High achiever! Always a high achiever?

A comparison of student achievements on mathematical tests with different aims and goals

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Faculty of Health, Science and Technology

Mathematics

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To all my students whose strength I have not discovered, acknowledged or encouraged

Abstract

This study explored changes in relative achievement over time. It also investigated differences in how two groups of students activate mathematical competencies. The aim of the study was to investigate students' relative achievement in mathematics over time, and how mathematical competencies can be used to explore differences between groups of students on a non-curriculum based test in mathematics. The study was divided in two parts. Study 1 compared students' (n=568) relative achievement in two national tests in mathematics (years 3 and 6). Study 1 explored changes in relative achievement between the two national tests as well as differences in relative achievement between the national test in year 6 and the mathematical kangaroo in year 7 (age 13). The study identified, from a sample (n=264) of study 1, two groups of students with high achievements in only one of the tests, the national test in year 6 or the mathematical kangaroo. Study 2 explored how differences between those students relative achievement on the mathematical kangaroo could be explained through activation of mathematical competencies. The results in study 1 show that students undergo large changes, both increases and decreases, in relative achievement between the national tests in years 3 and 6. Study 2 shows how the two identified groups activate the mathematical competencies differently on the mathematical kangaroo. 9% of the students achieve highly in the mathematical kangaroo although they do not in the national test. The study implicates the importance of using non-curriculum bounded tests to identify strength in mathematical competencies among students that not are able to show them through the national test.

Keywords: Achievement, alternative assessment, curriculum based assessment mathematical competency, mathematical kangaroo, mathematics tests, national tests.

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1 Introduction

In Sweden, one way to test the level of mathematical competence students achieve through school mathematics is by national tests in mathematics. Working as a teacher for more than 15 years, I have observed that some of my students were very good in another mathematical test, called the mathematical kangaroo, but achieved very low scores on the national test in mathematics. My observation was confirmed in discussions with other teachers of mathematics on different levels in Swedish schools. Mattsson (2013) writes about Swedish teachers who have made the same observation; those teachers identified these students as gifted. Students who achieve low in school mathematics get low grades in mathematics and will most certainly not continue university studies in mathematics, science or technology. Although my belief as a teacher is that, to achieve highly on a mathematical test, no matter what the purpose of the test is, cannot be done without possessing some mathematical competencies.

So how can this happen? Was it just sporadic observations made by me and other teachers of mathematics?

In Sweden, it is a legal right for each school student to be supported to develop their knowledge as far as possible (SFS 2010:800). There is a risk that there is a group of students with high mathematical competencies that not are made visible through the traditional assessment system. For example, teachers have noted that there are students who achieve highly on non-curriculum bounded tests. Despite the fact that they do not succeed on curriculum bounded tests some teachers suspect that those students are gifted in mathematics (Mattsson, 2013). With suitable support, those students might be able to succeed in school. Without suitable support, they might drop out of school for example for reasons such as boredom (Stamm, 2008). There is a possibility that those students are gifted in mathematics and not supporting them to develop their competencies as far as possible is a waste to society. It is therefore important for the individual and for the society to find ways to identify those students and to give them suitable support. It is also important to help teachers with tools that might provide a way to find some of those students.

It has been difficult to find research that compares students' achievement on curriculum bounded versus non curriculum bounded tests such as competitions. I have not found any in languages available to the author

(Swedish, English and German). Research investigating how mathematical competencies differ on different sorts of curriculum bounded tests have been done in Sweden, for example by Boesen (2006), who compares tasks in teacher-made tests with tasks in national tests according to mathematical competencies.

1.1 Main concepts used in the thesis

1.1.1 Relative achievement

In this study test results in three different mathematical tests are used as empirical data. The three tests are of different characters and have different aims. Here to use test results to compare achievement in the tests, despite the differences in the tests, relative achievement instead of actual achievement is used. An example is given to illustrate what is meant by relative achievement.

Example:

On a test it is possible to get a maximum of 50 points. 10 students participate in the test and their points on the test are shown in Table 1. The student with the lowest points will be ranked as 1, and the student with the highest point will be ranked as 10. The ranking number indicate the student's achievement in relation to the other students. This is how relative achievement is used in this study.

Table 1

Showing how points (actual achievement) are connected to ranking (relative achievement).

Student	Points	Ranking/ Relative achievement
S1	38	6
S2	48	10
S3	31	3
S4	13	1
S5	25	2
S6	44	8.5
S7	43	7
S8	36	5
S9	34	4
S10	44	8.5

1.1.2 Mathematical competence and a mathematical competency

The definition of the words competence and competency described in the Danish KOM project (Niss & Höjgaard, 2011) will be used in this thesis:

> A person possessing competence within a field is someone able to master the essential aspects of that field effectively, incisively, and with an overview and certainty of judgement (Niss & Höjgaard, 2011, p. 49).

In mathematics this means that a person possessing mathematical competence has knowledge of, understands, can do and use, and has an opinion about mathematics (Niss & Höjgaard, 2011). A mathematically competent person can act in mathematical activities in different contexts where mathematics plays or can play an important role. It implies factual and procedural knowledge as well as concrete skills within the mathematical field (Niss & Höjgaard, 2011). On an individual level, I interpret this to mean that there may be people who have some well-developed mathematical competencies but who still do not possess mathematical competence, for example, if he or she is unable to use the competencies in different contexts, or he or she lacks in other competencies. The difference between mathematical competence and mathematical competency is described by Niss:

> What then is *a* mathematical competency? It is an independent, relatively distinct major constituent in mathematical competence as described above. One could also say that *a mathematical competency is a wellinformed readiness to act appropriately in situations involving a certain type of mathematical challenge* (Niss & Höjgaard, 2011, p. 49).

A mathematically competent person possesses mathematical competencies that are distinct but intertwined. In the Danish KOM project (Niss & Höjgaard, 2011) eight mathematical competencies are distinguished as the content of mathematical competence, Figure 1.

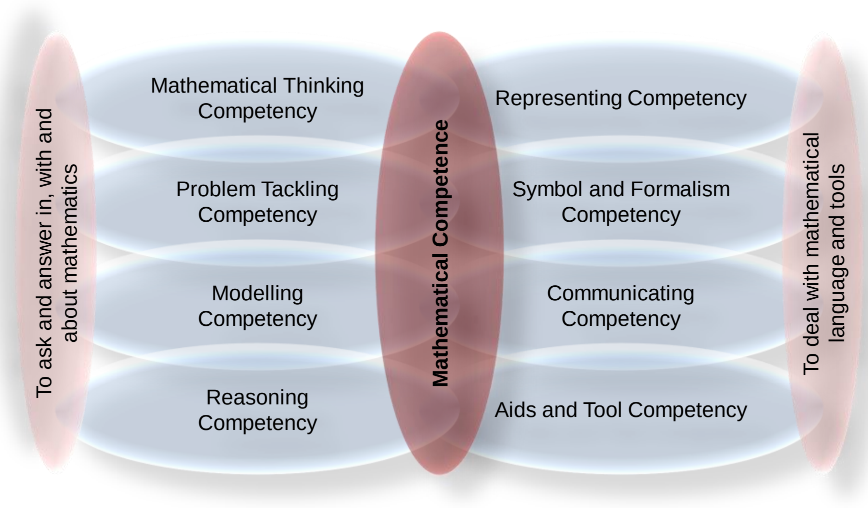


Figure 1. Mathematical Competence and Competency. Picture inspired by (Niss & Höjgaard, 2011, p. 51).

1.2 Aim

The overall aim was to investigate if there are students who possess good mathematical competencies although they fail in school mathematics. In more detail the aim is two folded.

One aim was to describe groups of students with similar movements in relative achievement by means of results on three different mathematical tests over a four-year period.

A further aim was to investigate a method used to explain differences in achievement on curriculum bounded and non-curriculum bounded tests.

1.3 Research questions

1. How does relative achievement in the national test change between year 3 and year 6?

2. How do students who are ranked highly through the mathematical kangaroo achieve in the national test (year 6)?

3. How do students who are ranked highly through the national test (year 6) achieve in the mathematical kangaroo?

Results from research questions 2 and 3 will be further investigated through research question 4.

4. How can differences in achievement on the mathematical kangaroo be explained by mathematical competencies?

1.4 Structure of the thesis

The thesis consists of eight chapters. The second chapter is a literature review that gives a background for the aims and research questions. The chapter has to do with assessment and achievement, with an extra discussion of high achievers. The tests included in the empirical data are also described.

In the third chapter, theories used in the process of working with the empirical material are discussed. The theory of mathematical competencies is given especially large room. Each mathematical competency that is used in the analysis is discussed in depth, both through the theoretical framework chosen for the study and also connected to other research involving mathematical competencies.

The fourth chapter describes the methods used in the thesis. The tests involved are described in more depth. The sample is compared with the population and the representativeness of the sample is described through statistical measurements and tests. Validity, reliability and ethical considerations are discussed in this chapter.

Analysis and results are described in chapter five. Chapter five starts with describing analysis and results concerning the descriptive part about movements in relative achievement. The last part of the chapter presents analysis and results of mathematical competencies in the tests and within groups of identified students.

Interpretations from both parts of the study are presented in chapter six, and a discussion of the interpretations connected to the aims of the thesis is included. Chapter seven gives a discussion of the contribution of knowledge together with suggestions for further research. The last chapter, chapter eight, is words ending the thesis.

2 Literature review

In this study curriculum bounded and non-curriculum bounded tests and students' results in the tests are used as empirical data. This chapter serves to give an overview of the literature and earlier research that have inspired and shaped this study. Several aspects must be discussed, such as curriculum, assessment and achievement, high achievement and the tests involved in the study.

2.1 Curriculum, mathematical competencies and national test

This section describes different levels of a curriculum and discusses how a standardised international test such as PISA, which uses concepts similar to mathematical competencies, influences national curricula like the one in Sweden. In the section the connection between mathematical competencies, the Swedish curriculum and the Swedish national tests are discussed and how the construction of different tests gives different opportunities for students is also mentioned.

A curriculum can be divided into three levels (Mesa, Gómez, & Cheah Hock, 2013), the intended, the implemented and the attained. As interpreted in the Swedish system:

- The intended is the curriculum that is the national curriculum,
- The implemented is the same as mentioned by Mesa et al. (2013) which is what happens in the classroom. What happens in the classroom is partly dependent on how the teachers interpret the curriculum,
- The attained is what the students have learned (Mesa et al., 2013).

How the attained curriculum is manifested is mainly measured through students' achievement on class assessment and external mandated tests, such as national tests or international standardised tests, for example PISA (Mesa et al., 2013, p. 866). The international test PISA has influence on national levels; the mathematical framework for PISA is based on capabilities (earlier named competencies) and not on content knowledge (OECD, 2013). For example the development of the Swedish curriculum (Skolverket, 2011a) is influenced by international measurements such as PISA (Skolverket, 2011b). In European countries, curricula in mathematics are today often related to mathematical competencies (Mesa et al., 2013), which in turn means that assessment must also relate to mathematical competencies.

From the year 2000 the Swedish curriculum has changed from earlier having a stronger focus on the content of mathematics to having a focus on what kinds of mathematical competencies are needed to work with the subject (Boesen, 2006). This shift also influenced the assessment system, and especially the national tests, since they are meant to guide Swedish teachers in assessments and grading (Skolverket, 2014). Because of the shift in the curriculum, the national tests now also aim to assess conceptual understanding instead of only factual knowledge.

The curriculum for compulsory school in Sweden today is goal oriented; students are graded in mathematics according to how well their mathematical abilities develop (Skolverket, 2011a). In education, the teacher is responsible for giving students opportunities to develop their abilities (Skolverket, 2011a) and also for judging how well those abilities are developed, finally giving a subject grade for each semester, starting in year 6 (age 12) (SFS 2010:800). It is common to use tests as a part of the judgement of students' grades and teachers both construct own tests and use national tests. These tests are not necessarily equal in terms of content or as to which competencies they require the student to succeed.

Boesen (2006) started filling the gap of research concerning the relation between national tests and teacher-made tests in the Swedish context. In one part of his research, he compared what kind of reasoning the students need to be able to solve the tasks in teacher-made tests and in national tests. He compares imitative reasoning versus mathematical creative reasoning. In short, imitative reasoning is a kind of reasoning the student has met before and has been trained in; the student does not need to invent anything new. Central in creative reasoning is "…the reasoning that goes beyond just following strict algorithmic paths or recalling ideas provided by others." (Boesen, 2006, p. 18). The national tests give tasks that cannot be solved by imitative reasoning and therefore give tasks that differ from textbook tasks (Boesen, 2006). These national tests, since they have a guiding position, should give students tasks that demand more than imitative reasoning. This was confirmed in Boesens (2006) study. The national tests give the possibility to use mathematically creative reasoning to a much higher degree than teacher-made tests, and, in the teacher-made tests, many of the tasks could be solved using only imitative reasoning (Boesen, 2006).

2.2 Assessment

This section indicates that assessment according to the Swedish curriculum is supposed to relate to mathematical abilities, similar to mathematical competencies. It also discusses how assessment is implemented and how it is used.

Assessment is supposed to be used *for* learning (A. Pettersson, 2004), meaning that assessment should be used in a formative way. There is an increased interest in external assessment (Mesa et al., 2013); external means that those tests are constructed outside the schools and can for example be national tests and/or international achievement tests like PISA or TIMSS. The externally constructed tests give each student a total mark that gives the students' achievement on that specific test. The Swedish national tests are given at the end of the school year; students either get a grade in that class (years 6 and 9) or a teacher opinion (year 3) of the students' knowledge (Skolverket, 2014). Since there is little time left of the school year it is difficult to work in a formative way with the students after the national tests. Achievement in those tests is therefore important, and it is partly used to decide students' subject grade.

When teachers are asked how they assess their students, they relate this to tests, portfolios etc., but if they are asked how they know that their students have learned something, they relate for example to classroom questions and group activities (Dorr-Bremme & Herman, 1986). One possible interpretation is that teachers do not completely think that assessments measure what students have learned. However, when using assessments, both formal and informal, the purpose of assessment is to determine the existing status of a student's knowledge (Wiliam, 2007). The results of the assessment can be used both summatively and formatively depending on the purpose of the assessment.

One aim of the Swedish national tests is to support the teacher in the assessment process (A. Pettersson, 2007). Sometimes both teachers and students focus on succeeding in the test instead of focusing on the learning (A. Pettersson, 2007), that is, to achieve highly on the tests. To get both teachers and students to focus on students' learning was one of the reasons for developing a new curriculum (Skolverket, 2011a) and new national tests. The national tests are supposed to be an assessment instrument *for* learning instead of *of* learning (A. Pettersson, 2007). Pettersson (2007) asks what it means to have knowledge in a subject; she divides the subject of knowledge into two parts – one personal and one official. Personal knowledge is how the individual looks at knowledge, for example in mathematics, while official knowledge is dictated by the curriculum. The national tests serve as the assessment of official knowledge, according to the intended curiculum. In the national test today, it is important to be able to apply knowledge to tasks that demand conceptual understanding, argumentation, communication and logical competency (A. Pettersson, 2007).

Assessment can be perceived positively or negatively to the student. Through the use of conventional tests in mathematics, some students achieve at the top of the class, getting good grades and teacher praise, while others achieve bottom results. Most students are aware of their place in this created hierarchy (Boaler, 2006). Assessment can be used to lift each student's positive sides and to help the student to develop those parts that can improve. For a teacher to observe all students' strength and weaknesses, it is important to observe and document students' knowledge in many ways (Jönsson & Svingby, 2008).

> Put simply, when there are many ways to be successful, many more students are successful. Students are aware of the different practices that are valued and they feel successful because they are able to excel at some of them (Boaler, 2006, p. 42).

Assessing each student individually is one of a teacher's difficult missions. In a mathematics classroom there are students without motivation, those with weak knowledge, those who are highly motivated and those with advanced knowledge (Boaler, 2006). Boaler (2006) showed that through a collaborative problem-solving approach, students achieved better in mathematics and also chose more advanced mathematics courses than students in comparison schools. When students asked for help, the teacher tried not to give the answers but tried to lead the groups into finding the solutions together. They also used more open-ended problems than usual (Boaler, 2006).

Using different practices, grouping, using open-ended problems can influence quality learning, and quality learning leads to higher achievement, also when externally mandated tests are used (Wiliam, 2007). Internationally, the use of summative tests constructed externally (i.e. by someone or some organisation outside the school) has increased. As a criticism, some say that that type of tests discriminates against certain groups of students owing to psychological issues. For example there are some qualities that can not be measured through tests, such as the metacognitive process (Gipps, 1999).

It is clear in the curriculum that it is the students' mathematical abilities that are to be assessed in mathematics (Skolverket, 2011a). For the curriculum between 1994 and 2011, Jönsson (2008) writes that many of the goals in the compulsory school are complex and difficult to assess and that there is a lack of models for how to assess those goals in the classroom. The present curriculum (Skolverket, 2011a) does not describe what is meant by all mathematical abilities. It is therefore plausible that teachers still think that the curriculum goals are complex and difficult to understand.

In school, students' performance is assessed, either comparing students with one another or according to goals in the curriculum. In the current Swedish curriculum for compulsory school (Skolverket, 2011a), students are assessed according to goals in the curriculum. What the purpose of assessment is does not matter; as long as students are being assessed, it is possible to study their relative achievement, rank their achievement and identify a top percentage population, for example the top 10%. The achievement can change as learning progresses; it may decrease as well as increase (Gagné, 2005). However, according to Gagné (2005), most talented (top 10%) students maintain their top-position through their formal schooling.

2.3 Achievement

Two studies that follow students' achievement in different ways in the Scandinavian context are in this section discussed. Both studies investigates movements in achievement over time and are therefore of interest for this study.

