeds it

This is a variant of a donkey-sentence, which is left as an exercise. The following is more interesting.

52. if a dog bites someone, s/he gets a rabies shot

This is ambiguous according to whether “s/he” is the dog or the person. Let's concentrate on the latter reading, which is computed as follows, in which we treat each INP as a wide-Π.

<table>
<tr><td>if</td><td>a dog +1</td><td>bites</td><td>someone +2 –1</td><td>then s/he (–1) gets-a-rabies-shot</td></tr>
<tr><td rowspan="3">$\lambda X \lambda Y[Y/X]$</td><td rowspan="2">$\Pi\{ x_1 \mid \mathbf{D}x \}$</td><td>$\lambda y_2 \lambda x_1 \mathbf{B}xy$</td><td>$\Pi\{ y_2 \times y_{-1} \mid \mathbf{P}y \}$</td><td rowspan="3"></td></tr>
<tr><td colspan="2">$\Pi\{ \lambda x_1 \mathbf{B}xy \times y_{-1} \mid \mathbf{P}y \}$</td></tr>
<tr><td colspan="3">$\Pi\{ \mathbf{B}xy \times y_{-1} \mid \mathbf{D}x \ \& \ \mathbf{P}y \}$</td></tr>
<tr><td colspan="4">$\Pi\{ \lambda Y[Y/\mathbf{B}xy] \times y_{-1} \mid \mathbf{D}x \ \& \ \mathbf{P}y \}$</td><td>$\lambda y_{-1} : \mathbf{R}y$</td></tr>
<tr><td colspan="5">$\Pi\{ \lambda Y[Y/\mathbf{B}xy] \times \mathbf{R}y \mid \mathbf{D}x \ \& \ \mathbf{P}y \}$
$\Pi\{ \mathbf{R}y/\mathbf{B}xy \mid \mathbf{D}x \ \& \ \mathbf{P}y \}$
$\Pi\{ \mathbf{R}y \mid \mathbf{D}x \ \& \ \mathbf{P}y \ \& \ \mathbf{B}xy \}$
$\forall x \forall y\{ \mathbf{D}x \ \& \ \mathbf{P}y \ \& \ \mathbf{B}xy \ .\!\rightarrow \mathbf{R}y \}$</td></tr>
</table>

We can also read either INP as a wide-Σ, and we can read ‘a dog’ as a narrow-Σ, but we cannot read ‘someone’ as a narrow-Σ [that binds ‘s/he’], and we cannot read ‘someone’ as a narrow-Π.[44]

---

[42] Note that these phrases should be distinguished from similar forms that have a slight pause before ‘one’. For example, ‘every…one’ is analogous to ‘this one’, as in the following example.

I own many dogs; every **one** is smart; this **one** is very smart

[43] Also, the general rule does not apply to plural quantifiers; for example, ‘several persons’ does not abbreviate as ‘several ones’. Also, the rule does not apply to numerical quantifiers; for example, ‘exactly one person’ does not abbreviate as ‘exactly one one’.

[44] Also, note the general problem of simplifying expressions of the form $\Sigma\{\Omega/\Psi \mid \Phi\}$.

2. Bare Pronouns

We next note that bare pronouns occasionally behave like bare common-nouns, and hence INPs. The following are examples.[45]

- **he** who hesitates is lost
- **he** who laughs last laughs best
- **he** who makes a beast of himself gets rid of the pain of being a man [46]
- **you** can lead a horse to water, but **you** cannot make it drink
- **you** can fool some of the people all of the time… [47]
- **you** can't make a silk purse out of a sow's ear
- **one** does not simply walk into Mordor [48]
- whereof **one** cannot speak, thereof **one** must be silent [49]
- **one** cannot think well, love well, sleep well, if **one** has not dined well [50]

The most natural semantic hypothesis is that these words have lexical entries that claim they are synonymous with ‘a person’,[51] but which dis-allow existential-readings.

B. *Any*

1. Introduction

There is a striking similarity between ‘a’ and ‘any’; for example, the following pairs are similar in meaning.

(1) do you own **a** dog?
do you own **any** dog?

(2) I do not own **a** dog
I do not own **any** dog

(3) if **a** wild animal comes into the house, we put it back outside[52]
if **any** wild animal comes into the house, we put it back outside

(4) not **a** creature was stirring, not even a mouse [53]
not **any** creature was stirring, not even a mouse

On the other hand, ‘a’ and ‘any’ are not interchangeable, as demonstrated by the following pairs.

Rex is **a** dog
≠ Rex is **any** dog

yes, I own **a** dog
≠ yes, I own **any** dog

a wild animal came into the house
≠ **any** wild animal came into the house

if Jay doesn't own **a dog**, he doesn't feed **it**
≠ if Jay doesn't own **any dog**, he doesn't feed **it**

if **a man** doesn't own **a dog**, **he** doesn't feed **it**

---

[45] There are also "perverbs" (short for ‘perverted proverbs’) such as:
he who hesitates laughs best
you can fool some of the people all of the time, but you can't make them drink

[46] Samuel Johnson (1887).

[47] Usually attributed to Abraham Lincoln, but the provenance is sketchy.
you can fool all of the people some of the time,
and some of the people all of the time,
but you cannot fool all of the people all of the time.

[48] The schema “one does not simply…” is a meme that derives from the movie *The Lord of the Rings*, based on a line in J.R.R. Tolkien's masterpiece by the same name.

[49] The concluding line of Wittgenstein's *Tractatus Logico-Philosophicus*.

[50] Virginia Woolf, “A Room of One's Own” (1929).

[51] Bear in mind that “person” may be broadly construed, especially in speculative fiction, to include non-humans.

[52] For example, a moth!

[53] From the poem “A Visit from Saint Nicholas” (1823), originally published anonymously, and later attributed to Clement Clarke Moore.

≠ if **any man** doesn't own **any dog**, he doesn't feed **it**

In particular, the former make sensible claims, whereas the latter seem odd, even bizarre.

Indeed, the word ‘any’ is grammatically quite eccentric, in a way similar to ‘ever’ and ‘either’. For example, if you are asked:

☺ does **any**one have a question?
☺ have you **ever** been to Paris?
☺ do you recognize **either** of these people?

you are *not* grammatically-permitted to answer:

☹ yes, **any**one has a question
☹ yes, I have **ever** been to Paris
☹ yes, I recognize **either** of these people

Also, one can say:

☺ **every** student is sitting

but not:

☹ **any** student is sitting

On the other hand, one can say **either** of the following.

☺ **every** student caught cheating will be punished
☺ **any** student caught cheating will be punished

Here the difference seems to be that the latter, but not the former, carries *modal force*.[54] This explains why the following is good or bad, according to whether it is modal or indicative in character.[55]

any pet of mine is neutered or spayed

2. The Proposed Account

By way of accounting for the behavior of ‘any’, we propose the following overall hypothesis.

Sentences of the form ‘any’ + CNP + VP are **NOT FUNDAMENTALLY ASSERTIONAL**; rather, they are **SUB-ASSERTIONAL**; they become assertional only when embedded in a syntactic **CONTEXT** that **PROMOTES** ‘any’ to a universal-quantifier.

In order to formalize this hypothesis, we propose yet another junction, Л,[56] called *subjunction*, with its own special properties, which are summarized as follows.

if	A	is a type	then	ЛA	is a type

Л$T \neq T$ (for any type T)

---

[54] By modal force, we mean that the domain is expanded to include possible objects or events. Whereas the ‘every’ statement is automatically true if no actual student is actually caught cheating, the ‘any’ statement is not automatically true under these circumstances.

[55] If it is modal, then I am talking about possible (past, present, and future) pets. In an earlier draft of this chapter, we were bereft of pets, so the sentence was modal in character. Now we have a new pet, Oscar Wildcat, who is neutered. Also, we only "fix" our *mammalian* pets; the others (snakes and spiders) remain intact!

