Heat Transfer Coefficient Correlation for Circular Fin Rods

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Abstract

The objective of this paper is to develop and present a correlation equation for the average heat transfer coefficient associated with long horizontally oriented circular fin rods that accounts for the effect of both modes of heat transfer: convection and radiation. Four aluminum circular cross-section test rods with diameters of 3.18, 6.35, 9.53, and 12.7 mm were used to develop such a correlation equation. This correlation will be supplied to the students to be used in the design of a fin attachment using the Design-Build-Test approach.

I. Introduction

Heat transfer is a very important subject and has long been an essential part of mechanical engineering curricula all over the world. Heat transfer is encountered in a wide variety of engineering applications where heating and cooling is required. Heat transfer plays an important role in the design of many devices, such as spacecrafts, radiators, heating and air conditioning systems, refrigerators, power plants, and others.

Traditional undergraduate heat transfer laboratories in mechanical engineering expose the students to heat transfer concepts learned in the classroom, but do not provide them with design experiences similar to what they might face as thermal engineers in industrial positions. In addition, the Accreditation Board for Engineering and Technology (ABET) accreditation criteria require that graduates of engineering programs possess “an ability to design and conduct experiments, as well as to analyze and interpret data” [1] and “an ability to design a system, component or process to meet desired needs” [1].

Very recently, the Design-Build-Test (DBT) concept was suggested by Abu-Mulaweh [2] to be used in creating an experiment for a junior-level heat transfer laboratory. In that experiment, student teams design, build, and test a fin attachment to increase the heat loss from a surface. Extended surfaces (fins) are used to enhance the heat transfer rate between a solid surface and adjoining fluid. The fin is a good application that involves combined conduction, convection, and radiation effects.

One important parameter that is involved in the design of such fin attachment is the heat transfer coefficient. However, values for the heat transfer coefficient associated with this kind of engineering application (i.e., a fin attachment) are not readily available in the literature. This is because the relations for heat transfer coefficient associated with circular rods that can be found in the literature are based on either uniform surface temperature or uniform surface heat flux. The fins by nature exhibit a decaying "exponential" temperature distribution (i.e., not uniform). In addition, those heat transfer coefficient relations available in the literature are applicable only for the case of pure natural convection in "still air." The fin attachment is designed for real life applications where the "still air" condition does not exist. Moreover, the heat transfer coefficient relations in the literature are for convection only. And the heat dissipated by the fin is through both convection and radiation. A lack of heat transfer coefficient correlations for this kind of application (i.e., the design of fin attachment) has motivated the current study.

The objective of this paper is to develop and present a correlation equation for the average heat transfer coefficient associated with horizontally oriented circular fin rods that accounts for the effects of two modes of heat transfer: convection and radiation. This correlation, in turn, will be supplied to the students to be utilized in the design of the fin attachment using the DBT approach.

II. Experimental Apparatus and Procedure

A portable fin experimental apparatus was designed and constructed. The experimental apparatus is relatively simple and inexpensive. It consists of constant-temperature energy source (heated aluminum plate) into which the end of circular cross-section rod is attached, as shown in Fig. 1. Aluminum circular cross-section test rods 90 cm in length with diameters of 3.18, 6.35, 9.53, and 12.7 mm were employed as fins. The 90 cm length was chosen, based on analytical calculations, to insure that the tip of the fin is at the same temperature of the adjacent fluid. Copper-constantan thermocouples were inserted into the rod to measure the axial temperature distributions. Thermal conductive adhesive was used to seal and secure the measuring junction of the thermocouple into the small hole in the fin rod. The heated aluminum plate was made of four composite layers that were held together by screws. The upper layer was an aluminum plate (15.24 cm x 15.24 cm, and 0.95 cm thick). The second layer consists of a heating pad that can be controlled for electrical energy input. The third layer is a 0.64 cm thick Transite insulating material. The bottom layer of the heated plate is a 1.6 cm thick aluminum plate that serves as backing and support for the heated plate structure.

