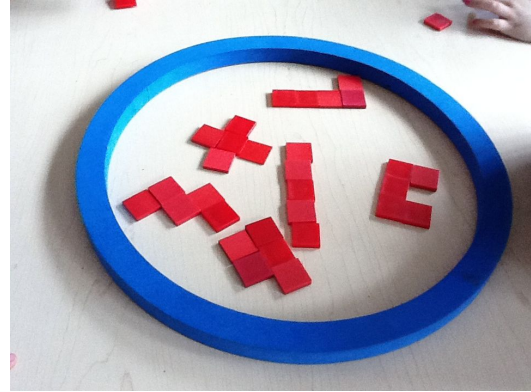
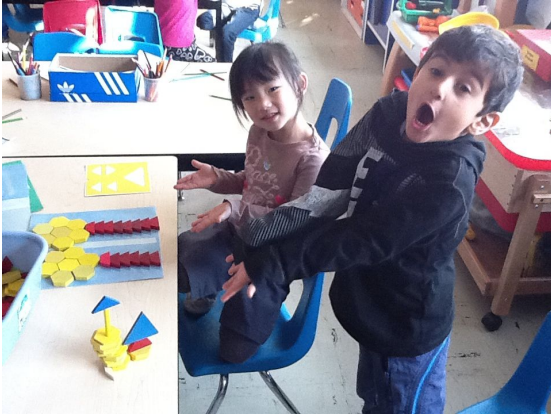


**HARRISON/THORNCLIFFE LESSON STUDY TEAM  
TDSB  
Kindergarten to Grade 1  
May 1 and 2, 2012**



Research Question(s):

***How do young children demonstrate and communicate their understanding of spatial reasoning during geometry tasks?***

***How can we use technology to support geometry teaching and learning?***

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## **Table of Contents**

- Background information
  - Number Knowledge Test
  - Clinical interviews and findings
  - Influential articles
  - Exploratory lessons
  - iPad apps
- Description of Public lesson
- Observation Questions

## **AGENDA**

8:30 to 9:40	Introductions and background provided by the planning team
9:45 to 10:30	Public Lesson
10:30 to 10:45	Break
10:45 to 11:45	Debrief: <ul style="list-style-type: none"><li>i. Teachers who taught lesson</li><li>ii. Observers from teacher planning group</li><li>iii. Comments from guests</li><li>iv. Discussant(s)</li></ul>

## Background Information

### Five sources of information:

1. Number Knowledge Test
2. Clinical Interviews
3. Literature Review
4. Exploratory Lessons
5. Use of Technology – iPad Apps

## 1. Number Knowledge Test

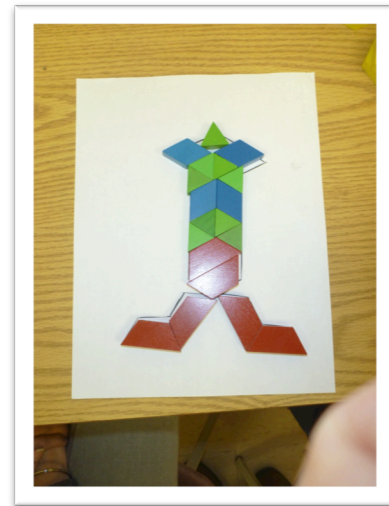
Six students from each class were selected by the teacher to participate in the Number Knowledge Test (Griffin & Case, 1997). Teachers, graduate students, and researchers administered the test. A sample of some of the test items is provided in Appendix A.

## 2. Clinical Interviews

The interview tasks were developed by the team in order to uncover students' early spatial, geometrical, and number sense. The teacher with the support of researchers and graduate students interviewed six students from each class. The interviews were videotaped and analyzed as a group. Our observations and findings motivated us to further explore children's spatial reasoning.

