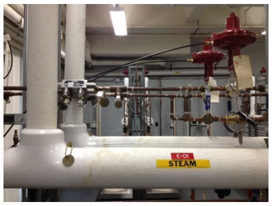
**CM3110**
**Transport I**
**Part II: Heat Transfer**

Michigan Tech



# *Applied Heat Transfer:*

## *Heat Exchanger Modeling, Sizing, and Design*

**Professor Faith Morrison**

Department of Chemical Engineering
Michigan Technological University

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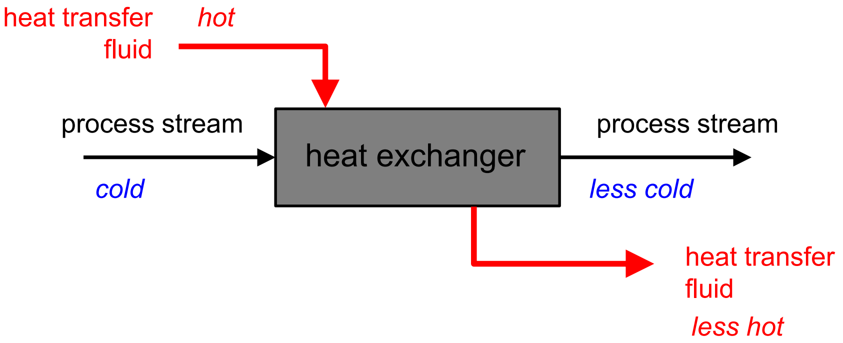
## Applied Heat Transfer

Before turning to radiation (last topic) we will discuss a few practical applications

How can we use Fundamental Heat Transfer to understand real devices like heat exchangers?



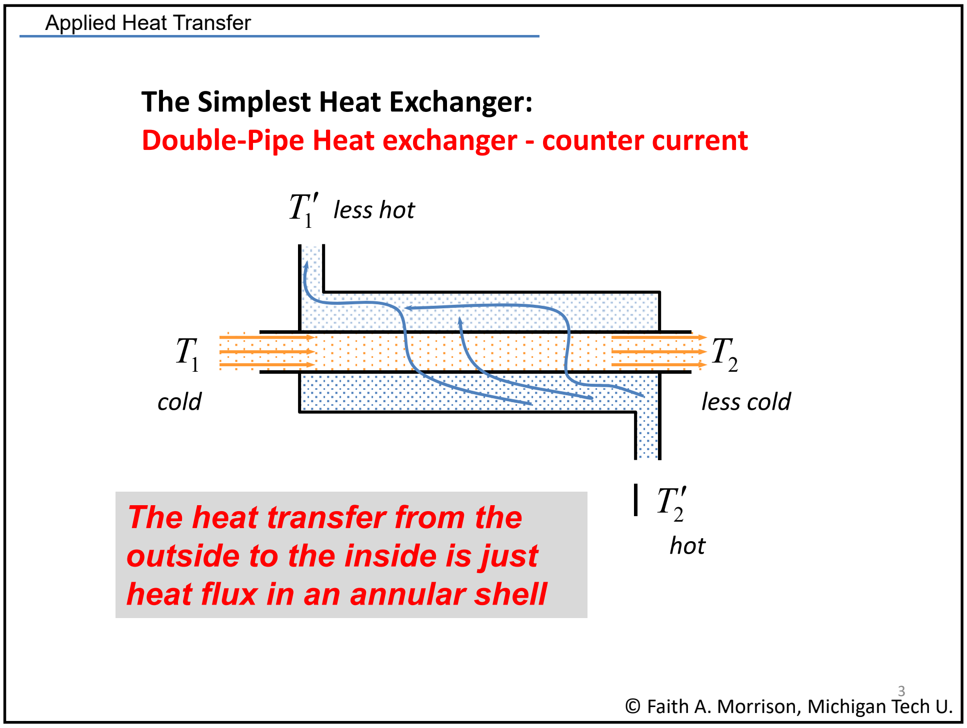
heat transfer fluid *hot*

process stream *cold*

heat exchanger

process stream *less cold*

heat transfer fluid *less hot*

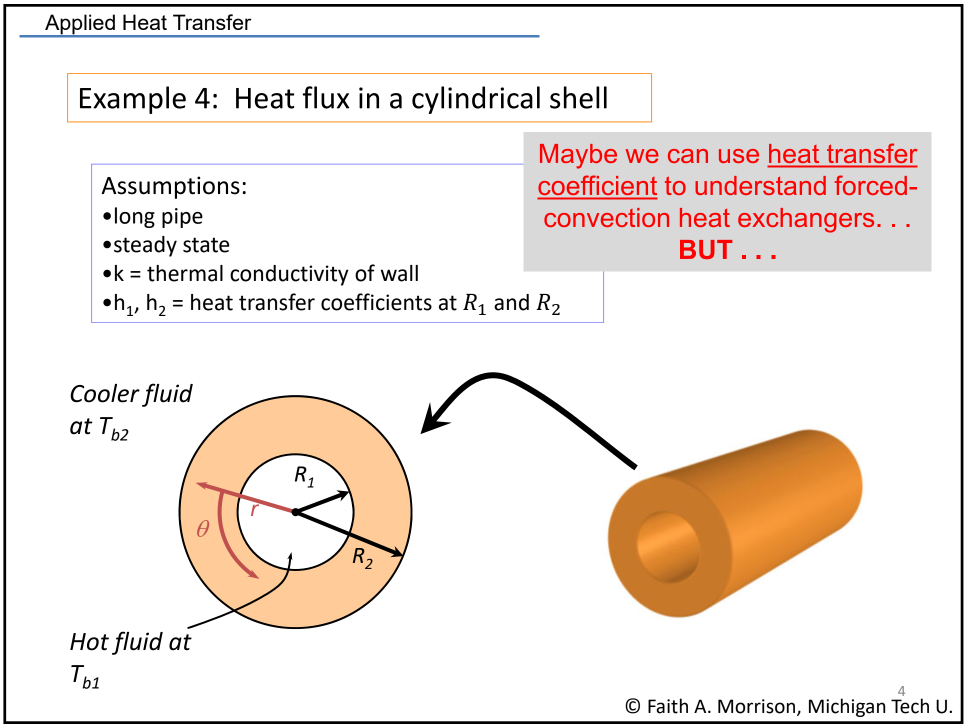


Applied Heat Transfer

**The Simplest Heat Exchanger:**

**Double-Pipe Heat exchanger - counter current**

***The heat transfer from the outside to the inside is just heat flux in an annular shell***

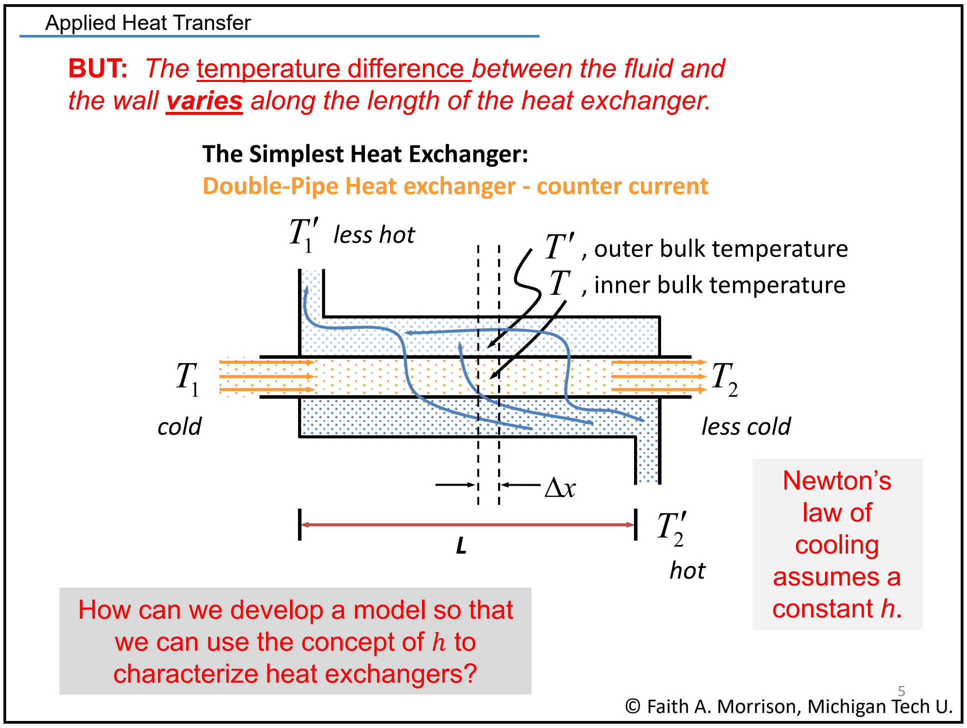


Applied Heat Transfer

Example 4: Heat flux in a cylindrical shell

Assumptions:
- long pipe
- steady state
- k = thermal conductivity of wall
- $h_1$, $h_2$ = heat transfer coefficients at $R_1$ and $R_2$

Maybe we can use heat transfer coefficient to understand forced-convection heat exchangers. . .
**BUT . . .**



Applied Heat Transfer

**BUT:** *The temperature difference between the fluid and the wall **varies** along the length of the heat exchanger.*

**The Simplest Heat Exchanger:**

**Double-Pipe Heat exchanger - counter current**

Newton's law of cooling assumes a constant *h*.

How can we develop a model so that we can use the concept of $h$ to characterize heat exchangers?

5

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Applied Heat Transfer

Let's look at the solution for radial conduction in an annulus

Example 4: Heat flux in a solid cylindrical shell

Solution:

$$\frac{q_r}{A} = \frac{c_1}{r}$$

**Flux is not constant**

$$T = -\frac{c_1}{k}\ln r + c_2$$

Boundary conditions?

