Harrison High School
AP Calculus AB Prerequisite Packet

To: AP Calculus AB Students and Parents
From: Monica Doriney, AP Calculus Instructor

The AP Course: AP Calculus AB is a college level course covering material traditionally taught in the first semester of college calculus. The course is taught in one semester consisting of 90 -minute classes. BC Calculus, corresponding to Calculus II at most universities, is taught during the second semester. Students are encouraged to take both semesters of AP Calculus.

The Prerequisite Packet: Students need a strong foundation to be ready for the rigorous work required throughout the term. Completing the prerequisite packet should prepare you for the material to be taught in the course. This packet consists of material studied during Algebra II and Pre-calculus. Students should anticipate working approximately 4 hours to complete it properly. The packet will be collected on the first day of class. In preparation for the AP test students need to begin showing all work with logical steps. Do not list only an answer. Work neatly and in an organized fashion.

Calculators: Students enrolled in AP Calculus AB will be using a graphing calculator throughout the course. A graphing calculator is required on the AP test. We will be using a TI nspire CX CAS in class; I recommend a TI-nspire CX CAS, but a TI 83, TI 84 or TI 89 can also be used.

Assistance with the packet: There is a lot of help on the internet. There will also be an optional help session as indicated at the top of this page. This is a help session to work on problems you are having difficulty with, not a session in which to do your packet. Please complete as much of the packet as possible before attending the session.

General Information: Students will have the opportunity to continue studying BC topics second semester through the BC Calculus course. The AB course is a prerequisite for BC. Students taking only the AB course and making a 3, 4, or 5 on the AB - AP test could receive 5 hours of credit for Calculus 1. Students taking both the AB and BC courses and passing the BC – AP test could earn up to 10 hours of college credit for Calculus 1 and Calculus 2. Be sure to check with your school of choice to determine their AP credit policy. Your success in this AP program will depend on the effort you put into the course. The work you do in the AB course is for you.

Since the AP test is offered only in May, students taking only the AB course should study for the test independently and attend any after school review sessions or practice test sessions that are offered. It is the student's responsibility to find out when these sessions will be held.

I anticipate a motivating and challenging year. Calculus is a stimulating and exciting field of mathematics and we look forward to sharing our excitement with you. I will be there to help and support you.

Formulas and Identities

Trigonometric Identities

Reciprocal Identities

$$\csc A = \frac{1}{\sin A} \qquad \sec A = \frac{1}{\cos A} \qquad \cot A = \frac{1}{\tan A}$$

