Heat Transfer Analysis of Fins with Spine Geometry Using Differential Transform Method

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***Abstract*—In this study, a general expression for heat distribution profile of a fin with spine geometry is found using Differential Transform Method (DTM). By using this general expression, fin efficiency, fin effectiveness, and entropy generation are calculated for different profiles. Based on efficiency, effectiveness and entropy generation values, cylindrical fin profile is found to have the highest heat transfer rate.**

***Index Terms*—Differential Transform Method, DTM, spine fin, heat distribution, fin efficiency, fin effectiveness, entropy generation**

I. Introduction

A wide array of engineering applications require efficient energy transform mechanisms for rapid movement of excessive heat. Extended surfaces are widely used for transportation of heat. Fins are a type of extended surface and they are used in various industries such as in cooling turbines, heat exchangers and electronic components.

There are a lot of extensive reviews about cooling by fins are presented [1]-[7].

The entropy generation method was first introduced by Bejan [8] as a measure of system performance and used as a general criterion for judging the thermodynamic performance of heat exchangers. He applied this method to extended surfaces with four different specific geometries, and used entropy generation minimization as a performance criterion [9].

Differential Transform Method is a semi-analytical numerical technique and it is used to solve differential equations where finding an analytical solution may not be possible. It is developed by Zhou in a study about electrical circuits [10], it makes possible to obtain highly accurate solutions for differential equations.

In the present study, DTM is used for calculating temperature profile for spine fins with various profiles. Fin efficiency and fin effectiveness are found to obtain the optimum profile. The results are validated by making an entropy generation minimization analysis.

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A. *Equations of Heat Transfer for a Fin with Spine Profile*

The geometry of a general spine type fin is shown in Fig. 1. Derivation of differential equation is carried out by Kraus et al [2] and rewritten here for the explanation of the problem.

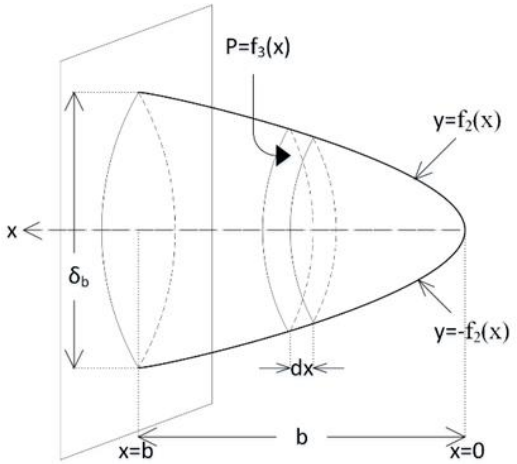

Figure 1. The geometry of a general spine fin.

Profile for the fin is given as follows:

$$f_2(x) = \frac{\delta_b}{2}\left(\frac{x}{b}\right)^{(1-2n)/(2-n)} \quad \text{where } n < 0.5. \tag{1}$$

The heat conduction into and out of the element dx through the cross section $f_1(x)$ is:

$$\frac{dq}{dx} = k\frac{d}{dx}\left[f_1(x)\frac{d\theta}{dx}\right] \tag{2}$$

The heat dissipation which occurs solely by convention is:

$$\frac{dq}{dx} = hf_3(x)\theta \tag{3}$$

Equaling right sides of these equations and rearranging yields:

$$f_1(x)\frac{d\theta}{dx} + \frac{df_1(x)}{dx}\frac{d\theta}{dx} - \frac{h}{k}f_3(x)\theta = 0 \tag{4}$$

By taking $f_1(x) = \pi[f_2(x)]^2$ and $f_3(x) = 2\pi f_2(x)$, the equation becomes:

$$[f_2(x)]^2\frac{d^2\theta}{dx^2} + \frac{d}{dx}[f_2(x)]^2\frac{d\theta}{dx} - \frac{h}{k}f_2(x)\theta = 0 \tag{5}$$

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The boundary conditions are:

$$\theta(b) = \theta_b, \ \left.\frac{d\theta}{dx}\right|_{x=0} = 0 \quad (6)$$

II. The Solution Method and Design Criteria

A. Differential Transform Method (DTM)

Differential Transform Method is a technique based on Taylor series expansion. A detailed analysis for the theorem and various examples can be found in several papers such as Arikoglu and Ozkol [11].

