Hierarchical Linear Models

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Overview and resources

- Overview
- Web site and links: www.uoregon.edu/~stevensj/HLM
- Software:
  - HLM
  - MLwinN
  - Mplus
  - SAS
  - R and S-Plus
  - WinBugs

Workshop Overview

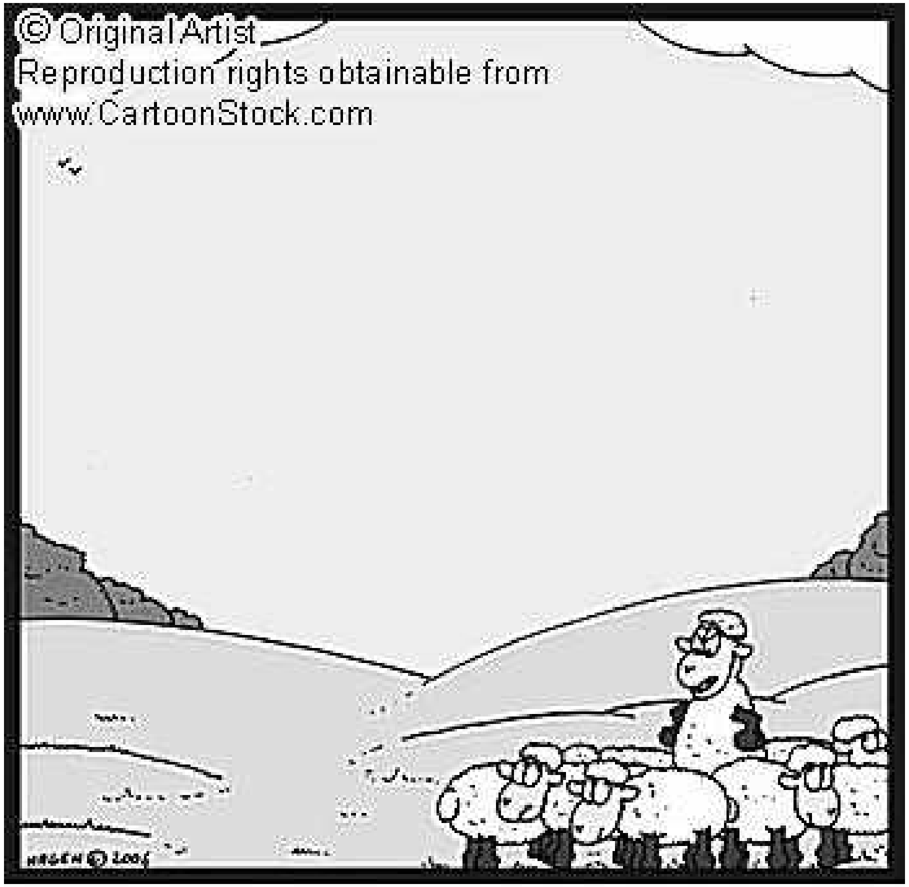
- Preparing data
- Two Level models
- Testing nested hierarchies of models
- Estimation
- Interpreting results
- Three level models
- Longitudinal models
- Power in multilevel models

Hierarchical Data Structures

Many social and natural phenomena have a nested or clustered organization:

- Children within classrooms within schools
- Patients in a medical study grouped within doctors within different clinics
- Children within families within communities
- Employees within departments within business locations

Grouping and membership in particular units and clusters are important



For goodness sake, this is a huge field!
Why do we need to huddle like this all the time?

Hierarchical Data Structures

More examples of nested or clustered organization:

- Children within peer groups within neighborhoods
- Respondents within interviewers or raters
- Effect sizes within studies within methods (meta-analysis)
- Multistage sampling
- Time of measurement within persons within organizations

Simpson's Paradox:

Clustering Is Important

Well known paradox in which performance of individual groups is reversed when the groups are combined

	Quiz 1	Quiz 2
Gina	60.0%	10.0%
Sam	90.0%	30.0%

	Quiz 1	Quiz 2	Total
Gina	60 / 100	1 / 10	61 / 110
Sam	9 / 10	30 / 100	39 / 110

Simpson's Paradox: Other Examples

2006 US School study:

1975 Berkeley sex bias case:

- UCB sued for bias by women applying to grad school
- Admissions figures showed men more likely to be admitted

"When the Oakies left Oklahoma and moved to California, it raised the IQ of both states."

– *Will Rogers*

- Men applied more to high admission rate departments

Hypothetical Data Example from Snijders & Bosker (1999), n = 2, j=5, nj = N = 10

Participant (i)	Cluster (j)	Outcome (Y)	Predictor (X)
1	1	5	1
2	1	7	3
3	2	4	1
4	2	6	4
5	3	3	3
6	3	5	5
7	4	2	4
8	4	4	6
9	5	1	5
10	5	3	7

All 10 cases analyzed without taking cluster membership into account:

Model Summary

Model	R	R Square	Adjusted R Square	Std. Error of the Estimate
1	.333[a]	.111	.000	1.826

a. Predictors: (Constant), X

Coefficients[a]

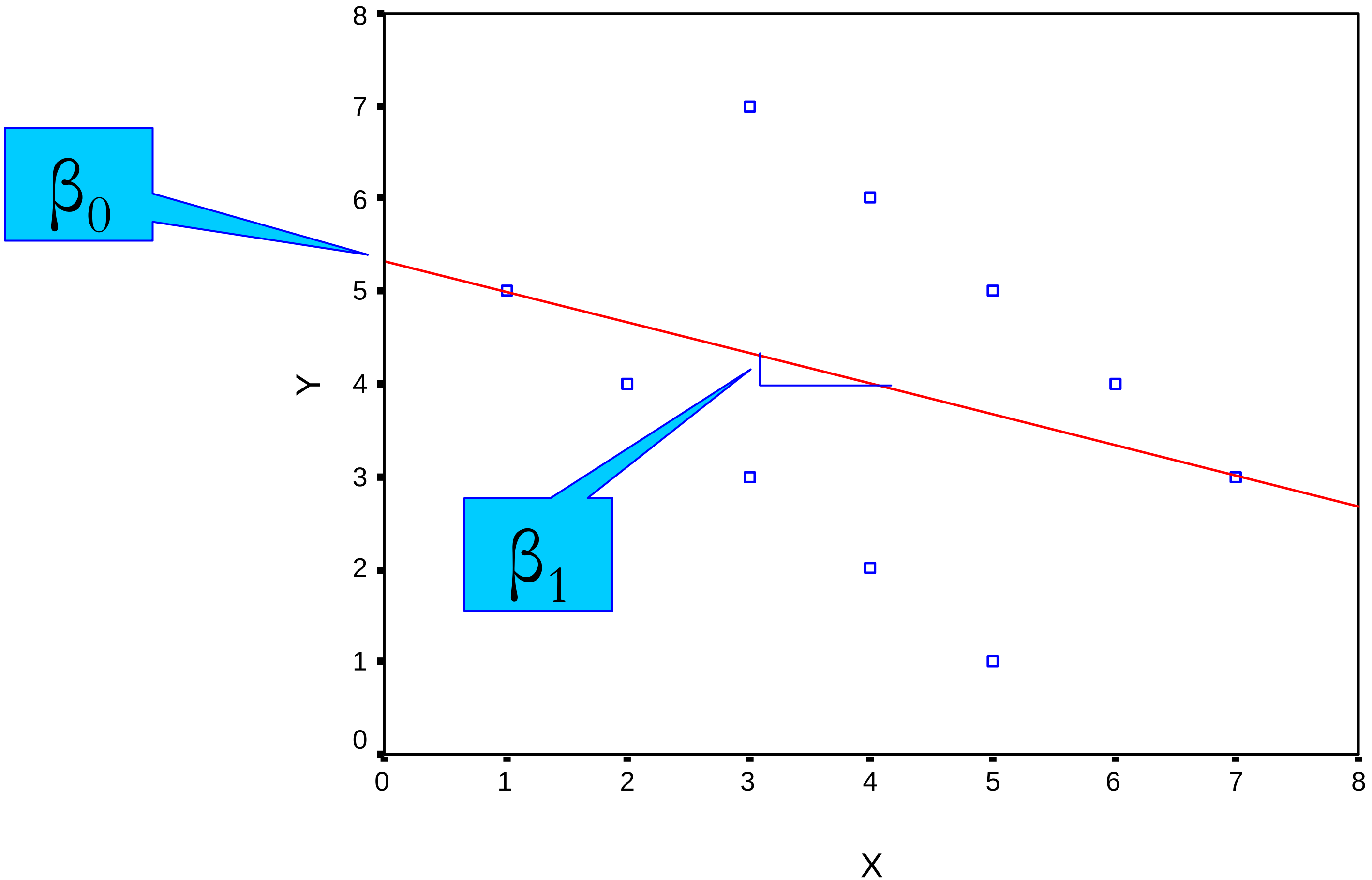
Model		Unstandardized Coefficients		Standardized Coefficients	t	Sig.
		B	Std. Error	Beta		
1	(Constant)	5.333	1.453		3.671	.006
	X	-.333	.333	-.333	-1.000	.347

a. Dependent Variable: Y

$$Y = 5.333 - .333(X) + r$$

Interpretation: There's a negative relationship between the predictor X and the outcome Y, a one unit increase in X results in .333 lower Y

$$Y = 5.333 - .333(X) + r$$



This is an example of a disaggregated analysis

Another alternative is to analyze data at the aggregated group level

Participant (i)	Cluster (j)	Outcome (Y)	Predictor (X)
1	1	5	1
2	1	7	3
3	2	4	2
4	2	6	4
5	3	3	3
6	3	5	5
7	4	2	4
8	4	4	6
9	5	1	5
10	5	3	7

Cluster (j)	Outcome (Y)	Predictor (X)
1	6	2
2	5	3
3	4	4
4	3	5
5	2	6

The clusters are analyzed without taking individuals into account:

Model Summary

Model	R	R Square	Adjusted R Square	Std. Error of the Estimate
1	1.000[a]	1.000	1.000	.000

a. Predictors: (Constant), MEANX

Coefficients[a]

Model		Unstandardized Coefficients		Standardized Coefficients	t	Sig.
		B	Std. Error	Beta		
1	(Constant)	8.000	.000		.	.
	MEANX	-1.000	.000	-1.000	.	.

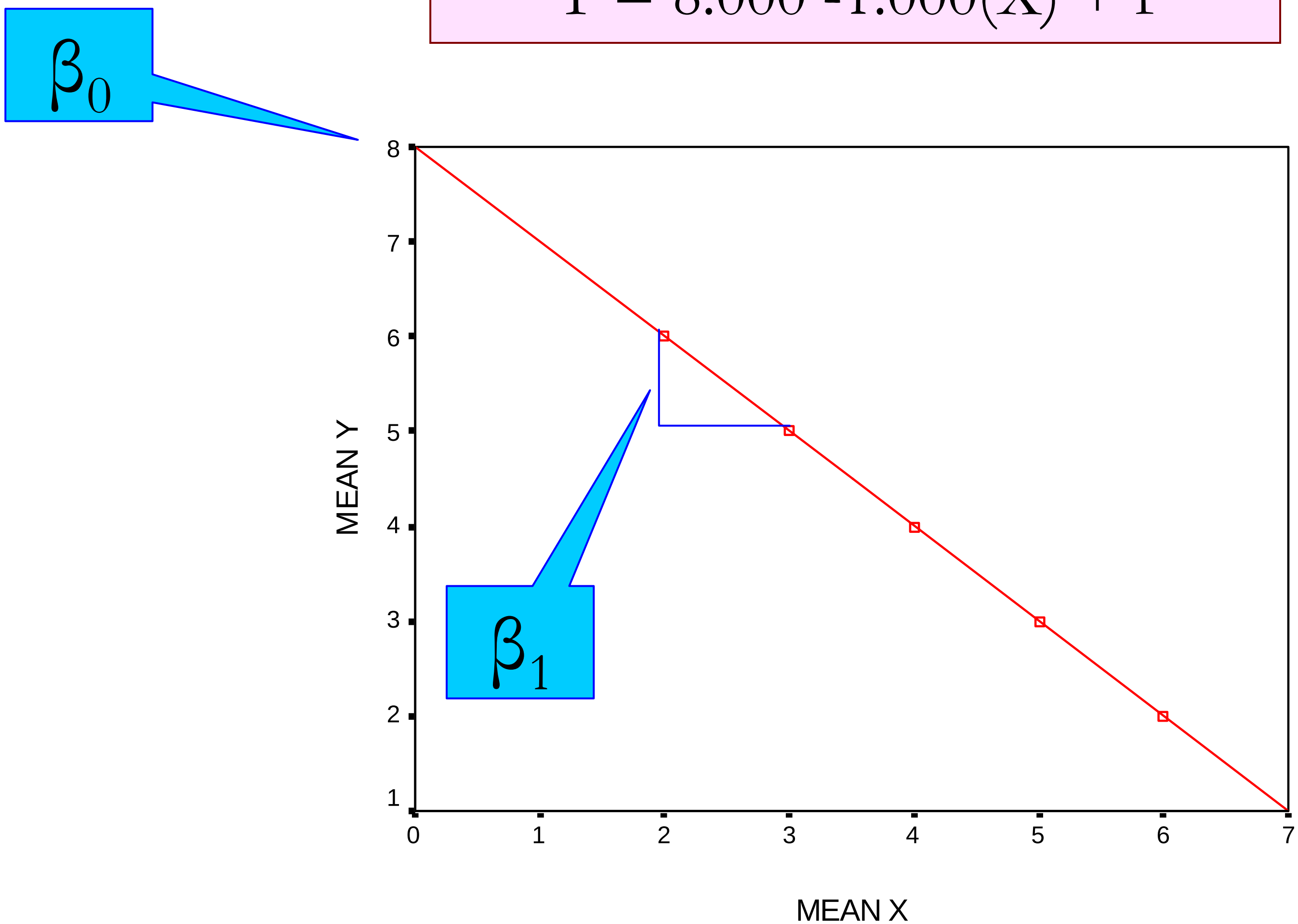
a. Dependent Variable: MEANY

$$Y = 8.000 - 1.000(X) + r$$

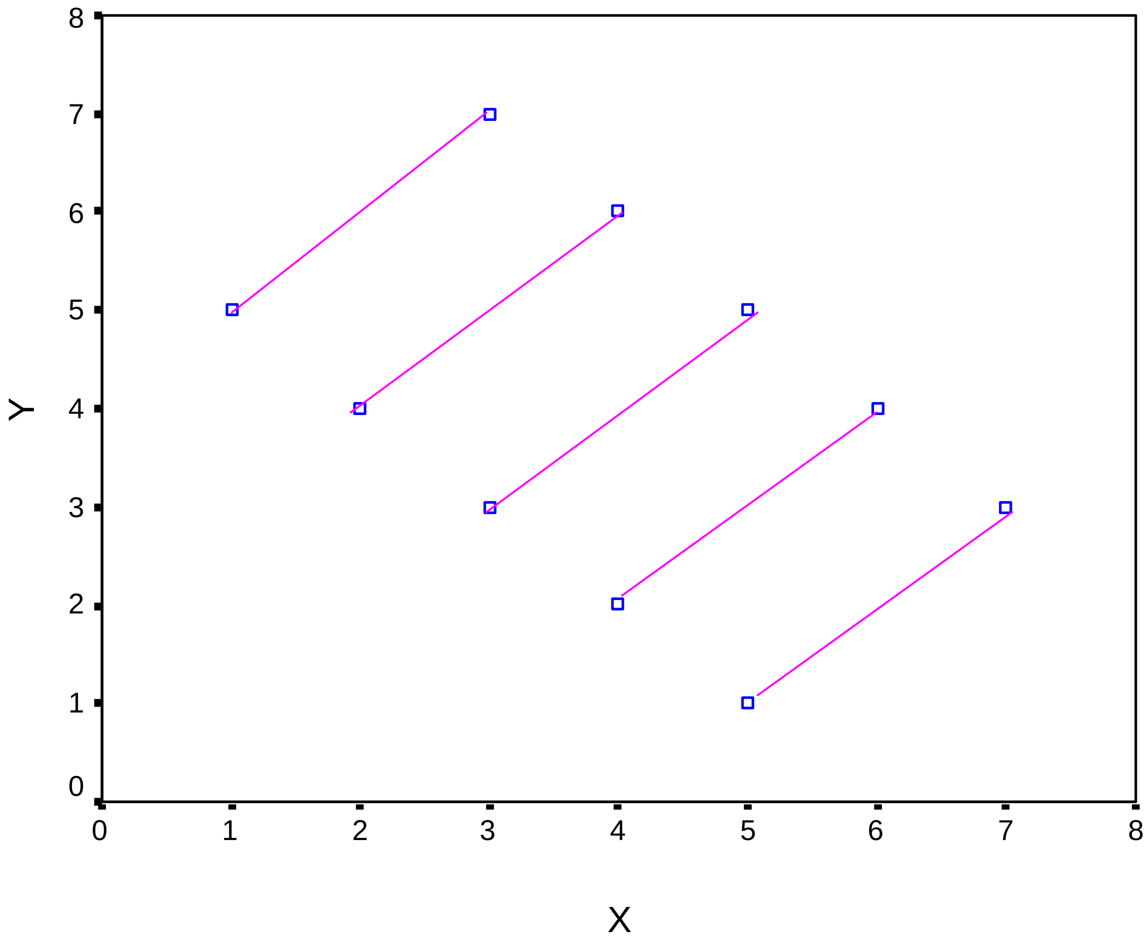
Interpretation: There's a negative relationship between the predictor X and the outcome Y, a one unit increase in X results in 1.0 lower Y

This is an example of a disaggregated analysis

$$Y = 8.000 - 1.000(X) + r$$



A third possibility is to analyze each cluster separately, looking at the regression relationship within each group



$$Y_{ij} = \overline{Y}_j + 1.00(X_{ij} - \overline{X}_j) + r_{ij}$$

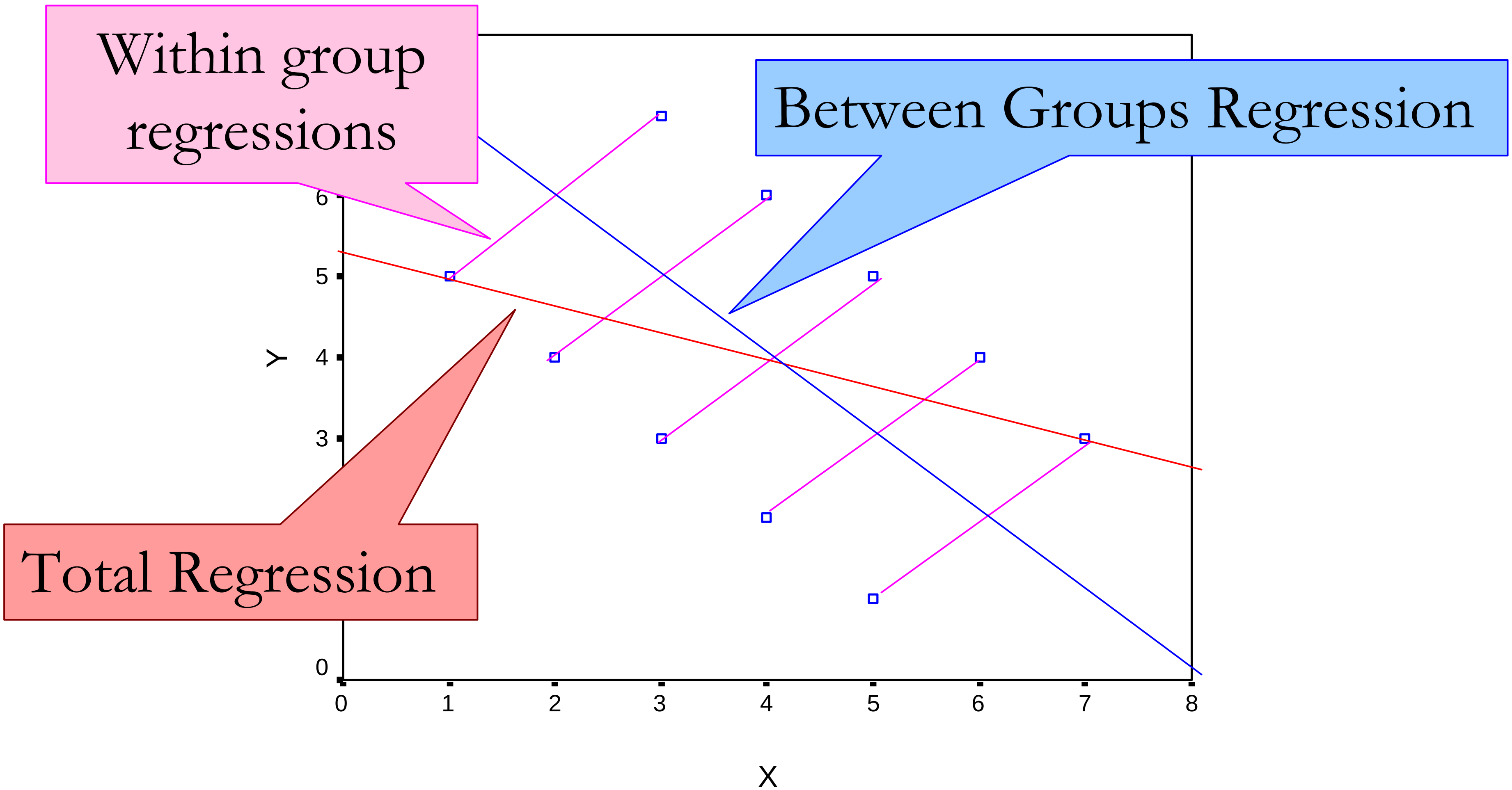
Multilevel regression takes both levels into account:

$$Y = 8.000 - 1.000(X) + r$$

$$Y_{ij} = \overline{Y}_j + 1.00(X_{ij} - \overline{X}_j) + r_{ij}$$

$$Y_{ij} = 8.00 - 1.00(\overline{X}_j) + 1.00(X_{ij} - \overline{X}_j) + r_{ij}$$

Taking the multilevel structure of the data into account:



Why Is Multilevel Analysis Needed?

