Steady Heat Transfer with Conduction and Convection

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Mechanical Engineering 375

Heat Transfer

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Outline

- Review last lecture
- Equivalent circuit analyses
  - Review basic concept
  - Application to series circuits with conduction and convection
  - Application to composite materials
  - Application to other geometries
- Two-dimensional shape factors

Review Steady, 1-D, $\dot{e}_{gen} = 0$

- Rectangular

$$\dot{q} = \frac{\dot{Q}}{A} = -\frac{\bar{k}(T_L - T_0)}{L}$$

- Cylindrical shell

$$\frac{\dot{Q}}{L} = -\frac{2\pi\bar{k}(T_2 - T_1)}{\ln(r_2/r_1)}$$

- Spherical shell

$$\dot{Q} = \frac{4\pi\bar{k}(T_2 - T_1)}{1/r_1 - 1/r_2}$$

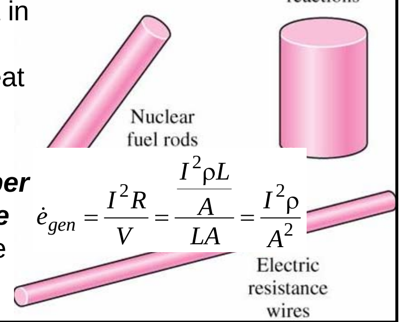
$\bar{k}$ is an average thermal conductivity (or a constant value) if k is constant

T_0, T_L = temperatures at x = 0,L; T_1, T_2 = temperatures at inner (r_1) and outer(r_2) radii

Review Heat Generation

- Various phenomena in solids can generate heat
- Define $\dot{e}_{gen}$ as the heat generated ***per unit volume*** per unit time

Figure 2-21 from Çengel, *Heat and Mass Transfer*



$$\dot{e}_{gen} = \frac{I^2R}{V} = \frac{\frac{I^2\rho L}{A}}{LA} = \frac{I^2\rho}{A^2}$$

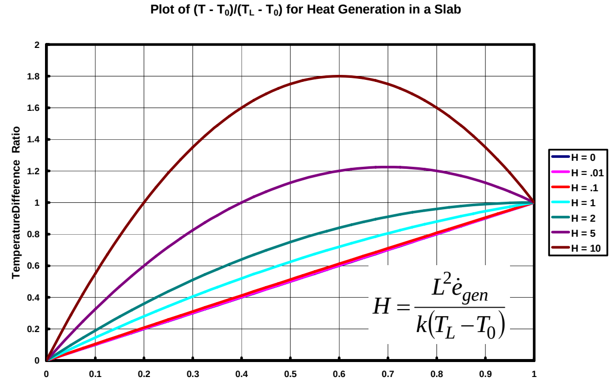
Review Heat Generation II

- Temperature and heat flux equations

$$T = T_0 - \frac{\dot{e}_{gen}x^2}{2k} + \frac{\dot{e}_{gen}xL}{2k} - \frac{(T_0 - T_L)x}{L}$$

$$\dot{q} = \frac{\dot{e}_{gen}(2x - L)}{2} + \frac{k(T_0 - T_L)}{L}$$

$$\dot{Q}_{x=0} + \dot{E}_{gen} = \dot{Q}_{x=L}$$

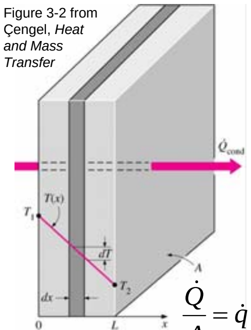


Steady Heat Transfer Definition

- In steady heat transfer the temperature and heat flux at any coordinate point do not change with time
- Both temperature and heat transfer can change with spatial locations, but not with time
- Steady energy balance (first law of thermodynamics) means that heat in plus heat generated equals heat out

Rectangular Steady Conduction

Figure 3-2 from Çengel, *Heat and Mass Transfer*



The heat transfer is constant in this 1D rectangle for both constant & variable k

$$\frac{\dot{Q}}{A} = \dot{q} = -k\frac{dT}{dx}$$

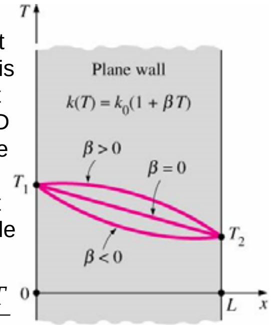
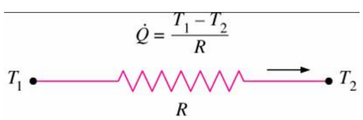


