Multimedia Computing: Algorithms, Systems, and Applications: Feature Extraction

By

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Harris Corner Detector

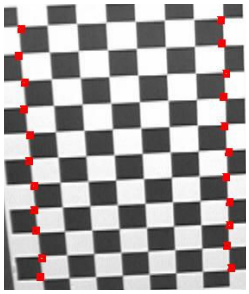
Finding Corners

Edge detectors perform poorly at corners.

Corners provide repeatable points for matching, so are worth detecting.

Idea:

- Exactly at a corner, gradient is ill defined.
- However, in the region around a corner, gradient has two or more different values.



3

The Harris corner detector

Form the second-moment matrix:

Sum over a small region around the hypothetical corner

Gradient with respect to x, times gradient with respect to y

$$C = \begin{bmatrix} \sum I_x^2 & \sum I_x I_y \\ \sum I_x I_y & \sum I_y^2 \end{bmatrix}$$

Matrix is symmetric

4

Simple Case

First, consider case where:

$$C = \begin{bmatrix} \sum I_x^2 & \sum I_x I_y \\ \sum I_x I_y & \sum I_y^2 \end{bmatrix} = \begin{bmatrix} \lambda_1 & 0 \\ 0 & \lambda_2 \end{bmatrix}$$

This means dominant gradient directions align with x or y axis

If either λ is close to 0, then this is **not** a corner, so look for locations where both are large.

5

General Case

It can be shown that since C is symmetric:

$$C = R^{-1} \begin{bmatrix} \lambda_1 & 0 \\ 0 & \lambda_2 \end{bmatrix} R$$

So every case is like a rotated version of the one on last slide.

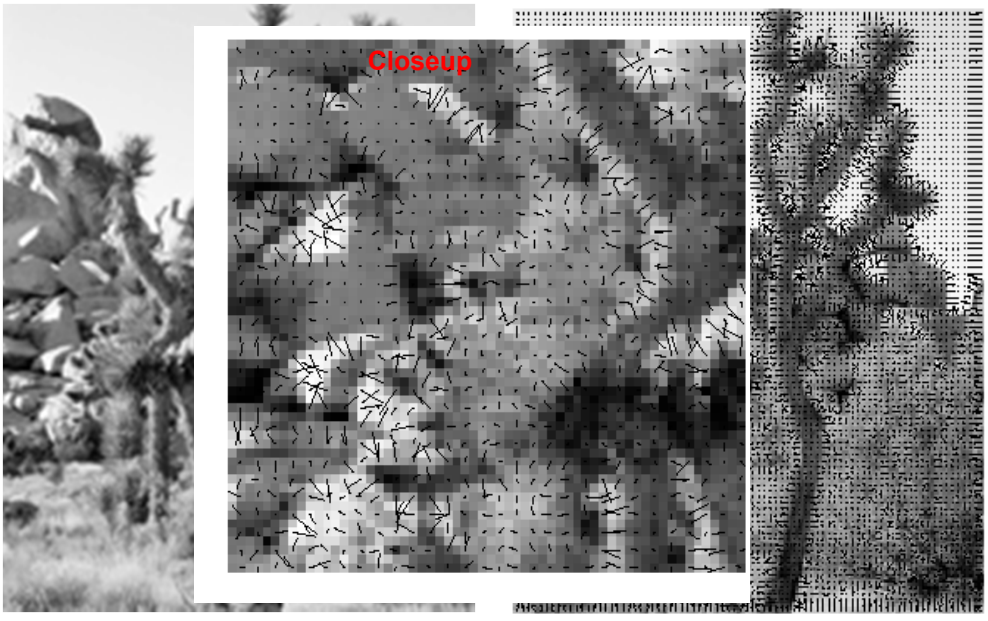
6

So, To Detect Corners

- Filter image
  - with Gaussian to reduce noise
- Compute magnitude of the gradient everywhere
  - magnitude of the x and y gradients at each pixel
- Construct C in a window
  - Construct C in a window around each pixel (Harris uses a Gaussian window – just blur)
- Use Linear Algebra to find $\lambda 1$ and $\lambda 2$
  - Solve for product of λs (determinant of C)
- If they are both big, we have a corner
  - If λs are both big (product reaches local maximum and is above threshold), we have a corner (Harris also checks that ratio of λs is not too high)

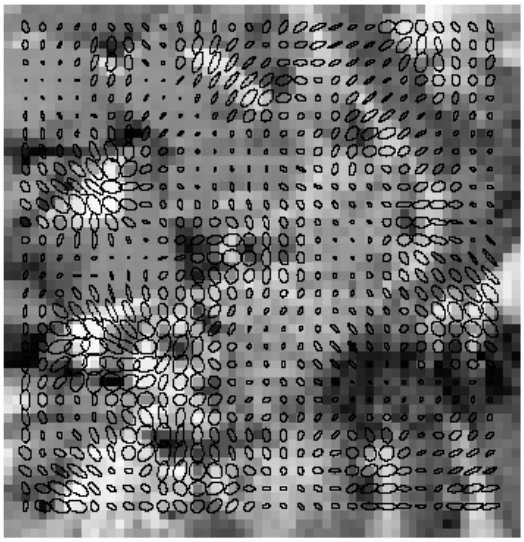
7

Gradient Orientation



8

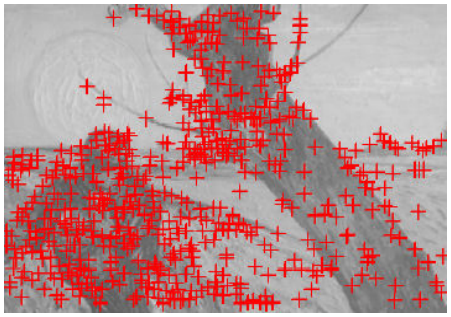
Corner Detection

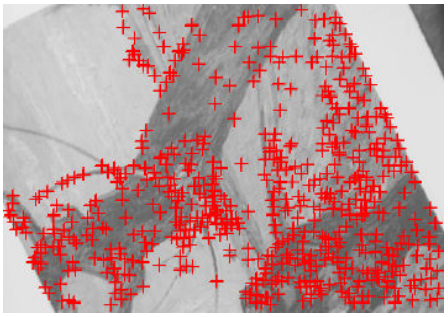


Corners are detected where the product of the ellipse axis lengths reaches a local maximum.

9

Harris Corners





- Originally developed as features for motion tracking
- Greatly reduces amount of computation compared to tracking every pixel
- Translation and rotation invariant (but not scale invariant)

