# HEAT TRANSFER FROM A ROTATING CIRCULAR CYLINDER IN THE STEADY REGIME: EFFECTS OF PRANDTL NUMBER 

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#### Abstract

In this work, effects of Prandtl number on the heat transfer characteristics of an unconfined rotating circular cylinder are investigated for varying rotation rate ( $\alpha=$ $=0-5$ ) in the Reynolds number range of $1-35$ and Prandtl numbers range of 0.7100 in the steady flow regime. The numerical calculations are carried out by using a finite volume method based commercial computational fluid dynamics solver FLUENT. The isotherm patterns are presented for varying values of Prandtl number and rotation rate in the steady regime. The variation of the local and the average Nusselt numbers with Reynolds number, Prandtl number, and rotation rate are presented for the range of conditions. The average Nusselt number is found to decrease with increasing value of the rotation rate for the fixed value of the Reynolds and Prandtl numbers. With increasing value of the Prandtl number, the average Nusselt number increases for the fixed value of the rotation rate and the Reynolds number; however, the larger values of the Prandtl numbers show a large reduction in the value of the average Nusselt number with increasing rotation rate.


Key words: steady regime, rotating circular cylinder, Prandtl number, Nusselt number, rotation rate

## Introduction

Forced convective heat transfer from rotating cylinders has been a subject of great interest for researchers due to its high applicability in various industrial processes. The applications may include cylindrical cooling devices in plastics and glass industries, contact cylinder driers in paper making, textile, food processing and chemical processing industries, etc. Despite the configuration being simple, the flow around a rotating cylinder involves complex transport phenomenon because of many factors such as the effect of cylinder rotation on the production of lift force and moment, evolution of surrounding temperature field, etc. This work is concerned with the heat transfer from a rotating cylinder for varying values of Prandtl (Pr) numbers in the steady regime.

In literature, sufficient amount of information is available regarding the effects of Pr number on the heat transfer from a stationary circular cylinder [1-7]. The effect of Pr number on the heat transfer from a cylinder in the cross-flow configuration has been investigated by

[^0]Dennis et al. [3] by varying Pr number up to $3 \cdot 3 \cdot 10^{4}$ and Reynolds (Re) number up to 40. Those ranges of values are extended by Chang et al. [4] for Re up to 150 and $\operatorname{Pr}$ up to $10^{4}$. The experimental results have been presented for uniform heat flux condition for different values of Pr number by varying air, water and ethylene glycol as mediums with varying concentrations and Re number in the range $10^{3}-10^{5}$ [5]. Subsequently, Sanitjai et al. [6] studied the local and average heat transfer characteristics around a circular cylinder for the Re number range of $2 \cdot 10^{3}-9 \cdot 10^{4}$ and Pr number range $0.7-176$. Three regions of flow: laminar boundary layer, reattachment of shear layer and periodic vortex regions are indicated around the cylinder for sub-critical flow. The Nusselt ( Nu ) number in each region strongly depends on the Re number and the Pr number with different power indices. Bharti et al. [7] investigated the effects of Re and Pr numbers and thermal boundary conditions (uniform heat flux and constant wall temperature) around the cylinder for the range $10 \leq \operatorname{Re} \leq 45$ and $0.7 \leq$ $\leq \operatorname{Pr} \leq 400$ in the steady regime. The rate of heat transfer is found to increase with an increase in Re and/or Pr numbers.