The transformation of a k^th derivative of a function at a point $x = x_0$ can be found by the given formula:

$$F(k) = \frac{1}{k!}\left[\frac{d}{d^k x} f(x)\right]_{x=x_0} \quad (7)$$

The inverse transformation can be found by the following formula:

$$f(x) = \sum_{k=0}^{N} F(k)(x - x_0)^k \text{ for } N \to \infty \quad (8)$$

The theorems that are used during the solution of fin problem can be seen below:

Theorem 1: If $f(x) = g(x) \pm h(x)$, then $F(k) = G(k) \pm H(k)$. (9)

Theorem 2: If $f(x) = cg(x)$, then $F(k) = cG(k)$. (10)

Theorem 3: If $f(x) = \frac{d^n}{dx^n} g(x)$, then $F(k) = \frac{(n+k)!}{k!} G(n+k)$. (11)

Theorem 4: If $f(x) = g(x)h(x)$, then $F(k) = \sum_{k_1=0}^{k} G(k_1)H(k - k_1)$. (12)

Theorem 5: If $f(x) = g(x + a)$, then $F(k) = \sum_{h_1=k}^{N} \binom{h_1}{k} a^{h_1-k} G(h_1)$ for $N \to \infty$. (13)

B. Fin Efficiency and Fin Effectiveness

Fin Efficiency is the ratio of the actual fin heat transfer rate to the heat transfer of the hypothetical fin if the entire fin were at the base temperature. This ratio will be always smaller than one. Its formula is given below:

$$\eta_{efficiency} = \frac{q_{base}}{hA_{fin}\theta_{base}} \quad (14)$$

Fin Effectiveness is the ratio of fin heat transfer rate to the heat transfer of the base area as if there isn't any fin. This ratio will be always bigger than one. Its formula is given below:

$$\eta_{effectiveness} = \frac{q_{base}}{hA_{base}\theta_{base}} \quad (15)$$

Total heat transfer of the fin through a cross section $A(x) = \pi[f_2(x)]^2$ is defined as follows:

$$q(x) = kA(x)\frac{d}{dx}\theta(x) = k\pi[f_2(x)]^2 \frac{d}{dx}\theta(x) \quad (16)$$

C. Entropy Generation

The formula for entropy generation in a static fluid is given as follows:

$$S_G = \frac{q_{base}\theta_{base}}{T_\infty^2(1+\theta_{base}/T_\infty)} \quad (17)$$

where q_{base} is the heat transfer rate at the base of the fin, θ_{base} is the temperature at the base of the fin, and T_∞ is the temperature of the medium.

III. Results and Discussion

Eq. 5 is solved using the boundary conditions given in Eq. 6 by DTM, and the results are compared to analytical solutions for cylindrical, concave, conical, and convex profiled spines. Taylor series for DTM are evaluated around b/2. Variables used for the evaluation of Eq. 5 are given as follows:

$$\begin{aligned} &\delta_b = 0.01\ (m), && b = 0.1\ (m), \\ &k = 32\left(\frac{W}{m}\cdot K\right), && h = 50\left(\frac{W}{m^2}\cdot K\right) \\ &T_{base} = 80\ (°C), && T_{medium} = 30 \end{aligned} \quad (18)$$

where δ_b is the base radius, b are the base radius the fin length, k is the thermal conductivity, h is the heat transfer coefficient in, and T_{base} and T_{medium} are the base and tip temperatures.

Comparison of the temperature excess for various profiles are shown at Table I and Table II. Here, $\bar{x} = x/b$, and temperature excess is defined as $\theta = T - T_{medium}$.

TABLE I. Comparison of DTM and Analytical Solution for Cylindrical and Convex Parabolic Spine

$\bar{x}$	Cylindrical Analytical	Cylindrical DTM	Convex Analytical	Convex DTM
0.1	8.4096	8.4096	8.4901	8.7269
0.2	9.1941	9.1941	9.8498	9.9795
0.3	10.5562	10.5562	11.7735	11.8505
0.4	12.5816	12.5815	14.3056	14.3529
0.5	15.3973	15.3973	17.5383	17.5668
0.6	19.1805	19.1805	21.5996	21.6150
0.7	24.1686	24.1686	26.6524	26.6580
0.8	30.6752	30.6752	32.8980	32.8955
0.9	39.1091	39.1091	40.5818	40.5735

TABLE II. Comparison of DTM and Analytical Solution for Conical and Concave Parabolic Spine

$\bar{x}$	Conical Analytical	Conical DTM	Concave Analytical	Concave DTM
0.1	6.9178	7.2819	1.9208	2.0742
0.2	9.0898	9.2066	5.1238	5.1463
0.3	11.7154	11.7532	9.0959	9.0825
0.4	14.8655	14.8695	13.6677	13.6316
0.5	18.6201	18.6043	18.7443	18.6864
0.6	23.0688	23.0383	24.2632	24.1833
0.7	28.3120	28.2683	30.1793	30.0775
0.8	34.4622	34.4056	36.4582	36.3358
0.9	41.6449	41.5818	43.0726	42.9446

As can be seen from the results, DTM solution is very accurate at the points closer to the fin base, however this is not the case for the points closer to the fin tip. It is worth mentioning that the total heat transfer from the fin is proportional to the heat gradient at the base. As a result, from an engineering perspective, it can be said that this method gives highly accurate results.