The aluminum plate can be heated and maintained at a constant temperature by adjusting and controlling the level of electrical energy input to the heating pad. This was accomplished by utilizing variable voltage transformer. The temperature of the aluminum plate is measured by a thermocouple that is inserted into the plate from the backside. The heated aluminum plate and the fin attachment assembly are rigidly mounted on a short stand as shown in Fig. 1. A portable thermocouple is also used to measure the ambient temperature (i.e., air temperature). Heat is dissipated by the fin to the adjacent air by convection and radiation.

The experimental procedure was very simple, quick, and straight forward. First, the energy source was turned on and the electrical energy input was adjusted to the desired heating level using the variable voltage transformer. Second, when the system reached steady-state conditions, the axial temperature distribution, *T(x)*, of the rod was measured. The surrounding air temperature, T_∞, was also measured using a portable thermocouple.

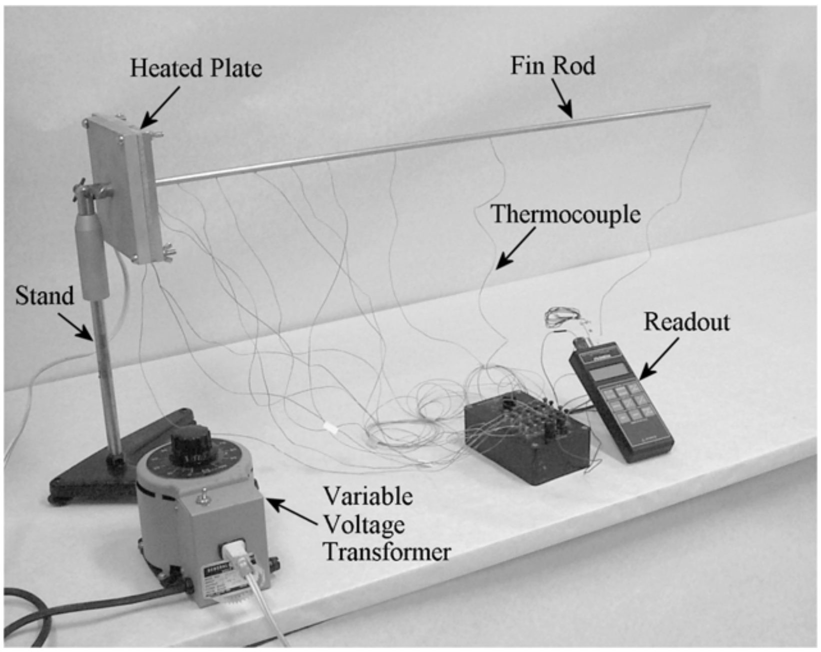

Figure 1: Fin Attachment Experimental Apparatus

III. Theory

The analytical analysis for fins of uniform cross-sectional area can be found in any standard heat transfer textbook (see e.g., Incropera and DeWitt [3] and Özisik [4]). In this experiment, the fins are assumed infinitely long (i.e., the tip of the fin is at the same temperature of the adjacent fluid) and the temperature at the base ($x = 0$) is constant, T_o. The analysis is simplified by the following assumptions: one-dimensional conduction in the x direction, steady-state conditions, constant thermal conductivity, no heat generation, and constant and uniform heat transfer coefficient over the entire surface. Under these assumptions, the energy equation and boundary conditions are given by:

$$\frac{d^2T}{dx^2} - m^2(T - T_\infty) = 0 \qquad (1)$$

and

$$T(0) = T_o \qquad \text{and} \qquad T(L \to \infty) = T_\infty\,, \tag{2}$$

where

$$m = \sqrt{\frac{hP}{kA_c}}\,. \tag{3}$$

In this relation, A_c is the cross sectional area of the fin rod, h is the total heat transfer coefficient (due to both convection and radiation), k is the thermal conductivity of fin, and P is the perimeter of the fin rod. See Figure 2 for a schematic of the fin.

