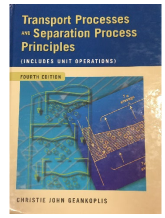
CM3110
Transport I
Part II: Heat Transfer



Heat Transfer Resistances
(Supplement)

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These slides are incorporated into the slides from lectures 14-16, but are assembled here to tell the *heat-transfer resistance* story all together.

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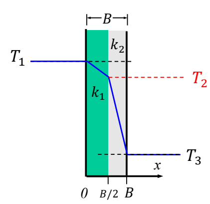
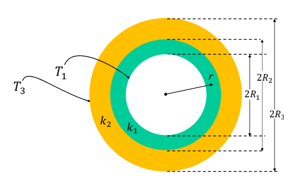
1D Heat Transfer – Resistance Supplement

Thermal conductivity k and heat transfer coefficient h may be thought of as sources of *resistance* to heat transfer.



These *resistances* stack up in a logical way, allowing us to quickly and accurately determine the effect of adding insulating layers, encountering pipe fouling, and other applications.



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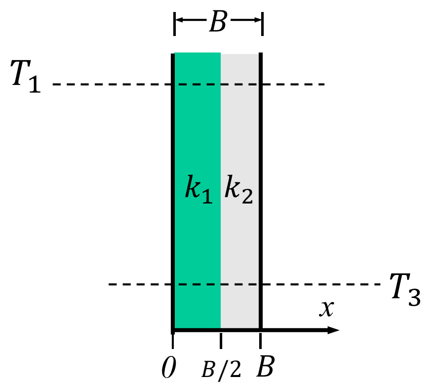
1D Heat Transfer

Using the solution: Composite Door:

For an outside door, a metal is used (k_1) for strength, and a cork k_2) is used for insulation. Both are the same thickness $B/2$. What is the temperature profile in the door at steady state? What is the flux? The inside temperature of the metal is T_1 and the outside temperature of the cork is T_3.

$k_1 \gg k_2$



Let's try.

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Note: *in the hand notes the temperatures from left to right are T_1, T_3, T_2.*

See handwritten notes.

https://pages.mtu.edu/~fmorriso/cm310/selected_lecture_slides.html

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1D Heat Transfer

Example 1b: Composite Door (two equal width layers)

SOLUTION:

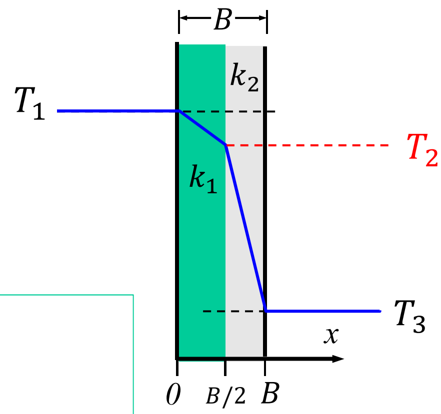
$$\frac{q_x}{A} = \frac{(T_1 - T_3)}{\left(\frac{\frac{B}{2}(k_1 + k_2)}{k_1 k_2}\right)}$$

k_1 material: $(0 \le x \le B/2)$

$$T(x) = \frac{(T_2 - T_1)}{B/2} x + T_1$$

k_2 material: $(B/2 \le x \le B)$

$$T(x) = \frac{(T_3 - T_2)}{B/2} x + (2T_2 - T_3)$$



$$T_2 = \frac{k_1 T_1 + k_2 T_3}{k_1 + k_2}$$

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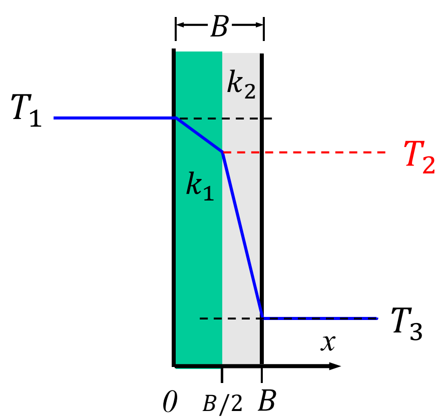
1D Heat Transfer

Example 1b: Composite Door (two equal width layers)

SOLUTION:

$$\frac{q_x}{A} = \frac{(T_1 - T_3)}{\frac{B/2}{k_1} + \frac{B/2}{k_2}}$$

Let: $\mathcal{R}_i \equiv \frac{\Delta x}{k_i}$



$$\frac{q_x}{A} = \frac{(T_1 - T_3)}{\mathcal{R}_1 + \mathcal{R}_2} = \frac{\text{driving force}}{\text{resistance}}$$

Each of the layers contributes a resistance, added in *series* (like in electricity).

