

Parent Packet

HAUPPAUGE MATH DEPARTMENT CCLS Grade 6 MODULE 1

<http://www.hauppauge.k12.ny.us/math>

Grade 6 Mathematics Module 1

Ratios and Unit Rates

Students begin their sixth grade year investigating the concepts of ratio and rate. They use multiple forms of ratio language and ratio notation, and formalize understanding of equivalent ratios. Students apply reasoning when solving collections of ratio problems in real world contexts using various tools (e.g., tape diagrams, double number line diagrams, tables, equations and graphs). Students bridge their understanding of ratios to the value of a ratio, and then to rate and unit rate, discovering that a percent of a quantity is a rate per 100. The 35 day module concludes with students expressing a fraction as a percent and finding a percent of a quantity in real world concepts, supporting their reasoning with familiar representations they used previously in the module.

Topic A

Representing and Reasoning About Ratios

In Topic A, students are introduced to the concepts of ratios (**6.RP.A.1**). In the first two lessons, students develop fluidity in using multiple forms of ratio language and ratio notation.

The complexity of problems increases as students are challenged to find values of quantities in a ratio given the total desired quantity or given the difference between the two quantities. *For example, “If the ratio of boys to girls in the school is 2:3, find the number of girls if there are 300 more girls than boys.”* As the first topic comes to a close, students develop a precise definition of the *value of a ratio $a:b$* , (**5.NF.B.3**). Students are then challenged to express their understanding of ratio equivalence using the newly defined term, value of a ratio. They conclude that equivalent ratios are ratios having the same value.

Topic B

Collections of Equivalent Ratios

With the concept of ratio equivalence formally defined, students explore collections of equivalent ratios in real-world contexts in Topic B. In Lessons 9 and 10, students build ratio tables and study and articulate their additive and multiplicative structure (**6.RP.A.3a**). In Lesson 11, students answer comparative questions about two distinct ratios using reasoning with ratio tables. Students continue to apply reasoning to solve ratio problems while they explore other representations of collections of equivalent ratios and relate those representations to their experience working with the ratio table (**6.RP.A.3**).

Building on their experience with number lines, students represent collections of equivalent ratios with a double number line model in Lesson 12. In Lesson 13, they relate ratio tables to equations using the value of a ratio defined in Topic A. (**5.G.A.1**, **5.G.A.2**) In the final Lesson of this topic, students begin to synthesize their experience of the various representations by working a variety of ratio problems and choosing the representation that best represents their thinking. They continue to apply their understanding of the representations as they apply them to rate and percent problems in Topics C and D.

Topic C

Unit Rates

In Topic C, students apply their understanding of ratios and the *value of a ratio* as they come to understand that a ratio relationship of 5 *miles to 2 hours* corresponds to a rate of 2.5 *miles per hour*, where the unit rate is the numerical part of the rate, 2.5, and miles per hour is the newly formed unit of measurement of the rate (**6.RP.A.2**). Throughout Topic C, students continue to make use of the representations and diagrams of Topics A and B. Students are asked to verbalize and depict the underlying ratio relationship as a collection of equivalent ratios.

In the remaining lessons of Topic C, students solve unit rate problems involving unit pricing, constant speed, and constant rates of work (**6.RP.A.3b**). They combine their new understanding of rate to connect and revisit concepts of converting among different-sized standard measurement units (**5.MD.A.1**). They then expand upon this background as they learn to manipulate and transform units when multiplying and dividing quantities (**6.RP.A.3d**). In Lessons 19–20, students are conscientious consumers, and comparison shop by comparing unit prices and converting measurement units as needed. For instance, when comparing a 10-ounce bag of salad that sells for \$2.25 to a 1-pound bag of salad that retails for \$3.50, students recognize that in addition to finding a unit price, they must convert pounds to ounces for an accurate comparison.

In Lessons 21–22, students conduct real-world simulations that generate rates related to speed and work. In doing so, students begin to view math as a tool for solving real-life problems. Topic C concludes with Lesson 23, in which students draw upon their experiences in previous modeling lessons to demonstrate their ability to problem-solve using rates, unit rates, and conversions.

Topic D

Percent

In the first lesson of Topic D, students are introduced to percent and find percent of a quantity as a rate per 100. Students understand that N percent of a quantity has the same value as $\frac{N}{100}$ of that quantity. In Lessons 24 and 25, students express a fraction as a percent and find a percent of a quantity in real-world contexts. In Lessons 26–28, students learn to express ratio using the language of percent and to solve percent problems selecting from familiar representations, such as tape diagrams and double number line diagrams or combinations of both (**6.RP.A.3c**).