In a large longitudinal (7517 students in 29 municipalities, over three years) study (A. Pettersson, 1990), students' achievement behaviour and their achievement development were investigated. Achievement behaviour (A. Pettersson, 2007) meant ways of solving tasks. She investigated whether there were differences in achievement behaviour between students who developed differently in achievement. Achievement was measured in a test with 15 tasks in year 3, with the addition of four new tasks in year 6. The tasks were dichotomously scored (either credit or no credit) and the students were grouped into five groups according to their achievement development (A. Pettersson, 1990).

A. Students who achieved highly (13 points or more), both in year 3 and in year 6,
B. Students who achieved low (less than 4 points), both in year 3 and in year 6,
C. Students who achieved better in year 6 compared to in year 3 (at least 9 points more),
D. Students who achieved lower in year 6 compared to in year 3 (a decrease of at least 2 points),
E. Students who achieved at an intermediate level both in year 3 and in year 6 (8 points in year 3, 11 points in year 6).

131 students belonged to group A, approximately 2% of the sample, and are called "good at computation". Group B called "weak at computation", consisted of 222 students, which is approximately 3%. Group C is an improvement group, containing 226 students, approximately 3%. Group D is a decreasing group and contains 180 students, approximately 2.5%. Group E is a group containing students that have average results both in years 3 and 6; this group consists of 118 students, which is approximately 1.5%.

Essential aspects for students' achievement are teaching, learning and the students' individual prerequisites. The students in the study were followed up in year 9. Those who achieved highly in both years 3 and 6 also achieved highly in year 9, and those who achieved low in both year 3 and 6 also achieved low in year 9 (A. Pettersson & Boistrup, 2010). Those students who had poor results in both school years (3 and 6) had difficulty understanding explanations given by the teachers and wanted more help than was given. In the study of Pettersson & Boistrup (2010), the students who achieved highly in all grades are not discussed.

By following mathematical achievement in the number sense, calculation skills and text tasks among children from the age of 6 to the age of 15 Häggblom

(2000) show that there are movements in relative achievement. Low achievers at age 6 can become high achievers at age 15 and high achievers at age 6 can become low achievers at age 15. When looking at number sense, the results show that movements among the high achievers are more common than among low achievers. However, as a summary, less than 20% of the children belong to the same achievement group throughout their time in school. Häggblom (2000) therefore concludes that mathematical achievement at the age of 6 says very little about how a child will achieve at the end of compulsory school.

2.4 High achievement, measured relatively

Most humans have different competence in different subjects. Some have competence to become a well-paid soccer player, some have competence to become an opera singer and some have competence to become a well-known (maybe not well-paid) mathematician. It is natural to believe that this is also the case for students in school; they are all different from each other and have different competencies in different subjects.

The aim of this section is to describe how some other studies have used relative achievement in the perspective of studying high achieving students. Those studies have guided percent limits that are used in the analysis in the present study. Using relative achievement means that there will always be students at the top and at the bottom. This study especially uses the perspective of those who achieve highly one way or another.

What does it mean to achieve highly or low on a test in school mathematics, or any other subject? It is possible to define a high achiever in a test as someone who scores above a certain number of points, and define that specific number of points for each test. An alternative is to say that someone who scores among the top percent of the participating students is a high-achiever, and to define that percent limit. In this study, a high achiever is someone who scores among the top percent of the participating population. The opposite is a low achiever, that is, someone who achieves among the bottom percent of the participating population, according to this study.

High achieving students are seen by some as gifted, although it is important to distinguish between giftedness and high achievement. Gifted students are not necessarily high achievers; and, vice versa, there are many high achievers who are not necessarily gifted (Bar-On & Maree, 2009). This study is not about giftedness; it uses relative achievement in three different mathematical tests as empirical data to describe and identify groups of students. Of special interest in this study are those students who achieve highly in one test but not on the test that is used in the comparison. It is therefore interesting to explore how high achievers and high achievement are discussed in earlier research and literature.

In mathematics, how do schools judge who is competent in mathematics and easily reach the goals of the curriculum? A note should be included here that, in teacher education in Sweden, there is very little, if any, information about how to identify and support students with a capacity to develop further than is stated in the curriculum (Mattsson, 2013).

Despite that high achievement is not equivalent to giftedness, the concept of high achiever is sometimes connected to talent, for example in Gagnés Differentiated Model for Giftedness and Talent, DMGT (Gagné, 2005). Very briefly the DMGT is a model that combines giftedness with talent. A gifted or talented individual, according to this model, possesses and uses outstanding natural abilities, aptitudes (Gagné, 2005). A talented person masters systematically developed knowledge and skills to an outstanding degree in a field of human activity (Gagné, 2005), for example mathematics or soccer. This outstanding degree is defined in the DMGT as the top 10% among age peers that are or have been active in that field (Gagné, 2005). DMGT mentions that some of the top 10% students will develop a talent with help of different internal and external support (Gagné, 2004). It is how those top 10% are measured that connects talent with high achievement.

In school, achievement is measured in assessments of different tests and/or teacher observation; the summary of the assessments finally gives a subject grade. Vialle (2007) made a longitudinal study to investigate relationships among personal factors, social support, emotional well-being and academic achievement. To identify the relevant students, she used the model of Gagné and selected those who scored among the top 10% in two standardised tests used in Australia: ELLA, which tests language and literacy, and SNAP, which measures numeracy skills in problem solving, number, measurements, data and space (Vialle et al., 2007).

In Ireland, the Irish Centre for Talented Youth (CTYI) identifies students for participating in their program that aims to challenge and encourage talented youth. To participate, the student must show, either through testing or in some other way that he or she is among the top 5% of the school population (Mönks & Pflüger, 2005).

In terms of giftedness in mathematics, up to 20% of students are in need of special education because of that specific giftedness (Advisory Committee on Mathematics Education, 2012; E. Pettersson, 2011). "Well above average ability" is a term used by Renzulli (2005) to describe those who perform or possess the potential for performance that belongs to the top 15 to 20 % in any given area.

As soon as percentages are used to form a group in a population, individuals are compared relative to each other, in one way or another. In the field of giftedness relative comparison is often made by testing, mostly by combining tests for example cognitive- and domain specific tests (Nolte, 2012b; Pitta-Pantazi, Christou, Kontoyianni, & Kattou, 2011; Vanderbilt University, 2014; Vialle et al., 2007). Individuals with the best results are chosen for further development or investigation.

In research, relative achievement is sometimes used to choose participants. One example is a large longitudinal study in the USA, the "Study of Mathematically Precocious Youth (SMPY)" (Vanderbilt University, 2014). This study uses high achievement as a criterion in choosing participants. SMPY is an ongoing study; it is planned as a 50-year study and started in the early 1970s (Gross, 2009). The participants enter the project in their adolescence on the basis of their scores on the math or verbal scale of the Scholastic Aptitude Test (SAT) that place them among the top 1% of the population (Gross, 2009). This means that the participants were chosen through their relative achievement on tests. In this project, it has been found that the participants seem to engage and manipulate math or language in ways that are more characteristic of students that are many years older. They also take great individual responsibility for their academic success or difficulties. They blame themselves and not external factors for difficulties. They accept that they have a math or verbal talent and this together with motivation and endeavour, contributes to their academic success (Gross, 2009). In an academic perspective, the project is successful; in their early 30s 90% had Bachelor's degrees and 25% held doctoral degrees (Benbow, Lubinski, Shea, & Eftekhari-Sanjani, 2000). The highest achievers among the participants tend to maintain their pre-eminence in adult life.

> However, Talent Searches have also identified that achievement and success are by no means built in for gifted students. Where schools have not provided structured opportunities for talent development, these students perform, in school, and in later life, at levels significantly below their true capacity. Even remarkably high ability is not by itself sufficient; exceptional ability does not develop into exceptional achievement unless the educational system accepts its responsibility to actively facilitate this process (Gross, 2009 p. 347).

When high achievers are mentioned in research in the Swedish context, it is common to measure achievement in traditional tests in school and/or grades for example (Hallesson, 2011; Szabo, 2013). In Sweden, as of 2009, there has been an opportunity for students with a special interest in mathematics to enter so called "spetsprogram", a gifted program in English, in upper secondary school. To get a place in one of those programs, the student has to do a special test for that specific program; together with the test result and with their average grade from compulsory school, they can be placed in one of those programs (Mattsson, 2013). In addition, in the next educational step, i.e. university, being accepted to a university is to the greatest extent based on students' average grades from upper secondary school. In order to be accepted in the most popular university programs through grades, a student must have the highest grade in each subject from upper secondary school, and, to get highest grade in any subject the student must achieve highly on almost every assessment during the time in upper secondary school[1].

Mattsson (2013) found that students with well-educated parents are over-represented in the mathematical tracks of the gifted programs and that females are under-represented. This is supported in an international perspective by Sivelman & Miller (2009), who stress that, when gifted programs are reserved for high achievers, they serve a primarily higher socioeconomic group. However, being gifted does not have to do with social class (Silverman & Miller, 2009). Mathematics is an academic subject, and academic achievement is correlated to social economic status (SES), ethnicity and language status; the relation is neither perfect nor deterministic (McCoach et al., 2010). Factors such as school, teacher and parent have also been associated with student achievement (McCoach et al., 2010).

[1] The author has been working as an upper secondary teacher for 15 years, on four different schools in two cities.

As described in this section, relative achievement connected to high achievement is used in research. In the Swedish school context, to gain a place in the programs in upper secondary school, mentioned gifted programs by Mattsson (2013), students must be high achievers in compulsory school. For example, it is very likely that they achieved highly on the national tests.

2.5 The Swedish national test in mathematics

Results in the national test in year 3 (2009) and year 6 (2012) are used as empirical data. One main goal of the national tests is to support equality and fairness in assessments and grading (Skolverket, 2014). The tests provide a basis for analysis of the extent to which the demands on knowledge are fulfilled on different levels - the school-, organisation-, and national levels. The national tests are not examination tests; they are meant to be one of teachers' collective information about each student's knowledge. The tests are summative; they shall function as a checkpoint at the end of a school year or in a subject course. The test should show what qualities the student has in his or her knowledge of the subject (Skolverket, 2014).

For compulsory school, the "PRIM-gruppen[2]" (Stockholm University, 2013) has the responsibility for developing the national test. Each test consists of several parts and aims to give opportunities to show as many aspects of knowledge as possible (A. Pettersson & Boistrup, 2010). The tests offer a variety of contexts and a range of ways to respond. The tasks shall be constructed to give students the opportunity to demonstrate different areas of knowledge and different levels of quality in their knowledge (A. Pettersson & Boistrup, 2010).

In Sweden students take national tests in mathematics in years 3, 6 and 9 in compulsory school. Students' knowledge in mathematics is assessed according to mathematical abilities and the level of these that each student shows (Skolverket, 2011a). The national tests in mathematics in Sweden have an influence on the student's grade in a subject. Korp (2006, p. 79) writes that, in

[2] PRIM-gruppen is a research group at Stockholm University; their main focus is on assessment of knowledge and competence. They develop different instruments for assessment and evaluation, for example national tests for compulsory school.

upper secondary school, it is most common for a student to either get a higher grade in mathematics or the same grade as was achieved in the national test. There are exceptions, however, when students are given a lower grade than the national test shows. It is reasonable to assume that the pattern between subject grade and results on the national test is the same in compulsory school.

2.6 The mathematical kangaroo

The non-curriculum bounded test used to collect empirical data is the mathematical kangaroo. The mathematical kangaroo is an international competition in which more than 50 countries and more than 6 million children participate (Wettbewerbsbedingungen, 2013). Each country constructs its own test from a selection of problems that an international group of researchers and well acknowledged teachers together construct[3]. In Sweden (2013), the tests were given on five school year levels, year 0—2, year 3—4, year 5—7, year 8—10 and year 10—12. The mathematical kangaroo is not connected to any curriculum; the aim of the mathematical kangaroo is to stimulate interest in mathematics and to arouse curiosity and a desire to learn mathematics (Nationellt centrum för matematikutbildning, 2013). Another aim is to offer interesting challenges (Wettbewerbsbedingungen, 2013).

Using kangaroo problems in mathematics education can be a part of a successful learning path; kangaroo problems are mentioned in discussions of problem solving (E. Pettersson, 2011), and problem solving is also mentioned in the context of challenging students (Krutetskii, 1976; Nolte, 2012a). The mathematical kangaroo has inspired a test used to measure mathematical competencies in the process of identifying mathematically gifted students (Pitta-Pantazi et al., 2011). The mathematical kangaroo, in comparison with other mathematical competitions given in Sweden for students at age 13, does not need training in mathematics, which is important for the choice of test. It is not natural to think that a student that achieves low in school mathematics would voluntarily participate in a training program aimed for competition in mathematics. Students who compete in competitions in advanced mathematics

[3] The researcher has asked people involved in the process of constructing the Mathematical Kangaroo from Romania and Sweden.

most likely achieve highly in school mathematics[4]; those students are not the focus of this study.

In the Swedish context it is therefore interesting to explore whether students who achieve highly on the mathematical kangaroo are given the opportunity to activate mathematical competencies that differ from the mathematical competencies they are given the opportunity to activate in the national test in mathematics.

2.7 Different aims in the two tests

The tests included in this study have different aims and are not constructed to measure the same things. However, they are mathematics tests, and in this study they will be analysed according to mathematical competencies and the distribution of those in the tests.

A student who achieves highly on a national test can be seen to have both a broad and deep understanding of the mathematical abilities in the Swedish curriculum, since the national test is constructed to broadly test goals and criteria given by the curriculum (Stockholm University, 2013). The mathematical content in the tests, for example what kind of geometry, what kind of equations, is chosen according to what the curriculum tells about the mathematical content in that special age group. It is therefore possible to say that the national tests measure mathematical abilities through the mathematical content determined by the current curriculum.

It is very important to offer students mathematical challenges, partly because of the Swedish Education Act (SFS 2010:800), which states that:

- Education in school shall promote all children's and students' development and learning,
- Education shall take into account that children and students have different needs,

[4] No research about which students who participate in mathematics competition has been found in Sweden. By asking teachers who are known in Sweden to participate in those competitions, the claim that those students are also high achievers in school mathematics is supported.

- Children and students shall be supported and stimulated in a way such that they develop as far as possible,
- There should be an ambition to compensate for differences in children's and students' abilities for being benefited by the education.

Internationally, supporting students who need more challenge than average is discussed as an important issue. This is shown in the following:

> Moreover, we should not forget the legal or ethical perspective. As clearly mentioned in article 26 of the UNESCO definition of the rights of children, all children – including intellectually gifted ones – have a right to an education that will foster the development of their abilities and personality to their fullest (Stoeger, 2004, p. 169).

Mathematically challenging tasks also give support and stimulation to students who are gifted in mathematics (Nolte, 2012a).

This literature review ends by saying that the Swedish curriculum in mathematics is connected to mathematical competencies, and that the national tests have a guiding position for the curriculum and should assess students' mathematical abilities. Further, different mathematical tests are used to assess mathematical competence, and how these tests are constructed gives students different opportunities to show different mathematical abilities, for example imitative or creative reasoning. A teacher-made test and a national test are both connected to the curriculum. This study compares a curriculum bounded test and a non-curriculum bounded test using mathematical competencies in its analysis.

3 Theoretical framework

The purpose of this chapter is to describe how mathematical competencies are to be interpreted in this study. A further purpose is to describe how percent limits are chosen in grouping students according to relative achievement.

Words such as competency, capability, proficiency, processes and ability are used in different texts for similar situations. It is probably possible to write a separate thesis in linguistics about these words, how they are connected and how they have been used through history. The words are not congruent although they are indeed related. Translations between different languages from one original language to a second can also complicate the meaning of these words. They can be used as synonyms in some situations but not in others. I choose not to go deeper into a discussion of their meaning.

Mathematical competence is sometimes divided into parts called mathematical competencies. To be mathematical competent, one must master not only one competency but all (Krutetskii, 1976; Niss & Höjgaard, 2011), although a lack of one competency may be offset by another (Krutetskii, 1976). In the second part of this study, the parts, the mathematical competencies, are considered, that is, not mathematical competence.

3.1 Mathematical competencies as physical and mental activities

Competence descriptions in mathematics education are used to describe the "aim" for teaching in mathematics (Niss & Höjgaard, 2011).

> …*mathematical competence* comprises having knowledge of, understanding, doing, using and having an opinion about mathematics and mathematical activity in a variety of contexts where mathematics plays or can play a role (Niss & Höjgaard, 2011, p. 49).

Mathematical competence includes procedural and factual knowledge, but mathematical competencies have more to do with mental and physical activities in how to treat mathematical challenges. The competencies are behavioural in nature; the focus is on the ability (as in being able to) to carry out relevant activities (Niss & Höjgaard, 2011).

All competencies are dual in nature; they have an "investigative" and a "productive" side. Both sides are behavioural because they are about people's competence in being able to carry out activities, mental or physical (Niss & Höjgaard, 2011). Using a mathematical competency in a mathematical activity therefore demands both mental and physical activity, Figure 2.