In contrast, for the problem under consideration, most of the available information in the open literature is limited to the flow and heat transfer for the working fluid as air (i.e., $\operatorname{Pr}=$ $=0.7$ ). Badr et al. [8] presented the limiting results for the steady flow at $\operatorname{Re}=5$ and 20 ( $\alpha=$ $=0.1-1.0$ ) and some unsteady results for $\mathrm{Re}=60,100$, and 200. Ingham et al. [9] obtained the numerical solutions for steady uniform flow past a rotating cylinder for the Re numbers of 5 and 20 for $0 \leq \alpha \leq 3$. The particular attention has been given to the quantities such as drag and lift coefficients since they are very sensitive to the method of solution. Kang et al. [10] investigated numerically the laminar flow round a rotating cylinder for $\operatorname{Re}=60,100,160$, and $\alpha=0-2.5$ and reported that an increase in the rotation rate leads to the decrease in the mean drag and the linear increase in the mean lift. Stojković et al. [11] studied the effect of rotation rate on the flow around a rotating cylinder in the range $0 \leq \alpha \leq 6$ with Re number varying from 0.01-45 in the steady regime; further, unsteady flow calculations are carried out for $\mathrm{Re}=$ $=100$ and $\alpha=0-2$. The rotation of the cylinder suppresses the vortex development in both steady and unsteady flow regimes and significantly changes the flow field close to the cylinder. For very low Re numbers, the drag force is not affected by rotation and the lift force is a linear function of rotation. Mittal et al. [12] examined the flow past a rotating cylinder for a fixed Re number of 200 and rotation rate varying from $0-5$. The vortex shedding is observed for $\alpha<1.91$. For higher rotation rates, the flow is found to achieve a steady-state, except for $4.34<\alpha<4.70$ where the flow becomes unstable again. It is proposed that for large rotation rates, very large lift coefficients can be obtained via the Magnus effect. Recently, Panda et al. [13] investigated the flow of non-Newtonian power-law fluids past a rotating cylinder in the range $\mathrm{Re}=0.1-40, \alpha=0-6$, and power-law index of $0.2-1$ in the steady regime.

The flow and heat transfer over an isothermal cylinder is investigated by Badr et al. [14] for the range $\mathrm{Re}=5-100$ for a fixed $\operatorname{Pr}$ number of 0.7 . They found a decrease in the laminar forced convection heat transfer on increasing $\alpha$ for $\alpha \leq 4$. Mahfouz et al. [15] investigated the problem of laminar heat convection from a circular cylinder performing steady rotation for Rayleigh (Ra) numbers up to $10^{4}$ and Re numbers (based on surface velocity) up to 400 for two $\operatorname{Pr}$ numbers of 0.7 and 7. The rate of heat transfer found to increase with the increase of Ra number and decrease with the increase of speed of rotation. The increase of Pr number resulted in an appreciable increase in the average Nu number only at low Re numbers. Yan et al. [16] proposed a lattice Boltzmann Method (LBM) approach for the numerical simulation of heat transfer and fluid flow past a rotating isothermal cylinder for $\operatorname{Re}=200$ and $\operatorname{Pr}=0.5$ and 1 . The numerical results, such as velocity and temperature
distributions and lift and drag coefficients, agreed well with those reported in the literature. Kendoush [17] employed a boundary layer approach in order to predict the value of the Nu number for a rotating cylinder. Recently, the flow and heat transfer around a rotating circular cylinder is studied by Paramane et al. [18] for the values of $\alpha$ varies from $0-6$ for Re number range 20-160 for a fixed value of the Pr number of 0.7 . The values of drag coefficient as well as average Nu number are found to decrease on increasing the rotation rate. Subsequently, Paramane et al. [19] investigated the free stream flow and forced convection heat transfer across a rotating cylinder, dissipating uniform heat flux, for identical range of $\mathrm{Re}, \alpha$, and Pr numbers [18]. At higher rotational velocity, the Nu number is almost independent of Re number and thermal boundary conditions. Nobari et al. [20] studied the convective heat transfer from a rotating cylinder with inline oscillation at Re numbers of 100, 200, and 300. Different rotational speeds of the cylinder ( $0-2.5$ ) are considered at various oscillating amplitudes and frequencies with three different Pr numbers of $0.7,6$, and 20. As the rotational speed of the cylinder increases, both the Nu number and the drag coefficient decrease rapidly. Nemati et al. [21] investigated the laminar flow and heat transfer from a rotating circular cylinder with uniform planar shear, where the free stream velocity varies linearly across the cylinder using Multi-Relaxation-Time (MRT) LBM. The effects of variation of Re number, rotational speed ratio at shear rate 0.1 , blockage ratio 0.1 , and $\operatorname{Pr}$ number 0.71 are studied. The Re number changing from 50 to 160 for three rotational speed ratios of $0,0.5$, and 1 is investigated. They reported that the rotation of the cylinder pushes forward the point of flow separation, hence, the minimum point of Nu number distribution on the cylinder surface shifts to the right.