Terminology

New or Recently Introduced Terms

- Ratio (A pair of nonnegative numbers, $A:B$, where both are not zero, and that are used to indicate that there is a relationship between two quantities such that when there are A units of one quantity, there are B units of the second quantity.)
- Rate (A rate indicates, for a proportional relationship between two quantities, how many units of one quantity there are for every 1 unit of the second quantity. For a ratio of $A: B$ between two quantities, the rate is A/B units of the first quantity per unit of the second quantity.)
- Unit Rate (The numeric value of the rate, e.g., in the rate 2.5 mph, the unit rate is 2.5.)
- Value of a Ratio (For the ratio $A: B$, the value of the ratio is the quotient A/B .)
- Equivalent Ratios (Ratios that have the same value.)
- Percent (Percent of a quantity is a rate per 100.)
- Associated Ratios (e.g., if a popular shade of purple is made by mixing 2 cups of blue paint for every 3 cups of red paint, not only can we say that the ratio of blue paint to red paint in the mixture is 2: 3, but we can discuss associated ratios such as the ratio of cups of red paint to cups of blue paint, the ratio of cups of blue paint to total cups of purple paint, the ratio of cups of red paint to total cups of purple paint, etc.)
- Double Number Line (See example under Suggested Tools and Representations.)
- Ratio Table (A table listing pairs of numbers that form equivalent ratios; see example below.)

Familiar Terms and Symbols⁴

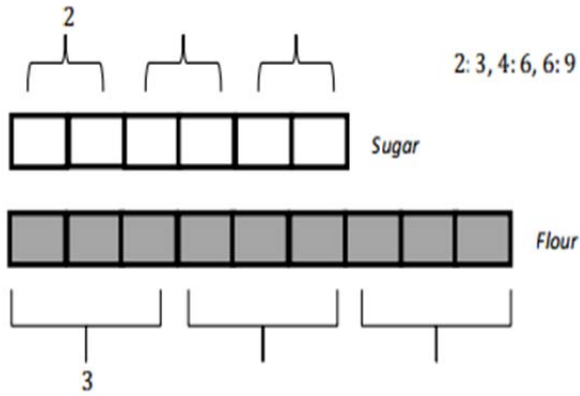
- Convert
- Tape Diagram
- Coordinate Plane
- Equation

Suggested Tools and Representations

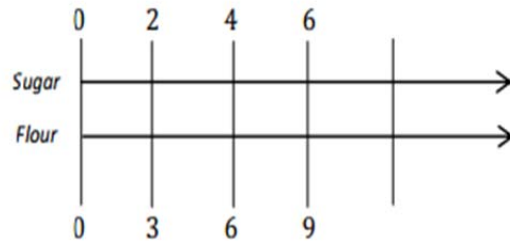
- Tape Diagrams (See example below.)
- Double Number Line Diagrams (See example below.)
- Ratio Tables (See example below.)
- Coordinate Plane (See example below.)

Representing Equivalent Ratios for a cake recipe
that uses 2 cups of sugar for every 3 cups of flour

Tape Diagram



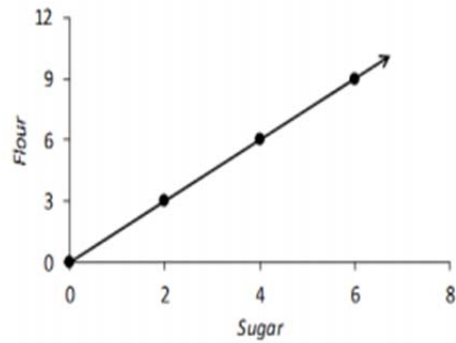
Double Number Line



Ratio Table

Sugar	Flour
2	3
4	6
6	9

Coordinate Plane



Lesson 5

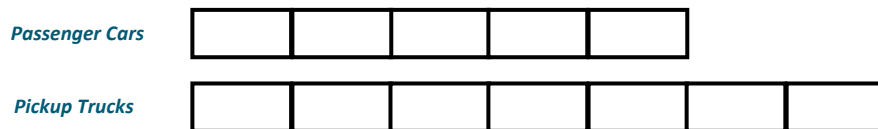
Objective: Solving Problems by Finding Equivalent Ratios

In the month of August, a total of **192** registrations were purchased for passenger cars and pickup trucks at the local Department of Motor Vehicles (DMV). The DMV reported that in the month of August, for every **5** passenger cars registered, there were **7** pickup trucks registered. How many of each type of vehicle were registered in the county in the month of August?