- Nesting creates dependencies in the data
  - Dependencies violate the assumptions of traditional statistical models (“independence of error”, “homogeneity of regression slopes”)
  - Dependencies result in inaccurate statistical estimates
- Important to understand variation at different levels

Decisions About Multilevel Analysis

- Properly modeling multilevel structure often matters (and sometimes a lot)
- Partitioning variance at different levels is useful
  - tau and sigma ($\sigma^2_Y = \tau + \sigma^2$)
  - policy & practice implications

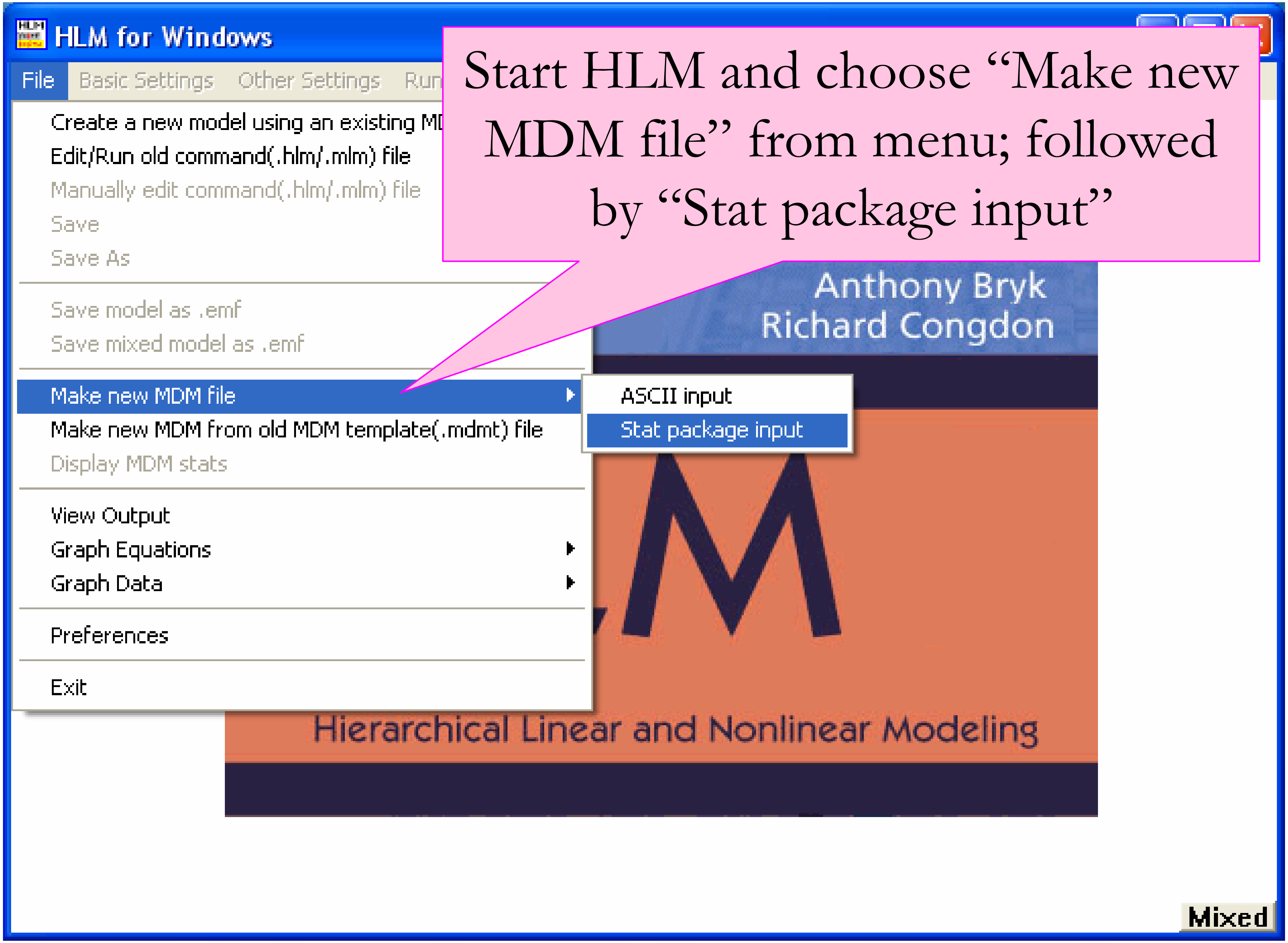
"Randomization by cluster accompanied by analysis appropriate to randomization by individual is an exercise in self-deception and should be discouraged" (Cornfield, 1978, pp.101-2)

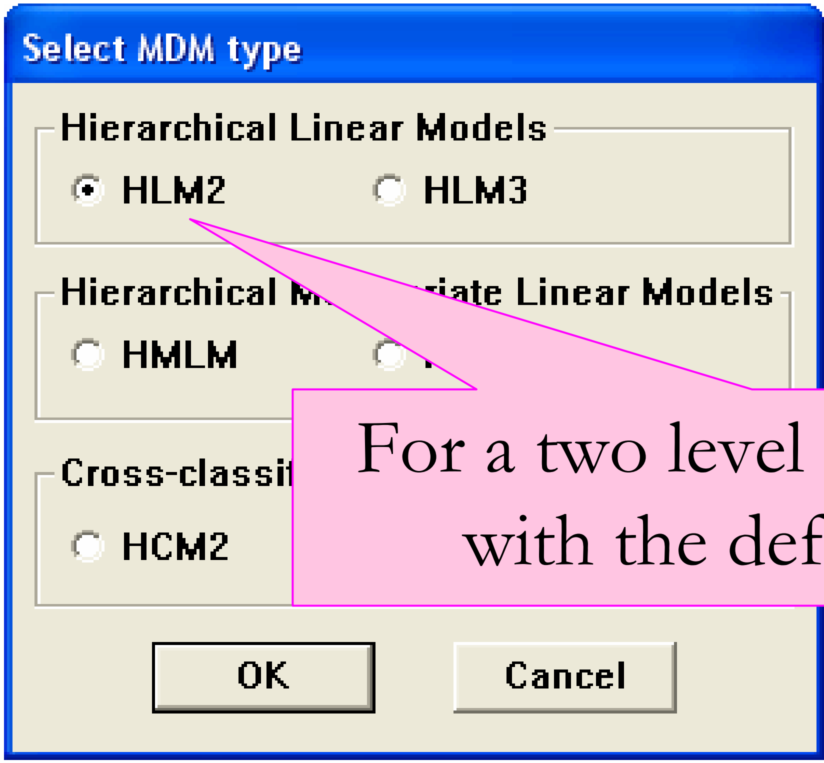
Preparing Data for HLM Analysis

- Use of SPSS as a precursor to HLM assumed
- HLM requires a different data file for each level in the HLM analysis
- Prepare data first in SPSS
  - Clean and screen data
  - Treat missing data
  - ID variables needed to link levels
  - Sort cases on ID
- Then import files into HLM to create an ".mdm" file

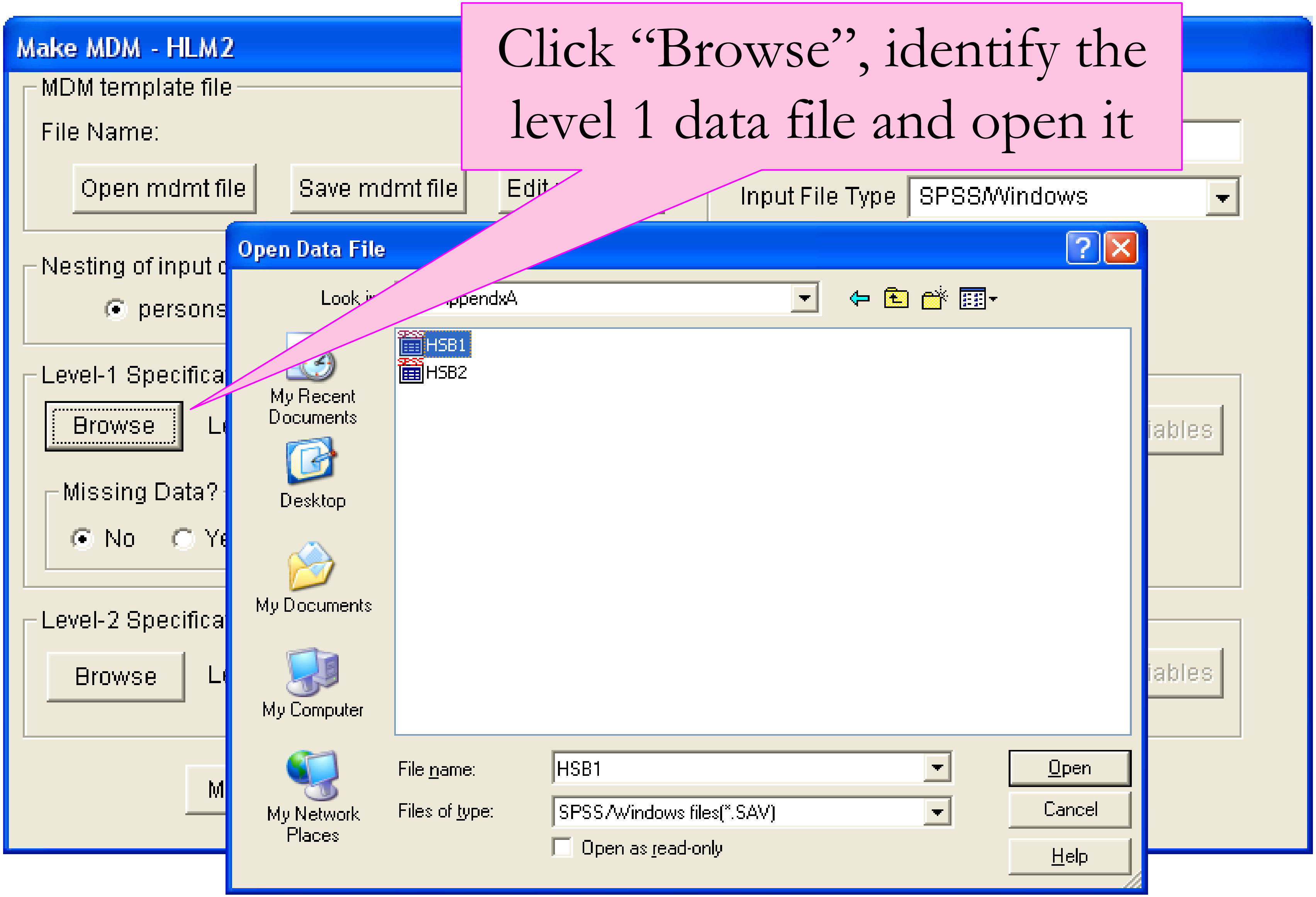
Creating an MDM file

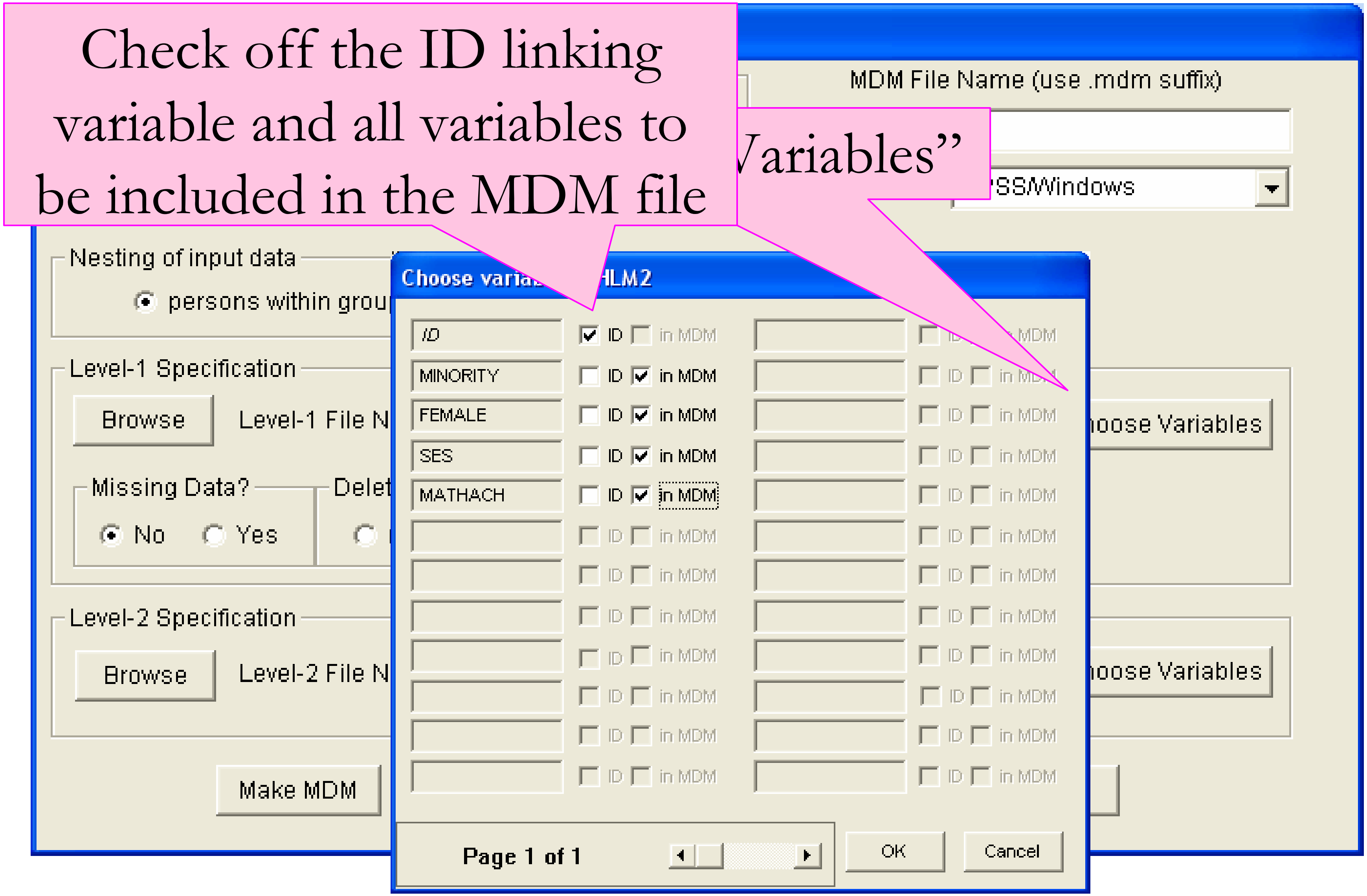
- Example:
  - Go to “Examples” folder and then “Appendix A” in the HLM directory
  - Open “HSB1.sav” and “HSB2.sav”

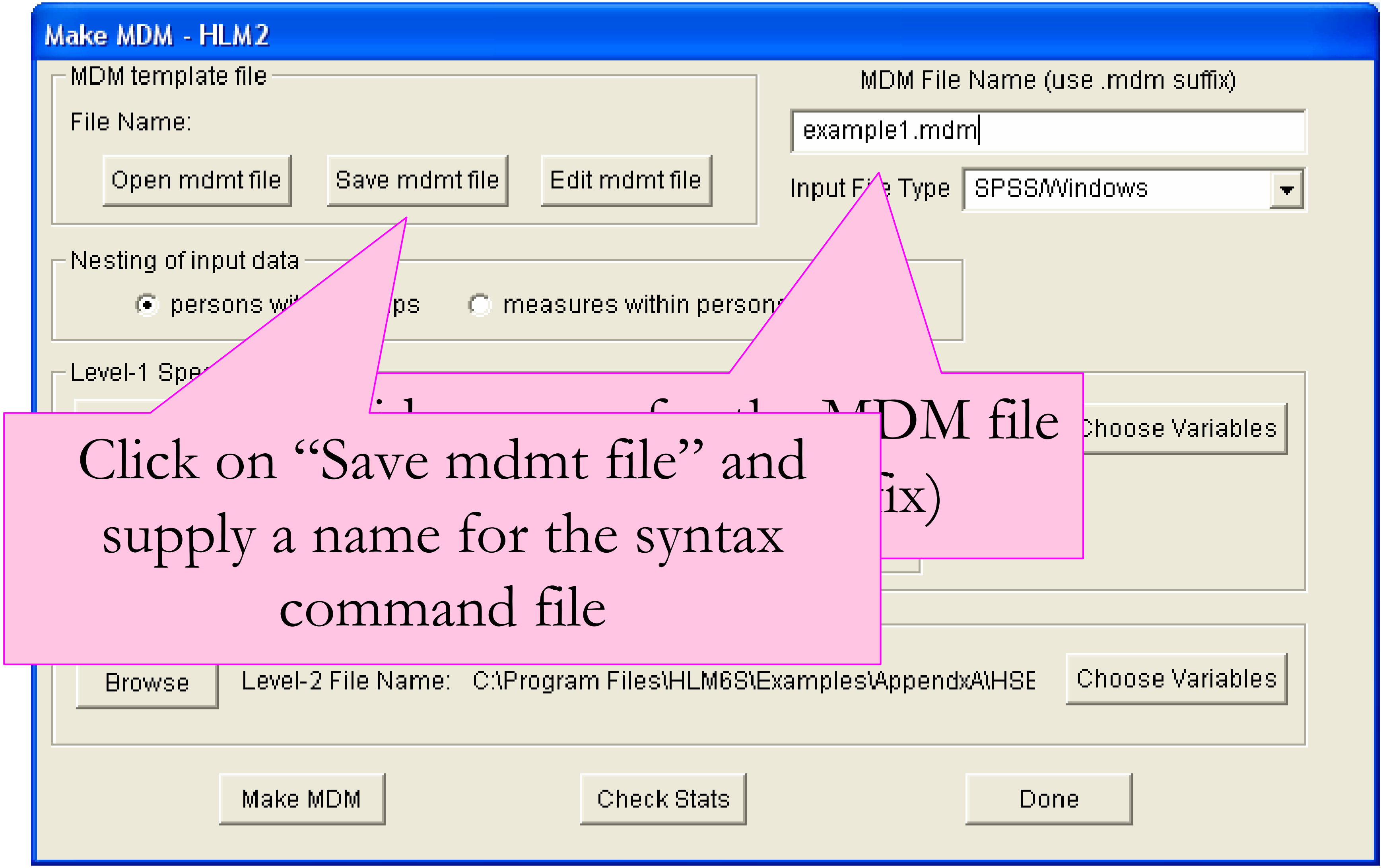

Start HLM and choose "Make new MDM file" from menu; followed by "Stat package input"
HLM for Windows
File
Basic Settings
Other Settings
Run
Create a new model using an existing MD
Edit/Run old command(.hlm/.mlm) file
Manually edit command(.hlm/.mlm) file
Save
Save As
Save model as .emf
Save mixed model as .emf
Make new MDM file
Make new MDM from old MDM template(.mdmt) file
Display MDM stats
View Output
Graph Equations
Graph Data
Preferences
Exit
ASCII input
Stat package input
Anthony Bryk
Richard Congdon
Hierarchical Linear and Nonlinear Modeling
Mixed

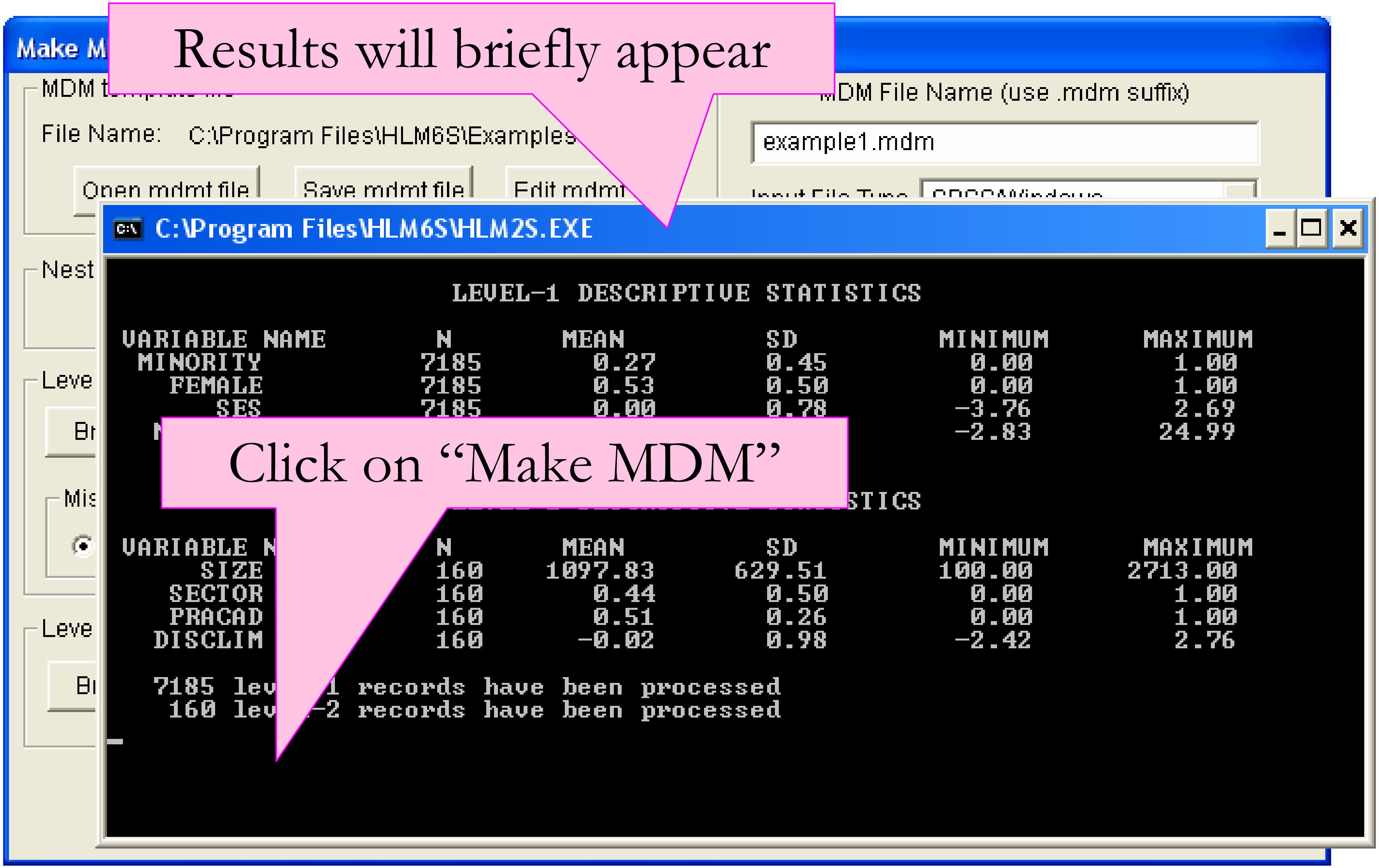


For a two level HLM model stay with the default “HLM2”

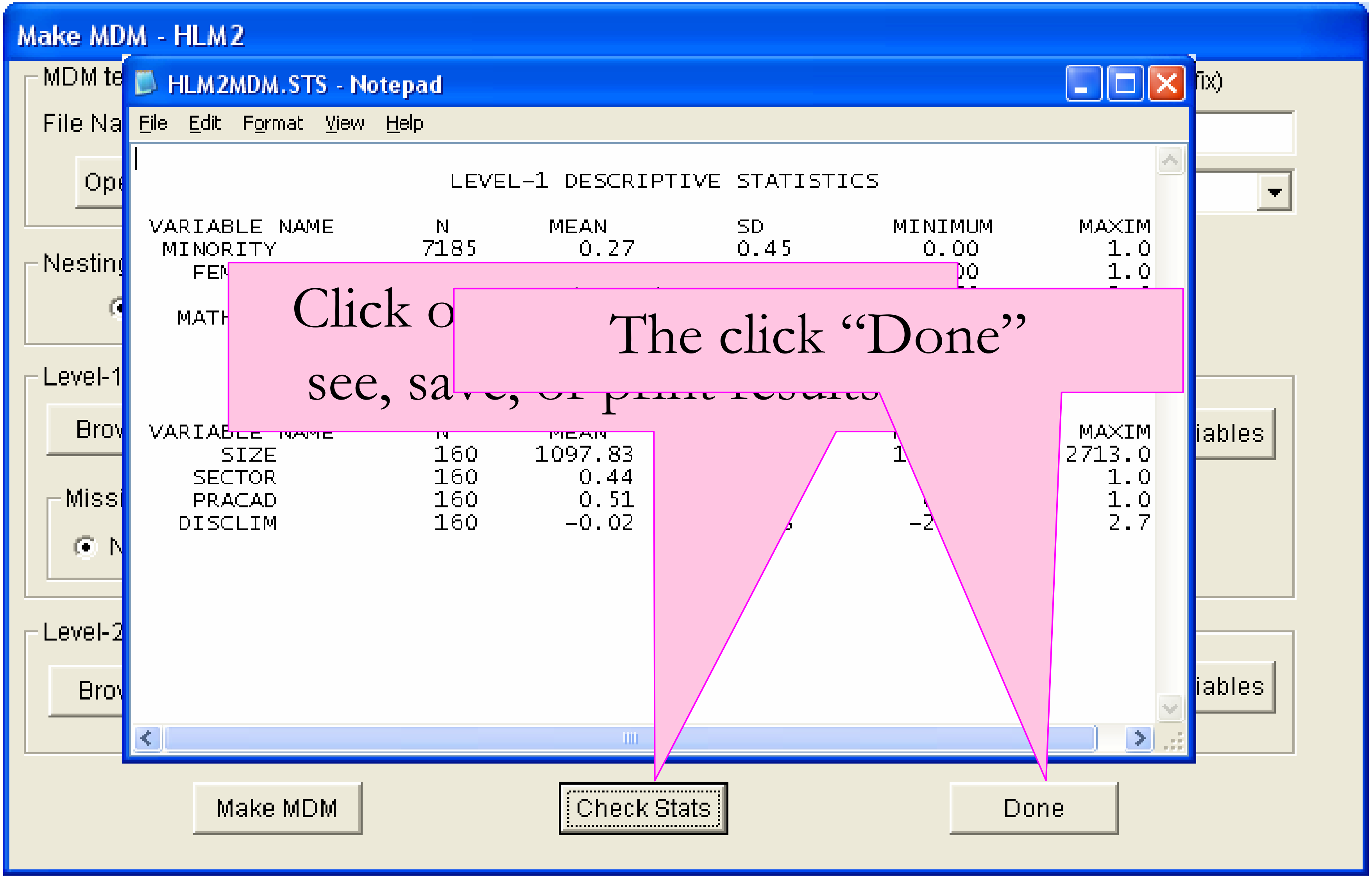

Click “Browse”, identify the level 1 data file and open it
Make MDM - HLM2
MDM template file
File Name:
Open mdmt file
Save mdmt file
Input File Type
SPSS/Windows
Nesting of input
persons
Level-1 Specifica
Browse
Missing Data?
No
Level-2 Specifica
Browse
Open Data File
HSB1
HSB2
My Recent Documents
Desktop
My Documents
My Computer
My Network Places
File name:
HSB1
Files of type:
SPSS/Windows files(*.SAV)
Open as read-only
Open
Cancel
Help

