Heat Transfer
Background Material
Chemical and Biological Engineering 346
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INTRODUCTION

In this experiment the resistances to heat flow from liquids to solids is examined. The coupling of convective heat transfer across a fluid-solid boundary, followed by conductive heat transfer through a solid will be compared for different solids and different fluid flow rates. The experimental test system is a simple cylindrical rod (and other simple shapes) with water flowing past. The temperature at the center of the rod depends on the rate at which heat is convected from the water to the outside of the solid cylinder followed by the heat conduction through the solid cylinder. By measuring the temperature at the center of the rod as a function of time it is possible to determine the thermal diffusivity of the rod or the convective heat transfer coefficient or both. The procedures to use in these measurements depend on which process dominates the heat transfer.

BACKGROUND

Thermal conductivity

The thermal conductivity (k, dimensions of energy/time-length-temperature-interval) of a material is the physical quantity that measures the rate at which heat moves through the material by conduction. It is a fundamental transport coefficient like viscosity and diffusivity (Bird et al., 2002). The defining equation for thermal conductivity is Fourier's law of heat conduction:

$$\frac{\mathrm{q_x}}{\mathrm{A}} = -\mathrm{k}\frac{\mathrm{dT}}{\mathrm{dx}} \tag{1}$$

where q_x/A is the heat flux in the x-direction, and $T(x)$ is the temperature as a function of position x. Good conductors have high thermal conductivities; poor conductors have low thermal conductivities. For example, iron has a thermal conductivity of 80 W/m-K, whereas a ceramic or glass such as silica will have a thermal conductivity of about 1 W/m-K. Gases have thermal conductivities much smaller than solids; for example, carbon dioxide at 300 K and 1 atm has a thermal conductivity of 0.0166 W/m-K (Green and Perry, 2007). Thermal conductivity is a material property.

Heat transfer coefficient

The heat transfer coefficient (h, dimensions of energy/time-length^2-temperature-interval) is a measure of the rate at which heat is transferred from a surface into a bulk fluid. The defining equation for heat transfer coefficient is Newton's law of cooling:

$$\left|\frac{q_x}{A}\right|_{surface} = h\left|T_{surface} - T_{bulk}\right| \tag{2}$$

where $q_x/A|_{surface}$ is the heat flux at the boundary, $T_{surface}$ and T_{bulk} are the surface and bulk phase temperatures, and h is the heat transfer coefficient. Heat transfer coefficient is not a material property; it is a property of a situation, that is, it reflects a particular surface and fluid and how they are in contact. The mechanism for heat transfer from the surface to the fluid may be convection (dominated by the flow of the fluid, natural or forced), conduction (due to the thermal conductivity of the fluid), or may be due to other mechanisms such as radiation (Bird et al., 2002; Geankoplis, 2003).

As an example of heat transfer from a surface to a bulk fluid, consider an automobile radiator: heat is transferred from the hot radiator fluid (antifreeze) to the inside of the radiator wall by a combination of convection and conduction, then conducted through the radiator wall and fins, to the outside wall of the radiator, is transferred from the outside wall to the outside air, and finally is convected away by the flow of air past the fins. There are different surface-to-bulk-fluid heat-transfer coefficients on the antifreeze side and on the air side. The value of h depends both on properties of the fluid (*e.g.*, antifreeze *vs.* air) as well as the system hydrodynamics—for example, the purpose of the fan in an automobile is to increase the value of h between the fins and the air.

While h can be calculated in a few special cases (*e.g.*, laminar flow in a tube with uniform heat flux through the wall, see Bird et al., 2002), h is usually determined by experiment. For many common geometries, correlations have been developed from experimental measurements which allow convenient estimation of h, although perhaps with limited accuracy. Correlations are generally expressed in terms of dimensionless variables (Reynolds, Nusselt and Prandtl numbers, as determined by non-dimensionalization of the governing equations; see Bird et al., 2002) so as to allow correlation of the behavior of fluids with a wide variety of physical properties.

Resistance to heat transfer and Biot number

Heat exchange between two working fluids is done in a heat exchanger where the two fluids are separated by a solid, such as in a shell-and-tube heat exchanger (Geankoplis, 2003). The overall heat transfer between the two bulk fluids is a coupling of a series of heat transfer steps. Heat transfer from one medium to another almost always involves the convection of heat from a fluid to a solid, followed by thermal conduction through the solid, and then heat transfer from the solid to another fluid. Each of these heat transfer steps can be associated with a resistance to heat transfer, and the overall resistance to heat transfer is the sum of the resistances. The Biot number Bi is a ratio of internal to external heat transfer resistances in a particular system.

$$Bi \equiv \frac{ha}{k} \tag{3}$$

where h is the heat transfer coefficient (characterizes transport from solid surface to bulk), k is the thermal conductivity of the solid, and a is a characteristic length of the solid object (in our case, the radius of the cylinder).

Consider the case of heat transfer to a cylinder. Initially the cylinder is at a uniform temperature. Suddenly the cylinder is dropped into a well mixed liquid that is maintained at a constant bulk temperature that is higher than the initial cylinder temperature. If the cylinder has a low thermal conductivity, the value of Bi is high. As $\mathrm{Bi}\rightarrow\infty$, heat can transfer easily from the fluid to the surface of the cylinder, but the overall rate of heat transfer is limited by the slow conduction within the cylinder. This situation is usually referred to as being *internally limited* or *internally controlled*. In this case there will be a significant radial temperature gradient internal to the cylinder when the system is not at equilibrium.

Conversely, if the cylinder has a very high thermal conductivity, then $Bi \rightarrow 0$; in this case heat transfer is facile within the cylinder, and most of the resistance to heat transfer occurs in transferring heat from the fluid to the cylinder (governed by the heat transfer coefficient, h). This situation is usually referred to as being *externally limited* or *externally controlled*. In this case the radial temperature gradient internal to the cylinder will be negligible, while a significant temperature drop will occur across the boundary layer between the fluid and the surface of the cylinder.

We can measure heat transfer properties of a system (h) or of a material (k) by performing the cylinder experiments described above. Appendix D describes five methods that may be used to obtain the values from experiments. The methods are not all applicable or equally accurate in all cases (internally controlled versus eternally controlled, for example). Experimenters need to choose wisely from among the available methods.

Experimental guidelines

Small cylindrical samples of the materials of construction of the test pieces are available for you to directly measure density and heat capacity.

Consider how to maintain the initial condition of the test pieces (room temp)

Consider how long it takes to regain the initial condition after a test.

Try to space out the Biot numbers you investigate

Externally controlled systems (Small Biot numbers): Measurement of h

- Vary diameter, flow rate
- Perform replicates
- Suggestion: if you block the runs by flow rate (i.e., run four different rods at one flow rate, then all four at the next flow rate, etc.) the time between runs on a given rod may be sufficient for it to cool completely to room temperature.
- Once you have values of h for the externally controlled case, use it to evaluate the dependence of h on system hydrodynamics, and estimate Bi for all other test pieces

Internally controlled systems (large Biot numbers): Measurement of h

- These experiments are more time consuming
- Estimate time to steady state before coming to lab
- Pick flow rates that will yield large Biot number, Perform replicates

Neither externally nor internally controlled systems (Bi≈1)

- Look for test conditions for this range
- Perform replicates

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APPENDICES

Appendix A: Heat Transfer Lab Equipment Inventory

Appendix B: Heat Transfer Standard Operating Procedure

Appendix C: Exact Solution to Transient Radial Heat Conduction in a Cylinder

Appendix D: Analysis Methods to Determine Heat Transfer Coefficient from Transient Data on a Cylinder Suddenly Immersed in a Fluid

Appendix E: Material Data

Appendix F: Convective Heat Transfer Correlations

Appendix A: Heat Transfer Lab Equipment Inventory

Omegatherm 201:.High temperature high thermal conductivity paste
Omega Engineering Inc One Omega Drive
Stamford,CT 06907
(203) 359-1660

GE Silicone II Clear RTV Sealant: Wal-Mart- approximately $2.00

Jeweler's screwdrivers: for repairing thermocouples,
purchased at Home Depot (approximately $5.00)

Appendix B: Heat Transfer Operating Procedure (CBE346)

Apparatus Description

A schematic of the Heat Transfer experimental apparatus is shown in Figure B1. The apparatus consists of a large thermostated reservoir containing hot water, which is pumped through a smaller vessel ("test reservoir") that is able to hold test pieces of various shapes and compositions. The test pieces have a dedicated thermocouple embedded along the axis of cylindrical symmetry. The hot water flow rate through the test reservoir is controlled by two ball valves (the metering valves) and measured using two rotameters in parallel (this design was dictated by the available capacity of the rotameters). In the test reservoir, hot water flows coaxially with the test shape, which is mounted such that it is centered in the cylindrical test reservoir. The test reservoir has an inner diameter of 95.0 mm. A bypass path diverts water from the pump back to the main reservoir to modulate water flow rate and to prevent pump dead heading during sample loading.