Using the information in the problem, write four different ratios and describe the meaning of each.

The ratio of cars to trucks is **5: 7** and is a part-to-part ratio. The ratio of trucks to cars is **7: 5** and that is a part-to-part ratio. The ratio of cars to total vehicles is **5** to **12** and that is a part-to-whole ratio. The ratio of trucks to total vehicles is **7** to **12** and that is a part-to-whole ratio.

Make a tape diagram that represents the quantities in the part-to-part ratios that you wrote.



1. How many equal-sized parts does the tape diagram consist of?

12

2. What total quantity does the tape diagram represent?

192 vehicles

3. What value does each individual part of the tape diagram represent?

Divide the total quantity into **12** equal-sized parts:

$$\frac{192}{12} = 16$$

4. How many of each type of vehicle were registered in August?

$$5 \cdot 16 = 80 \text{ passenger cars}$$

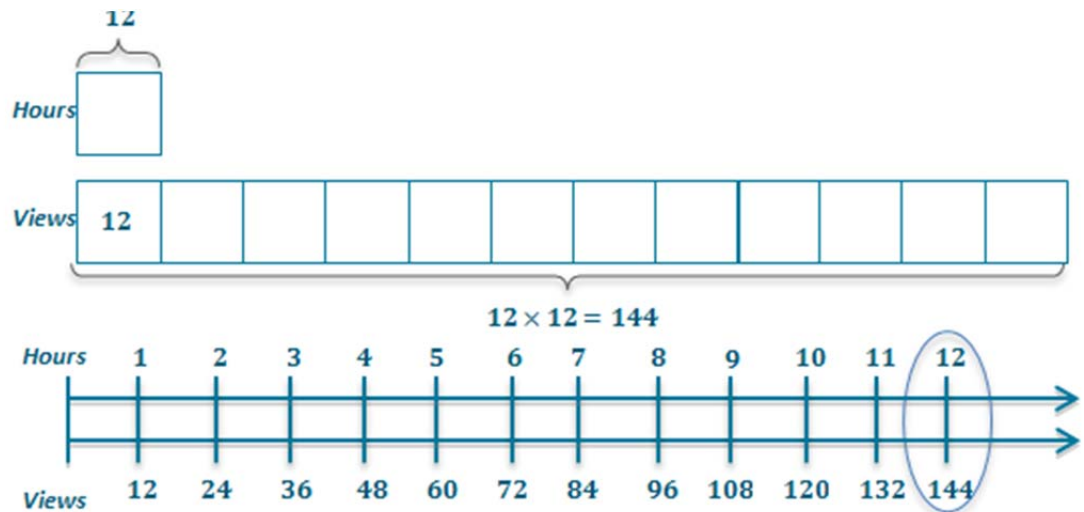
$$7 \cdot 16 = 112 \text{ pickup trucks}$$

Lesson 15

Objective: A Synthesis of Representations of Equivalent Ratio Collections

Predict how many views the website will have after twelve hours. Use at least two representations (e.g., tape diagram, table, double number line diagram) to justify your answer.

Hours	Views
1	12
2	24
3	36
4	48
5	60
6	72
7	84
8	96
9	108
10	120
11	132
12	144



Lesson 29

Objective: Solving Percent Problems

Use at least one model to solve each problem (e.g., tape diagram, table, double number line diagram, 10×10 grid).

Make a prediction. Do you think the claim is true or false?

Explain why.

Answers will vary. One could think the claim is true because 10% as a fraction is $\frac{1}{10}$. The same thing happens when one divides by 10 or multiplies by $\frac{1}{10}$. A student may think the claim is false because it depends on what whole amount represents the number from which the percentage is taken.

a. Determine 10% of 300. 30

$$300 \times \frac{1}{10} = \frac{300}{10} = 30$$

b. Find 10% of 80. 8

$$80 \times \frac{1}{10} = \frac{80}{10} = 8$$

c. Determine 10% of 64. 6.4

$$64 \times \frac{1}{10} = 6.4$$

d. Find 10% of 5. $\frac{1}{2}$

$$5 \times \frac{1}{10} = \frac{5}{10} = \frac{1}{2}$$

e. 10% of 480 is 48.