[56] The symbol is the Cyrillic letter ‘el’, which derives from Greek lambda (Λ), which is short for ‘любой’ [‘liuboi’], which is Russian for (approximately) ‘any one’. This symbol is chosen also because it is graphically intermediate between ‘∧’ and ‘Π’, and ‘any’ is between ∧ (conjunction) and Π (product) with respect to scope. Some occurrences of ‘Л’ even look exactly like ‘Λ’; for example, Lenin's Tomb has the following inscribed on it – ΛЕНИН.

if	α	is an expression of type	A
and	Φ	is a formula	
then	$\text{Л}\{\alpha \mid \Phi\}$	is an expression of type	$\text{Л}A$
	reads:	the **subjunction** of all α such that Φ	

$\text{Л}\nu\Phi$	$=_{df}$	$\text{Л}\{\nu \mid \Phi\}$	ν is a variable of any type; Φ is any formula

Л-Composition	
α	α, β, γ are any expressions
$\text{Л}\{ \beta \mid \Phi \}$	Φ is any formula
$\alpha \,;\, \beta \hookrightarrow \gamma$	any sub-derivation of γ from $\{\alpha,\beta\}$
$\wedge\{ \gamma \mid \Phi \}$	if α is Л-promoting
$\text{Л}\{ \gamma \mid \Phi \}$	otherwise

Л *is* promoted* to ∧ by:

1. no
2. not
3. if-clauses…
4. nomic/modal contexts
5. question contexts

* Promotion is *obligatory*.

Л-Simplification	
$\text{Л}\{ \Psi \mid \Phi \}$	$\text{Л}\{ \Psi \mid \Phi \}$ is **sub-assertional**;
※	Л **never simplifies**.

3. *Not-Any* and *If-Any*

53. Jay does not respect any woman

Jay +1	does-not	respect	any	woman	+2
			$\lambda P_0\, \text{Л}y Py$	$\mathbf{W}_0$	
			$\text{Л}y\mathbf{W}y$		$\lambda x.x_2$
		$\lambda y_2 \lambda x_1 \mathbf{R}xy$	$\text{Л}\{ y_2 \mid \mathbf{W}y \}$		
	$\lambda X{\sim}X$	$\text{Л}\{ \lambda x_1 \mathbf{R}xy \mid \mathbf{W}y \}$			
J_1	$\wedge\{ \lambda x_1{\sim}\mathbf{R}xy \mid \mathbf{W}y \}$				
$\wedge\{ {\sim}\mathbf{R}\text{J}y \mid \mathbf{W}y \}$ $\forall y \{ \mathbf{W}y \rightarrow {\sim}\mathbf{R}xy \}$					

Note that, unlike ∧, Л admits ‘not’, which moreover promotes it to ∧, which enables us to treat the resulting phrase as genuinely assertional.

54. if Jay respects any woman, Jay respects Kay

<table>
<tr><td>if</td><td>Jay +1</td><td>respects</td><td>any woman +2</td><td>Jay respects Kay</td></tr>
<tr><td rowspan="3">$\lambda X \lambda Y[Y/X]$</td><td rowspan="2">J_1</td><td>$\lambda y_2 \lambda x_1 \mathbf{R}xy$</td><td>$Л\{ y_2 \mid \mathbf{W}y \}$</td><td rowspan="4">$\mathbf{R}\mathrm{JK}$</td></tr>
<tr><td colspan="2">$Л\{ \lambda x_1 \mathbf{R}xy \mid \mathbf{W}y \}$</td></tr>
<tr><td colspan="3">$Л\{ \mathbf{R}\mathrm{J}y \mid \mathbf{W}y \}$</td></tr>
<tr><td colspan="4">ⓟ $\wedge\{ \lambda Y[Y/\mathbf{R}\mathrm{J}y] \mid \mathbf{W}y \}$</td></tr>
<tr><td colspan="5">$\wedge\{ \mathbf{R}\mathrm{JK}/\mathbf{R}\mathrm{J}y \mid \mathbf{W}y \}$
$\wedge\{ \mathbf{R}\mathrm{JK} \mid \mathbf{W}y \,\&\, \mathbf{R}\mathrm{J}y \}$
$\forall y\{ \mathbf{W}y \,\&\, \mathbf{R}\mathrm{J}y \,.\!\rightarrow \mathbf{R}\mathrm{JK} \}$</td></tr>
<tr><td colspan="5">$[\, \exists y\{\mathbf{W}y \,\&\, \mathbf{R}\mathrm{J}y\} \rightarrow \mathbf{R}\mathrm{JK} \,]$</td></tr>
</table>

Note that the optional final formula suggests, once again, that ‘any’ is like ‘a’, which is like ‘some’. But not *exactly like*! Since the following only makes sense with the wide-universal reading.

55. if Jay respects any woman, he [i.e. Jay] talks to her

<table>
<tr><td>if</td><td>Jay +1</td><td>respects</td><td>any woman +2 –1</td><td>Jay talks-to (–1) her</td></tr>
<tr><td rowspan="3">$\lambda X \lambda Y[Y/X]$</td><td rowspan="2">J_1</td><td>$\lambda y_2 \lambda x_1 \mathbf{R}xy$</td><td>$Л\{ y_2 \times y_{-1} \mid \mathbf{W}y \}$</td><td rowspan="4">$\lambda y_{-1} : \mathbf{T}\mathrm{J}y$</td></tr>
<tr><td colspan="2">$Л\{ \lambda x_1 \mathbf{R}xy \times y_{-1} \mid \mathbf{W}y \}$</td></tr>
<tr><td colspan="3">$Л\{ \mathbf{R}\mathrm{J}y \times y_{-1} \mid \mathbf{W}y \}$</td></tr>
<tr><td colspan="4">ⓟ $\wedge\{ \lambda Y[Y/\mathbf{R}\mathrm{J}y] \times y_{-1} \mid \mathbf{W}y \}$</td></tr>
<tr><td colspan="5">$\wedge\{ \lambda Y[Y/\mathbf{R}\mathrm{J}y] \times \mathbf{T}\mathrm{J}y \mid \mathbf{W}y \}$
$\wedge\{ \mathbf{T}\mathrm{J}y/\mathbf{R}\mathrm{J}y \mid \mathbf{W}y \}$
$\wedge\{ \mathbf{T}\mathrm{J}y \mid \mathbf{W}y \,\&\, \mathbf{R}\mathrm{J}y \}$
$\forall y\{\mathbf{W}y \,\&\, \mathbf{R}\mathrm{J}y \,.\!\rightarrow \mathbf{T}\mathrm{J}y\}$</td></tr>
</table>

Note that, unlike $\wedge$, Л admits ‘if’, which moreover promotes it to $\wedge$.

4. *No-Any*

We treat ‘any’ as sub-assertional, as becoming fully-assertional only when embedded in a context that promotes Л to $\wedge$. So far we have looked at examples involving ‘if’ and ‘not’, which both promote Л to $\wedge$.

We next look at examples involving ‘no’, which also promotes Л to $\wedge$. As we discover, however, this is not the whole story!

First consider the following simple example.

56. no man respects any woman
[granting wide-scope to ‘any woman’]

<table>
<tr><td>no man +1</td><td>respects</td><td>any woman +2</td></tr>
<tr><td rowspan="2">$\bigcirc\{ x_1 \mid \mathbf{M}x \}$</td><td>$\lambda y_2 \lambda x_1 \mathbf{R}xy$</td><td>$Л\{ y_2 \mid \mathbf{W}y \}$</td></tr>
<tr><td colspan="2">$Л\{ \lambda x_1 \mathbf{R}xy \mid \mathbf{W}y \}$</td></tr>
<tr><td colspan="3">ⓟ $\wedge\{ \bigcirc\{\mathbf{R}xy \mid \mathbf{M}x\} \mid \mathbf{W}y \}$
$\forall y\{ \mathbf{W}y \rightarrow {\sim}\exists x\{\mathbf{M}x \,\&\, \mathbf{R}xy\} \}$</td></tr>
</table>

Note that Л admits $\bigcirc$, which promotes it to $\wedge$. So simply treating ‘any’ as wide-scope ‘every’ appears promising – **except** when we face examples that involve pronoun-binding such as the following.