Cylindrical test pieces are available in two rod diameters (either 1 or 2" outer diameter) and

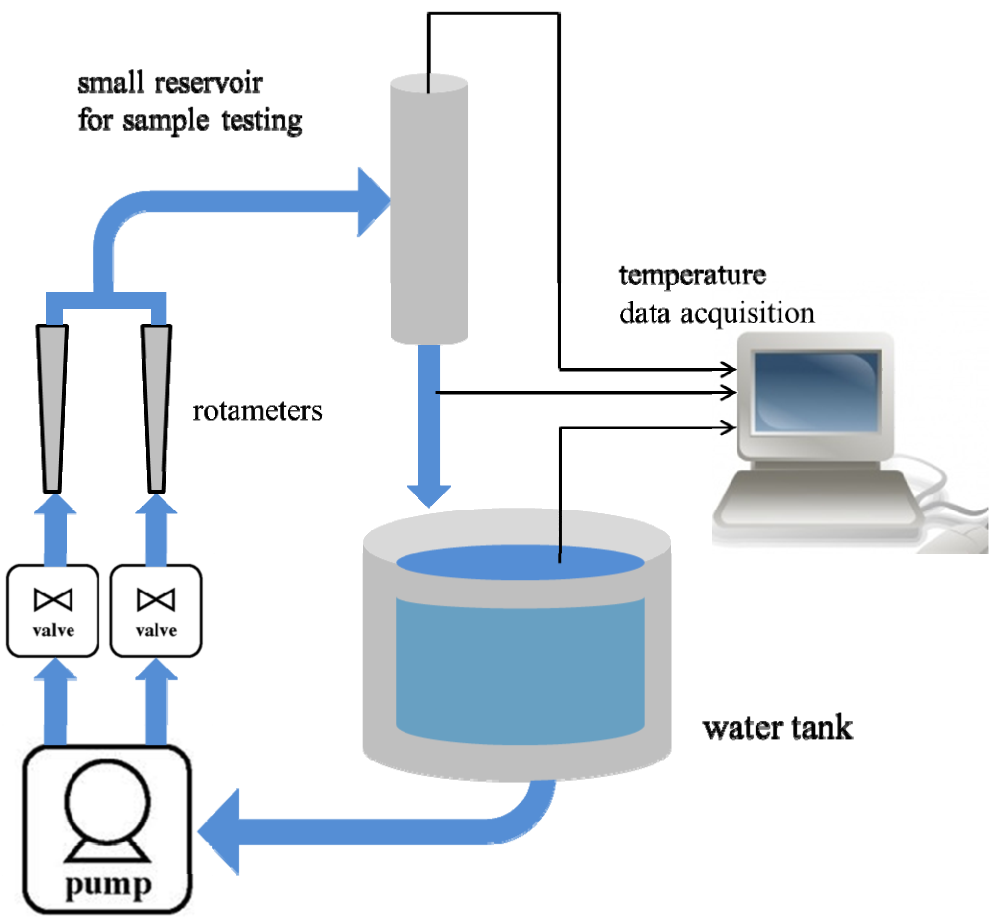

Figure B1: Equipment diagram for Heat Transfer experiment in CBE346 CBE Laboratory

Table B1: Materials of Construction of Test Pieces
1. Copper
2. Aluminum
3. Polymethylmethacrylate (PMMA)
4. Stainless Steel
5. Machinable Ceramic
6. Phenolic Resin-Cloth Composite
7. Aluminum over Plexiglas
8. Plexiglas over Aluminum

are made of a variety of materials (Table B1). There are also test pieces that are not simple cylinders.

Standard Operating Procedure:

1. **Prepare the system for operation**. Seal the empty test reservoir with a cap clamped securely, and verify that the two metering valves (ball valves with yellow grips) are closed. You will not need to touch any other valves for your experiments. Make sure the water level in the water bath is at least an inch above the liquid-level safety shut-off switch (a white cylindrical device mounted inside the tank for safety measures—the power of the apparatus will automatically shut off if the liquid level falls below the switch).

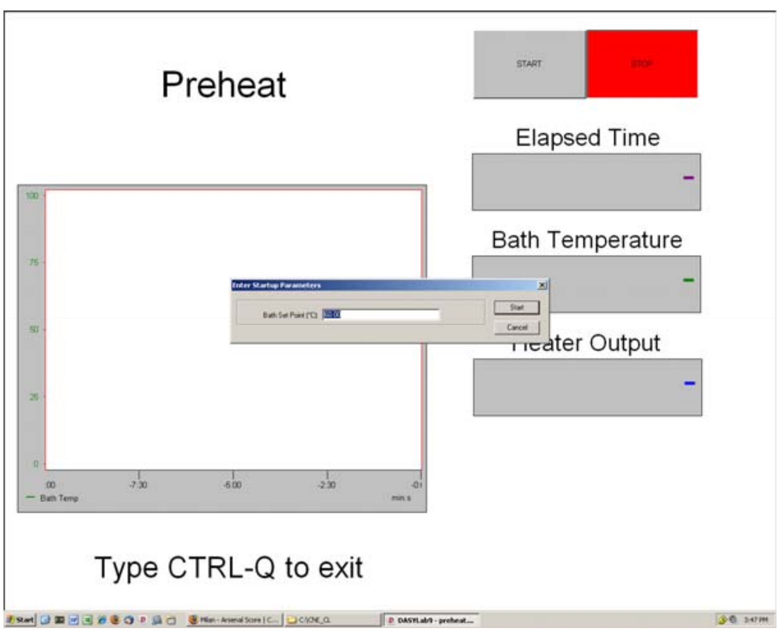

Figure B2: Screen shot whenPreheat.dsb is first started up.

2. **Heat the water in the main reservoir**. To start up the heating system, push the Main Power START button located in the middle of the control panel, and the red power indicator will light up. The fault light should also blink once. The fault light indicate either a low water level in the reservoir or a high reservoir tem condition (above 75^{o}C). If the fault light is on at any time during the experiment and neither of these conditions is met, notify your AI immediately. Switch on the Heater Enable and Stirrer for the main reservoir. Do not turn on the pump at this time. To preheat your bath, open up PREHEAT.DSB in directory C:\ChE_CL. Click on the Start button. The program will prompt you to set bath temperature. Set the heater bath temperature controller to the desired set point (for example 60^{o}C). It takes about 40 minutes for the bath temperature to completely stabilize. (You may wish to come to the lab a half an hour before the lab start time to preheat the water; see the Safety Manual for the procedure for unattended operation of laboratory equipment). Due to evaporative losses, check the water level often throughout the day and top off the water as need.

3. **Preheat the test reservoir and flow system**. When the bath temperature is at temperature or getting close to the set point (say within 2-3 degrees), turn on the pump. Adjust the metering valves to set the flow rate to 1 gal/min in order to preheat the test reservoir. Never leave a running pump unattended. Allow the test reservoir and flow system to come to thermodynamic equilibrium at the test temperature. It is up to you to validate that you have chosen this thermal soak time wisely.