The resulting temperature distribution is given by

$$T(x) - T_\infty = (T_o - T_\infty)\exp(-mx) \qquad \text{or} \qquad \theta(x) = \theta_o \exp(-mx)\,, \tag{4}$$

where $\theta = T(x) - T_\infty$ and $\theta_0 = T_o - T_\infty$. The heat transfer by conduction along the fin is given by

$$Q(x) \ = (T_o - T_\infty)\sqrt{hPkA_c}\exp(-mx)\ , \tag{5}$$

while total heat transfer from the fin is given by

$$Q_f = \sqrt{hPkA_c}\,(T_o - T_\infty)\,. \tag{6}$$

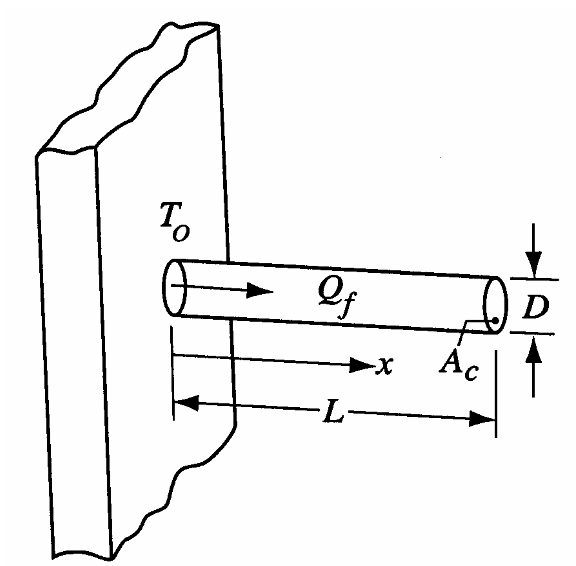

Figure 2: Schematic of the fin.

IV. Results and Discussion

Temperature Distributions:

The experiment was carried out using four aluminum (k = 120 W/m°C) fin rods of diameters D = 3.18, 6.35, 9.53, and 12.7 mm. When steady state conditions were reached, the axial temperature distribution, $T(x)$, of the fin was measured along with the base, T_o, and ambient, T_∞, temperatures. For each fin rod, the experiments were carried out for different base temperatures. The temperature range was 40°C ≤ θ_o ≤ 90°C. A sample of dimensionless axial temperature distributions for the four different rod diameters (θ_o = 80°C) and different base temperatures (D = 6.35 mm) are shown in Fig. 3.