10

Harris Corner in Matlab

```
% Harris Corner detector - by Kashif Shahzad
sigma=2; thresh=0.1; sze=11; disp=0;

% Derivative masks
dy = [-1 0 1; -1 0 1; -1 0 1];
dx = dy'; %dx is the transpose matrix of dy

% Ix and Iy are the horizontal and vertical edges of image
Ix = conv2(bw, dx, 'same');
Iy = conv2(bw, dy, 'same');

% Calculating the gradient of the image Ix and Iy
g = fspecial('gaussian',max(1,fix(6*sigma)), sigma);
Ix2 = conv2(Ix.^2, g, 'same');                    % Smoothed squared image derivatives
Iy2 = conv2(Iy.^2, g, 'same');
Ixy = conv2(Ix.*Iy, g, 'same');

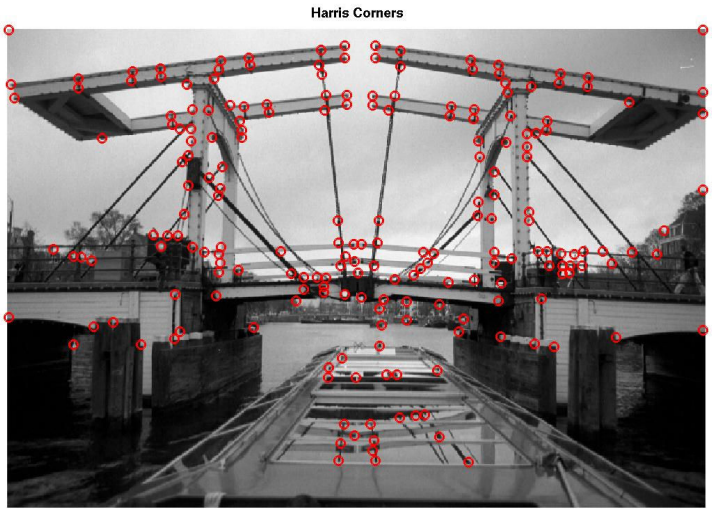
% My preferred  measure according to research paper
cornerness = (Ix2.*Iy2 - Ixy.^2)./(Ix2 + Iy2 + eps);

% We should perform nonmaximal suppression and threshold
mx = ordfilt2(cornerness,sze^2,ones(sze));        % Grey-scale dilate
cornerness = (cornerness==mx)&(cornerness>thresh); % Find maxima
[rws,cols] = find(cornerness);                    % Find row,col coords.

clf ; imshow(bw);
hold on;
p=[cols rws];
plot(p(:,1),p(:,2),'or');
title('\bf Harris Corners')
```

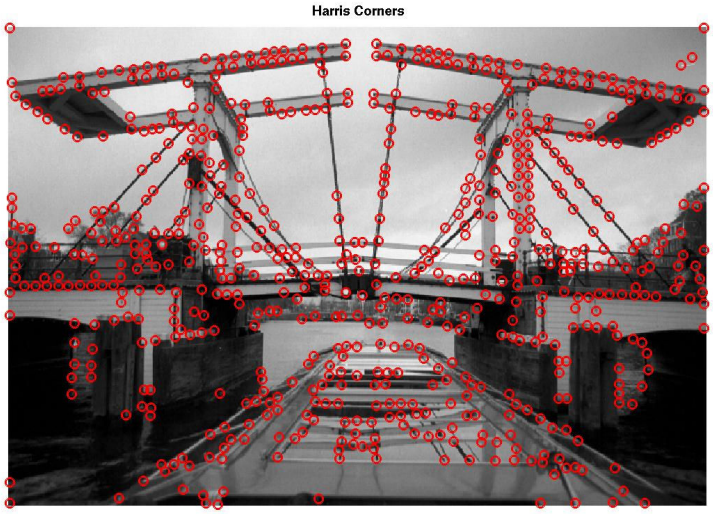
11

Example (σ=0.1)



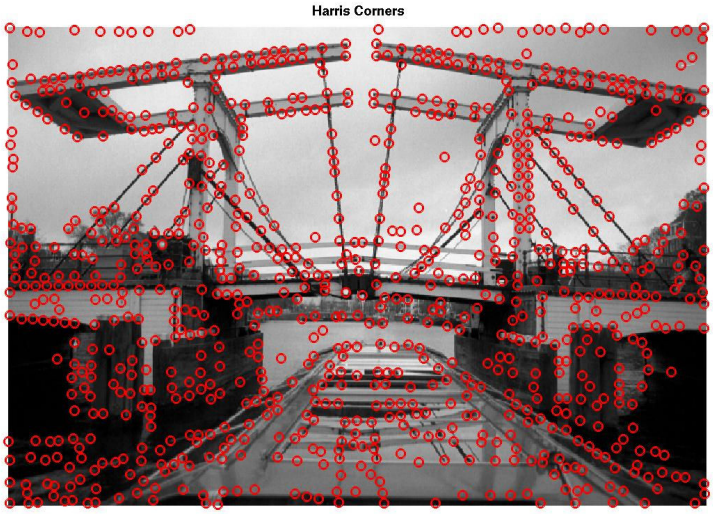
12

Example (σ=0.01)



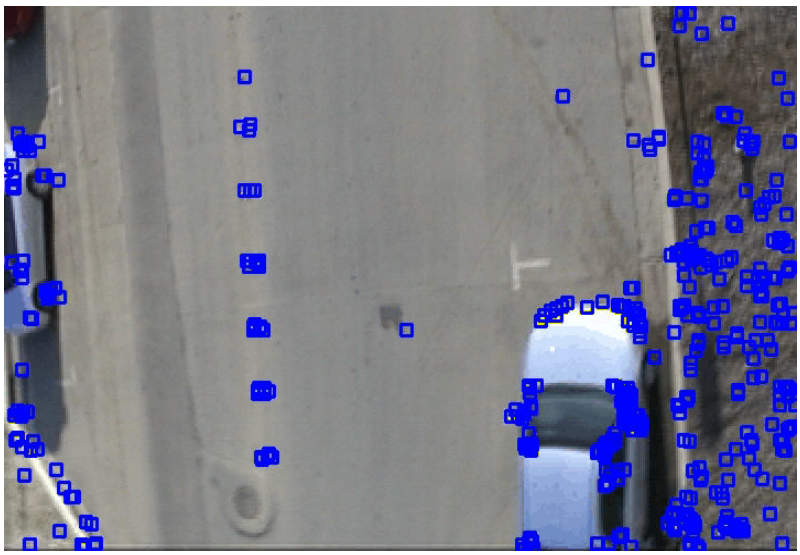
13

Example (σ=0.001)



14

Harris: OpenCV Implementation

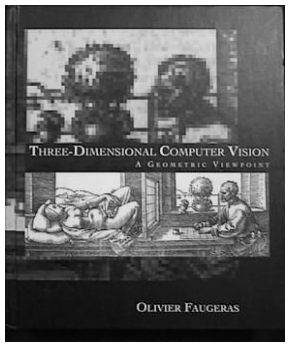


15

Template Matching

16

Problem: Features for Recognition



17

Templates

- Find an object in an image!
- Want Invariance!
  - Scaling
  - Rotation
  - Illumination
  - Deformation

18

Convolution with Templates

```
% read image
im = imread('bridge.jpg');
bw = double(im(:,:,1)) ./ 256;
imshow(bw)

% apply FFT
FFTim = fft2(bw);
bw2 = real(ifft2(FFTim));
imshow(bw2)

% define a kernel
kernel=zeros(size(bw));
kernel(1, 1) = 1;
kernel(1, 2) = -1;
FFTkernel = fft2(kernel);

% apply the kernel and check out
    the result
FFTresult = FFTim .*
    FFTkernel;
result = real(ifft2(FFTresult));
imshow(result)
```

```
% select an image patch
patch = bw(221:240,351:370);
imshow(patch)
patch = patch - (sum(sum(patch)) / size(patch,1) /
    size(patch, 2));

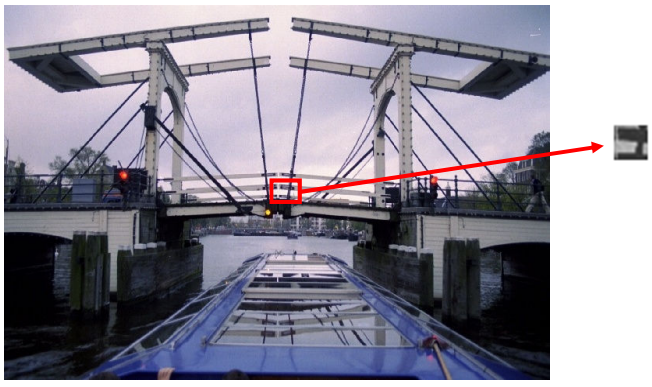
kernel=zeros(size(bw));
kernel(1:size(patch,1),1:size(patch,2)) = patch;
FFTkernel = fft2(kernel);

% apply the kernel and check out the result
FFTresult = FFTim .* FFTkernel;
result = max(0, real(ifft2(FFTresult)));
result = result ./ max(max(result));
result = (result .^ 1 > 0.5);
imshow(result)

% alternative convolution
imshow(conv2(bw, patch, 'same'))
```