4. **Set up the data acquisition program**. Once the temperature of the reservoirs is fully stabilized, quit PREHEAT.DSB and open the data acquisition program HOTROD.DSB. This program logs the data from the centerline temperature of the test piece when it is installed. You will be prompted for a bath temperature and for a file location where data will be saved. Data will be acquired at a rate fixed by the program (1 Hz). It is strongly suggested that you store the data on a flash drive, or FTP it to your PU account at the end of the lab period. The data acquisition rate in HOTROD.DSB is set at 1 Hz, which is adequate for most runs. If you wish to take more frequent data points for test pieces that exhibit fast equilibration there is another program called “HOTROD 10Hz” in the same folder with a data acquisition rate of 10 Hz.

5. **Install the test piece and begin a run.** Plug the thermocouple of the test piece to be examined into the control board. Wait until you get a good steady temperature reading for the cylinder on the HOTROD program prior to beginning your run. To begin a run, one partner shuts any open metering valves and then plunges the sample cylinder into the test reservoir and seals the top while the other partner records the starting time reading in the notebook. It is important to insert the test piece into the reservoir quickly so that the initial condition in the modeling analysis is accurately reflected in the experimental process. *Warning: inserting the test piece too rapidly may result in slight overflow of water; take appropriate safety precaution to avoid slip hazards and electrical hazards.* Make sure the cylinder is tightly clamped in place before directing water flow through the test reservoir with the metering valves.

6. **Record the flow rate using the scales on the rotameters**. The rotameter should be read at the top of the “cap” on the float

7. **End a run**. Take data until the dimensionless test piece temperature is within 2% of the bath temperature, or reaches steady state (the test piece and bath temperatures are the same). Because of small differences in the thermocouples and the amplifier circuits the temperature measurements are only accurate to +/- 1ºC. When you are satisfied that you have taken sufficient data for your purposes, terminate the data logging by pressing the STOP button.
8. **Remove the test piece**. At the end of the run, close the metering valves and turn off the pump before removing the test piece. Clean up all water spills.

Shut-Down Procedure:

1. Close the metering valves and turn off the pump and remove any test piece that is present.
2. Close the test reservoir with the cap and turn off the system.
3. Before you leave, check the water level in the tank to make sure there is adequate water to cover the automatic shut-off switch.
4. Turn off the Stirrer and Heater Enable.
5. Power off the system with the Main Power STOP button.
6. Log off of the computer.

Emergency Shut-Down Procedure:

1. Power off the system with the Main Power STOP button. This will cut power to the pump, heaters, and thermocouples.

Appendix C: Exact Solution to Transient Radial Heat Conduction in a Cylinder

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The governing equation for heat conduction in an infinitely-long solid cylinder, as shown schematically in Figure C1, is the microscopic energy balance ($\underline{v} = 0, \theta$ symmetry, no axial conduction; Carslaw and Jaeger, 1946; Bird et al., 2002):

$$\frac{\partial T}{\partial t} = \frac{\alpha}{r} \frac{\partial}{\partial r}\left(r \frac{\partial T}{\partial r} \right) \tag{C1}$$

where T is the temperature in the cylinder at any radial distance r and time t, and α is the thermal diffusivity, $\alpha = k/\rho C_p$, where k is thermal conductivity, ρ is density, and C_p is heat capacity of the rod. To obtain a solution to this partial differential equation we need an initial condition and two boundary conditions.

Consider the solution to this equation corresponding to an idealized experiment. The rod is initially at uniform temperature T_i, and it is submerged at time $t = 0$ into an infinite constant-temperature bath that is perfectly mixed. The initial condition is thus written as:

$$T = T_i \text{ for } 0 \le r \le a \text{ at } t = 0 \tag{C2}$$

The first boundary condition for our idealized experiment is Newton's Law of Cooling, namely that the heat flux across the fluid-solid interface is proportional to the temperature difference between the temperature of the solid surface and the bulk fluid temperature T_0:

Boundary condition 1: $$-k \frac{\partial T}{\partial r} = h\left(T - T_o\right) \qquad at \quad r = a \tag{C3}$$

where a is the radius of the cylinder, and this equation serves as the definition of the heat transfer coefficient h. The second boundary condition is symmetry at the centerline of the cylinder:

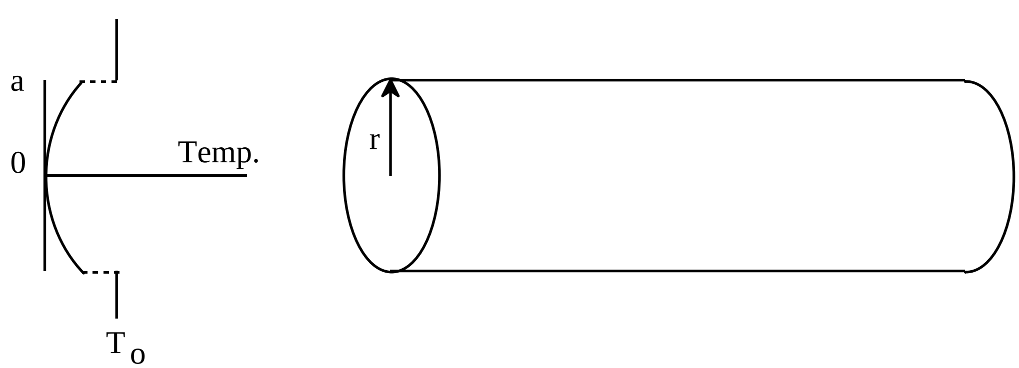


Figure C1: Schematic of heat conduction in a long solid cylinder of radius a. Unsteady-state radial temperature profile is sketched at left. Fluid surrounding the cylinder is assumed to be at a uniform bulk temperature T_0.

Boundary condition 2: $$\frac{\partial T}{\partial r} = 0 \qquad at \;\; r = 0 \tag{C4}$$

The partial differential equation (C1) can be solved by the method of separation of variables. This is a classic problem, so the complete solution can be found in a number of references on heat transfer (*e.g.*, Carslaw and Jaeger, 1946; Arpaci, 1966). The solution is an infinite series given in dimensionless form as:

$$\Theta(R,\tau) = \sum_{i=1}^{\infty} e^{-\beta_i^2 \tau} \frac{2Bi J_0(R\beta_i)}{\{\beta_i^2 + Bi^2\} J_0(\beta_i)} \tag{C5}$$

where: Θ = $\frac{(T - T_o)}{(T_i - T_o)}$, dimensionless temperature at any R and τ

τ = $\alpha t/a^2$, Fourier number, a dimensionless time (characteristic time for this conduction problem is a^2/α)

α = $k/\rho c_p$, thermal diffusivity, dimensions of length^2/time

ρ = mass density of rod material, dimensions of mass/length^3

c_p = heat capacity of rod material, dimensions of energy/mass-temperature

R = r/a, dimensionless radial distance

J_0 and J_1 are Bessel functions of the first kind (of order zero and one, respectively)

$Bi = ha/k$, Biot number for this problem; Biot number is the ratio of the heat-transfer resistances inside of and at the surface of a body

β_i are the eigenvalues to this heat transfer problem, and are roots to the characteristic equation:

$$f(\beta) = \beta J_1(\beta) - Bi\, J_0(\beta) = 0 \tag{C6}$$

Bessel functions may be thought of simply as tabulated functions, just like trigonometric functions. A good discussion of Bessel functions may be found in Hildebrand (1976). Bessel functions of the first kind (and integral order) typically arise in problems having cylindrical symmetry, as is the case here. Qualitatively, Bessel functions resemble sine or cosines functions multiplied by a decaying exponential. Tables of $J_0(x)$ and $J_1(x)$ may be found in the literature (Abramowitz and Stegun, 1964) or values may be obtained in MS Excel with the function $J_n = \text{BESSELJ}(\text{x}, \text{n})$.

To visualize the roots of equation C6, we plot the function $f(\beta)$ and note where the function crosses the x-axis. For Bi=0, the characteristic equation is shown in Figure C2. The roots of equation C6 may be calculated by numerically solving the equation $f(\beta) = 0$ for the various β_i, and these are also tabulated in the literature; the first six roots are given in Table C1.