Figure 2-63 from Çengel, *Heat and Mass Transfer*

Thermal Resistance

- Heat flow analogous to current
- Temperature difference analogous to potential difference
- Both follow Ohm's law with appropriate resistance term



Thermal Resistance II

- Conduction

$$\dot{Q} = \frac{\bar{k}A(T_1 - T_2)}{L} \Rightarrow \dot{Q} = \frac{T_1 - T_2}{R_{cond}} \Rightarrow R_{cond} = \frac{L}{kA}$$

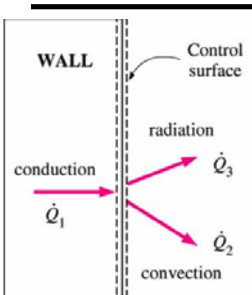
- Convection

$$\dot{Q} = hA(T_s - T_f) \Rightarrow \dot{Q} = \frac{T_s - T_f}{R_{conv}} \Rightarrow R_{conv} = \frac{1}{hA}$$

- Radiation

$$R_{rad} = \frac{1}{A_1\mathfrak{F}_{12}\sigma\left(T_1^3 + T_2^3 + T_2^2T_1 + T_1^2T_2\right)} = \frac{1}{A_1h_{rad}}$$

Where Does the Heat Go?



Energy conservation requires that conduction heat through wall equals the heat leaving the wall by convection and radiation

$$\dot{Q}_1 = \dot{Q}_2 + \dot{Q}_3$$

Figure 1-18 from Çengel, *Heat and Mass Transfer*

Where Does the Heat Go? II

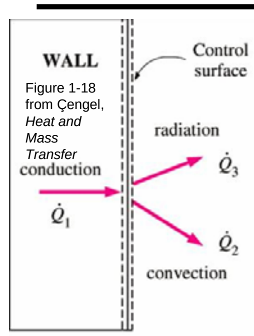


Figure 1-18 from Çengel, *Heat and Mass Transfer*

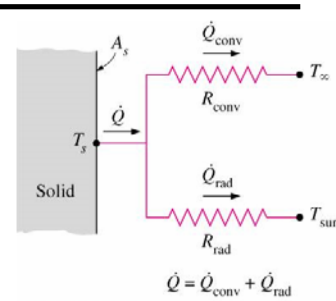


Figure 3-5 from Çengel, *Heat and Mass Transfer*

Parallel Resistances ($T_\infty = T_{surr}$)

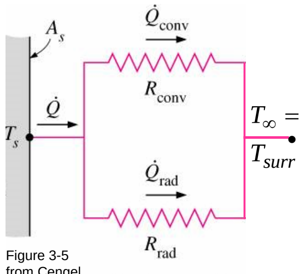


Figure 3-5 from Çengel, *Heat and Mass Transfer*

$$\frac{1}{R_{total}} = \frac{1}{R_{conv}} + \frac{1}{R_{rad}}$$

$$\frac{1}{R_{total}} = A_s h_{conv} + A_s h_{rad}$$

Define total heat transfer coefficient, h_{total}

$$\dot{Q} = \dot{Q}_{conv} + \dot{Q}_{rad} \qquad h_{total} = \frac{1}{A_s R_{total}} = h_{conv} + h_{rad}$$

Combined Modes

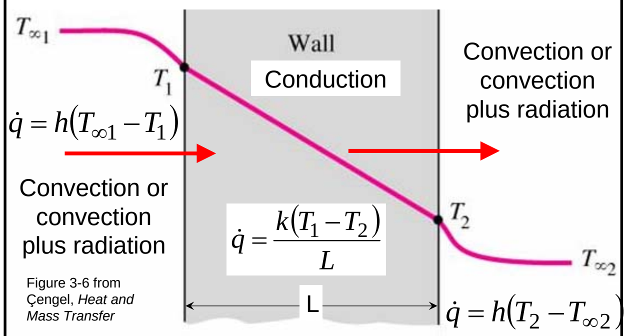


Figure 3-6 from Çengel, *Heat and Mass Transfer*

All $\dot{q}$ values are the same

Combined Modes II

A is area normal to heat flow

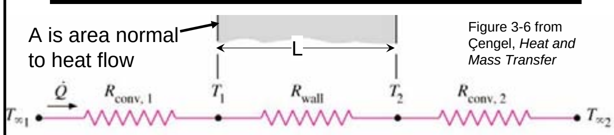


Figure 3-6 from Çengel, *Heat and Mass Transfer*

$$\dot{Q} = \frac{T_{\infty 1} - T_{\infty 2}}{R_{total}} = \frac{T_{\infty 1} - T_{\infty 2}}{R_{conv,1} + R_{wall} + R_{conv,1}}$$