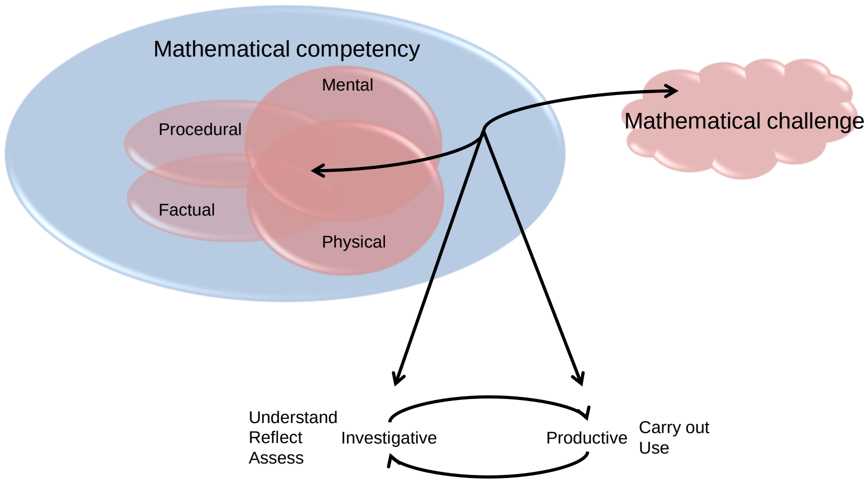


Figure 2. Components of a mathematical competency and the parts activated when working with a mathematical challenge.

3.2 Mathematical competencies

Mathematical competencies: A research framework, MCRF (Lithner et al., 2010) is used here as a framework in analysing tasks in tests. The framework has been developed by a group of researchers at Umeå University, Sweden, and constructed to analyse empirical data with a focus on students' opportunities to activate mathematical competencies (Lithner et al., 2010). It has been developed for a project called "National tests in mathematics as a catalyst for implementing educational reforms" (Lithner, 2011). Swedish national tests in mathematics are analysed with this framework in one part of the project. The framework is not used to analyse what students actually learn but to analyse opportunities to learn (Lithner et al., 2010). As the study involves an analysis of what is needed to come to a solution of a mathematical task, it is possible to use this framework.

It is not only in the framework chosen here that mathematical competence is divided into smaller parts that together build competence. The MCRF was developed in Sweden and has been used for analyses of mathematical competencies in the Swedish context (Boesen et al., 2014; Säfström, 2013) . To show that the framework is not restricted to the Swedish context this chapter will discuss two other frameworks that use parts to describe mathematical skills, knowledge or competence as a wholeness. The comparative frameworks are chosen because they have inspired or are related to the MCRF. Those are "Principal and standards for school mathematics" (NCTM, 2000) and the Danish KOM-project (Niss & Höjgaard, 2011). The frameworks chosen for comparison are well known and are used in research in Western culture.

Another framework (Krutetskii, 1976) is chosen for comparison because it deals with mathematical giftedness and is a well-known framework in this field of research. The frameworks will be used in the discussion of the study, and it is therefore important to discuss similarities and differences in the framework used in the analysis. This study deals with high achieving students in different ways. Teachers' suspect that students in one of the groups studied might be gifted (Mattsson, 2013). The framework of Krutetskii will not be used in the analysis, partly because giftedness is a complex phenomenon that, above domain specific competencies, also involves cognitive and affective factors, for example (Nolte, 2012a; Pitta-Pantazi et al., 2011), factors that are not involved in this study. It is important to note that there is not an equivalence between high achievement and giftedness (Bar-On & Maree, 2009).

3.2.1 Mathematical competencies – MCRF, KOM, NCTM

NCTM (2000) is intended to be a resource for all involved in mathematics education in the US. The standard defines 10 curriculum standards, 5 related to content goals and 5 related to processes. The 5 processes are: problem solving, reasoning and proof, communication, connections and representation. Content goals and processes are thoroughly described for education from prekindergarten to grade 12. Those processes together with content knowledge, build mathematical competence (NCTM, 2000).

The aim of the KOM project (Niss & Höjgaard, 2011) is to contribute to a coherence and progression of mathematics education in the Danish school system, lengthwise and crosswise. Eight overlapping mathematical competencies divided into two groups together build up mathematical competence in the KOM project, see Figure 1. The competencies are, in one group "To ask and answer in, with, about mathematics", reasoning-, modelling-, problem tackling and mathematical thinking competency. The other group consists of, "To deal with mathematical language and tools", representing-, symbol and formalism-, communicating- and aids and tools competency.

The MCRF is mainly inspired by the NCTM and the KOM-project, although it was developed for research and not for education. The main difference is that the competencies used are made more distinct and are differentiated from each other. The framework defines 6 mathematical competencies:

- Applying procedures,
- Reasoning,
- Communication,
- Representation,
- Connection,
- Problem solving.

The competencies will each be discussed in section 3.3.

3.2.2 Mathematical competencies – Krutetskii and MCRF

Krutetskii (1976) uses the word ability which is used here when linking to his work. Mathematical ability and the possibility to make progress in mathematical activities are seen as a complex set of mathematical abilities (Krutetskii, 1976). The combination of mathematical abilities in a mathematical activity is a condition for high achievement. However, weakness in one ability can be compensated for by another ability so that successful or high achievement is still possible (Krutetskii, 1976).

Krutetskii (1976, p. 87-88) lists nine component mathematical abilities.

> 1. An ability to formalize mathematical material, to isolate form from content, to abstract oneself from concrete numerical relationships and spatial forms, and to operate with formal structure – with structures of relationships and connections.
>
> 2. An ability to generalize mathematical material, to detect what is of chief importance, abstracting oneself from the irrelevant, and to see what is common in what is externally different.
>
> 3. An ability to operate with numerals and other symbols.
>
> 4. An ability for "sequential, properly segmented logical reasoning" (Kolmogrov, 180, p. 10), which is related to the need for proof, substantiation, and deductions.
>
> …
>
> 5. An ability to shorten the reasoning process, to think in curtailed structures.
>
> 6. An ability to reverse a mental process (to transfer from a direct to a reverse train of thought).
>
> 7. Flexibility of thought – an ability to switch from one mental operation to another; freedom from the binding influence of the commonplace and the hackneyed. This characteristic of thinking is important for the creative work of a mathematician.
>
> 8. A mathematical memory. It can be assumed that its characteristics also arise from the specific features of the mathematical sciences, that this is a memory for generalizations, formalized structures, and logical schemes.
>
> 9. An ability for spatial concepts, which is directly related to the presence of a branch of mathematics such as geometry (especially the geometry of space).

Krutetskii (1976) addresses the abilities as mental activities. However, I believe that when someone operates with something it is also a physical act, meaning that, to be able to observe those abilities, the student needs to do a physical act – verbally through thinking aloud and sometimes also in combination with a writing process. Figure 3 shows a comparison of the abilities defined by Krutetskii (1976) with the competencies defined in MCRF (Lithner et al., 2010).

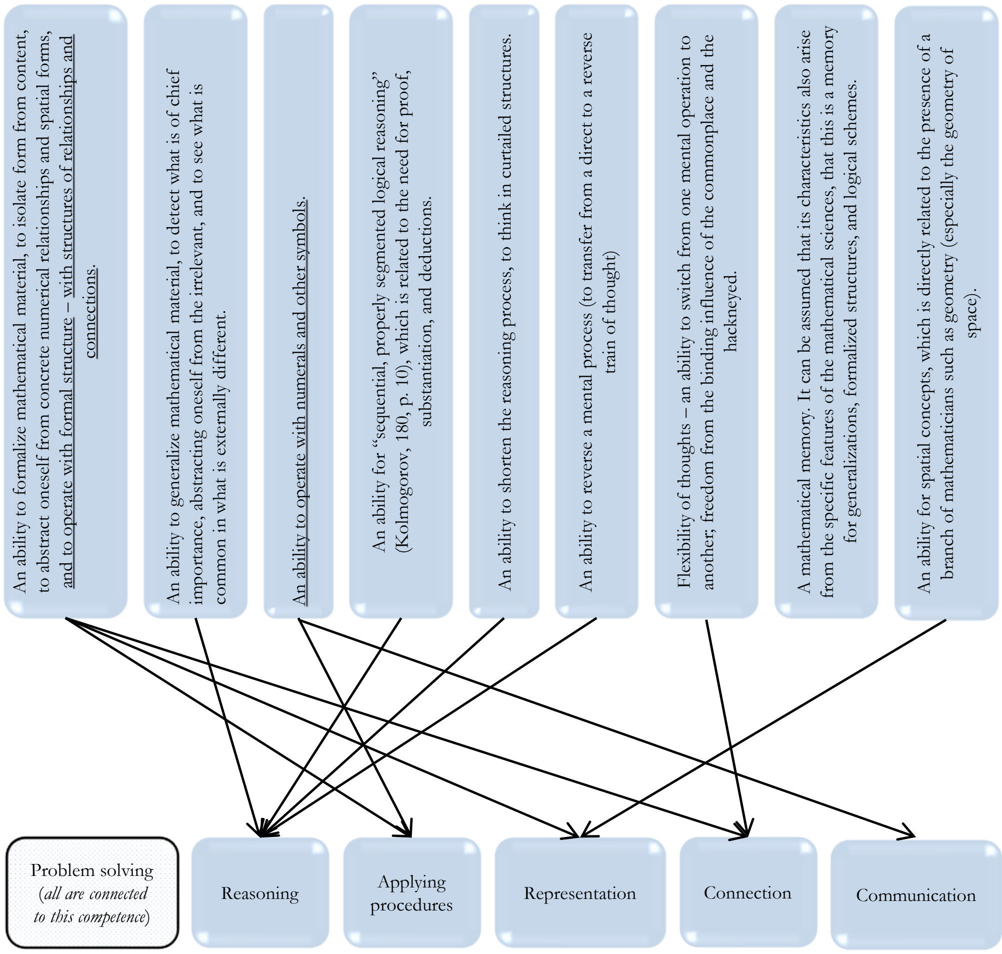


Figure 3. The abilities defined by Krutetskii connected to the competencies in MCRF. The underlined parts are those connected to physical acts. The text of the abilities is taken from Krutetskii (1976, p. 87-88).

Figure 3 shows that there are some abilities described by Krutetskii that partly or fully can be connected to physical activity. The method used in Krutetskii's work, when letting the students think aloud during the problem solving process shows that students' communication competence is also of importance. The comparison of MCRF (Lithner et al., 2010) with Krutetskii (1976) shows that both frameworks divide the mathematical competence or ability into parts. Some of these parts are connected, for example the reasoning competency in the MCRF can be interpreted in four of the abilities defined by Krutetskii. As a comparison, the communication competency in the MCRF is only connected to one of the abilities in Krutetskii's work. The problem solving competency is special, since Krutetskii uses problem solving as an activity and analyses all other abilities in that activity. The problem solving competency is therefore marked in another way. The reasoning competency and the problem solving competency in the MCRF are those that recur most in Krutetskii's work.

The four frameworks discussed in this chapter are compared with the MCRF as a base in Table 2. The comparison shows that all competencies defined in the MCRF (Lithner et al., 2010) can also be found in the framework of Krutetskii, although not evenly distributed as Figure 3 shows. When the NCTM (NCTM, 2000) is combined with the Danish KOM project (Niss & Höjgaard, 2011), they together cover all competencies defined in the MCRF.

Table 2

Comparison of the frameworks on the basis of the competencies defined in the MCRF.

	Problem Solving	Reasoning	Procedure	Representation	Connection	Communi-cation
MCRF	Solution method is not known in advanced.	Justify choices and conclusions.	Accepted (math) actions used to solve the task.	Concrete replacements of abstract mathematical entities.	Connecting or linking between for example relationships and/or representa-tions.	Exchange of mathematical information between a sender and a receiver.
Krutet-skii	An activity	Generalise, to detect what is of chief importance. Logical reasoning. Shorten reasoning. Reverse a mental process.	Operate with formal structures. Operate with numerals.	To abstract from the concrete. Spatial concepts.	Operate with structures of connections. Flexibility in thoughts	Operate with mathematical symbols. Clear, short, rational solutions
NCTM	An activity, the solver is unaware of the solution method.	Proof is the ultimate form. Use mathematical conjectures in all areas.		External observable and internal in people's mind.	Make connections between different topics.	Communicate and understand mathematical knowledge.
KOM	Detect, formulate, delineate and specify different kinds of math problems.	Follow and assess mathematical reasoning. Devise informal and formal mathematical reasoning.	Carry out informal and formal mathematics.	Understand and use different kinds of mathematical representations.		Symbol: to decode symbol and formal language. Communica-tion: to study and interpret others and to express oneself.

3.3 Conceptualising the mathematical competencies

In this study mathematical competencies are used to investigate a method to explain differences in achievement on mathematical tests. The following section aims to explain how the competencies are interpreted in the analysis of the tests in this study. Each competency is presented and discussed – this starts with presenting how the competencies are described in the MCRF. The reason for including this comprehensive section is to make it possible to fully understand how each competency is interpreted in the analysis.

When the word ability is used, it is in the sense of being able to, the same way as it is used in the Danish KOM project (Niss & Höjgaard, 2011).

> In the characterisations of the individual competencies below, the word "ability" is sometimes used. It must be pointed out that this is merely a linguistic substantivation of "being able to", and by no means a psychological term aimed at referring to a person's mental personality traits general mental faculties (Niss & Höjgaard, 2011, p. 50).

In the theory of mathematical competencies used, extra attention is given to whether it is necessary to activate a competency or not. In the MCRF, it says that "…a competency is the ability to handle *something*…"(Lithner et al., 2010, p. 161). This something is defined for each competency in the MCRF. The purpose is to analyse the opportunities a task gives to activate a competency; to handle *something* is defined in this study as an action (mental or physical) that is necessary and cannot be avoided if the task is going to be solved.

3.3.1 Applying procedures competency

In the MCRF (Lithner et al., 2010), applying procedures competency is defined as "a sequence of mathematical actions that is an accepted way of solving a task." In the analysis conceptual and procedures knowledge are both interpreted as parts of the applying procedures competency.

Procedures are seldom studied in research in mathematics education (Säfström, 2013). In the Swedish curriculum one aim in mathematics education is to give students opportunities to develop the ability to use and analyse mathematical concepts (Skolverket, 2011a). Another ability the students should be given an opportunity to develop is the ability to choose and use mathematical methods in order to calculate and to solve routine tasks (Skolverket, 2011a). I interpret those two abilities as conceptual and procedural competencies. Procedural and conceptual competencies can be seen as intertwined where deep understanding is thought to be reached through connecting those two competencies (Baroody, Feil, & Johnson, 2007). The two types are developed together, each type interacts with and influences and is influenced by the other (Voutsina, 2012), Figure 4.

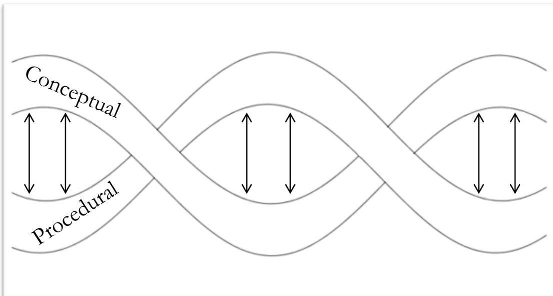


Figure 4. The iterative process between conceptual and procedural competency.

Voutsina (2012) means that the application of procedures strengthens the conceptual knowledge and that it is important to justify and explain procedures both to get more effective procedures and for the conceptual understanding.

3.3.2 Reasoning competency

Reasoning is "the explicit act of justifying choices and conclusions by mathematical arguments." (Lithner et al., 2010, p. 161). Reasoning can be connected to mathematical proof (NCTM, 2000; Niss & Höjgaard, 2011). Reasoning competency can be seen as the ability to follow and understand mathematical proof and the difference between proofs and intuition and/or special cases. The competency is also about the ability to form valid proofs when people "devise and carry out informal and formal arguments (on the basis of intuition)" (Niss & Höjgaard, 2011, p. 60).

Lithner defines reasoning as "the line of thought adopted to produce assertions and reach conclusion in task solving" (Lithner, 2008, p. 257). The ultimate form of mathematical reasoning is mathematical proofs (NCTM, 2000). To reason mathematically is essential to understand mathematics and it includes developing ideas, exploring phenomena and using mathematical conjectures in all areas. To reason mathematically "is a habit of mind, and like all habits, it must be developed through consistent use in many contexts." (NCTM, 2000, p. 56). According to Lithner (2008) the line of thought does not need to be based on formal logic and does not need to be restricted to proofs; it can even include incorrect arguments, as long as the one who is doing the reasoning has reasons to back it up.

Mathematical arguments are seen to be mathematical if they motivate why conclusions are true or plausible based on mathematics, or if they are anchored in mathematical properties (Lithner, 2008).

Reasoning is tightly connected to problem solving and modelling; Lithner (2010) calls it the "juridical counterpart". In the situation of working with tasks, I interpret this as when activating the reasoning competency, it validates the chosen strategy for the solver. Since a mathematical problem in this framework is seen as a problem with an unknown mathematical solution, it is natural to believe that reasoning is a part of the solution strategy. It is likely that reasoning is an implication of problem solving, but it is not necessarily that problem solving is an implication of reasoning.

3.3.3 Communication competency

To communicate is "to engage in a process where information is exchanged between individuals through a common system of symbols, signs, or behaviour" (Lithner et al., 2010, p. 165). The definition involves knowledge of the mathematical language (symbols, signs and behaviour) and the ability to use the language. The definition demands that individuals are involved, but it is not necessarily a direct communication, the communication can for example be from an author - through a book - to the reader. Another example is, from a student - through a solution or an answer - to the teacher.