We can plot the solution to the cylinder heat transfer model equation (equation C5) using computer software (Excel, Matlab, Mathematica, for example). The result is $\Theta(R,\tau)$ in dimensionless form or $T(r,t)$ in dimensional form and is a complex three-dimensional function. The material response to the proposed experiments fall into two categories: a response that

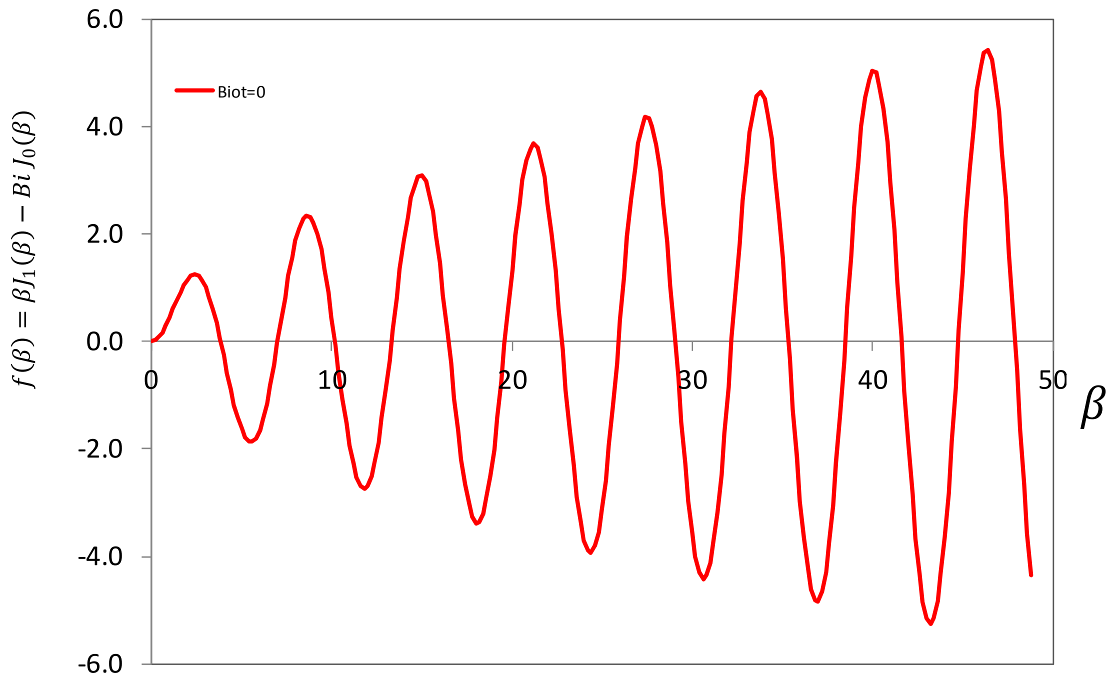


Figure C2: The characteristic equation $f(\beta)$ for the eigenvalues of the problem of heat conduction from a rod with Newton's law of cooling boundary conditions (Bi=0). The roots of the equation are where it crosses the x-axis; these roots correspond to the first row in Table C1.

exhibits internal resistance (internally controlled); and a response that exhibits no internal resistance (externally controlled).

When the heat transfer exhibits internal resistance (high Biot number), this means that the thermal conductivity of the rod is sufficiently low (relative to h) that temperature varies within the rod (Figure C3a) and the wall temperature is always equal to the bulk fluid temperature. If thermal conductivity is high, however (low Biot number), the temperature equilibrates rapidly within the rod and everywhere in the rod the temperature is equal to the wall temperature (Figure C3b) and the wall temperature varies with time depending on the heat transfer coefficient h. At intermediate Biot number, the behavior exhibits sensitivity to both internal and external resistances and the temperature varies within the rod and at the wall (Figure C4).

When analyzing $T(0,t)$ data to obtain h, we can always fit the complete model to the data and obtain a best fit for h; this is unnecessarily complex, however. Approximations to the exact solution that are appropriate to our experiments are described in Appendix D.

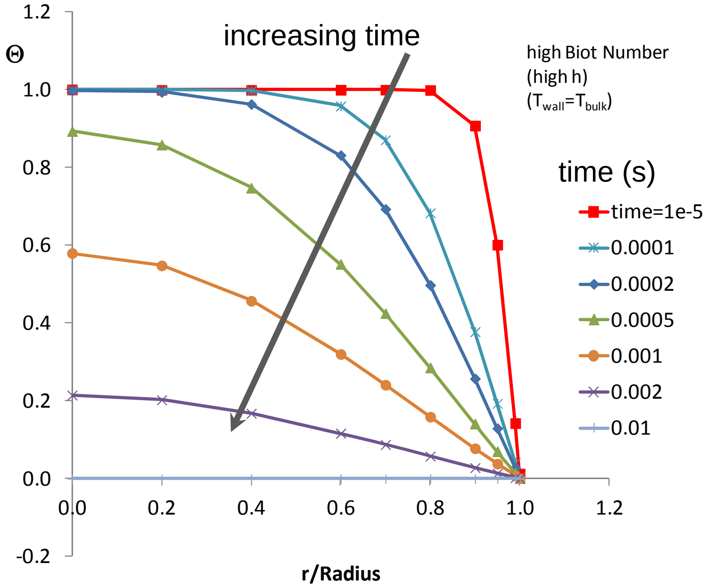
Table C1: The first six roots of $\beta_i J_1(\alpha) - Bi J_0(\beta_i) = 0$ as tabulated in the literature (Carslaw and Jaeger, 1946). Note that α_n in this table corresponds to β_i in this document and C corresponds to the Biot number, Bi.

The first six roots, α_n, of

$$\alpha J_1(\alpha) - C J_0(\alpha) = 0.$$

C	α_1	α_2	α_3	α_4	α_5	α_6
0	0	3·8317	7·0156	10·1735	13·3237	16·4706
0·01	0·1412	3·8343	7·0170	10·1745	13·3244	16·4712
0·02	0·1995	3·8369	7·0184	10·1754	13·3252	16·4718
0·04	0·2814	3·8421	7·0213	10·1774	13·3267	16·4731
0·06	0·3438	3·8473	7·0241	10·1794	13·3282	16·4743
0·08	0·3960	3·8525	7·0270	10·1813	13·3297	16·4755
0·1	0·4417	3·8577	7·0298	10·1833	13·3312	16·4767
0·15	0·5376	3·8706	7·0369	10·1882	13·3349	16·4797
0·2	0·6170	3·8835	7·0440	10·1931	13·3387	16·4828
0·3	0 7465	3·9091	7·0582	10·2029	13·3462	16·4888
0·4	0·8516	3·9344	7·0723	10·2127	13·3537	16·4949
0·5	0·9408	3·9594	7·0864	10·2225	13·3611	16·5010
0·6	1·0184	3·9841	7·1004	10·2322	13·3686	16·5070
0·7	1·0873	4·0085	7·1143	10·2419	13·3761	16·5131
0·8	1·1490	4·0325	7·1282	10·2516	13·3835	16·5191
0·9	1·2048	4·0562	7·1421	10·2613	13·3910	16·5251
1·0	1·2558	4·0795	7·1558	10·2710	13·3984	16·5312
1·5	1·4569	4·1902	7·2233	10·3188	13·4353	16·5612
2·0	1·5994	4·2910	7·2884	10·3658	13·4719	16·5910
3·0	1·7887	4·4634	7·4103	10·4566	13·5434	16·6499
4·0	1·9081	4·6018	7·5201	10·5423	13·6125	16·7073
5·0	1·9898	4·7131	7·6177	10·6223	13·6786	16·7630
6·0	2·0490	4·8033	7·7039	10·6964	13·7414	16·8168
7·0	2·0937	4·8772	7·7797	10·7646	13·8008	16·8684
8·0	2·1286	4·9384	7·8464	10·8271	13·8566	16·9179
9·0	2·1566	4·9897	7·9051	10·8842	13·9090	16·9650
10·0	2·1795	5·0332	7·9569	10·9363	13·9580	17·0099
15·0	2·2509	5·1773	8·1422	11·1367	14·1576	17·2008
20·0	2·2880	5·2568	8·2534	11·2677	14·2983	17·3442
30·0	2·3261	5·3410	8·3771	11·4221	14·4748	17·5348
40·0	2·3455	5·3846	8·4432	11·5081	14·5774	17·6508
50·0	2·3572	5·4112	8·4840	11·5621	14·6433	17·7272
60·0	2·3651	5·4291	8·5116	11·5990	14·6889	17·7807
80·0	2·3750	5·4516	8·5466	11·6461	14·7475	17·8502
100·0	2·3809	5·4652	8·5678	11·6747	14·7834	17·8931
∞	2·4048	5·5201	8·6537	11·7915	14·9309	18·0711

a)



b)