57. no man respects any woman who does not respect **him**

<table>
<tr><td>no man +1 –1</td><td>respects</td><td>any</td><td>woman</td><td>who +1</td><td>does not respect (–1) him</td><td>+2</td></tr>
<tr><td rowspan="6">$○\{ x_1 \times x_{-1} \mid \mathbf{M}x \}$</td><td rowspan="5">$\lambda y_2 \lambda x_1 \mathbf{R}xy$</td><td rowspan="3">$\lambda P_0 \text{Л} y P y$</td><td rowspan="2">$\lambda x_0 \mathbf{W}x$</td><td>$\lambda x_0 . x_1$</td><td>$\lambda z_{-1} \lambda x_1 {\sim}\mathbf{R}xz$</td><td rowspan="4">$\lambda x . x_2$</td></tr>
<tr><td colspan="2">$\lambda z_{-1}\, \lambda x_0\, \{\mathbf{W}x \,\&\, {\sim}\mathbf{R}xz\}$</td></tr>
<tr><td colspan="3">$\lambda z_{-1}\, \lambda x_0\, \{\mathbf{W}x \,\&\, {\sim}\mathbf{R}xz\}$</td></tr>
<tr><td colspan="4">$\lambda z_{-1}\, \text{Л}\{ y \mid \mathbf{W}y \,\&\, {\sim}\mathbf{R}yz \}$</td></tr>
<tr><td colspan="5">$\lambda z_{-1}\, \text{Л}\{ y_2 \mid \mathbf{W}y \,\&\, {\sim}\mathbf{R}yz \}$</td></tr>
<tr><td colspan="6">$\lambda z_{-1}\, \text{Л}\{ \lambda x_1 \mathbf{R}xy \mid \mathbf{W}y \,\&\, {\sim}\mathbf{R}yz \}$</td></tr>
<tr><td colspan="7">ⓟ $\lambda z_{-1}\, \wedge\{ ○\{ \mathbf{R}xy \times x_{-1} \mid \mathbf{M}x \} \mid \mathbf{W}y \,\&\, {\sim}\mathbf{R}yz \}$</td></tr>
<tr><td colspan="7">does not simplify; ‘him’ is left dangling</td></tr>
</table>

What happens if we assign wide-scope to ‘no man’? First, let's go back and do the following, simpler, example.

58. no man respects any woman

[granting wide-scope to ‘no man’]

<table>
<tr><td>no man +1</td><td>respects</td><td>any woman +2</td></tr>
<tr><td rowspan="2">$○\{ x_1 \mid \mathbf{M}x \}$</td><td>$\lambda y_2 \lambda x_1 \mathbf{R}xy$</td><td>$\text{Л}\{ y_2 \mid \mathbf{W}y \}$</td></tr>
<tr><td colspan="2">$\text{Л}\{ \lambda x_1 \mathbf{R}xy \mid \mathbf{W}y \}$</td></tr>
<tr><td colspan="3">$○\{ \text{Л}\{\mathbf{R}xy \mid \mathbf{W}y\} \mid \mathbf{M}x\}$
??? there is no man who respects any woman **???**</td></tr>
</table>

This does not compute, as it stands, since the Л-expression does not simplify, being sub-assertional. In order to solve this problem, we propose a new composition rule, according to which ○ *absorbs* Л.

<table>
<tr><td>**○Л-Absorption**</td><td rowspan="3">Φ, Ψ, Ω are formulas</td></tr>
<tr><td>$○\{ \text{Л}\{ \Omega \mid \Psi \} \mid \Phi \}$</td></tr>
<tr><td>$○\{ \Omega \mid \Phi \,\&\, \Psi \}$</td></tr>
</table>

Then the derivation proceeds as follows.

<table>
<tr><td>no man +1</td><td>respects</td><td>any woman +2</td></tr>
<tr><td rowspan="2">$○\{ x_1 \mid \mathbf{M}x \}$</td><td>$\lambda y_2 \lambda x_1 \mathbf{R}xy$</td><td>$\text{Л}\{ y_2 \mid \mathbf{W}y \}$</td></tr>
<tr><td colspan="2">$\text{Л}\{ \lambda x_1 \mathbf{R}xy \mid \mathbf{W}y \}$</td></tr>
<tr><td colspan="3">$○\{ \text{Л}\{\mathbf{R}xy \mid \mathbf{W}y\} \mid \mathbf{M}x\}$
$○\{ \mathbf{R}xy \mid \mathbf{M}x \,\&\, \mathbf{W}y \}$
${\sim}\exists x \exists y\{ \mathbf{M}x \,\&\, \mathbf{W}y \,\&\, \mathbf{R}xy \}$</td></tr>
</table>

Note that the resulting formula is equivalent to the earlier computation, so we can treat either ‘no man’ or ‘any woman’ as wide scope, and the results are equivalent. What about the problematic example involving pronoun-binding? We can use ○Л-absorption to construct the following derivation.

no man +1 –1	respects any woman who does not respect (–1) him
$\bigcirc\{ x_1 \times x_{-1} \mid \mathbf{M}x \}$	$\lambda z_{-1}\, \text{Л}\{ \lambda x_1 \mathbf{R}xy \mid \mathbf{W}y \;\&\; {\sim}\mathbf{R}yz \}$
$\bigcirc\{ x_1 \times \text{Л}\{ \lambda x_1 \mathbf{R}xy \mid \mathbf{W}y \;\&\; {\sim}\mathbf{R}yx \} \mid \mathbf{M}x \}$ $\bigcirc\{ \text{Л}\{ \mathbf{R}xy \mid \mathbf{W}y \;\&\; {\sim}\mathbf{R}yx \} \mid \mathbf{M}x \}$ $\bigcirc\{ \mathbf{R}xy \mid \mathbf{M}x \;\&\; \mathbf{W}y \;\&\; {\sim}\mathbf{R}yx \}$ ${\sim}\exists x \exists y\{ \mathbf{M}x \;\&\; \mathbf{W}y \;\&\; {\sim}\mathbf{R}yx \;\&\; \mathbf{R}xy \}$	
there are no *x*,*y* **:** *x* is a man, and *y* is a woman, and *y* does not respect *x*, and *x* respects *y*	

This approach also works with examples involving two occurrences of ‘any’.

59. no man gives any book to any woman

no man +1	gives	any book +2	to any woman
	$\lambda y_2 \lambda z_3 \lambda x_1 \mathbf{G}xyz$	$\text{Л}\{ y_2 \mid \mathbf{B}y \}$	
	$\lambda x_1\, \text{Л}\{ \lambda z_3 \mathbf{G}xyz \mid \mathbf{B}y \}$		$\text{Л}\{ z_3 \mid \mathbf{W}z \}$
$\bigcirc\{ x_1 \mid \mathbf{M}x \}$	$\lambda x_1\, \text{Л}\{ \mathbf{G}xyz \mid \mathbf{B}y \;\&\; \mathbf{W}z \}$		
$\bigcirc\{ \text{Л}\{ \mathbf{G}xyz \mid \mathbf{B}y \;\&\; \mathbf{W}z \} \mid \mathbf{M}x \}$ $\bigcirc\{ \mathbf{G}xyz \mid \mathbf{M}x \;\&\; \mathbf{B}y \;\&\; \mathbf{W}z \}$			
${\sim}\exists x \exists y \exists z \{ \mathbf{M}x \;\&\; \mathbf{B}y \;\&\; \mathbf{W}z \;\&\; \mathbf{G}xyz \}$			

Notice that we combine the two Л's by parallel-composition into a big Л, and we combine the latter with ○ into an even bigger ○.