What does it mean to communicate in mathematics? It could mean that you use a language, I would like to call it *Mathematish*, which means that you use words and connect them grammatically correctly. Mathematicians, mathematics educators, teachers in mathematics, all need to have something in common in their view of mathematics; otherwise it would not be possible to talk about mathematics together (Maier & Schweiger, 1999). Also, students being taught mathematics need to be given an adequate picture of mathematics to enable a discussion of the importance of mathematics for culture and society (Maier & Schweiger, 1999). Mathematical texts are special; they endeavour unambiguously; all objects, actions and relations should be clarified without any doubt for misunderstanding (Maier & Schweiger, 1999). The mathematical language also involves symbols, for example:

$$\int_a^b f(x)dx \qquad \lim_{n\to\infty}\left(1+\frac{1}{n}\right)^n \qquad \sum_0^i g(x)$$

Those symbols are impossible to understand without being taught in some way. The special terminology is used in the mathematical language, which means three things that are clearly distinguished from each other (Maier & Schweiger, 1999):

- Words that do not occur in everyday language, such as Prime numbers, Divisor, Logarithms, Orthogonal …
- Words used in everyday language that have the same or a similar meaning in measurements, such as Even, Triangle, Quadrate …
- Words used in everyday language in the first glimpse with a different meaning; at a closer analysis they often have a common origin. An example is the word Product, meaning in math the result of a multiplication. The meaning in everyday life is the result of a production process.

The goal of education in mathematics is to reach an understanding of mathematics. Communication can either be seen as part of the treatment to reach the goal – understanding mathematics or learning to communicate in mathematics can be seen as the goal where the instructional intervention is the treatment (Lampert & Cobb, 2003). Those two views of communication cannot be separated. To participate in the activities of mathematics a student both needs to be able to understand, for example, the mathematical symbols, which he/she ought to have learned - and learn to communicate as a goal. The mathematical language is also fundamental to come further and to engage in certain areas of mathematics, for example calculus, and is used as part of the treatment to gain deeper understanding.

Communication competency requires both a sender and a receiver therefore "communication is about reaching shared meaning" (Säfström, 2013). In education the sender is usually a teacher, student or a textbook and the receiver is often a student or a teacher (Lithner et al., 2010). Communication also requires a medium within which the sender and the receiver can understand the communicated information (Lithner et al., 2010). The medium is often physical, such as writing and, gesturing or auditory, such as listening and talking.

In the Swedish curriculum (Skolverket, 2011a) both the learning of communication and the use of mathematical communication are goals that the student should be given possibilities to develop. In the Danish KOM-project (Niss & Höjgaard, 2011) and in NCTM (2000) communication competency is the ability to express one's own mathematical knowledge and to understand others' mathematical communication.

3.3.4 Representation competency

In the MCRF (Lithner et al., 2010, p. 163) representation competency is "the concrete replacements (substitutes), mental or real, of abstract mathematical entities".

Dörfler (2006) puts the mathematical signs at the centre of mathematical activities and includes both writing and reading in this activity. The need to use external representations for mathematical objects derives from the abstraction in them. "They are not accessible to the senses, not palpable, not perceivable, they cannot be shown directly…" (Dörfler, 2006, p. 98). The representations are either used to learn or to investigate the abstract object; they serve predominantly a mediating role between the learner (or researcher) and the abstract object. Dörfler means that learning through representations could help students who believe they are incapable of learning abstract objects.

> The basic idea is that the learner, by the use of external representations, constructs or develops in his/her mind a mental representation (cognitive structure, schema, or the like) which then permits him/her to think with and about the respective mathematical concept (object) (Dörfler, 2006, p. 100).

According to Dörfler (2006) representations are important for the process of learning mathematics; therefore the ability to interpret, use and judge mathematical representations are important in school mathematics. Representations can be external, such as symbols, graphs, diagrams, tables and concrete material, or internal, such as mental pictures (Dörfler, 2006; NCTM, 2000; Niss & Höjgaard, 2011). The representation competency includes the use of mathematical forms in expressions (Skolverket, 2011a). It also includes the ability to understand, use and compare different mathematical representations, to choose the best suited representation for specific situations (Niss & Höjgaard, 2011).

3.3.5 Connection competency

Connection competency is "the process *to use something that connects or makes a link between two things*, e.g. a relationship in fact or a causal or logical relation or sequence" (Lithner et al., 2010, p. 163). Connection ability is about being able to find meaningful relationships/connections between mathematical entities and their representations, Figure 5.

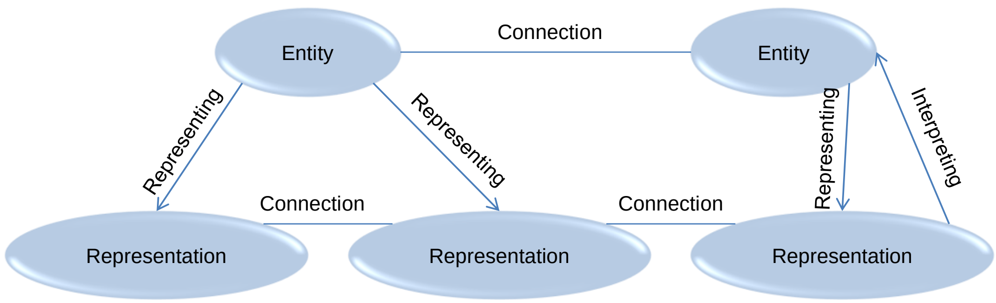


Figure 5. Relation between representations and connections. The picture is an adaptation from Lithner et al. (2010).

One way to speak about connections is with the word flexibility. Flexibility can refer to the ability to adopt a known procedure to meet new demands (Baroody et al., 2007) or perhaps to transfer knowledge from one mathematical entity to another. To do this, it is necessary to make connections between the entities.

According to Kilpatrick et al (2001), flexibility is a major cognitive requirement for solving non-routine problems. Krutetskii (1976) also mentioned flexibility, as I interpret it in the meaning of the ability to change solution strategies for the same problem.

The connection competency can also be seen as the ability to go back and forth between different mathematical representations such as: symbols, formal language and natural language (Niss & Höjgaard, 2011). The competency can also include the ability to make connections between different contexts to get deeper and long lasting knowledge (NCTM, 2000).

3.3.6 Problem solving competency

Lithner et al. (2010, p. 161) defines the problem solving competency as "engaging in a task for which the solution method is not known in advance".

What is a mathematical problem? Today, at least in Sweden a common way to define a mathematical problem is a mathematical task where the solution method is not known in advance by the solver (Skolverket, 2011b). This means that a mathematical problem is not a mathematical problem for every student, since all students do not have the same mathematical knowledge (Skolverket, 2011b). A similar definition is given in the NCTM (2000, p. 52) for the activity of problem solving: "engaging in a task for which the solution method is not known in advance."

What defines a mathematical problem solving competency? In the Swedish curriculum (Skolverket, 2011a) it involves both mental and physical activities, it involves the ability to formulate and solve the problem and to assess the selected strategies and methods. This is also supported by the educational frameworks of the Danish KOM project (Niss & Höjgaard, 2011) and the NCTM (2000): problem solving competency is about the ability to interpret, solve and assess mathematical problems. Niss (2011, p. 55) also adds "if necessary or desirable in different ways".

In some educational frameworks such as the Danish KOM project (Niss & Höjgaard, 2011), modelling competency is set as a separate competency. Modelling competency is closely related to problem solving competency, which can be seen when comparing the definition of a mathematical model with the definition of a mathematical problem.

Mathematical problem: A mathematical problem is a problem for which the solution method is not known in advanced by the solver.

Mathematical model: A mathematical model is a connection between the real world and the mathematical world. Using a mathematical model either means to de-mathematise mathematical models and interpret them into the real world, or to mathematise real-life situations (beyond mathematics) and make mathematical models that explain them (Niss & Höjgaard, 2011). These situations are most often situations for students where the solution methods are not known in advance, which makes mathematical models mathematical problems. In NCTM (2000) it is written about contextualising mathematics, which can be interpreted as using mathematics in mathematical models.

Problem solving can be used as a general activity in which the object of study is engaged. This was how Krutetskii (1976) used problem solving. He assumed that a student has to be in a situation containing mathematical problem solving to be able to observe the student's mathematical abilities. He also meant that the abilities are primarily individual psychological characteristics of mental activity (Krutetskii, 1976).

3.4 Competency Related Activities, CRA

Each competency has an investigative and a productive side (Lithner et al., 2010; Niss & Höjgaard, 2011). This duality can be divided into three competency related activities (CRA:s) (Lithner et al., 2010) that can be used to describe the aspects of mastering each competency (Säfström, 2013). These aspects are: Interpret, Do and use, and Judge (Lithner et al., 2010). Some parts of the MCRF are questioned by Säfström (2013), for example the competency related activity - interpret. Säfström feels that this activity can not be separated from the other two, *do and use* and *judge* (Säfström, 2013). She believes that the definition of *interpret*, that is taking in information, relates to building knowledge, understanding, identification and recognition. The CRAs, *do and use* and *judge* rely on interpretations and cannot be implemented without them. Also the other way around, through the activities *do and use*, and *judge*, interpretations are manifested. Säfström therefore decided to remove the first CRA *interpret* and to use *do and use* as the productive aspect, and *judge* as the analytical aspect and merge the *interpret* activity into both.

I chose to exclude all the three CRAs. No students' solutions are available in this study, and no other interaction with students has been attempted. The lack of information about how students actually act with the tasks in the empirical data makes it difficult to separate the CRAs from each other.

3.5 Situations for studying mathematical competencies

Mathematical competencies can be studied through observing and/or interacting with students. Opportunities for activating the competencies can be studied for example through investigating material (textbook tasks, tests et.) that students meet. Examples of both are given in this section.

To be able to study the actual competencies a student possesses, Krutetskii (1976) felt that the student must be in a mathematical activity. Some qualitative studies involving mathematical competencies in Sweden, for example (E. Pettersson, 2011; Szabo, 2013), have chosen problem solving as a mathematical activity, as Krutetskii did.

Studying whether tasks give students the opportunity for imitative and/or creative reasoning has been done in Sweden both through textbooks and task analysis and through more students' interactive studies (Boesen, Lithner, & Palm, 2010). In one part of the project "National tests in mathematics as a catalyst for implementing educational reforms" (Lithner, 2011), tasks in national tests were analysed with the MCRF (Lithner et al., 2010) to investigate which mathematical competencies the tasks gave the students the opportunity to use.

3.6 Master a competency

Analysis of abilities is about the qualities or traits of the person who is performing the activity; in an analysis of skills or habits it has to do with analysing the features of the activity a person is carrying out (Krutetskii, 1976). Ability has to do with the psychological traits of a person, skills or habits are about something that proceeds from the concept of an operation. Krutetskii stresses that, regardless of whether the analysis deals with skills and habits or abilities, it is about analysing an activity. "Therefore the investigation of a pupil´s mathematical ability is also an *investigation of his mathematical activity*, but from a certain standpoint" (Krutetskii, 1976p. 72).

The competencies can be measured in three dimensions: *degree of coverage*, *radius of action* and *technical level* (Niss & Höjgaard, 2011). By observing those three dimensions, the progression of the competencies can be measured and a person's mastery of a mathematical competency can be assessed.

The Swedish grading system in each subject ranges from F to A; F means fail, E is the passing level and A is the highest grade. Each mathematical competency has knowledge requirements for the grades of E, C and A. To get the subject grade E, C or A, all knowledge requirements for <u>each</u> competency must be fulfilled for the knowledge requirement. To get the subject grade of D or B, all knowledge requirements for the lower levels (E and C) must be fulfilled as well as the majority of the knowledge requirements for the higher levels (C and A) (Skolverket, 2011a). Connected to Niss (2011), it means that a student must master each competency to a certain extent in a technical level, in the radius of action and in the degree of coverage. Tests are a necessary tool for a teacher to be able to assess each student in a classroom with many other students. Results of tests are part of the assessment; a student who succeeds and achieves well in tests will have better possibilities to get a high subject grade than a student who achieves low on tests. In the spring semester in years 3, 6 and 9 of compulsory school, the students also take a national test in mathematics. The national test has a large influence on the subject grade, although it is only supposed to be one test among others.

3.7 Achievement

3.7.1 The national test in year 3 vs the national test in year 6

In this study students are categorised according to relative achievement. In the first part of the study, comparing results in national tests from the same individuals in year 3 and in year 6, achievement categories are inspired by the work of Pettersson (1990) and of Gagné's model, DMGT (Gagné, 2004), although changes are made to make categorise more suitable for the purpose of this study.

Pettersson (1990) defines an increase in points that are more than four times larger than the decrease (an increase of 9 points compared to a decrease of 2 points). However she works with actual scores and I work with relative achievement.

I define a large increase or a decrease in achievement with the same size of measure; here a large movement is defined in the following way. A large increase or decrease has occurred if the ranking position changes up or down more than 40% of the total number of ranking positions. There are 568 ranking positions; 568 students took the national test both in year 3 and in year 6; 40% of 568 is 227.2. A movement of more than 227.2 ranking positions up or down is defined as a large movement. Only students that in year 3 ranked at the bottom 60% or the top 60% have the possibility to make an increase or decrease defined in this way.

Due to the statistical phenomenon "regression towards the mean", measuring changes in achievement will result in an increase of achievement for a group of students with bottom results from the beginning. This means that, if the bottom group had a lower average result than the average of the population on the first test (year 3), the average of the group on the second test (year 6) will approach the average of the population on the second test (year 6), see Test 1a and Test 2a in Figure 6. An increase of the average for the group has occurred. The opposite will happen for the group that were top achievers on the first test.

If an increase or a decrease is to be studied that cannot be explained by "the regression towards the mean", the increase or decrease has to be chosen such that the identified group breaks the pattern of regression towards the mean, resulting in that the group who were bottom achievers in the first test (year 3), achieved in the second test (year 6) so that their average will be higher than the average of the population, see Test 1b and Test 2b in Figure 6. The definitions of a large increase and a large decrease are chosen because they break the pattern of regression towards the mean.

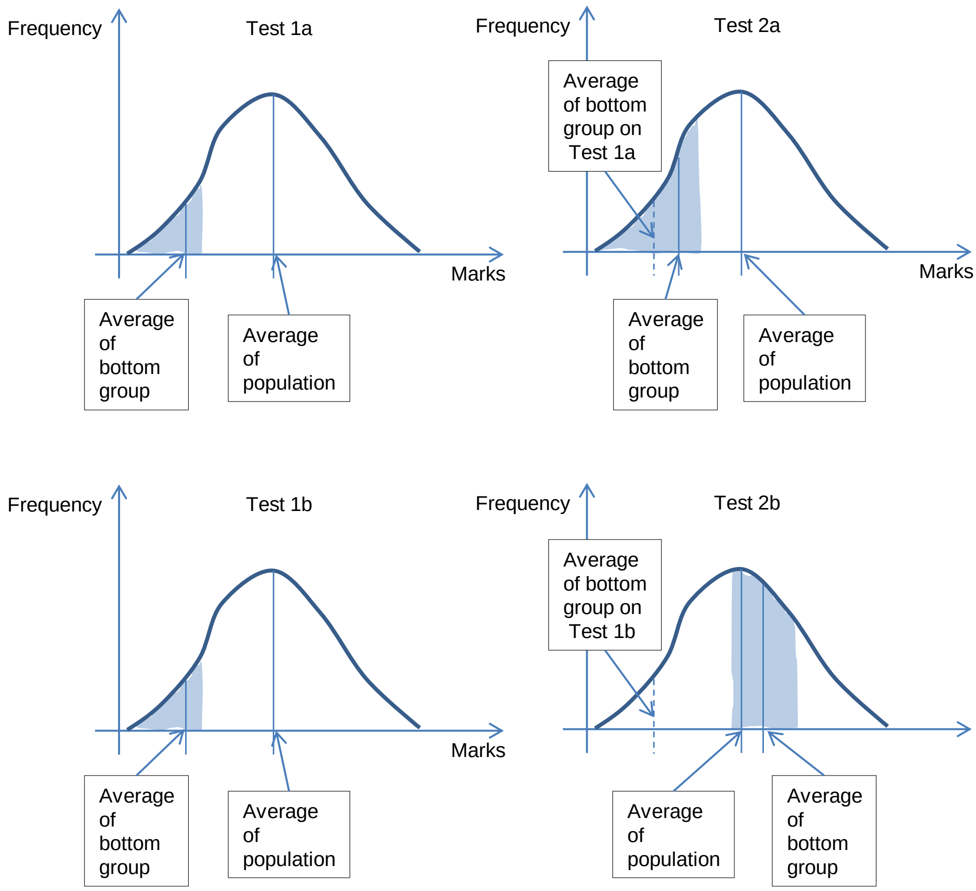


Figure 6. Test 1a and 2a show a movement of achievement that follows the pattern of regression towards the mean. Test 1b and 2b show a movement of achievement that breaks the pattern.

The second difference is that Pettersson (1990) has a category for those who were intermediate achievers. This category is removed in this study as there is no focus on intermediate students who do not change their relative achievement.

To decide who is a high achiever and who a low achiever, Pettersson (1990) uses points. Since I use relative achievement, I use percentages limits; those limits are chosen from the work of Gagné (2004).

The combination of the work of Pettersson (1990) and Gagné (2004) gives the following categories:

I. Students highly ranked, top 10 %, in both years 3 and 6,
II. Students low ranked, bottom 10%, in both years 3 and 6,
III. Students who showed a large increase in ranking position from year 3 to year 6,
IV. Students who showed a large decrease in ranking position from year 3 to year 6.

3.7.2 The national test in year 6 vs the mathematical kangaroo

The aim of comparing relative achievement in the national test in year 6 with the relative achievement in the mathematical kangaroo is to explore how those who achieve highly on one test achieve on the other. To define who is a high achiever in this study, percentage limits are used. Those percentage limits are inspired by other research that in some way mentions a part of a population defined by percentage limits (Advisory Committee on Mathematics Education, 2012; Gagné, 2004; Mönks & Pflüger, 2005; E. Pettersson, 2011; Renzulli, 2005; Vialle et al., 2007). The aim of those percentage limits is different; for more detail see chapter 2.4. Some mention a limit indicating that a certain percentage of students need more challenges (Advisory Committee on Mathematics Education, 2012). Others mention a percentage limit to indicate how many students are gifted according to different models (Gagné, 2004; Mönks & Pflüger, 2005; E. Pettersson, 2011; Renzulli, 2005), while some use top percentage limits to choose participants for further investigation (Vialle et al., 2007).