### Developmental Levels for Composing Geometric Shapes

- Children move through levels in the composition and decomposition of two-dimensional figures.
- Very young children cannot compose shapes but then gain the ability to combine shapes into pictures, synthesize combinations of shapes into new shapes and eventually build new shapes.
- Children typically follow an observable developmental progression in learning about shapes. This developmental path is often described as part of a learning trajectory.
  - ➔ The first 2D task which the Kindergarten students took part in during the clinical interviews was an activity using Tangrams. Students were shown a picture and asked what they thought it resembled. Students then had to fill in the outline picture using the Tangram pieces.
  - ➔ Students were highly motivated to participate in the task.
  - ➔ Most students identified the outline pictures as a cat and a rocket.
  - ➔ Some students especially were very meticulous in which shapes they chose and how they placed the shapes inside the outline picture.
  - ➔ Although some students faced difficulties, most students were persistent as they attempted to fill in the whole picture.
  - ➔ The student you will see in this video clip engaged in self talk and the use of gestures while trying to figure out where the shapes should be placed.
  - ➔ Overall we found many students used mathematical language



### 3. Literature Background

**Clements, D. H., & Sarama, J. (2011). Early childhood teacher education: The case of geometry. *Journal of Mathematics Teacher Education*, 14, 133-148.**

Geometry is essential as both a math content area and as a focal point for teacher professional development. The authors argue that although geometry underlies all aspects of math (since mathematical ideas are inherently spatial) and other disciplines such as the sciences, it is neglected in early childhood education and in the professional development of early childhood teachers. Most elementary school math teachers admit to having a lack of knowledge of geometric concepts and to the scarcity of professional development opportunities in this area and math in general. The authors introduce a professional development program known as the Building Blocks/TRIAD Model that serves to enrich teachers' content knowledge of geometry along with their understanding of children's learning trajectories in this area. Specifically, teachers build common content knowledge, clarify misconceptions about geometric concepts and explore best practices for teaching geometry. They also examine children's mathematical thinking, and design lessons to advance children's geometric understanding.

**Newcombe, N. (2010). Picture this: Increasing math and science learning by improving spatial thinking. *American Educator*, Summer 2010, 29-43.**

Strong spatial thinking ability has been demonstrated by research to be essential for pursuing careers in sciences, technology, engineering, and mathematics (STEM). Spatial thinking generally involves the location of objects and our ability to manipulate them in different ways. It also includes our capacity to relate to and navigate the wider world around us. The article further provides specific examples of spatial thinking questions used for academic assessment. The ability to think spatially is not immutable and can be improved through effective school programming starting from the early years. A list of suggestions and strategies are provided to help teachers enhance students' spatial thinking in the classroom. For example, early years teachers are encouraged to introduce spatially challenging books, teach spatial words, and to introduce students to both standard and non-standard geometric shapes. Gestures are also emphasized to play a key role in helping students improve their ability to think spatially. Thus, teachers should encourage young children to gesture when explaining how they have located or manipulated certain objects. Finally, the author also touches on sex differences in spatial thinking by noting that although they do exist, they do not reflect individual performance. Both boys and girls can become better at spatial thinking through high quality programming.

**Newcombe, N.S. & Frick, A. (2010). Early education for spatial intelligence: Why, what, and how. *Mind, Brain, and Education*, 4, 102-111.**

Spatial thinking has been regarded in the extant literature as a fundamentally important skill from both an evolutionary and adaptive standpoint. Furthermore, the development of this skill is seen as foundational for the pursuit of careers in the science, technology, engineering, and mathematics (STEM) disciplines. The two types of spatial thinking most relevant to the STEM disciplines are mental rotation and perspective taking. Research studies examining these two types have generally found that they are improvable through intervention. Since studies have shown that children have a natural predisposition for mental rotation and perspective taking in the early years, effective intervention can increase children's spatial reasoning skills allowing more of them to pursue careers in the STEM disciplines. The authors introduce three primary forms of intervention: 1) spatial reasoning activities in the preschool setting; 2) the usage of media to practice spatial thinking; 3) free play and active experiences. Adults can facilitate all of these experiences by introducing the relevant spatial language.

### **Additional publications of influence:**

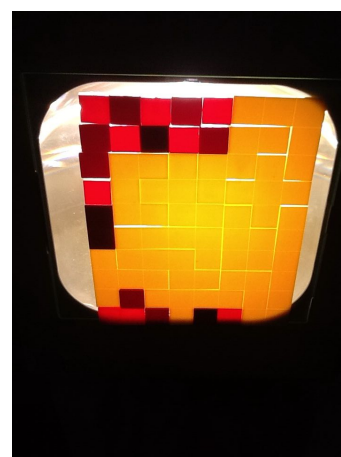
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### 3. Exploratory Lessons