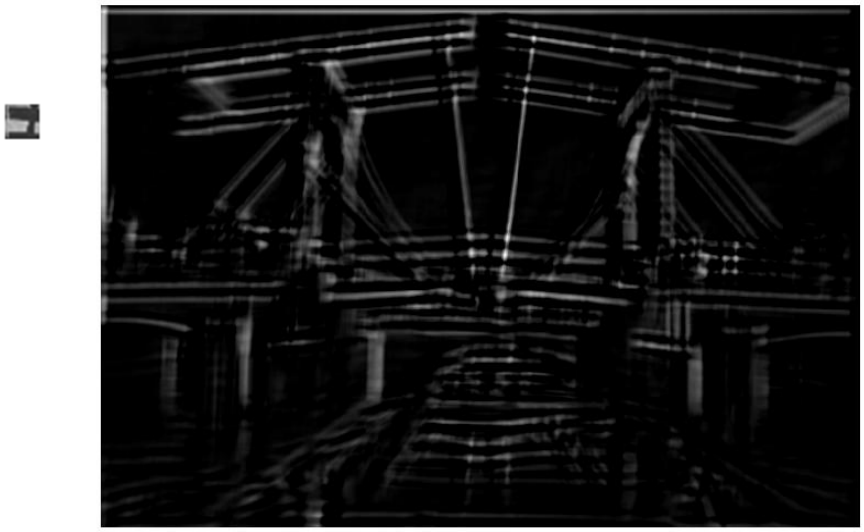
19

Template Convolution



20

Template Convolution



21

Recap: Convolution Theorem

$$F(I \otimes g) = F(I) \cdot F(g)$$

$$F(g(x,y))(u,v) = \int_{\Re^2} \int g(x,y) \ \exp\{-i2\pi(ux+vy)\} \ dx\,dy$$

22

Convolution with Templates

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kernel=zeros(size(bw));
kernel(1:size(patch,1),1:size(patch,2)) = patch;
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result = result ./ max(max(result));
result = (result .^ 1 > 0.5);
imshow(result)