6

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Applied Heat Transfer

Example 4: Heat flux in a cylindrical shell, Newton's law of cooling boundary Conditions

## Results: Radial Heat Flux in a Solid Cylindrical Shell

$$T - T_{b2} = \frac{(T_{b1} - T_{b2})\left(\frac{1}{k}\ln\left(\frac{R_2}{r}\right) + \frac{1}{h_2 R_2}\right)}{\frac{1}{h_2 R_2} + \frac{1}{k}\ln\left(\frac{R_2}{R_1}\right) + \frac{1}{h_1 R_1}}$$

$$\frac{q_r}{A} = \frac{(T_{b1} - T_{b2})}{\frac{1}{h_2 R_2} + \frac{1}{k}\ln\left(\frac{R_2}{R_1}\right) + \frac{1}{h_1 R_1}}\left(\frac{1}{r}\right)$$

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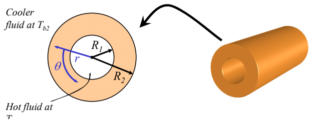
Applied Heat Transfer



**Example 4: Heat flux in a solid cylindrical shell**

**Solution for Heat Flux:**

$$\frac{q_r}{A} = \frac{(T_{b1} - T_{b2})}{\frac{1}{h_2 R_2} + \frac{1}{k}\ln\left(\frac{R_2}{R_1}\right) + \frac{1}{h_1 R_1}}\left(\frac{1}{r}\right)$$

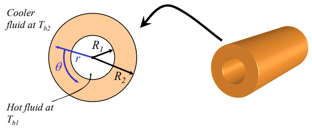
**Calculate Total Heat flow through any chosen $r$:**

**(including $r = R_1$ and $r = R_2$)**

$$Q = \frac{q_r}{A}(2\pi r L) = \frac{(T_{b1} - T_{b2})(2\pi L)}{\frac{1}{h_2 R_2} + \frac{1}{k}\ln\left(\frac{R_2}{R_1}\right) + \frac{1}{h_1 R_1}}$$

**Note that** total heat flow is proportional to bulk $\Delta T$ and (almost) area of heat transfer

Applied Heat Transfer



**Total Heat flow through any chosen $r$:**

**(including $r = R_1$ and $r = R_2$)**

$$\boxed{Q = \frac{q_r}{A}(2\pi r L) = \frac{(T_{b1} - T_{b2})(2\pi L)}{\frac{1}{h_2 R_2} + \frac{1}{k}\ln\left(\frac{R_2}{R_1}\right) + \frac{1}{h_1 R_1}}} \quad \boldsymbol{R_1}\left(\frac{\boldsymbol{1}}{\boldsymbol{R_1}}\right)$$

**Note that** total heat flow is proportional to bulk $\Delta T$ and (almost) area of heat transfer

9

Applied Heat Transfer—Define **Overall** Heat-Transfer Coefficient, U

Overall Heat Transfer Coefficient, U

$$\boxed{Q = UA\Delta T}$$
$$= UA(T_{b1} - T_{b2})$$

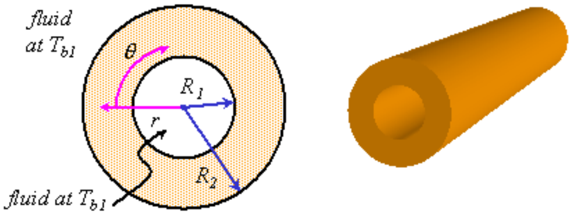
this equation serves as the definition of U

$A$ = *area of heat transfer (not always unambiguous)*

$\Delta T$ = *driving temperature difference*

Example: in a pipe

Do we use inner or outer area?



10

**Overall** heat transfer coefficients in pipe

**Area must be specified when $U$ is reported**

$$Q = U_1 A_1 \Delta T$$

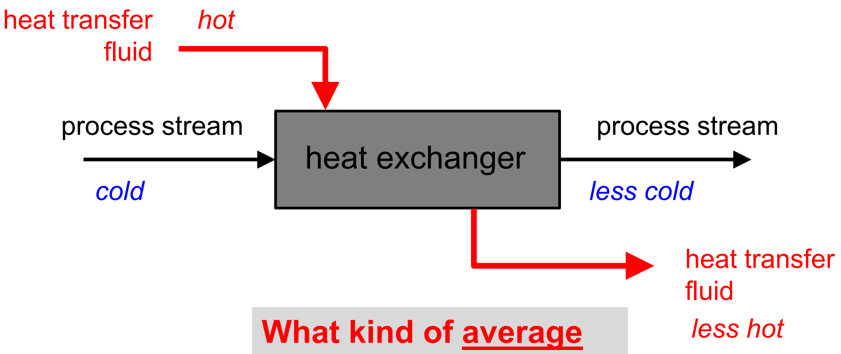
$$= \left( \frac{\frac{1}{R_1}}{\frac{1}{h_2 R_2} + \frac{1}{k} \ln \frac{R_2}{R_1} + \frac{1}{h_1 R_1}} \right) (2\pi R_1 L)(T_{b1} - T_{b2})$$

$$Q = U_2 A_2 \Delta T$$

$$= \left( \frac{\frac{1}{R_2}}{\frac{1}{h_2 R_2} + \frac{1}{k} \ln \frac{R_2}{R_1} + \frac{1}{h_1 R_1}} \right) (2\pi R_2 L)(T_{b1} - T_{b2})$$

**Heat flux in a cylindrical shell:** $Q = UA(T_{b1} - T_{b2})$

**But,** in an actual heat exchanger, $T_{b1}$ and $T_{b2}$ vary along the length of the heat exchanger



heat transfer fluid *hot*

process stream *cold*

heat exchanger

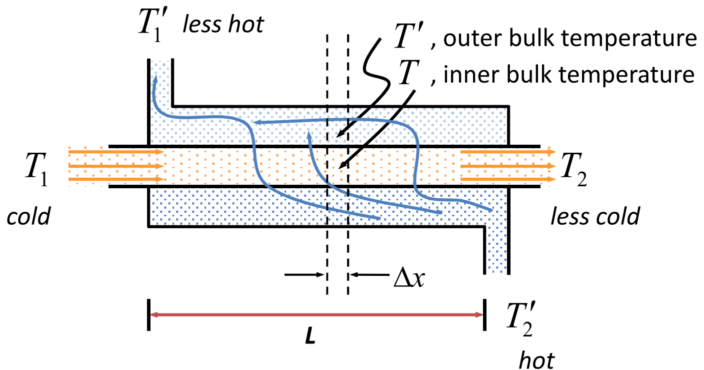
process stream *less cold*

heat transfer fluid *less hot*

**What kind of average $\Delta T$ do we use?**

**The Simplest Heat Exchanger:**

**Double-Pipe Heat exchanger - counter current**



***We will do an open-system energy balance on a <u>differential section</u> to determine the correct average temperature difference to use.***

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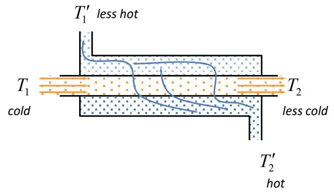
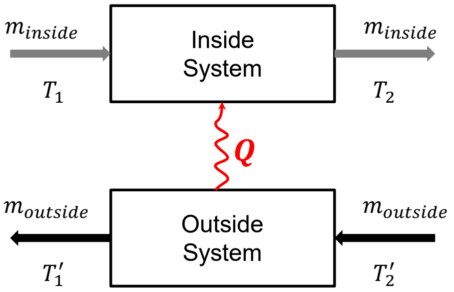
**The Simplest Heat Exchanger:**

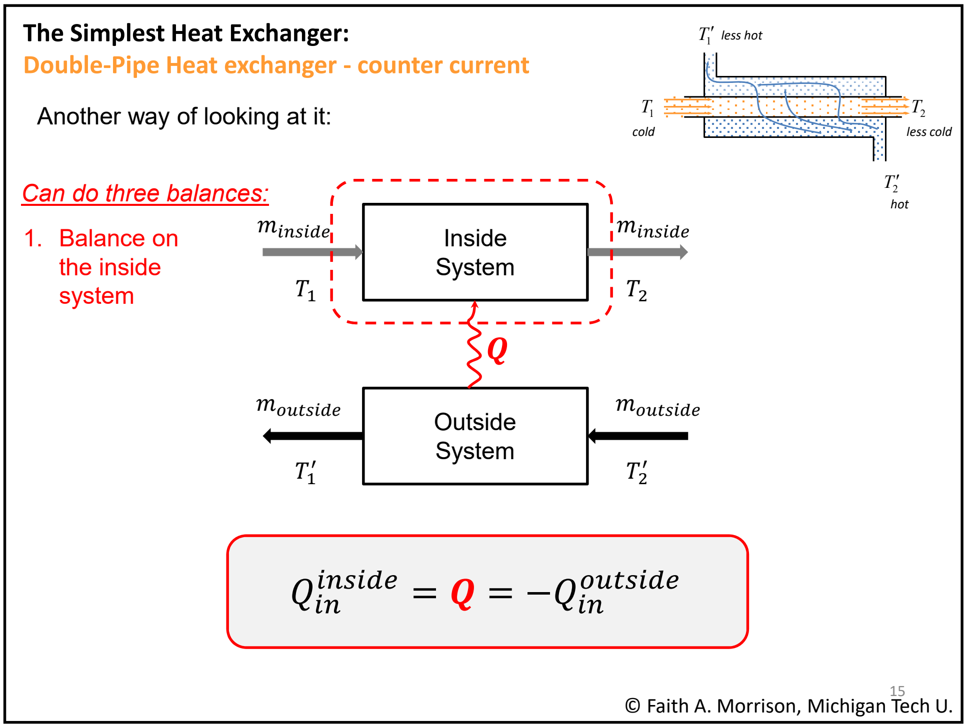
**Double-Pipe Heat exchanger - counter current**



Another way of looking at it:



$$Q_{in}^{inside} = \boldsymbol{Q} = -Q_{in}^{outside}$$



**The Simplest Heat Exchanger:**

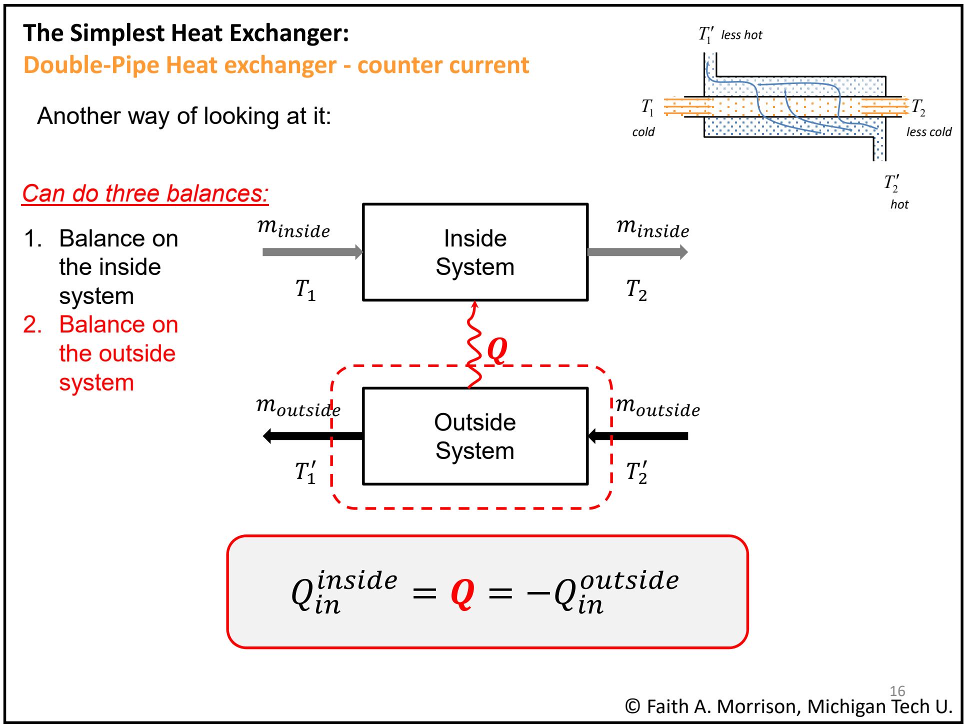
**Double-Pipe Heat exchanger - counter current**

Another way of looking at it:

*Can do three balances:*

1. Balance on the inside system

$$Q_{in}^{inside} = \boldsymbol{Q} = -Q_{in}^{outside}$$



**The Simplest Heat Exchanger:**

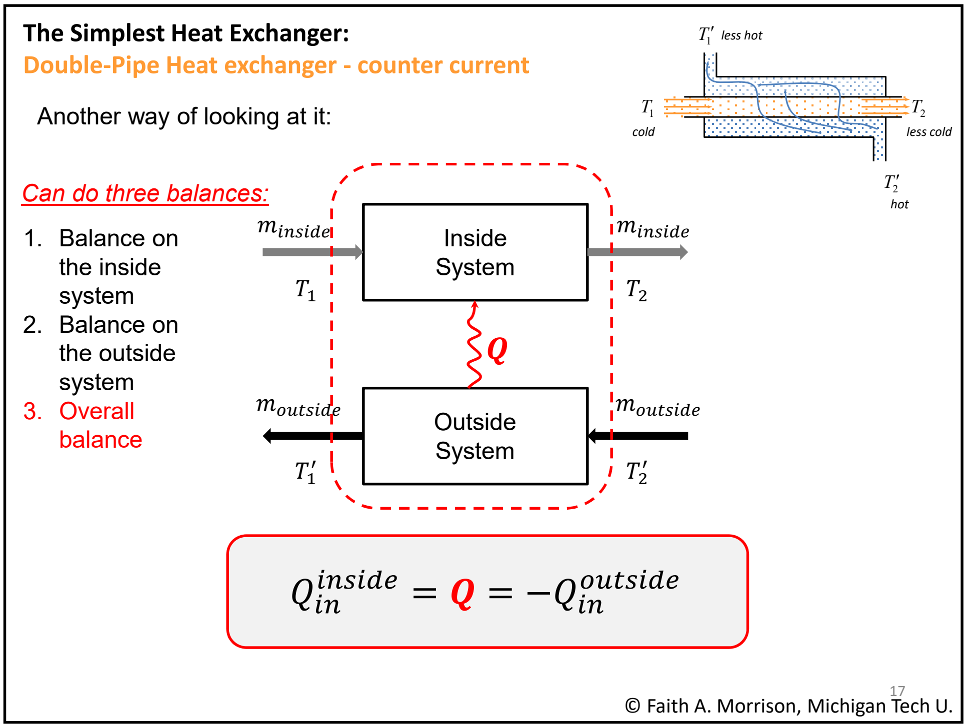
**Double-Pipe Heat exchanger - counter current**

Another way of looking at it:

*Can do three balances:*

1. Balance on the inside system
2. Balance on the outside system

$$Q_{in}^{inside} = \boldsymbol{Q} = -Q_{in}^{outside}$$



**The Simplest Heat Exchanger:**

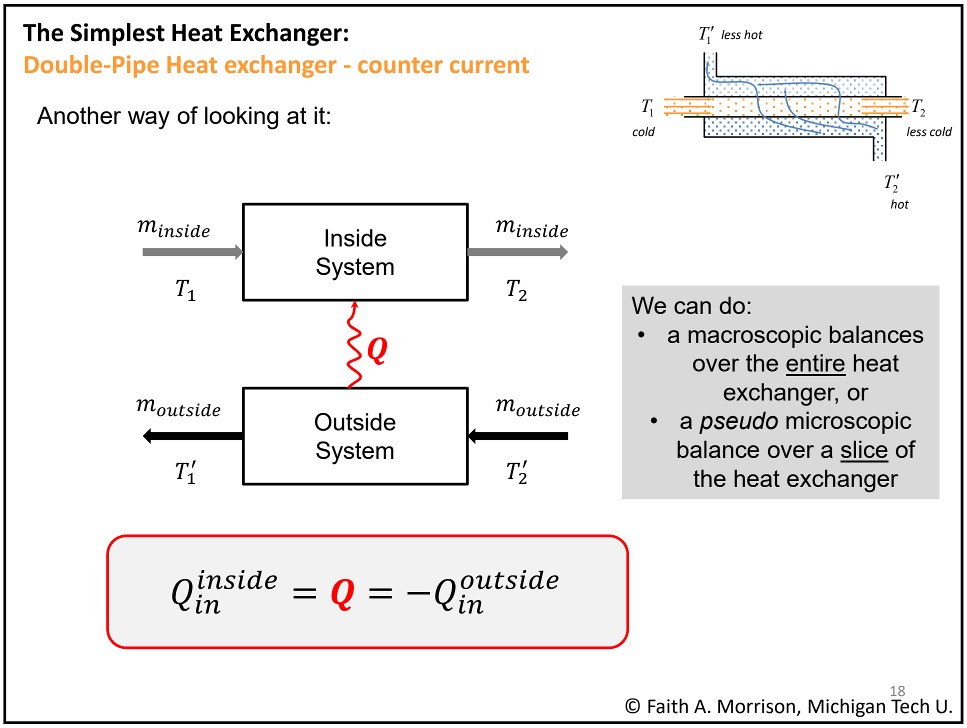
**Double-Pipe Heat exchanger - counter current**

Another way of looking at it:

*Can do three balances:*

1. Balance on the inside system
2. Balance on the outside system
3. Overall balance

$$Q_{in}^{inside} = \boldsymbol{Q} = -Q_{in}^{outside}$$



**The Simplest Heat Exchanger:**

**Double-Pipe Heat exchanger - counter current**

Another way of looking at it:

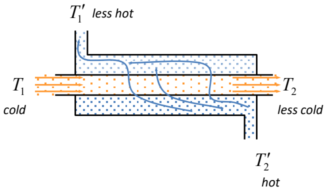
We can do:

- a macroscopic balances over the entire heat exchanger, or
- a *pseudo* microscopic balance over a slice of the heat exchanger

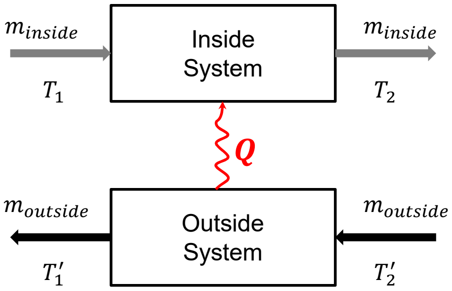
$$Q_{in}^{inside} = \boldsymbol{Q} = -Q_{in}^{outside}$$

**The Simplest Heat Exchanger:**

**Double-Pipe Heat exchanger - counter current**



Another way of looking at it:



We can do:

- a macroscopic balances over the entire heat exchanger, or
- a *pseudo* microscopic balance over a slice of the heat exchanger

All the details of the algebra are here:
www.chem.mtu.edu/~fmorriso/cm310/double_pipe.pdf

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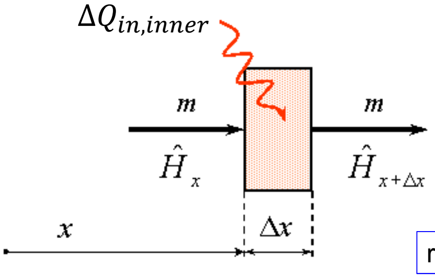
Pseudo Microscopic Energy Balance on a slice of the heat exchanger

Open system energy balance on a differential volume:

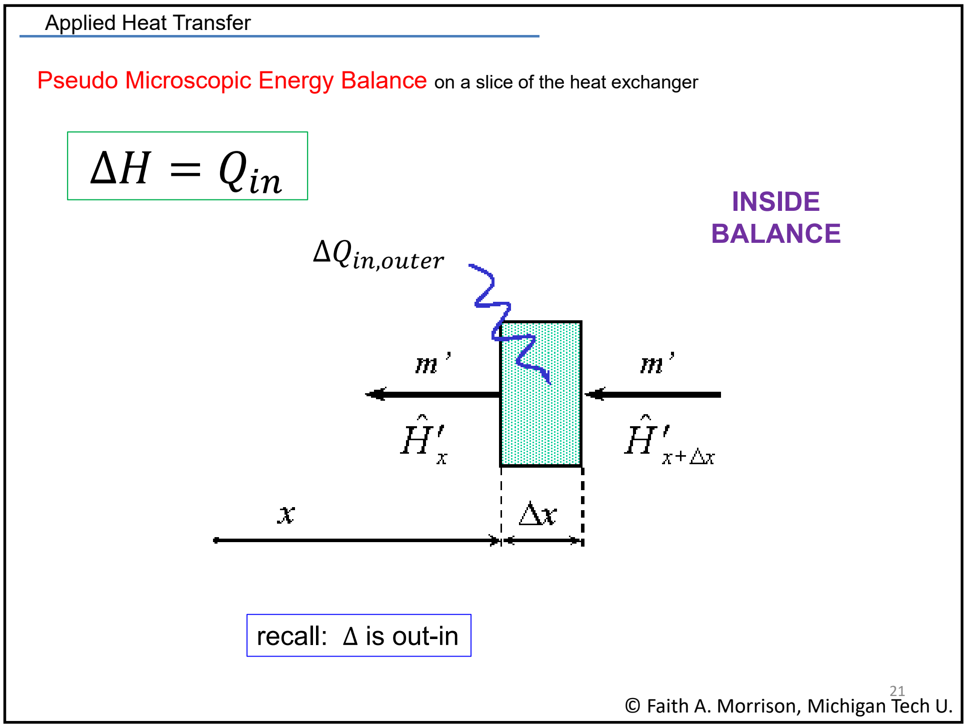
$$\cancel{\Delta E_p} + \cancel{\Delta E_k} + \Delta H = Q_{in} + \cancel{W_{s,on}}$$

$$\boxed{\Delta H = Q_{in}}$$



**INSIDE BALANCE**

recall: Δ is out-in

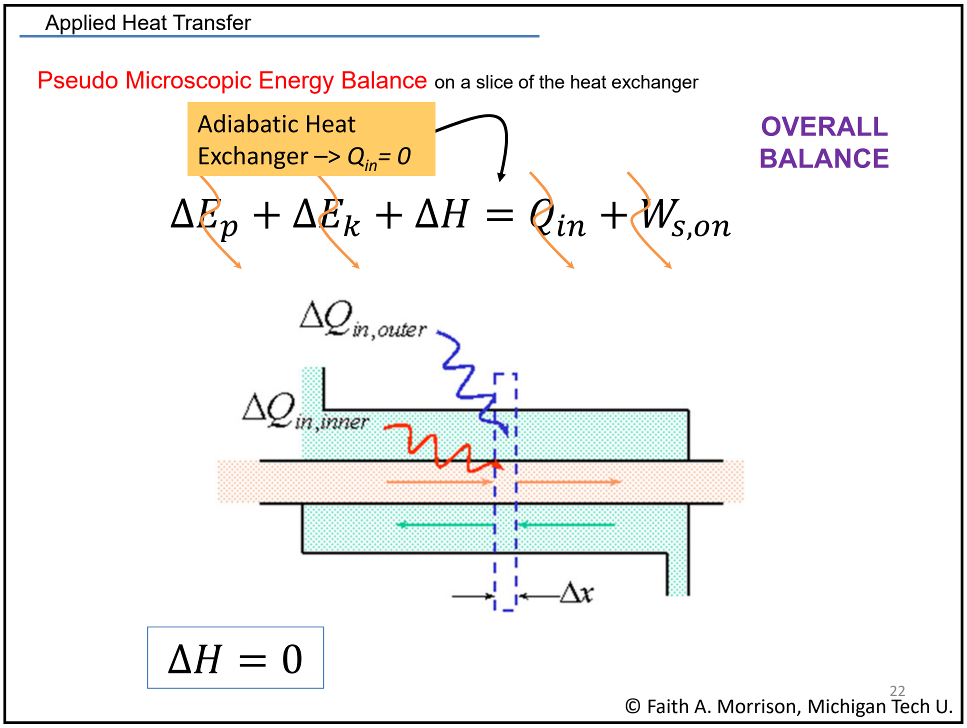


## Pseudo Microscopic Energy Balance on a slice of the heat exchanger

$$\boxed{\Delta H = Q_{in}}$$

**INSIDE BALANCE**

recall: Δ is out-in



## Pseudo Microscopic Energy Balance on a slice of the heat exchanger

Adiabatic Heat Exchanger –> $Q_{in} = 0$

**OVERALL BALANCE**

$$\cancel{\Delta E_p} + \cancel{\Delta E_k} + \Delta H = \cancel{Q_{in}} + \cancel{W_{s,on}}$$