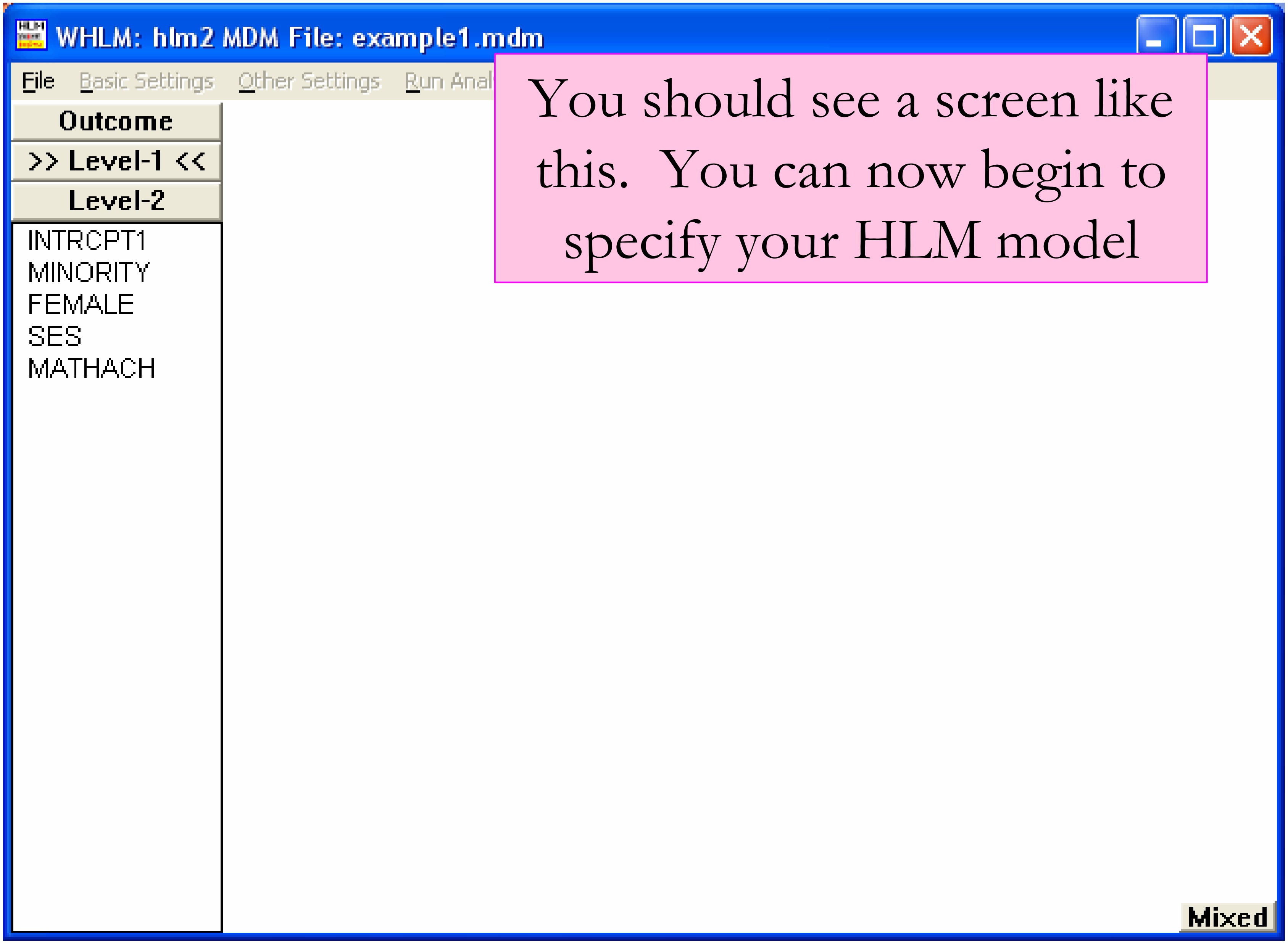

Check off the ID linking variable and all variables to be included in the MDM file
Variables”
MDM File Name (use .mdm suffix)
SS/Windows
Nesting of input data
persons within grou
Level-1 Specification
Browse
Level-1 File N
Missing Data?
No
Yes
Delet
Level-2 Specification
Browse
Level-2 File N
Make MDM
hoose Variables
hoose Variables
Choose varia
HLM2
ID
MINORITY
FEMALE
SES
MATHACH
ID
in MDM
Page 1 of 1
OK
Cancel


Make MDM - HLM2
MDM template file
File Name:
Open mdmt file
Save mdmt file
Edit mdmt file
MDM File Name (use .mdm suffix)
example1.mdm
Input File Type
SPSS/Windows
Nesting of input data
persons within groups
measures within persons
Level-1 Spec
Choose Variables
MDM file
ix)
Click on "Save mdmt file" and supply a name for the syntax command file
Browse
Level-2 File Name: C:\Program Files\HLM6S\Examples\AppendxA\HSE
Choose Variables
Make MDM
Check Stats
Done



Results will briefly appear
Click on “Make MDM”
C:\Program Files\HLM6S\HLM2S.EXE
LEVEL-1 DESCRIPTIVE STATISTICS
VARIABLE NAME N MEAN SD MINIMUM MAXIMUM
MINORITY 7185 0.27 0.45 0.00 1.00
FEMALE 7185 0.53 0.50 0.00 1.00
SES 7185 0.00 0.78 -3.76 2.69
-2.83 24.99
STICS
VARIABLE N N MEAN SD MINIMUM MAXIMUM
SIZE 160 1097.83 629.51 100.00 2713.00
SECTOR 160 0.44 0.50 0.00 1.00
PRACAD 160 0.51 0.26 0.00 1.00
DISCLIM 160 -0.02 0.98 -2.42 2.76
7185 lev 1 records have been processed
160 lev -2 records have been processed
MDM File Name (use .mdm suffix)
example1.mdm
File Name: C:\Program Files\HLM6S\Examples
Open mdmt file
Save mdmt file


Make MDM - HLM2
HLM2MDM.STS - Notepad
File Edit Format View Help
LEVEL-1 DESCRIPTIVE STATISTICS
VARIABLE NAME N MEAN SD MINIMUM MAXIM
MINORITY 7185 0.27 0.45 0.00 1.0
VARIABLE NAME
SIZE 160 1097.83
SECTOR 160 0.44
PRACAD 160 0.51
DISCLIM 160 -0.02
Click o
see, sa
The click "Done"
Make MDM
Check Stats
Done



You should see a screen like this. You can now begin to specify your HLM model

Two-Level HLM Models

The Single-Level, Fixed Effects Regression Model

$$Y_i = \beta_0 + \beta_1 X_{1i} + \beta_2 X_{2i} + \ldots + \beta_k X_{ki} + r_i$$

- The parameters β_{kj} are considered fixed
  - One for all and all for one
  - Same values for all i and j; the single level model
- The r_i 's are random: $r_i \sim N(0, \sigma)$ and independent

The Multilevel Model

- Takes groups into account and explicitly models group effects
- How to conceptualize and model group level variation?
- How do groups vary on the model parameters?
- Fixed versus random effects

Fixed vs. Random Effects

- Fixed Effects represent discrete, purposefully selected or existing values of a variable or factor
  - Fixed effects exert constant impact on DV
  - Random variability only occurs as a within subjects effect (level 1)
  - Can only generalize to particular values used
- Random Effects represent more continuous or randomly sampled values of a variable or factor
  - Random effects exert variable impact on DV
  - Variability occurs at level 1 and level 2
  - Can study and model variability
  - Can generalize to population of values

Fixed vs. Random Effects?

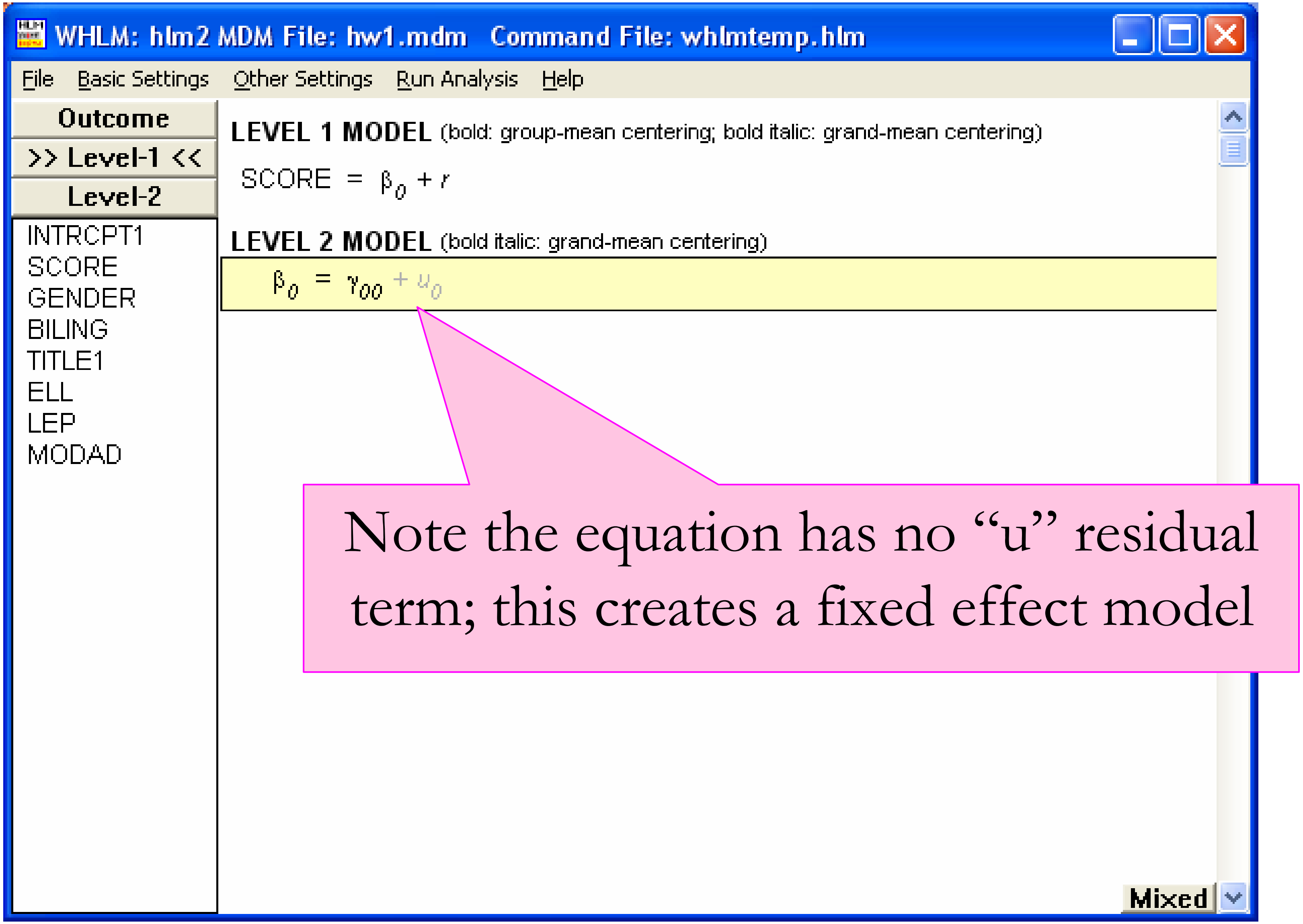
- Use fixed effects if
  - The groups are regarded as unique entities
  - If group values are determined by researcher through design or manipulation
  - Small j (< 10); improves power
- Use random effects if
  - Groups regarded as a sample from a larger population
  - Researcher wishes to test effects of group level variables
  - Researcher wishes to understand group level differences
  - Small j (< 10); improves estimation

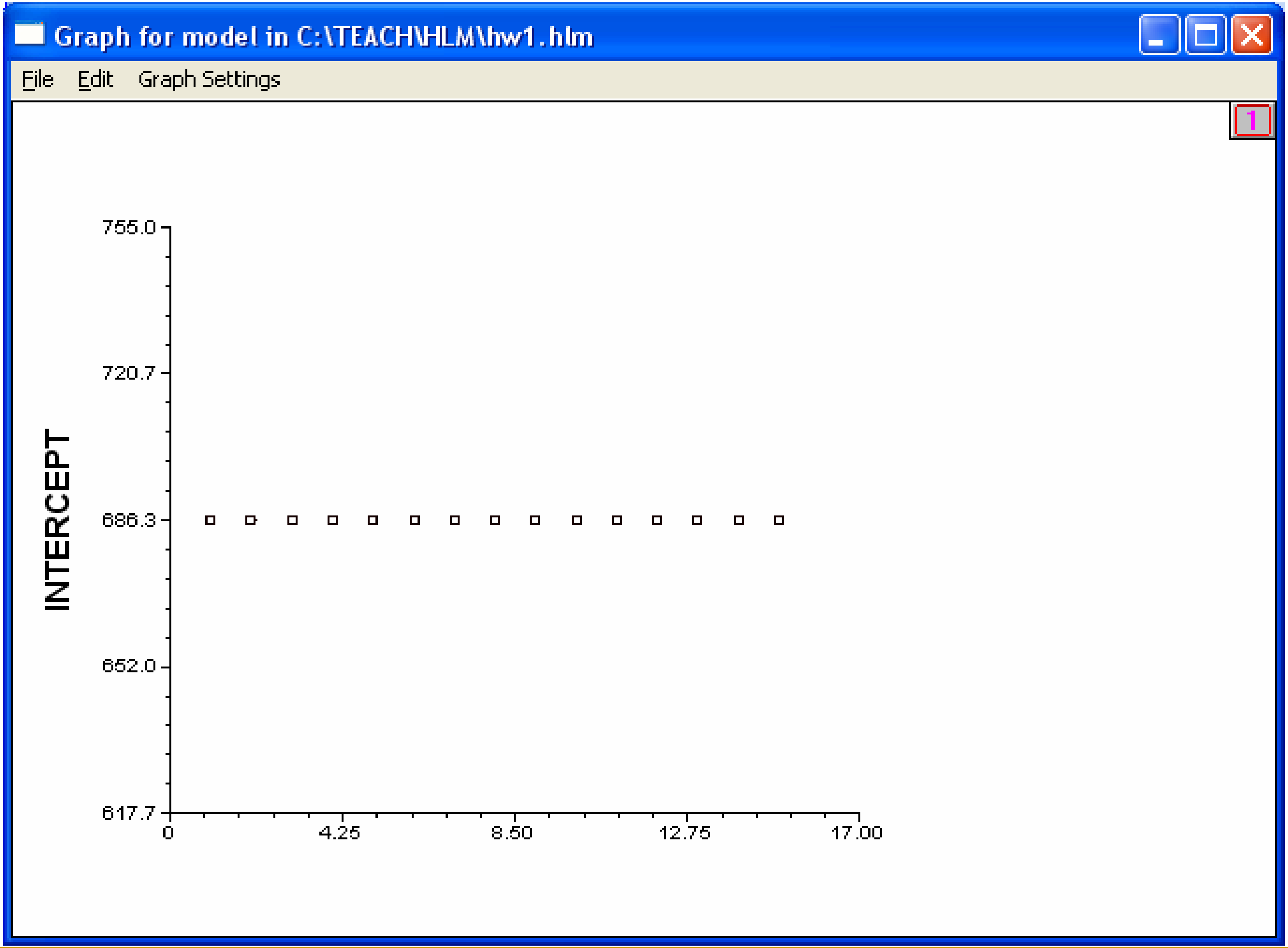
Fixed Intercepts Model

- The simplest HLM model is equivalent to a one-way ANOVA with fixed effects:

$$Y_{ij} = \gamma_{00} + r_{ij}$$

- This model simply estimates the grand mean (γ_{00}) and deviations from the grand mean (r_{ij})
- Presented here simply to demonstrate control of fixed and random effects on all parameters


WHLM: hlm2 MDM File: hw1.mdm Command File: whlmtemp.hlm
File Basic Settings Other Settings Run Analysis Help
Outcome
>> Level-1 <<
Level-2
INTRCPT1
SCORE
GENDER
BILING
TITLE1
ELL
LEP
MODAD
LEVEL 1 MODEL (bold: group-mean centering; bold italic: grand-mean centering)
SCORE = β0 + r
LEVEL 2 MODEL (bold italic: grand-mean centering)
β0 = γ00 + u0
Mixed
Note the equation has no "u" residual term; this creates a fixed effect model



Graph for model in C:\TEACH\HLM\hw1.hlm
File Edit Graph Settings
1
INTERCEPT
755.0
720.7
686.3
652.0
617.7
0
4.25
8.50
12.75
17.00

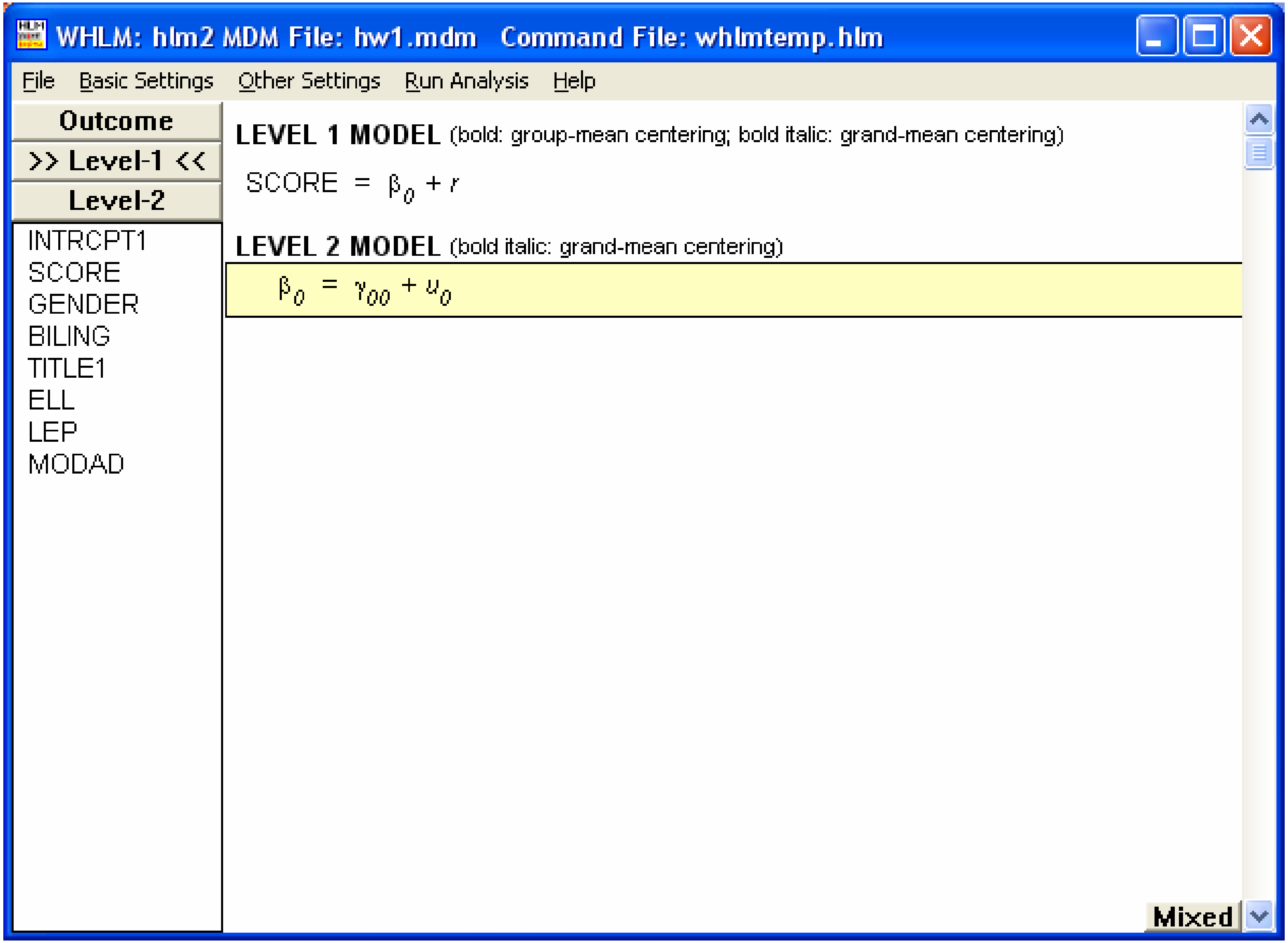
ANOVA Model (random intercepts)

- A simple HLM r… intercepts
- Equivalent to a c… effects:

$$Y_{ij} = \beta_{0j} + r_{ij}$$
$$\beta_{0j} = \gamma_{00} + u_{0j}$$

$$Y_{ij} = \gamma_{00} + u_{0j} + r_{ij}$$

Note the addition of u_{0j} allows different intercepts for each j unit, a random effects model


WHLM: hlm2 MDM File: hw1.mdm Command File: whlmtemp.hlm
File Basic Settings Other Settings Run Analysis Help
Outcome
>> Level-1 <<
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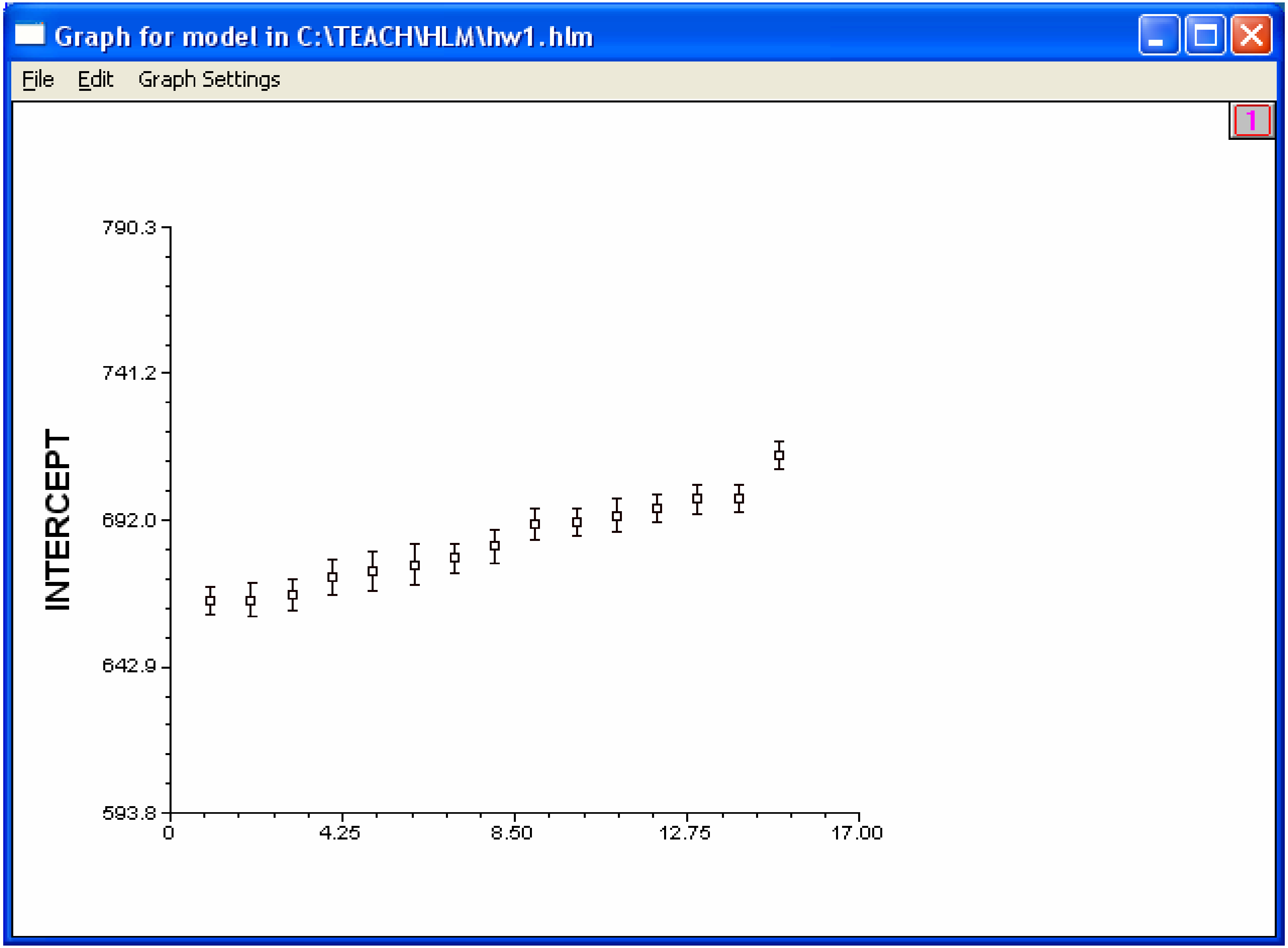
INTRCPT1
SCORE
GENDER
BILING
TITLE1
ELL
LEP
MODAD
LEVEL 1 MODEL (bold: group-mean centering; bold italic: grand-mean centering)
SCORE = $\beta_0 + r$
LEVEL 2 MODEL (bold italic: grand-mean centering)
$\beta_0 = \gamma_{00} + u_0$
Mixed