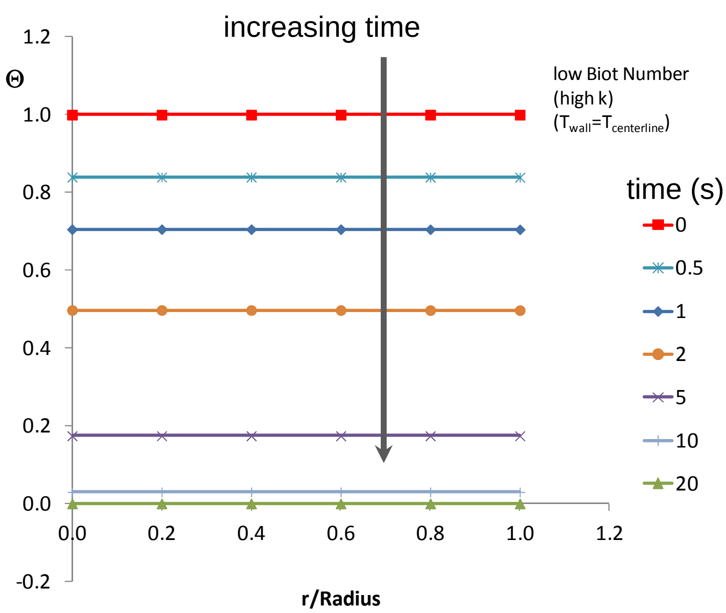


Figure C3: If the heat transfer to a rod is internally controlled (top, a, $Bi = 1000$), the temperature varies within the rod; for this case the wall temperature ($r/a = 1$) is equal to the bulk temperature of the surrounding fluid. If the heat transfer is externally controlled ($Bi = 0.001$), the temperature is uniform within the rod and the wall temperature varies with time as heat moves between the fluid and the rod. All points shown were calculated from the exact solution (Equation C5).

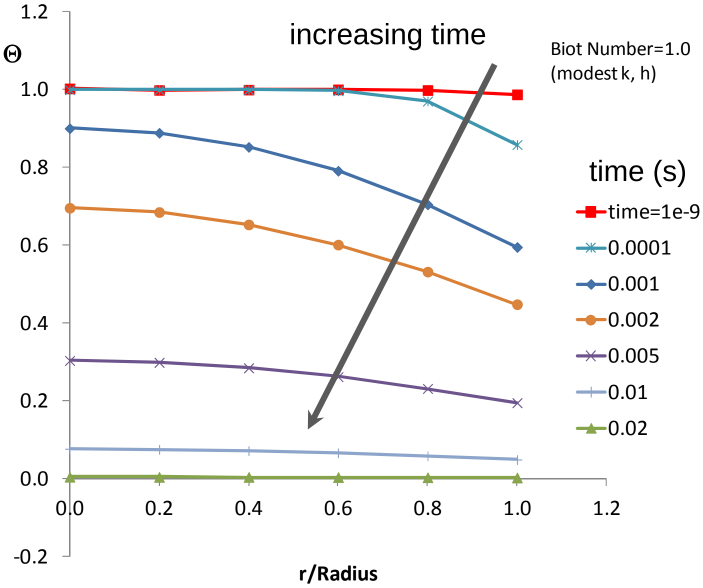


Figure C4: When the Biot number is neither high nor low ($Bi = 1.0$ shown), both the temperature profile shape and the wall temperature vary with time. All points shown were calculated from the exact solution (Equation C5).

Appendix D: Five Analysis Methods to Determine Heat Transfer Coefficient from Transient Data on a Cylinder Suddenly Immersed in a Fluid

Method 1: Long-Time Behavior

(Faith Morrison, 2013; includes some text from Ilhan Aksay, 2012)

The solution to the heat-transfer problem that is represented by the laboratory experiments is given in Appendix C. In the experimental apparatus, the temperature is measured at the rod centerline (R = 0), so the PDE solution (infinite sum) simplifies to:

$$\Theta(0,\tau) = \theta(\tau) = \sum_{i=1}^{\infty} e^{-\beta_i^2 \tau} \frac{2Bi}{\{\beta_i^2 + Bi^2\} J_0(\beta_i)}, \tag{D1}$$

$$\beta_i J_1(\beta_i) - Bi J_0(\beta_i) = 0 \tag{D2}$$

where $\theta(\tau)$ refers to the centerline temperature. There are an infinite number of roots, β_i, to equation D2, each corresponding to one term in the infinite series. All the roots are positive and increase with an approximate spacing of π. A tabulation of the first six roots to equation (D2) as a function of the Biot number, Bi, is given in Appendix C (Table C1) (from Carslaw and Jaeger, 1946).

We can use Excel (or similarly capable software) to plot the solution $\theta(\tau)$; the first five terms in the series are plotted in Figure D1 as a function of Fourier number τ. The first term is positive and is by far the largest term; the terms alternate in sign. As shown in Figure D1, for Fourier number $Fo = \alpha t/a^2$ greater than 0.2, the leading order term dominates.

If we discard all terms higher than i = 1 and take the logarithm of both sides of equation (D1), we obtain:

$$\ln\theta = -\beta_1^2\left(\frac{\alpha t}{a^2}\right) + \left\{\ln(2Bi) - \ln\left(\beta_1^2 + Bi^2\right) - \ln J_o(\beta_1)\right\} \tag{D3}$$

This is the equation of a straight line for $\ln\theta$ *vs.* time with slope $S = -\beta_1^2\alpha/a^2$, and intercept I equal to the quantity contained between { }. The slope is a function of Biot number (through β_1) and thermal diffusivity α; the intercept is only a function of Biot number. One can envision the following procedure for the determination of h:

1) Produce the flow we are modeling (rapid immersion of a cylinder in a well mixed fluid) and measure cylinder centerline temperature *vs.* time (to obtain θ(t)), taking data out to long time $Fo > 0.2$), where $\ln\theta$ *vs.* time is a straight line and equation D3 holds.

2) Fit the experimental data to equation D3 to obtain the slope S.

3) If $\alpha = k/\rho C_p$ is known, obtain β_1. From tables or a correlation (see Figure D2) obtain the Biot number $Bi = ha/k$ from β_1 and from Bi calculate the heat transfer coefficient.

Uncertainty in the experimental starting conditions introduces a large amount of error into a determination of the intercept I of equation D3. If the intercept could be accurately measured, we could determine both Bi and α from the data.