Yoon et al. [22] examined the laminar forced convection heat transfer past two rotating circular cylinders in a side-by-side arrangement at various range of absolute rotational speeds $(|\alpha| \leq 2)$ for different gap spacings at the Re number of 100 and a fixed $\operatorname{Pr}$ number of 0.7. As $|\alpha|$ increases, the behavior of time- and space- averaged Nu number has the decaying pattern for all the gap spacings considered. Moshkin et al. [23] investigated the 2-D heat transfer from two rotating circular cylinders in cross-flow of incompressible fluid under isothermal boundary condition for Re numbers up to 40 , while $\operatorname{Pr}$ number ranges between 0.7 and 50. The increase of Pr number resulted in an appreciable increase in the average Nu number.

Thus, based upon the above discussion and as far as known to us, it can be concluded here that no work is available regarding the effects of Pr numbers on the heat transfer across a rotating circular cylinder with varying Pr numbers in the steady regime. Hence, the objectives of the present study are to investigate the effects of Pr number on the heat transfer across a rotating circular cylinder and to fill this gap in the literature.

## Problem statement, governing equations, and boundary conditions

The system here consists of a 2-D infinite long circular cylinder (fig. 1) having diameter $D$ which is maintained at a constant temperature of $T_{\mathrm{w}}^{*}$ and is rotating in a counter-clockwise direction with a con-


Figure 1. Schematics of the unconfined flow and heat transfer around a rotating (counter-clock wise) circular cylinder
stant angular velocity of $\omega$. It is exposed to a constant free stream velocity of $U_{\infty}$ at a uniform temperature of $T_{\infty}$ at the inlet. As the temperature difference between the streaming liquid and the surface of the cylinder is small $(=2 \mathrm{~K})$, therefore the variation of physical properties, particularly density and viscosity, with temperature could be neglected.

The governing partial differential equations here are the dimensionless form of Navier-Stokes and energy equations in two dimensions for the incompressible flow around a rotating circular cylinder:

- continuity equation

$$
\begin{equation*}
\frac{\partial U}{\partial X}+\frac{\partial V}{\partial Y}=0 \tag{1}
\end{equation*}
$$

- X-momentum equation

$$
\begin{align*}
& \frac{\partial U}{\partial \tau}+\frac{\partial(U U)}{\partial X}+\frac{\partial(V U)}{\partial Y}= \\
= & -\frac{\partial p}{\partial X}+\frac{1}{\operatorname{Re}}\left(\frac{\partial^{2} U}{\partial X^{2}}+\frac{\partial^{2} U}{\partial Y^{2}}\right) \tag{2}
\end{align*}
$$

- Y-momentum equation

$$
\begin{equation*}
\frac{\partial V}{\partial \tau}+\frac{\partial(U V)}{\partial X}+\frac{\partial(V V)}{\partial Y}=-\frac{\partial p}{\partial Y}+\frac{1}{\operatorname{Re}}\left(\frac{\partial^{2} V}{\partial X^{2}}+\frac{\partial^{2} V}{\partial Y^{2}}\right) \tag{3}
\end{equation*}
$$

- energy equation

$$
\begin{equation*}
\frac{\partial \theta}{\partial \tau}+\frac{\partial(U \theta)}{\partial X}+\frac{\partial(V \theta)}{\partial Y}=\frac{1}{\operatorname{Re} \operatorname{Pr}}\left(\frac{\partial^{2} \theta}{\partial X^{2}}+\frac{\partial^{2} \theta}{\partial Y^{2}}\right) \tag{4}
\end{equation*}
$$

The dimensionless boundary conditions for the flow across a rotating circular cylinder can be written as (fig. 1):

- at the inlet boundary: $U=1, V=0$, and $\theta=0$,
- at the exit boundary: $\partial U / \partial X=0, \partial V / \partial X=0$ and $\partial \theta / \partial X=0$, and
- on the surface of the cylinder: $U=-\alpha \sin (\phi), V=-\alpha \cos (\phi)$, and $\theta=1$.