% alternative convolution
imshow(conv2(bw, patch, 'same'))
```

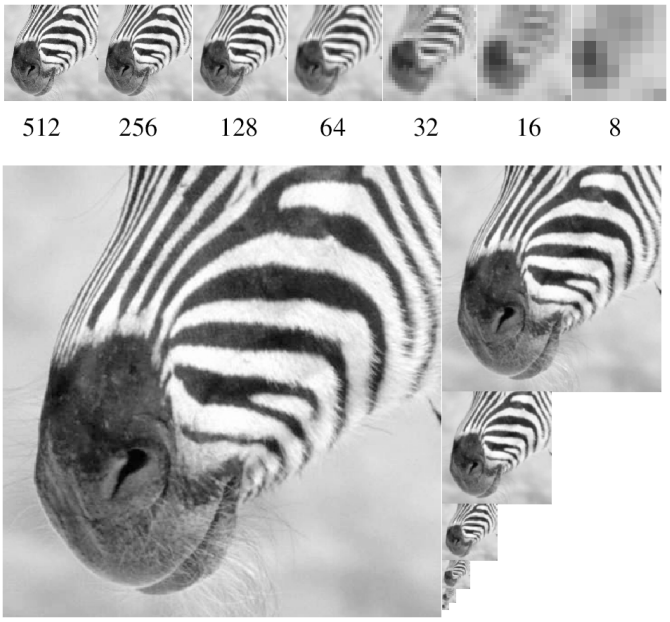
23

Convolution with Templates

- Invariances:
  - Scaling — No
  - Rotation — No
  - Illumination — No
  - Deformation — Maybe
- Provides
  - Good localization — No

24

Scale Invariance: Image Pyramid



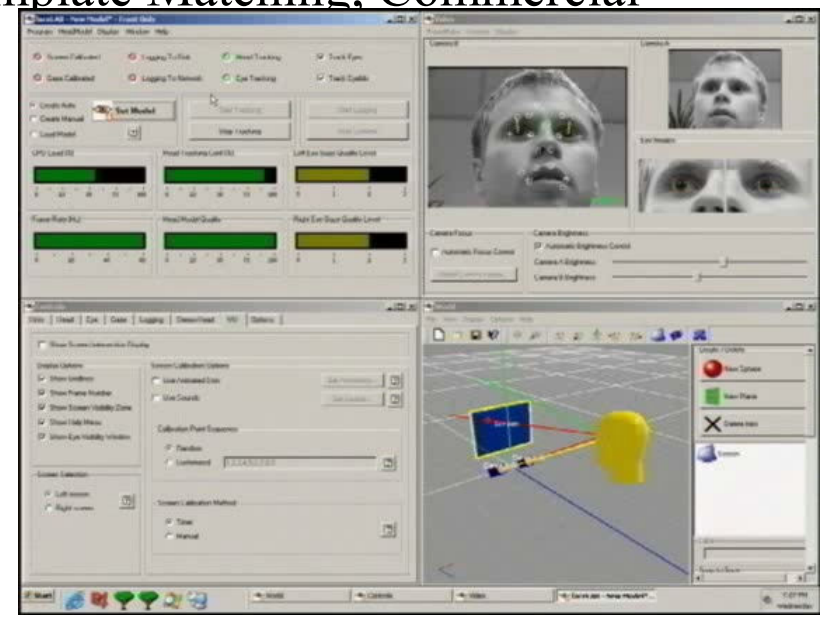
25

Templates with Image Pyramid

- Invariances:
  - Scaling — Yes
  - Rotation — No
  - Illumination — No
  - Deformation — Maybe
- Provides
  - Good localization — No

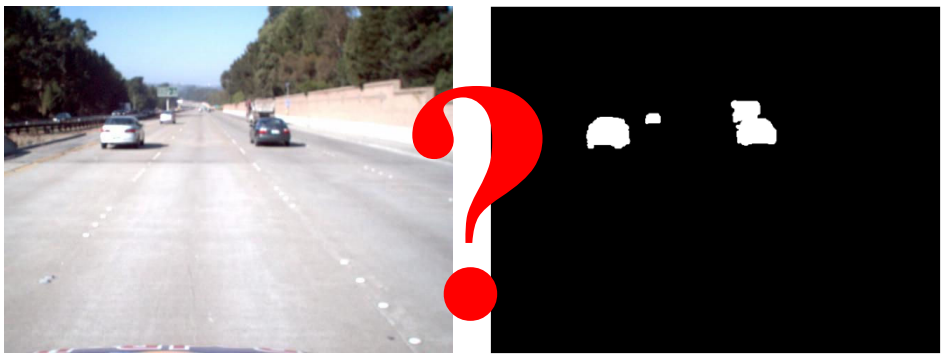
26

Template Matching, Commercial



27

Templates

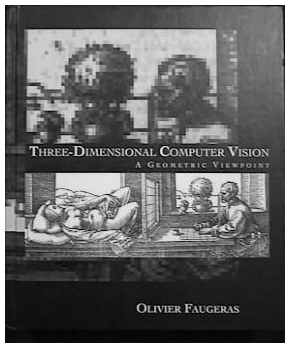


28

Scale-invariant feature transform (SIFT)

29

Let's Return to this Problem…



30

SIFT

- Invariances:
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  - Illumination — Yes
  - Deformation — Maybe
- Provides
  - Good localization — Yes

31

SIFT Reference

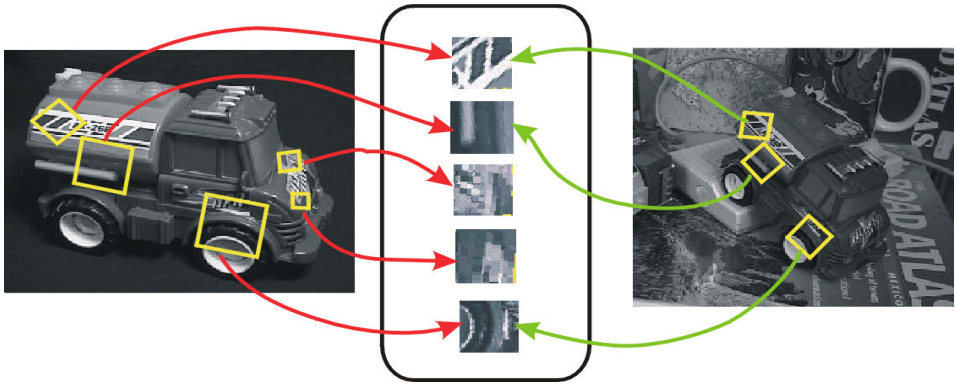
Distinctive image features from scale-invariant keypoints. David G. Lowe, International Journal of Computer Vision, 60, 2 (2004), pp. 91-110.

SIFT = Scale Invariant Feature Transform

32

Invariant Local Features

- Image content is transformed into local feature coordinates that are invariant to translation, rotation, scale, and other imaging parameters



33

Advantages of invariant local features

- **Locality:** features are local, so robust to occlusion and clutter (no prior segmentation)
- **Distinctiveness:** individual features can be matched to a large database of objects
- **Quantity:** many features can be generated for even small objects
- **Efficiency:** close to real-time performance
- **Extensibility:** can easily be extended to wide range of differing feature types, with each adding robustness

34

SIFT On-A-Slide

1. **Enforce invariance to scale: Compute Gaussian difference max, for may different scales; non-maximum suppression, find local maxima: keypoint candidates**
2. **Localizable corner: For each maximum fit quadratic function. Compute center with sub-pixel accuracy by setting first derivative to zero.**
3. **Eliminate edges: Compute ratio of eigenvalues, drop keypoints for which this ratio is larger than a threshold.**
4. **Enforce invariance to orientation: Compute orientation, to achieve scale invariance, by finding the strongest second derivative direction in the smoothed image (possibly multiple orientations). Rotate patch so that orientation points up.**
5. **Compute feature signature: Compute a "gradient histogram" of the local image region in a 4x4 pixel region. Do this for 4x4 regions of that size. Orient so that largest gradient points up (possibly multiple solutions). Result: feature vector with 128 values (15 fields, 8 gradients).**
6. **Enforce invariance to illumination change and camera saturation: Normalize to unit length to increase invariance to illumination. Then threshold all gradients, to become invariant to camera saturation.**

35

SIFT On-A-Slide

Step 1: Scale-space extrema detection

1. **Enforce invariance to scale: Compute Gaussian difference max, for may different scales; non-maximum suppression, find local maxima: keypoint candidates**
2. **Localizable corner: For each maximum fit quadratic function. Compute center with sub-pixel accuracy by setting first derivative to zero.**
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36

Find Invariant Corners

1. **Enforce invariance to scale: Compute Gaussian difference max, for may different scales; non-maximum suppression, find local maxima: keypoint candidates**

   - Keypoints are detected using scale-space extrema in difference-of-Gaussian function *D*
   - *D* definition:

$$D(x,y,\sigma) = (G(x,y,k\sigma) - G(x,y,\sigma)) * I(x,y)$$
$$= L(x,y,k\sigma) - L(x,y,\sigma)$$

   - Efficient to compute

37

Finding "Keypoints" (Corners)

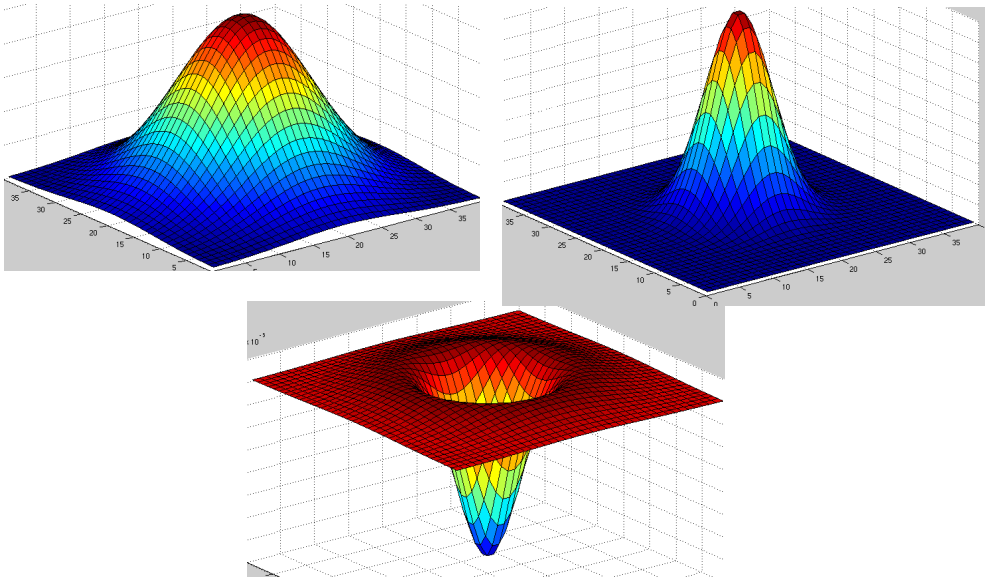
Idea: Find Corners, but scale invariance

Approach:

- Run linear filter (diff of Gaussians)
- At different resolutions of image pyramid

38

Difference of Gaussians



39

Difference of Gaussians

surf(fspecial('gaussian',40,4))
surf(fspecial('gaussian',40,8))
surf(fspecial('gaussian',40,8) - fspecial('gaussian',40,4))

40

Find Corners with DiffOfGauss

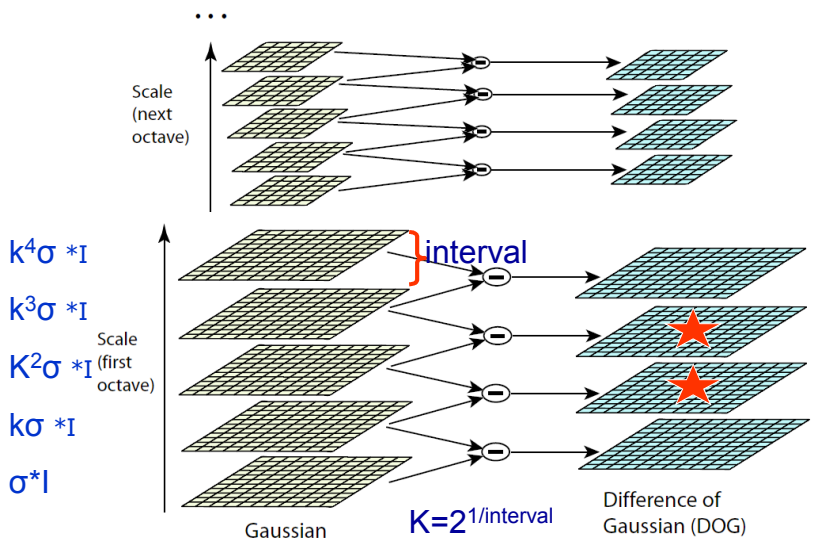
```
im =imread('bridge.jpg');
bw = double(im(:,:,1)) / 256;