Inspired by those references, three percentage limits are used in this study to define high achievers, top 5%, top 10%, and top 20%. Those who are not seen as high achievers therefore make up the bottom 80%.

4 Method

This study starts with comparing results in national tests in mathematics over a three-year period in a whole population, and comparing results in the national test and the mathematical kangaroo in a large part of the same population. The quantitative approach gives answers to the correctness of the observation by a teacher with 15 years of experience. The statistical analysis of the quantitative material identifies students that were further investigated. The second part of the study investigates tasks and what mathematical competencies they give the opportunity to activate. Further, a method will be investigated that explores how mathematical competencies can explain differences in achievement between two identified groups.

Empirical data in this study are results in three mathematical tests all of them with different purposes.

- The national tests in mathematics given in year 3 (2009) tests the passing level according to the curriculum (Skolverket, 2010),
- The national test in mathematics given in year 6 (2012) tests mathematical knowledge according to the curriculum, focusing on the passing level (Skolverket, 2012),
- The mathematical kangaroo given in year 7 (2013) is not connected to the syllabus. The aim of the mathematical Kangaroo is to stimulate interest in mathematics and to arouse curiosity and lust to learn mathematics (Nationellt centrum för matematikutbildning, 2013).

4.1 Design of the study

The full study contains two parts, study 1 and study 2.

Study 1 is descriptive, where movements in relative achievement are the object of study. It is relative achievement on the individual level that is compared between the tests, although they are analysed as groups. Movements over a four-year period (2009—2013) through the results in three tests are analysed, two curriculum bounded and one non-curriculum bounded. The same individuals are followed through the years; all students in public schools are involved in the national test since results are official data. To employ students' results in the mathematical kangaroo, students and guardians needed to sign an informed consent form, which reduced the number of participants to 264 from 611. In study 1, two comparisons are made:

- Relative achievement on the national test in year 3 (2009) is compared with relative achievement on the national test in year 6 (2012), n=568,
- Relative achievement on the national test in year 6 (2012) is compared with relative achievement on the mathematical kangaroo in year 7 (2013), n=264.

Study 2 aims to investigate a method that explores how mathematical competencies can be used to explain why some students are high achievers in one test but not in another. The two tests compared are the national test in year 6 and the mathematical kangaroo in year 7. Test competency profiles are constructed through analysis of the tests on the task level, exploring which mathematical competencies a student is considered to be in need of activating to solve the task. Individual results in the tests, on the task or part level, produces individual competency profiles, in turn leading to the group of students identified in study 1 being able to be analysed. Study 2 involves five steps:

- Competency analysis of the two tests involved on the task level,
- Producing competency profiles for the tests and individual competency profiles based on results in each task in the mathematical kangaroo,
- Comparison of competency profiles in the mathematical kangaroo for the identified groups,
- Investigating differences between the two identified groups in activated competencies by means of the mathematical kangaroo,
- Investigating tasks in the mathematical kangaroo that are of special interest.

4.1.1 The national test in year 3 (2009)

The national test in year 3 (2009) is connected to the curriculum that existed before 2011. It was the first national test given in year 3 in Sweden (Skolverket, 2008). The aim of the test was to assess students' knowledge and to support teachers in the process of assessing students' fulfilment of goals in the curriculum. The test contains nine parts, A—J; three of them (B, C and J) are meant to be solved in pairs or in groups, and one part (J) is a game (Skolverket, 2010). One part of the test (part A) is about self-assessment; the students are to tell how secure they feel in different mathematical situations. What can be said about the content and maximum points on each part is given in Table 3. Getting full points on each part results in a test result of 93 points. Public data on the test from the municipality office gives each student's points on each part except part A of the test.

Table 3

The national test in year 3 (2009); mathematical content and maximum points for each part.

Part	Content	Maximum points
A (*individual*)	Self-assessment	No points
B (*group*)	Follow instruction to build a 3D object	Together with C, 8 points
C (*pair*)	2D and 3D geometrical object. Positions measurements	Together with B, 8 points
D (*individual*)	Properties of geometrical objects, 2D and 3D	7 points
E (*indicidual*)	Mental arithmetic, addition and subtraction, without context	Together with F, 39 points
F (*individual*)	Mental arithmetic, addition and subtraction, with context	Together with E, 39 points
G (*individual*)	Written calculation, addition and subtraction	14 points
H (*individual*)	The meaning of the four basic operations	9 points
I (*individual*)	Length, measurements and comparisons	8 points
J (*group*)	A game about number sense	8 points

4.1.2 The national test in year 6 (2012)

The national test in year 6 (2012) was obligatory for all students and, was the first national test for year 6 connected to the curriculum Lgr 11 (Skolverket, 2011a). The test focuses on the lowest passing level according to the curriculum (Skolverket, 2012). There was one oral part on the test (part A) in which the content is geometry. In the three written parts (B—D), the content is: number sense, algebra, geometry, problem solving, statistics and relationships and changes. For parts B and C, some tasks demand written solutions in the test booklet and some "only" an answer. Part D demands written solutions on a separate paper. At the time of this writing the test is still classified, and thus the description of this test must be limited. What can be said about the content and maximum points on each part is given in Table 4. Getting full points on each part results in a test result of 106 points. Public data on the test from the municipality office indicate each student's points on each part of the test.

Table 4

The national test in year 6 (2012); mathematical content and maximum points for each part.

Part	Content	Maximum points
A *(oral in groups of 3-4 students)*	Geometry	Credit (1) or no credit (0)
B *(individual, no calculator)*	Number sense, Algebra, Geometry and Problem Solving	38, of which 9 points are of higher level
C *(individual, no calculator)*	Number sense, Algebra, Geometry	37, of which 3 points are of higher level
D *(individual, with calculator)*	Number sense, Probability and statistics, Relationships and changes, Problem solving	30, of which 9 points are of higher level

4.1.3 The mathematical kangaroo (2013)

The mathematical kangaroo in Sweden was given on five levels in 2013, *Milou* for preschool and years 1—2, *Ecolier* for years 3—4, *Benjamin* for years 5—7, *Cadet* for years 8—9 and one track, *Junior,* for upper secondary school. Students in year 7 participated in this study, and therefore *Benjamin* is the choice here.

The mathematical kangaroo is a multiple choice test. For *Benjamin* there are five choices in each task. All tasks are dichotomously scored, either credit or no credit, although there are three levels in the scoring. The first seven tasks give 3 points, task numbers 8—14 give 4 points and task numbers 15—21 give 5 points.

The multiple choice tasks give an opportunity to test whether one of the five options is a possible solution, but the tasks can also be solved mentally or in writing. No solutions are required of the students. Getting full points on each part results in a test result of 84 points.

The national centre of mathematics (NCM) in Sweden distributes the mathematical kangaroo to the schools that want to participate in the competition. After the implementation of the competition, it is possible to download all material, the test, mark scheme and suggestions for how to continue the work with the students (Nationellt centrum för matematikutbildning, 2013). In this material, NCM has also divided the task into four topics, numbers, geometry, time, problem solving and logical reasoning, although it is not described how this division of the content is made. The division of the contents according to NCM is shown in Table 5.

Table 5

The mathematical kangaroo (Benjamin) in year 7 (2013); mathematical content and maximum points for each part, described by NCM.

Part	Content	Maximum points
3 mark part	Numbers – 3 tasks Geometry – 2 tasks Time – 2 tasks	9 points 6 points 6 points
4 mark part	Number – 1 task Geometry – 3 tasks Problem/Logic – 1 task	4 points 12 points 4 points
5 mark part	Number – 2 tasks Geometry 1 task Problem/Logic – 4 tasks	10 points 5 points 20 points
Task numbers 8 and 13	These two tasks are not placed in any of the four contents	8 points

4.2 Sample

The sample consists of the students who participated in the study; those are a part of the whole population of students in year 7 in the spring of 2013 in a municipality in Sweden.

For all students in public schools in a municipality, the results of the national tests in mathematics, given in year 3 (2009, n=654) and in year 6 (2012, n=611), were collected. The results are official data. Most students (n=568) had results on both tests. The gap in the data is explained by some students moving to or from the municipality and some students having attended private schools before year 6 or choosing a private school after year 3. Results of national tests from students attending private schools are not official.

All students in public schools (8 schools) were asked to do the mathematical kangaroo in year 7 (spring 2013). The students and their guardians were asked to allow the results on the mathematical kangaroo be used in this study; 264 students and guardians agreed to do so, and 247 of those had results in all three tests involved in the study. The sample used in the study is thereby approximately 43 % of the population.

4.2.1 Representativeness of the sample

When using statistics and talking about the whole population, one must specify what is meant by the whole population. Is it all the humans in the world, or is it all 7-year old children in a specific school? If a whole population participates in a study, it must nevertheless be seen as a sample; for example, the study is done in a specific time range or at a specific geographic place. This means that the results can not be generalised for something that will happen ten years later or happened ten years earlier or at another geographic place (Lisper & Lisper, 2005).

In this study, results of different mathematical tests from a sample of a whole population are compared. It is important to analyse how well the sample represents the population.

The statistical program R (R Core Team, 2013) was used for statistical analysis of the representativeness of the sample, and a significance level of 5% was used in the statistical tests. The whole population is counted as those students with results in the national test in year 6 who went in year 7 to public schools in the spring of 2013. The whole population was invited to participate in the study, although naturally not everyone did. The sample group is those who chose to participate in the mathematical kangaroo and also agreed to let their results be used in this study.

When a sample is chosen from a population, it is important to investigate how well the sample pictures the population. This can be done by comparing the statistical parameters m (arithmetic mean value for the selection) with μ (arithmetic mean value for the population), and s (standard deviation for the selection) with σ (standard deviation for the population) (Lisper & Lisper, 2005).

4.2.1.1 Statistical parameters

To statistically investigate whether the sample is a representative sample, mean, maximum, minimum, lower and upper quartile, median values and standard deviation of the results in the national tests in year 3 and year 6 were compared between the whole population and the sample group, Table 6. Boxplots were drawn to visualise the statistical parameters, Figure 7 and Figure 8.

Table 6

Comparison of statistical parameters between the population and the sample.

	National test grade 3 population	National test grade 3 sample	National test grade 6 population	National test grade 6 sample
Minimum	19	56	9	23
Lower quartile	75	78	61	68.75
Median	81	83	74	79
Upper quartile	86	87	85	88.25
Maximum	93	93	102	101
Mean	79.28	81.80	71.57	76.92
Standard deviation	9.25	7.00	18.20	15.33

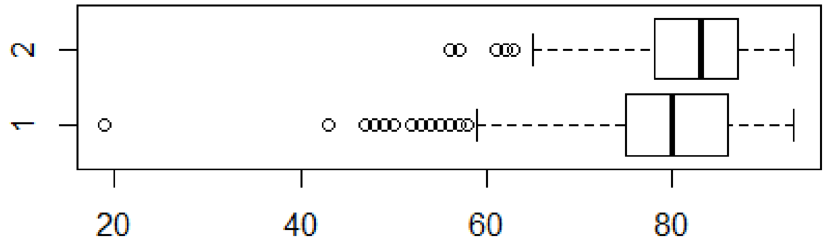


Figure 7. National test year 3; population no 1, sample no 2.

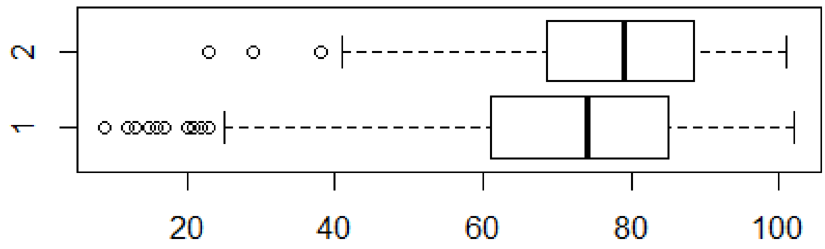


Figure 8. National test year 6; population no 1, sample no 2.

In Table 6, Figure 7, and Figure 8 it is shown that the range of variation is larger in the population than in the sample. The reason for this is that low achieving students are less represented in the sample as compared to in the population; this also has the effect that the median for both tests is higher in the sample than in the population.

4.2.1.2 Frequency distribution graph

If a test is constructed so that most students pass it, the test has a ceiling effect. A test that is very difficult, resulting in most students failing it, has a floor effect, Figure 9 (Statistik för samhällsvetare, Lisper & Lisper2005).

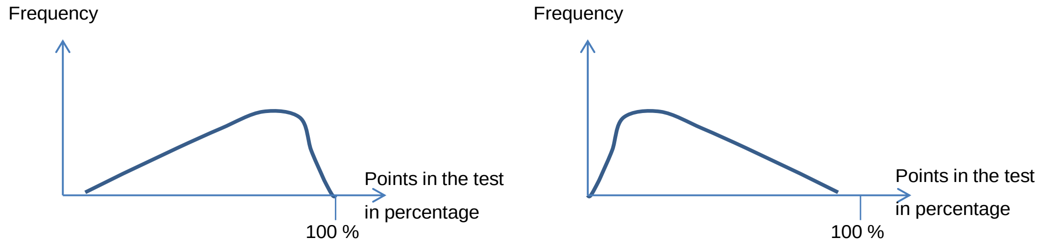


Figure 9. Left graph illustrates the ceiling effect and the right graph illustrates the floor effect.

One way to judge whether empirical data are normal or skewed in any direction is to draw a frequency density graph of the data. The frequency distribution graph of the sample also has to be compared with the frequency distribution graph of the population, with the aim to show whether the sample describes the population (Lisper & Lisper, 2005). Differences and similarities as well as ceilings effects are visualised through frequency distribution graphs. To compare the distribution of the sample and the population, frequency density graphs were drawn for the results in the population and the sample for each test, Figure 10 and Figure 11.

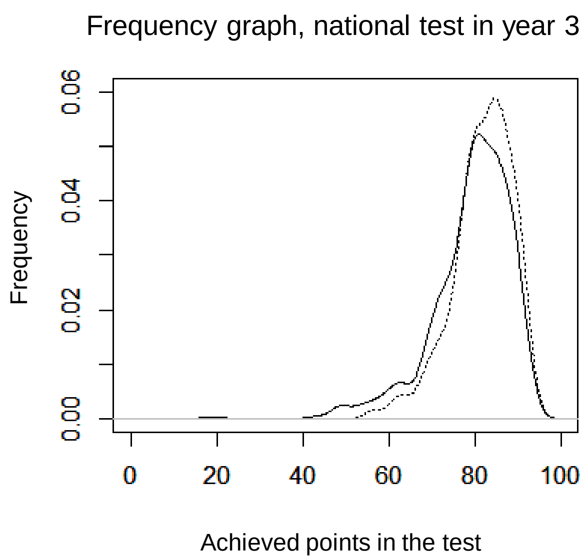


Figure 10. Frequency graph of results in the national test in year 3; the population solid line, the sample dotted line.

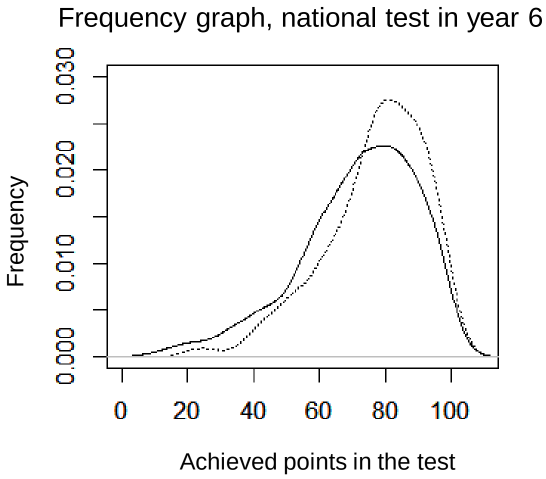


Figure 11. Frequency graph of results in the national test in year 6; the population solid line, the sample dotted line.

In the frequency graphs, Figure 10 and Figure 11, it can be seen that the sample represents those students who achieved highly on both national tests more as compared to those that had low achievement. It is also visualised that both national tests have a ceiling effect, which means that data (actual results on tests) must be transformed, for example to ranked data, to be able to do statistics (Polit, 2008).

4.2.1.3 Scatter plot

Plotting the actual results in the national test in grade 6 against the actual results in the national test in grade 3, and marking (in black) those students who belong to the sample, gives a picture of the distribution of the sample over the population, Figure 12.