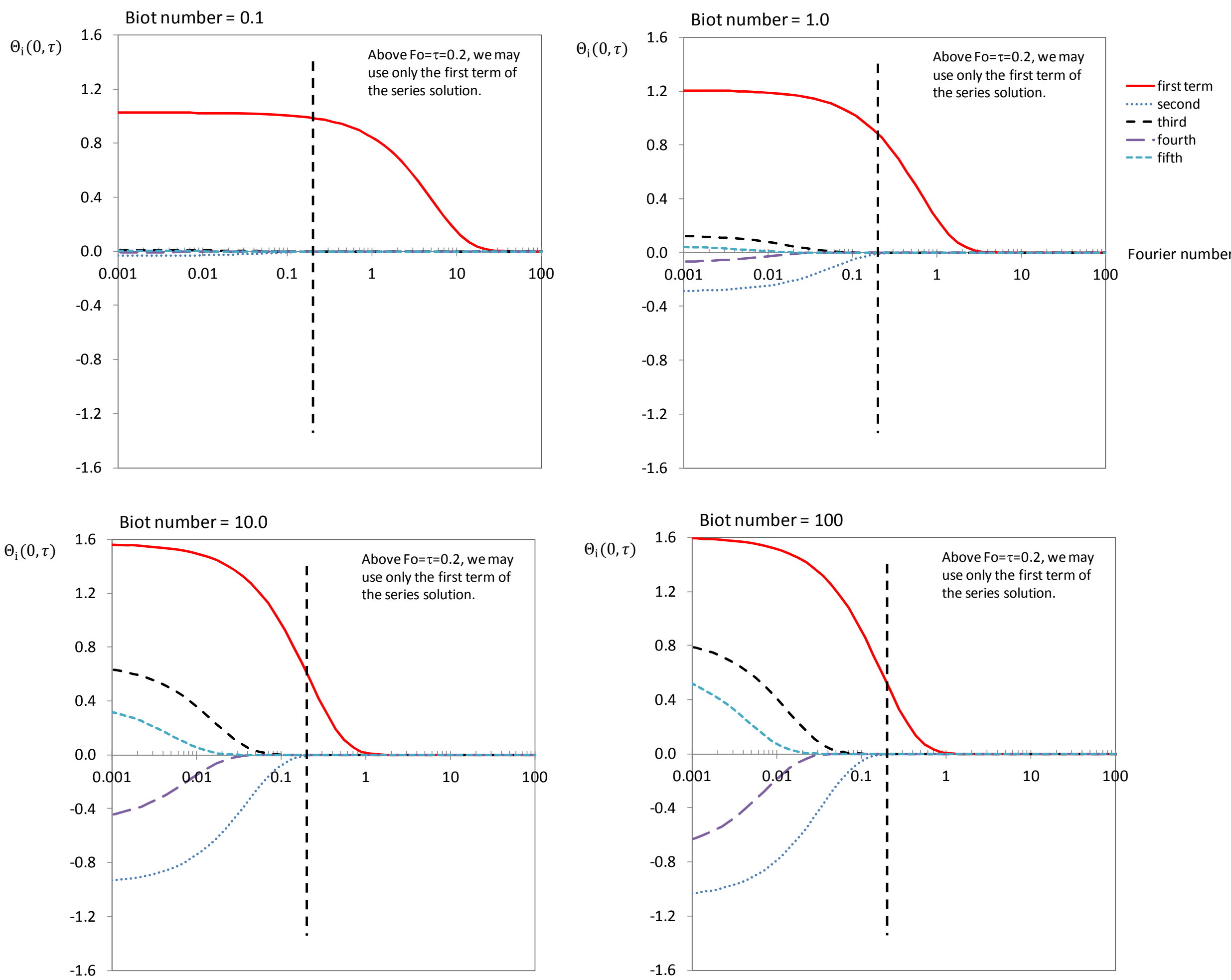


Figure D1: The contribution of the higher-order terms increases with Biot number, but their significance decreases with Fourier number, $Fo = \alpha t/a^2$ (scaled time). For Fourier number greater than 0.2 we need not consider the higher order terms. All points shown were calculated from the exact solution (Equation C5).

The slope $S(Bi, \alpha)$, which measures the overall rate of heat transfer to the rod center, is governed predominantly by which process (conduction within the rod or convection in the fluid to the rod surface) is limiting. That is, if the system is externally controlled, then the slope predominantly reflects h and should not depend strongly on k; conversely, if the system is internally controlled, the slope is predominantly sensitive to k and not h.

References:

Faith Morrison, "*Empirical fit of first eigenvalue of characteristic equation for heat transfer to a rod as a function of Biot number*," CBE346 Chemical Engineering Laboratory Handout for Heat Transfer, 11 March 2013, Princeton University, NJ; unpublished.

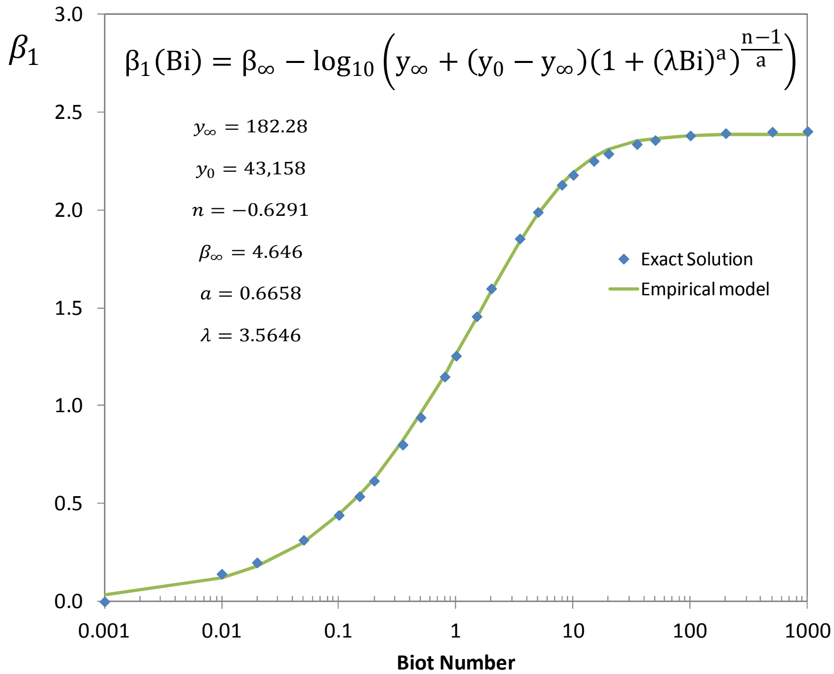


Figure D2: We can numerically solve the characteristic equation (Equation C6) for β_1 as a function of Biot number as was done to produce the published data in Table C1. With the data for $\beta_1(Bi)$ in hand, we can empirically fit an arbitrary function to the data to make it easier to use the $\beta_1(Bi)$ data. Morrison (2013) performed such a fit, which is shown above.

Method 2: Asymptotic Expressions (Good for low and high Bi)

(I. Aksay and prior CBE346 faculty, 2012)

External control

Consider first the case of external control ($Bi \to 0$). An inspection of the table of β_1 values (Appendix C) reveals that, in this limit, β_1 approaches zero as well. Examination of the Bessel function values (Table D1) reveals the following limiting behavior for $J_0(\beta_1)$ and $J_1(\beta_1)$ as $\beta_1 \to 0$:

$$J_0(\beta_1) \to 1 - (\beta_1/2)^2 \qquad \text{(D4)}$$

$$J_1(\beta_1) \to \beta_1/2 \qquad \text{(D5)}$$

Therefore, from the characteristic equation (C6):

$$Bi = \beta_1 J_1(\beta_1)/J_0(\beta_1) \approx \beta_1^2/2 \qquad \text{(D6)}$$

or:

$$\beta_1 = \sqrt{(2Bi)} \qquad \text{(D7)}$$

It then follows that the intercept approaches zero as $Bi \to 0$, so there is a large relative uncertainty in the intercept, and its value is unlikely to be significant. The slope S, however, can be determined accurately, and is given by:

$$S = -\beta_1^2 \alpha / a^2 \qquad \text{(D8)}$$

so, if α is known (*e.g.*, from independent experiments reported in the literature), a good value of Bi (and hence h) can be obtained (from equation D7) in this limit from:

$$Bi = -Sa^2 / 2\alpha \qquad \text{(D9)}$$

Any error in the values for ρ and c_p will propagate into the absolute values of h so calculated. However, if we are generally looking for *changes* in h (with flowrate, cylinder size, *etc.*), a systematic error of a few percent is not of concern. Using equation D9 we can find 'h' even without knowing 'k'.