for i = 1 : 10
  gaussD = fspecial('gaussian',40,2*i) - fspecial('gaussian',40,i);
  res = abs(conv2(bw, gaussD,  'same'));
  res = res / max(max(res));
  imshow(res) ; title(['\bf i = ' num2str(i)]); drawnow
end
```

41

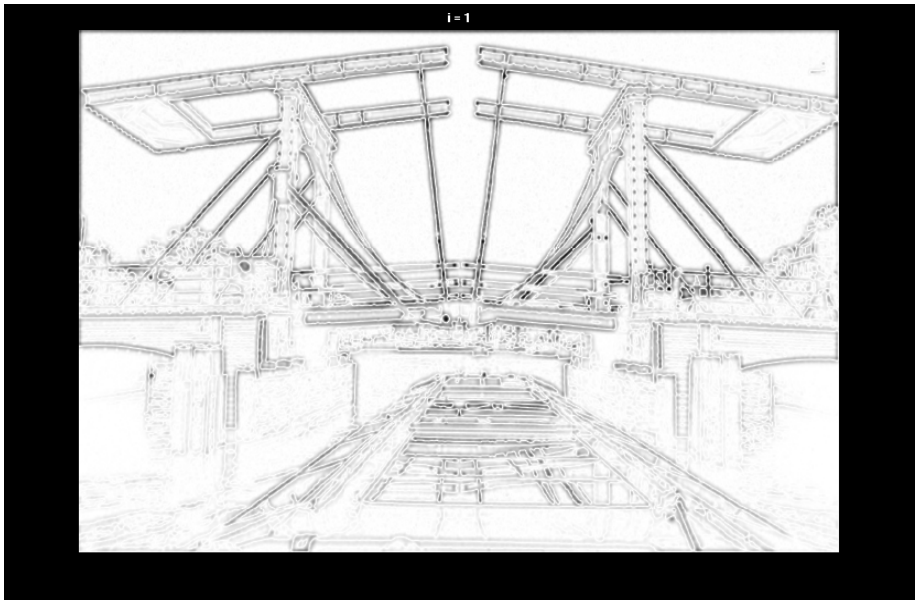
Scale-space extrema detection

- Scale space construction



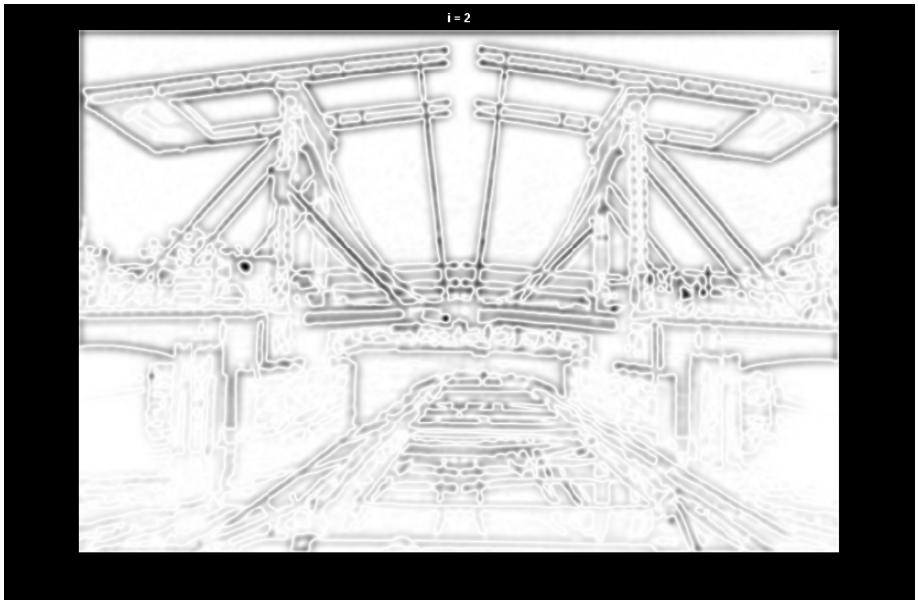
42

Gaussian Kernel Size i=1



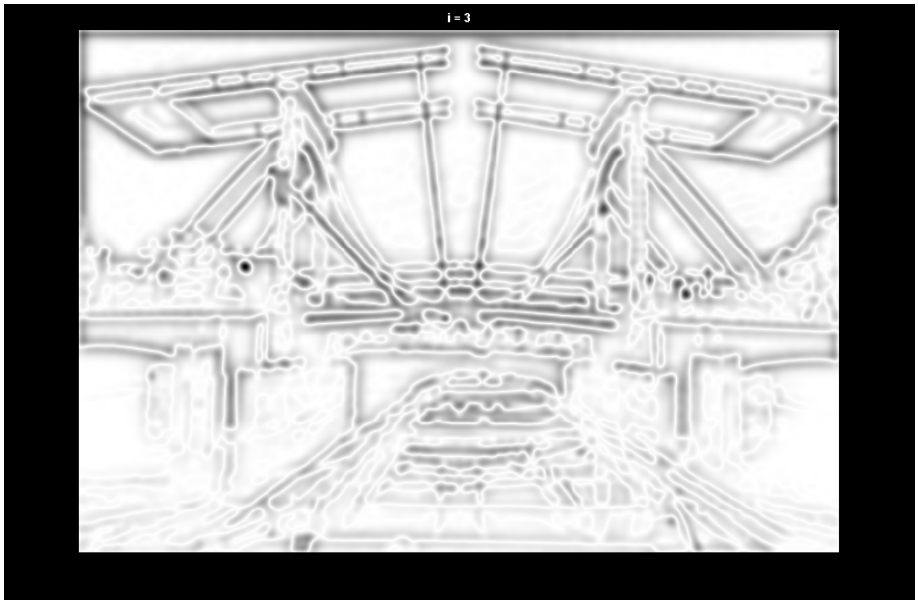
43

Gaussian Kernel Size i=2



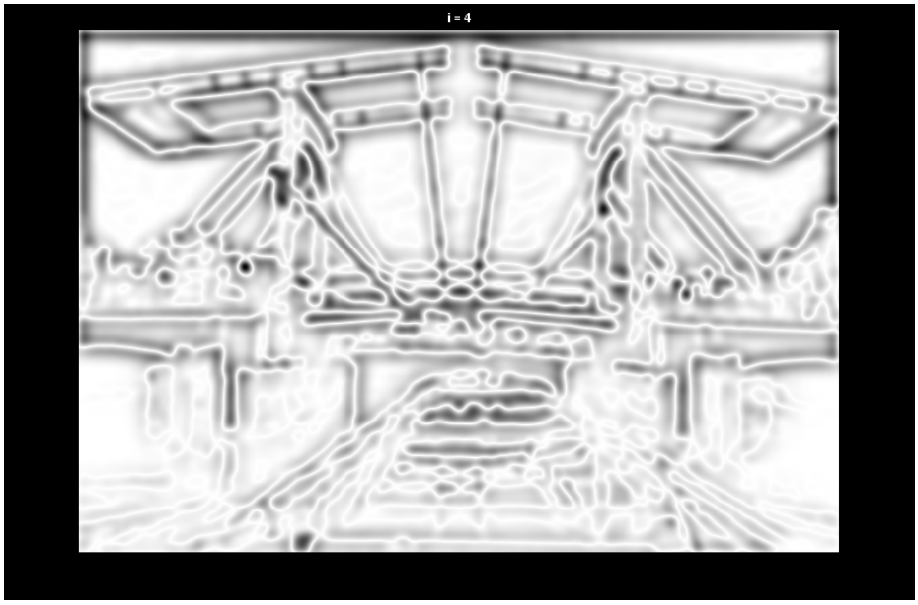
44

Gaussian Kernel Size i=3



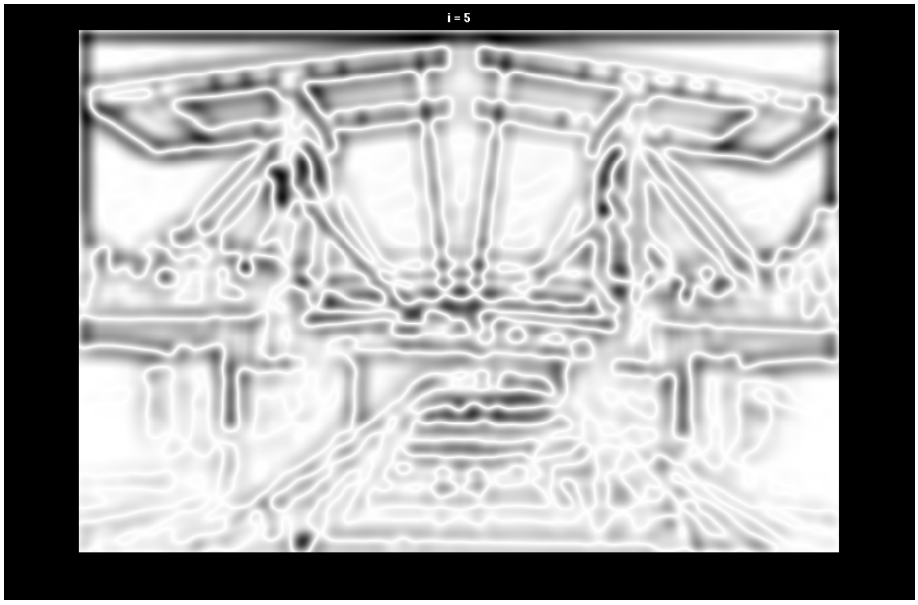
45

Gaussian Kernel Size i=4



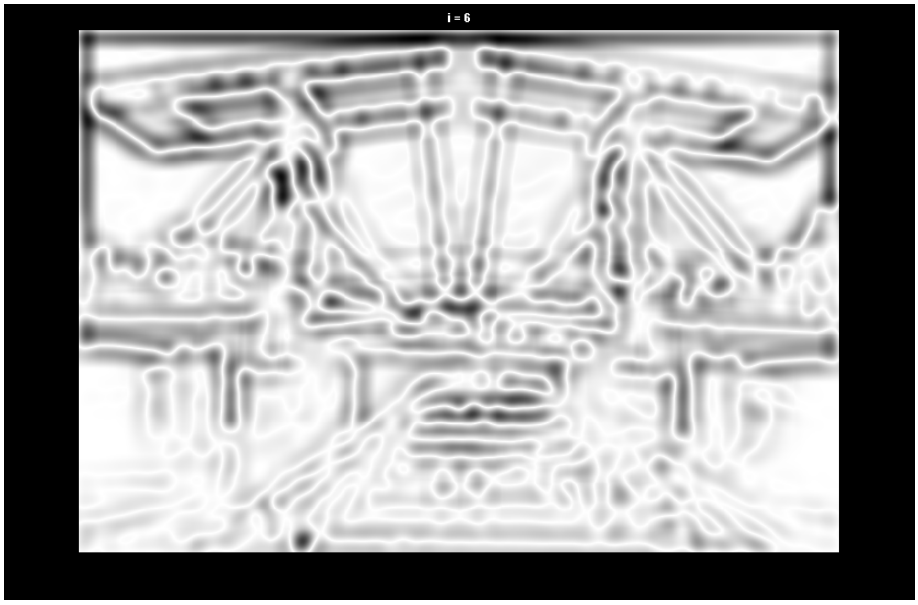
46

Gaussian Kernel Size i=5



47

Gaussian Kernel Size i=6



48

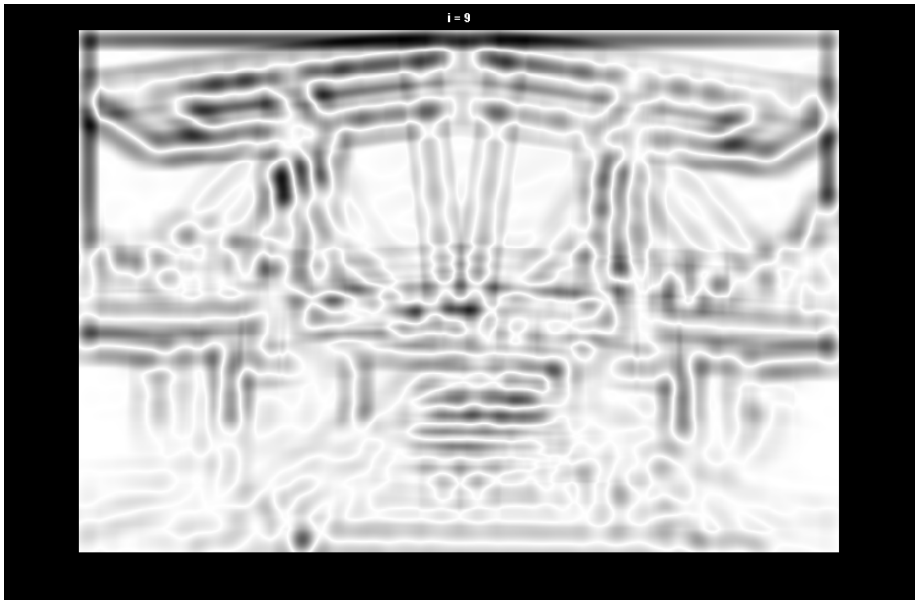
Gaussian Kernel Size i=7

49

Gaussian Kernel Size i=8

50

Gaussian Kernel Size i=9



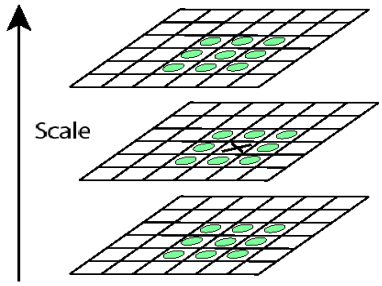
51

Gaussian Kernel Size i=10

52

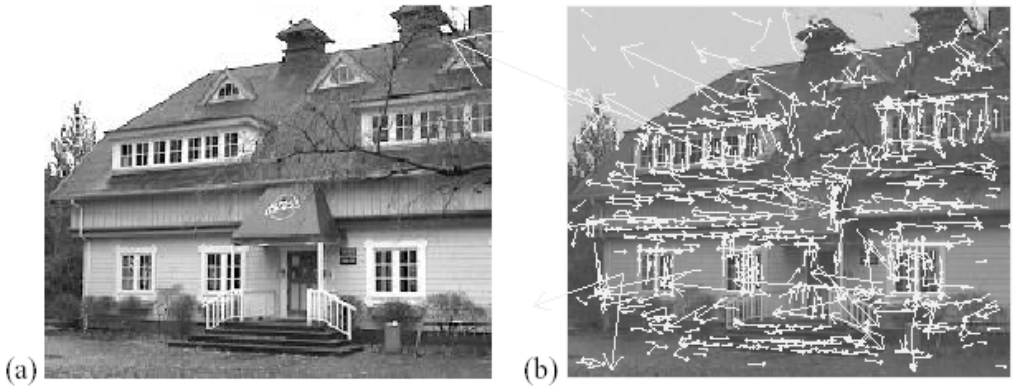
Key point localization

- Detect maxima and minima of difference-of-Gaussian in scale space
- Sample point is selected only if it is a minimum or a maximum of these points



53

Example of keypoint detection



(a) 233x189 image
(b) 832 DOG extrema
(c) 729 above threshold

54

SIFT On-A-Slide

Step 2: Key point localization

1. **Enforce invariance to scale: Compute Gaussian difference max, for may different scales; non-maximum suppression, find local maxima: keypoint candidates**
2. **Localizable corner: For each maximum fit quadratic function. Compute center with sub-pixel accuracy by setting first derivative to zero.**
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55

Localization

- 3D quadratic function is fit to the local sample points