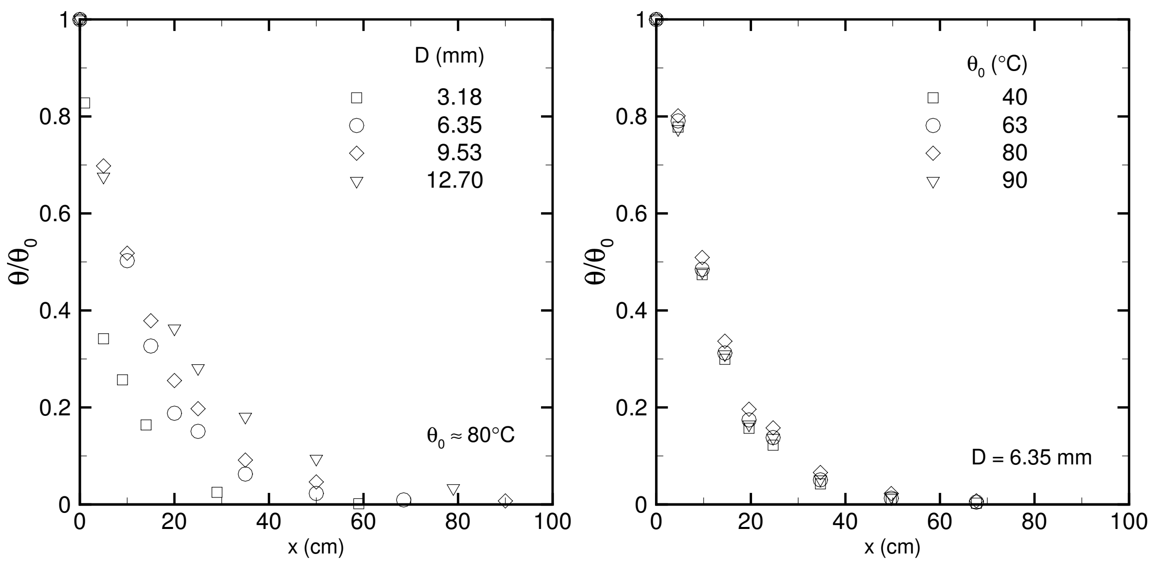

Figure 3: Effect of diameter (left) and base temperature (right) on the temperature distributions along the fin rods.

Figure 3 illustrates the exponential decay of the temperature along the rod as predicted by equation (4). Moreover, all of the fins can be considered to be infinitely long because near the tip θ approaches zero. As the diameter of the rod decreases the fin temperature approaches the ambient temperature more rapidly. From equation (4), this implies that the exponent m is increasing. Figure 3 also shows that for the base temperature range considered in this study (40°C ≤ θ_o ≤ 90°C), the effect on the non-dimensional temperature distribution is minimal and no clear trend is observed.

Average Heat Transfer Coefficient Determination:

The heat transfer coefficient, in general, varies along the fin as well as its circumference. For convenience in the mathematical analysis, it is assumed that the heat transfer coefficient is constant and uniform over the entire surface of the fin. The heat transfer coefficients in this study represent average values that account for both convection and radiation heat transfer.

The method to determine the average heat transfer coefficient makes use of equation (4) and the measured temperature distribution along the fin. This is accomplished by rearranging equation (4) and taking the natural logarithm of both sides, which yields

$$y = \ln\left(\frac{T - T_\infty}{T_o - T_\infty}\right) = -mx \qquad \text{or} \qquad y = \ln\left(\frac{\theta}{\theta_o}\right) = -mx\,. \tag{6}$$

The measured temperature distributions are used to plot $\ln[\theta/\theta_o]$ versus x as shown in Fig. 4. A least squares method is then used to find the slope of the line which is equal to m. In Fig. 4, the measured data are represented with symbols and the solid lines represent the best fit from a linear least squares analysis.

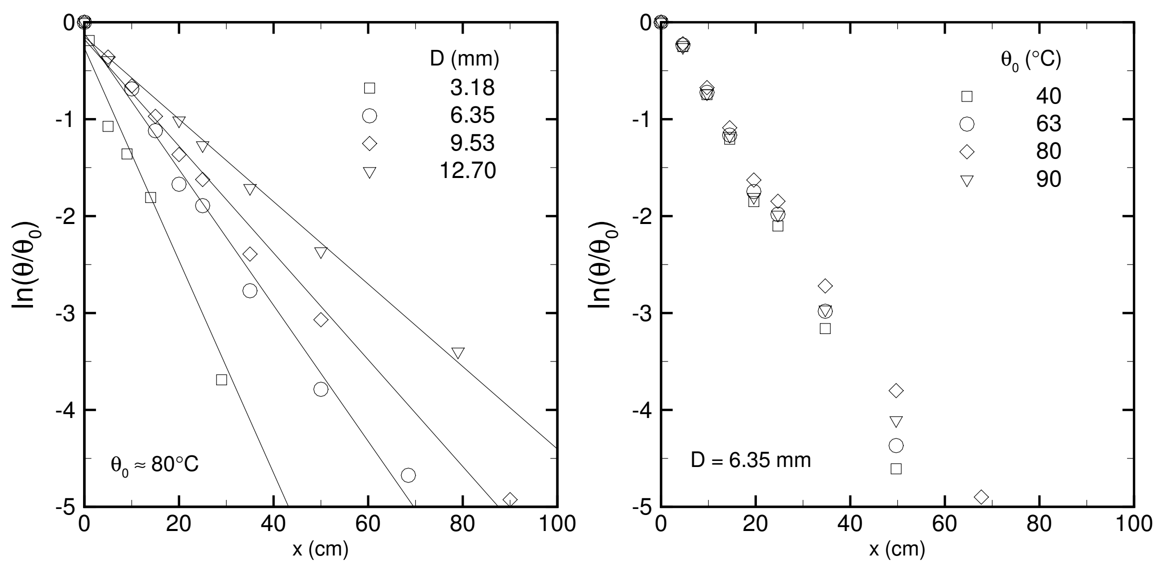

Figure 4: Least squares method for the estimation the convection heat transfer coefficient.