Series Resistance Formula

$$\dot{Q} = \frac{T_{\infty 1} - T_{\infty 2}}{\frac{1}{Ah_1} + \frac{L}{kA} + \frac{1}{Ah_2}} \Rightarrow \dot{q} = \frac{\dot{Q}}{A} = \frac{T_{\infty 1} - T_{\infty 2}}{\frac{1}{h_1} + \frac{L}{k} + \frac{1}{h_2}}$$

Combined Modes III

A is area normal to heat flow

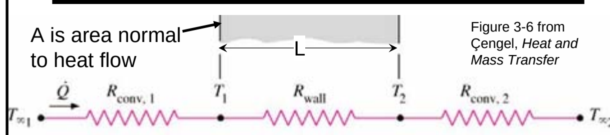


Figure 3-6 from Çengel, *Heat and Mass Transfer*

$$\dot{q} = \frac{\dot{Q}}{A} = \frac{T_{\infty 1} - T_{\infty 2}}{\frac{1}{h_1} + \frac{L}{k} + \frac{1}{h_2}}$$

If you know h_1, h_2, L, k, $T_{\infty 1}$, and $T_{\infty 2}$, but you do not know T_1 and T_2, can you find the heat flux?

Once you found the heat flux from the information give, can you find T_1 and T_2?

$$T_1 = T_{\infty 1} - \frac{\dot{q}}{h_1} \qquad T_2 = T_{\infty 2} + \frac{\dot{q}}{h_2}$$

Problem

A house has a 4 in thick brick wall with k = 0.6 Btu/hr·ft·°F. The interior temperature is 70°F and the exterior temperature is 0°F. The inside and outside convection plus radiation coefficients are 3 Btu/hr·ft²·°F and 4 Btu/hr·ft²·°F, respectively. Find the heat flux through the wall.

Given: Wall with L = 4 in = 4/12 ft and k =0.6 Btu/hr·ft·°F has convection on two sides. $T_{\infty 1}$ = 70°F, $T_{\infty 2}$ = 0°F, h_1 = 3 Btu/hr·ft²·°F and h_2 = 4 Btu/hr·ft²·°F.

Find: $\dot{q} = \frac{\dot{Q}}{A}$

Solution

A is area normal to heat flow

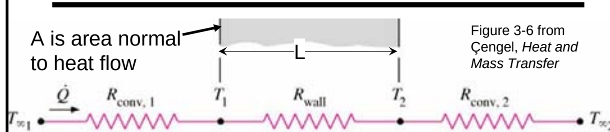


Figure 3-6 from Çengel, *Heat and Mass Transfer*

$$\dot{q} = \frac{\dot{Q}}{A} = \frac{T_{\infty 1} - T_{\infty 2}}{\frac{1}{h_1} + \frac{L}{k} + \frac{1}{h_2}} = \frac{70^oF - 0^oF}{\frac{hr \cdot ft^2 \cdot {}^oF}{3\,Btu} + \frac{4}{12}ft\frac{hr \cdot ft \cdot {}^oF}{0.6\,Btu} + \frac{hr \cdot ft^2 \cdot {}^oF}{4\,Btu}}$$

$$\dot{q} = \frac{\dot{Q}}{A} = \frac{61.5\,Btu}{hr \cdot ft^2}$$

Find values of T_1 and T_2. Can you check these values?

Solution II

A is area normal to heat flow

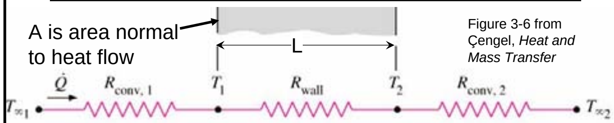


Figure 3-6 from Çengel, *Heat and Mass Transfer*

$$\dot{q} = \frac{T_{\infty 1} - T_1}{\frac{1}{h_1}} \Rightarrow T_1 = T_{\infty 1} - \frac{\dot{q}}{h_1} = 70^oF - \frac{\frac{61.5\ Btu}{hr \cdot ft^2}}{\frac{3\ Btu}{hr \cdot ft^2 \cdot {}^oF}} = 49.5^oF$$

$$\dot{q} = \frac{T_2 - T_{\infty 2}}{\frac{1}{h_2}} \Rightarrow T_2 = T_{\infty 2} + \frac{\dot{q}}{h_1} = 0^oF - \frac{\frac{61.5\ Btu}{hr \cdot ft^2}}{\frac{4\ Btu}{hr \cdot ft^2 \cdot {}^oF}} = 15.4^oF$$