The boundary condition on the surface of the cylinder can be implemented by considering wall motion: moving wall and rotational for a particular rotational speed.

The governing eqs. (1)-(4) when solved using the above boundary conditions yield the primitive variables, i.e., velocity ( $U$ and $V$ ), pressure $(p)$ and temperature $(\theta)$ fields. The local and global characteristics such as local and average Nu numbers are obtained using these flow and thermal fields.

## Numerical details

The computational grid for the problem under consideration is generated by using a commercial grid generator GAMBIT and the numerical calculations are performed in the full computational domain using FLUENT for varying conditions of Re number, Pr number, and rotation rate. In particular, the O-type grid structure is created here and it consists of nonuniform quadrilateral cells having a total of 24000 grid points in the full computational domain. The grid near the surface of the cylinder is sufficiently fine to resolve the boundary
layer around the cylinder. The nearest grid point from the circular obstacle is taken to be at a distance of $0.0015 D$, where $D$ is the diameter of the cylinder. The outer boundary is taken to be at a distance of 300 D . The 2-D, steady, laminar, segregated solver is employed here to solve the incompressible flow on the collocated grid arrangement. The second order upwind scheme is used to discretize convective terms of momentum equation, whereas the diffusive term is discretized by central difference scheme. The semi implicit method for the pressure linked equations (SIMPLE) has been used to solve Navier-Stokes and energy equations along with the above noted boundary conditions. A convergence criterion of $10^{-10}$ is used for continuity, the $x$ - and $y$-components of Navier-Stokes and energy equations.

## Choices of numerical parameters

In the present study, three non-uniform grids of 18000,24000 , and 32000 cells (with 50,100 , and 200 grid points on the surface of the cylinder and having a first grid point at distances of $0.0018 D, 0.0015 D$, and $0.001 D$, respectively) have been tested for the domain size of $300 D$ for the grid resolution study. The percentage relative differences in the values of the total drag coefficient and the average cylinder Nu number are found to be less than $4.85 \%$ and $4.6 \%$, respectively for the highest value of the Re number of 35 and the rotation rate of 5 for the Pr number of 100 used in this work. The grid structure of 24000 cells with 200 grid points on the surface of the cylinder and having a first grid point at a distance of $0.0015 D$ is used in all the computations reported in this work. Similarly, the domain dependence study is carried out for the two values of the computational domain, i.e., 200 D and 300 D for the grid size of 24000 cells with 200 grid points prescribed on the surface of the cylinder and having a first grid point at a distance of 0.0015 D . The percentage differences in the values of the drag coefficient and the average Nu number are found to be less than $3.5 \%$ and $3 \%$ for the Re number of 35 and the rotation rate of 5 for the Pr number of 100 . Thus, the computational domain of 300 times the diameter of the cylinder is used in this work.

## Results and discussion

In this study, effects of Re and Pr numbers around a rotating circular cylinder are examined for $\operatorname{Re}=1-35$ (in the steps of 5) for $\operatorname{Pr}=0.7,1,10,50$, and 100 for $\alpha$ varying from $0-5$ (in the steps of 1 ). The validation of present results, representative isotherm patterns, variation of the local Nu number on the surface of the cylinder and the average Nu number of the cylinder for the above range of conditions are discussed in the next sub-sections in the steady unconfined flow regime.

## Validation of results

Extensive benchmarking of the present heat transfer results with literature values is carried out here for $\operatorname{Re}=10,20$, and 40 for a stationary cylinder $(\alpha=0)$ for $\operatorname{Pr}=0.7,1,50$, and 100 (tab. 1) and for a rotating cylinder $(\alpha \square 0)$ for $\operatorname{Re}=20$ and 40 for $\operatorname{Pr}=0.7$ (tab. 2) in the steady flow regime. The present results of the average Nu number for the stationary cylinder are in excellent agreement with the literature values.

For instance, the maximum relative change between the present results and that of the literature values is found to be about $1.75 \%$. For a rotating cylinder, the maximum change between the present results and that of Paramane et al. [18] is found to be less than $4.65 \%$ for
varying rotation rates for $\mathrm{Re}=20$ and 40. Thus, this validates the present numerical solution procedure.