Graph for model in C:\TEACH\HLM\hw1.hlm
File Edit Graph Settings
1
INTERCEPT
790.3
741.2
692.0
642.9
593.8
0
4.25
8.50
12.75
17.00

ANOVA Model

- In addition to providing parameter estimates, the ANOVA model provides information about the presence of level 2 variance (the ICC) and whether there are significant differences between level 2 units
- This model also called the Unconditional Model (because it is not “conditioned” by any predictors) and the “empty” model
- Often used as a baseline model for comparison to more complex models

Variables in HLM Models

- Outcome variables
- Predictors
  - Control variables
  - Explanatory variables
- Variables at higher levels
  - Aggregated variables (Is n sufficient for representation?)
  - Contextual variables

Conditional Models: ANCOVA

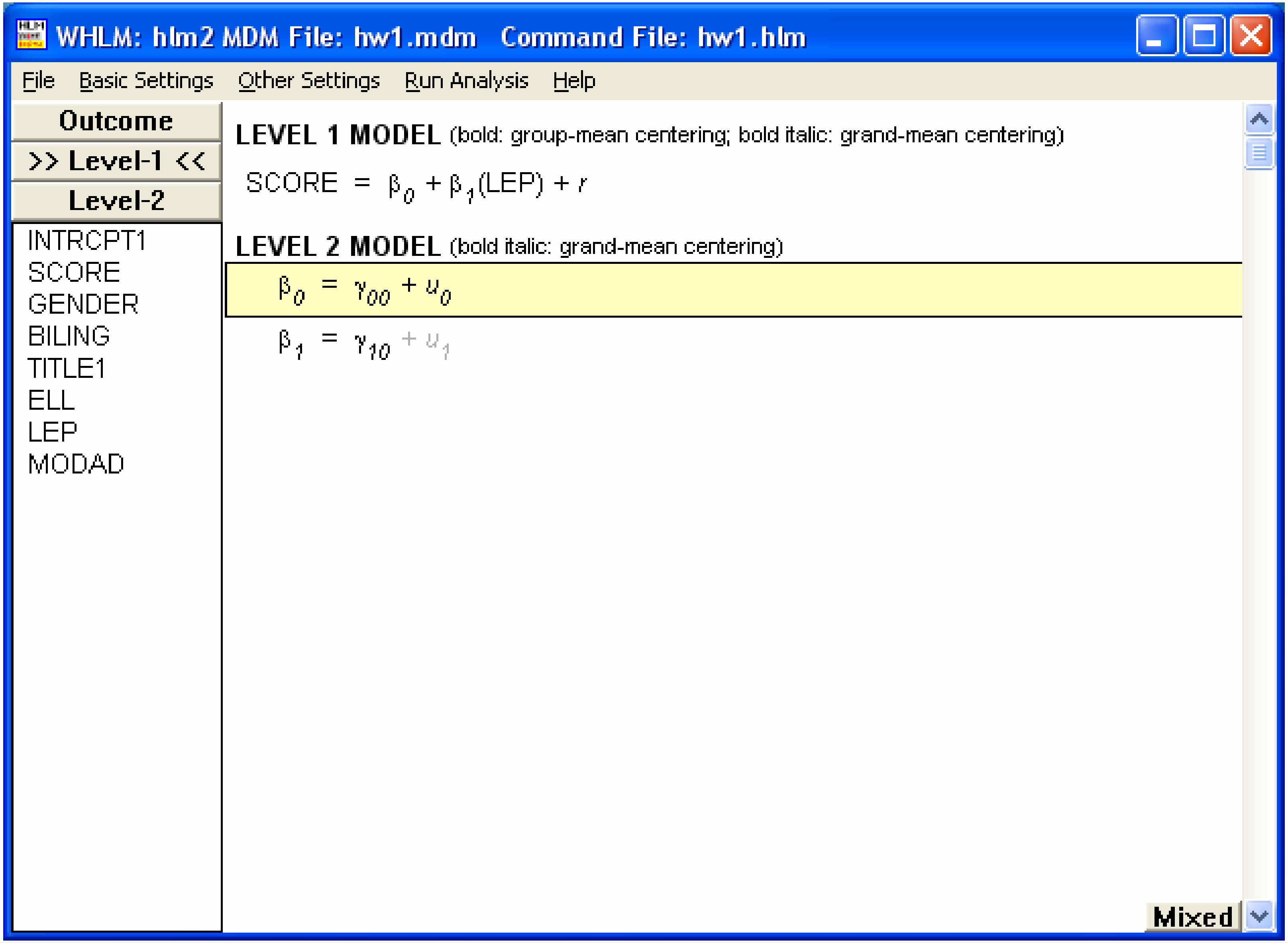
- Adding a predictor to the ANOVA model results in an ANCOVA model with random intercepts:

$$Y_{ij} = \beta_{0j} + \beta_1(X_1) + r_{ij}$$

$$\beta_{0j} = \gamma_{00} + u_{0j}$$

$$\beta_1 = \gamma_{10}$$

- Note that the effect of X is constrained to be the same fixed effect for every j unit (homogeneity of regression slopes)


WHLM: hlm2 MDM File: hw1.mdm Command File: hw1.hlm
File Basic Settings Other Settings Run Analysis Help
Outcome
>> Level-1 <<
Level-2
INTRCPT1
SCORE
GENDER
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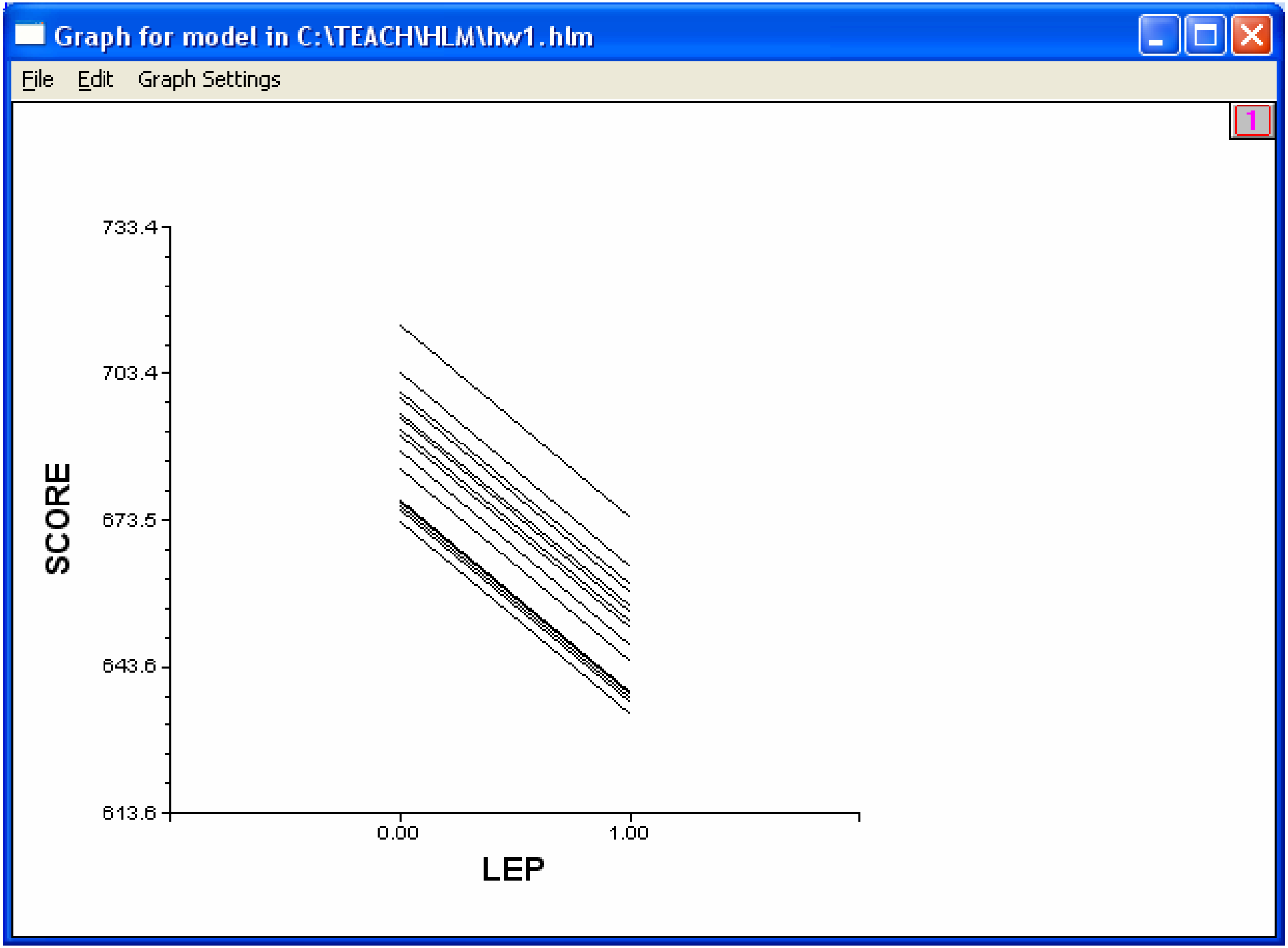
TITLE1
ELL
LEP
MODAD
LEVEL 1 MODEL (bold: group-mean centering; bold italic: grand-mean centering)
SCORE = β0 + β1(LEP) + r
LEVEL 2 MODEL (bold italic: grand-mean centering)
β0 = γ00 + u0
β1 = γ10 + u1
Mixed



Graph for model in C:\TEACH\HLM\hw1.hlm
File Edit Graph Settings
1
733.4
703.4
673.5
643.6
613.6
SCORE
0.00
1.00
LEP

Conditional Models: Random Coefficients

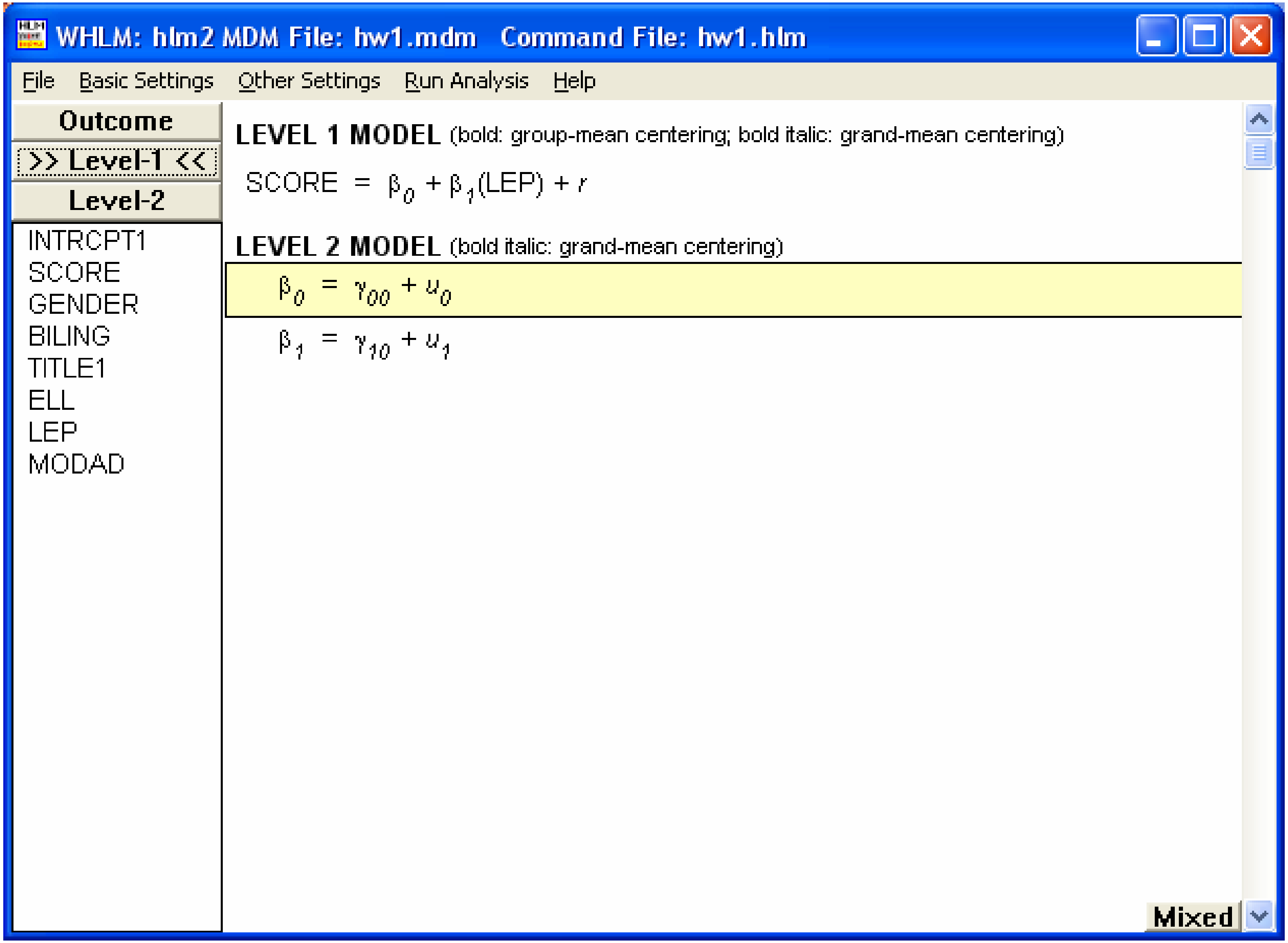
- An additional parameter results in random variation of the slopes:

$$Y_{ij} = \beta_{0j} + \beta_1(X_1) + r_{ij}$$

$$\beta_{0j} = \gamma_{00} + u_{0j}$$

$$\beta_{1j} = \gamma_{10} +$$

- Both intercepts and slopes now vary from group to group


WHLM: hlm2 MDM File: hw1.mdm Command File: hw1.hlm
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Outcome
>> Level-1 <<
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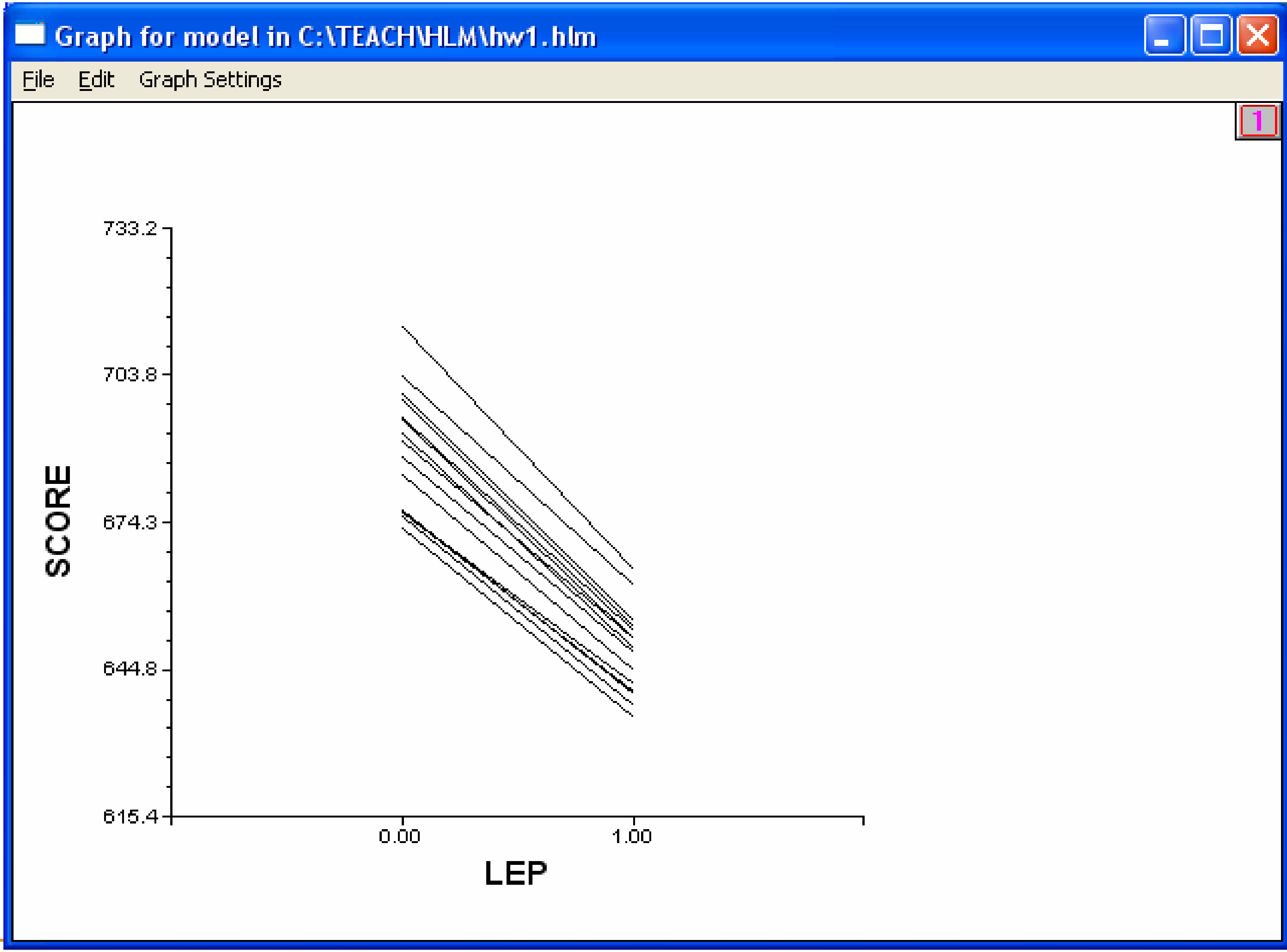
LEVEL 1 MODEL (bold: group-mean centering; bold italic: grand-mean centering)
SCORE = β0 + β1(LEP) + r
LEVEL 2 MODEL (bold italic: grand-mean centering)
β0 = γ00 + u0
β1 = γ10 + u1
Mixed


Graph for model in C:\TEACH\HLM\hw1.hlm
File
Edit
Graph Settings
1
733.2
703.8
674.3
644.8
615.4
SCORE
0.00
1.00
LEP

Standardized coefficients

Standardized coefficient at level 1:

$$\beta_{0j}\ (SD_X\ /\ SD_Y)$$

Standardized coefficient at level 2:

$$\gamma_{00}\ (SD_X\ /\ SD_Y)$$

Modeling variation at Level 2: Intercepts as Outcomes

$$Y_{ij} = \boxed{\phantom{\beta_{0j}}} + \beta_{1j}X_{1ij} + r_{ij}$$

$$\boxed{\phantom{\beta_{0j} = \gamma_{00} + \gamma_{01}W_{j} + u_{0j}}}$$

$$\beta_{1j} = \gamma_{10} + u_{1j}$$

- Predictors (W's) at level 2 are used to model variation in intercepts between the j units

Modeling Variation at Level 2: Slopes as Outcomes

$$Y_{ij} = \beta_{0j} + \quad X_{1ij} + r_{ij}$$

$$\beta_{0j} = \gamma_{00} + \gamma_{0j} W_j + u_{0j}$$

- Do slopes vary from one j unit to another?
- W's can be used to predict variation in slopes as well

Variance Components Analysis

- VCA allows estimation of the size of random variance components
  - Important issue when unbalanced designs are used
  - Iterative procedures must be used (usually ML estimation)
- Allows significance testing of whether there is variation in the components across units

Estimating Variance Components: Unconditional Model

$$\mathrm{Var}(Y_{ij}) = \mathrm{Var}(u_{0j}) + \mathrm{Var}(r_{ij})$$

$$= \boldsymbol{\tau_0 + \sigma^2}$$

HLM Output

Final estimation of variance components:

Random Effect		Standard Deviation	Variance Component	df	Chi-square	P-value
INTRCPT1,	U0	14.38267	206.86106	14	457.32201	0.000
level-1,	R	32.58453	1061.75172			

Statistics for current covariance components model

Deviance = 21940.853702
Number of estimated parameters = 2

Variance explained

R^2 at level 1 =

$$1 - (\sigma^2_{\mathbf{cond}} + \boldsymbol{\tau}_{\mathbf{cond}}) / (\sigma^2_{\mathbf{uncond}} + \boldsymbol{\tau}_{\mathbf{uncond}})$$

R^2 at level 2 =

$$1 - [(\sigma^2_{\mathbf{cond}} / n_h) + \boldsymbol{\tau}_{\mathbf{cond}}] / [(\sigma^2_{\mathbf{uncond}} / n_h) + \boldsymbol{\tau}_{\mathbf{uncond}}]$$

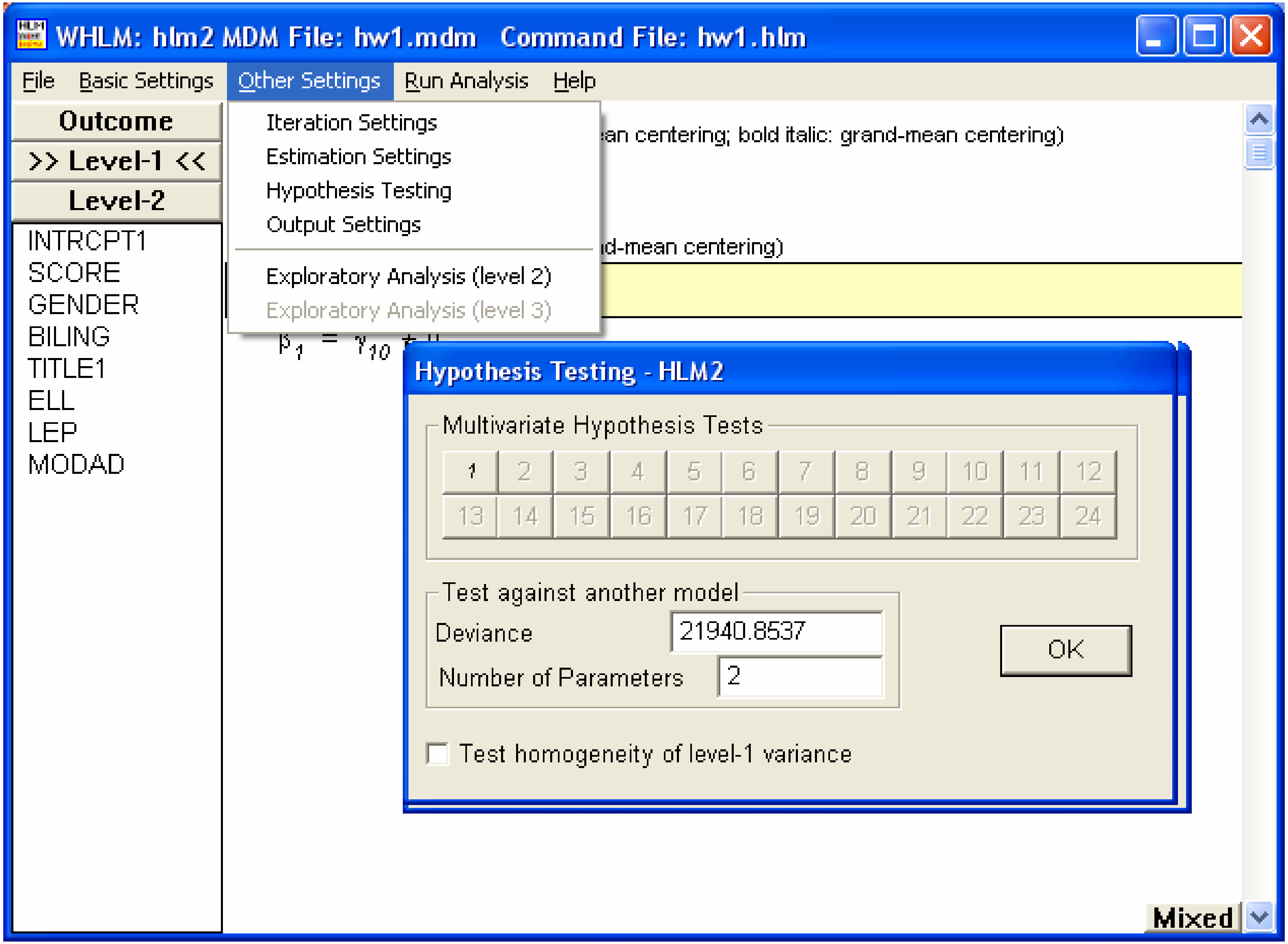
Where n_h = the harmonic mean of n for the level 2 units

$(k / [1/n_1 + 1/n_2 + \ldots 1/n_k])$

Comparing models

- Deviance tests
  - Under "Other Settings" on HLM tool bar, choose "hypothesis testing"
  - Enter deviance and number of parameters from baseline model
- Variance explained
  - Examine reduction in unconditional model variance as predictors added, a simpler level 2 formula:

$$R^2 = (\tau_{\mathbf{baseline}} - \tau_{\mathbf{conditional}}) \,/\, \tau_{\mathbf{baseline}}$$


WHLM: hlm2 MDM File: hw1.mdm Command File: hw1.hlm
File
Basic Settings
Other Settings
Run Analysis
Help
Outcome
>> Level-1 <<
Level-2
INTRCPT1
SCORE
GENDER
BILING
TITLE1
ELL
LEP
MODAD
Iteration Settings
Estimation Settings
Hypothesis Testing
Output Settings
Exploratory Analysis (level 2)
Exploratory Analysis (level 3)
an centering; bold italic: grand-mean centering)
d-mean centering)
Hypothesis Testing - HLM2
Multivariate Hypothesis Tests
1 2 3 4 5 6 7 8 9 10 11 12
13 14 15 16 17 18 19 20 21 22 23 24
Test against another model
Deviance
21940.8537
Number of Parameters
2
OK
Test homogeneity of level-1 variance
Mixed

Deviance Test Results

```
Statistics for current covariance components model
------------------------------------------------------------
Deviance                          =   21615.283709
Number of estimated parameters =   2

Variance-Covariance components test
------------------------------------------------------------
Chi-square statistic          =    325.56999
Number of degrees of freedom =    0
P-value                        =   >.500
```

Testing a Nested Sequence of HLM Models

1. Test unconditional model
2. Add level 1 predictors
   - Determine if there is variation across groups
   - If not, fix parameter
   - Decide whether to drop nonsignificant predictors
   - Test deviance, compute R^2 if so desired
3. Add level 2 predictors
   - Evaluate for significance
   - Test deviance, compute R^2 if so desired

Example

- Use the HSB MDM file previously created to practice running HLM models:
  - Unconditional
  - Level 1 predictor fixed, then random
  - Level 2 predictor

Statistical Estimation in HLM Models

- Estimation Methods
  - FML
  - RML
  - Empirical Bayes estimation
- Parameter estimation
  - Coefficients and standard errors
  - Variance Components
- Parameter reliability
- Centering
- Residual files

Estimation Methods: Maximum Likelihood Estimation (MLE) Methods

- MLE estimates model parameters by estimating a set of population parameters that maximize a likelihood function
- The likelihood function provides the probabilities of observing the sample data given particular parameter estimates
- MLE methods produce parameters that maximize the probability of finding the observed sample data

Estimation Methods

RML – Restricted Maximum Likelihood, only

FML – Full Maximum Likelihood, both the regression coefficients and the variance components are included in the likelihood function

components

Goodness of fit statistics (deviance tests) apply only to the random effects

RML only tests hypotheses about the VCs (and the models being compared must have identical fixed effects)

fixed effects and the variance components.