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Solution III

A is area normal to heat flow

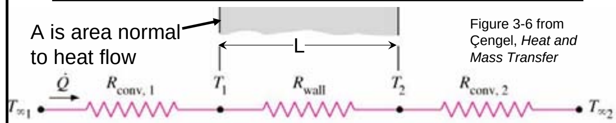

Figure 3-6 from Çengel, *Heat and Mass Transfer*

How can we check results below found from analysis of overall problem and convection processes?

$$\dot{q} = \frac{61.5\ Btu}{hr \cdot ft^2} \qquad T_1 = 49.5^oF \qquad T_2 = 15.4^oF$$

Analyze conduction step for consistency.

$$\dot{q} = \frac{61.5\ Btu}{hr \cdot ft^2} = \frac{\dot{Q}}{A} = \frac{k(T_1 - T_2)}{L} = \frac{\frac{0.6\ Btu}{hr \cdot ft \cdot {}^oF}\left(49.5^oF - 15.4^oF\right)}{\frac{4}{12}ft}$$

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Composite Materials

How would you analyze this problem?

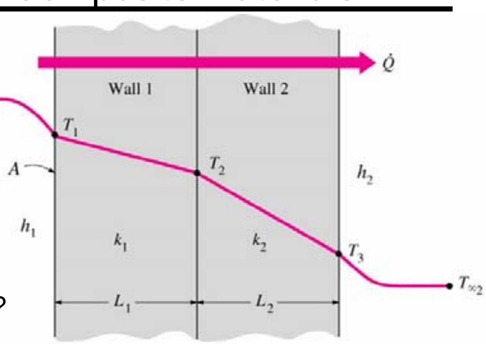

Figure 3-9 from Çengel, *Heat and Mass Transfer*

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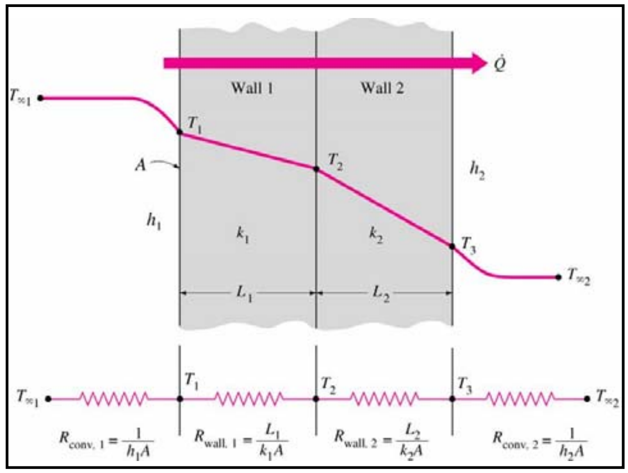

Review Cylindrical Shell

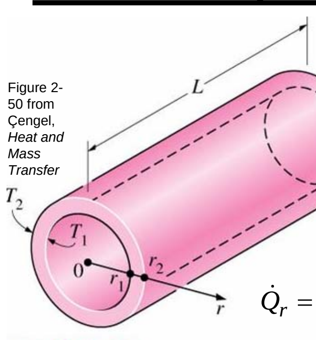

Figure 2-50 from Çengel, *Heat and Mass Transfer*

For constant k

$$\frac{\dot{Q}_r}{L} = \frac{2\pi k(T_1 - T_2)}{\ln\left(\frac{r_2}{r_1}\right)}$$

$$R = \frac{1}{2\pi kL}\ln\left(\frac{r_2}{r_1}\right)$$

$$\dot{Q}_r = \frac{T_1 - T_2}{\frac{1}{2\pi kL}\ln\left(\frac{r_2}{r_1}\right)} = \frac{T_1 - T_2}{R}$$

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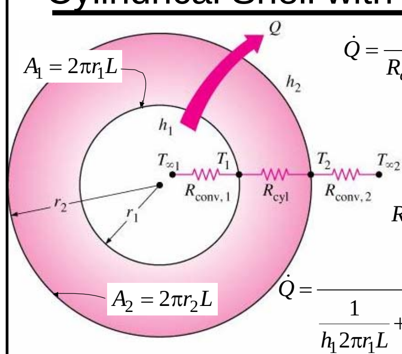
Cylindrical Shell with Convection



$$\dot{Q} = \frac{T_{\infty 1} - T_{\infty 2}}{R_{conv,1} + R_{cond} + R_{conv,2}}$$

$$R_{conv,1} = \frac{1}{h_1 A_1} = \frac{1}{h_1 2\pi r_1 L}$$

$$R_{conv,2} = \frac{1}{h_2 A_2} = \frac{1}{h_2 2\pi r_2 L}$$

$$\dot{Q} = \frac{T_{\infty 1} - T_{\infty 2}}{\frac{1}{h_1 2\pi r_1 L} + \frac{1}{2\pi kL}\ln\left(\frac{r_2}{r_1}\right) + \frac{1}{h_2 2\pi r_2 L}}$$