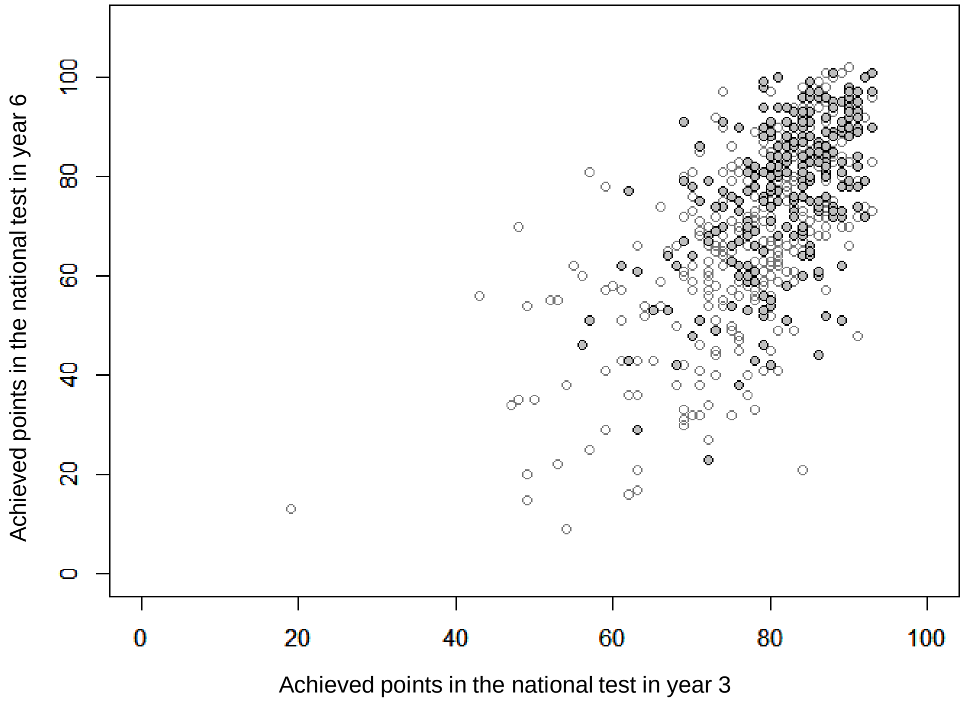


Figure 12. Results in the national test in year 6 versus results in the national test in year 3.

There is a lack in the sample of students who had low achievement in both national tests. However, those students are not very many (12—14 students), which can be seen in Figure 12.

Some students (n=43) have results in the national test in year 6 but not in the national test in year 3. Of those, some (n=17) are included in the sample, i.e. they completed the national test in year 6 and agreed to participate in the study with their results in the mathematical kangaroo. The distribution of their points in the national test in year 6 is shown in Table 7.

Table 7

Distribution of points in the national test in year 6 for students who do not have results in the national test in year 3.

Statistical measures	Results from year 6 but not from year 3 (n=47)	The sample - with results from year 6 but not from year 3 (n=17)
Minimum	12	21
Lower quartile (25%)	50	32
Median (50%)	66	39
Upper quartile (75%)	86	49
Maximum	100	71

4.2.1.4 Godness of fit test

To find out, statistically, how well the sample represents the population, a "goodness of fit" test, the $\chi 2$ test, was used (Wackerly, Mendenhall, & Scheaffer, 2002). With the null hypothesis, H_0:

> There is no significant difference between the distribution of the sample and the population.

The $\chi 2$ test is suitable to use since it is possible to compare two groups even if data not are normally distributed. The test compares the sample with the population through their results on the national test in year 3 and their results on the national test in year 6, data are given in appendix 1 and appendix 2.

Comparing the sample with the population through the national test in year 3, shown in appendix 1, gives a calculated $\chi 2$-value of 28.68615. The degree of freedom, 23, with the critical $\chi 2$ value ($\alpha = 0.05$) 35.172, says that the null hypothesis is retained.

The calculated $\chi 2$ value for year 6 is 36.99813. The degree of freedom is 24 and has the critical $\chi 2$ value ($\alpha = 0.05$) of 36.415. Since the calculated value is greater than the critical one, there is a significant difference between the sample and the population. Looking at the table in appendix 2, it is very clear that the reason is mainly the three groups with low points on the test, 0—32 points, 33—40 points and 56—58 points, meaning that the reason for why the sample is not representative of the population is primarily because there is a lack of students who participated who were low achievers in the national test in year 6.

The sample is not a perfect representation of the population, which the $\chi 2$ test shows. The discrepancy can be explained by there being a lack of students who were low achievers in the national test in year 6. In the study, students will be identified by different criteria; if they are identified by their low achievement in the national test, there is a chance that those students are underrepresented in the population.

4.2.1.5 Change in ranking position

Statistical parameters describe the population's location and shape in the frequency distribution graph. For some measurements, knowledge of the location on the scale of measurements is arbitrary (Wackerly et al., 2002). It is more important to compare the location in comparison to other data. In this case, data can be ranked and analysed with nonparametric statistical methods. For example, if a group of people is defined by a specific percentage of a population, it is only interesting to compare that group of people relatively to each other, which ends up in ranking them. For example, in Gagné's differentiated model of giftedness and talents (Gagné, 2000), the gifted and talented are seen as the top 10 % of a population, and this top is defined by means of individual performance relative to the population.

If the distribution of data is not symmetric, for example there is extreme skewness or the presence of a ceiling or floor effect, data must be transformed to meet the requirements of certain statistical tests (Polit, 2008). One transformation can be to rank the data; ranked data leads to the choice of nonparametric statistical methods. Nonparametric tests shall be used if the distribution of the population is unknown and if no assumptions are made about the distribution. For a sample with large amount of data, nonparametric tests should also be used if the data are ranked (Lisper & Lisper, 2005). An example is used to describe the difference between empirical data best suited for nonparametric methods and empirical data suited for parametric methods.

Example

In a sprint race among 8-year old children, there is a winner, a second place and a third place and so on, in the same way as in a sprint race among elite athletes. Those numbers do not say anything about the performance or about the differences in performance. The distributions of the positions of winner to tenth place look the same for the children as for the elites. This kind of data is more suited for nonparametric methods. In the same sprint race, individual times for the races can be measured, which gives individuals' actual performance. Parametric methods are more suitable for analysing this kind of data.

For ranked paired data, where pair represents "pre-treatment" and "post-treatment" observations, shifts in location can be analysed with a Wilcoxon signed rank test (Hollander, Wolfe, & Chicken, 2014). This also leads to making it possible to use a Wilcoxon signed rank test to compare individual ranking positions for two measurement points, for example test A and test B. The null hypothesis will in that case be: the ranking position is the same for each individual on both tests. If the null hypothesis is retained, the (pseudo)median is equal or close to zero, meaning that there has not been a shift in location. A large p-value, $p \geq 0.05$, says that there has not been a shift in location. The changes measured by the Wilcoxon test is in the individuals and how those changes are placed (in ranking). It is a measurement of "individual movement" from test A to test B.

The tests used in this study are said to be not comparative because of the different aim in each test. The purpose of the national test in year 3 is to support formative assessment and it only tests whether the student has reached the curriculum goal for year 3, the passing level (A. Pettersson & Boistrup, 2010). The national test in year 6 tests students' achievement according to the curriculum with some possibility to show deeper mathematical knowledge (Skolverket, 2012). The mathematical kangaroo is not connected to any curriculum; the aim of the mathematical kangaroo is to stimulate interest in mathematics and to arouse curiosity and lust to learn mathematics (Nationellt centrum för matematikutbildning, 2013). The Wilcoxon signed rank test is used to statistically investigate whether students' ranking position changes between the tests. To be able to make a Wilcoxon signed rank test, values from the two groups that will be compared must be in the same interval. The results of the tests were therefore recalculated to percentages of the maximum value.

If the Wilcoxon test says that there has been a change, one must consider what the cause of the change is. For top achieving students, Gagné (2005) stresses that most of them maintain their ranking position.

To investigate whether movements in ranking positions occurred, the Wilcoxon signed rank test was carried out first comparing the national test in year 3 and the national test in year 6, Table 8, and second comparing the national test in year 6 and the mathematical kangaroo in year 7, Table 9.

Table 8

Wilcoxon test – the national test in year 3 and the national test in year 6, data from students who have results in the national test in year 3 (2009) and in year 6 (2012).

Wilcoxon values	Percentage values
V	158038
p-value	<2.2e-16
95 percent confidence interval	[15.14599,∞[
(pseudo)median	16.06755

Table 9

Wilcoxon test – the national test in year 6 and the Kangaroo test; data from students who had results in the national test in year 6 (2012) and the mathematical kangaroo in year 7 (2013).

Wilcoxon values	Percentage values
V	34595.5
p-value	<2.2e-16
95 percent confidence interval	[23.2143,∞[
(pseudo)median	24.76195

A small p-value indicates that there has been a shift in the location of ranking positions among the students. The value of the (pseudo)median is 16.1 when the two national tests are compared and 24.8 when the national test in year 6 is compared with the mathematical kangaroo. Since the percentage results of all tests were used in the Wilcoxon signed rank test, the values of the (pseudo)median indicate that there have been large movements in the ranking position between the tests among the students.

There may be many reasons for a change in ranking position: trauma, change of teacher, being ill on the test day, different kind of test etc. People have told me "of course you get different results when using different test" but since it surprises not only me but also other teachers (Mattsson, 2013) that there are students who are low achievers in the national test and high achievers in the mathematical kangaroo, it is interesting to analyse differences. Some teachers suspect that those students are gifted in mathematics (Mattsson, 2013). In this study, reasons for changes in ranking position between the national test in year 6 and the mathematical kangaroo are investigated by means of the opportunities offered by the tasks to activate mathematical competencies; the MCRF framework (Lithner et al., 2010) is used in the analysis.

4.3 Procedure of the study

The design of the study is visualised in Figure 13. This chapter discusses the practical parts of the study.

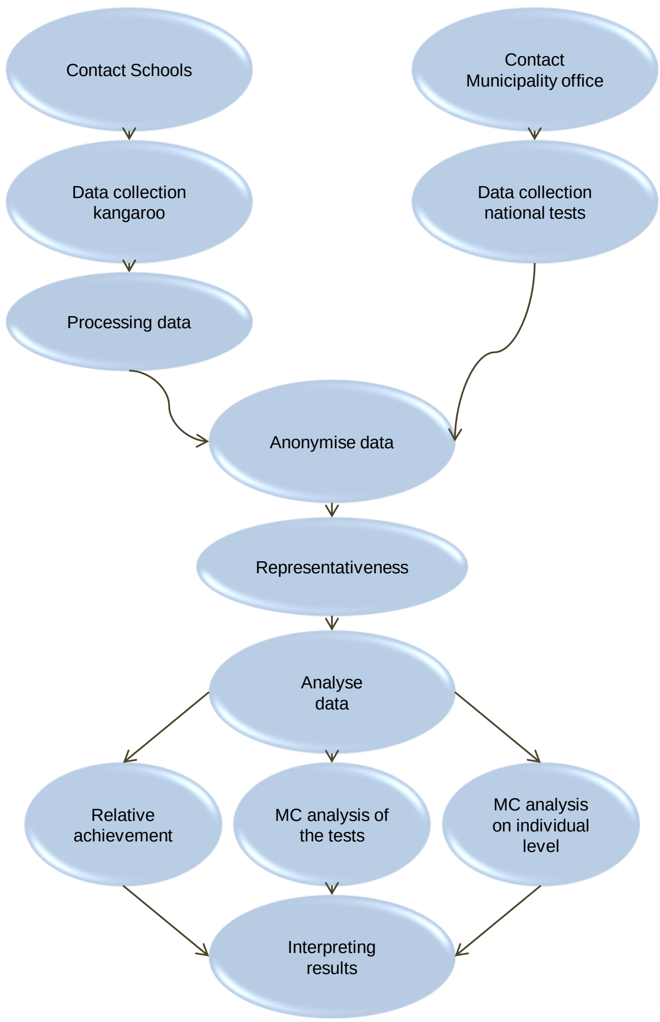


Figure 13. The process of the study, not related to time.

The practical process started with planning the study and having it inspected by the ethics committee at the university.

All secondary public schools were contacted to ask whether the schools wanted to participate. Contact with a teacher responsible for mathematics education at each school was then established.

Informed consent asking permission to connect results of the national tests with results on the mathematical kangaroo was delivered, through the teachers, to all students and their guardians, see appendix 3 and appendix 4. Here, it was necessary for the students to agree without the influence of their guardians, and thus the guardians and the students received informed consent forms.

All material for the implementation of the mathematical kangaroo was distributed to the schools by the author. After the implementation, which was done by the teachers at the schools, the author collected all the material. All tests were corrected by the author; after collecting the data needed for the study, the tests were returned to the schools and the teachers returned them to the students. Data needed for the study were results in each task among the participating students.

The municipality office was contacted to get the results of the national test among the population of students who were asked to do the mathematical kangaroo. The municipality office was informed about the ethical considerations and that the study had been reviewed by the ethics committee at the university. National test results in public schools are official data in Sweden, however.

The national test from 2012 among students in year 6 was classified at the time of the study; a special agreement was made with the national agency of education to gain access to the test.

When all empirical data were collected, all the individuals were anonymised before the analysis started.

The representativeness of the sample was explored through statistical analysis. An analysis of data aiming to answer the research questions was then made; this analysis was done in three separate steps: *Step 1*, analysis of relative achievement between the national test in year 3 and in year 6; *Step 2*, analysis of relative achievement in the national test in year 6 versus the mathematical kangaroo;

Step 3, analysis of the mathematical competencies the tests gave opportunity to activate and analysis of the distribution of activated mathematical competencies in the national test in year 6 and in the mathematical kangaroo for students of special interest.

The last step in the study is to interpret and discuss the results.

4.4 Empirical data collection

The national test is obligatory and individual results of the test are recorded at the municipality office. Results of the national test from public schools are official data; therefore those were collected from the municipality office at a visit. The data of individuals with a secret identity are not public, and their data are thus not included in the study.

It is optional to participate in the mathematical kangaroo. In this study, the researcher contacted the headmasters at each school, who first gave permission to carry out the study at their school and second gave the researcher the name of a teacher for discussions. In order not to increase the work load for teachers too much, most of the work with the implementation of the mathematical kangaroo was done by the author. All material needed for the implementation was distributed to each school by the author. The students' ordinary mathematics teachers received instructions about the implementation of the mathematical kangaroo, both written, appendix 5, and verbal. The author corrected all the tests and returned them to each school after collecting all the necessary data, that is: results in each task connected to each student who participated in the study. The mathematical kangaroo did not require more than one hour of the teachers and the students. Since it is a multiple choice test, it was also possible to gather a large amount of data and be handled in a one-woman project.

4.5 Data analysis

All data were made anonymous: both the official results in the national tests and the results in the mathematical kangaroo. A computer program (Wireflow AB) produced a SHA-256 hash string of each identification number combined with cryptographic salt (comparable to stretching passwords) to make it impossible for an unauthorised to go back and identify someone.

The students were ranked according to their results to get their relative achievement in each test, and an analysis was carried out in three steps:

Step 1: Comparing movements in relative achievement in two national tests (year 3, 2009 and year 6, 2012) over a three-year period. The movements are measured by categorising students into four groups, I-IV, defined in chapter 3.7.1.

Step 2: Comparing relative achievement in two different tests, one non curriculum-bounded – the mathematical kangaroo – and one curriculum-bounded – the national test. The mathematical kangaroo was given to the students in year 7 (2013), while the national test had already been done in year 6 (2012). Analysing relative achievement in those two tests identifies groups of high achieving students using percentage limits 5%, 10% and 20%, see chapter 3.7.2.

Students' relative achievements in the national test in year 6 and in the mathematical kangaroo in year 7 are compared. The focus is on students among the top 20% on one test but not on the other. The overlapping group with students among the top 20% on both tests is not included in the analysis.

It is investigated how high achievers on one test achieved on the second. The students identified in this part of the study were explored by means of the mathematical competencies they are given the opportunity to activate in the tasks and the tests in the third step of the study.

Step 3: Analysis of what mathematical competencies the tasks give the opportunity to activate in the national test in year 6 and in the mathematical kangaroo. The analysis results in competency profiles for each test and for the students identified in *step 2*; comparing those profiles investigates whether differences in achievement can be explained by mathematical competencies.

4.6 Guide to analysing mathematical competencies in tasks

Analyse of the tasks is based on an unpublished analysis document used in Boesen et al. (2014)[5]. The analysis of the tasks uses the perspective of what is *required and reasonable* (RR) to expect of a student in year 7. The key idea in this analysis is to show what is necessary, working through the so called exclusion principle. In analysing the tasks for each competency, asking – *in a required and reasonable solution* – is it necessary to activate the competency.

The guide in Table 10 is used to analyse tasks in the mathematical kangaroo and in the national test in year 6. The results of the analysis indicate the opportunity the test gives to activate the six competencies in the MCRF. To simplify, the analysis will judge what is required of and is reasonable to believe that a student in year 7 uses in the solution process of the task (Boesen et al., 2014). Through experience, Boesen et al. (2014) have noticed that students do what is required to solve a task but not much more; this is verified in the 15 years of teacher experience of the author. In the analysis of the task, there is an underlying question for each competency.

In a required and reasonable solution that solves a task correctly, is it necessary to activate the competency? By necessary it is meant: expected to be necessary for a student in year 7.

The analysis procedure was:

First, to find a reasonable solution or solutions. Reasonableness is determined by teacher experience of how students normally solve tasks. The solutions made for the analysis are possible solutions and were produced by the author.
A task that does not demand a written solution but only an answer is still seen to have been solved when a correct answer is given. The student could have solved the task mentally or physically. The empirical data do not include any student solutions.

Working through each of the six competencies in the MCRF, try to show that it is necessary to activate the competency in a reasonable solution (that fulfils the task conditions).

[5] This study had access to written analysis documents that are used but not published in the referenced article.

- If it is not necessary to activate, for this task, the competency is given the classification '0'.
- If it is necessary to activate the competency, for this task, the competency is given the classification '1'.

There is one exception from this analyse procedure in this study, although, the problem solving competency was not analysed in this way. To get an answer as to whether the tasks in the mathematical kangaroo are seen as problems, eight experienced teachers used to teaching mathematics in year 7 were asked to mark the tasks they interpreted as problems according to the following definition.

> A mathematical problem is a problem where the solution method is not known in advance for the solver (NCTM, 2000, p. 52).

Those tasks that the majority (five or more) of the teachers agreed to be problems were seen in this study as giving an opportunity to activate the problem solving competency and were given the classification '1'.