Table 1. Comparison of present $\overline{\mathrm{Nu}}$ results ( $\alpha=0$ ) with literature values

| Source | $\operatorname{Pr}=0.7$ | $\operatorname{Pr}=1$ | $\operatorname{Pr}=50$ | $\operatorname{Pr}=100$ |  |
| :--- | ---: | :---: | :---: | :---: | :---: |
|  | $\operatorname{Re}=10$ |  |  |  |  |
|  | 1.8371 | 2.0575 | 7.0991 | 8.8611 |  |
| Bharti et al. [7] | 1.8623 | 2.0942 | 7.1942 | 8.9942 |  |
|  | $\operatorname{Re}=20$ |  |  |  |  |
| Present work | 2.4317 | 2.7313 | 9.7518 | 12.3032 |  |
| Bharti et al. $[7]$ | 2.4653 | 2.7242 | 9.8642 | 12.4442 |  |
|  | $\operatorname{Re}=40$ |  |  |  |  |
| Present work | 3.2374 | 3.6506 | 14.1345 | 17.9347 |  |
| Bharti et al. [7] | 3.2825 | 3.6842 | 14.1242 | 17.9044 |  |

Table 2. Comparison of present $\overline{\mathrm{Nu}}$ results with Paramane et al. [18]

| $\alpha$ | Paramane <br> et al. $[18]$ | Present <br> work | Paramane <br> et al. [18] | Present <br> work |
| :---: | :---: | :---: | :---: | :---: |
|  | $\mathrm{Re}=20$ |  | $\operatorname{Re}=40$ |  |
| 0 | 2.4189 | 2.4317 | 3.2465 | 3.2374 |
| 1 | 2.3883 | 2.3994 | 3.1954 | 3.1968 |
| 2 | 2.2861 | 2.3031 | 3.0115 | 3.0266 |
| 3 | 2.2248 | 2.2460 | 2.9502 | 2.9642 |
| 4 | 2.2554 | 2.2768 | 3.0422 | 3.0512 |
| 5 | 2.2759 | 2.3010 | 3.0115 | 3.0156 |
| 6 | 2.2248 | 2.2185 | 2.4802 | 2.3658 |

## Isotherm patterns

The representative isotherm profiles around the rotating cylinder for various values of rotation rates for the $\operatorname{Re}=1,20$, and 35 for $\operatorname{Pr}=0.7,50$, and 100 are shown in figs. 2-4 (a-$-i)$. It is worth mentioning that no reverse flow in the simulation is observed for the range of conditions studied here. For a stationary cylinder, isotherms have maximum density close to the front surface of the cylinder. This indicates high temperature gradients or in other words, the higher values of local Nu number near the front stagnation point on the front surface as compared to other points on the cylinder surface. For a fixed Pr number, the density of isotherms close to the rear surface of the cylinder increases on increasing the Re number. This effect can be explained as on increasing the Re number, the recirculation region increases. Similar trend is observed on increasing Pr number keeping Re number constant. As the value of Pr number increases, the thermal boundary layer becomes thinner which leads to an increase in the temperature gradients close to the rear end. On increasing the value of the rotation rate, the maximum density of isotherms shifts from front surface towards the bottom surface of the rotating cylinder (rotating counter-clock wise) for the fixed Pr and Re numbers. It is also observed that isotherms shifts in the direction of rotation of the cylinder and becomes almost vertical at higher values of the rotation rate. However, at $\operatorname{Re}=1$ and $\operatorname{Pr}=0.7$, slight change in isotherm patterns occurs on increasing the rotation rate. The decay of temperature fields around the cylinder can also be seen in these figures as the value of the Pr number increases due to the thinning of the thermal boundary layer. The temperature distributions presented by way of isotherms can be used to interpret the variation in the local and average heat transfer characteristics with Re number, Pr number, and rotation rate.