48	48	48	48	48	48	48	48	48	48
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$$48 \times 10 = 480$$

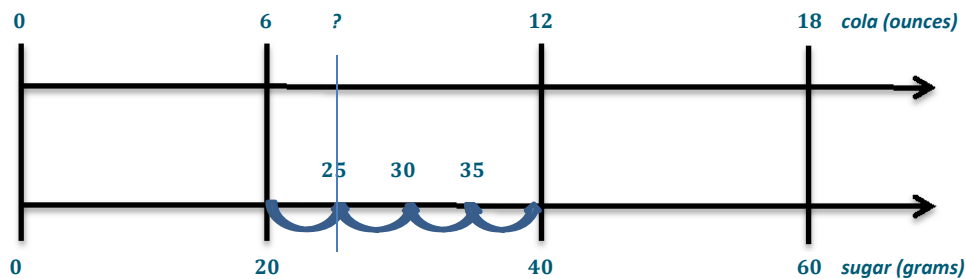
f. 10% of 60 is 6.

$$6 \times 10 = 60$$

Lesson 12

Objective: From Ratio Tables to Double Number Line Diagrams

Ex. 1) A school cafeteria has a restriction on the amount of sugary drinks available to students. Drinks may not have more than 25 grams of sugar. Based on this restriction, what is the largest size cola (in ounces) the cafeteria can offer to students?



My estimate is between 6 and 12 ounces but closer to 6 ounces. I need to find $\frac{1}{4}$ of 6 and add it to 6.

$$\frac{1}{4} \times \frac{6}{1} = \frac{6}{4} = 1\frac{1}{2}$$

$$6 + 1\frac{1}{2} = 7\frac{1}{2}$$

A $7\frac{1}{2}$ ounce cola is the largest size that the school cafeteria can offer to students.

Ex. 2) Shontelle has soccer practice on Thursday evening. She has a half hour before practice to work on her math homework and to talk to her friends. She has **20** math skill-work questions for homework, and she wants to complete them before talking with her friends. How many minutes will Shontelle have left after completing her math homework to talk to her friends?

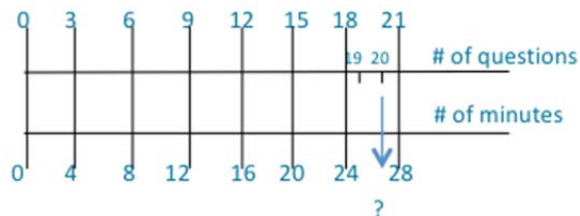
Use a double number line diagram to support your answer, and show all work

Step 1: $\frac{2}{3} \times 4 = \frac{8}{3} = 2\frac{2}{3}$

Step 2: $24 + 2\frac{2}{3} = 26\frac{2}{3}$

Step 3: $30 - 26\frac{2}{3} = 3\frac{1}{3}$

Shontelle can talk to her friends for $3\frac{1}{3}$ minutes.



Technology Resources

www.k-5mathteachingresources.com -This site provides an extensive collection of free resources, math games, and hands-on math activities aligned with the Common Core State Standards for Mathematics.

www.parccgames.com – fun games to help kids master the common core standards.

<http://www.mathplayground.com> –common core educational math games and videos.

www.learnzillion.com – math video tutorials.

www.ixl.com – practice common core interactive math skills practice.

www.mathnook.com –common core interactive math skill practice/ games, worksheets and tutorials.

www.adaptedmind.com – common core interactive practice, video lessons and worksheets

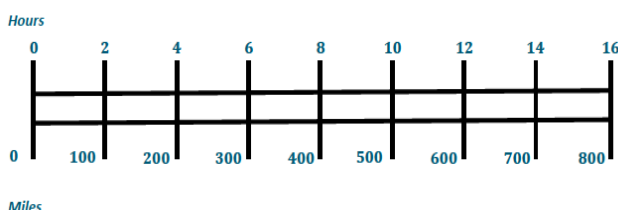
www.brainpop.com – animated tutorials of curriculum content that engages students. Can use a limited free version or buy a subscription.

Ratios and Unit Rates

In this 29-lesson module, students are introduced to ratios and rates and use ratio language and notation to solve real-world problems. Students will explore tape diagrams, double number line diagrams, ratio tables, equations and the coordinate plane to develop deep understanding, enabling them to become proficient and confident mathematicians in middle school.

Two Models Seen in the Module:

The Double Number Line Diagram



The Ratio Table

Number of Cups of Sugar	Number of Cups of Flour
2	3
4	6
6	9

Key Words

Ratio: A pair of nonnegative numbers, $A:B$, where both are not zero, and describes a relationship between the quantities.

Rate: Indicates how many units of one quantity there are for every 1 unit of the second quantity.