Quotient Identities

$$\tan A = \frac{\sin A}{\cos A} \qquad \cot A = \frac{\cos A}{\sin A}$$

Double Angle Identities

$$\cos(2A) = \cos^2 A - \sin^2 A \qquad \cos(2A) = 2\cos^2 A - 1$$

$$\sin(2A) = 2\sin A \cos A \qquad \cos(2A) = 1 - 2\sin^2 A$$

Polar Formulas

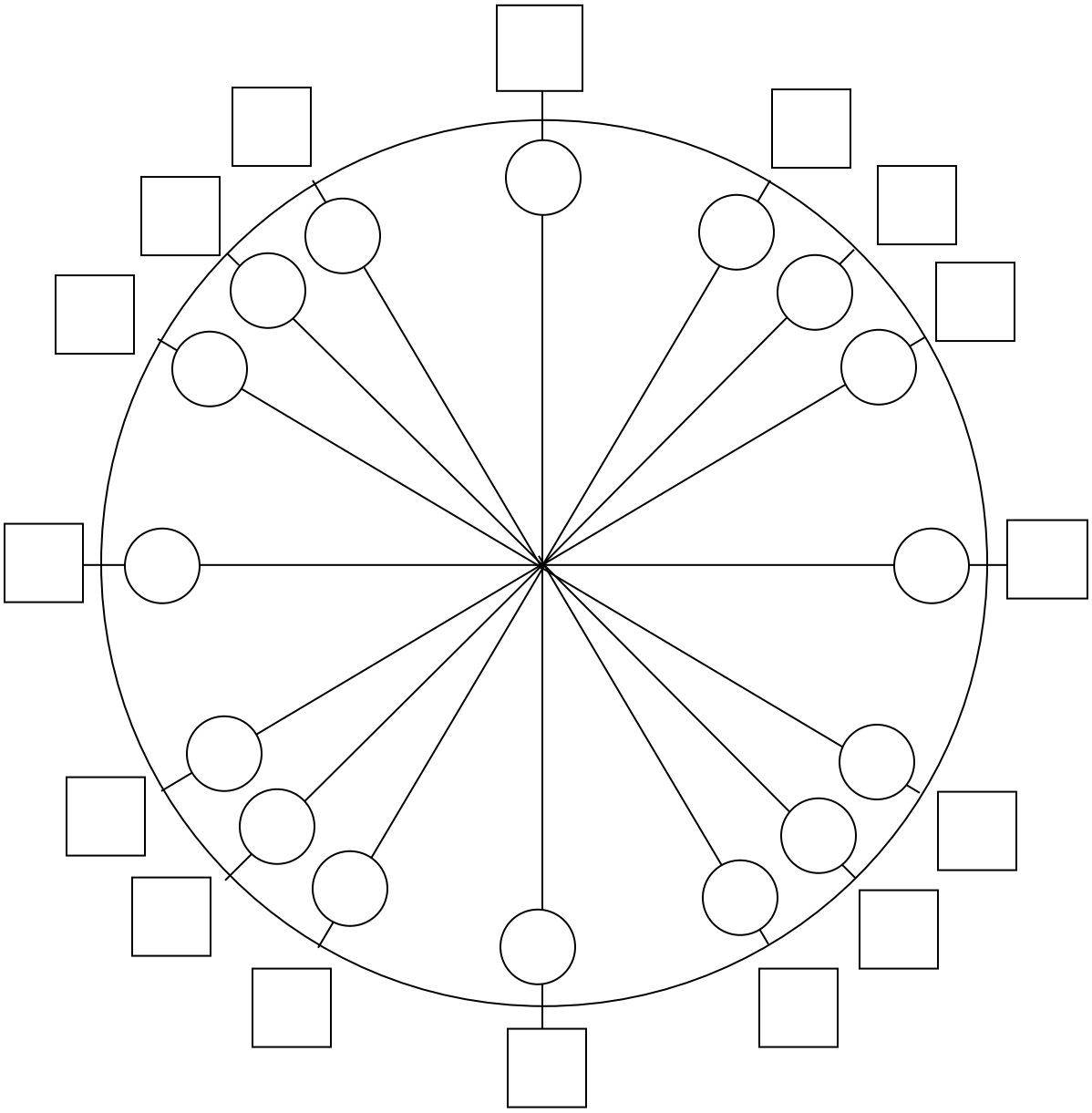
$$x^2 + y^2 = r^2 \qquad x = r \cdot \cos\theta \qquad y = r \cdot \sin\theta$$

$$\tan^{-1}\frac{y}{x} = \theta, \quad x > 0 \qquad \tan^{-1}\frac{y}{x} = \theta + \pi, \quad x < 0$$

Geometric Formulas

Area of a Triangle	$A = \frac{1}{2}bh$
Area of a Trapezoid	$A = \frac{1}{2}h(b_1 + b_2)$
Area of an Equilateral Triangle	$A = \frac{\sqrt{3}}{4}s^2$
Area of a Circle	$A = \pi r^2$
Circumference of a Circle	$C = 2\pi r \ \ or \ C = \pi d$

The Unit Circle



Place degree measures in the circles.

Place radian measure in the squares.

Place $(\cos\theta, \sin\theta)$ in parenthesis outside the square.

Place $\tan\theta$ outside the parenthesis.

$\tan\theta$ = _____

$\cot\theta$ = _____

$\csc\theta$ = _____

$\sec\theta$ = _____

SKILLS NEEDED FOR CALCULUS

1) ***Algebra***
   a) Exponents (operations with integer, fractional, and negative exponents)
   b) Factoring (GCF, trinomials, difference of squares and cubes, sum of cubes, grouping)
   c) Rationalizing (numerator and denominator)
   d) Simplifying rational expressions
   e) Solving algebraic equations and inequalities (linear, quadratic, rational, radical, and absolute value equations)
   f) Simultaneous equations

2) ***Graphing and Functions***
   a) Lines (intercepts, slopes, write equations using point-slope and slope intercept, parallel, perpendicular, distance and midpoint formulas)
   b) Functions (definition, notation, domain, range, inverse, composition)
   c) Basic shapes and transformations of the following functions (absolute value, rational, root, higher order curves, log, ln, exponential, trigonometric, piece-wise, inverse functions)

3) Geometry
   a) Pythagorean Theorem
   b) Area Formulas (Circle, polygons, surface area of solids)
   c) Volume formulas
   d) Similar Triangles

4) Logarithmic and Exponential Functions
   a) Simplify Expressions (Use laws of logarithms and exponents)
   b) Solve exponential and logarithmic equations (include ln as well as log)
   c) Sketch graphs
   d) Inverses

5) Trigonometry
   a) Unit Circle (definition of functions, angles in radians and degrees)
   b) Use of Pythagorean Identities and formulas to simplify expressions and prove identities
   c) Solve equations
   d) Inverse Trigonometric functions
   e) Right triangle trigonometry
   f) Graphs

6) Limits
   a) Concept of a limit
   b) Find limits as x approaches a number and as x approaches ∞

A solid working foundation in ALL of these areas is very important.