Treating no-any as a special kind of parallel-quantification is further supported by the active-passive transformation of no-any sentences.

no man respects any woman [active]
⇒ ☺ no woman is respected by any man [passive]
versus ☹ any woman is respected by no man

We conclude this section by noting that *no-any* may also be *understood* as a quirky variant of *no-no* understood via parallel composition.[57] Consider the following derivation.

60. no man respects no woman
[parallel-scope]

no man +1	respects	no woman +2
	$\lambda y_2 \lambda x_1 \mathbf{R}xy$	$\bigcirc\{ y_2 \mid \mathbf{W}y \}$
$\bigcirc\{ x_1 \mid \mathbf{M}x \}$	$\bigcirc\{ \lambda x_1 \mathbf{R}xy \mid \mathbf{W}y \}$	
$\bigcirc\{ \mathbf{R}xy \mid \mathbf{M}x \;\&\; \mathbf{W}y \}$ ${\sim}\exists x \exists y\{ \mathbf{M}x \;\&\; \mathbf{W}y \;\&\; \mathbf{R}xy \}$		

5. Relative Pronouns and *Any*

Recall that ‘any’ admits ‘not’ and ‘if’, which promote it to ∧. By contrast, ∧ does not admit these items, nor does ∧ admit relative pronoun phrases.[58] We naturally wonder whether ‘any’ admits relative pronoun phrases. The following are examples.

- every man who respects any woman is virtuous
- some man who respects any woman is virtuous
- no man who respects any woman is virtuous

---

[57] This turns the tables on the received view in *prescriptive grammar*, which proclaims that ‘no A respects no B’ is a grammatically-dubious rendering of ‘no A respects any B’.

[58] Recall that a relative-pronoun-phrase includes a relative pronoun plus case-marker(s) plus adjunct phrase(s); an excellent example is ‘whose mother’.

These are difficult to read unless we take ‘any’ to carry modal force, in which case ‘any woman’ means “every *possible* woman”. The oddness is perhaps even more clear if we add a potential bound pronoun.

- every man who respects any woman respects her mother
- some man who respects any woman respects her mother
- no man who respects any woman respects her mother

These make perfectly good sense if we replace ‘any’ by ‘a’, but as they stand they seem quite strange. The following derivation shows us exactly where the computation fails.

<table>
<tr><th>every</th><th>man</th><th>who +1</th><th>respects</th><th>any woman +2 –1</th><th>+1</th><th>respects (–1) her mother</th></tr>
<tr><td></td><td></td><td></td><td>$\lambda y_2\lambda x_1\mathbf{R}xy$</td><td>Л$\{y_2 \times y_{-1} \mid \mathbf{W}y\}$</td><td></td><td></td></tr>
<tr><td></td><td></td><td>$\lambda x_0{:}x_1$</td><td colspan="2">Л$\{ \lambda x_1\mathbf{R}xy \times y_{-1} \mid \mathbf{W}y \}$</td><td></td><td></td></tr>
<tr><td></td><td>$\lambda x_0\mathbf{M}x$</td><td colspan="3">Л$\{ \lambda x_0\mathbf{R}xy \times y_{-1} \mid \mathbf{W}y \}$</td><td></td><td></td></tr>
<tr><td>$\lambda P_0\wedge xPx$</td><td colspan="4">Л$\{ \lambda x_0(\mathbf{M}x \ \&\mathbf{R}xy) \times y_{-1} \mid \mathbf{W}y \}$</td><td></td><td></td></tr>
<tr><td colspan="5">Л$\{ \wedge x(\mathbf{M}x \ \&\mathbf{R}xy) \times y_{-1} \mid \mathbf{W}y \}$</td><td>$\lambda x.x_1$</td><td></td></tr>
<tr><td colspan="6">Л$\{ \wedge\{x_1 \mid \mathbf{M}x \ \&\mathbf{R}xy\} \times y_{-1} \mid \mathbf{W}y \}$</td><td>$\lambda z_{-1} : \lambda x_1 \ \mathbf{R}[x,\mathrm{M}(z)]$</td></tr>
<tr><td colspan="7">Л$\{ \wedge\{ x_1 \mid \mathbf{M}x \ \&\mathbf{R}xy \} \times \lambda x_1 \ \mathbf{R}[x,\mathrm{M}(y)] \mid \mathbf{W}y \}$
Л$\{ \wedge\{ \mathbf{R}[x,\mathrm{M}(y)] \mid \mathbf{M}x \ \&\ \mathbf{R}xy \} \mid \mathbf{W}y \}$
Л$\{ \forall x\{ \mathbf{M}x \ \&\ \mathbf{R}xy \ .\!\to \mathbf{R}[x,\mathrm{M}(y)] \} \mid \mathbf{W}y \}$
does not simplify [sub-assertational]</td></tr>
</table>

Notice that whether Л admits ‘who’ is not the problem here. The problem is that ‘every’ does not promote ‘any’.

Nevertheless, let's consider the following restraint on Л-composition.

(P) Л does not admit any relative pronoun phrase.

Since we are fond of self-referring principles, let's replace this principle by:

(P*) Л does not admit any phrase **that** is headed by **any** relative pronoun.

Notice that if (P*) is true, then (P*) does not compute! In particular, consider the following analysis, which fails if (P*) holds.

<table>
<tr><th>Л +1</th><th>does not</th><th>admit</th><th>any phrase</th><th>that +1</th><th>is</th><th>headed</th><th>by any R</th><th>+2</th></tr>
<tr><td></td><td></td><td></td><td></td><td></td><td></td><td>$\lambda z_5\lambda x_0\mathbf{H}zx$</td><td>Л$\{ z_5 \mid \mathbf{R}z \}$</td><td></td></tr>
<tr><td></td><td></td><td></td><td></td><td></td><td>$\lambda P_0{:}P_1$</td><td colspan="2">Л$\{ \lambda x_0\mathbf{H}zx \mid \mathbf{R}z \}$</td><td></td></tr>
<tr><td></td><td></td><td></td><td></td><td>$\lambda x_0{:}x_1$</td><td colspan="3">Л$\{ \lambda x_1\mathbf{H}zx \mid \mathbf{R}z \}$</td><td></td></tr>
<tr><td>A_1</td><td>$\lambda X{:}{\sim}X$</td><td>$\lambda y_2\lambda x_1\mathbf{A}xy$</td><td>$\lambda P_0$Л$yPy$</td><td colspan="4">※ [because of Principle (P*)] ※</td><td>$\lambda x.x_2$</td></tr>
<tr><td colspan="9">※</td></tr>
</table>

On the other hand, if we reject (P*), and instead propose that Л admits relative pronoun phrases, then the following derivation works.

Л +1	does not	admit	any phrase	that +1	is	headed	by any **R**	+2
						$\lambda z_5\lambda x_0\mathbf{H}zx$	$\text{Л}\{ z_5 \mid \mathbf{R}z \}$	
					$\lambda P_0{:}P_1$	$\text{Л}\{ \lambda y_0\mathbf{H}zy \mid \mathbf{R}z \}$		
				$\lambda y_0{:}y_1$	$\text{Л}\{ \lambda x_1\mathbf{H}zy \mid \mathbf{R}z \}$			
			$\lambda P_0\text{Л}yPy$	$\text{Л}\{ \lambda y_0\mathbf{H}zy \mid \mathbf{R}z \}$				
			$\text{Л}\{ \text{Л}y\mathbf{H}zy \mid \mathbf{R}z \}$ $\text{Л}\{ y \mid \mathbf{R}z \;\&\; \mathbf{H}zy \}$					$\lambda y.y_2$
		$\lambda y_2\lambda x_1\mathbf{A}xy$	$\text{Л}\{ y_2 \mid \mathbf{R}z \;\&\; \mathbf{H}zy \}$					
	$\lambda X{:}{\sim}X$	$\lambda x_1\, \text{Л}\{ \mathbf{A}xy \mid \mathbf{R}z \;\&\; \mathbf{H}zy \}$						
L1	$\lambda x_1 \wedge\{ {\sim}\mathbf{A}xy \mid \mathbf{R}z \;\&\; \mathbf{H}zy \}$							
$\wedge\{ {\sim}\mathbf{A}\text{L}y \mid \mathbf{R}z \;\&\; \mathbf{H}zy \}$								
$\forall y\forall z\{ \mathbf{R}z \;\&\; \mathbf{H}zx \;.\!\!\rightarrow {\sim}\mathbf{A}\text{L}x) \}$								

By way of conclusion, we reject (P*), and instead maintain that Л, like Π and Σ, admits all phrases.