Goodness of fit statistics apply to the entire model

(both fixed and random effects)

Check on software default

Estimation Methods

- RML expected to lead to better estimates especially when j is small
- FML has two advantages:
  - Computationally easier
  - With FML, overall chi-square tests both regression coefficients and variance components, with RML only variance components are tested
  - Therefore if fixed portion of two models differ, must use FML for nested deviance tests

Computational Algorithms

- Several algorithms exist for existing HLM models:
  - Expectation-Maximization (EM)
  - Fisher scoring
  - Iterative Generalized Least Squares (IGLS)
  - Restricted IGLS (RIGLS)
- All are iterative search and evaluation procedures

Model Estimation

- Iterative estimation methods usually begin with a set of start values
- Start values are tentative values for the parameters in the model
  - Program begins with starting values (usually based on OLS regression at level 1)
  - Resulting parameter estimates are used as initial values for estimating the HLM model

Model Estimation

- Start values are used to solve model equations on first iteration
- This solution is used to compute initial model fit
- Next iteration involves search for better parameter values
- New values evaluated for fit, then a new set of parameter values tried
- When additional changes produce no appreciable improvement, iteration process terminates (convergence)
- Note that convergence and model fit are very different issues

Intermission

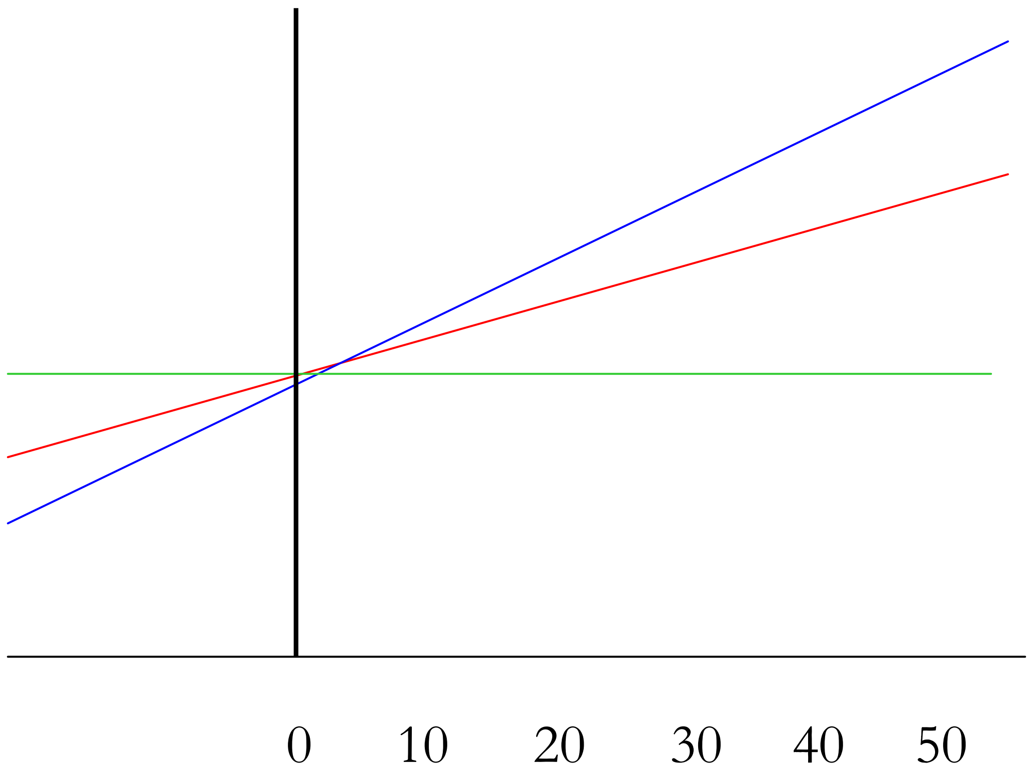
"Frankly, Harold, you're beginning to bore everyone with your statistics."

Centering

- No centering (common practice in single level regression)
- Centering around the group mean ($\bar{X}_j$)
- Centering around the grand mean (M)
- A known population mean
- A specific meaningful time point

Centering: The Original Metric

- Sensible when 0 is a meaningful point on the original scale of the predictor
  - For example, amount of training ranging from 0 to 14 days
  - Dosage of a drug where 0 represents placebo or no treatment
- Not sensible or interpretable in many other contexts, i.e. SAT scores (which range from 200 to 800)

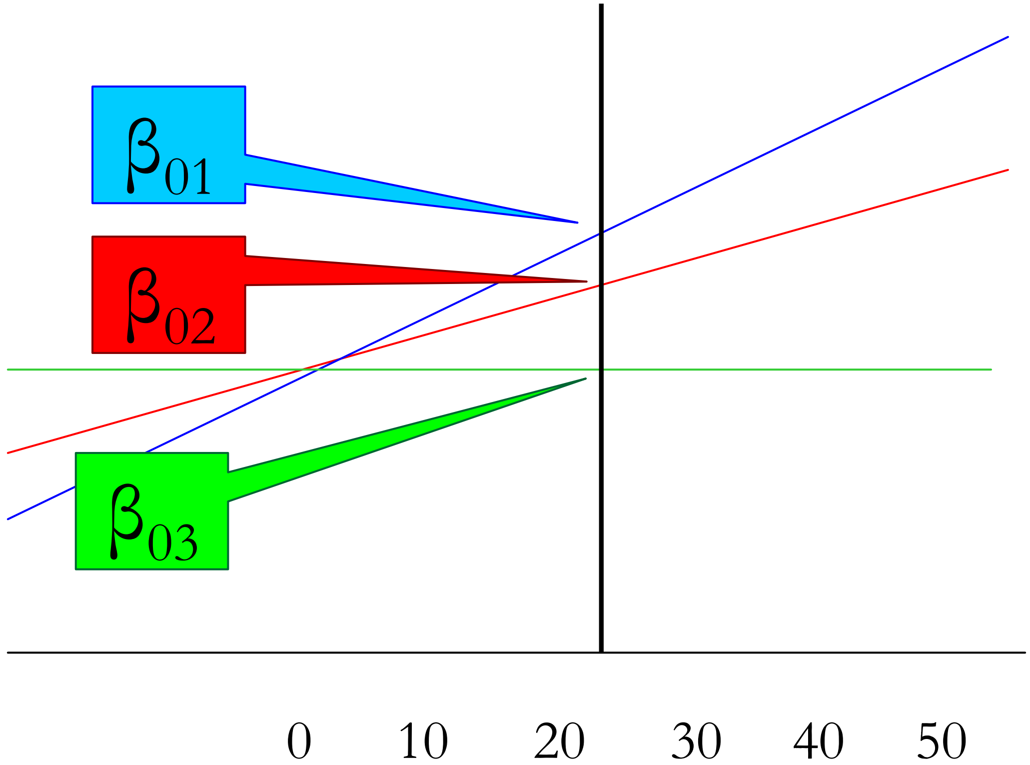


$$\beta_{0j} = E(Y_{ij} \mid X_{ij} = 0)$$

Centering Around the Grand Mean

- Predictors at level 1 (X's) are expressed as deviations from the grand mean (M): $(X_{ij} – M)$
- Intercept now expresses the expected outcome value (Y) for someone whose value on predictor X is the same as the grand mean on that predictor
- Centering is computationally more efficient
- Intercept represents the group mean adjusted for the grand mean $\overline{X}_j - M$
- Variance of $\beta_{0j} = \tau_{00}$, the variance among the level-2 unit means adjusted for the grand mean

Centering Around the Grand Mean



$$\beta_{0j} = E(Y_{ij} \mid X_{ij} = \gamma_{00})$$

Centering Around the Group Mean

- Individual scores are interpreted relative to their group mean $(X_{ij} - \overline{X})$
- The individual deviation scores are orthogonal to the group means
- Intercept represents the unadjusted mean achievement for the group
- Unbiased estimates of within-group effects
- May be necessary in random coefficient models if level 1 predictor affects the outcome at both level 1 and level 2
- Can control for unmeasured between group differences
- But can mask between group effects; interpretation is more complex

Centering Around the Group Mean

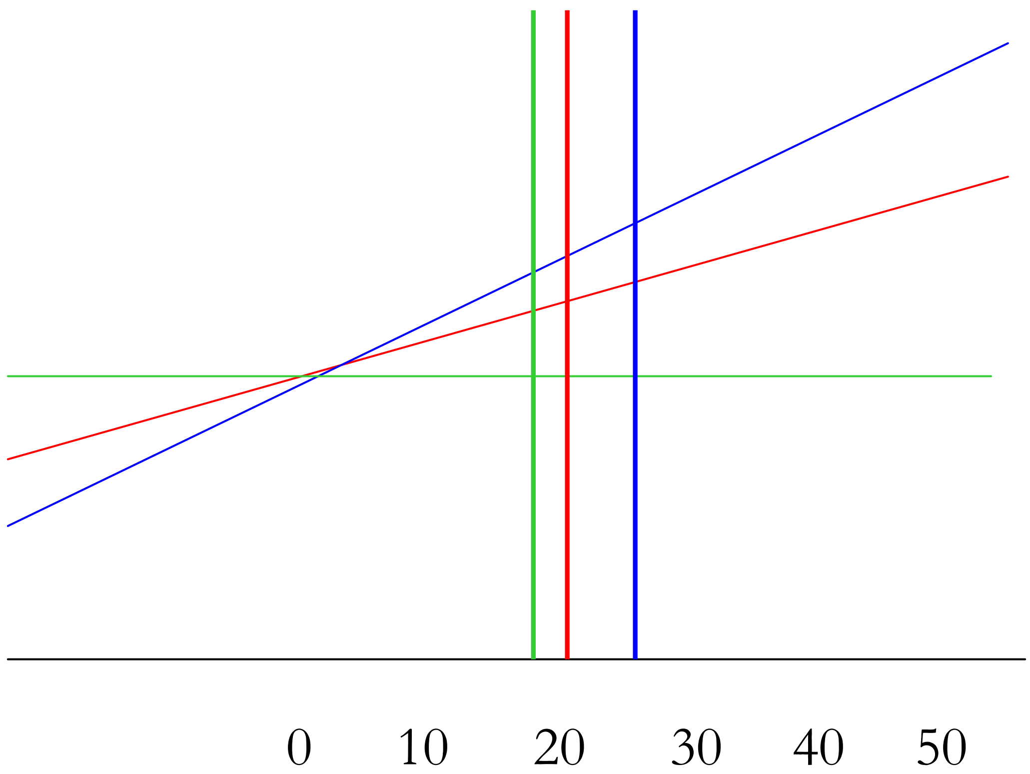
- Level 1 results are relative to group membership
- Intercept becomes the unadjusted mean for group j
- Should include level 2 mean of level 1 variables to fully disentangle individual and compositional effects
- Variance β_{0j} is now the variance among the level 2 unit means

Centering Around the Group Mean

If $\overline{X} = 25$, then $\beta = E(Y|X = 25)$

If $\overline{X} = 20$, then $\beta = E(Y|X = 20)$

If $\overline{X}_3 = 18$, then $\beta_{03} = E(Y_{i3}|X_{i3} = 18)$



Parameter estimation

- Coefficients and standard errors estimated through maximum likelihood procedures (usually)
  - The ratio of the parameter to its standard error produces a Wald test evaluated through comparison to the normal distribution (z)
  - In HLM software, a more conservative approach is used:
    - t-tests are used for significance testing
    - t-tests more accurate for fixed effects, small n, and nonnormal distributions)
- Standard errors
- Variance components

Parameter reliability

- Analogous to score reliability: ratio of true score variance to total variance (true score + error)
- In HLM, ratio of true parameter variance to total variability
- For example … , par… , is:

True variance of the sample means (estimated)

Variance of error of the sample means

$$\lambda_j = Var(\beta_{0j}) / Var(\bar{Y}_j) = \tau_{00}^2 / (\tau_{00}^2 + \sigma^2 / n_j)$$

True variance of the sample means (estimated)

Total variance of the sample means (observed)

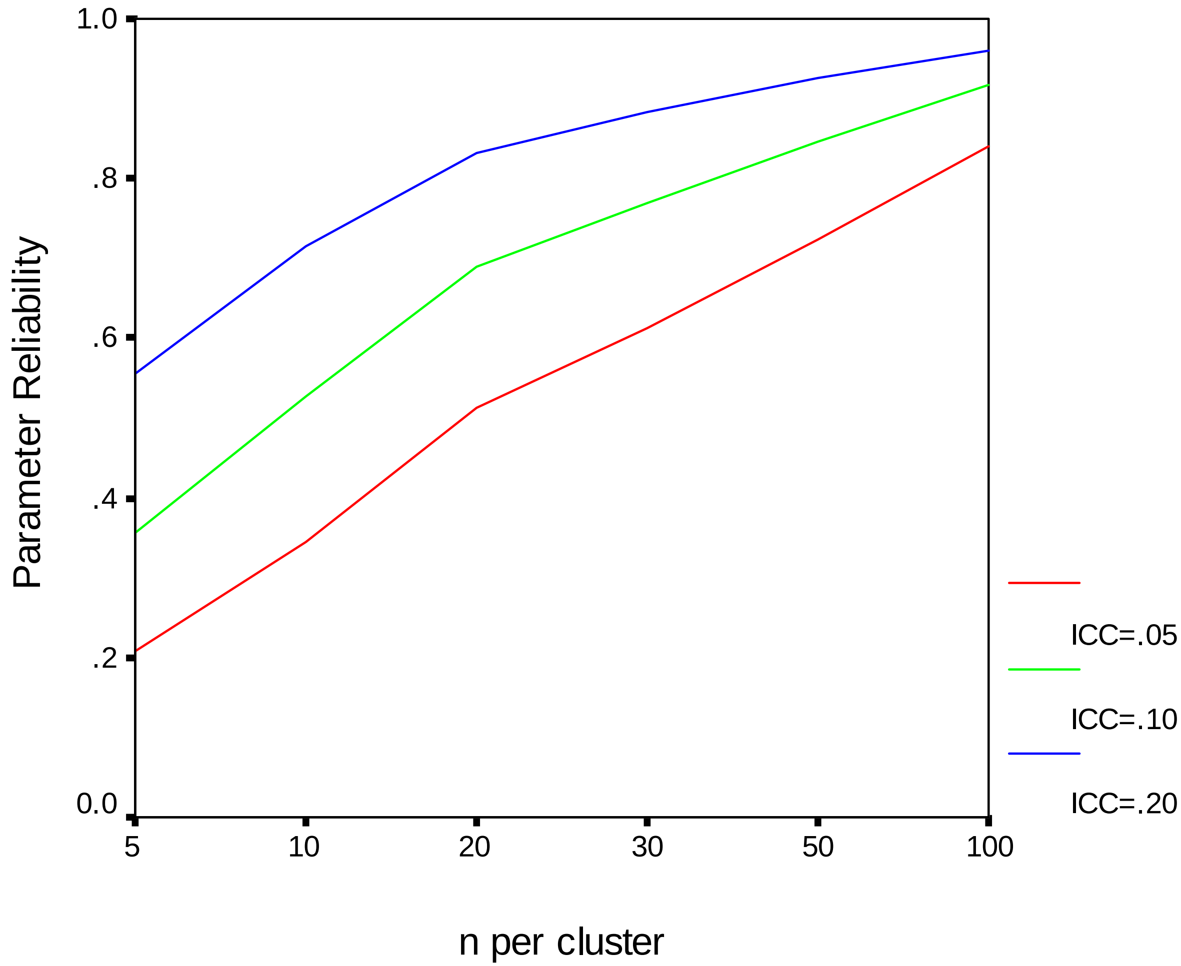
Parameter reliability

$$\lambda_j = \frac{n_j \rho_I}{1 + (n_j - 1)\rho_I}$$

	ICC (ρ_I)		
n_j	**.05**	**.10**	**.20**
5	.21	.36	.56
10	.34	.53	.71
20	.51	.69	.83
30	.61	.77	.88
50	.72	.85	.93
100	.84	.92	.96

Parameter reliability

$$\lambda_j = \frac{n_j \rho_I}{1 + (n_j - 1)\rho_I}$$



Predicting Group Effects

- It is often of interest to estimate the random group effects (β_{0j}, β_{1j})
- This is accomplished using Empirical Bayes (EB) estimation
- The basic idea of EB estimation is to predict group values using two kinds of information:
  - Group j data
  - Population data obtained from the estimation of the regression model

Empirical Bayes

- If information from only group j is used to estimate then we have the OLS estimate:

$$\beta_{0j} = \bar{Y}_j$$

- If information from only the population is used to estimate then the group is estimated from the grand mean:

$$\gamma_{00} = \bar{Y}_{..} = \sum_{j=1}^{N} \frac{n_j}{N} \bar{Y}_j$$

Empirical Bayes

The smaller the reliability, the greater the weight of the grand mean

The larger the reliability, the greater the weight of the group mean

- A third possibility is to co
  pop
- The ar erage weighted
  the param reliability:

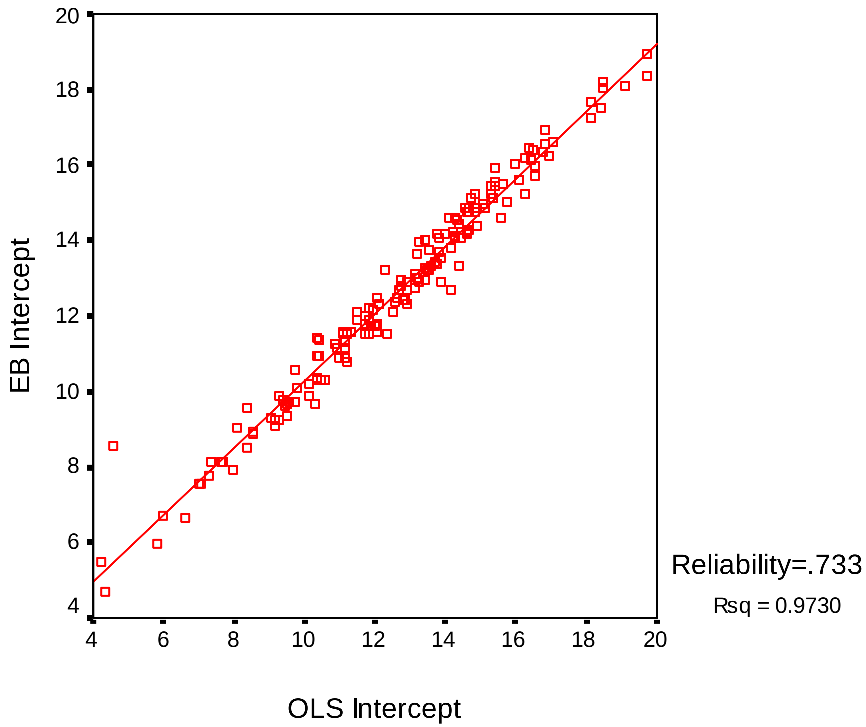
$$\beta_{0j}^{EB} = \lambda_j \beta_{0j} + (1-\lambda_j)\gamma_{00}$$

- The results in the "posterior means" or EB estimates

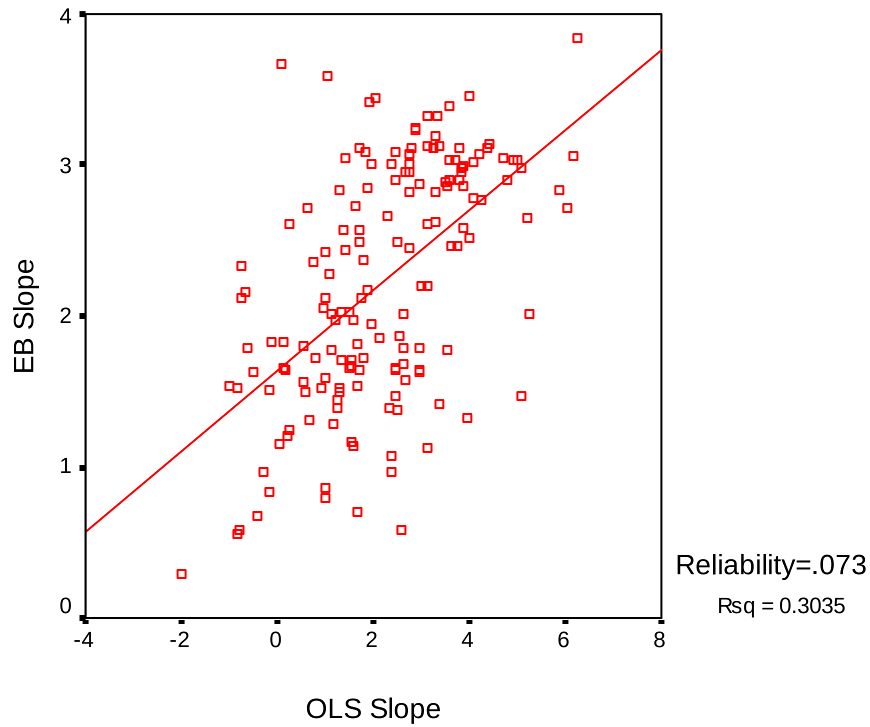
Bayesian Estimation

- Use of prior and posterior information improves estimation (depending on purpose)
- Estimates "shrink" toward the grand mean as shown in formula
- Amount of shrinkage depends on the "badness" of the unit estimate
  - Low reliability results in greater shrinkage (if $\lambda = 1$, there is no shrinkage; if $\lambda = 0$, shrinkage is complete, γ_{00})
  - Small n-size within a j unit results in greater shrinkage, "borrowing" from larger units