$$\boxed{\Delta H = 0}$$

energy balance on overall differential system

$$\boxed{\Delta H = 0}$$

$$= \Delta H_{\substack{inner\\system}} + \Delta H_{\substack{outer\\system}}$$

$$= \underbrace{\Delta Q_{in,inner}}_{\text{heat into inner differential system}} + \underbrace{\Delta Q_{in,outer}}_{\text{heat into outer differential system}} = 0$$

divide by Δx and take the limit as Δx goes to zero:

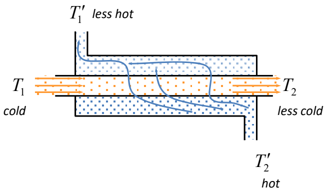
$$\left(\frac{dQ_{in,inner}}{dx}\right) = -\left(\frac{dQ_{in,outer}}{dx}\right)$$

$$\equiv \frac{dQ_{in}}{dx}$$

This expression characterizes the rate of change of heat transferred with respect to distance down the heat exchanger

## The Simplest Heat Exchanger:

## Double-Pipe Heat exchanger - counter current



Result of inside balance:

$$\frac{dQ_{inner}}{dx} = m\hat{C}_p\left(\frac{dT}{dx}\right)$$

Result of outside balance:

$$-\frac{dQ_{outer}}{dx} = m'\hat{C}_p'\left(\frac{dT'}{dx}\right)$$

Result of overall balance:

$$-\frac{dQ_{outer}}{dx} = \frac{dQ_{inner}}{dx} \equiv \frac{dQ_{in}}{dx}$$

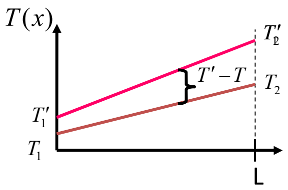
Solve for temperature derivatives, and subtract:

$$\underbrace{\frac{dQ_{in}}{dx}}_{\text{This depends on } T'-T}\left(\frac{1}{m'\hat{C}_p'} - \frac{1}{m\hat{C}_p}\right) = \left(\frac{dT'}{dx} - \frac{dT}{dx}\right)$$

$$= \frac{d(T'-T)}{dx}$$

All the details of the algebra are here:
www.chem.mtu.edu/~fmorriso/cm310/double_pipe.pdf

## Analysis of double-pipe heat exchanger



**Question:** How can we write $\overbrace{\frac{dQ_{in}}{dx}}^{\text{Rate of change of heat transferred with respect to distance down the heat exchanger}}$ in terms of $\overbrace{T'-T}^{\text{Driving force for heat transfer}}$ ?

**Answer:** Define an "overall" heat transfer coefficient, U

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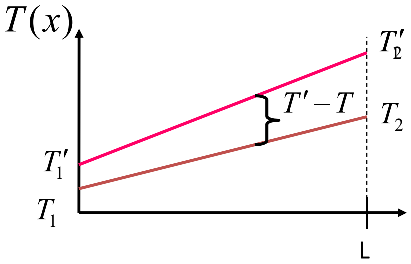
## Analysis of double-pipe heat exchanger

$$\underbrace{\frac{d(T'-T)}{dx}}_{\text{Want to integrate to solve for } T'-T,} = \frac{dQ_{in}}{dx}\left(\frac{1}{m'\hat{C}_p'} - \frac{1}{m\hat{C}_p}\right)$$

but this is a function of $T' - T$

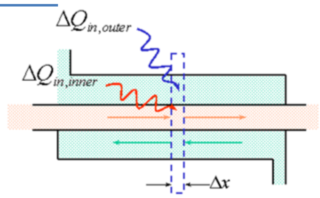


For the **differential slice of the heat exchanger** that we are considering (modeling our ideas on Newton's law of cooling),

$$\frac{dQ_{in}}{dA} = (?)(T' - T)$$

## Analysis of double-pipe heat exchanger

For the **differential slice of the heat exchanger** that we are considering (modeling our ideas on Newton's law of cooling),



$$\frac{dQ_{in}}{dA} = (?)(T' - T)$$

$$dQ_{in} = (U)dA(T' - T)$$
$$= U(2\pi R dx)(T' - T)$$

$$\frac{dQ_{in}}{dx} = U(2\pi R)(T' - T)$$

This is the missing piece that we needed.

We can write $\frac{dQ_{in}}{dx}$ in terms of $T' - T$ if we define an "overall" heat transfer coefficient, $U$

## Analysis of double-pipe heat exchanger

$$\frac{d(T' - T)}{dx} = \frac{dQ_{in}}{dx}\left(\frac{1}{m'\hat{C}_p'} - \frac{1}{m\hat{C}_p}\right)$$

$$\frac{dQ_{in}}{dx} = 2\pi R U(T' - T)$$

$$\frac{d(T' - T)}{dx} = 2\pi R U(T' - T)\left(\frac{1}{\hat{C}_p' m'} - \frac{1}{\hat{C}_p m}\right)$$

$$\frac{d(T' - T)}{(T' - T)} = \left[2\pi R U\left(\frac{1}{\hat{C}_p' m'} - \frac{1}{\hat{C}_p m}\right)\right] dx$$

## Analysis of double-pipe heat exchanger

$$\frac{d(T'-T)}{(T'-T)} = \left[2\pi R U\left(\frac{1}{\hat{C}'_p m'} - \frac{1}{\hat{C}_p m}\right)\right] dx$$

$$\Phi \equiv T' - T$$

$$\alpha_0 \equiv 2\pi R U\left(\frac{1}{\hat{C}'_p m'} - \frac{1}{\hat{C}_p m}\right)$$

(we'll *assume* *U* is constant)

$$\frac{d\Phi}{\Phi} = \alpha_0 dx$$

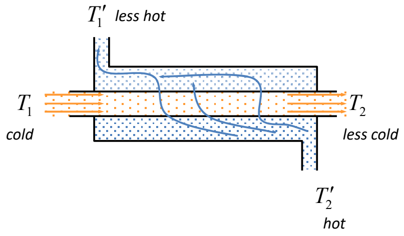
$$\int \frac{d\Phi}{\Phi} = \alpha_0 \int dx$$

$$\ln\Phi = \alpha_0 x + \text{constant}$$

$$\Phi = \Phi_0 e^{\alpha_0 x}$$

B.C:
$x = 0, T - T' = T_1 - T_1'$

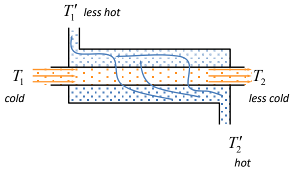
## Analysis of double-pipe heat exchanger



**Temperature profile in a double-pipe heat exchanger:**

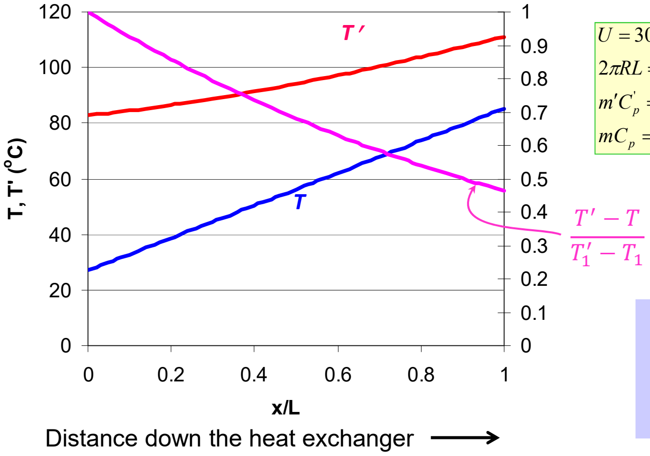
$$\frac{T' - T}{T_1' - T_1} = e^{\alpha_0 x} \qquad \alpha_0 = 2\pi R U\left(\frac{1}{\hat{C}'_p m'} - \frac{1}{\hat{C}_p m}\right)$$

## Analysis of double-pipe heat exchanger



$$\frac{T'-T}{T_1'-T_1} = e^{\alpha_0 L\left(\frac{x}{L}\right)}$$

$$\alpha_0 L = 2\pi RLU\left(\frac{1}{\hat{C}_p' m'} - \frac{1}{\hat{C}_p m}\right)$$



$U = 300\ W/mK$
$2\pi RL = 15.4 m^2$
$m'C_p' = 6\ kW/K$
$mC_p = 3\ kW/K$

Note that the temperature curves are not linear.

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## Analysis of double-pipe heat exchanger

### Temperature profile in a double-pipe heat exchanger:

$$\boxed{\frac{T'-T}{T_1'-T_1} = e^{\alpha_0 x}}$$

$$\alpha_0 = 2\pi RU\left(\frac{1}{\hat{C}_p' m'} - \frac{1}{\hat{C}_p m}\right)$$

Useful result, but what we **REALLY** want is an easy way to relate $Q_{in,\ overall}$ to inlet and outlet temperatures.