This linear least squares approach, as described, allows for two degrees-of-freedom. Thus, the lines do not pass exactly through the origin. Moreover, the least squares approach does not minimize the sum of the squares of the deviations of the non-dimensional temperature curve, but rather the deviations of the natural logarithm of the non-dimensional temperature curve. This amounts to minimizing the squares of the percentage errors [5], and thus gives greater influence to the lower temperatures near the tip of the fin.

The calculation of y depends directly on the temperature measurements, i.e. $y = f(T, T_o, T_\infty)$. Following the approach in Refs. [6,7], the uncertainty in the calculation of y can be estimated by

$$u_y = \left[\left(\frac{\partial y}{\partial T} u_T \right)^2 + \left(\frac{\partial y}{\partial T_o} u_{T_o} \right)^2 + \left(\frac{\partial y}{\partial T_\infty} u_{T_\infty} \right)^2 \right]^{1/2}, \tag{7}$$

where u_y, u_T, u_{T_o}, and u_{T_∞} represent the uncertainties in y and the temperatures. The partial derivatives in equation (7) can be found from equation (6). With the assumption that $u_T = u_{T_o} = u_{T_\infty}$ and conservative assumptions for the temperature differences, the uncertainty in y can be simplified and estimated by

$$u_y \approx \frac{\sqrt{3}\, u_T}{T - T_\infty}. \tag{8}$$

This analysis indicates that the calculation of y is sensitive to errors in the temperature measurements, especially near the tip of the fin. For example, if the uncertainty in the temperature measurement is taken to be 1^oC and the temperature difference between the fin and the ambient is 5^oC, the uncertainty in y is 0.35. For the case of $\theta_o = 80^oC$, $y = -2.77$ which corresponds to a percent uncertainty in y of 13%. When the temperature difference between the fin and the ambient is 20^oC, the percent uncertainty in y is 6%. This analysis indicates that temperature measurements near the base minimize the uncertainty.

Once the value of m is known, the average heat transfer coefficient value, h, can be calculated from the rearrangement of equation (3), i.e.

$$h = m^2 kD/4. \tag{9}$$

The uncertainty in the calculation of h can also be estimated according to Refs. [6,7], i.e.

$$u_h = \left[\left(\frac{mkD}{2} u_m \right)^2 + \left(\frac{m^2 D}{4} u_k \right)^2 + \left(\frac{m^2 k}{4} u_D \right)^2 \right]^{1/2}. \tag{10}$$

Values for the quantities in equation (10) were assumed (at the 95% confidence level) and the percent uncertainty in m was estimated to be 5% based on numerical studies of several typical data sets. This resulted in a percent uncertainty of the average heat transfer coefficients of approximately ±10%. Note that first term in equation (10) dominates the uncertainty calculation.

As mentioned, the calculation of h is dependent on the location of the axial position of the data. Table 1 shows the effect of the location of points on the calculation of h for the 6.35 mm diameter fin with $\theta_o = 80^oC$. Also shown in the table is the dimensionless quantity mx. This quantity is useful to interpret the results. As mx becomes large the heat transfer along the fin goes to zero according to equation (5). Thus, perhaps points near the tip of the fin should not be included in the calculation of h for physical reasons. Therefore, h was calculated for the four fins using five or six temperature measurements closest to the base.