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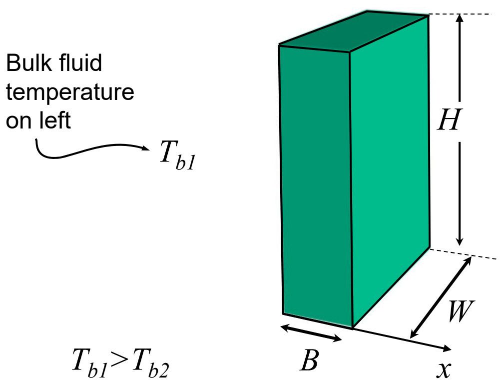
1D Heat Transfer

Example 2: Heat flux in a rectangular solid – Newton's law of cooling BC

Assumptions:
- wide, tall slab
- steady state
- *h_1 and h_2 are the heat transfer coefficients of the left and right walls*

What is the steady state temperature profile in a rectangular slab if the fluid on one side is held at T_{b1} and the fluid on the other side is held at T_{b2}?



Newton's law of cooling boundary conditions

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See handwritten notes (in class, also on web).

https://pages.mtu.edu/~fmorriso/cm310/selected_lecture_slides.html

https://pages.mtu.edu/~fmorriso/cm310/algebra_details_N_law_cooling.pdf

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1D Heat Transfer

Example 2: Heat flux in a rectangular solid – Newton's law of cooling BC

Solution: **(temp profile, flux)**

Temperature profile:
(linear)

$$\frac{T_{b1}-T}{T_{b1}-T_{b2}}=\frac{\frac{x}{k}+\frac{1}{h_1}}{\left(\frac{1}{h_1}+\frac{B}{k}+\frac{1}{h_2}\right)}$$

Flux:
(constant)

$$\frac{q_x}{A}=\frac{T_{b1}-T_{b2}}{\frac{1}{h_1}+\frac{B}{k}+\frac{1}{h_2}}$$

Rectangular slab with Newton's law of cooling BCs

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1D Heat Transfer

Example 2: Heat flux in a rectangular solid – Newton's law of cooling BC

Solution: **(temp profile, flux)**

Temperature profile:
(linear)

$$\frac{T_{b1}-T}{T_{b1}-T_{b2}}=\frac{\frac{x}{k}+\frac{1}{h_1}}{\left(\frac{1}{h_1}+\frac{B}{k}+\frac{1}{h_2}\right)}$$

$$T=T_{b1}-\left(\frac{(T_{b1}-T_{b2})\frac{1}{k}}{\left(\frac{1}{h_1}+\frac{B}{k}+\frac{1}{h_2}\right)}\right)x+\left(\frac{(T_{b1}-T_{b2})\frac{1}{h_1}}{\left(\frac{1}{h_1}+\frac{B}{k}+\frac{1}{h_2}\right)}\right)$$

Resistance due to heat transfer at boundary
Resistance due to finite thermal conductivity

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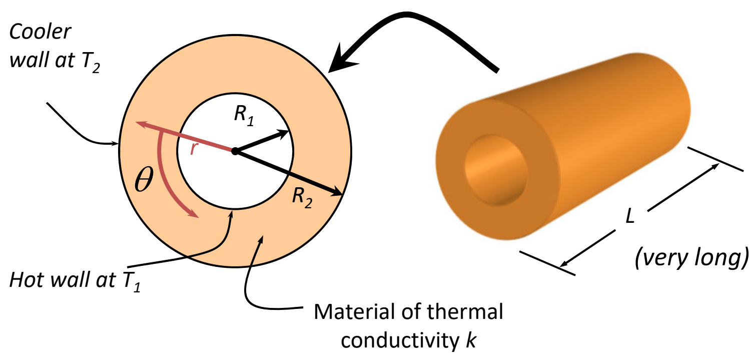
1D Heat Transfer – Radial

Example 3: Heat flux in a cylindrical shell – Temp BC

Assumptions:
- long pipe
- steady state
- *k* = thermal conductivity of wall

*What is the steady state temperature profile in a cylindrical shell (pipe) if the **inner wall** is at T_1 and the **outer wall** is at T_2? ($T_1>T_2$)*



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See handwritten notes in class.

https://pages.mtu.edu/~fmorriso/cm310/selected_lecture_slides.html

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1D Heat Transfer – Radial

Example 3: Heat flux in a cylindrical shell – Temp BC

Solution for Cylindrical Shell:

NOT constant

$$\frac{q_r}{A} = \left(\frac{(T_1 - T_2)}{\frac{1}{k}\ln\frac{R_2}{R_1}} \right) \frac{1}{r}$$