- Start with Taylor expansion with sample point as the origin $D(\mathrm{X}) = D + \frac{\partial D^T}{\partial \mathrm{X}}\mathrm{X} + \frac{1}{2}\mathrm{X}^T\frac{\partial^2 D}{\partial \mathrm{X}^2}\mathrm{X}$
  - where $\hat{\mathrm{X}} = -\frac{\partial^2 D^{-1}}{\partial \mathrm{X}^2}\frac{\partial D}{\partial \mathrm{X}}$
- Take the derivative with respect to X, and set it to 0, giving $0 = \frac{\partial D}{\partial X} + \frac{\partial^2 D}{\partial X^2}\hat{X}$
- $\mathrm{X} = (x, y, \sigma)^T$ is the location of the keypoint
- This is a 3x3 linear system

56

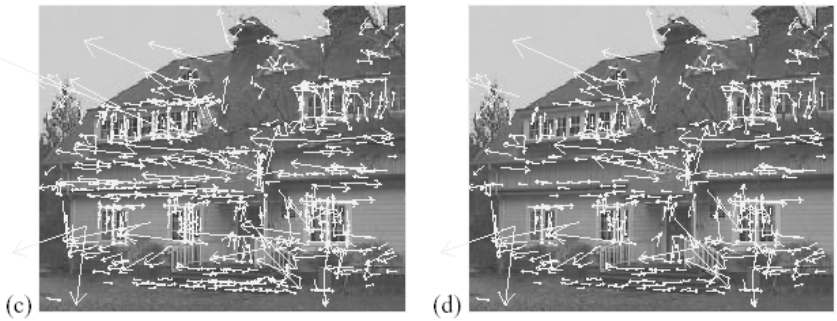
Filtering

- Contrast (use prev. equation): $D(\hat{X}) = D + \frac{1}{2}\frac{\partial D^T}{\partial X}\hat{X}$
  - If | *D(X)* | < 0.03, throw it out
- Edginess:
  - Use ratio of principal curvatures to throw out poorly defined peaks
  - Curvatures come from Hessian: $H = \begin{bmatrix} D_{xx} & D_{xy} \\ D_{xy} & D_{yy} \end{bmatrix}$
  - Ratio of *Trace(H)*^{2} and *Determinant(H)*

    $Tr(H) = D_{xx} + D_{yy}$

    $Det(H) = D_{xx}D_{yy} - (D_{xy})^2$
  - If ratio > $(r+1)^2/(r)$, throw it out (SIFT uses r=10)

Example of keypoint detection



(c) 729 left after peak value threshold (from 832)
(d) 536 left after testing ratio of principle curvatures

SIFT On-A-Slide

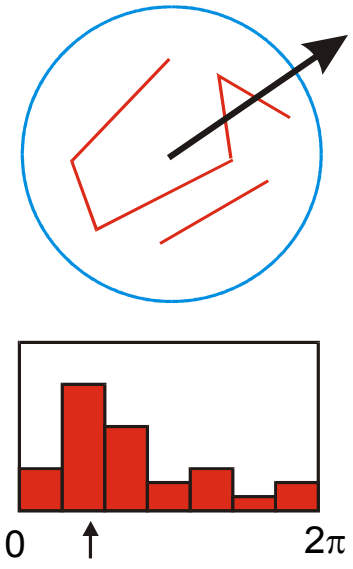
Step 3: Orientation assignment

1. **Enforce invariance to scale: Compute Gaussian difference max, for may different scales; non-maximum suppression, find local maxima: keypoint candidates**
2. **Localizable corner: For each maximum fit quadratic function. Compute center with sub-pixel accuracy by setting first derivative to zero.**
3. **Eliminate edges: Compute ratio of eigenvalues, drop keypoints for which this ratio is larger than a threshold.**