Fig. 2 through Fig. 5 shows the temperature distribution for DTM solutions for spine fins with cylindrical profile, convex profile, conical profile, and concave profile respectively.

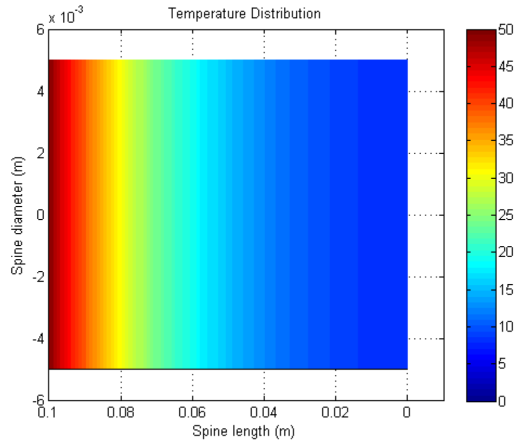

Figure 2. Temperature distribution for a cylindrical profiled fin.

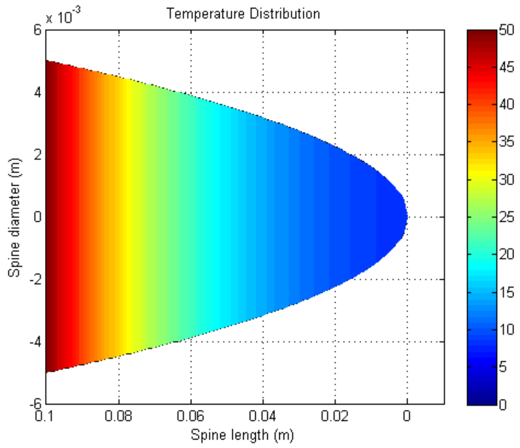

Figure 3. Temperature distribution for a convex profiled fin.

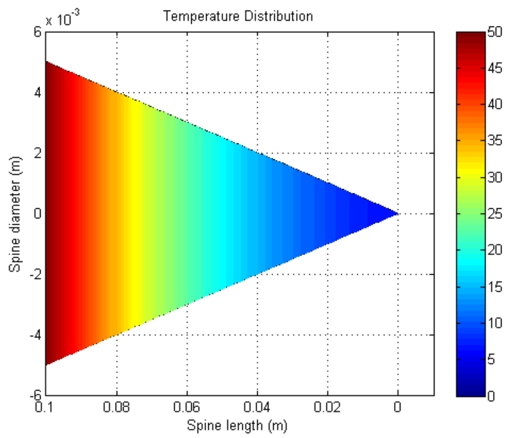

Figure 4. Temperature distribution for a conical profiled fin.

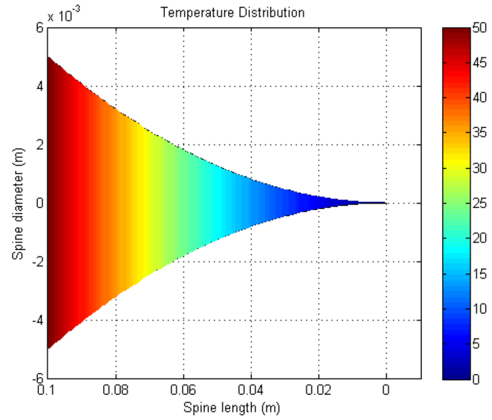

Figure 5. Temperature distribution for a concave profiled fin.

Comparisons of temperature distributions and total heat flux distributions are given at Fig. 6 and Fig. 7 respectively. Total heat transfer can be measured by the heat flux at the base. As can be seen from Fig. 6, cylindrical fin has the highest total heat flux at the base, thus it is the best design between those four types of fins for heat removal. Also, total heat flux at the base decreases as the n increases, and concave profiled fin is the worst design considering only the total heat flux at the base.

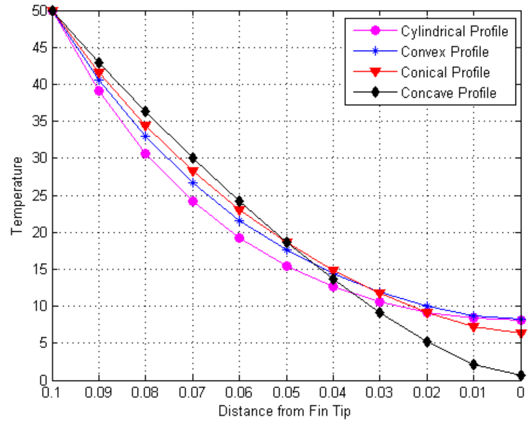

Figure 6. Temperature distribution for four different types of fins.