At the exit: $x = L,\ (T - T') = (T_2 - T_2')$

$$\boxed{\ln\left(\frac{T_2'-T_2}{T_1'-T_1}\right) = U(2\pi RL)\left(\frac{1}{\hat{C}_p' m'} - \frac{1}{\hat{C}_p m}\right)}$$

Analysis of double-pipe heat exchanger

$$\ln\left(\frac{T_2'-T_2}{T_1'-T_1}\right) = U\underbrace{(2\pi RL)}\left(\frac{1}{\hat{C}_p' m'} - \frac{1}{\hat{C}_p m}\right)$$

The $m\hat{C}_p$ terms appear in the overall macroscopic energy balances. We can therefore rearrange this equation by replacing the $m\hat{C}_p$ terms with $Q_{in}$:

$$Q_{in} = m\hat{C}_p(T_2 - T_1)$$

$$\Rightarrow \frac{1}{m\hat{C}_p} = \frac{T_2 - T_1}{Q_{in}}$$

$$-Q_{in} = m\hat{C}_p'(T_1' - T_2')$$

$$\Rightarrow \frac{1}{m\hat{C}_p'} = \frac{-(T_1' - T_2')}{Q_{in}}$$

$$\underbrace{Q_{in}}_{\text{total heat transferred in exchanger}} = UA\underbrace{\frac{(T_2'-T_2)-(T_1'-T_1)}{\ln\left(\frac{T_2'-T_2}{T_1'-T_1}\right)}}_{\text{average temperature driving force}}$$

total heat transferred in exchanger

**average temperature driving force**

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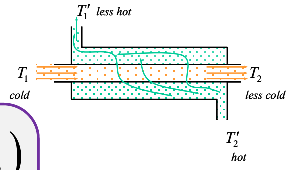
Analysis of double-pipe heat exchanger



**FINAL RESULT:**

$$\boxed{Q = U\underbrace{(2\pi RL)}_{A}\frac{(T_1'-T_1)-(T_2'-T_2)}{\ln\frac{(T_1'-T_1)}{(T_2'-T_2)}}}$$

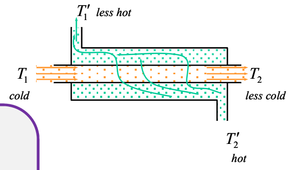
$$\equiv \Delta T_{lm}$$

=log-mean temperature difference

$$\boxed{Q = UA\Delta T_{lm}}$$

**$\Delta T_{lm}$ is the correct average temperature to use for the overall heat-transfer coefficients in a double-pipe heat exchanger.**

Analysis of double-pipe heat exchanger



**FINAL RESULT:**

$$Q = UA \underbrace{\left[\frac{\left(\Delta T_{left} - \Delta T_{right}\right)}{\ln\left(\frac{\Delta T_{left}}{\Delta T_{right}}\right)}\right]}_{\equiv \Delta T_{lm}}$$

$$\boxed{Q = UA\Delta T_{lm}}$$

=log-mean temperature difference

**$\Delta T_{lm}$ is the correct average temperature to use for the overall heat-transfer coefficients in a double-pipe heat exchanger.**

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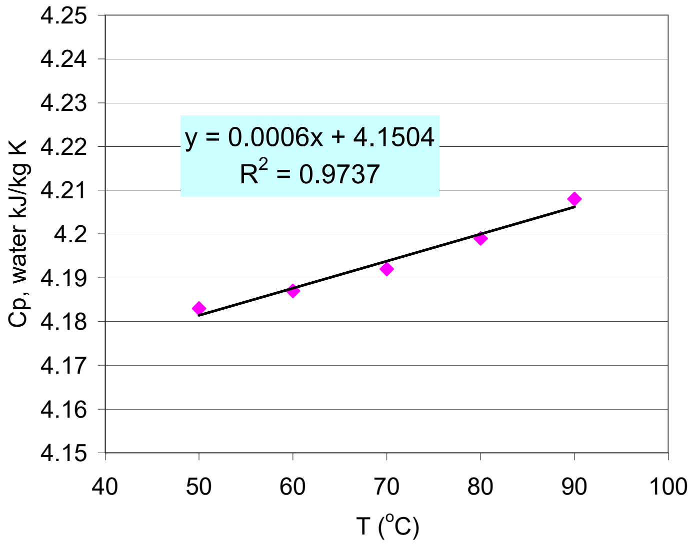
Analysis of double-pipe heat exchanger

## Example: Heat Transfer in a Double-Pipe Heat Exchanger: *Geankoplis 4th ed. 4.5-4*

Water flowing at a rate of 13.85 kg/s is to be heated from 54.5 to 87.8°C in a double-pipe heat exchanger by 54,430 kg/h of hot gas flowing counterflow and entering at 427°C ($\hat{C}_{pm} = 1.005\ kJ/kg\ K$). The overall heat-transfer coefficient based on the outer surface is $U_o$ =69.1 W/m$^2$ K. Calculate the exit-gas temperature and the heat transfer area needed.

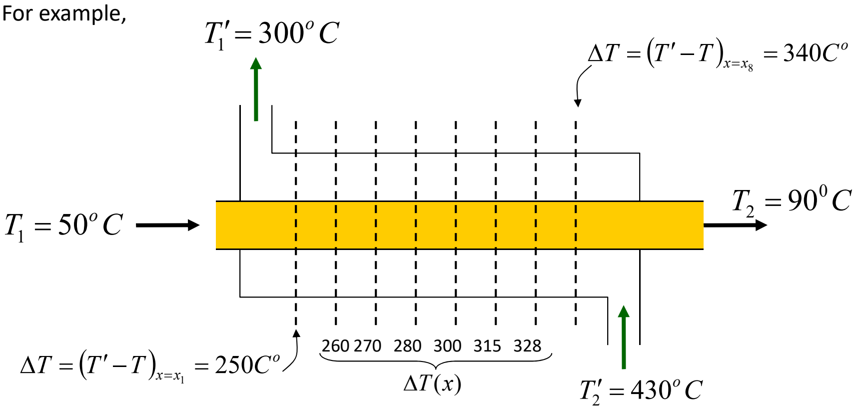
## Analysis of double-pipe heat exchanger



**Summary:**

**Double-Pipe Heat Exchanger** – the driving force for heat transfer changes along the length of the heat xchanger

For example,

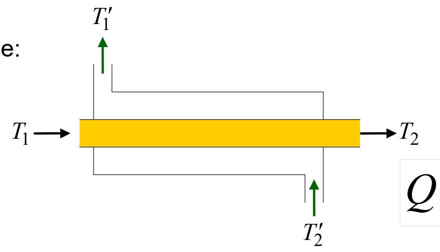


The correct *average* driving force for the whole xchanger is the log-mean temperature difference $= \Delta T_{lm}$

Optimizing heat exchangers

## Optimizing Heat Exchangers

double-pipe:



$$Q = UA\Delta T_{lm}$$

To increase $Q$ appreciably, we must increase A, i.e. $R_i$

***But:***
- only small increases possible
- increasing $R_i$ decreases $h$

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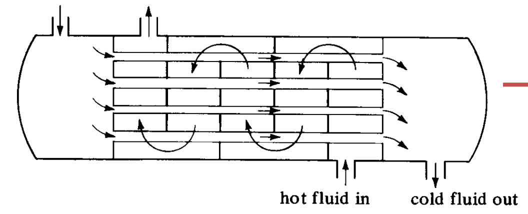
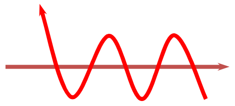
Optimizing heat exchangers

### 1-1 Shell and Tube Heat Exchanger

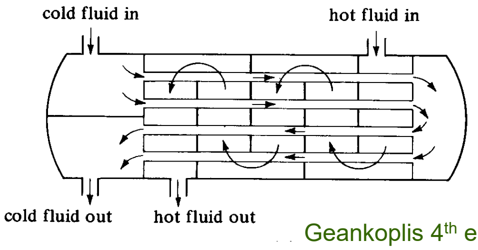
(Same as double pipe H.E.)

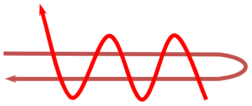




1 shell
1 tube

### 1-2 Shell and Tube Heat Exchanger



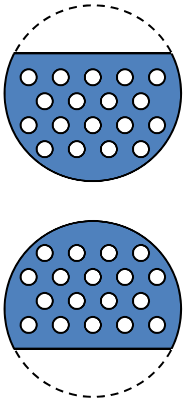


1 shell
2 tube

Geankoplis 4th ed., p292

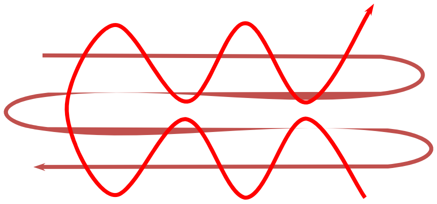
Optimizing heat exchangers

## Cross Baffles in Shell-and-Tube Heat Exchangers



Optimizing heat exchangers

And other more complex arrangements:



2 shell
4 tube

Optimizing heat exchangers

For double-pipe heat exchanger:

$$Q = UA\Delta T_{lm}$$

For shell-and-tube heat exchangers:

$$Q = UA\underbrace{\left[\Delta T_{lm}\left(F_T\right)\right]}_{\equiv \Delta T_m}$$

calculated correction factor (obtain from experimentally determined charts)

correct mean temperature difference for shell-and-tube heat exchangers

---

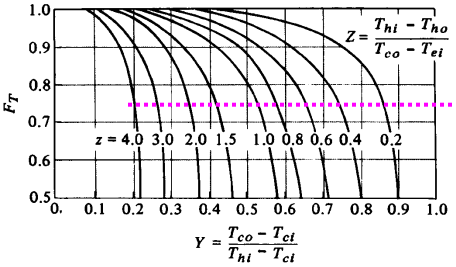
Optimizing heat exchangers

$F_T$

Shell-and-Tube Heat Exchangers

(1-1 exchanger, $F_T = 1$)

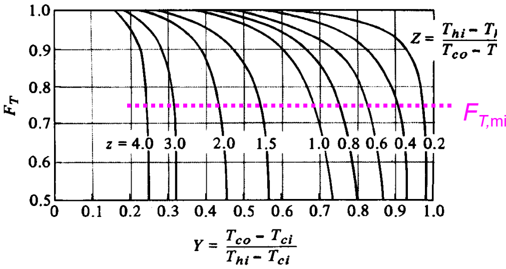
1-2 Shell and Tube H.E.



$F_{T,min}$=0.75

Efficiency is low when $F_T$ is below $F_{T,min}$

2-4 Shell and Tube H.E.



$F_{T,min}$=0.75

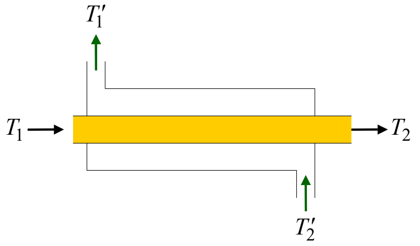
$T_{hi}$ = hot, in
$T_{ho}$ = hot, out
$T_{ci}$ = cold, in
$T_{co}$ = cold, out

Geankoplis 4th ed., p295

# Heat Exchanger Design

To calculate $Q$, we need both inlet and outlet temperatures:

$$Q = UA\Delta T_m = UA\left(F_T \Delta T_{lm}\right)$$



$$Q = UA\left[\frac{\left(\Delta T_{left} - \Delta T_{right}\right)}{\ln\left(\frac{\Delta T_{left}}{\Delta T_{right}}\right)}\right]$$

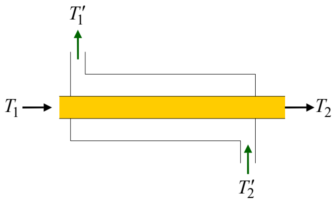
**What if the outlet temperatures are unknown?**
i.e. the design/spec problem.