Internal control

For the other extreme, internal control (Bi >>1), by inspection of the table of β_1 it is evident that at large Bi:

$$\beta_1 \approx 2.4048(1 - Bi^{-1}) \qquad \text{(D10)}$$

The limiting value, approximately 2.4048 (denoted as A below for convenience), is also the first root of J_0 (*i.e.*, $J_0(A) = 0$). Taking the large Bi limit, the slope S becomes:

$$S = -A^2 \alpha / a^2 \qquad \text{(D11)}$$

or:

$$\alpha = -Sa^2/A^2 \qquad \text{(D12)}$$

so, α can be obtained from the slope directly, although no information on *h* is obtained. In this case, the intercept is not small; however, it approaches a finite limit as *Bi* increases. By inspection of the table of J_0 in Table D1, the local slope $dJ_0(x)/dx$ in the vicinity of x = A is approximately 0.520. Therefore, for $Bi \to \infty$, from equation (15):

$$J_0(\beta_1) \approx J_0(A(1-Bi^{-1})) \approx 0.520A/Bi \tag{D13}$$

$$\ln(\beta_1^2 + Bi^2) \approx 2\ln(Bi) \tag{D14}$$

and the intercept I then becomes:

$$I \approx \ln(2/(0.520*2.4048)) \approx 0.470 \tag{D15}$$

All the information in *h* is thus tied up in how much the intercept I deviates from this limiting value of 0.47. When *Bi* is large, the extrapolation from the linear region of $\ln\theta$ to time zero is so long (in time) that it can introduce a substantial error in the intercept (relative to the difference between the real I and the limiting value of I), thus making an accurate determination of *h* difficult. If *h* is known from other sources (*e.g.*, measurements under identical conditions but with rods made from material such that the heat transfer is externally controlled), then it is possible to refine the estimate of α by calculating *Bi*, then calculating β_1 from equation (D10), then substituting this value (which will be slightly less than 2.4048) into equation D12).

All these approximations are, of course, asymptotically valid (exact only for *Bi* = 0 and ∞). If we consider a 10% departure from the exact result to be the limit of validity of these approximations, then the $Bi \to 0$ limit holds for $Bi < 0.4$; the extreme high *Bi* limit ($\beta_1 = 2.4048$) holds for $Bi > 20$; and the high *Bi* limit (equation (D10)) holds for $Bi > 4$. One should always check for self-consistency when using an asymptotic expression, i.e. one should check that the values of *h* and *k* which are obtained place the results in the appropriate regime of *Bi*.

Intermediate control

For $0.4 < Bi < 4$, there is no valid limiting approximation, and the full solution needs to be considered. There are two possibilities involving the asymptotic approach:

1) use the values of *S* and I, solving for both *h* and *k*;

2) if either *h* or *k* is known, use only the value of S to solve for the other (*h* or *k*).

To do either of these, simply use the tables provided (Table C1 and Table D1) and use an interpolation method.

To choose among the various methods we need to consider the uncertainty associated with the asymptotic approach.

Table D1: Bessel functions of orders 0, 1, 2; from Abramowitz and Stegun (1964). The same values may be generated with Excel's function BESSELJ(x,n).

Table 9.1 BESSEL FUNCTIONS—ORDERS 0, 1 AND 2

x	$J_0(x)$	$J_1(x)$	$J_2(x)$
0.0	1.00000 00000 00000	0.00000 00000	0.00000 00000
0.1	0.99750 15620 66040	0.04993 75260	0.00124 89587
0.2	0.99002 49722 39576	0.09950 08326	0.00498 33542
0.3	0.97762 62465 38296	0.14831 88163	0.01116 58619
0.4	0.96039 82266 59563	0.19602 65780	0.01973 46631
0.5	0.93846 98072 40813	0.24226 84577	0.03060 40235
0.6	0.91200 48634 97211	0.28670 09881	0.04366 50967
0.7	0.88120 08886 07405	0.32899 57415	0.05878 69444
0.8	0.84628 73527 50480	0.36884 20461	0.07581 77625
0.9	0.80752 37981 22545	0.40594 95461	0.09458 63043
1.0	0.76519 76865 57967	0.44005 05857	0.11490 34849
1.1	0.71962 20185 27511	0.47090 23949	0.13656 41540
1.2	0.67113 27442 64363	0.49828 90576	0.15934 90183
1.3	0.62008 59895 61509	0.52202 32474	0.18302 66988
1.4	0.56685 51203 74289	0.54194 77139	0.20735 58995
1.5	0.51182 76717 35918	0.55793 65079	0.23208 76721
1.6	0.45540 21676 39381	0.56989 59353	0.25696 77514
1.7	0.39798 48594 46109	0.57776 52315	0.28173 89424
1.8	0.33998 64110 42558	0.58151 69517	0.30614 35353
1.9	0.28181 85593 74385	0.58115 70727	0.32992 57277
2.0	0.22389 07791 41236	0.57672 48078	0.35283 40286
2.1	0.16660 69803 31990	0.56829 21358	0.37462 36252
2.2	0.11036 22669 22174	0.55596 30498	0.39505 86875
2.3	0.05553 97844 45602	0.53987 25326	0.41391 45917
2.4	+0.00250 76832 97244	0.52018 52682	0.43098 00402
2.5	−0.04838 37764 68198	0.49709 41025	0.44605 90584
2.6	−0.09680 49543 97038	0.47081 82665	0.45897 28517
2.7	−0.14244 93700 46012	0.44160 13791	0.46956 15027
2.8	−0.18503 60333 64387	0.40970 92469	0.47768 54954
2.9	−0.22431 15457 91968	0.37542 74818	0.48322 70505
3.0	−0.26005 19549 01933	0.33905 89585	0.48609 12606
3.1	−0.29206 43476 50698	0.30092 11331	0.48620 70142
3.2	−0.32018 81696 57123	0.26134 32488	0.48352 77001
3.3	−0.34429 62603 98885	0.22066 34530	0.47803 16865
3.4	−0.36429 55967 62000	0.17922 58517	0.46972 25683
3.5	−0.38012 77399 87263	0.13737 75274	0.45862 91842
3.6	−0.39176 89837 00798	0.09546 55472	0.44480 53988
3.7	−0.39923 02033 71191	0.05383 39877	0.42832 96562
3.8	−0.40255 64101 78564	+0.01282 10029	0.40930 43065
3.9	−0.40182 60148 87640	−0.02724 40396	0.38785 47125
4.0	−0.39714 98098 63847	−0.06604 33280	0.36412 81459
4.1	−0.38866 96798 35854	−0.10327 32577	0.33829 24809
4.2	−0.37655 70543 67568	−0.13864 69421	0.31053 47010
4.3	−0.36101 11172 36535	−0.17189 65602	0.28105 92288
4.4	−0.34225 67900 03886	−0.20277 55219	0.25008 60982
4.5	−0.32054 25089 85121	−0.23106 04319	0.21784 89837
4.6	−0.29613 78165 74141	−0.25655 28361	0.18459 31052
4.7	−0.26933 07894 19753	−0.27908 07358	0.15057 30295
4.8	−0.24042 53272 91183	−0.29849 98581	0.11605 03864
4.9	−0.20973 83275 85326	−0.31469 46710	0.08129 15231

Method 3: Heisler Chart

(Faith A. Morrison, 2013)

A traditional approach to one-dimensional heat conduction is to use the Heisler charts (Heisler, 1947). These plots (see Figure D2, which was taken from Geankoplis, 2003) were created using a one-term truncation of the exact solution (see Appendix C). Heisler charts are only applicable for Fourier number $Fo = \alpha t / a^2$ greater than 0.2. The parameter α is the thermal diffusivity and a is the characteristic lengthscale for conduction (the cylinder radius).

Data from the lab may be re-cast and plotted on the same type of axes as used in the Heisler chart. The value of $m = k/hx_1 = k/ha$ is deduced by comparing the data with the chart: the m of the experiment is that associated with the line that most closely matches the measurements (note that m is not the slope of the line; it is the inverse Biot number, $m = 1/Bi$.). Once m is known, h may be deduced.