6. Key Difference between *A* and *Any*

Both 'a' and 'any' can be promoted to a junction [respectively, Π and ∧] that gets simplified to a universal quantifier. What is the difference? Consider the following examples.

61. if Jay doesn't own **a** dog, then Jay doesn't feed it

if	Jay +1	doesn't	own	a dog +2 –1	then Jay doesn't feed (–1) it
			$\lambda y_2\lambda x_1\mathbf{O}xy$	$\Pi\{ y_2 \times y_{-1} \mid \mathbf{D}y \}$	
		$\lambda X{:}{\sim}X$	$\Pi\{ \lambda x_1\mathbf{O}xy \times y_{-1} \mid \mathbf{D}y \}$		
	J1	$\Pi\{ \lambda x_1{\sim}\mathbf{O}xy \times y_{-1} \mid \mathbf{D}y \}$			
$\lambda X\lambda Y[Y/X]$	$\Pi\{ {\sim}\mathbf{O}\text{J}y \times y_{-1} \mid \mathbf{D}y \}$				
$\Pi\{ \lambda Y[Y/{\sim}\mathbf{O}\text{J}y] \times y_{-1} \mid \mathbf{D}y \}$					$\lambda y_{-1}{\sim}\mathbf{F}\text{J}y$
$\Pi\{ \lambda Y[Y/{\sim}\mathbf{O}\text{J}y] \times {\sim}\mathbf{F}\text{J}y \mid \mathbf{D}y \}$ $\Pi\{ {\sim}\mathbf{F}\text{J}y/{\sim}\mathbf{O}\text{J}y \mid \mathbf{D}y \}$ $\Pi\{ {\sim}\mathbf{O}\text{J}y \mid \mathbf{D}y \;\&\; {\sim}\mathbf{F}\text{J}y \}$ $\forall x \{ \mathbf{D}x \;\&\; {\sim}\mathbf{O}\text{J}x \;.\!\!\rightarrow {\sim}\mathbf{F}\text{J}x \}$					

62. if Jay doesn't own **any** dog, then Jay doesn't feed it

if	Jay +1	doesn't	own	any dog +2 –1	then Jay doesn't feed (–1) it
			$\lambda y_2\lambda x_1\mathbf{O}xy$	$\text{Л}\{ y_2 \times y_{-1} \mid \mathbf{D}y \}$	
		$\lambda X{:}{\sim}X$	$\text{Л}\{ \lambda x_1\mathbf{O}xy \times y_{-1} \mid \mathbf{D}y \}$		
	J1	$\wedge\{ \lambda x_1{\sim}\mathbf{O}xy \times y_{-1} \mid \mathbf{D}y \}$			
$\lambda X\lambda Y[Y/X]$	$\wedge\{ {\sim}\mathbf{O}\text{J}y \times y_{-1} \mid \mathbf{D}y \}$				
※					$\lambda z_{-1}{\sim}\mathbf{F}\text{J}z$
※					

The most important difference between these two sentences is that, whereas 'a dog' succeeds in binding 'it', 'any dog' does not. According to our account, this is because Σ is promoted to Π, which admits 'if', but Л is promoted to ∧, which does not admit 'if'. So although the two sentences

Jay doesn't own a dog
Jay doesn't own any dog

have the same truth-conditions, they are not semantically equivalent.[59]

[59] This is further evidence that the meaning of a sentence is not (merely) its truth-conditions.

C. Expanded Account of Quantifiers

1. Introduction

Originally, we treated quantifier-phrases as second-order predicates, as in the following categorial rendering of ‘every’.

every	$C \to [(D \to S) \to S]$	$\lambda P_0 \lambda Q \forall x\{Px \to Qx\}$

More recently, we have treated quantifier-phrases as entity-junctions, as in the following categorial rendering of ‘every’.

every	$C \to \wedge D$	$\lambda P_0 \wedge x Px$

Even more recently, we have proposed that items of type C can be alternatively rendered as entity-sums, according to the following inference principles.

$\lambda \nu_0 \Phi$	$\dashv\vdash$	$\Sigma \nu \Phi$
C		ΣD

In the current unit, we combine these ideas, and expand them even further, producing a greatly expanded account of quantifiers.

2. Re-Rendering Case-Marking and Quantifier Phrases

By way of developing the new account, we begin by considering the phrase

63. every woman's mother

which has previously been analyzed as follows.

<table>
<tr><td>every</td><td>woman</td><td>'s</td><td>mother-DEF</td></tr>
<tr><td>$\lambda P_0 \wedge x Px$</td><td>$\lambda x_0 \mathbf{W}x$</td><td></td><td rowspan="2"></td></tr>
<tr><td colspan="2">$\wedge x \mathbf{W}x$</td><td>$\lambda x.x_6$</td></tr>
<tr><td colspan="3">$\wedge\{ x_6 \mid \mathbf{W}x \}$</td><td>$\lambda x_6{:}\mathbf{M}(x)$</td></tr>
<tr><td colspan="4">$\wedge\{ \mathbf{M}(x) \mid \mathbf{W}x \}$</td></tr>
</table>

The analysis seems odd because *apostrophe-s* attaches grammatically to ‘every woman’, even though it attaches morphologically to ‘woman’,[60] This is because a case-marker attaches to an NP, such as ‘every woman’, and not a CNP, such as ‘woman’.

But with our new apparatus, we can re-render ‘woman’ as an entity-sum (type ΣD), and apply *apostrophe-s* to ‘woman’, as in the following reworking.

<table>
<tr><td>every</td><td>woman</td><td>'s</td><td>mother-DEF</td></tr>
<tr><td rowspan="2"></td><td>$\lambda x_0 \mathbf{W}x$
$\Sigma x \mathbf{W}x$</td><td>$\lambda x.x_6$</td><td></td></tr>
<tr><td colspan="2">$\Sigma\{ x_6 \mid \mathbf{W}x \}$</td><td>$\lambda x_6 \mathbf{M}(x)$</td></tr>
<tr><td>$\lambda P_0 \wedge x Px$</td><td colspan="3">$\Sigma\{ \mathbf{M}(x) \mid \mathbf{W}x \}$</td></tr>
<tr><td colspan="4">**???**</td></tr>
</table>

Unfortunately, this derivation does not complete as it stands, since ‘woman's mother’ does not have the proper type to combine with ‘every’.

Fortunately, however, we can manipulate $\Sigma\{\mathbf{M}(x)|\mathbf{W}x\}$ into proper form, using some mathematical trickery. First, we note the following set-theoretic definition.

[60] This phenomenon is usually described by saying that *apostrophe-s* is a **clitic**. A very well-known example is the Latin ‘que’, as in the widely inscribed ‘SPQR’ [Senatus Populus**que** Romanus; the Senate **and** the people of Rome].