The heat flux $\frac{q_r}{A}$ **DOES** depend on, k; also $\frac{q_r}{A}$ decreases as $1/r$

NOT linear

$$\frac{T_2 - T}{T_2 - T_1} = \frac{\ln\frac{R_2}{r}}{\ln\frac{R_2}{R_1}}$$

Note that $T(r)$ does not depend on the thermal conductivity, k (steady state)

Pipe with temperature BCs

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1D Heat Transfer – Radial

Example 3: Heat flux in a cylindrical shell – Temp BC

Solution for Cylindrical Shell:

NOT constant

$$\frac{q_r}{A} = \left(\frac{(T_1 - T_2)}{\frac{1}{k}\ln\frac{R_2}{R_1}} \right) \frac{1}{r}$$

Let: $\mathcal{R}_i \equiv \frac{1}{k_i}\ln\frac{R_{i+1}}{R_i}$

$$\frac{q_r}{A} = \left(\frac{(T_1 - T_2)}{\mathcal{R}_1} \right) \frac{1}{r} = \frac{\text{driving force}}{\text{resistance}}$$

Resistance due to finite thermal conductivity, radial

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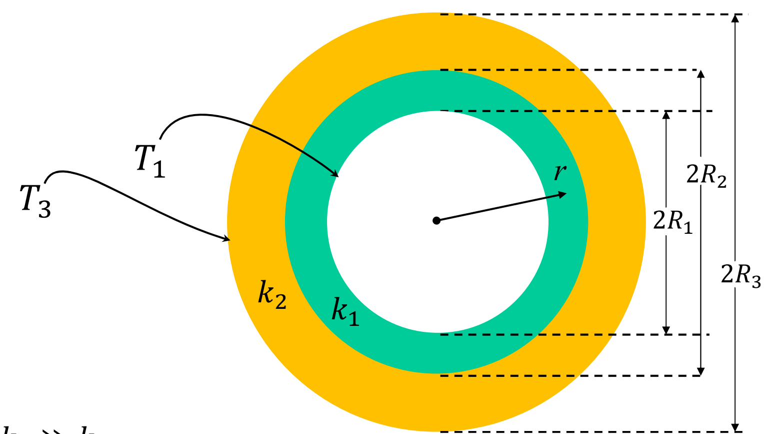
1D Radial Heat Transfer

Using the solution: Insulated Pipe (Composite, radial conduction)

For a metal pipe carrying a hot liquid (k_1) an insulation layer is added with thermal conductivity k_2. What is the temperature profile in the composite pipe at steady state? What is the flux? The inside temperature of the metal pipe is T_1 and the outside temperature of the insulation is T_3.



$k_1 \gg k_2$

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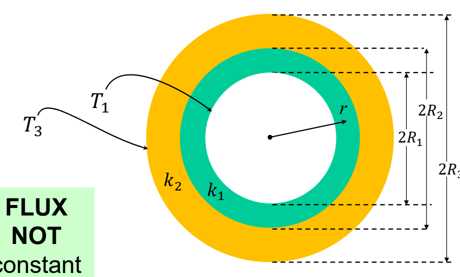
1D Heat Transfer – Radial

Example 3b: Insulated Pipe (Composite, radial conduction)

SOLUTION:



$$\frac{q_r}{A} = -k_i\left(\frac{dT}{dr}\right) = (\text{constant})\frac{1}{r}$$

FLUX NOT constant

k_1 material: $(R_1 \le r \le R_2)$

$$T(r) = a_1 \ln r + \mathrm{b}_1$$

k_2 material: $(R_2 \le r \le R_3)$

$$T(r) = a_2 \ln r + \mathrm{b}_2$$

$\boldsymbol{T(r)}$ **NOT** linear

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See Lecture 16 Slides

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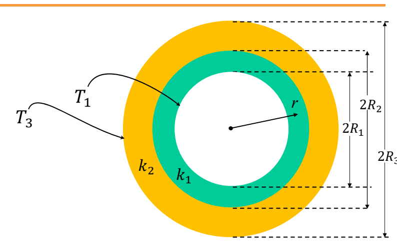
1D Heat Transfer – Radial

Example 3b: Insulated Pipe (Composite, radial conduction)

SOLUTION:



$$\frac{q_r}{A} = \left(\frac{(T_1 - T_3)}{\frac{1}{k_1}\ln\frac{R_2}{R_1} + \frac{1}{k_2}\ln\frac{R_3}{R_2}} \right) \frac{1}{r}$$

Let: $\mathcal{R}_i \equiv \frac{1}{k_i}\ln\frac{R_{i+1}}{R_i}$

$$\frac{q_r}{A} = \left(\frac{(T_1 - T_3)}{\mathcal{R}_1 + \mathcal{R}_2} \right) \frac{1}{r} = \frac{\text{driving force}}{\text{resistance}}$$

Note that we can continue to add layers in terms of resistance

Each of the layers contributes a resistance, added in series (like in electricity).