Calculus Prerequisite Problems

Work all problems without a calculator unless indicated otherwise.

1. Simplify: $\frac{\left((8x^3yz)^{\frac{1}{3}}(2x)^3\right)}{4x^{\frac{1}{3}}\left(yz^{\frac{2}{3}}\right)^{-1}}$

2. Factor: $9x^2 + 3x - 3xy - y$

3. Factor: $64x^6 - 1$

4. Factor: $42x^4 + 35x^2 - 28$

5. Factor: $15x^{\frac{5}{2}} - 2x^{\frac{3}{2}} - 24x^{\frac{1}{2}}$

6. Factor: $x^{-1} - 3x^{-2} + 2x^{-3}$

7. Rationalize: $\frac{3-x}{1-\sqrt{x-2}}$

8. Rationalize: $\frac{\sqrt{x+1}+1}{x}$

9. Simplify: $\frac{(x+1)^3(x-2)+3(x+1)^2}{(x+1)^4}$

10. Solve: $(x+3)^2 > 4$

11. Solve: $\frac{x+5}{x-3} \leq 0$

12. Solve: $3x^3 - 14x^2 - 5x \leq 0$

13. Solve: $x < \frac{1}{x}$

14. Solve: $\frac{x^2-9}{x+1} \geq 0$

15. Solve: $\frac{1}{x-1} + \frac{4}{x-6} > 0$

16. Solve: $x^2 < 4$

17. Solve: $|2x + 1| < \frac{1}{4}$

18. Solve the system: $\begin{matrix} x - y + 1 = 0 \\ y - x^2 = -5 \end{matrix}$

19. Solve the system: $\begin{matrix} x^2 - 4x + 3 = y \\ -x^2 + 6x - 9 = y \end{matrix}$

20. Write the equation of the line that passes through $(2, -1)$ and has slope $-\frac{1}{3}$.

21. Write the equation of the line that passes through $(4, -3)$ and is perpendicular to $3x + 2y = 4$.

22. Write the equation of the line that passes through $(-1, -2)$ and is parallel to $y = \frac{3}{5}x - 1$.

23. Find the domain: $\boldsymbol{y = \frac{3}{x-2}}$

24. Find the domain: $\boldsymbol{y = \log(x-3)}$

25. Find the domain: $\boldsymbol{y = \sqrt{2x-3}}$

26. Find the domain: $\boldsymbol{y = \frac{\sqrt{x+1}}{x^2-1}}$

27. Sketch the graph of:

$$f(x) = \begin{cases} x, & x \geq 0 \\ 1, & -1 \leq x < 0 \\ x-2, & x < -1 \end{cases}$$

28. Sketch the graph of:

$$\boldsymbol{f(x) = \begin{cases} \sqrt{25-x^2}, & x < 0 \\ \dfrac{x^2-25}{x-5}, & x \geq 0, x \neq 5 \\ 0, & x = 5 \end{cases}}$$

Questions 29-37:

Let $\boldsymbol{f(x) = x^2 + 3x - 2}$ $\boldsymbol{g(x) = 4x - 3}$ $\boldsymbol{h(x) = \ln x}$ $\boldsymbol{w(x) = \sqrt{x-4}}$