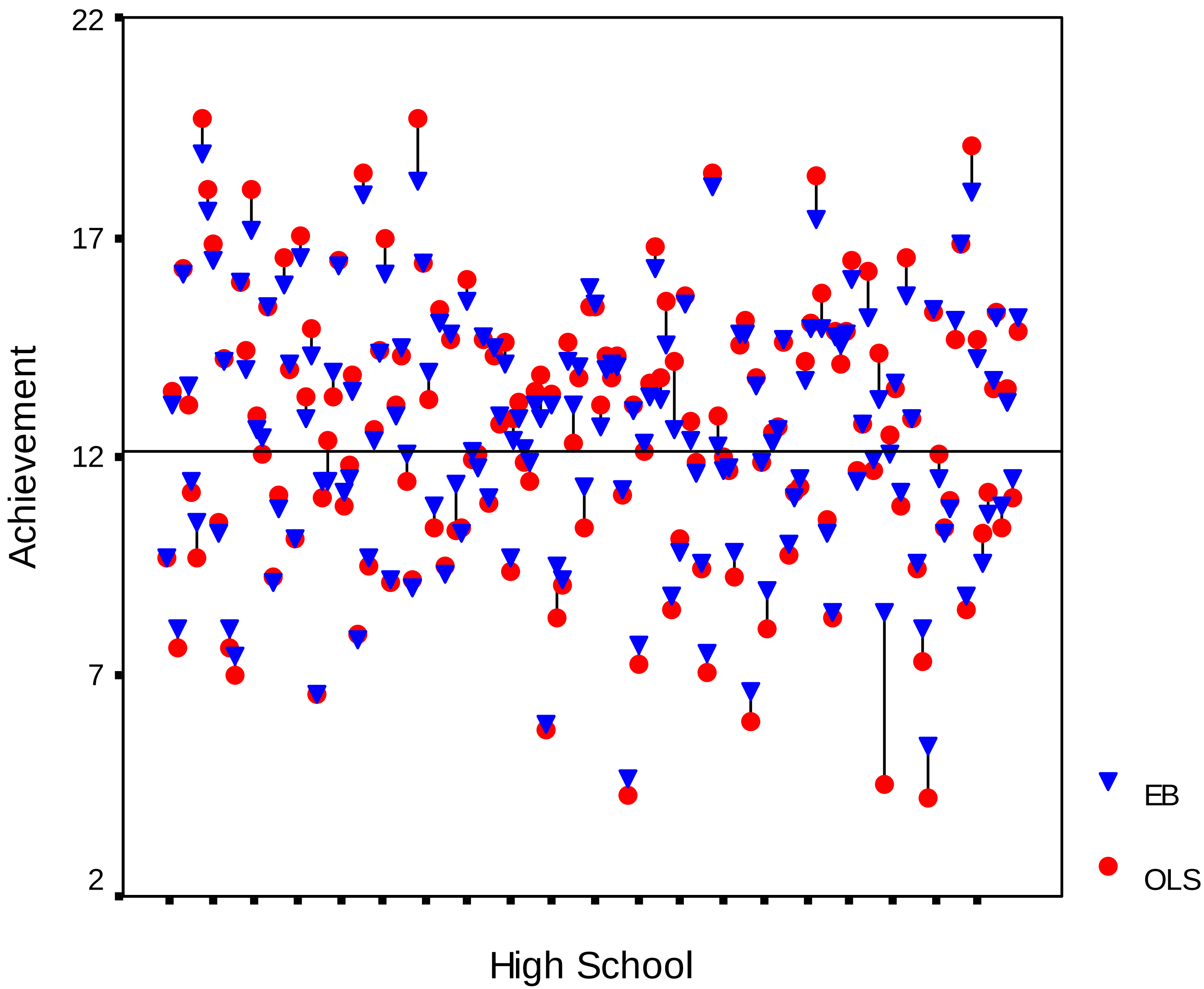
$$\beta_{0j}^{EB} = \lambda_j \beta_{0j} + (1 - \lambda_j)\gamma_{00}$$



20
18
16
14
12
10
8
6
4
EB Intercept
4
6
8
10
12
14
16
18
20
OLS Intercept
Reliability=.733
Rsq = 0.9730



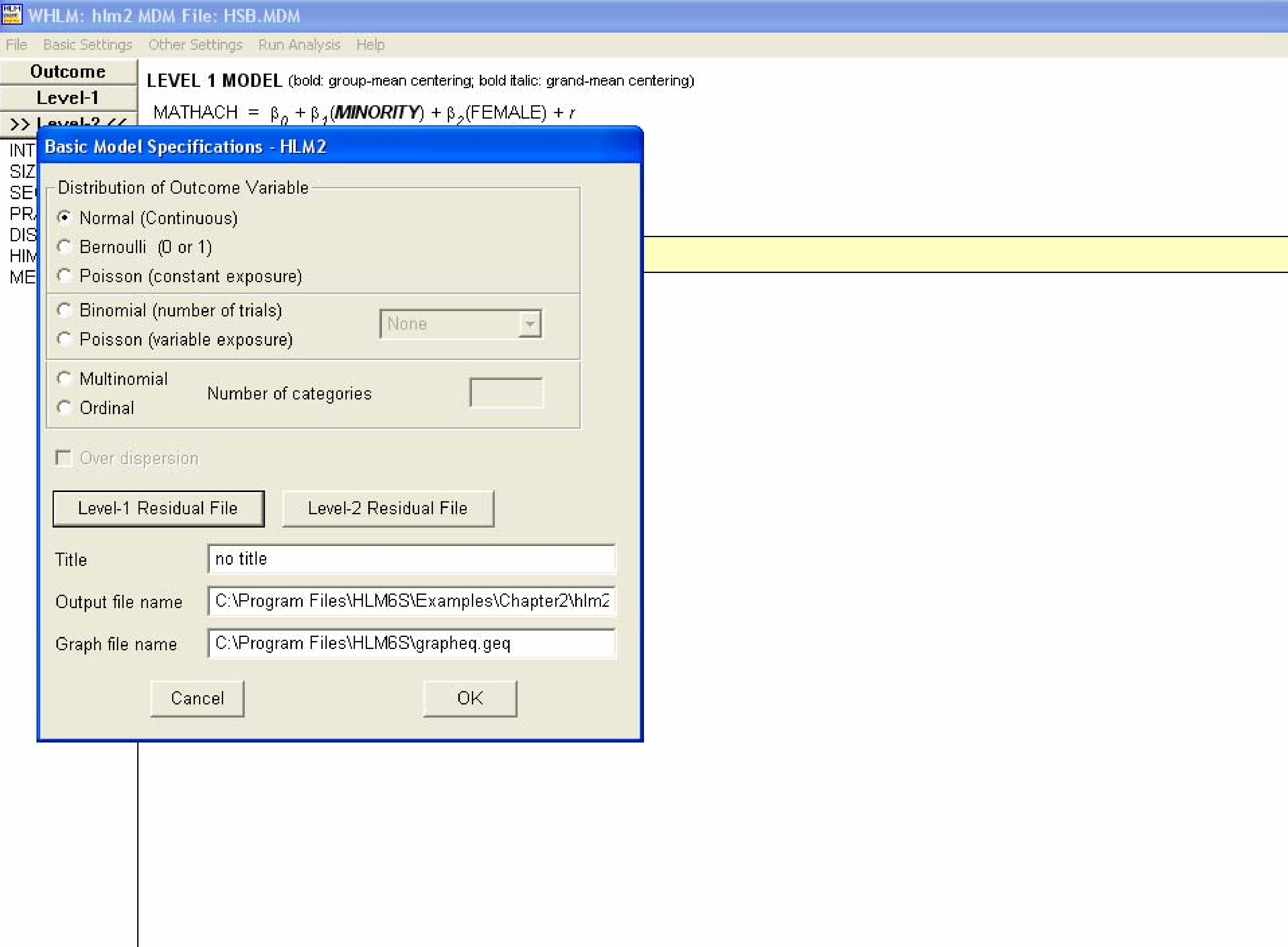
4
3
2
1
0
EB Slope
-4
-2
0
2
4
6
8
OLS Slope
Reliability=.073
Rsq = 0.3035

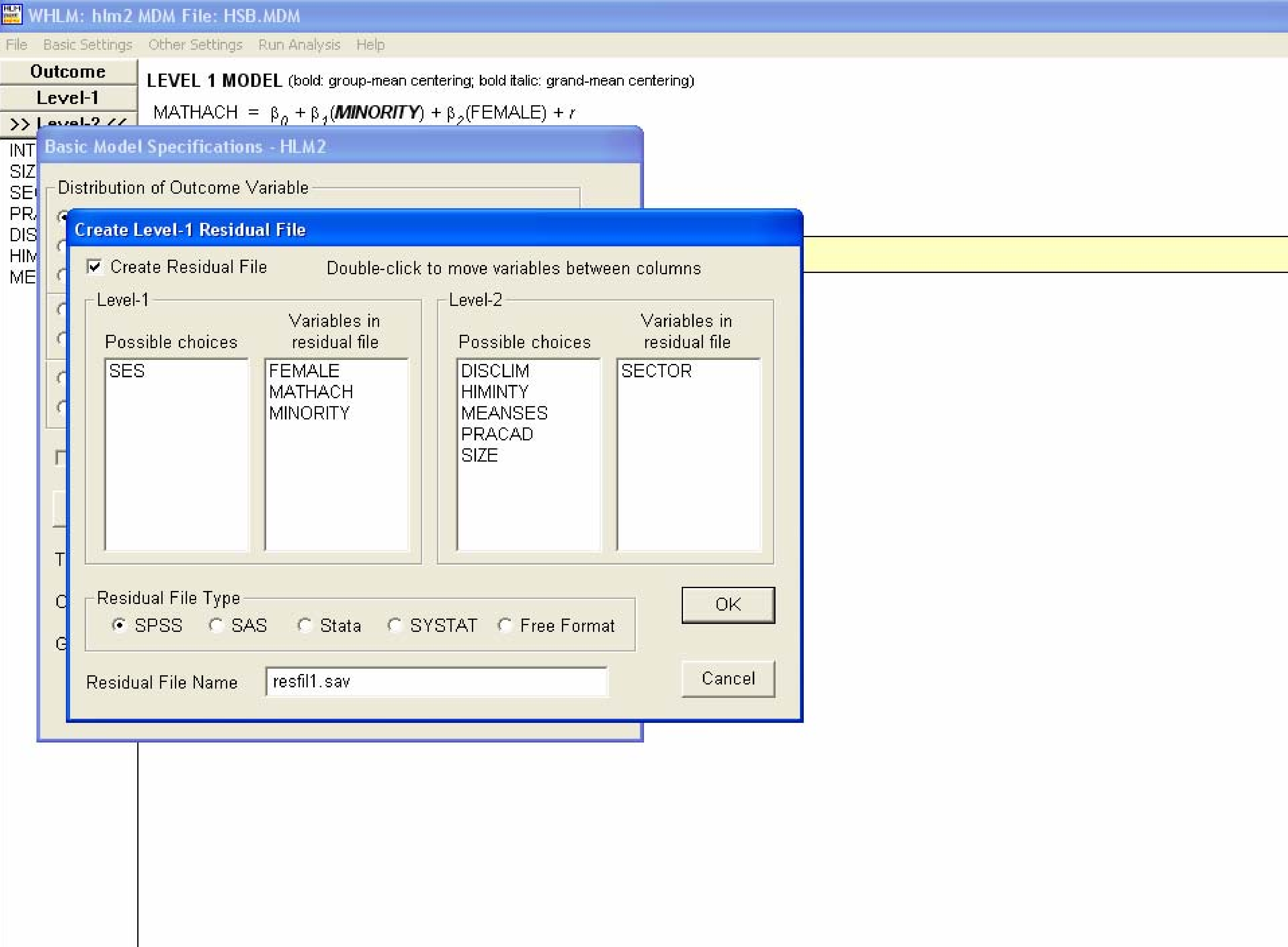


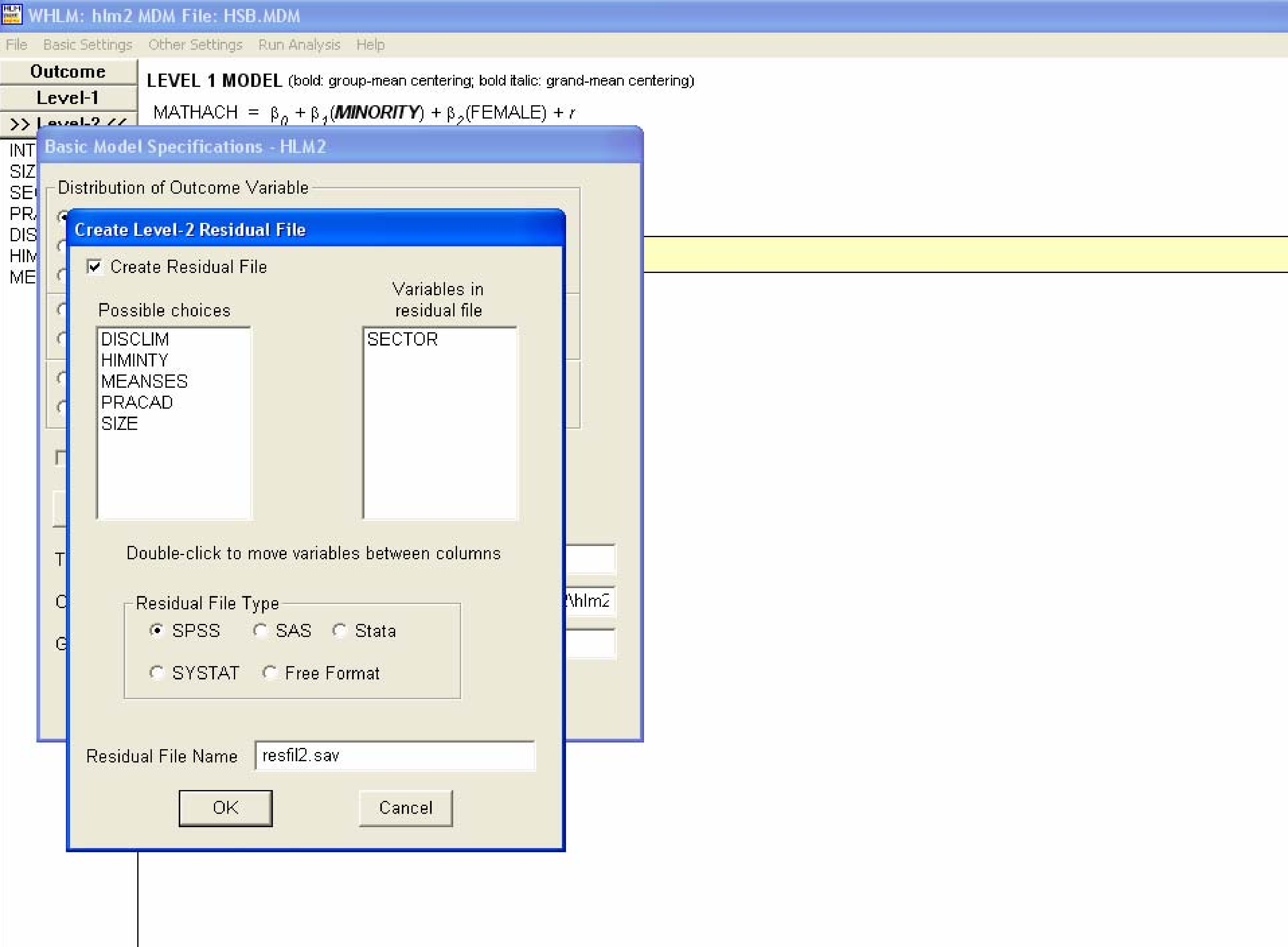
22
17
12
7
2
Achievement
High School
EB
OLS

HLM Residual Files

- Important outcome information from an HLM analysis can be saved for each level of the analysis in a "residual file"
  - Residual files contain parameter estimates and other variables from the analysis
  - Residual files can be save in statistical package format (SPSS, SAS, etc.)
- Residual files can be used for diagnostic evaluation of statistical model assumptions
- Residual files can be used to estimate and further describe or analyze effects among the units at each level


WHLM: hlm2 MDM File: HSB.MDM
File
Basic Settings
Other Settings
Run Analysis
Help
Outcome
Level-1
LEVEL 1 MODEL (bold: group-mean centering; bold italic: grand-mean centering)
MATHACH = β0 + β1(MINORITY) + β2(FEMALE) + r
Basic Model Specifications - HLM2
Distribution of Outcome Variable
Normal (Continuous)
Bernoulli (0 or 1)
Poisson (constant exposure)
Binomial (number of trials)
Poisson (variable exposure)
None
Multinomial
Ordinal
Number of categories
Over dispersion
Level-1 Residual File
Level-2 Residual File
Title
no title
Output file name
C:\Program Files\HLM6S\Examples\Chapter2\hlm2
Graph file name
C:\Program Files\HLM6S\grapheq.geq
Cancel
OK


WHLM: hlm2 MDM File: HSB.MDM
File
Basic Settings
Other Settings
Run Analysis
Help
Outcome
Level-1
>> Level-2 <<
LEVEL 1 MODEL (bold: group-mean centering; bold italic: grand-mean centering)
MATHACH = β0 + β1(MINORITY) + β2(FEMALE) + r
Basic Model Specifications - HLM2
Distribution of Outcome Variable
Create Level-1 Residual File
Create Residual File
Double-click to move variables between columns
Level-1
Possible choices
Variables in residual file
SES
FEMALE
MATHACH
MINORITY
Level-2
Possible choices
Variables in residual file
DISCLIM
HIMINTY
MEANSES
PRACAD
SIZE
SECTOR
Residual File Type
SPSS
SAS
Stata
SYSTAT
Free Format
OK
Cancel
Residual File Name
resfil1.sav

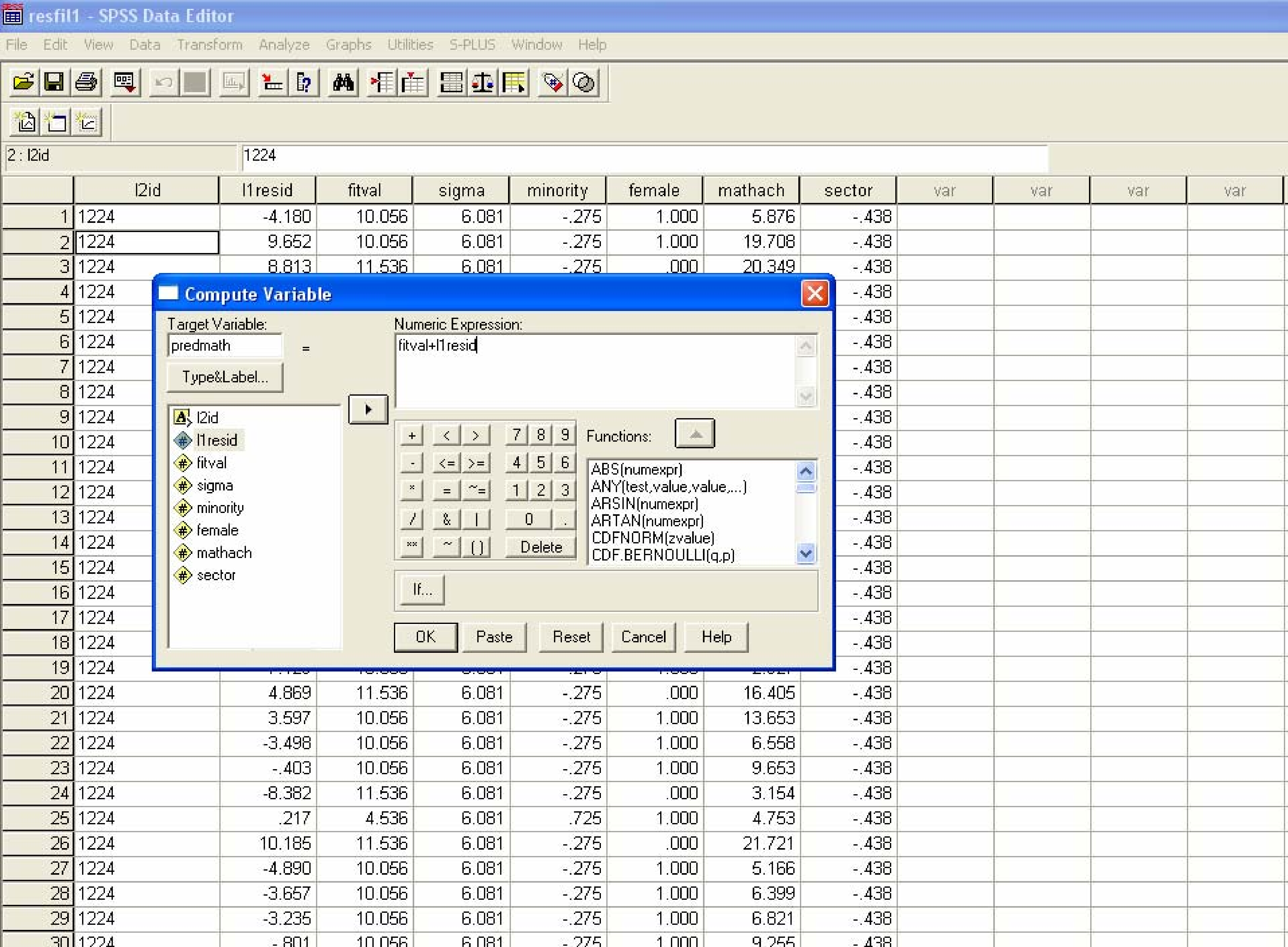

WHLM: hlm2 MDM File: HSB.MDM
File Basic Settings Other Settings Run Analysis Help
Outcome
Level-1
>> Level-2 <<
LEVEL 1 MODEL (bold: group-mean centering; bold italic: grand-mean centering)
MATHACH = β0 + β1(MINORITY) + β2(FEMALE) + r
Basic Model Specifications - HLM2
Distribution of Outcome Variable
Create Level-2 Residual File
Create Residual File
Possible choices
Variables in residual file
DISCLIM
HIMINTY
MEANSES
PRACAD
SIZE
SECTOR
Double-click to move variables between columns
Residual File Type
SPSS
SAS
Stata
SYSTAT
Free Format
Residual File Name
resfil2.sav
OK
Cancel

resfil1 - SPSS Data Editor

File Edit View Data Transform Analyze Graphs Utilities S-PLUS Window Help

1 : l2id 1224

	l2id	l1resid	fitval	sigma	minority	female	mathach	sector	var	var	var	var
1	1224	-4.180	10.056	6.081	-.275	1.000	5.876	-.438				
2	1224	9.652	10.056	6.081	-.275	1.000	19.708	-.438				
3	1224	8.813	11.536	6.081	-.275	.000	20.349	-.438				
4	1224	-2.755	11.536	6.081	-.275	.000	8.781	-.438				
5	1224	6.362	11.536	6.081	-.275	.000	17.898	-.438				
6	1224	-6.953	11.536	6.081	-.275	.000	4.583	-.438				
7	1224	-12.888	10.056	6.081	-.275	1.000	-2.832	-.438				
8	1224	-11.013	11.536	6.081	-.275	.000	.523	-.438				
9	1224	-8.529	10.056	6.081	-.275	1.000	1.527	-.438				
10	1224	9.985	11.536	6.081	-.275	.000	21.521	-.438				
11	1224	-.581	10.056	6.081	-.275	1.000	9.475	-.438				
12	1224	6.001	10.056	6.081	-.275	1.000	16.057	-.438				
13	1224	9.642	11.536	6.081	-.275	.000	21.178	-.438				
14	1224	10.122	10.056	6.081	-.275	1.000	20.178	-.438				
15	1224	8.813	11.536	6.081	-.275	.000	20.349	-.438				
16	1224	10.452	10.056	6.081	-.275	1.000	20.508	-.438				
17	1224	7.802	11.536	6.081	-.275	.000	19.338	-.438				
18	1224	-1.872	6.017	6.081	.725	.000	4.145	-.438				
19	1224	-7.129	10.056	6.081	-.275	1.000	2.927	-.438				
20	1224	4.869	11.536	6.081	-.275	.000	16.405	-.438				
21	1224	3.597	10.056	6.081	-.275	1.000	13.653	-.438				
22	1224	-3.498	10.056	6.081	-.275	1.000	6.558	-.438				
23	1224	-.403	10.056	6.081	-.275	1.000	9.653	-.438				
24	1224	-8.382	11.536	6.081	-.275	.000	3.154	-.438				
25	1224	.217	4.536	6.081	.725	1.000	4.753	-.438				
26	1224	10.185	11.536	6.081	-.275	.000	21.721	-.438				
27	1224	-4.890	10.056	6.081	-.275	1.000	5.166	-.438				
28	1224	-3.657	10.056	6.081	-.275	1.000	6.399	-.438				
29	1224	-3.235	10.056	6.081	-.275	1.000	6.821	-.438				
30	1224	-.801	10.056	6.081	-.275	1.000	9.255	-.438				