Figure 2. Isotherm profiles at $\operatorname{Pr}=\mathbf{1}$ for varying values of Reynolds number and rotation rate

## Local Nusselt number

Figures 5 (a-l) show the variation of the local Nu number on the surface of the circular cylinder for different rotation rates $(\alpha=0,1,3$, and 5) for $\operatorname{Pr}=0.7,50$, and 100 for $\operatorname{Re}=1,20$, and 35 . For a stationary cylinder, i. e., at $\alpha=0$, figs. 5 (a, e, i), the variation of the local Nu number around the cylinder is found to be symmetrical at $\phi=180^{\circ}$. The maximum value of the local Nu number occurs at the front stagnation point for all Re and Pr numbers studied; whereas the least value occurs at the rear stagnation point for $\operatorname{Re}=1$. A kink is also observed at the rear end of the cylinder for higher values of Pr numbers ( $>0.7$ ) and the size of the kink increases as the value of the Pr number increases for the fixed value of the Re number. For a rotating circular cylinder, for the low value of the Pr number (= 0.7 ), the maximum in the value of the local Nu number shifts in the direction of the rotation and the


Figure 3. Isotherm profiles at $\operatorname{Pr}=\mathbf{5 0}$ for varying values of Reynolds number and rotation rate
kink also shifts in the direction of rotation at $\alpha=1$ (fig. 5b); however, on further increasing the value of the rotation rate $(\alpha>1)$, the kink disappears (figs. 5c, d). On the other hand, for the higher value of the Pr number ( $>0.7$ ), the shift in the maximum value of the local Nu number is more than that of $\operatorname{Pr}=0.7$ and the slight shift in the kink in the direction of rotation is observed at $\alpha=1$ (figs. $5 \mathrm{f}, \mathrm{j}$ ); however, on further increasing the value of the rotation rate ( $\alpha>1$ ), the local Nu number curve becomes smooth and the kink disappears (figs. $5 \mathrm{~g}, \mathrm{~h}, \mathrm{k}, \mathrm{l}$ ). On increasing the value of the Pr number, the local Nu number curve becomes smooth and almost independent of rotation rates at higher rotation rates (figs. $5 \mathrm{~g}, \mathrm{~h}, \mathrm{k}, \mathrm{l}$ ). It can also be observed that for the fixed value of the Pr number, the value of the local Nu number increases with increasing Re number for a particular $\alpha$; however, the value of the local Nu number decreases with increasing rotation rate for the fixed value of the Re number. With increasing value of the Pr number, the value of the local Nu number increases for the fixed value of the Re number and the rotation rate in the steady regime.


Figure 4. Isotherm profiles at $\mathbf{P r}=100$ for varying values of Reynolds number and rotation rate

## Average Nusselt number

The average Nu number variation is presented in figs. 6(a-d) for the Pr number range $0.7-100$ in the steady regime. On increasing the value of the rotation rate, for the fixed value of the Pr number, the value of the average Nu number decreases for all Re numbers studied here. This is due to the enveloping vortex containing the fluid trapped inside for the decrease in the average Nu number with increase in the rotation rate as also explained in [18] for the fixed Pr number of 0.7. This enveloping vortex acts as a buffer for heat transfer and limits the heat transfer to only conduction. Further, as the size of this vortex continuously increases with increasing rotation rate, conduction heat transfer decreases thereby reducing


Figure 5. Local Nusselt number variation for $\operatorname{Pr}=\mathbf{0 . 7}, 50$, and 100 for varying values of Reynolds number and rotation rate
the average Nu number. For a fixed rotation rate, the value of the average Nu number increases with increasing Re number for a fixed Pr number. This can be explained as when Re number increases the inertia of flow increases thereby increasing the heat transfer. The decrease in the average Nu number is more at higher values of Re and Pr numbers as compared to lower values of Re and Pr numbers. In other words, at low Re and Pr numbers, the decrease in the average Nu number is very slow. On increasing the value of the Pr number, the average Nu number increases for the fixed value of the Re number for a particular $\alpha$.