---

## Example Problem:
## How will this heat exchanger perform?

Water flowing at a rate of $0.723\ kg/s$ enters the inside of a countercurrent, double-pipe heat exchanger at 300.K and is heated by an oil stream that enters at $385\ K$ at a rate of $3.2\ kg/s$. The heat capacity of the oil is $1.89\ kJ/kgK$, and the average heat capacity of the water of the temperature range of interest is $4.192\ kJ/kgK$. The overall heat-transfer coefficient of the exchanger is $300.\ W/m^2K$, and the area for heat transfer is $15.4\ m^2$. What is the total amount of heat transferred?

## Heat Exchanger Design



**Example Problem:**
**How will this heat exchanger perform?**

Water flowing at a rate of $0.723\ kg/s$ enters the inside of a countercurrent, double-pipe heat exchanger at 300.K and is heated by an oil stream that enters at $385\ K$ at a rate of $3.2\ kg/s$. The heat capacity of the oil is $1.89\ kJ/kgK$, and the average heat capacity of the water of the temperature range of interest is $4.192\ kJ/kgK$. The overall heat-transfer coefficient of the exchanger is $300.\ W/m^2K$, and the area for heat transfer is $15.4\ m^2$. What is the total amount of heat transferred?

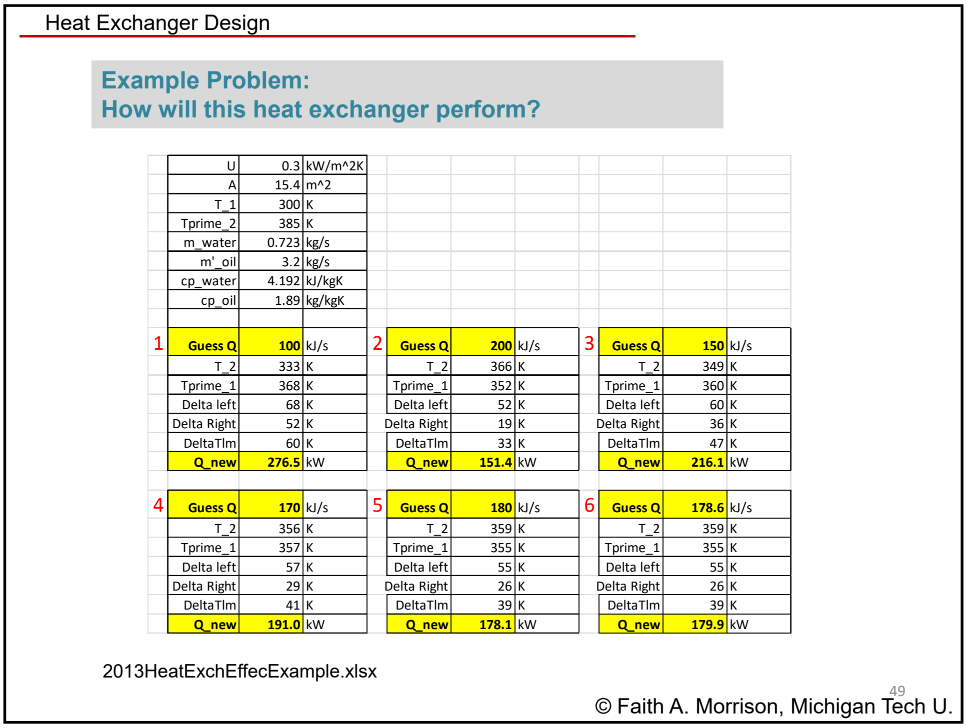
You try.

## Heat Exchanger Design

**Example Problem:**
**How will this heat exchanger perform?**

To calculate unknown outlet temperatures:

Procedure:
1. Guess Q
2. Calculate outlet temperatures
3. Calculate $\Delta T_{lm}$
4. Calculate Q
5. Compare, adjust, repeat



Heat Exchanger Design

## Example Problem:
## How will this heat exchanger perform?

| | | |
|---|---|---|
| U | 0.3 | kW/m^2K |
| A | 15.4 | m^2 |
| T_1 | 300 | K |
| Tprime_2 | 385 | K |
| m_water | 0.723 | kg/s |
| m'_oil | 3.2 | kg/s |
| cp_water | 4.192 | kJ/kgK |
| cp_oil | 1.89 | kg/kgK |

1

| **Guess Q** | **100** | kJ/s |
|---|---|---|
| T_2 | 333 | K |
| Tprime_1 | 368 | K |
| Delta left | 68 | K |
| Delta Right | 52 | K |
| DeltaTlm | 60 | K |
| **Q_new** | **276.5** | kW |

2

| **Guess Q** | **200** | kJ/s |
|---|---|---|
| T_2 | 366 | K |
| Tprime_1 | 352 | K |
| Delta left | 52 | K |
| Delta Right | 19 | K |
| DeltaTlm | 33 | K |
| **Q_new** | **151.4** | kW |

3

| **Guess Q** | **150** | kJ/s |
|---|---|---|
| T_2 | 349 | K |
| Tprime_1 | 360 | K |
| Delta left | 60 | K |
| Delta Right | 36 | K |
| DeltaTlm | 47 | K |
| **Q_new** | **216.1** | kW |

4

| **Guess Q** | **170** | kJ/s |
|---|---|---|
| T_2 | 356 | K |
| Tprime_1 | 357 | K |
| Delta left | 57 | K |
| Delta Right | 29 | K |
| DeltaTlm | 41 | K |
| **Q_new** | **191.0** | kW |

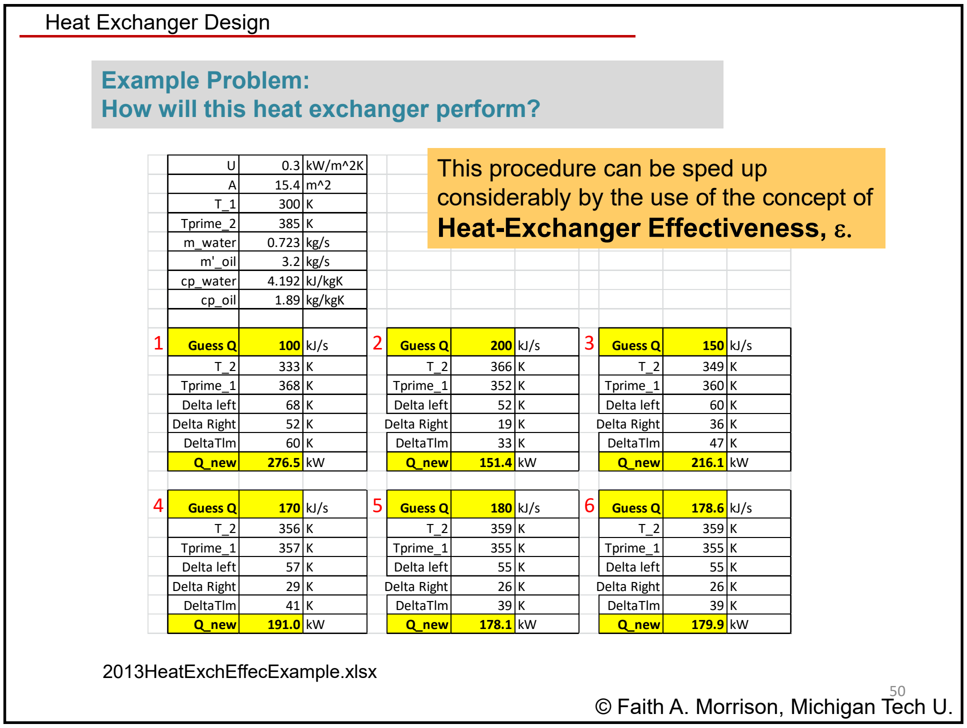
5

| **Guess Q** | **180** | kJ/s |
|---|---|---|
| T_2 | 359 | K |
| Tprime_1 | 355 | K |
| Delta left | 55 | K |
| Delta Right | 26 | K |
| DeltaTlm | 39 | K |
| **Q_new** | **178.1** | kW |

6

| **Guess Q** | **178.6** | kJ/s |
|---|---|---|
| T_2 | 359 | K |
| Tprime_1 | 355 | K |
| Delta left | 55 | K |
| Delta Right | 26 | K |
| DeltaTlm | 39 | K |
| **Q_new** | **179.9** | kW |

2013HeatExchEffecExample.xlsx



Heat Exchanger Design

## Example Problem:
## How will this heat exchanger perform?

This procedure can be sped up considerably by the use of the concept of **Heat-Exchanger Effectiveness, ε.**

| | | |
|---|---|---|
| U | 0.3 | kW/m^2K |
| A | 15.4 | m^2 |
| T_1 | 300 | K |
| Tprime_2 | 385 | K |
| m_water | 0.723 | kg/s |
| m'_oil | 3.2 | kg/s |
| cp_water | 4.192 | kJ/kgK |
| cp_oil | 1.89 | kg/kgK |

1

| **Guess Q** | **100** | kJ/s |
|---|---|---|
| T_2 | 333 | K |
| Tprime_1 | 368 | K |
| Delta left | 68 | K |
| Delta Right | 52 | K |
| DeltaTlm | 60 | K |
| **Q_new** | **276.5** | kW |

2

| **Guess Q** | **200** | kJ/s |
|---|---|---|
| T_2 | 366 | K |
| Tprime_1 | 352 | K |
| Delta left | 52 | K |
| Delta Right | 19 | K |
| DeltaTlm | 33 | K |
| **Q_new** | **151.4** | kW |

3

| **Guess Q** | **150** | kJ/s |
|---|---|---|
| T_2 | 349 | K |
| Tprime_1 | 360 | K |
| Delta left | 60 | K |
| Delta Right | 36 | K |
| DeltaTlm | 47 | K |
| **Q_new** | **216.1** | kW |

4

| **Guess Q** | **170** | kJ/s |
|---|---|---|
| T_2 | 356 | K |
| Tprime_1 | 357 | K |
| Delta left | 57 | K |
| Delta Right | 29 | K |
| DeltaTlm | 41 | K |
| **Q_new** | **191.0** | kW |

5

| **Guess Q** | **180** | kJ/s |
|---|---|---|
| T_2 | 359 | K |
| Tprime_1 | 355 | K |
| Delta left | 55 | K |
| Delta Right | 26 | K |
| DeltaTlm | 39 | K |
| **Q_new** | **178.1** | kW |

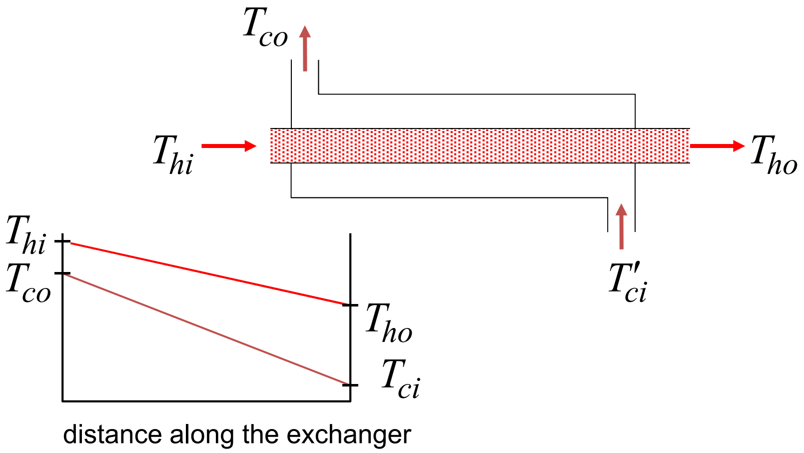
6

| **Guess Q** | **178.6** | kJ/s |
|---|---|---|
| T_2 | 359 | K |
| Tprime_1 | 355 | K |
| Delta left | 55 | K |
| Delta Right | 26 | K |
| DeltaTlm | 39 | K |
| **Q_new** | **179.9** | kW |