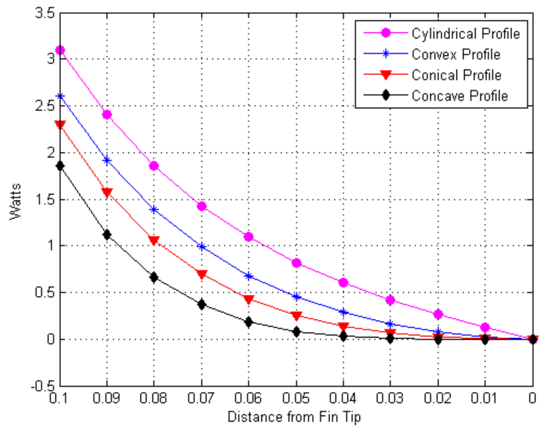

Figure 7. Heat flux distribution for four different types of fins.

Fin effectiveness and fin efficiency graphs can be seen at Fig. 8 and Fig. 9 respectively. As can be seen from the figures, fin effectiveness increases with higher values of *n*, and fin efficiency increases with lower values of *n*. Having a higher value for both effectiveness and efficiency for heat removal is desirable for a fin. Thus, none of the designs satisfy both of these criteria at the same time.

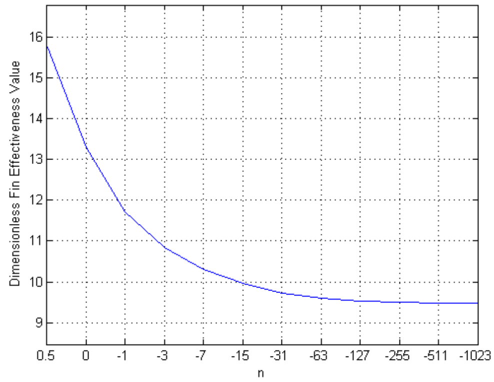

Figure 8. Fin effectiveness versus values of n.

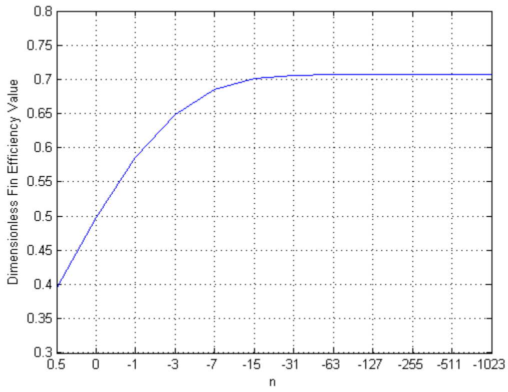

Figure 9. Fin efficiency versus values of n.

In Fig. 10, Entropy Generation for various values of n can be seen. For maximum heat dissipation from surrounding medium, entropy generation value must be maximized. As can be seen from the graph, entropy generation increases as the value of *n* increases. Thus, cylindrical profile has the highest entropy generation.

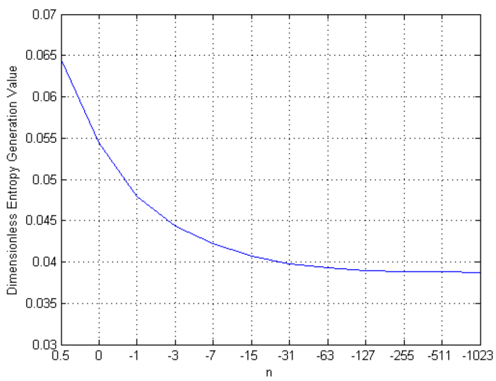

Figure 10. Entropy generation for various values of n.

IV. Conclusion

A wide range of applications such as air-cooled automotive engines, air-conditioning systems, oil industries, computer processors and other electronic devices require extended surfaces for fast removal of excessive heat.

In this study, a generalized solution for spine fins with different geometries is sought. The DTM results for four different types of fins are compared to analytical solutions, and a good agreement is found at the base of the fins. Fin efficiency, fin effectiveness and entropy generation rates are found for various profiles. The results show that, with increasing *n*, fin effectiveness and entropy generation increases, and fin efficiency decreases. Although cylindrical profile is found to be the best at heat dissipation, further studies are required for profile optimization, and fin array optimization with constraints such as the amount of material used and the total base area.

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