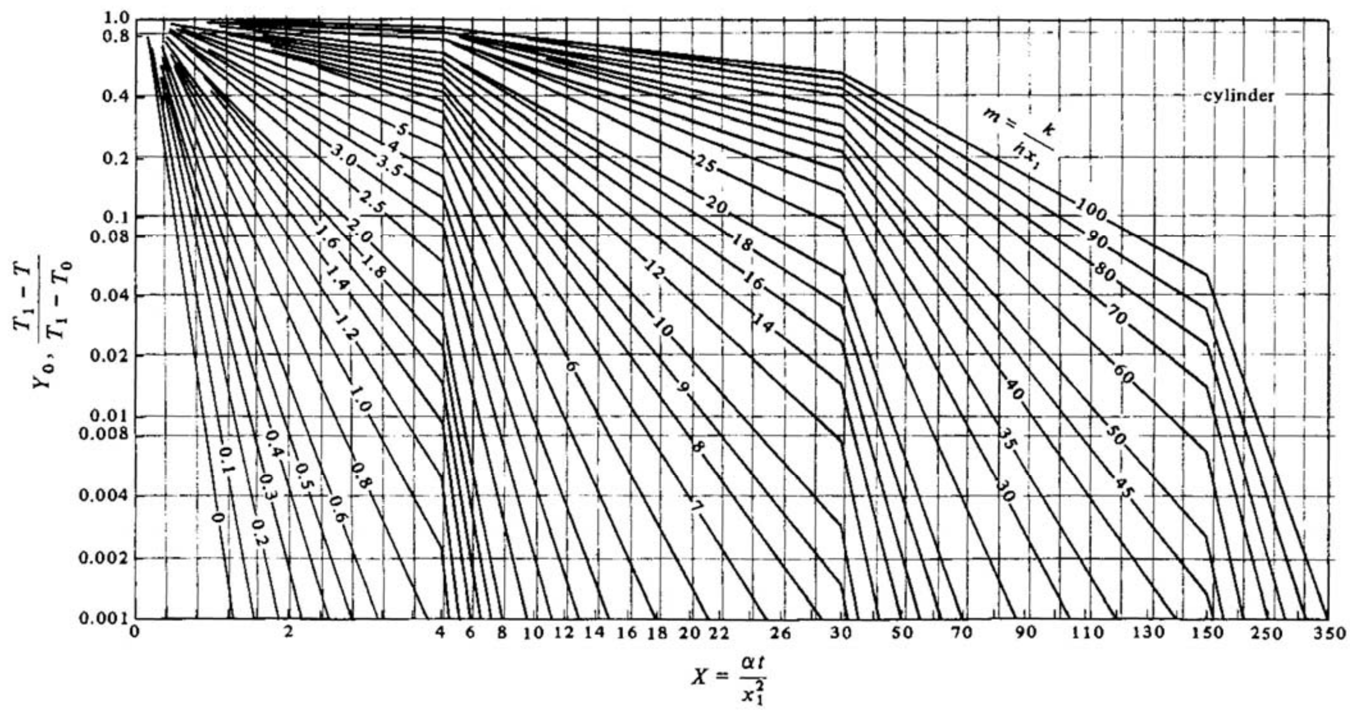


Figure D2: Heisler chart for determining temperature at the center of a long cylinder for unsteady-state heat conduction (Heisler, 1947); x_1 is the cylinder radius, and thus the abscissa is the Fourier number. Reproduced from Geankoplis (2003).

Method 4: Use general computer software to plot the solution to the governing equation

Laboratory group members who are sufficiently proficient in Excel, Mathematica, or Matlab, may choose to use these programs to fit the exact solution to the data directly. Comsol Multiphysics may also be used to solve the problem numerically.

Appendix E: Material Data

Suggested values of ρ, c_p, and k for Al, Cu, stainless steel, and PMMA are given in Table E1, with sources. Be aware that the Al and Cu rods may not necessarily be the pure element—that is, they may be alloyed with a minor amount of another metal to improve material properties (usually hardness). Moreover, "stainless steel" is actually a whole class of materials, basically steel (iron plus carbon) alloyed with minor quantities of Ni and Cr (and perhaps other elements as well). The exact compositions of all these metal rods are unknown, at least to the instructor. Alloying (at low levels of the alloying element) generally has only a small effect (a few percent at most) on ρ and c_p, but it can have a much larger effect on k; see, for example, the tables in Holman Appendix A-2.

The "ceramic" is some type of machinable ceramic, meaning that it is an unfired (hydrated) aluminosilicate. The "resin" rod is a laminate of cloth and a thermosetting resin, probably phenol-formaldehyde (like Formica). Values of ρ, c_p and k for materials with similar compositions are given in Table E2.

These values should be helpful to you in analyzing your data, and in estimating what flow rate-cylinder diameter combinations to use so as to place Bi within a certain range.

Table E1: Physical Properties of Selected Materials.[a]

Material	ρ (kg/m^3)	c_p (kJ/kg-K)	k (W/m-K)	Comments and Source
Al	2707	0.896	204	pure Al (Holman, p.535)
Cu	8954	0.3831	386	pure Cu (Holman, p.535)
type 316 stainless steel	7865	0.46	16	type 316 has 16-18% Cr; 10-14% Ni; 2-3% Mo; 1% Si; and 2% Mn (Liley, p. 3-262; Holman, p. 536)
PMMA	1190 1150[b]	1.255[c] 1.42[d] 1.72[e]	0.193[f] 0.250[e]	(Wunderlich, p.V-79)

[a]values at 20^{o}C unless otherwise noted
[b]at 105^{o}C
[c]at 0^{o}C
[d]at 25^{o}C
[e]at 100^{o}C
[f]average over 0-50^{o}C

Table E2: Material properties for two composite test pieces.

Material	ρ (kg/m^3)	c_p (kJ/kg-K)	k (W/m-K)	Comments and Source
"resin"	1000	1.38	0.15	values are for *hardboard*, which is a composite of phenol-formaldehyde or urea-melamine resin with sawdust (Liley, p.3-263)
"ceramic"	2600[b]	0.96[b]	1.00[b]	values are for *Missouri firebrick*, which is a fired (dehydrated) aluminosilicate (Holman, p.538)

[a]values at 20^{o}C unless otherwise noted
[b]at 200^{o}C

Appendix F: Convective Heat Transfer Correlations

(I. Aksay and prior CBE346 faculty, 2012)

Some forced-convection heat transfer relationships which you may find useful (taken from Holman (1981)) are given below. Note that the relevant thermal conductivity is that of the convected fluid, denoted for clarity by k_f: Geankoplis (2003) also has correlations for Nu.

<u>Laminar flow in a smooth tube of diameter *d*, constant wall temperature:</u>

$$Nu_d = hd/k_f = 48/11 \qquad \text{Holman 5-106 (21)}$$

where Nu_d is the Nusselt number.

<u>Turbulent flow in a smooth tube of diameter *d*, constant wall heat flux:</u>

$$Nu_d = 0.0395 Re_d^{3/4} Pr^{1/3} \qquad \text{Holman 5-115a (22)}$$

where Re is the Reynolds number and Pr is the Prandtl number. This equation was developed from the analogy between heat transfer and fluid friction, using an empirical expression for the friction factor developed from data up to $Re \approx 200{,}000$.

<u>Turbulent flow in a smooth tube of diameter *d*, constant wall temperature:</u>

$$Nu_d = 0.023 Re_d^{0.8} Pr^n \qquad \text{Holman 6-4 (23)}$$

Here $n = 0.4$ if the fluid is being heated and it is equal to 0.3 if the fluid is being cooled.

This is an empirical relation developed directly from heat transfer measurements. Measurements show that this equation can correlate the data to ±25% for $5000 < Re < 500{,}000$ and $0.6 < Pr < 100$.

Since your flow geometry is not that of a tube, it is clear that these correlations are not going to be directly (quantitatively) applicable. However you should study these correlations for suggestions as to how your own data would best be examined.