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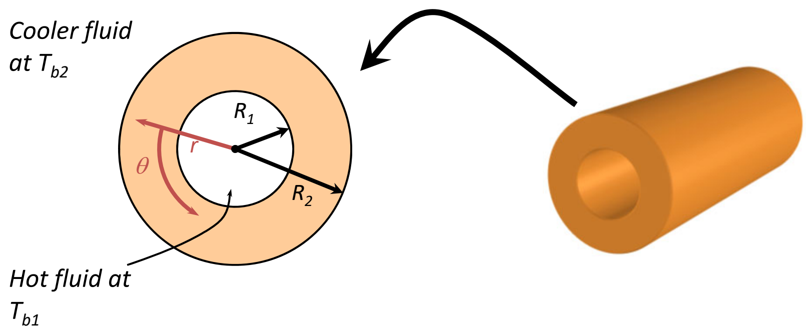
1D Heat Transfer – Radial

Example 4: Heat flux in a cylindrical shell – Newton's law of cooling

Assumptions:
- long pipe
- steady state
- k = thermal conductivity of wall
- h_1, h_2 = heat transfer coefficients

What is the steady state temperature profile in a cylindrical shell (pipe) if the ***fluid on the inside*** *is at T_{b1} and the* ***fluid on the outside*** *is at T_{b2}? (T_{b1}>T_{b2})*



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1D Heat Transfer – Radial

Example 4: Heat flux in a cylindrical shell

Newton's law of cooling boundary conditions

Solution: Radial Heat Flux in an Annulus

$T(r)$

$$T - T_{b2} = \frac{(T_{b1} - T_{b2})\left(\ln\left(\frac{R_2}{r}\right) + \frac{k}{h_2 R_2}\right)}{\frac{k}{h_2 R_2} + \ln\left(\frac{R_2}{R_1}\right) + \frac{k}{h_1 R_1}}$$

$q_r(r)$

$$\frac{q_r}{A} = \frac{(T_{b1} - T_{b2})}{\frac{1}{h_2 R_2} + \frac{1}{k}\ln\left(\frac{R_2}{R_1}\right) + \frac{1}{h_1 R_1}}\left(\frac{1}{r}\right)$$

Resistance $\mathcal{R}$ due to heat transfer coefficients, radial
Resistance $\mathcal{R}$ due to finite thermal conductivity, radial

1D Heat Transfer – Radial

Solution: Radial Heat Flux in an Annulus

$$\frac{q_r}{A} = \frac{(T_{b1} - T_{b2})}{\frac{1}{h_2 R_2} + \frac{1}{k}\ln\left(\frac{R_2}{R_1}\right) + \frac{1}{h_1 R_1}}\left(\frac{1}{r}\right)$$

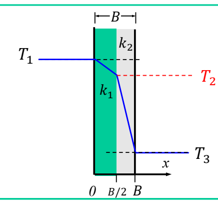
Note that we can continue to add layers in terms of *resistance*

Resistance $\mathcal{R}$ due to heat transfer coefficients, radial
Resistance $\mathcal{R}$ due to finite thermal conductivity, radial

1D Heat Transfer – Composite Structures

Let: $\mathcal{R}_i \equiv \frac{\Delta x}{k_i}$

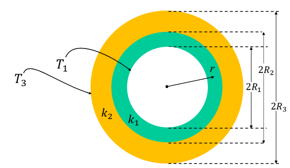
$$\frac{q_x}{A} = \frac{(T_1 - T_3)}{\mathcal{R}_1 + \mathcal{R}_2} = \frac{\text{driving force}}{\text{resistance}}$$



Note: Geankoplis uses a different resistance. For rectangular heat flux:
$R_{Geankoplis} = \mathcal{R}/LW$

Let: $\mathcal{R}_i \equiv \frac{1}{k_i} \ln \frac{R_{i+1}}{R_i}$

$$\frac{q_r}{A} = \left(\frac{(T_1 - T_3)}{\mathcal{R}_1 + \mathcal{R}_2}\right)\frac{1}{r} = \frac{\text{driving force}}{\text{resistance}}$$



Note: Geankoplis uses a different resistance. For radial heat flux:
$R_{Geankoplis} = \mathcal{R}/2\pi L$

Michigan Tech

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Back to regular thread