Figure 3-25 from Çengel, *Heat and Mass Transfer*

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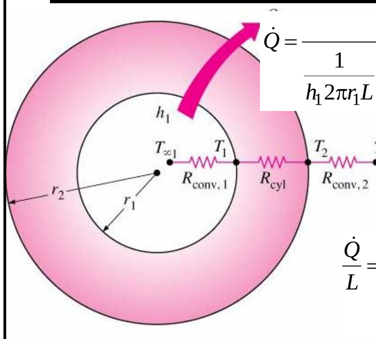
Cylinder plus Convection Result

$$\dot{Q} = \frac{T_{\infty 1} - T_{\infty 2}}{\frac{1}{h_1 2\pi r_1 L} + \frac{1}{2\pi k L}\ln\left(\frac{r_2}{r_1}\right) + \frac{1}{h_2 2\pi r_2 L}}$$



We can rearrange this equation as shown below

$$\frac{\dot{Q}}{L} = \frac{2\pi(T_{\infty 1} - T_{\infty 2})}{\frac{1}{h_1 r_1} + \frac{1}{k}\ln\left(\frac{r_2}{r_1}\right) + \frac{1}{h_2 r_2}}$$

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Figure 3-25 from Çengel, *Heat and Mass Transfer*

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Problem

- A hot-water pipe (k = 35 Btu/hr·ft·°F) in a house, made of ¾ inch schedule 40 pipe (OD = 1.050 in; ID = 0.824 in) is 40 ft long and contains water at 120°F. The air around the pipe is at 60°F. The heat transfer coefficients inside and outside the pipe are, respectively, 200 and 3 Btu/hr·ft²·°F. Determine the heat loss from the pipe.

$$\frac{\dot{Q}}{L} = \frac{2\pi(T_{\infty 1} - T_{\infty 2})}{\frac{1}{h_1 r_1} + \frac{1}{k}\ln\left(\frac{r_2}{r_1}\right) + \frac{1}{h_2 r_2}}$$

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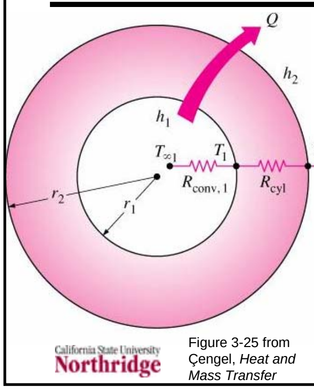
Solution



Given: $T_{\infty 2}$ = 60°F, $T_{\infty 1}$ = 120°F, r_1 = ID/2 = 0.412 in, r_2 = OD/2 = 0.525 in, k =35 Btu/hr·ft·°F, L = 40 ft, h_1= 200 Btu/hr·ft²·°F, h_2= 3 Btu/hr·ft²·°F

Find: $\dot{Q}$

$$\frac{\dot{Q}}{L} = \frac{2\pi(T_{\infty 1} - T_{\infty 2})}{\frac{1}{h_1 r_1} + \frac{1}{k}\ln\left(\frac{r_2}{r_1}\right) + \frac{1}{h_2 r_2}}$$

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Figure 3-25 from Çengel, *Heat and Mass Transfer*

Solution II

Given: $T_{\infty 2}$ = 60°F, $T_{\infty 1}$ = 120°F, r_1 = ID/2 = 0.412 in, r_2 = OD/2 = 0.525 in, k =35 Btu/hr·ft·°F, L = 40 ft, h_1= 200 Btu/hr·ft²·°F, h_2= 3 Btu/hr·ft²·°F

Find: $\dot{Q} = \frac{1{,}940\ Btu}{hr}$

$$\frac{1}{h_1 r_1} = \frac{hr\cdot ft^2\cdot{}^oF}{200\ Btu}\frac{1}{0.412\ in}\frac{12\ in}{ft}$$

$$\frac{1}{k}\ln\left(\frac{r_2}{r_1}\right) = \frac{hr\cdot ft\cdot{}^oF}{35\ Btu}\ln\left(\frac{0.525\ in}{0.412\ in}\right)$$

$$\frac{1}{h_2 r_2} = \frac{hr\cdot ft^2\cdot{}^oF}{3\ Btu}\frac{1}{0.525\ in}\frac{12\ in}{ft}$$

$$\dot{Q} = \frac{2\pi(T_{\infty 1} - T_{\infty 2})L}{\frac{1}{h_1 r_1} + \frac{1}{k}\ln\left(\frac{r_2}{r_1}\right) + \frac{1}{h_2 r_2}} = \frac{2\pi(120^oF - 60^oF)(40\ ft)}{(0.146 + 0.007 + 7.619)\frac{hr\cdot ft\cdot{}^oF}{Btu}}$$