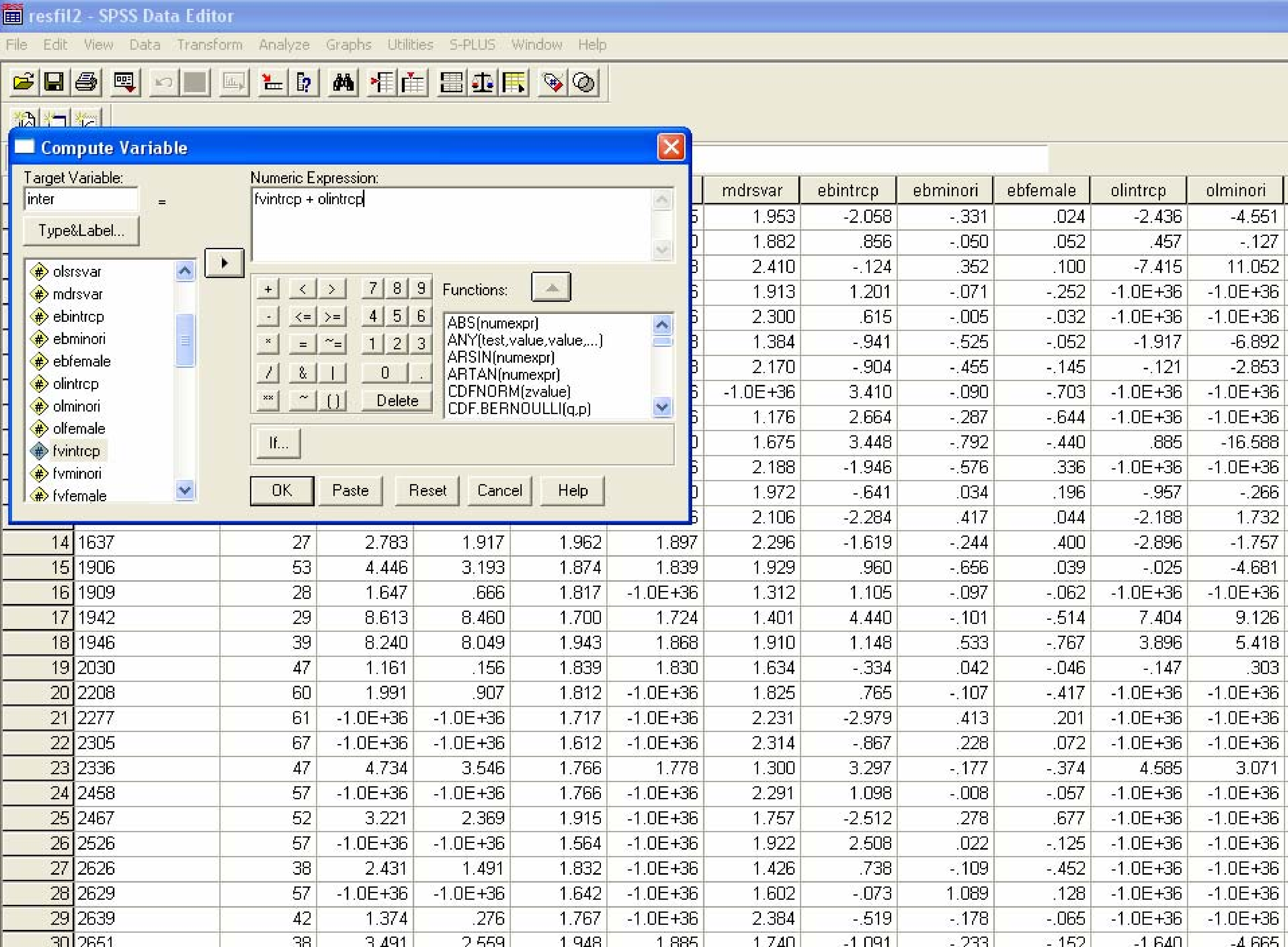
resfil1 - SPSS Data Editor
File Edit View Data Transform Analyze Graphs Utilities S-PLUS Window Help
2 : l2id
1224
l2id l1resid fitval sigma minority female mathach sector var var var var
1 1224 -4.180 10.056 6.081 -.275 1.000 5.876 -.438
2 1224 9.652 10.056 6.081 -.275 1.000 19.708 -.438
3 1224 8.813 11.536 6.081 -.275 .000 20.349 -.438
Compute Variable
Target Variable:
predmath
=
Type&Label...
Numeric Expression:
fitval+l1resid
l2id
l1resid
fitval
sigma
minority
female
mathach
sector
+ < > 7 8 9
- <= >= 4 5 6
* = ~= 1 2 3
/ & | 0 .
** ~ ( ) Delete
Functions:
ABS(numexpr)
ANY(test,value,value,...)
ARSIN(numexpr)
ARTAN(numexpr)
CDFNORM(zvalue)
CDF.BERNOULLI(q,p)
If...
OK Paste Reset Cancel Help
20 1224 4.869 11.536 6.081 -.275 .000 16.405 -.438
21 1224 3.597 10.056 6.081 -.275 1.000 13.653 -.438
22 1224 -3.498 10.056 6.081 -.275 1.000 6.558 -.438
23 1224 -.403 10.056 6.081 -.275 1.000 9.653 -.438
24 1224 -8.382 11.536 6.081 -.275 .000 3.154 -.438
25 1224 .217 4.536 6.081 .725 1.000 4.753 -.438
26 1224 10.185 11.536 6.081 -.275 .000 21.721 -.438
27 1224 -4.890 10.056 6.081 -.275 1.000 5.166 -.438
28 1224 -3.657 10.056 6.081 -.275 1.000 6.399 -.438
29 1224 -3.235 10.056 6.081 -.275 1.000 6.821 -.438

resfil2 - SPSS Data Editor

File Edit View Data Transform Analyze Graphs Utilities S-PLUS Window Help

1 : l2id 1224

	l2id	nj	chipct	mdist	lntotvar	olsrsvar	mdrsvar	ebintrcp	ebminori	ebfemale	olintrcp	olminori
1	1224	47	4.812	3.572	2.027	1.975	1.953	-2.058	-.331	.024	-2.436	-4.551
2	1288	25	2.051	.923	1.949	1.960	1.882	.856	-.050	.052	.457	-.127
3	1296	48	4.068	2.969	1.678	1.678	2.410	-.124	.352	.100	-7.415	11.052
4	1308	20	-1.0E+36	-1.0E+36	1.811	-1.0E+36	1.913	1.201	-.071	-.252	-1.0E+36	-1.0E+36
5	1317	48	-1.0E+36	-1.0E+36	1.698	-1.0E+36	2.300	.615	-.005	-.032	-1.0E+36	-1.0E+36
6	1358	30	4.974	3.871	1.771	1.513	1.384	-.941	-.525	-.052	-1.917	-6.892
7	1374	28	3.095	2.291	2.123	2.033	2.170	-.904	-.455	-.145	-.121	-2.853
8	1433	35	-1.0E+36	-1.0E+36	1.356	-1.0E+36	-1.0E+36	3.410	-.090	-.703	-1.0E+36	-1.0E+36
9	1436	44	3.136	2.293	1.515	-1.0E+36	1.176	2.664	-.287	-.644	-1.0E+36	-1.0E+36
10	1461	33	10.373	10.322	1.939	1.800	1.675	3.448	-.792	-.440	.885	-16.588
11	1462	57	-1.0E+36	-1.0E+36	1.842	-1.0E+36	2.188	-1.946	-.576	.336	-1.0E+36	-1.0E+36
12	1477	62	1.267	.184	1.968	1.980	1.972	-.641	.034	.196	-.957	-.266
13	1499	53	4.379	3.141	1.848	1.806	2.106	-2.284	.417	.044	-2.188	1.732
14	1637	27	2.783	1.917	1.962	1.897	2.296	-1.619	-.244	.400	-2.896	-1.757
15	1906	53	4.446	3.193	1.874	1.839	1.929	.960	-.656	.039	-.025	-4.681
16	1909	28	1.647	.666	1.817	-1.0E+36	1.312	1.105	-.097	-.062	-1.0E+36	-1.0E+36
17	1942	29	8.613	8.460	1.700	1.724	1.401	4.440	-.101	-.514	7.404	9.126
18	1946	39	8.240	8.049	1.943	1.868	1.910	1.148	.533	-.767	3.896	5.418
19	2030	47	1.161	.156	1.839	1.830	1.634	-.334	.042	-.046	-.147	.303
20	2208	60	1.991	.907	1.812	-1.0E+36	1.825	.765	-.107	-.417	-1.0E+36	-1.0E+36
21	2277	61	-1.0E+36	-1.0E+36	1.717	-1.0E+36	2.231	-2.979	.413	.201	-1.0E+36	-1.0E+36
22	2305	67	-1.0E+36	-1.0E+36	1.612	-1.0E+36	2.314	-.867	.228	.072	-1.0E+36	-1.0E+36
23	2336	47	4.734	3.546	1.766	1.778	1.300	3.297	-.177	-.374	4.585	3.071
24	2458	57	-1.0E+36	-1.0E+36	1.766	-1.0E+36	2.291	1.098	-.008	-.057	-1.0E+36	-1.0E+36
25	2467	52	3.221	2.369	1.915	-1.0E+36	1.757	-2.512	.278	.677	-1.0E+36	-1.0E+36
26	2526	57	-1.0E+36	-1.0E+36	1.564	-1.0E+36	1.922	2.508	.022	-.125	-1.0E+36	-1.0E+36
27	2626	38	2.431	1.491	1.832	-1.0E+36	1.426	.738	-.109	-.452	-1.0E+36	-1.0E+36
28	2629	57	-1.0E+36	-1.0E+36	1.642	-1.0E+36	1.602	-.073	1.089	.128	-1.0E+36	-1.0E+36
29	2639	42	1.374	.276	1.767	-1.0E+36	2.384	-.519	-.178	-.065	-1.0E+36	-1.0E+36
30	2651	38	3.491	2.559	1.948	1.885	1.740	-1.091	-.233	-.152	-1.640	-4.665


resfil2 - SPSS Data Editor
File Edit View Data Transform Analyze Graphs Utilities S-PLUS Window Help
Compute Variable
Target Variable:
inter
=
Type&Label...
Numeric Expression:
fvintrcp + olintrcp
olsrsvar
mdrsvar
ebintrcp
ebminori
ebfemale
olintrcp
olminori
olfemale
fvintrcp
fvminori
fvfemale
Functions:
ABS(numexpr)
ANY(test,value,value,...)
ARSIN(numexpr)
ARTAN(numexpr)
CDFNORM(zvalue)
CDF.BERNOULLI(q,p)
Delete
If...
OK
Paste
Reset
Cancel
Help
mdrsvar
ebintrcp
ebminori
ebfemale
olintrcp
olminori

Example: Creating Residual Files

Three level models

Level-1 (p students)

$$Y_{ijk} = \pi_{0jk} + \pi_{1jk}(a_{pijk}) + e_{ijk}$$

Level-2 (j classrooms)

$$\pi_{0jk} = \beta_{p0k} + \beta_{p1k}(X_{qjk}) + r_{p0k}$$

$$\pi_{1jk} = \beta_{p1kj} + \beta_{p1k}(X_{qjk}) + r_{p1k}$$

Level-3 (k schools)

$$\beta_{p0k} = \gamma_{pq0} + \gamma_{pqs}(W_{sk}) + u_{pqk}$$

$$\beta_{p1k} = \gamma_{pq1} + \gamma_{pqs}(W_{sk}) + u_{pqk}$$

Partitioning variance in the three level model

Proportion of variance within classrooms (individual student differences) = $\sigma^2 / (\sigma^2 + \tau_\pi + \tau_\beta)$

Proportion of variance between classrooms within schools = $\tau_\pi / (\sigma^2 + \tau_\pi + \tau_\beta)$

Proportion of variance between schools = $\tau_\beta / (\sigma^2 + \tau_\pi + \tau_\beta)$

Three level example

- Example:
  - Go to “Examples” folder and then “Chapter 4” in the HLM directory
  - Open “EG1.sav”, “EG2.sav”, and “EG3.sav”

Longitudinal models

- Level 1 defined as repeated measurement occasions
- Levels 2 and 3 defined as higher levels in the nested structure
- For example, longitudinal analysis of student achievement

  Level 1 = achievement scores at times 1 – t

  Level 2 = student characteristics

  Level 3 = school characteristics

Longitudinal models

- Two important advantages of the MLM approach to repeated measures:
  - Times of measurement can vary from one person to another
  - Data do not need to be complete on all measurement occasions

Longitudinal models

Level-1

$$Y_{tij} = \pi_{0ij} + \pi_{1ij}(time) + e_{tij}$$

Level-2

$$\pi_{0ij} = \beta_{00j} + \beta_{01j}(X_{ij}) + r_{0ij}$$

$$\pi_{1ij} = \beta_{10j} + \beta_{11j}(X_{ij}) + r_{1ij}$$

Level-3

$$\beta_{00j} = \gamma_{000} + \gamma_{001}(W_{1j}) + u_{00j}$$

$$\beta_{10j} = \gamma_{100} + \gamma_{101}(W_{1j}) + u_{10j}$$

level1long - SPSS Data Editor

File Edit View Data Transform Analyze Graphs Utilities S-PLUS Wi

1 : id 63

A variable is also needed to indicate the time of occurrence of each measurement occasion

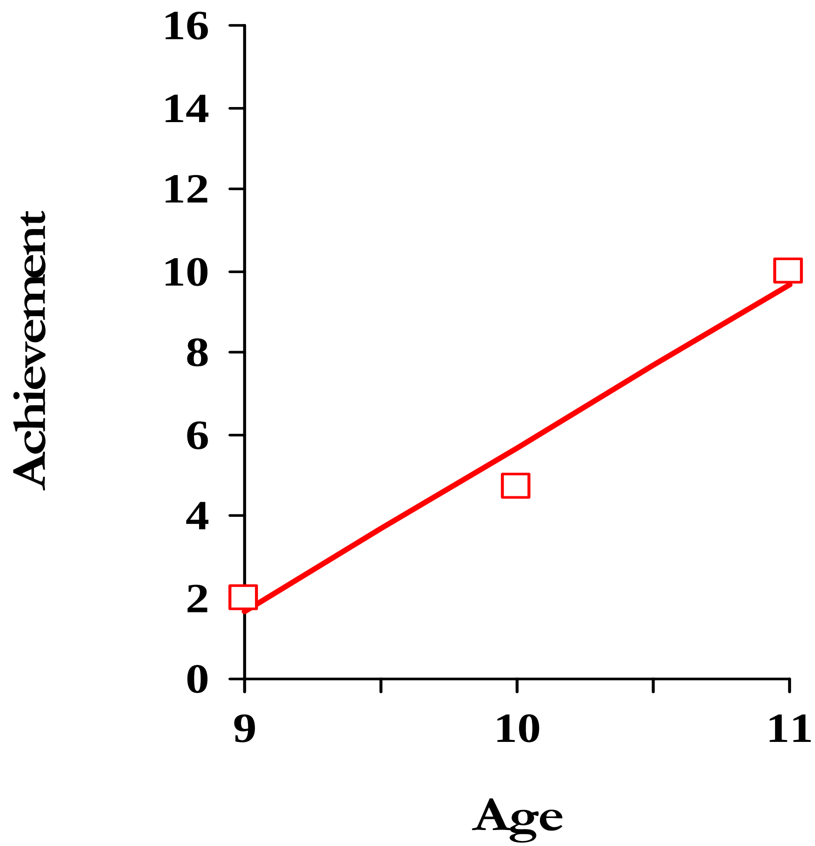
An link

link the level 2 file to level 3

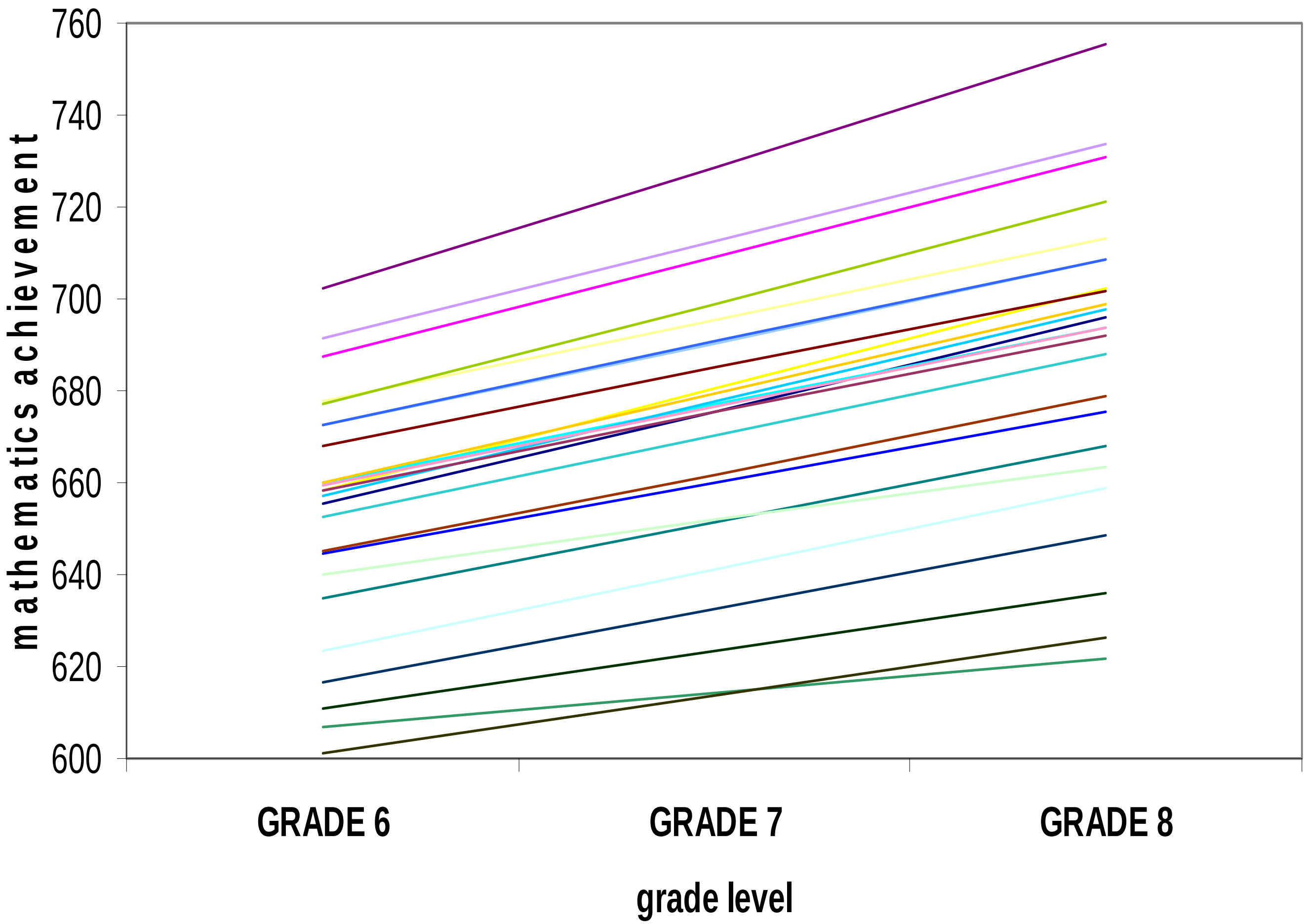
The set of rows represent the repeated measures for one participant

	id	schnumb	math	ye
1	63	1417	692	
2	63	1417	671	1
3	63	1417	707	2
4	171	1417	646	0
5	171	1417	660	1
6	171	1417	689	2
7	190	1417	652	0
8	190	1417	674	1
9	190	1417	691	2
10	246	1417	644	0
11	246	1417	650	1
12	246	1417	680	2
13	267	1417	648	0
14	267	1417	645	1
15	267	1417	651	2
16	279	1417	654	0
17	279	1417	661	1
18	279	1417	680	2
19	332	1417	639	0
20	332	1417	673	1
21	332	1417	723	2
22	451	1417	669	0
23	451	1417	685	1

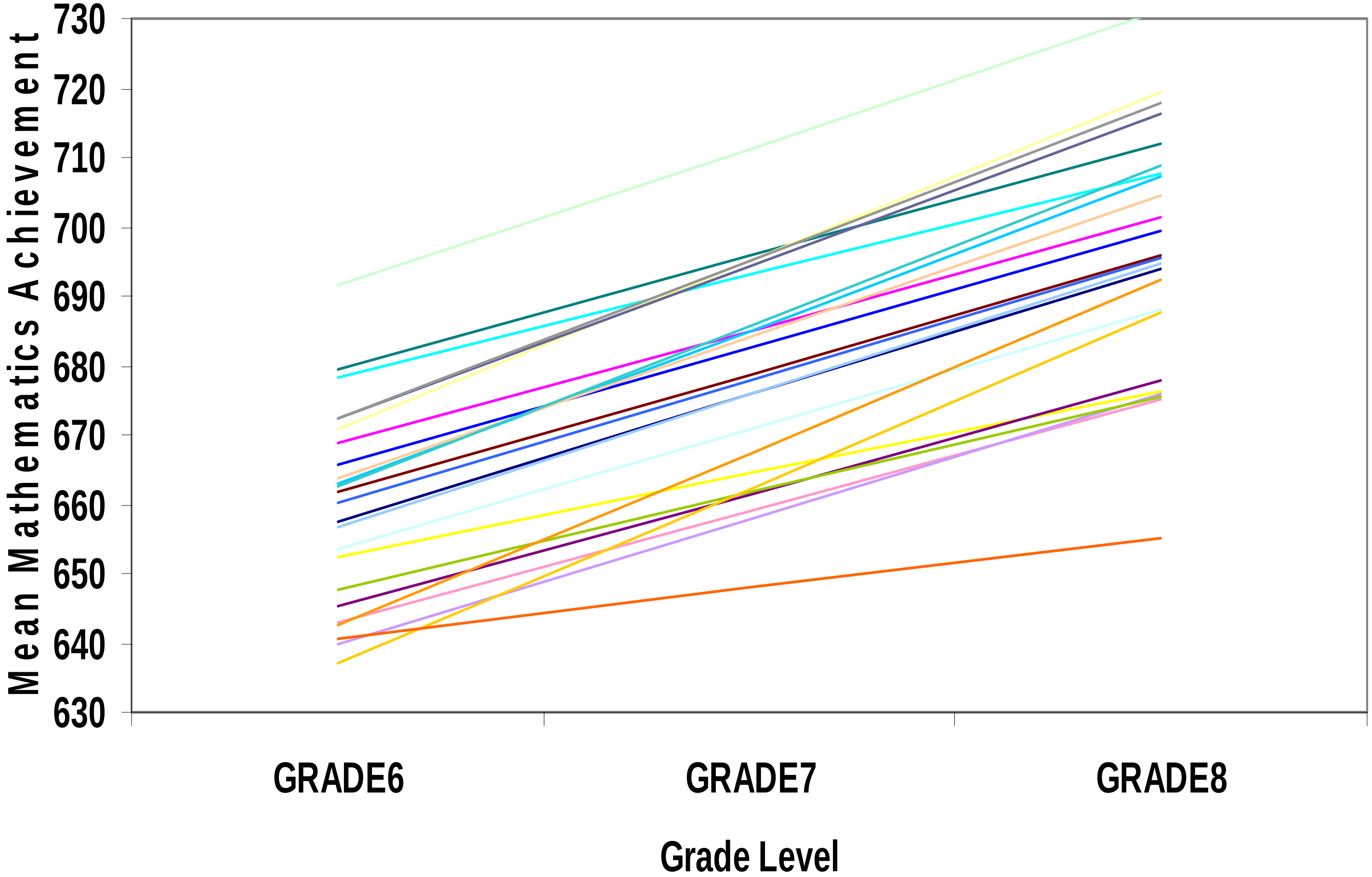
Fitting a growth trajectory



Linear growth for individual students



Average linear growth by school



Curvilinear Longitudinal models

Level-1

$$Y_{tij} = \pi_{0ij} + \pi_{1ij}(\text{time}) + \quad + e_{tij}$$

Level-2

$$\pi_{0ij} = \beta_{00j} + \beta_{01j}(X_{ij}) + r_{0ij}$$

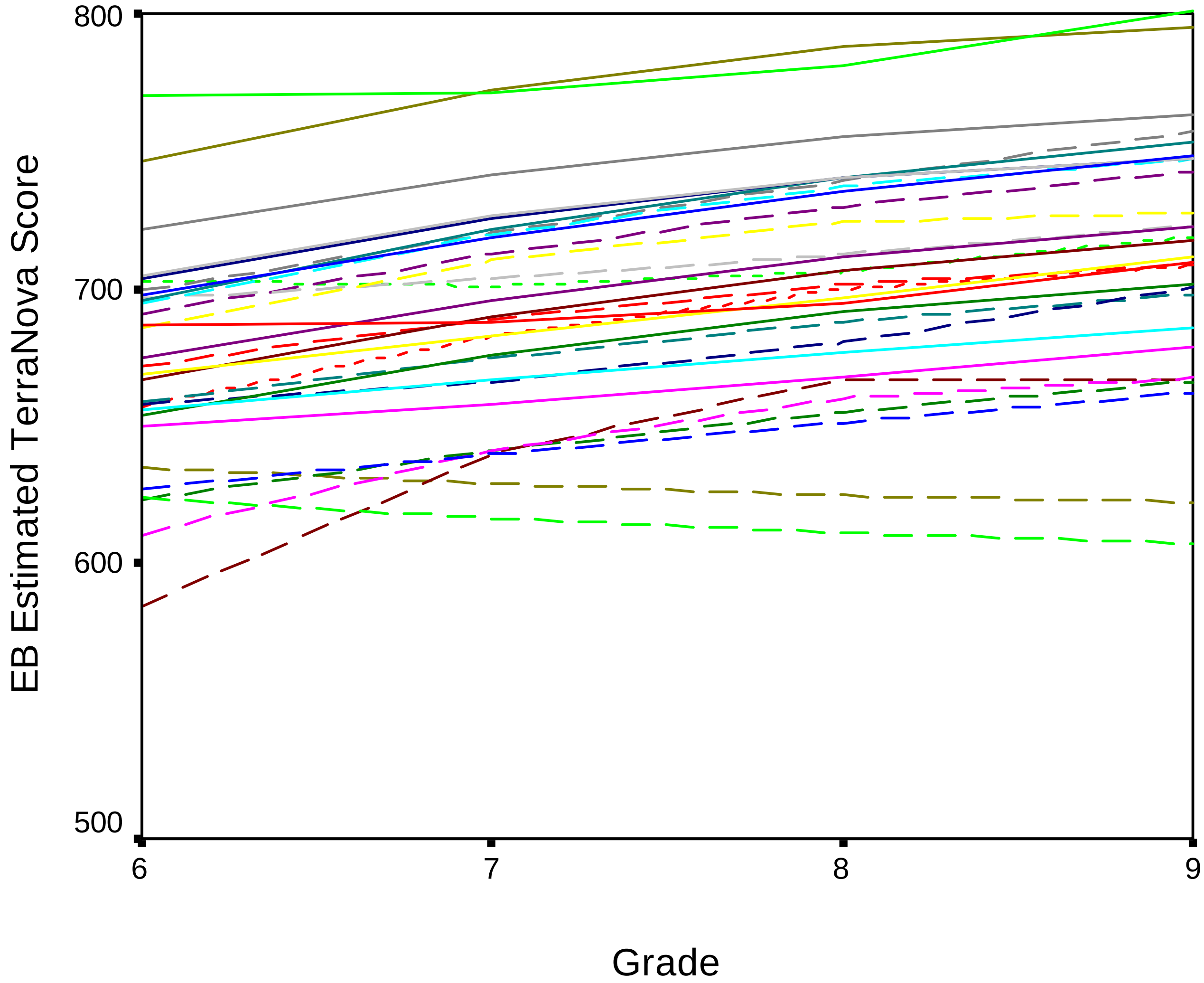
$$\pi_{1ij} = \beta_{10j} + \beta_{11j}(X_{ij}) + r_{1ij}$$