29. Find: $\boldsymbol{g^{-1}(x)}$

30. Find: $\boldsymbol{h^{-1}(x)}$

31. **Find $\boldsymbol{w^{-1}(x)}$, for $\boldsymbol{x \geq 4}$**

32. Find: $\boldsymbol{f(g(x))}$

33. Find $h\big(g\big(f(1)\big)\big)$

34. Does $\boldsymbol{f(x)}$ have an inverse function? Explain your answer

35. Find: $(f \circ w)(x)$

36. Find $(f \circ g \circ h)(x)$

37. Find the domain of $w(g(x))$

38. Sketch the graph: $y = \sqrt{x}$

39. Sketch the graph: $y = \ln x$

40. Sketch the graph: $y = \frac{1}{x}$

41. Sketch the graph: $y = |x - 2|$

42. Sketch the graph: $y = \frac{1}{x-2}$

43. Sketch the graph: $y = \frac{x}{x^2-4}$

44. Sketch the graph: $y = e^{-x}$

45. Simplify: $\log_4\left(\frac{1}{16}\right)$

46. Simplify: $\log_9 27$

47. Simplify: $\mathbf{3\log_3 3 - \frac{3}{4}\log_3 81 + \frac{1}{3}\log_3 \frac{1}{27}}$

48. Simplify: $\mathbf{\log_{125}\left(\frac{1}{5}\right)}$

49. Simplify: $\mathbf{\log_w W^{45}}$

50. Simplify: $\mathbf{\ln e}$

51. Simplify: $\mathbf{\ln e^2}$

52. Simplify: $\mathbf{\ln 1}$

53. Solve: $\mathbf{\log_6(x+3) + \log_6(x+4) = 1}$

54. Solve: $\mathbf{x^2 - \log 100 = \log 1}$

55. Solve: $\mathbf{3^{x+1} = 15}$

56. State the domain, range, and period for each function: $\mathbf{f(x) = \sin x}$, $\mathbf{f(x) = \cos x}$, $\mathbf{f(x) = \tan x}$

57. Solve: $\mathbf{\cos^2 x = \cos x + 2}$, $\mathbf{0 \le x \le 2\pi}$

58. Solve: $\mathbf{2\sin(2x) = \sqrt{3}}$, $\mathbf{0 \le x \le 2\pi}$

59. Solve: $\cos^2 x + \sin x + 1 = 0,\ 0 \le x \le 2\pi$

60. Evaluate: $\arcsin 1$

61. Evaluate: $\arcsin -\frac{\sqrt{2}}{2}$

62. Evaluate: $\arccos \frac{\sqrt{3}}{2}$

63. Evaluate: $\sin\left(arccos \frac{\sqrt{3}}{2}\right)$

64. State the domain and range of $\arcsin x, \arccos x, and \arctan x$

65. Find all roots of $f(x) = 2x^4 - 11x^3 - x^2 + 30x$

66. Find the local maxima of $f(x) = 2x^4 - 11x^3 - x^2 + 30x$ (calculator)

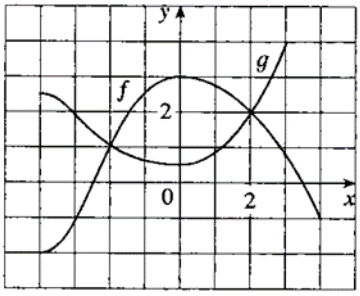
67. Find the local minima of $f(x) = 2x^4 - 11x^3 - x^2 + 30x$ (calculator)

68. Find the values of $f(-1), f(2), f(0), f\left(\frac{1}{2}\right)$ for the function $f(x) = 2x^4 - 11x^3 - x^2 + 30x$

69. Find the point of intersection using your calculator:

$$y = x^3 + 5x^2 - 7x + 2$$
$$y = 0.2x^2 + 10$$

70. The graphs of f and g are given.



a) State the values of f(-4) and g(3).
b) For what values of x if f(x)=g(x)?
c) Estimate the solution of the equation f(x) = -1.
d) On what interval is f decreasing?
e) State the domain and range of f.
f) State the domain and range of g.

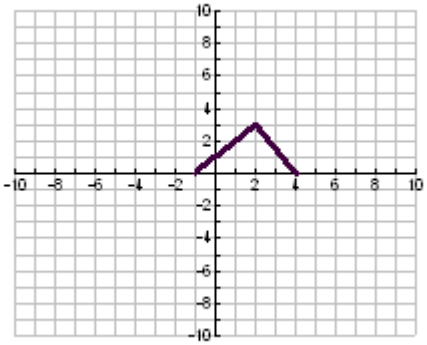
71. If $f(x) = 3x^2 - x + 2$, find $f(2)$, $f(-2)$, $f(a)$, $f(-a)$, $f(a+1)$, $2f(a)$, $f(a^2)$, $[f(a)]^2$, and $f(a+h)$.

72. Find the domain: $f(x) = \frac{x}{3x-1}$

73. Find the domain: $g(u) = \sqrt{u} + \sqrt{4-u}$

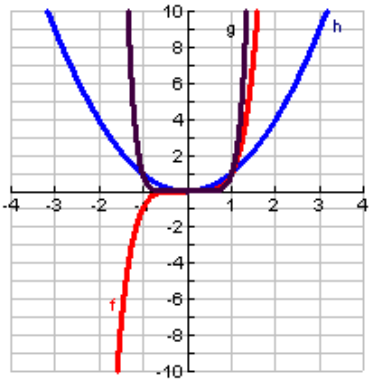
74. Find an expression for the bottom half of the parabola $x + (y-1)^2 = 0$.