4. **Enforce invariance to orientation: Compute orientation, to achieve scale invariance, by finding the strongest second derivative direction in the smoothed image (possibly multiple orientations). Rotate patch so that orientation points up.**
5. **Compute feature signature: Compute a "gradient histogram" of the local image region in a 4x4 pixel region. Do this for 4x4 regions of that size. Orient so that largest gradient points up (possibly multiple solutions). Result: feature vector with 128 values (15 fields, 8 gradients).**
6. **Enforce invariance to illumination change and camera saturation: Normalize to unit length to increase invariance to illumination. Then threshold all gradients, to become invariant to camera saturation.**

59

Select canonical orientation

- Create histogram of local gradient directions computed at selected scale
- Assign canonical orientation at peak of smoothed histogram
- Each key specifies stable 2D coordinates (x, y, scale, orientation)



60

SIFT On-A-Slide

Step 4: Generation of key point descriptors

1. **Enforce invariance to scale: Compute Gaussian difference max, for may different scales; non-maximum suppression, find local maxima: keypoint candidates**
2. **Localizable corner: For each maximum fit quadratic function. Compute center with sub-pixel accuracy by setting first derivative to zero.**
3. **Eliminate edges: Compute ratio of eigenvalues, drop keypoints for which this ratio is larger than a threshold.**
4. **Enforce invariance to orientation: Compute orientation, to achieve scale invariance, by finding the strongest second derivative direction in the smoothed image (possibly multiple orientations). Rotate patch so that orientation points up.**
5. **Compute feature signature: Compute a "gradient histogram" of the local image region in a 4x4 pixel region. Do this for 4x4 regions of that size. Orient so that largest gradient points up (possibly multiple solutions). Result: feature vector with 128 values (15 fields, 8 gradients).**