- Geometric shapes can be used to represent and understand objects.
- Analyzing, comparing, and classifying shapes helps create new knowledge of shapes and their relationships. Shapes can be decomposed into other shapes.
- Through their everyday activities children build both intuitive and explicit knowledge of geometric figures.
- We recognize that most children can recognize and name basic two-dimensional shapes around the age of four. However, children can learn richer concepts about shapes if they have varied examples and non-examples of shape, discussions about classes, and interesting tasks.
- In order to help students develop a better understanding of 2D and 3D shapes, we tried to expose the students to as much mathematical language and concepts related to geometry as we thought we could at the time. Our mathematical journey often led us in to different directions. For example, our exploration of 2D shapes led us into discussions and activities

#### Exploratory lessons:

- a) Explored 2D shapes using strings
- b) Students used 2D shapes to create pictures and then began to explore how to create pictures that were symmetrical
- c) Sang a song and played a mystery game trying to identify 2D shapes
- d) Made 3D and 2D shapes using toothpicks and marshmallows
- e) Explored tiles on the Smartboard (tried to make as many shapes using 3 and 4 tiles)
- f) Pentominoes activity on Smartboard and in small group settings
- g) Students explored tiles on the overhead
- h) Students explored a pentominoes puzzle on the overhead



	Exploratory Lesson	Description	Geometry concepts involved
1	Carpet Activity: Build My Structure	Students were asked to build a structure with 5 blocks while sitting back to back in pairs. One of the students (speaker) builds and explains his/her structure and the other student recreates the speaker's structure without looking.	Visualization Location Orientation
2	Math Centre: Composing and Decomposing 2-D Shapes	Students were presented with different 2-D shapes. Students were asked to decompose shapes and fit shapes within shapes.	Shape recognition Decomposing Shapes
3	Smart Board activity	Students were asked to recognize rotations and flips of objects from memory. These objects were initially rotated or flipped animals; later we used rotated or flipped pentominoes (which the students built).	Transformations Visualization Visual Memory skills Exploring motions of slides, flips, turns.

			Altering objects' location or orientation but not its size or shape
4	Math Centre, recognizing 3-D shapes	5 shapes were created with 4 snap cubes and students had to find the matching shapes from a basket which held many different shapes.	Same as above
5	Smart Board activity	Students were asked to rotate and flip objects like bears and pentominoes and explain and show their thinking on the Smart Board.	Same as above

## 5. Use of Technology – iPad Apps

Our team has been exploring the use of technology to support students' spatial reasoning. Here are some of the Spatial/Geometry apps that we've been using with students. Thank you Latoya for recommending our favourites: Cube It 3D and Block Master.

Cube It 3D: <http://itunes.apple.com/ca/app/cube-it-3d-free/id440476902?mt=8>

Block Master: <http://itunes.apple.com/ca/app/block-master/id478746812?mt=8>

Toy Block: <http://itunes.apple.com/us/app/toy-block-2.0/id500974600?mt=8>

Dare Block: <http://itunes.apple.com/ca/app/dare-block/id447746213?mt=8>

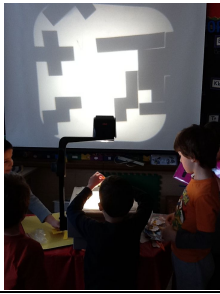
Block Builder: <http://itunes.apple.com/us/app/block-builder-3d/id512521242?mt=8>


Pentix: <http://itunes.apple.com/ca/app/pentix-free/id345382715?mt=8>

BlockArt: <http://itunes.apple.com/ca/app/blockart-free/id418540521?mt=8>



## More Exploratory Lessons



Activity	Description
Traffic light activity	<p>Students were given three snap cubes (red, green and yellow).</p> <p>Students were asked to sit back to back. One student was asked to build their own traffic light and without looking at their partner describe to their partner how to make their traffic light.</p> <p>This task was primarily to observe positional language used by the students.</p>
Battle Ship activity	<p>Students were given a 3x3 chart and a cup of five snap cubes all the same color. Students sat next to a partner with a book as a divider. One student created a ship by placing cubes in the squares on the chart. The student then described the positions of each cube to their partner so that they could create a “ship” in the same position.</p>
Where is Teddy?	<p>This was a cut and paste booklet students created using various 2D shapes and using a Teddy Bear that could be moved onto each page within the book. On each page there was a different shape with instructions to place the Teddy in different places. This booklet encouraged students to use positional language such as (on top, behind, in front of, above, below).</p>
Sang a song and played a mystery game trying to identify 2D shapes.	<p>Students given a shape to hide behind their back. Teacher then sang: “Where is _____? Where is _____?” Students who had the shape requested held out their shape and sang back “Here I am. Here I am.”</p>
String Shapes	<p>Students used yarn and tape to create 2D shapes.</p>
Shape Museum	<p>Made 3D and 2D shapes using toothpicks and marshmallows.</p>
 <p>Symmetry</p>	<p>Students used 2D shapes to create pictures and then began to explore how to create pictures that were symmetrical.</p>
Tile Exploration	<p>Students explored tiles on the overhead.</p>