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Composite Cylindrical Shell

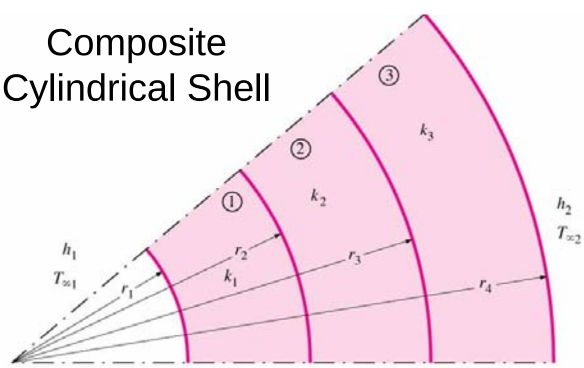

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Figure 3-26 from Çengel, *Heat and Mass Transfer*

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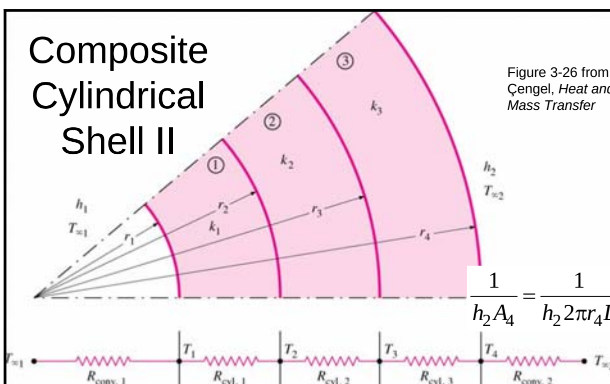
Composite Cylindrical Shell II

Figure 3-26 from Çengel, *Heat and Mass Transfer*



$$\frac{1}{h_2 A_4} = \frac{1}{h_2 2\pi r_4 L}$$

$$\frac{1}{h_1 A_1} = \frac{1}{h_1 2\pi r_1 L} \quad \frac{1}{k_1 L}\ln\left(\frac{r_2}{r_1}\right) \quad \frac{1}{k_2 L}\ln\left(\frac{r_3}{r_2}\right) \quad \frac{1}{k_3 L}\ln\left(\frac{r_4}{r_3}\right)$$

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Composite Cylindrical Shell III

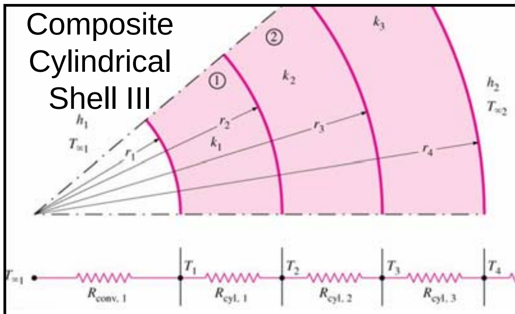

Figure 3-26 from Çengel, *Heat and Mass Transfer*

$$\dot{Q} = \frac{T_{\infty 1} - T_{\infty 2}}{\frac{1}{h_1 2\pi r_1 L} + \frac{1}{2\pi k_1 L}\ln\left(\frac{r_2}{r_1}\right) + \frac{1}{2\pi k_2 L}\ln\left(\frac{r_3}{r_2}\right) + \frac{1}{2\pi k_3 L}\ln\left(\frac{r_4}{r_3}\right) + \frac{1}{h_2 2\pi r_4 L}}$$

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Another Problem

- Insulation with k = 0.2 Btu/hr·ft·°F is to be added to the pipe in the previous example problem. Determine the heat transfer if the insulation is one inch thick.

$$\dot{Q} = \frac{T_{\infty 1} - T_{\infty 2}}{\frac{1}{h_1 2\pi r_1 L} + \frac{1}{2\pi k_1 L}\ln\left(\frac{r_2}{r_1}\right) + \frac{1}{2\pi k_2 L}\ln\left(\frac{r_3}{r_2}\right) + \frac{1}{h_2 2\pi r_3 L}}$$

$$\dot{Q} = \frac{2\pi L(T_{\infty 1} - T_{\infty 2})}{\frac{1}{h_1 r_1} + \frac{1}{k_1}\ln\left(\frac{r_2}{r_1}\right) + \frac{1}{k_2}\ln\left(\frac{r_3}{r_2}\right) + \frac{1}{h_2 r_3}}$$