6. **Enforce invariance to illumination change and camera saturation: Normalize to unit length to increase invariance to illumination. Then threshold all gradients, to become invariant to camera saturation.**

SIFT vector formation

- Thresholded image gradients are sampled over 16x16 array of locations in scale space
- Create array of orientation histograms
- 8 orientations x 4x4 histogram array = 128 dimensions

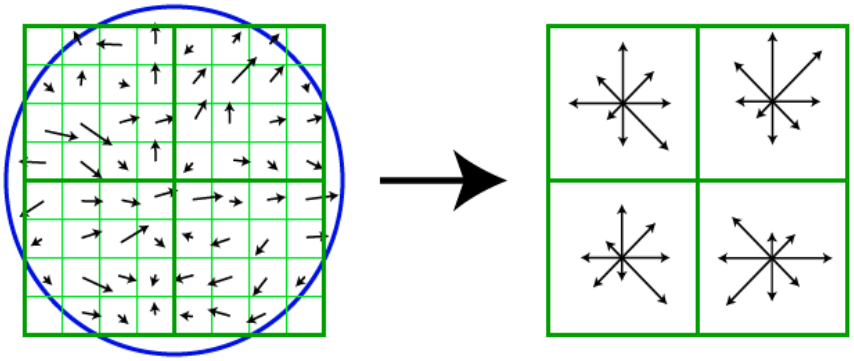


Image gradients

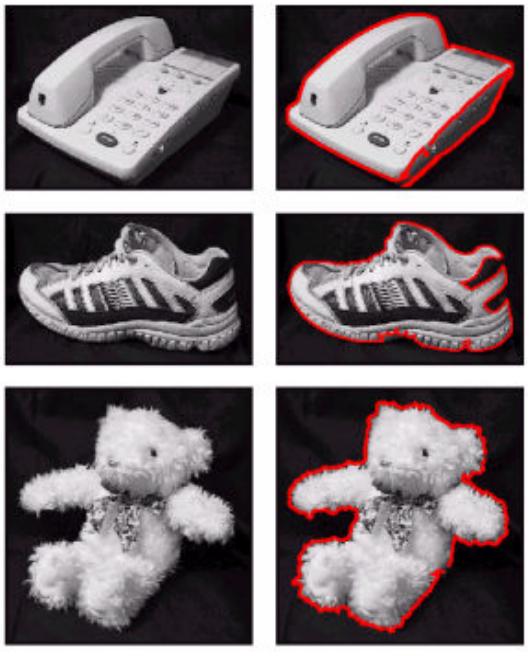
Keypoint descriptor

Nearest-neighbor matching to feature database

- Hypotheses are generated by **approximate nearest neighbor** matching of each feature to vectors in the database
  - SIFT use best-bin-first (Beis & Lowe, 97) modification to k-d tree algorithm
  - Use heap data structure to identify bins in order by their distance from query point
- **Result:** Can give speedup by factor of 1000 while finding nearest neighbor (of interest) 95% of the time

63

3D Object Recognition



- Extract outlines with background subtraction

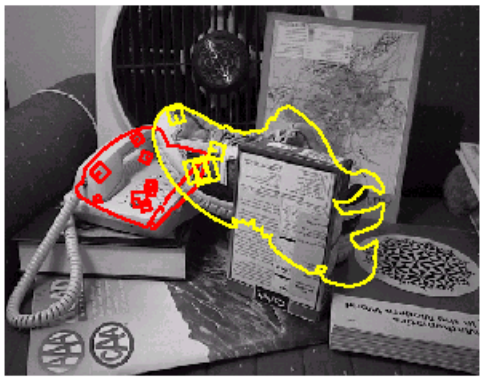
64

3D Object Recognition

- Only 3 keys are needed for recognition, so extra keys provide robustness
- Affine model is no longer as accurate

65

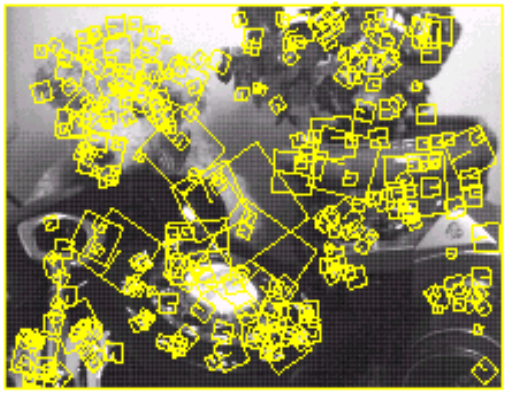
Recognition under occlusion



66

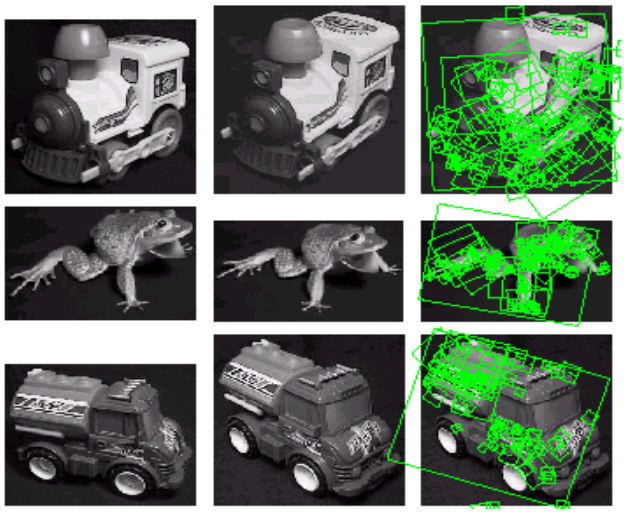
Test of illumination invariance

- Same image under differing illumination



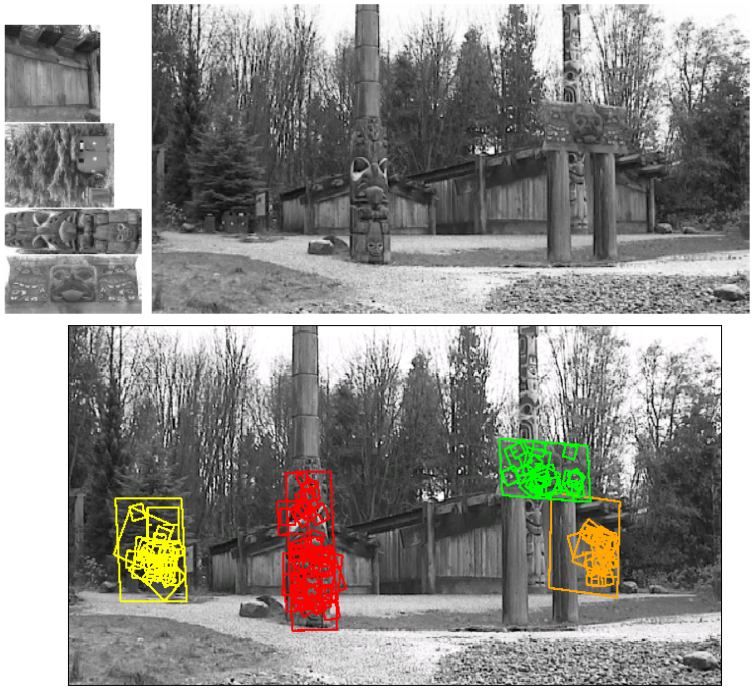
67

Examples of view interpolation



68

Location recognition



69

SIFT

- Invariances:
  - Scaling — Yes
  - Rotation — Yes
  - Illumination — Yes
  - Deformation — Maybe
- Provides
  - Good localization — Yes

70

Questions?

71