The national test, which was classified material at the time of this study, could not be discussed with teachers in the same way. The tasks that the constructors (Stockholm University, 2013) defined as problem solving tasks are classified as giving an opportunity to activate the problem solving competency and are given the classification '1'.

Table 10 is used as a guide for analysis of the competencies in a task, for each task, and the reasonable solution. The guide is used and finally offers a classification of '1' or '0' for each competency and task.

Table 10
Competency analysis guide used for each task.

Competency	Question to ask in the analysis of the task. *For all competencies, if the answer is:* *Yes – the classification is '1'* *No – the classification is '0'*	Classification '0'	Classification '1'
Applying procedures (App)	Is it necessary to activate the applying procedures competency in an *RR* solution? That is, is it necessary to involve a sequence of mathematical actions in an *RR* solution?		
Reasoning (Rea)	Is it necessary to activate the reasoning competency in an *RR* solution? That is, is it necessary to involve an explicit act of using a mathematical argument to justify choices and conclusions in an *RR* solution?		
Communication (Com)	Is it necessary to activate the communication competency in an *RR* solution? That is, is it necessary to involve a process where mathematical symbols, signs and/or language are exchanged between the sender (the test) to the receiver (the student) in an *RR* solution?		
Representation (Rep)	Is it necessary to activate the representation competency in an *RR* solution? That is, is it necessary to use representations in an *RR* solution?		
Connection (Con)	Is it necessary to activate the connection competency in an *RR* solution? That is, is it necessary to draw a connection between different entities and/or different representations in an *RR* solution?		
Problem solving (Pro)	No question For the mathematical kangaroo - statement of experienced teachers decides. For the national test – constructors' choice decides.		

The analysis results in a reduced competency matrix for each task, Figure 14.

Task 1	class
App	1
Rea	0
Com	1
Rep	0
Con	1
Pro	0

Figure 14. Reduced matrix showing the competency classification for task no. 1

Examples of the analysis of tasks from mathematical kangaroo are shown in appendix 6.

4.7 Competency profile

The mathematical kangaroo has 21 tasks, 7 tasks give 3 points, 7 tasks give 4 points and 7 tasks give 5 points. Each student identified will be given a matrix showing their results, Figure 15. Row one represents three-point tasks, row two four-point tasks, and row three five-point tasks. Number 1 in a cell indicates credit; 0 indicates no credit.

KT-S1							
3 m	1	0	1	1	0	1	1
4 m	1	1	0	1	1	0	0
5 m	0	1	1	1	0	1	1

Figure 15. Results on the task level in the mathematical kangaroo for student S1.

The analysis will result in one summary competency matrix for each identified student that tells how often that student used each competency and how often each competency is used in relation to the other that is used, Figure 16. ∑ class is the sum of all times the students received a point in a task that gave the opportunity to activate the competency. Rel class is the relative distribution of the competency.

Sum KT-S1	$\sum$ class		Rel class
Pro	2		0.095
Rea	6		0.286
App	1		0.048
Rep	3		0.143
Con	4		0.190
Com	5		0.238
∑	21		1

Figure 16. Student matrix of activated competencies for student S1.

A competency profile for the tests and for each student identified in study 1 is made. The competency profile indicates how much each competency is activated in relation to the total amount of activated competencies. An example is given to explain how a competency profile is created.

Example

A test has 10 tasks and the competencies are distributed among the tasks in the example test shown in Figure 17. To solve all 10 tasks, it is necessary to activate the competencies as summarised in column 2 in Table 11. Relative values for each competency are shown in column 3. In this example, there is a specific student that has failed in tasks no. 2, 7 and 8. The student is therefore seen to have activated the reasoning competency two times less, the communication competency one time less, the representation competency two times less and the problem solving competency two times less than possible, summarised in columns 4 and 5 in Table 11. Data from column 3 result in the competency profile for the test; data from column 5 result in the competency profile for the specific student, Figure 18.

Task 1	Class
App	1
Rea	0
Com	1
Rep	0
Con	0
Pro	0

Task 2	Class
App	0
Rea	0
Com	0
Rep	1
Con	0
Pro	0

Task 3	Class
App	1
Rea	1
Com	1
Rep	0
Con	0
Pro	0

Task 4	Class
App	0
Rea	0
Com	0
Rep	0
Con	0
Pro	1

Task 5	Class
App	1
Rea	0
Com	1
Rep	1
Con	0
Pro	0

Task 6	Class
App	1
Rea	0
Com	0
Rep	0
Con	1
Pro	0

Task 7	Class
App	0
Rea	1
Com	1
Rep	0
Con	0
Pro	1

Task 8	Class
App	0
Rea	1
Com	0
Rep	1
Con	0
Pro	1

Task 9	Class
App	1
Rea	0
Com	1
Rep	0
Con	1
Pro	0

Task 10	Class
App	1
Rea	0
Com	0
Rep	0
Con	0
Pro	0

Figure 17. Competency distribution in the task in an example test.

Table 11

Example of data used to produce a competency profile for one test and one specific student.

Competency	No. of times activated *Maximum*	Relative values *Maximum*	No. of times activated *By the student*	Relative values *By the student*
Applying procedures (App)	6	0.27	6	0.40
Reasoning (Rea)	3	0.14	1	0.07
Communication (Com)	5	0.23	4	0.27
Representation (Rep)	3	0.14	1	0.07
Connection (Con)	2	0.09	2	0.13
Problem solving (Pro)	3	0.14	1	0.07
Total amount of competencies	*22*	*1.01*	*15*	*1.01*

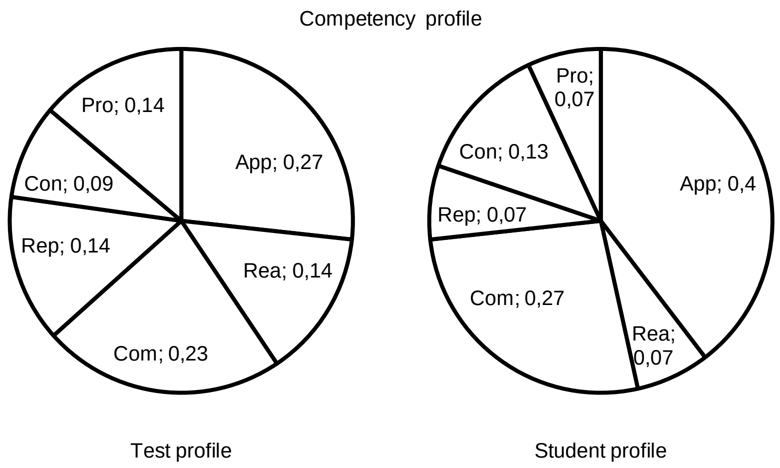


Figure 18. Example of a competency profile for one test and one student.

A competency profile for a test shows the possibilities that have existed to activate the six competencies in relation to each other. The numbers in Figure 18 show how much a competency is given opportunity to be activated by the tasks in the test in relation to the total amount of possible competencies on the test.

A competency profile for a student shows how the competencies are activated in relation to each other in the tasks solved by the student. The numbers in Figure 18 show how much a competency is activated by the student on the test in relation to the total amount of activated competencies by the student on the test.

The test profiles give possibilities to make comparisons between opportunities the test gives to activate the competencies and how students activated the competencies, both on an individual level and group level. The profiles are not correlated to achievement; a student profile can look exactly or almost exactly like the test profile without there being any demand that the student has full points in the test.

4.8 Validity and reliability

4.8.1 Sample

The whole population of students from a school year in a municipality was invited to participate in the part of the study that includes the mathematical kangaroo; those who participated make up the sample. Statistical measurements were made to validate whether the sample was representative for the population; these are presented in chapter 4.2.1. The results of the statistical measurements (chi square test) are that the sample is representative, with p = 0.025. The p-value is very close to 0.05; it is obvious in the data (appendix 2) that the reason for not achieving a better representativeness is a lack of students that were low achievers in the two national tests.

4.8.2 Non-participants

The students that did not participate in the study are primarily those who achieved low on the national test in both years 3 and 6, this is visualised in Figure 12. This affects the results in that there could be more students that achieve highly in the mathematical kangaroo and low in the national test in year 6 than are identified in the sample.

4.8.3 Error in identification

Empirical data in this study are students' results in three mathematical tests. There is a risk that there are individuals that did not achieve the way they should have under normal conditions. Factors such as stress, illness, social circumstances etc. can influence individual achievement. It is therefore possible that some students should have been identified that were not owing to factors like those mentioned above. In turn, this leads to there being a possibility that some of the students identified should not have been identified, since ranking and percentages are used as identification criteria. When using identification criteria, one must always chose a limit, which gives a risk of some identification errors. Although students are treated as a group in this study, which reduces the identification factors, the probability that all students had a bad day is considered low.

4.8.4 Coding

The coding of personal numbers to hash codes was done by a coding expert (Wireflow AB). The expert constructed a program that coded original data. The program added cryptographic salt to the personal numbers before the SHA-256 hash was calculated. This makes it impossible for any unauthorised person to go back to and identify individuals. Use of an expert for coding and security in the program makes the coding reliable and safe.

4.8.5 Required and reasonable solution

The best way to find solutions for the task analysis would have been to let a large group of students in year 7 produce solutions that could be used for the analysis. This would have been time consuming, however. To make it possible to manage the study, solutions were created by the author. The solutions were compared with authentic students' solutions in the classified material from the national test in year 6. The solutions produced by the researcher and the authentic student solutions were judged to be of the same complexity and on the same mathematical level. The solutions used in the task analysis are therefore seen to be comparable to solutions that students in year 7 would have produced.

4.8.6 Task analysis

The analysis procedure of the tasks was discussed in a seminar with researchers and research students in mathematics education. Comments given at the seminar resulted in the analysis guide described in chapter 4.6. The analyser is supposed to consider what a student in year 7 should need in order to solve a task. The author has worked as a teacher in mathematics for more than 15 years, although in the upper secondary school. The author also has five years of experience in teaching situations in mathematics with students at younger ages, years 2—9. To test the analysis guide further, the author had a one-hour seminar with six teachers from a lower secondary school (year 7—9). The author first briefly introduced the analysis guide, and the teachers analysed 6 tasks in the mathematical kangaroo working in pairs. After the analysis of the task, the analysis procedure was summarised in a discussion between the teachers and the author. After the analysis done by the teachers working in pairs, it was found that the results of the analyses differed between the teachers and the author. After the discussion, however, the author and the teachers, except for one, agreed upon how the analyses of the competencies should be interpreted.

The discussion of the analysis tool with research colleagues, the teachers' experience of the author, the seminar with the lower secondary school teachers, the control of the solutions with the authentic solutions given in the material for the national test in year 6 make the analysis process reliable and valid enough to test the method, which aims to explore differences between the groups that were identified. Nevertheless, it could have been even better if the whole analysis process had been done in collaboration with a larger group of lower secondary school teachers in mathematics, and/or more in depth with research colleagues in mathematics education.

4.8.7 Problem solving competency

The definition of a mathematical problem that is used says that a mathematical problem is an individual phenomenon. One student might be aware of a solution method while another student is not. The task is therefore a problem for one of them but not for the other. A good approximation is made by asking teachers who are experienced in teaching students of this age, but this is still a source of failure. The only way to find out whether the task is a problem for a student is to interact with the student in some way, but this makes it difficult to handle large groups of students.

There is also a complication that problem solving competency is not analysed in the same way in the both tests. Problem solving competency on the national test is judged by the constructors (Stockholm University, 2013). However, the constructors use experienced teachers in the process of constructing the tests, and this process involves categorising tasks according to which competency they test.

The National Centre of Mathematics (NCM) in Sweden has categorised tasks on the mathematical kangaroo that they judge to measure problem solving competency. I choose not to use their classification because it is not described whether they use experienced teachers in this process. The ways that are chosen have one thing in common, i.e. it is the participation of teachers experienced in teaching students for whom the test is aimed.

In the analysis, the identified groups are compared in one test at a time. It is therefore not a dilemma that the competency is not defined in exactly the same way.

The analysis of the problem solving competency is a weak point in this study, but the conclusions are valid because the groups are compared in one test at a time. There is no comparison that mixes problem solving competency between the two tests.

4.9 Ethical considerations

The ethical perspective of the study has been discussed in a seminar with senior researchers and research colleagues. The study has also been examined by the ethics committee at the University without requiring additional review. There has been no interaction between the researcher and the participants of the study. Data connected to the national tests are public data. To be able to include results from the mathematical kangaroo, it was necessary to obtain a signed informed consent form from the students and their guardians.

All data were connected to the individuals' personal number; the data were anonymised and coded through a program that produced a SHA-256 hash string. An expert in coding (Wireflow AB) implemented the coding procedure. The security in the coding process is high.

Considering the seminar with colleagues and the advice of the ethics committee, it is reasonable to draw the conclusion that no individual was injured psychologically or physically by participation in this study.

5 Results

Chapters 5.1 to 5.4 give the results of study 1, the descriptive part comparing relative achievement. Chapters 5.5 to 5.8 give the results of study 2, involving the analysis with mathematical competencies.

5.1 Comparison of relative achievement on the national test in years 3 and 6, categories I and II

When comparing the change in ranking position between the national tests in year 3 and the national test in year 6, it was required that the students had taken both tests. There were 568 students with results on both tests. Since students can have the same ranking position it means that the groups of the top 10% on each test and the groups of the bottom 10% on each test do not need to contain the same number of students.

Category I - Students highly ranked, top 10%, in both years 3 and 6

49 students were ranked among the top 10% in year 3, and 55 students were among the top 10% in year 6; of those, 15 students, 10 girls and 5 boys, were among the top 10% on both tests. Ranking positions for the top 10% students in year 3 are placed to the right of the vertical solid line in Figure 19, and ranking positions for the top 10% students in year 6 are placed above the horizontal solid line in Figure 19. The circles inside the box in the upper right corner in Figure 19 therefore represent students who are ranked among the top 10% on both tests.

Category II - Students ranked low, bottom 10%, in both years 3 and 6

56 students were among the bottom 10% in year 3, and 57 students were among the bottom 10% in year 6; of those, 26 students, 10 girls and 16 boys, were among the bottom 10% on both tests. Ranking positions for the bottom 10% students in year 3 are placed to the left of the vertical dotted line in Figure 19, and ranking positions for the bottom 10% students in year 6 are placed below the horizontal dotted line in Figure 19. The circles inside the box in the

bottom left corner in Figure 19 therefore represent students who are ranked among the bottom 10% on both tests.

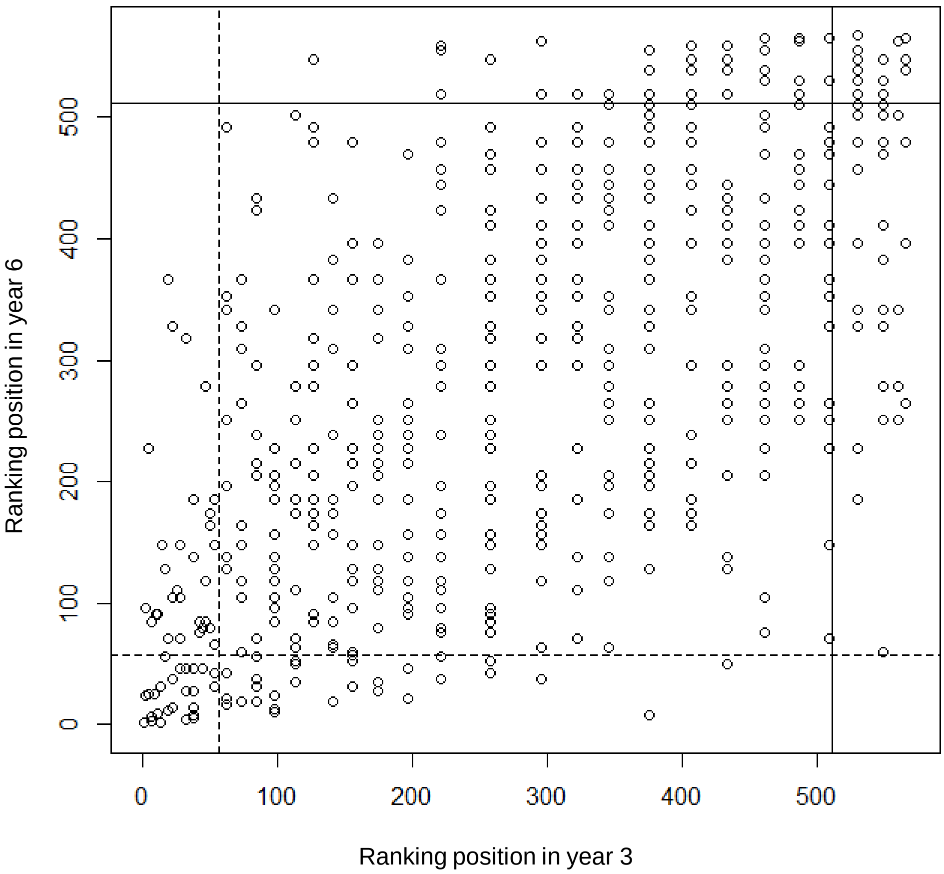


Figure 19. Ranking position on the national test in year 6 versus the ranking position on the national test in year 3.