At $\operatorname{Pr}=0.7$ : the percentage maximum change in the value of the average Nu number (taking reference values to be at $\alpha=0$ ) for the case $\alpha=1$ is in the range of $0.06(\operatorname{Re}=1)$ to $1.34(\operatorname{Re}=25)$, for the case $\alpha=2$ is in the range of $0.24(\operatorname{Re}=1)$ to $6.3(\operatorname{Re}=35)$, for $\alpha=3$ is in the range of $0.5(\mathrm{Re}=1)$ to $8.34(\operatorname{Re}=35)$, and for $\alpha=4$ is in the range of $0.7(\operatorname{Re}=1)$ to 6.4 $(\operatorname{Re}=20)$, and for $\alpha=5$ is in the range of $0.9(\operatorname{Re}=1)$ to $6.4(\operatorname{Re}=$ $=35$ ).

At $\operatorname{Pr}=1$ : the percentage change in the average Nu number for the case $\alpha=1$ is in the range of $0.2(\operatorname{Re}=1)$ to $1.5(\operatorname{Re}=15)$, for the case $\alpha=2$ is in the range of 7 $(\operatorname{Re}=20)$ to $15(\operatorname{Re}=15)$, for $\alpha=$ $=3$ is in the range of $1.2(\operatorname{Re}=1)$ to $11.8(\operatorname{Re}=35)$, and for $\alpha=4$ is in the range of $1.8(\operatorname{Re}=1)$ to 9.76


Figure 6. The average Nusselt number variation with increasing $\alpha$ for $\operatorname{Pr}=0.7,10,50$, and 100 for varying Reynolds number ( $\mathrm{Re}=20$ ), and for $\alpha=5$ is in the range of $2.3(\operatorname{Re}=1)$ to $10(\operatorname{Re}=35)$.

At $\operatorname{Pr}=10$ : the percentage change in the average Nu number for the case $\alpha=1$ is in the range of $3.24(\operatorname{Re}=1)$ to $11.9(\operatorname{Re}=15)$, for the case $\alpha=2$ is in the range of $10.24(\operatorname{Re}=1)$ to $30(\operatorname{Re}=15)$, for $\alpha=3$ is in the range of $15.86(\operatorname{Re}=1)$ to $39.6(\operatorname{Re}=15)$, for $\alpha=4$ is in the range of $20(\operatorname{Re}=1)$ to $42(\operatorname{Re}=15)$, and for $\alpha=5$ is in the range of $23(\operatorname{Re}=1)$ to $42.3(\operatorname{Re}=15)$.

At $\operatorname{Pr}=50$ : the percentage change in the average Nu number for the case $\alpha=1$ is in the range of $12.44(\operatorname{Re}=1)$ to $20.7(\operatorname{Re}=35)$, for the case $\alpha=2$ is in the range of $26.4(\operatorname{Re}=1)$ to $49.3(\operatorname{Re}=35)$, for $\alpha=3$ is in the range of $35(\operatorname{Re}=1)$ to $59(\operatorname{Re}=35)$, for $\alpha=4$ is in the range of $40.5(\operatorname{Re}=1)$ to $60(\operatorname{Re}=35)$, and for $\alpha=5$ is in the range of $44(\operatorname{Re}=1)$ to $62.3(\operatorname{Re}=$ 35).

At $\operatorname{Pr}=100$ : the corresponding percentage change in the average cylinder Nu number for the case $\alpha=1$ is in the range of $18.8(\operatorname{Re}=1)$ to 29.9 ( $\operatorname{Re}=35$ ), for the case $\alpha=2$ is in the range of $35(\operatorname{Re}=1)$ to $57.8(\operatorname{Re}=35)$, for $\alpha=3$ is in the range of $44(\operatorname{Re}=1)$ to 66.3 $(\operatorname{Re}=35)$, for $\alpha=4$ is in the range of $49.4(\operatorname{Re}=1)$ to $67.2(\operatorname{Re}=35)$, and for $\alpha=5$ is in the range of $52.4(\operatorname{Re}=1)$ to $69(\operatorname{Re}=35)$.