Know all terms from previous example except these two

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Another Problem II

Unchanged resistances from previous example $\frac{1}{h_1 r_1} + \frac{1}{k_1}\ln\left(\frac{r_2}{r_1}\right) = \frac{0.153\,hr\cdot ft\cdot{}^oF}{Btu}$

New and modified resistances

$$\frac{1}{k_2}\ln\left(\frac{r_3}{r_2}\right) = \frac{hr\cdot ft\cdot{}^oF}{0.2\,Btu}\ln\left(\frac{1.525\,in}{0.525\,in}\right) = \frac{5.332\,hr\cdot ft\cdot{}^oF}{Btu}$$

$$\frac{1}{h_2 r_3} = \frac{hr\cdot ft^2\cdot{}^oF}{3\,Btu}\,\frac{1}{1.525\,in}\,\frac{12\,in}{ft} = \frac{2.623\,hr\cdot ft\cdot{}^oF}{Btu}$$

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Another Problem III

$$\dot{Q} = \frac{2\pi L(T_{\infty 1} - T_{\infty 2})}{\frac{1}{h_1 r_1} + \frac{1}{k_1}\ln\left(\frac{r_2}{r_1}\right) + \frac{1}{k_2}\ln\left(\frac{r_3}{r_2}\right) + \frac{1}{h_2 r_3}}$$

$$= \frac{2\pi(120^oF - 60^oF)(40\ ft)}{(0.146 + 0.007 + 5.332 + 2.623)\frac{hr\cdot ft\cdot{}^oF}{Btu}} = \frac{1860\,Btu}{hr}$$

- Insulation and outer convection resistances are largest
  - Inner convection and pipe conduction negligible
  - Outer convection resistance less with insulation

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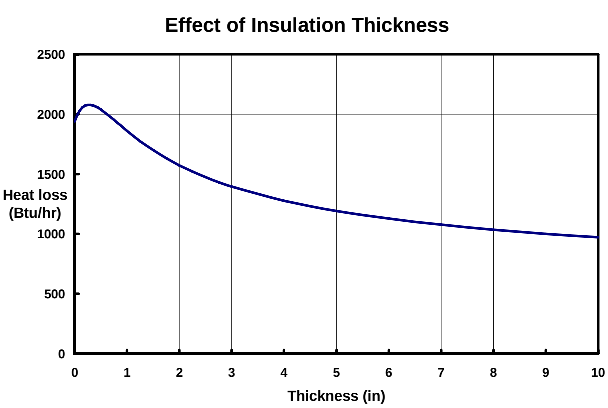

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Insulation Increases $\dot{Q}$?

- Why does initial amount of insulation increase heat transfer?
  - Tradeoff of two resistances
  - Added insulation adds conduction resistance
  - Added insulation also increases outer radius which decreases the outer convection resistance $1/(h_{outer}A_{outer}) = 1/(h_{outer}2\pi r_{outer}L)$

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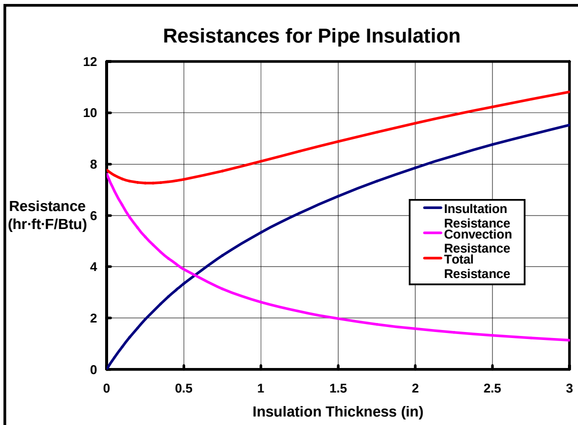

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Radius for Maximum $\dot{Q}$

$$\dot{Q} = \frac{2\pi L(T_{\infty 1} - T_{\infty 2})}{R_{other} + \frac{1}{k_o}\ln\left(\frac{r_o}{r_i}\right) + \frac{1}{h_2 r_o}}$$

r_o = outer radius

k_o = thermal conductivity of outer layer

For maximum $\dot{Q}$: $$\frac{d\dot{Q}}{dr_o} = -\frac{2\pi L(T_{\infty 1} - T_{\infty 2})\left(\frac{1}{k_o}\frac{1}{r_o} - \frac{1}{h_2 r_o^2}\right)}{\left(R_{other} + \frac{1}{k_o}\ln\left(\frac{r_o}{r_i}\right) + \frac{1}{h_2 r_o}\right)^2} = 0$$