$\{\, \alpha \mid \Phi \,\}$	$=_{df}$	$\{\, \nu \mid \exists\underline{\omega}(\nu{=}\alpha \;\&\; \Phi) \,\}$
α is an expression of any type ν is a variable with the same type as α [61] ν is not free in α $\underline{\omega}$ are all the variables C-free in α		

Applying this principle to the example above, we have:

$$\{\, \mathbf{M}(x) \mid \mathbf{W}x \,\} =_{df} \{\, y \mid \exists x[\mathbf{W}x \;\&\; y{=}\mathbf{M}(x)] \,\}$$

so

$$\Sigma\{\, \mathbf{M}(x) \mid \mathbf{W}x \,\} = \Sigma\{\, y \mid \exists x[\mathbf{W}x \;\&\; y{=}\mathbf{M}(x)] \,\}$$

Next, we can apply CNP-duality to

$$\Sigma\{\, y \mid \exists x[\mathbf{W}x \;\&\; y{=}\mathbf{M}(x)] \,\}$$

by which we obtain:

$$\lambda y_0 \exists x[\mathbf{W}x \;\&\; y{=}\mathbf{M}(x)]$$

The latter submits to $\lambda P_0 \wedge y P y$, which produces:

$$\wedge\{\, y \mid \exists x[\mathbf{W}x \;\&\; y{=}\mathbf{M}(x)] \,\}$$

Finally, we can once again apply the set theoretic definition above, by which we obtain:

$$\wedge\{\, \mathbf{M}(x) \mid \mathbf{W}x \,\}$$

This computational maneuver demonstrates that we can re-render ‘every’ as follows.[62]

every	$\Sigma D \rightarrow \wedge D$	$\Sigma\tau\Phi \rightarrow \wedge\tau\Phi$	abbreviation: $\Sigma{\rightarrow}\wedge$
τ is any expression of type D			

This writes the function using schematic notation [Greek letters!] instead of ordinary object-language variables, and using ‘→’ instead of lambda.[63] The idea is that ‘every’ acts as a function that takes an item of type ΣD, and delivers an item of type ∧D, which symbolically involves simply replacing ‘Σ’ by ‘∧’. So when we redo our extant example, we obtain the following.[64]

every	woman	's	mother-DEF
	$\Sigma x \mathbf{W}x$	$\lambda x.x_6$	
	$\Sigma\{\, x_6 \mid \mathbf{W}x \,\}$		$\lambda x_6{:}\mathbf{M}(x)$
$\Sigma{\rightarrow}\wedge$	$\Sigma\{\, \mathbf{M}(x) \mid \mathbf{W}x \,\}$		
$\wedge\{\, \mathbf{M}(x) \mid \mathbf{W}x \,\}$			

[61] This requires that we have at our disposal a list of variables for every type.
[62] We expand it further in a later section. We also expand the other quantifiers – ‘some’, ‘no’, ‘any’.
[63] In particular, $\lambda\alpha{:}\beta$ becomes $\alpha{\rightarrow}\beta$, which is often how mathematicians (in effect!) write lambda-abstracts.
[64] Alas, the clitic behavior of *apostrophe-s* can't be entirely eliminated, since this trick doesn't work on the following example – the Queen of England's mother. Presumably, this is not the queen of the mother of England.

3. Further Expansion of Quantifiers

We could stop here, but we don't! Rather, noting the suggestiveness of the expression '$\Sigma\!\rightarrow\!\wedge$', we expand our account of quantifiers even further, as follows.

every	$\Sigma\mathsf{T} \rightarrow \wedge\mathsf{T}$	$\Sigma\alpha\Phi \rightarrow \wedge\alpha\Phi$	abbr: $\Sigma\!\rightarrow\!\wedge$
some	$\Sigma\mathsf{T} \rightarrow \vee\mathsf{T}$	$\Sigma\alpha\Phi \rightarrow \vee\alpha\Phi$	abbr: $\Sigma\!\rightarrow\!\vee$
no	$\Sigma\mathsf{T} \rightarrow \bigcirc\mathsf{T}$	$\Sigma\alpha\Phi \rightarrow \bigcirc\alpha\Phi$	abbr: $\Sigma\!\rightarrow\!\bigcirc$
any	$\Sigma\mathsf{T} \rightarrow$ ЛT	$\Sigma\alpha\Phi \rightarrow$ Л$\alpha\Phi$	abbr: $\Sigma\!\rightarrow$Л
T is **any** type; α is any expression of type T			

The original account is then a special case of the new account, obtained by setting $\mathsf{T}{=}\mathsf{D}$.

While we are at it, we take this opportunity to formally introduce some further useful algebraic principles governing junctions.[65]

1. Distribution of × over Ж

Ж{ $\alpha \mid \Phi$ } $\times\, \beta$	$\vdash$	Ж{ $\alpha \times \beta \mid \Phi$ }
$\alpha \times$ Ж{ $\beta \mid \Phi$ }	$\vdash$	Ж{ $\alpha \times \beta \mid \Phi$ }

2. Associativity

Ж{ Ж{ $\alpha \mid \Psi$ } $\mid \Phi$ }	$\vdash$	Ж{ $\alpha \mid \Phi\ \&\ \Psi$ }
Ж $= \wedge, \vee, \Pi, \Sigma$, Л $[\neq \bigcirc, \nabla, \S]$[66]		

3. Contraction

Ж{ $\alpha \mid \Phi$ }	$\vdash$	Ж{ $\alpha \mid \exists\underline{\omega}\Phi$ }
$\underline{\omega}$ are all the variables C-free in Φ but not C-free in α		

4. Examples of New Scheme

Earlier, we analyzed

every woman's mother

treating 'mother' as a function-sign, and hence definite. What happens if we instead treat 'mother' as indefinite? The following two derivations illustrate.

[65] These principles are infinitary versions of standard finitary algebraic notions, which are discussed more fully in Chapter 10 [Finitary Junctions].

[66] ∇ is exclusive-disjunction, which officially first appears in Chapter 9 [Number Words]. The oddity of exclusive-disjunction is further discussed in Chapter 10 [Finitary Junctions].

<table>
<tr><th>every</th><th>woman</th><th>'s</th><th>mother</th><th>+1</th><th>is-kind</th></tr>
<tr><td rowspan="3">$\Sigma\!\to\!\wedge$</td><td>$\Sigma x\mathbf{W}x$</td><td>$\lambda x.x_6$</td><td rowspan="2">$\lambda x_6\lambda y_0\mathbf{M}yx$
$\lambda x_6\Sigma y\mathbf{M}yx$</td><td rowspan="3"></td><td rowspan="4"></td></tr>
<tr><td colspan="2">$\Sigma\{\, x_6 \mid \mathbf{W}x \,\}$</td></tr>
<tr><td colspan="3">$\Sigma\{\, \Sigma y\mathbf{M}yx \mid \mathbf{W}x \,\}$
ⓐ $\Sigma\{\, y \mid \mathbf{W}x \;\&\; \mathbf{M}yx \,\}$</td></tr>
<tr><td colspan="4">$\wedge\{\, y \mid \mathbf{W}x \;\&\; \mathbf{M}yx \,\}$</td><td>$\lambda y.y_1$</td></tr>
<tr><td colspan="5">$\wedge\{\, y_1 \mid \mathbf{W}x \;\&\; \mathbf{M}yx \,\}$</td><td>$\lambda y_1\mathbf{K}y$</td></tr>
<tr><td colspan="6">$\wedge\{\, \mathbf{K}y \mid \mathbf{W}x \;\&\; \mathbf{M}yx \,\}$
$\forall x\forall y\{\, \mathbf{W}x \;\&\; \mathbf{M}yx \;.\!\!\to \mathbf{K}y \,\}$</td></tr>
</table>

<table>
<tr><th>every</th><th>woman</th><th>'s</th><th>mother</th><th>+1</th><th>is-kind</th></tr>
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<tr><td colspan="3">$\Sigma\{\, \Sigma y\mathbf{M}yx \mid \mathbf{W}x \,\}$
ⓐ $\Sigma\{\, y \mid \mathbf{W}x \;\&\; \mathbf{M}yx \,\}$
ⓒ $\Sigma\{\, y \mid \exists x(\mathbf{W}x \;\&\; \mathbf{M}yx) \,\}$</td></tr>
<tr><td colspan="4">$\wedge\{\, y \mid \exists x(\mathbf{W}x \;\&\; \mathbf{M}yx) \,\}$</td><td>$\lambda y.y_1$</td></tr>
<tr><td colspan="5">$\wedge\{\, y_1 \mid \exists x(\mathbf{W}x \;\&\; \mathbf{M}yx) \,\}$</td><td>$\lambda y_1\mathbf{K}y$</td></tr>
<tr><td colspan="6">$\wedge\{\, \mathbf{K}y \mid \mathbf{W}x \;\&\; \mathbf{M}yx \,\}$
$\forall y\{\, \exists x(\mathbf{W}x \;\&\; \mathbf{M}yx) \to \mathbf{K}y \,\}$</td></tr>
</table>

ⓐ associativity ⓒ contraction

Note that the two formulas are logically equivalent.