Table 1. Effect of the location of points on the calculation of h (D=6.35 mm and $\theta_o = 80^oC$).

number of points	x (m)	h (W/m^{2o}C)	$m\,x$
4	0.15	11.22	1.15
5	0.20	13.79	1.70
6	0.25	12.70	2.04
7	0.35	12.72	2.86
8	0.50	11.72	3.92
9	0.69	9.73	4.89

A numerical experiment was performed to examine the effect of the error in ambient temperature measurement on the calculation of h. In the above calculations, the ambient temperature was perturbed by 1^oC. This resulted in a 6% change in h for typical cases.

Finally, it should be noted that for the temperature range considered in this study ($40^oC \leq \theta_o \leq 90^oC$), it was observed that the effect of base temperature on the heat transfer coefficient is minimal and not noticeable. Sparrow and Abraham [8] reported a similar observation for the convective heat transfer coefficient. In their study, they found that the convective heat transfer coefficients are insensitive to the oven set-point.

Average Heat Transfer Coefficient Empirical Correlation Development:

The heat transfer coefficients for the four different fin rods that were determined from the measured data using the scheme described above were plotted versus the diameter, as shown in Fig. 5. A least squares method was then used to obtain the following empirical correlation equation for the heat transfer coefficient for horizontally oriented fin rods:

$$h = 17.1 - 0.664D, \tag{11}$$

where D is in mm and h is in $W/m^{2\,o}C$. In Fig. 5 the measured data are represented with symbols and the solid line represents the best fit using linear least squares. The correlation coefficient for the fit was 0.988, which indicates a linear relationship with the data.

The fin rod diameter range used in establishing the correlation equation (11) is 3.18 mm $\leq D \leq$ 12.7 mm. The empirical correlation equation (11) predicts the measured average heat transfer coefficient within 10%.

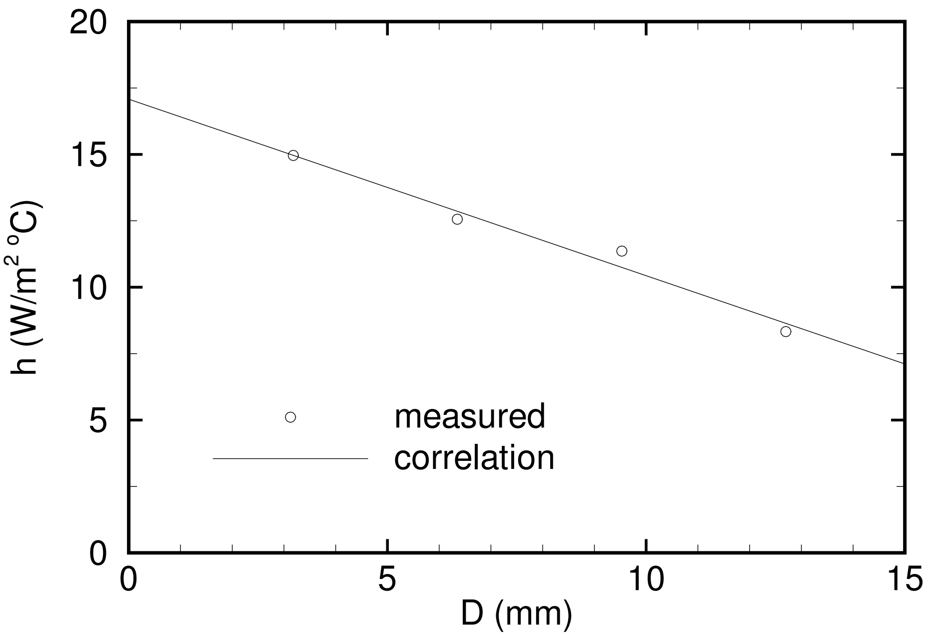

Figure 5: Effect of diameter on the average heat transfer coefficient.

V. Conclusion

The objective of this paper has been achieved. An empirical correlation equation for the average heat transfer coefficient associated with long horizontal circular fin rods has been developed. The correlation depends on the fin diameter. For the temperatures considered, the effect of the base temperature was not significant. It should be noted that the average heat transfer coefficient values deduced from the correlation equation reported in this paper accounts for the effects of convection and radiation.

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