2013HeatExchEffecExample.xlsx

## Heat Exchanger Effectiveness

Consider a *counter-current* double-pipe heat exchanger:



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**Energy balance cold side:**

$$Q_{in,cold}=Q=\left(mC_p\right)_{cold}\left(T_{co}-T_{ci}\right)$$

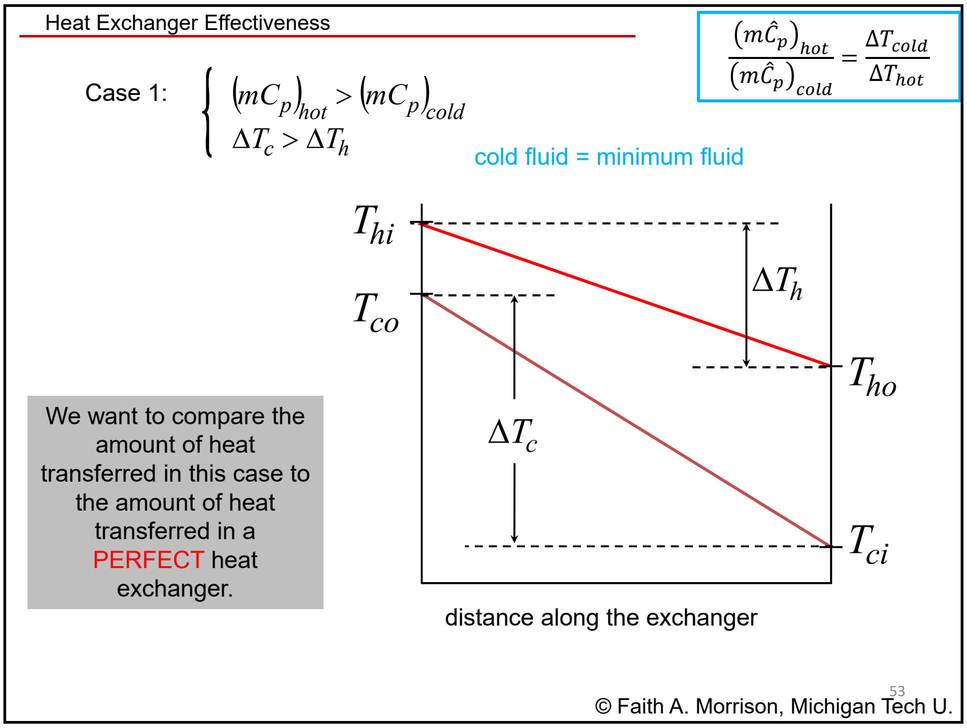
**Energy balance hot side:**

$$Q_{in,hot}=-Q=\left(mC_p\right)_{hot}\left(T_{ho}-T_{hi}\right)$$

**Equate:**

$$\left(m\hat{C}_p\right)_{cold}\left(T_{co}-T_{ci}\right)=-\left(m\hat{C}_p\right)_{hot}\left(T_{ho}-T_{hi}\right)$$

$$\boxed{\frac{\left(mC_p\right)_{hot}}{\left(mC_p\right)_{cold}}=\frac{\left(T_{co}-T_{ci}\right)}{-\left(T_{ho}-T_{hi}\right)}=\frac{\Delta T_c}{\Delta T_h}}$$



## Heat Exchanger Effectiveness

$$\frac{(m\hat{C}_p)_{hot}}{(m\hat{C}_p)_{cold}} = \frac{\Delta T_{cold}}{\Delta T_{hot}}$$

Case 1: $\begin{cases}(mC_p)_{hot} > (mC_p)_{cold} \\ \Delta T_c > \Delta T_h\end{cases}$

cold fluid = minimum fluid

We want to compare the amount of heat transferred in this case to the amount of heat transferred in a PERFECT heat exchanger.

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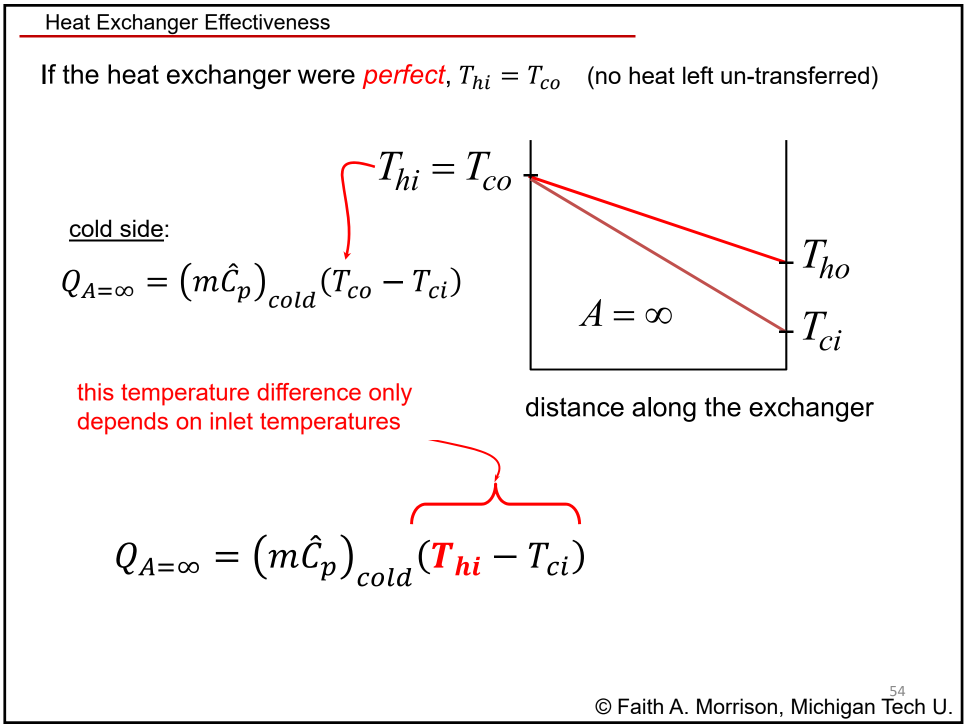

## Heat Exchanger Effectiveness

If the heat exchanger were *perfect*, $T_{hi} = T_{co}$ (no heat left un-transferred)

cold side:

$$Q_{A=\infty} = (m\hat{C}_p)_{cold}(T_{co} - T_{ci})$$

this temperature difference only depends on inlet temperatures

$$Q_{A=\infty} = (m\hat{C}_p)_{cold}\overbrace{(T_{hi} - T_{ci})}$$

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Heat Exchanger Effectiveness

## Heat Exchanger Effectiveness, ε

$$\boxed{\varepsilon \equiv \frac{Q}{Q_{A=\infty}}}$$

$$\boxed{\Rightarrow Q = \varepsilon \left(mC_p\right)_{cold}\left(T_{hi} - T_{ci}\right)}$$

cold fluid = minimum fluid

if ε is known, we can calculate Q <u>without iterations</u>

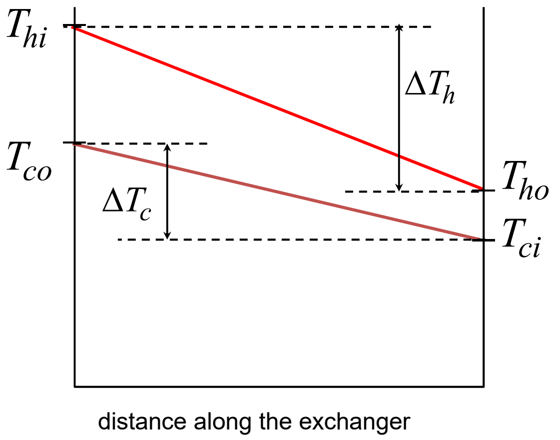
---

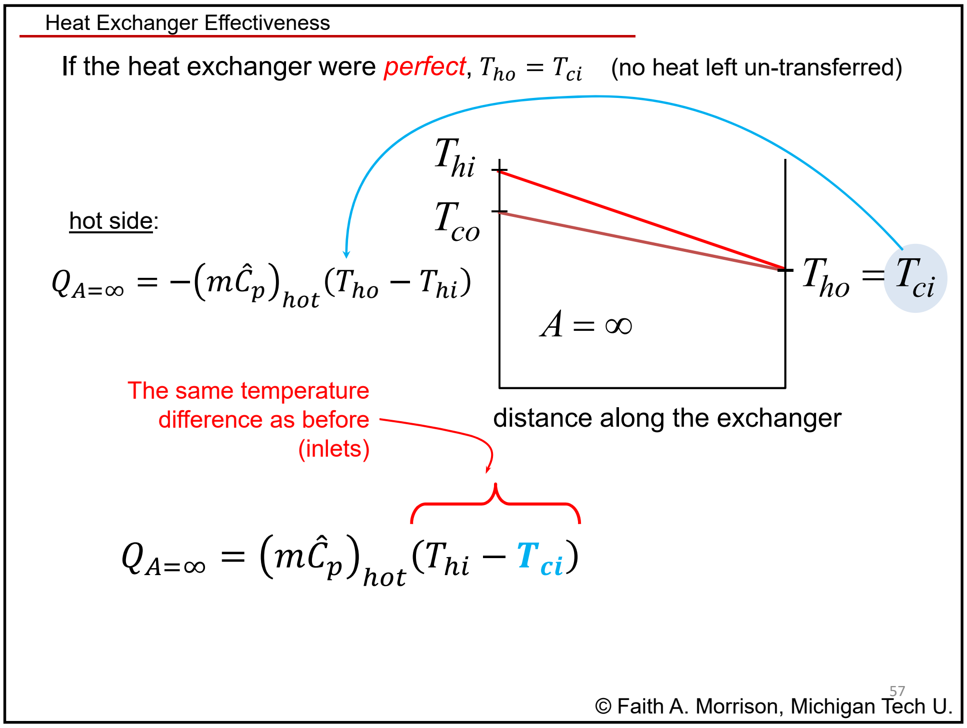
Heat Exchanger Effectiveness

$$\boxed{\frac{(m\hat{C}_p)_{hot}}{(m\hat{C}_p)_{cold}} = \frac{\Delta T_{cold}}{\Delta T_{hot}}}$$

Case 2: $\left\{ \begin{array}{l} (mC_p)_{hot} < (mC_p)_{cold} \\ \Delta T_c < \Delta T_h \end{array} \right.$