The following is a more lofty example with basically the same form.

64. every mother of a slain soldier receives a purple heart [67]

We can treat 'mother' as definite or indefinite, as follows.

<table>
<tr><th>every</th><th>mother-DEF</th><th>of</th><th>a-slain-soldier</th><th>+1</th><th>receives-a-purple-heart</th></tr>
<tr><td rowspan="3">$\Sigma\!\to\!\wedge$</td><td rowspan="2">$\lambda x_6{:}\textsc{m}(x)$</td><td>$\lambda x.x_6$</td><td>$\Sigma x\mathbf{S}x$</td><td rowspan="3"></td><td rowspan="4"></td></tr>
<tr><td colspan="2">$\Sigma\{\, x_6 \mid \mathbf{S}x \,\}$</td></tr>
<tr><td colspan="3">$\Sigma\{\, \textsc{m}(x) \mid \mathbf{S}x \,\}$</td></tr>
<tr><td colspan="4">$\wedge\{\, \textsc{m}(x) \mid \mathbf{S}x \,\}$</td><td>$\lambda x.x_1$</td></tr>
<tr><td colspan="5">$\wedge\{\, \textsc{m}(x)_1 \mid \mathbf{S}x \,\}$</td><td>$\lambda x_1\mathbf{R}x$</td></tr>
<tr><td colspan="6">$\wedge\{\, \mathbf{R}[\textsc{m}(x)] \mid \mathbf{S}x \,\}$
$\forall x\{\, \mathbf{S}x \to \mathbf{R}[\textsc{m}(x)] \,\}$</td></tr>
<tr><td colspan="6">every slain soldier's mother receives a purple heart</td></tr>
</table>

<table>
<tr><th>every</th><th>mother</th><th>of</th><th>a-slain-soldier</th><th>+1</th><th>receives-a-purple-heart</th></tr>
<tr><td rowspan="3">$\Sigma\!\to\!\wedge$</td><td rowspan="2">$\lambda y_6\lambda x_0\mathbf{M}xy$</td><td>$\lambda y.y_6$</td><td>$\Sigma y\mathbf{S}y$</td><td rowspan="3"></td><td rowspan="4"></td></tr>
<tr><td colspan="2">$\Sigma\{\, y_6 \mid \mathbf{S}y \,\}$</td></tr>
<tr><td colspan="3">$\Sigma\{\, \lambda x_0\mathbf{M}xy \mid \mathbf{S}y \,\}$
$\Sigma\{\, \Sigma x\mathbf{M}xy \mid \mathbf{S}y \,\}$
ⓐ $\Sigma\{\, x \mid \mathbf{S}y \;\&\; \mathbf{M}xyx \,\}$ │ ⓒ $\Sigma\{\, x \mid \exists y(\mathbf{S}y \;\&\; \mathbf{M}xy) \,\}$</td></tr>
<tr><td colspan="4">$\wedge\{\, x \mid \mathbf{S}y \;\&\; \mathbf{M}xy \,\}$ │ $\wedge\{\, x \mid \exists y(\mathbf{S}y \;\&\; \mathbf{M}xy) \,\}$</td><td>$\lambda x.x_1$</td></tr>
<tr><td colspan="5">$\wedge\{\, x_1 \mid \mathbf{S}y \;\&\; \mathbf{M}xy \,\}$ │ $\wedge\{\, x_1 \mid \exists y(\mathbf{S}y \;\&\; \mathbf{M}xy) \,\}$</td><td>$\lambda x_1\mathbf{R}x$</td></tr>
<tr><td colspan="6">$\wedge\{\, \mathbf{R}x \mid \mathbf{S}y \;\&\; \mathbf{M}xy \,\}$ │ $\wedge\{\, \mathbf{R}x \mid \exists y(\mathbf{W}y \;\&\; \mathbf{M}xy) \,\}$
$\forall x\forall y\{\, \mathbf{S}y \;\&\; \mathbf{M}xy \;.\!\!\to \mathbf{R}x \,\}$ │ $\forall y\{\, \exists y(\mathbf{S}y \;\&\; \mathbf{M}xy) \to \mathbf{R}x \,\}$</td></tr>
<tr><td colspan="6">every one who mothers any (at least one) slain soldier receives a purple heart</td></tr>
</table>

Notice that in these examples, 'every' [$\Sigma\!\to\!\wedge$] acts on phrases of type ΣD. The following variants apply $\Sigma\!\to\!\wedge$ to a phrase of type $\Sigma[\mathsf{D}_2]$.

[67] Also known as Gold Star Mothers. The Purple Heart is a U.S. military medal, established by George Washington, and bearing his resemblance. Personal note: each of my grandmothers received a Purple Heart for a son who died in combat in World War II, one in France (buried there), the other in the South China Sea ("buried" there).

every	mother	of	a-slain-soldier	+1
	$\lambda y_6\lambda x_0\mathbf{M}xy$	$\lambda y.y_6$	$\Sigma y\mathbf{S}y$	
	$\Sigma\{\, x \mid \mathbf{S}y \;\&\; \mathbf{M}xyx \,\}$ ┃ $\Sigma\{\, x \mid \exists y(\mathbf{S}y \;\&\; \mathbf{M}xy) \,\}$			$\lambda x.x_1$
$\Sigma\to\wedge$	$\Sigma\{\, x_1 \mid \mathbf{S}y \;\&\; \mathbf{M}xyx \,\}$ ┃ $\Sigma\{\, x_1 \mid \exists y(\mathbf{S}y \;\&\; \mathbf{M}xy) \,\}$			
$\wedge\{\, x_1 \mid \mathbf{S}y \;\&\; \mathbf{M}xyx \,\}$ ┃ $\wedge\{\, x_1 \mid \exists y(\mathbf{S}y \;\&\; \mathbf{M}xy) \,\}$				

The following donkey-example is a considerably more interesting application of the new scheme. Notice once again that 'every' in effect simply replaces 'Σ' by '$\wedge$'.

every	mother	of a-slain-soldier –1	+1	reveres (–1) him/her
	$\lambda y_6\lambda x_0\mathbf{M}xy$	$\Sigma\{\, y_6 \times y_{-1} \mid \mathbf{S}y \,\}$		
	$\Sigma\{\, \lambda x_0\mathbf{M}xy \times y_{-1} \mid \mathbf{S}y \,\}$ $\Sigma\{\, \Sigma x\mathbf{M}xy \times y_{-1} \mid \mathbf{S}y \,\}$ ⓓ $\Sigma\{\, \Sigma\{x \times y_{-1} \mid \mathbf{M}xy\} \mid \mathbf{S}y \,\}$ ⓐ $\Sigma\{\, x \times y_{-1} \mid \mathbf{S}y \;\&\; \mathbf{M}xy \,\}$			
$\Sigma\to\wedge$				
$\wedge\{\, x \times y_{-1} \mid \mathbf{S}y \;\&\; \mathbf{M}xy \,\}$			$\lambda x.x_1$	
$\wedge\{\, x_1 \times y_{-1} \mid \mathbf{S}y \;\&\; \mathbf{M}xy \,\}$				$\lambda y_{-1}\lambda x_1\mathbf{R}xy$
$\wedge\{\, \mathbf{R}xy \mid \mathbf{S}y \;\&\; \mathbf{M}xy \,\}$ $\forall x\forall y\{\mathbf{S}y \;\&\; \mathbf{M}xy \;.\!\!\to \mathbf{R}xy\}$				