Level-3

$$\beta_{00j} = \gamma_{000} + \gamma_{001}(W_{1j}) + u_{00j}$$

$$\beta_{10j} = \gamma_{100} + \gamma_{101}(W_{1j}) + u_{10j}$$

Curvilinear growth for individual students



Testing a Nested Sequence of HLM Longitudinal Models

1. Test unconditional model
2. Test Level 1 growth model
3. After establishing the level 1 growth model, use it as the baseline for succeeding model comparisons
4. Add level 2 predictors
   - Determine if there is variation across groups
   - If not, fix parameter
   - Decide whether to drop nonsignificant predictors
   - Test deviance, compute R^2 if so desired
5. Add level 3 predictors
   - Evaluate for significance
   - Test deviance, compute R^2 if so desired

Regression Discontinuity and Interrupted Time Series Designs: Change in Intercept

$$Y_{ij} = \pi_{0i} + \pi_{1i}Time_{ij} + \pi_{2i}Treatment_{ij} + \varepsilon_{ij}$$

Treatment is coded 0 or 1

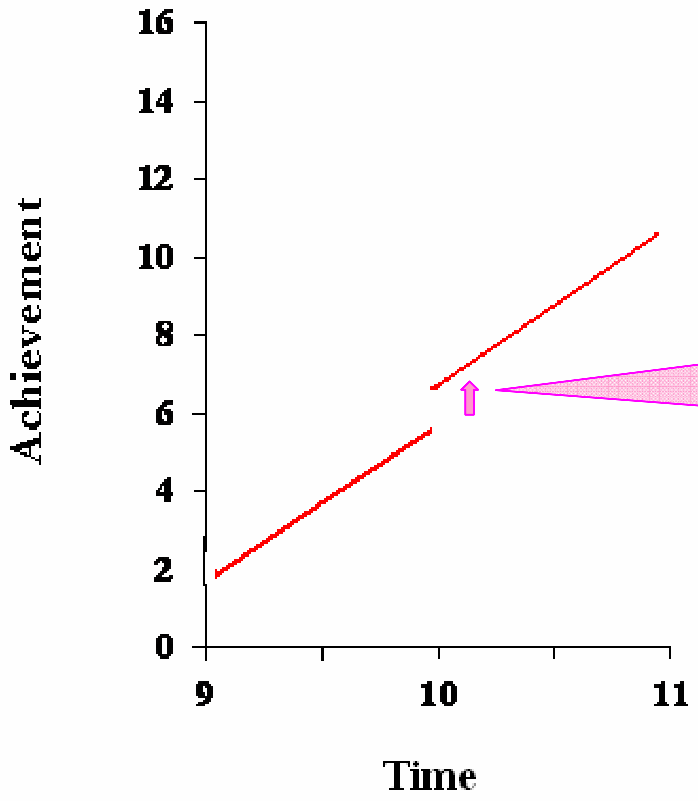
When Treatment = 0:

$$Y_{ij} = \pi_{0i} + \pi_{1i}Time_{ij} + \varepsilon_{ij}$$

When Treatment = 1:

$$Y_{ij} = (\pi_{0i} \quad\quad) + \pi_{1i}Time_{ij} + \varepsilon_{ij}$$

Regression Discontinuity and Interrupted Time Series Designs



Treatment effect on level: $(\pi_{0i} + \pi_{2i})$

Regression Discontinuity and Interrupted Time Series Designs: Change in Slope

When Treatment = 1:

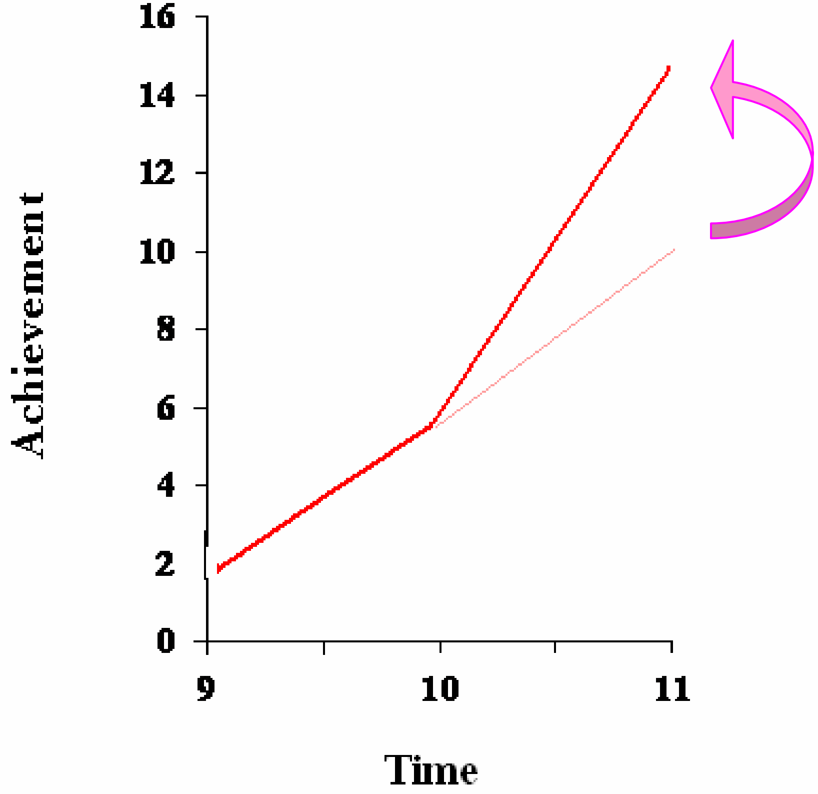
$$Y_{ij} = \pi_{0i} + \pi_{1i}Time_{ij} + \pi_{3i}TreatmentTime_{ij} + \varepsilon_{ij}$$

When Treatment = 0:

$$Y_{ij} = \pi$$

Treatment time expressed as 0's before treatment and time intervals post-treatment (i.e., 0, 0, 0, 1, 2, 3

Regression Discontinuity and Interrupted Time Series Designs



Treatment effect on slope: $(+\pi_{3i})$

Change in Intercept and Slope

$$Y_{ij} = \pi_{0i} + \pi_{1i}Time_{ij} + \pi_{2i}Treatment + \pi_{3i}TreatmentTime_{ij} + \varepsilon_{ij}$$

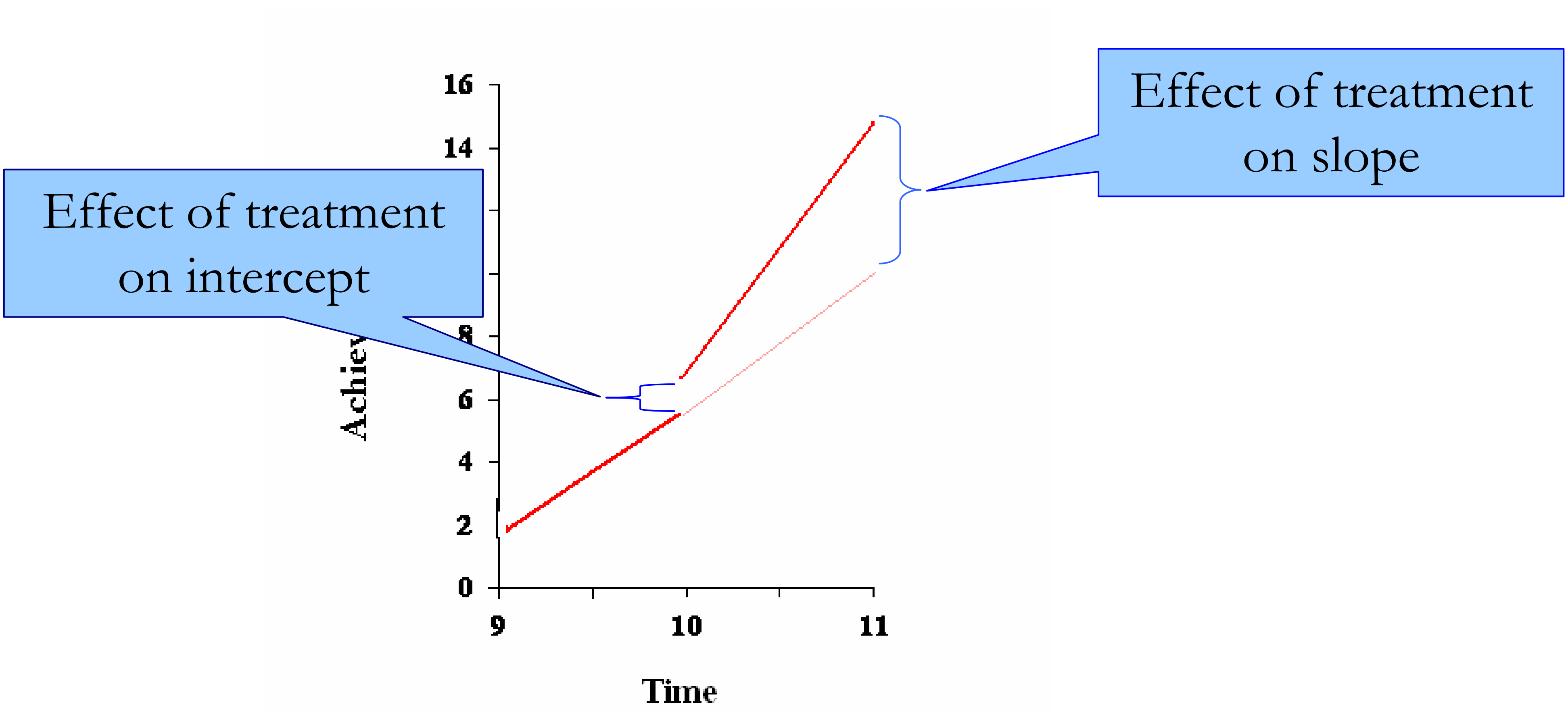
Effect of treatment on intercept

Effect of treatment on slope

When Treatment = 0:

$$Y_{ij} = \pi_{0i} + \pi_{1i}Time_{ij} + \varepsilon_{ij}$$

Regression Discontinuity and Interrupted Time Series Designs



Analyzing Randomized Trials (RCT) with HLM

- When random assignment is accomplished at the participant level, treatment group is dummy coded and included in the participant level data file
- When random assignment is accomplished at the cluster level, treatment group is dummy coded and included in the cluster level data file
  - Treatment can be used to predict intercepts or slopes as outcomes
  - Another strength of this approach is the ability to empirically model treatment variation across clusters (i.e., replication)

Power in HLM Models

Using the Optimal Design Software

- The Optimal Design Software can also be used to estimate power in a variety of situations
- The particular strength of this software is its application to multilevel situations involving cluster randomization or multisite designs
- Available at:

  http://sitemaker.umich.edu/group-based/optimal_design_software
- Optimal Design

Factors Affecting Power in CRCT

- Sample Size
  - Number of participants per cluster (N)
  - Number of clusters (J)
- Effect Size
- Alpha level
- Unexplained/residual variance
- Design Effects
  - Intraclass correlation (ICC)
  - Between vs. within cluster variance
  - Treatment variability across clusters
  - Repeated measures
  - Blocking and matching
- Statistical control

Effect of Unexplained Variance on Power

- Terminology: “error” versus unexplained or residual
- Residual variance reduces power
  - Anything that decreases residual variance, increases power (e.g., more homogeneous participants, additional explanatory variables, etc.)
- Unreliability of measurement contributes to residual variance
- Treatment infidelity contributes to residual variance
- Consider factors that may contribute to residual between cluster variance

Effect of Design Features on Statistical Power

- Multicollinearity (and restriction of range)

$$s_{b_{y1.2}} = \sqrt{\frac{s^2_{y12}}{\qquad}}$$

- Statistical model misspecification
  - Linearity, curvilinearity,…
  - Omission of relevant variables
  - Inclusion of irrelevant variables

The number of clusters has a stronger influence on power than the cluster size as ICC departs from 0

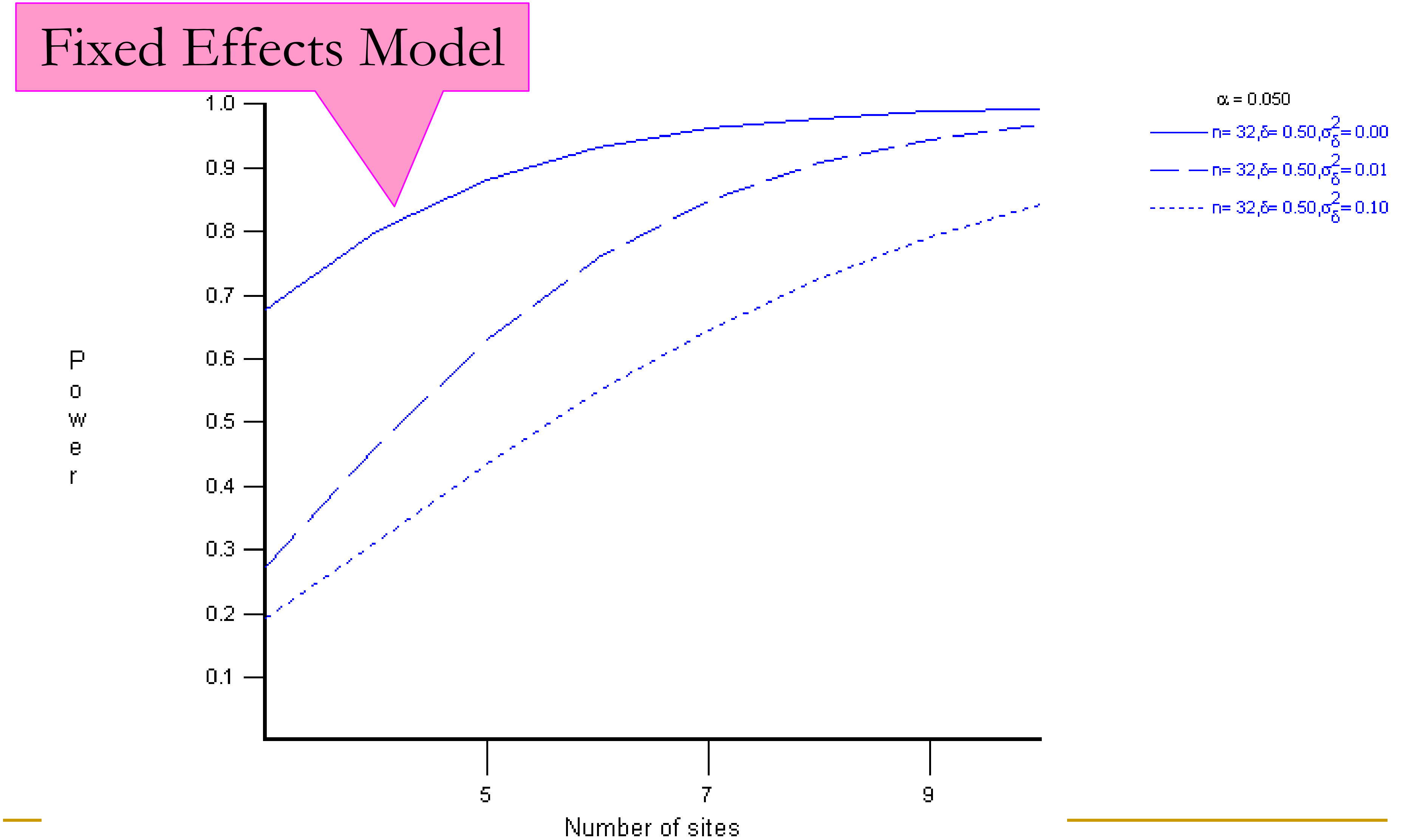
- The standard error of the main effect of treatment is:

$$SE(\hat{\gamma}_{01}) = \sqrt{\frac{4(\rho + (1-\rho)/n)}{J}}$$

- As ρ increases, the effect of n decreases
- If clusters are variable (ρ is large), more power is gained by increasing the number of clusters sampled than by increasing n

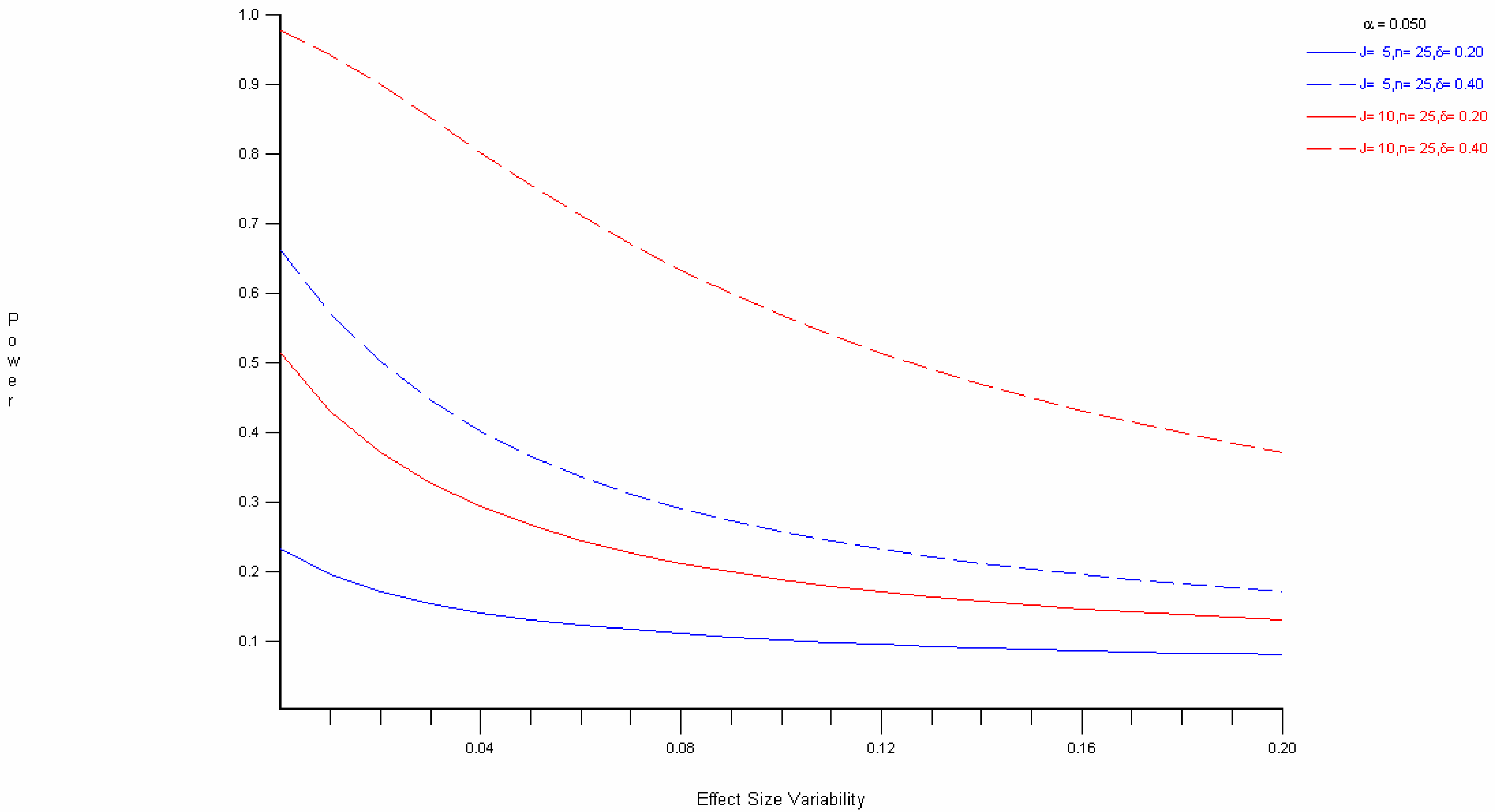
Power in Studies with a Small Number of Clusters

Fixed vs. Random Effects



Effect of Effect Size Variability (σ_δ^2)

Variance of the treatment effect across clusters



Randomization as a Control Tactic

- What does randomization accomplish?
  - Controls bias in assignment of treatment (works OK with small J)
  - Turns confounding factors into randomly related effects (equivalence vs. randomness; does not work well with small J)
- Applying an underpowered, small CRCT may not be sufficient to achieve rigor
  - Consider other design approaches (e.g., interrupted time series, regression discontinuity designs)
  - Aggressively apply other tactics for experimental or statistical control
- Not all designs are created equal
- No one design is best (e.g., randomized trials)

Improving Power Through Planned Design

- Evaluate the validity of inferences for the planned design
- Design to address most important potential study weaknesses
- Realistic appraisal of study purpose, context, and odds of success
- Importance of fostering better understanding of the factors influencing power
- Planning that tailors design to study context and setting
  - Strategies for cluster recruitment
  - Prevention of missing data
  - Planning for use of realistic designs and use of other strategies like blocking, matching, and use of covariates

Design For Statistical Power

- Stronger treatments!
- Treatment fidelity
- Blocking and matching
- Repeated measures
- Focused tests ($df = 1$)
- Intraclass correlation
- Statistical control, use of covariates
- Restriction of range (IV and DV)
- Measurement reliability and validity (IV and DV)

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