75. Find an expression for the function whose graph is the given curve.



76. Match each equation with its graph. Explain your choices. (Don't use a computer or graphing calculator).

a) $y = x^2$
b) $y = x^5$
c) $y = x^8$



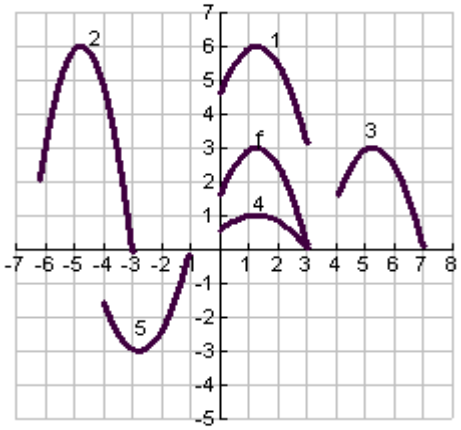
77. Suppose the graph of f is given. Write equations for the graphs that are obtained from the graph of f as follows.

(a) Shift 3 units upward.
(b) Shift 3 units downward.
(c) Shift 3 units to the right.
(d) Shift 3 units to the left.
(e) Reflect about the x-axis.
(f) Reflect about the y-axis.
(g) Stretch vertically by a factor of 3.

(h) Shrink vertically by a factor of 3.

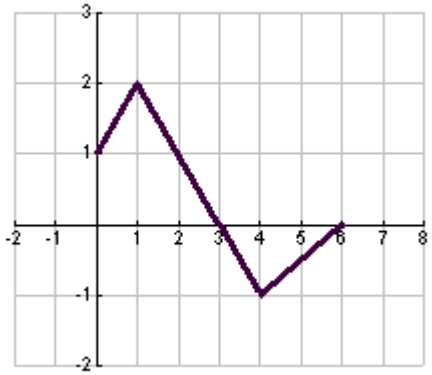
78. The graph of y = f(x) is given. Match each equation with its graph and give reasons for your choices.

(a) $y = f(x - 4)$
(b) $y = f(x) + 3$
(c) $y = 1/3\, f(x)$
(d) $y = -f(x + 4)$
(e) $y = 2f(x + 6)$



79. The graph of f is given. Use it to graph the following functions.

(a) $y = f(2x)$
(b) $y = f(\frac{1}{2}x)$
(c) $y = f(-x)$
(d) $y = -f(-x)$



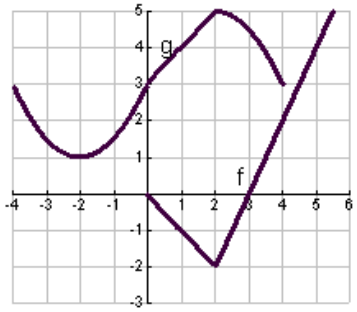
80. Find the functions $f \circ g, g \circ f, f \circ f, g \circ g$ and their domains.

$$f(x) = \sin x, g(x) = 1 - \sqrt{x}$$

81. Express the function in the form of $f \circ g$:

$F(x) = (x^2 + 1)^{10}$

82. Use the given graphs of f and g to evaluate each expression, or explain why it is undefined.



a) $f(g(2))$
b) $g(f(0))$
c) $(f \circ g)(0)$
d) $(g \circ f)(6)$
e) $(g \circ g)(-2)$
f) $(f \circ f)(4)$

83. Graph the ellipse $4x^2 + 2y^2 = 1$ by graphing the functions whose graphs are the upper and lower halves of the ellipse.

84. Use your calculator to find all solutions of the equation correct to three decimal places. $x^3 - 9x^2 - 4 = 0$

85. Starting with the graph of $y = e^x$, write the equation of the graph that results from
    a) Shifting 2 units downward
    b) Shifting 2 units to the right
    c) Reflecting about the x-axis
    d) Reflecting about the y-axis
    e) Reflecting about the x-axis and then about the y-axis

86. Find the Inverse: $f(x) = \sqrt{10 - 3x}$

87. Find the Inverse: $f(x) = e^{x^3}$

88. Find the Inverse: $f(x) = \ln(x + 3)$

89. Evaluate: $\log_2 64$

90. Evaluate: $\log_6 \frac{1}{36}$

91. Evaluate: $\log_{10} 1.25 + \log_{10} 80$

92. Evaluate: $\log_5 10 + \log_5 20 - 3\log_5 2$

93. Condense: $2\ln 4 - \ln 2$