ⓓ distribution

The following are the usual donkey examples.[68]

65. every man who owns a dog feeds it

every	man	who +1	owns	a dog +2 –1	+1	feeds	(–1) it +2
			$\lambda y_2\lambda x_1\mathbf{O}xy$	$\Sigma\{\, y_2 \times y_{-1} \mid \mathbf{D}y \,\}$		$\lambda y_2\lambda x_1\mathbf{F}xy$	$\lambda y_{-1}{:}y_2$
		$\lambda x_0{:}x_1$	$\Sigma\{\, \lambda x_1\mathbf{O}xy \times y_{-1} \mid \mathbf{D}y \,\}$				
	$\lambda x_0\mathbf{M}x$	$\Sigma\{\, \lambda x_0\mathbf{O}xy \times y_{-1} \mid \mathbf{D}y \,\}$					
	$\Sigma\{\, \lambda x_0(\mathbf{M}x \;\&\; \mathbf{O}xy) \times y_{-1} \mid \mathbf{D}y \,\}$ $\Sigma\{\, \Sigma x(\mathbf{M}x \;\&\; \mathbf{O}xy) \times y_{-1} \mid \mathbf{D}y \,\}$ ⓓ $\Sigma\{\, \Sigma\{x \times y_{-1} \mid \mathbf{M}x \;\&\; \mathbf{O}xy\} \mid \mathbf{D}y \,\}$ ⓐ $\Sigma\{\, x \times y_{-1} \mid \mathbf{M}x \;\&\; \mathbf{D}y \;\&\; \mathbf{O}xy \,\}$						
$\Sigma\to\wedge$							
$\wedge\{\, x \times y_{-1} \mid \mathbf{M}x \;\&\; \mathbf{D}y \;\&\; \mathbf{O}xy \,\}$					$\lambda x.x_1$		
$\wedge\{\, x_1 \times y_{-1} \mid \mathbf{M}x \;\&\; \mathbf{D}y \;\&\; \mathbf{O}xy \,\}$						$\lambda y_{-1}\lambda x_1\mathbf{F}xy$	
$\wedge\{\, \mathbf{F}xy \mid \mathbf{M}x \;\&\; \mathbf{D}y \;\&\; \mathbf{O}xy \,\}$ $\forall x\forall y\{\, \mathbf{M}x \;\&\; \mathbf{D}y \;\&\; \mathbf{O}xy \;.\!\!\to \mathbf{F}xy \,\}$							

66. no man who owns a dog feeds it

no	man	who +1	owns	a dog +2 –1	+1	feeds	(–1) it +2
$\Sigma\to\bigcirc$	$\Sigma\{\, x \times y_{-1} \mid \mathbf{M}x \;\&\; \mathbf{D}y \;\&\; \mathbf{O}xy \,\}$						
$\bigcirc\{\, x \times y_{-1} \mid \mathbf{M}x \;\&\; \mathbf{D}y \;\&\; \mathbf{O}xy \,\}$					$\lambda x.x_1$		
$\bigcirc\{\, x_1 \times y_{-1} \mid \mathbf{M}x \;\&\; \mathbf{D}y \;\&\; \mathbf{O}xy \,\}$						$\lambda y_{-1}\lambda x_1\mathbf{F}xy$	
$\bigcirc\{\, \mathbf{F}xy \mid \mathbf{M}x \;\&\; \mathbf{D}y \;\&\; \mathbf{O}xy \,\}$ $\sim\exists x\exists y\{\, \mathbf{M}x \;\&\; \mathbf{D}y \;\&\; \mathbf{O}xy \;\&\; \mathbf{F}xy \,\}$							

[68] We also perhaps need to consider a wide-scope *general* reading of 'a dog'.

67. some man who owns a dog feeds it

<table>
<tr><td>some</td><td>man</td><td>who +1</td><td>owns</td><td>a dog +2 –1</td><td>+1</td><td>feeds</td><td>(–1) it +2</td></tr>
<tr><td>$\Sigma\!\rightarrow\!\vee$</td><td colspan="4">$\Sigma\{\ x \times y_{-1} \mid \mathbf{M}x \,\&\, \mathbf{D}y \,\&\, \mathbf{O}xy\ \}$</td><td></td><td colspan="2" rowspan="2"></td></tr>
<tr><td colspan="5">$\vee\{\ x \times y_{-1} \mid \mathbf{M}x \,\&\, \mathbf{D}y \,\&\, \mathbf{O}xy\ \}$</td><td>$\lambda x.x_1$</td></tr>
<tr><td colspan="6">$\vee\{\ x_1 \times y_{-1} \mid \mathbf{M}x \,\&\, \mathbf{D}y \,\&\, \mathbf{O}xy\ \}$</td><td colspan="2">$\lambda y_{-1}\lambda x_1\mathbf{F}xy$</td></tr>
<tr><td colspan="8">$\vee\{\ \mathbf{F}xy \mid \mathbf{M}x \,\&\, \mathbf{D}y \,\&\, \mathbf{O}xy\ \}$
$\exists x\exists y\{\ \mathbf{M}x \,\&\, \mathbf{D}y \,\&\, \mathbf{O}xy \,\&\, \mathbf{F}xy\ \}$</td></tr>
</table>

Notice that compositional-ambiguity still governs junction-composition. For example, we also have the following derivation.

68. every man who owns a dog feeds it

<table>
<tr><td>every</td><td>man</td><td>who +1</td><td>owns</td><td>a dog +2 –1</td><td>+1</td><td>feeds</td><td>(–1) it +2</td></tr>
<tr><td>$\Sigma\!\rightarrow\!\wedge$</td><td colspan="4">$\Sigma\{\ \lambda x_0(\mathbf{M}x \,\&\, \mathbf{O}xy) \times y_{-1} \mid \mathbf{D}y\ \}$
$\Sigma\{\ \Sigma x(\mathbf{M}x \,\&\, \mathbf{O}xy) \times y_{-1} \mid \mathbf{D}y\ \}$</td><td></td><td colspan="2" rowspan="2"></td></tr>
<tr><td colspan="5">$\Sigma\{\ \wedge x(\mathbf{M}x \,\&\, \mathbf{O}xy) \times y_{-1} \mid \mathbf{D}y\ \}$</td><td>$\lambda x.x_1$</td></tr>
<tr><td colspan="6">$\Sigma\{\ \wedge\{x_1 \mid \mathbf{M}x \,\&\, \mathbf{O}xy\} \times y_{-1} \mid \mathbf{D}y\ \}$</td><td colspan="2">$\lambda y_{-1}\lambda x_1\mathbf{F}xy$</td></tr>
<tr><td colspan="8">$\Sigma\{\ \wedge\{x_1 \mid \mathbf{M}x \,\&\, \mathbf{O}xy\} \times \lambda x_1\mathbf{F}xy \mid \mathbf{D}y\ \}$
$\Sigma\{\ \wedge\{\mathbf{F}xy \mid \mathbf{M}x \,\&\, \mathbf{O}xy\} \mid \mathbf{D}y\ \}$
$\exists x\{\mathbf{D}y \,\&\, \forall x\{\mathbf{M}x \,\&\, \mathbf{O}xy \,.\!\!\rightarrow \mathbf{F}xy\}\ \}$</td></tr>
</table>

5. Comparison with Discourse Representation Theory

The structures we have introduced recently are formally parallel to discourse representation structures originally introduced by Kamp (1981) and Heim (1982).

+++ FORTHCOMING +++