**hot fluid = minimum fluid**





Heat Exchanger Effectiveness

If the heat exchanger were *perfect*, $T_{ho} = T_{ci}$ (no heat left un-transferred)

hot side:

$$Q_{A=\infty} = -\left(m\hat{C}_p\right)_{hot}\left(T_{ho} - T_{hi}\right)$$

The same temperature difference as before (inlets)

$$Q_{A=\infty} = \left(m\hat{C}_p\right)_{hot}\overbrace{\left(T_{hi} - T_{ci}\right)}$$

57

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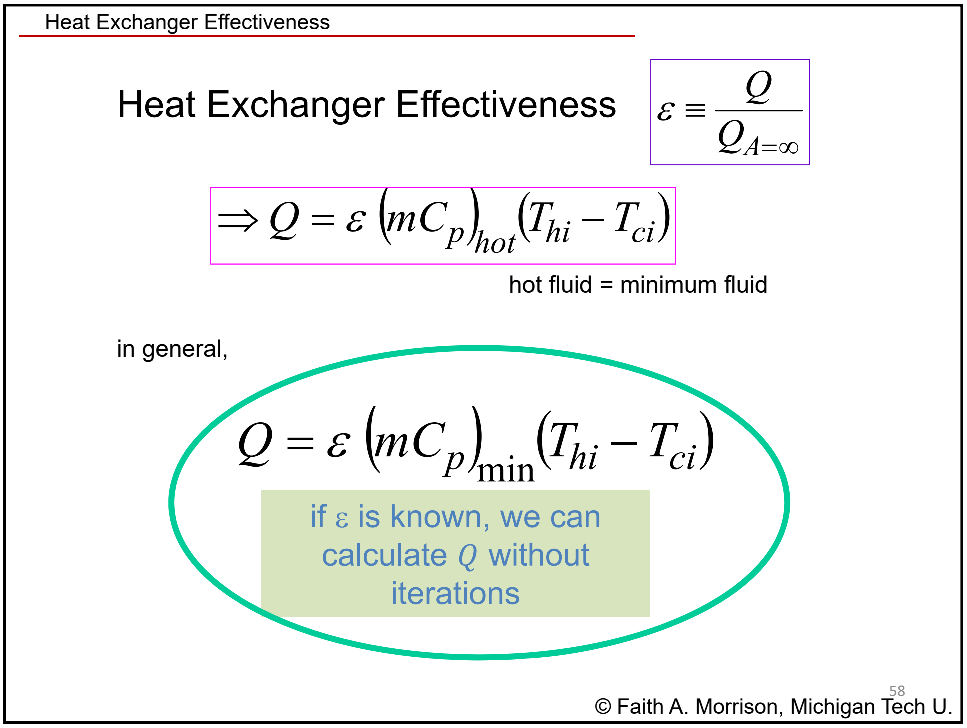

Heat Exchanger Effectiveness

## Heat Exchanger Effectiveness

$$\boxed{\varepsilon \equiv \frac{Q}{Q_{A=\infty}}}$$

$$\boxed{\Rightarrow Q = \varepsilon\left(mC_p\right)_{hot}\left(T_{hi} - T_{ci}\right)}$$

hot fluid = minimum fluid

in general,

$$Q = \varepsilon\left(mC_p\right)_{\min}\left(T_{hi} - T_{ci}\right)$$

if ε is known, we can calculate $Q$ without iterations

58

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Heat Exchanger Effectiveness

## But where do we get $\varepsilon$ ?

The same equations we use in the trial-and-error solution can be combined algebraically to give $\varepsilon$ as a function of $(mC_p)_{min}$, $(mC_p)_{max}$.

counter-current flow:

$$\varepsilon = \left( \frac{1 - e^{\frac{-UA}{(mC_p)_{\min}}\left(1 - \frac{(mC_p)_{\min}}{(mC_p)_{\max}}\right)}}{1 - \frac{(mC_p)_{\min}}{(mC_p)_{\max}} e^{\frac{-UA}{(mC_p)_{\min}}\left(1 - \frac{(mC_p)_{\min}}{(mC_p)_{\min}}\right)}} \right)$$

This relation is plotted in Geankoplis, as is the relation for co-current flow.

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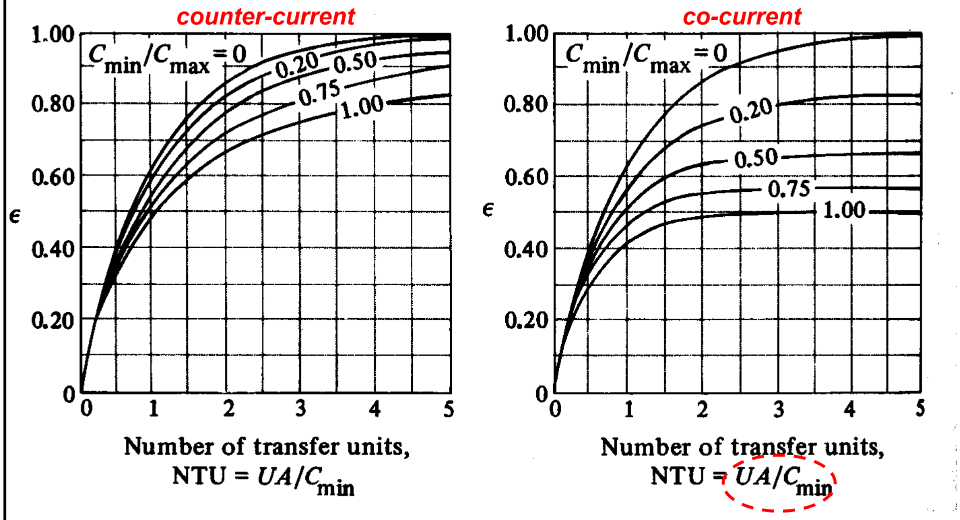
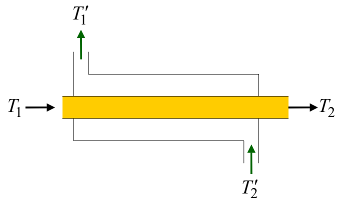
*Heat Exchanger Effectiveness for Double-pipe or 1-1 Shell-and-Tube Heat Exchangers*



note: Geankoplis' $C_{min}=(mC_p)_{min}$

Geankoplis 4th ed., p299

## Heat Exchanger Effectiveness



**Example Problem:**
**How will this heat exchanger perform?**

Water flowing at a rate of $0.723\ kg/s$ enters the inside of a countercurrent, double-pipe heat exchanger at 300.K and is heated by an oil stream that enters at $385\ K$ at a rate of $3.2\ kg/s$. The heat capacity of the oil is $1.89\ kJ/kgK$, and the average heat capacity of the water of the temperature range of interest is $4.192\ kJ/kgK$. The overall heat-transfer coefficient of the exchanger is $300.\ W/m^2K$, and the area for heat transfer is $15.4\ m^2$. What is the total amount of heat transferred?

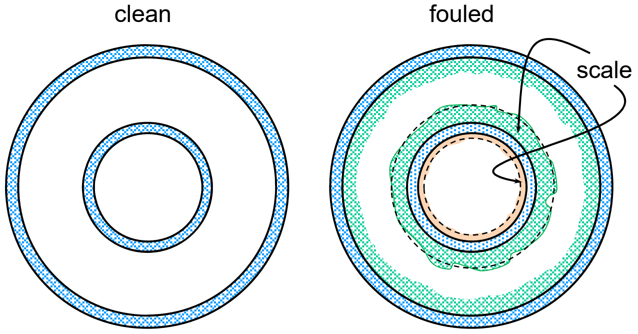
You try.

## Heat Exchanger Effectiveness

### Heat Exchanger Fouling

- material deposits on hot surfaces
- rust, impurities
- strong effect when boiling occurs

scale adds an additional resistance to heat transfer



## Heat Exchanger Effectiveness

Heat transfer resistances

$$U_{i\,or\,o} = \frac{\dfrac{1}{R_{i\,or\,o}}}{\dfrac{1}{h_iR_i} + \dfrac{1}{k}\ln\left(\dfrac{R_o}{R_i}\right) + \dfrac{1}{h_oR_o}}$$

resistance due to interface

resistance due to limited thermal conductivity

add effect of fouling

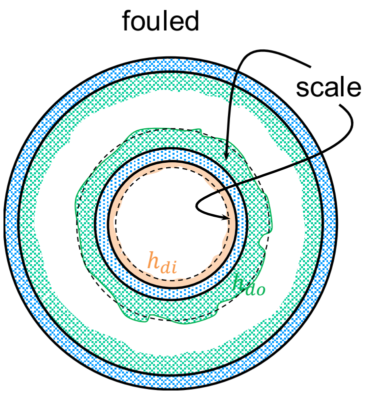
$$U_{i\,or\,o} = \frac{\dfrac{1}{R_{i\,or\,o}}}{\dfrac{1}{h_iR_i} + \dfrac{1}{h_{di}R_i} + \dfrac{1}{k}\ln\left(\dfrac{R_o}{R_i}\right) + \dfrac{1}{h_{do}R_o} + \dfrac{1}{h_oR_o}}$$

Measured by calibration against measured effect of fouling.

Fouling, inside surface

Fouling, outside surface

## Heat Exchanger Fouling



Geankoplis, 4th edition, p300 also in Perry's

Table 4.9-1. *Typical Fouling Coefficients (P3, N1)*

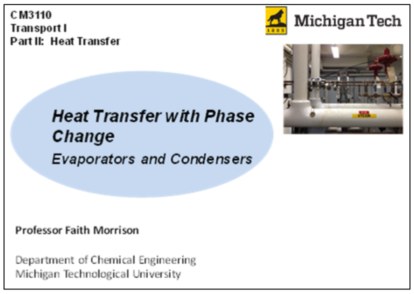
| | $h_d$ (W/m² · K) |
|---|---|
| Distilled and seawater | 11 350 |
| City water | 5680 |
| Muddy water | 1990–2840 |
| Gases | 2840 |
| Vaporizing liquids | 2840 |
| Vegetable and gas oils | 1990 |

Table 4.9-2. *Typical Values of Overall Heat-Transfer Coefficients in Shell-and-Tube Exchangers (H1, P3, W1)*

| | $U$ (W/m² · K) |
|---|---|
| Water to water | 1140–1700 |
| Water to brine | 570–1140 |
| Water to organic liquids | 570–1140 |
| Water to condensing steam | 1420–2270 |
| Water to gasoline | 340–570 |
| Water to gas oil | 140–340 |
| Water to vegetable oil | 110–285 |
| Gas oil to gas oil | 110–285 |
| Steam to boiling water | 1420–2270 |
| Water to air (finned tube) | 110–230 |
| Light organics to light organics | 230–425 |
| Heavy organics to heavy organics | 55–230 |

## Next:

- Heat transfer with phase change
- Evaporators
- Radiation
- *DONE*



CM3110
Transport I
Part II: Heat Transfer

Michigan Tech

***Heat Transfer with Phase Change***

*Evaporators and Condensers*

Professor Faith Morrison

Department of Chemical Engineering
Michigan Technological University