## Conclusions

In the present study, forced convection heat transfer for varying values of Prandtl number has been studied across a long rotating circular cylinder. The cylinder is rotating at a rotation rate $(\alpha)$ varying from $0-5$ in the Reynolds number range $1-35$ and Prandtl number range of $0.7-100$. The representative isotherm patterns are presented and analyzed for the range of conditions. For the fixed value of the Prandtl number, the value of the local Nusselt number increases with increasing Reynolds number for the fixed $\alpha$; however, the value of the local Nusselt number decreases with increasing rotation rate for the fixed value of the Reynolds number. With increasing value of the Prandtl number, the value of the local Nusselt number increases for the fixed value of the Reynolds number and the rotation rate in the steady regime. The average Nusselt number increases with increasing Prandtl number for the fixed value of the Reynolds number for the particular value of $\alpha$.

## Nomenclature

| $c_{p}$ | - specific heat of the fluid, $\left[\mathrm{Jkg}^{-1} \mathrm{~K}^{-1}\right]$ | $X$ | - non-dimensional stream-wise |
| :---: | :---: | :---: | :---: |
| D | - diameter of the circular cylinder, [m] |  | dimension of co-ordinates $(=x / D),[-]$ |
| $\underline{h}$ | - local heat transfer coefficient, [ $\mathrm{Wm}^{-2} \mathrm{~K}^{-1}$ ] | X | - stream-wise dimension of co-ordinates, [m] |
| $\bar{h}$ | - average heat transfer coefficient, $\left[\mathrm{Wm}^{-2} \mathrm{~K}^{-1}\right]$ | $Y$ | - non-dimensional cross stream dimension |
| $k$ | - thermal conductivity of the fluid, $\left[\mathrm{Wm}^{-1} \mathrm{~K}^{-1}\right]$ - local Nusselt number $(=h D / k),[-]$ |  | of co-ordinates $(=y / D),[-]$ <br> - cross-steam dimension of |
| $\overline{\mathrm{Nu}}$ | - average Nusselt number, ( $=\bar{h} D / k],[-]$ | $y$ | co-ordinates, [m] |
| Pr $p$ | - Prandtl number, $\left(=\mu c_{p} k^{-1}\right),[-]$ - non-dimensional pressure $\left(=p^{*} / \rho U_{\infty}^{2}\right),[-]$ | Greek symbols |  |
| $p^{*}$ | - pressure, $[\mathrm{Pa}]$ | $\alpha$ | - non-dimensional rotation |
| Re | - Reynolds number ( $=\rho U_{\infty} D / \mu$ ), [-] |  | rate ( $=D \omega / 2 U_{\infty}$ ), [-] |
| $T_{\infty}$ | - temperature of the fluid at the inlet, [K] | $\mu$ | - viscosity of the fluid, [Pa s ] |
| $T_{\mathrm{w}}^{*}$ | - constant wall temperature at the surface of the cylinder, [K] | $\begin{aligned} & \rho \\ & \phi \end{aligned}$ | - density of the fluid, $\left[\mathrm{kgm}^{-3}\right]$ <br> - angle measured from the |
| $t$ | - time, [s] |  | front stagnation point, [deg.] |
| $U$ | - non-dimensional stream-wise velocity $\left(=u / U_{\infty}\right),[-]$ | $\begin{gathered} \tau \\ \theta \end{gathered}$ | - non-dimensional time $\left(=t U_{\alpha} / D\right),[-]$ <br> - non-dimensional temperature, |
| $U_{\infty}$ | - free-steam velocity, $\left[\mathrm{ms}^{-1}\right]$ |  | (= $\left(T^{*}-T_{\infty}\right) /\left(T_{\mathrm{w}}^{*}-\mathrm{T}_{\infty}\right)$ ), [-] |
| $u$ | - stram-wise velocity, [ $\mathrm{ms}^{-1}$ ] | $\omega$ | - constant angular velocity of |
| $V$ | - non-dimensional cross stream |  | cylinder rotation, $\left[\mathrm{rad} \cdot \mathrm{s}^{-1}\right]$ |
| $v$ | $\begin{aligned} & \text { velocity }\left(=v / U_{\infty}\right),[-] \\ - & \text { cross steam velocity, }\left[\mathrm{ms}^{-1}\right]\end{aligned}$ | Superscripts |  |
|  |  |  | - dimensional variable |

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