## **Description of Public Lesson: Finding the Castle Keys**

### **Goal of public lesson:**

- a. Students will recognize rotations and flips of pentominoes and translations.  
Math Curriculum: describe, sort, classify, and compare two-dimensional shapes and three-dimensional figures, and describe the location and movement of objects through investigation;
- b. Students will verbalize similarities and differences between various 2D arrangements of 5 squares.  
Language: Use language to talk about their thinking, to reflect, and to solve problems  
Math: Demonstrate an understanding of basic spatial relationships and movements (e.g., use above/below, near/far, in/out).
- c. Children begin to develop and apply problem-solving strategies, and persevere when solving problems and conducting mathematical investigations.
- d. Apply developing reasoning skills (e.g., pattern recognition, classification) to make and investigate conjectures (e.g., through discussion with others).

### **Specific Learning Goal:**

By the end of this lesson, students will visualize and verbalize shapes as turns, flips, and movements.

### **Materials:**

- Smart board
- Centers
- Pentominoes
- 1 inch square tiles
- Pencils
- Scissors

### **Activation of student thinking –Whole group**

Begin lesson with whole group on carpet in front of SmartBoard. Introduction of narrative accompanied by images on SmartBoard.

Narrative: Picture of princess (Kate) and prince (William). Witch appears on screen and puts spell on prince and locks him in castle. (Teacher touches screen and prince disappears). Good fairy appears and says that the princess can help free the prince by meeting the following challenge: unlocking 12 of the castle doors by creating 12 special keys (which will be pentominoes).

Review the challenge with the students - for each of the 12 pentominoe shapes they create, they will receive a key (this is a plastic pentominoe form).

- Make a new design – does it follow the rules?
  - Squares must touch side to side.
- Have some 4 piece designs on side
- Is the new piece the same or different?
- How do you know? (Encourage vocabulary: if you turn/ flip it, is it the same? )

- Shapes are the same if they can be flipped turned or moved to be the same
- Can you find all 12 different keys?
- How will you know if two keys are different or the same?

### **Development of ideas- 2-3 pairs working in each center**

1. Center One: Smart board file
  - a. Students will take turns building Pentominoes on SmartBoard.
  - b. Possible teacher prompts:
    - i. Is this key the same as one that was already built?
    - ii. How can you prove that?
  - c. Unique keys will be moved across the board to a “storage” area.
  
2. Center Two, Three, and Four
  - a. Each student or pair will have 5 square tiles to build a “key”
  - b. When a student thinks they have a new key, they will compare it to the keys in the center of the table.
  - c. Possible teacher prompts:
    - i. Is this key the same as one that was already built?
    - ii. How can you prove that?
  - d. Teacher will give the matching pentomino key to the student who will place it in the circular ring at center of the table.
  - e. Unique keys will be traced on 1” chart paper and cut out and placed in the center of the table (the key design storage area)

### **Consolidation of Student Thinking: whole group**

Students bring Pentominoes to circle around smart board.

- Class counts unique keys on SmartBoard,
- Ask groups to compare their keys to those on the board – ask for names for the shapes
- If they have any different keys, ask questions to pull language and gesture, (flip turn, move)
- The teacher congratulates the students on finding the keys
- Extensions: Teacher asks class to sort Pentominoes (? 1 straight five , 2 with 4 straight, 8 with a 3 block longest section, 1 with a 2 block longest section- the w often the last one found) or asks the kids to try at home with their families

## Observation Questions

1. How do students demonstrate and communicate their mathematical understanding?
2. What role does visualization play in this lesson?
3. Do students use **spatial language** during the tasks to express their reasoning? (Examples: turn, flip, rotate, lines of symmetry)
4. How do students use **gestures and actions** to express their mathematical thinking?