$r_o = k_o/h_2$ for maximum $\dot{Q}$

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Radius for Maximum $\dot{Q}$

- $r_o = k_o/h_2$ for maximum $\dot{Q}$
- In the example problem $h_2 = 3$ Btu/hr·ft²·°F, and $k_o = 0.2$ Btu/hr·ft·°F so $r_o = 0.0667$ ft = 0.8 in for maximum $\dot{Q}$
- Pipe radius was 0.525 in; $r_o = 0.8$ in gives an insulation thickness of 0.275 in
- Note that $r_o = k_o/h_2$ does not depend on r_i and is usually larger than r_i
- There is no radius for minimum $\dot{Q}$

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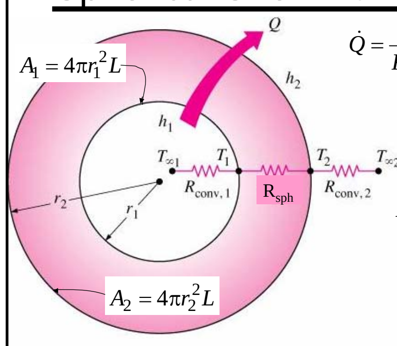
Spherical Shell with Convection



$$\dot{Q} = \frac{T_{\infty 1} - T_{\infty 2}}{R_{conv,1} + R_{sph} + R_{conv,2}}$$

$$R_{conv,1} = \frac{1}{h_1 A_1} = \frac{1}{h_1 4\pi r_1^2}$$

$$R_{conv,2} = \frac{1}{h_2 A_2} = \frac{1}{h_2 4\pi r_2^2}$$

$$R_{sph} = \frac{\frac{1}{r_1} - \frac{1}{r_2}}{4\pi k}$$

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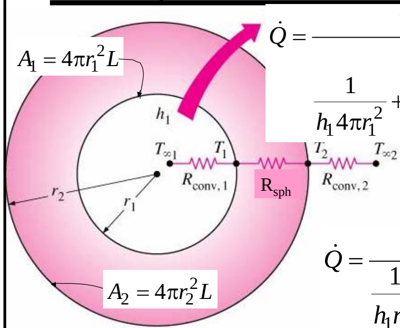
Figure 3-25 from Çengel, *Heat and Mass Transfer* 40

Spherical Shell Result



$$\dot{Q} = \frac{T_{\infty 1} - T_{\infty 2}}{\frac{1}{h_1 4\pi r_1^2} + \frac{\frac{1}{r_1} - \frac{1}{r_2}}{4\pi k} + \frac{1}{h_2 4\pi r_2^2}}$$

$$\dot{Q} = \frac{4\pi(T_{\infty 1} - T_{\infty 2})}{\frac{1}{h_1 r_1^2} + \frac{r_2 - r_1}{k r_1 r_2} + \frac{1}{h_2 r_2^2}}$$

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Figure 3-25 from Çengel, *Heat and Mass Transfer* 41

Conduction Shape Factors

- Simplified analysis
  - for multidimensional geometries with each surface at a uniform temperature
  - Use shape factor, S, whose equation is found from tables like Çengel Table 3-7
  - Basic equation: $\dot{Q} = kS(T_1 - T_2)$
  - S must have dimensions of length
    - Equations for S depend on parameters in the different geometries

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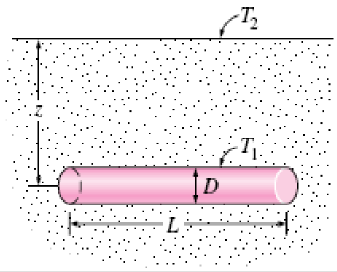
Example Shape Factor

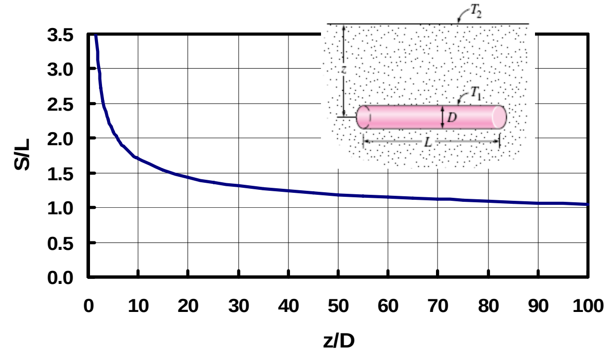
(1) Isothermal cylinder of length L buried in a semi-infinite medium ($L >> D$ and $z > 1.5D$)

$$S = \frac{2\pi L}{\ln(4z/D)}$$



From Table 7-1 in Çengel, *Heat and Mass Transfer*

Buried Pipe Shape Factor



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