Unit Rate: The numeric value of the rate, e.g. in the rate 2.5 mph, the unit rate is 2.5.

Rate unit: The unit of measure of the rate. (e.g. in the rate 2.5 mph, the rate unit is miles per hour)

Equivalent Ratios: Ratios that have the same value.

Percent: Percent of a quantity is a rate per 100.

Associated Ratios: Ratios that are related, e.g. if the ratio of the number of boys to the number of girls is 1:2, we can also determine the ratio of the number of girls to the total number of children is 2:3. We can further determine the ratio of the number of girls to the number of boys is 2:1.

Double Number Line Diagrams: A math model that is often used when quantities have different units and is a visual representation for showing multiple equivalent ratios.

Ratio Table: A table listing pairs of numbers that represent equivalent ratios.

Tape Diagrams: Drawings that look like a segment of tape, used to illustrate number relationships. Also known as strip diagrams, bar model, fraction strip, or length model.

+ How Can You Help At Home?

- Ask your child what they learned in school today and ask them to show you an example.
- Reinforce fast recall of multiplication and division facts by playing math games using flashcards. See how many facts your child can answer in 20 seconds. Then, see if they can answer more questions the next time by playing again!

What Came Before this Module:

In Grade 5, students studied multiplicative comparisons, which is the foundation for their understanding of comparing quantities in a ratio relationship. In addition, students worked to develop fluency in operations with whole numbers, fractions and decimals.

What Comes After this Module:

Students will divide fractions using various strategies and arithmetic operations.

Key Common Core Standards:

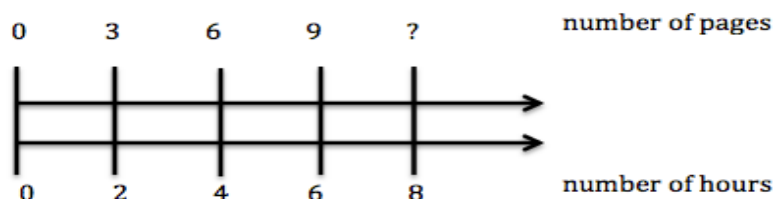
- **Understand ratio concepts and use ratio reasoning to solve problems.**
 - Use ratio language to describe the relationship between two quantities.
 - Use rate language in the context of a ratio relationship.
 - Solve real-world problems using ratio and rate reasoning.

Model in the Spotlight!

Double Number Line

Suppose you want to determine how many pages of homework Megan will complete in 8 hours if she can complete 3 pages in 2 hours, assuming she completes the homework at a constant rate.

You can represent the situation using a double number line diagram to visibly show the ratio relationship of 3 pages for every 2 hours. If you extend the double number line diagram, you can see that in 8 hours, Megan will complete 12 pages of homework.



Welcome to A
Story of Ratios!

There are several models used in A *Story of Ratios* that will foster deep knowledge of important concepts in middle school mathematics.

Read on to learn a little bit about *Eureka Math*, the creators of *A Story of Ratios*:

Eureka Math is a complete, PreK–12 curriculum and professional development platform. It follows the focus and coherence of the Common Core State Standards (CCSS) and carefully sequences the progression of mathematical ideas into expertly crafted instructional modules.

This curriculum is distinguished not only by its adherence to the CCSS; it is also based on a theory of teaching math that is proven to work. That theory posits that mathematical knowledge is conveyed most effectively when it is taught in a sequence that follows the “story” of mathematics itself. This is why we call the middle school portion of *Eureka Math* “A Story of Ratios.” The sequencing has been joined with methods of instruction that have been proven to work, in this nation and abroad. These methods drive student understanding beyond process, to deep mastery of mathematical concepts.

The goal of *Eureka Math* is to produce students who are not merely literate, but fluent, in mathematics. Your student has an exciting year of discovering the story of mathematics ahead!

Sample Problem from Module 1: (Example taken from Module 1, Lesson 8)

Your middle school has 900 students. $\frac{1}{3}$ of the students bring their lunch instead of buying lunch at school. What is the value of the ratio of the number of students who do bring their lunch to the number of students who do not?



First I created a tape diagram. In the tape diagram $\frac{1}{3}$ of the students bring their lunch instead of buying lunch at school. I determined that 300 students bring their lunch, leaving 600 students who buy their lunch. One unit of the tape diagram represents 300, and 2 units of the tape diagram represent 600. This creates a ratio of 1:2. As such, the value of the ratio of the number of students who bring their lunch to the number of students who buy their lunch is $\frac{1}{2}$.