5.1.1 Interpretations of the results in categories I and II

The results in this study show that there are more students who stay among the bottom 10% than those who stay among the top 10%. One explanation for the results can be the existing ceiling effects in both tests. Considering the ceiling effect in the two national tests, Figure 20 and Figure 21, there are more students who achieve high (actual results) in the national test than who achieve low (actual results). The students who achieved among the top 10% in one of the two national tests are shaded in Figure 20 and Figure 21. The ceiling effect means that a change by only a few points can change the ranking position remarkably among the high achievers but does not have the same affect among the low achievers. With the results it can be concluded that it is more likely that a student stays among the poorer achievers than it is that he or she stays among the best achievers. In this study relative achievement is used in the comparison, the frequency graphs based on actual results on the two tests, Figure 20 and Figure 21, show that students among the bottom 10% have a wide distribution of their actual results, while students belonging to the top 10% in any of the tests have all achieved highly in actual points on the tests.

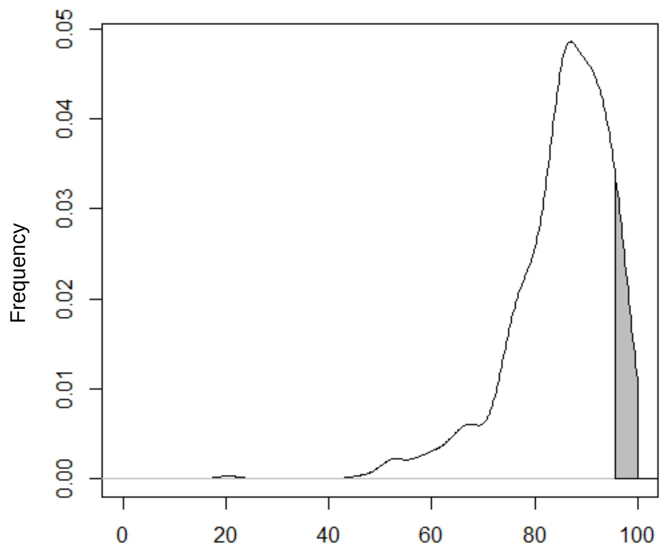


Figure 20. Frequency distribution in the national test in year 3, top 10% shaded. Percentage results on the x-axis

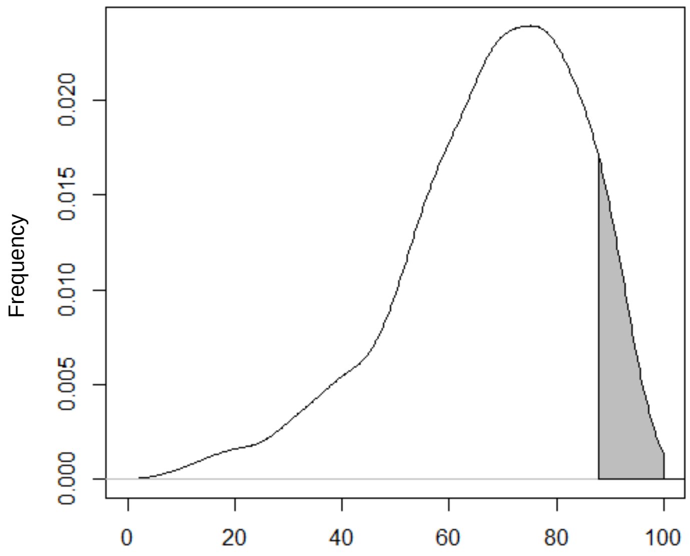


Figure 21. Frequency distribution in the national test in year 6, top 10% shaded. Percentage results on the x-axis

5.2 Comparison of relative achievement on the national test in years 3 and 6, categories II and IV

Category III – large increase in ranking position from year 3 to year 6

For a student to be categorised in group III the student must increase the ranking position by at least 40 percentage points, i.e. by at least 227 ranking positions, from year 3 to year 6, as illustrated in Figure 22. This means that students that have a ranking position between 1 and 341 in year 3 have the possibility to increase their ranking position by 40 percentage points and, after the increase, they will have a ranking position between 228 and 568 in year 6.

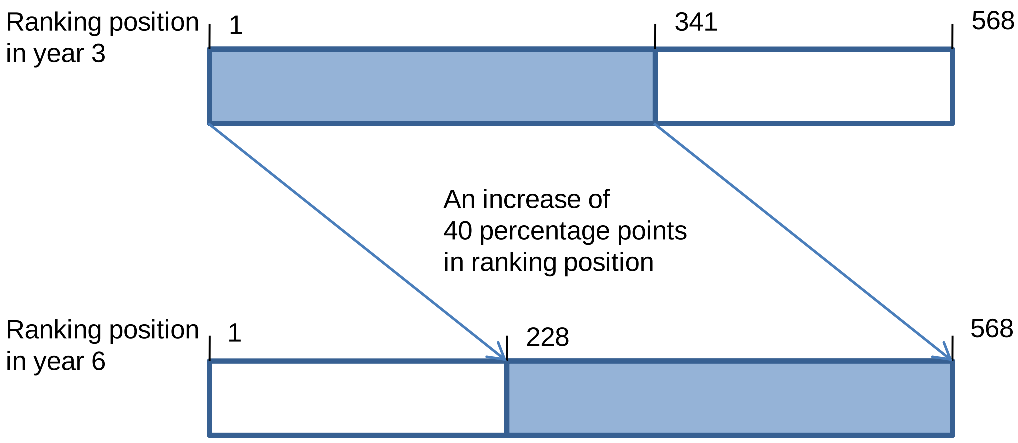


Figure 22. Illustration of possible movements of ranking position for students in category III.

The students that are ranked among the bottom 60% (below 341 in ranking position) on the national test in year 3 are those who theoretically have the possibility to increase their ranking position by at least 40 percentage points. They are 333 in total: of those, 33 students, 17 girls and 16 boys, increased their ranking position by at least 40 percentage points on the national test from year 3 to year 6, marked with filled circles in Figure 23.

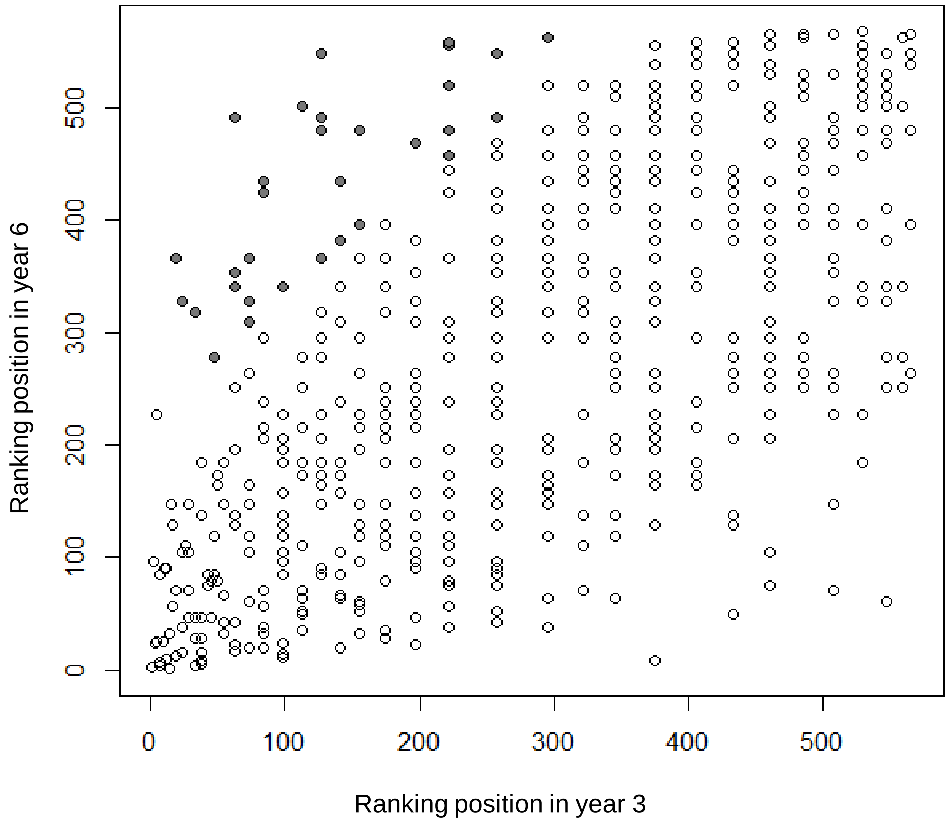


Figure 23. Group III, students with a large increase in ranking position from year 3 to year 6.

To compare students that belong to category III, with those who had the possibility to belong to the category, statistical measures of minimum, quartiles and maximum values, for actual results on the two national tests were analysed. The comparison of the statistical measures for different groups of students show which students are most likely to perform a large increase in ranking position. The groups of interest are:

- Group R is defined as the group of students (n=333) with ranking positions 1 to 341 by their actual results on the national test in year 3, that is, those who had the possibility to perform a large increase in ranking position from year 3 to year 6, see figure 22,

- Group S is defined as the group of students (n=334) with ranking positions 228 to 568 by their actual results on the national test in year 6, that is, the maximum range of ranking positions for students in Category III, see figure 22,

- Category III are the students (n=33) who performed an increase of at least 40 percentage points from year 3 to year 6, and are therefore included in both Group R and Group S.

The statistical measures for the groups of interest based on their actual total sum on the national test in year 3 and/or year 6 are shown in Table 12.

Table 12
Statistical measures based on the actual total sum on the national test in year 3 and/or year 6 for the students in category III, Group R and Group S.

Statistical measures	Group R Actual results on the national test in year 3	Category III Actual results on the national test in year 3	Group S Actual results on the national test in year 6	Category III Actual results on the national test in year 6
Minimum	19	57	71	74
Lower quartile	71	70	77	81
Median	77	74	83	88
Upper quartile	80	79	90	91
Maximum	82	81	102	100

Comparing the distribution of actual results between category III and Group R shows that students in category III are not among the lowest achievers on the national test in year 3, due to the difference in minimum value in the two groups, see Table 12. However, the distribution of actual results from lowest quartile to the maximum are similar for category III and Group R. Comparing category III and Group S shows that the distribution of actual results are similar for both groups, see Table 12. These results indicate that among students that perform a large increase from year 3 to year 6, however, low achievers in year 3 are not represented.

Category IV - large decrease in ranking position from year 3 to year 6

For a student to be categorised in group IV, he or she must have decreased in ranking position by at least 40 percentage points, i.e. by at least 227 ranking positions, from year 3 to year 6. This means that students that have a ranking position between 228 and 568 in year 3 have the possibility to decrease their ranking position by 40 percentage points and, after the decrease, they can have a ranking position between 1 and 341 in year 6.

The students that are ranked among the top 60% on the national test in year 3 are those who theoretically have the possibility to decrease their ranking position by at least 40 percentage points. They are 333 in total; 35 students, 15 girls and 20 boys, decreased their ranking position by at least 40 percentage points in the national test from year 3 to year 6, marked with filled circles in Figure 24.

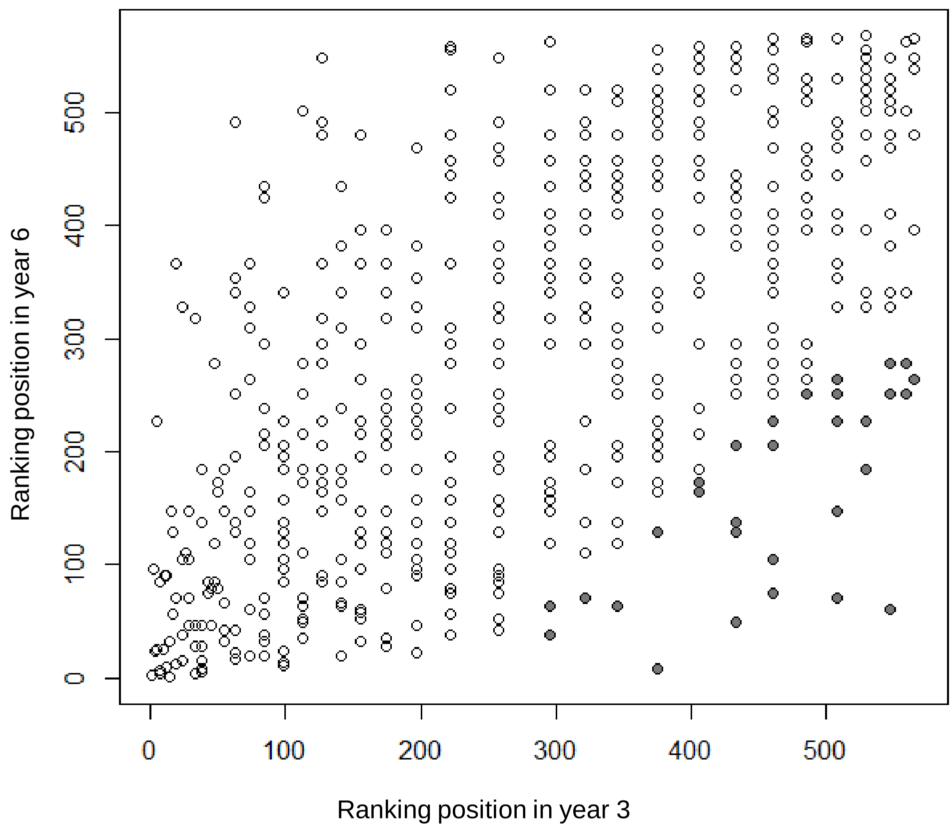


Figure 24. Group IV, students with a large decrease in ranking position from year 3 to year 6.

To compare students that belong to category IV, with those who had the possibility to belong to the category, statistical measures: minimum, quartiles and maximum values, for actual results on the two national tests were analysed. Comparing the statistical measures for different groups of students, shows which students who are most likely to make a large decrease in ranking position. The groups of interest are:

- Group X is defined as the group of students (n=333) with ranking positions 228 to 568 by their actual results on the national test in year 3, that is those who had the possibility to make a large decrease in ranking position from year 3 to year 6,

- Group Y is defined as the group of students (n=333) with ranking positions 1 to 341 by their actual results on the national test in year 6, that is the maximum range of ranking positions for students in Category IV,

- Category IV are the students (n=35) who made a decrease of at least 40 percentage points from year 3 to year 6, and are therefore included in both Group X and Group Y.

The statistical measures for the groups of interest based on their actual total sum on the national test in year 3 and/or year 6 are shown in Table 13.

Table 13

Statistical measures based on the actual total sum on the national test in year 3 and/or year 6 for the students in category IV, Group X and Group Y.

Statistical measures	Group X Actual results on the national test in year 3	Category IV Actual results on the national test in year 3	Group Y Actual results on the national test in year 6	Category IV Actual results on the national test in year 6
Minimum	80	81	9	21
Lower quartile	82	85	54	54.5
Median	85	87	64	64
Upper quartile	88	89	72	72
Maximum	93	93	78	74

Comparing category IV and Group X shows that the distribution of actual results are similar for both groups, see Table 13. Comparing the distribution of actual results between category IV and Group Y shows that students in category IV are not among the lowest achievers on the national test in year 6, due to the difference in minimum value in the two groups, see Table 13. However, the distribution of actual results from lowest quartile to the maximum are similar for category IV and Group Y. These results indicate that students that make a large decrease in ranking position from year 3 to year 6 are not placed among the lowest achievers in year 6.

5.2.1 Interpretations of the results in categories III and IV

There are an equal number of students who decrease in relative achievement as students who increase in relative achievement. The percentage of students who did those large movements is 10% both for those that performed an increase in achievement and for those who made a decrease in achievement.

The results of the analysis of category III and IV show that low achievers in any of the tests did not belong to the groups that performed large movements in relative achievement. Therefore it can be deduced that, as low achievers in the national test in year 3 did not improve their results remarkable, and also students belonging to category IV did not end up among the lowest achievers in the national test in year 6, the results verify that low achievers continue to be low achievers.

5.3 Comparison of the national test in year 6 with the mathematical kangaroo in year 7

Students involved in the comparison of relative achievement on the national test in year 6 and on the mathematical kangaroo in year 7 are those with results in the national test in year 6 and in the mathematical kangaroo in year 7, 264 students, 109 boys and 155 girls. These 264 students are the sample discussed in chapter 4.2. The results of the analysis are to answer research question 2 and 3.

The analysis aims to investigate the number of, and the distribution, of those students who ranked among the top 20%, top 10% and top 5% in one of the tests, but among the bottom 80% in the other test. The students who are among the top 20% in both tests (n=23) are not investigated because one aim of the study is to explore movements in relative achievement, and another aim is to explain why students are high achievers in one test but not in another. To fulfil those aims, comparable groups must be distinct and must not overlap. In addition, to explore differences or movements in achievement, students in an identified group must have made some sort of change in achievement between the measuring points. Those who are among the top 20% in both tests can only have made minor movements in achievement. It is not the aim in this study to explore those students.

Two non-overlapping groups of students are identified for the comparison between the national test in year 6 and the mathematical kangaroo in year 7. Students are either identified to be among the top 20% achievers in the mathematical kangaroo and among the bottom 80% achievers in the national test in year 6, those belongs to what will be called Group 1. The alternative is that students are identified to be among the top 20% achievers in the national test in year 6 and among the bottom 80% achievers in the mathematical kangaroo, those belongs to what will be called Group 2. Group 1 and 2 will be further investigated in study 2.

Table 14 and Table 15 show detailed information of the identified students. The tables show their actual results in the national test in year 6 (Sum6) and in the mathematical kangaroo in year 7 (SumK). The tables also show each student ranking position in each test (Rank6, RankK) and the change in ranking position between the tests (Change in ranking). In the study ranking positions are used, the tables make it possible to compare the ranking positions with actual results for the identified students. In Table 14, students in Group 1 are shown and in Table 15, students in Group 2 are shown.

Table 14

Students ranked top 5% (white), top 10% (light shaded) and top 20% (dark shaded) in the mathematical kangaroo and the bottom 80% in the national test in year 6, 13 boys and 12 girls.

ID	Gender	Sum6	SumK	Rank6	RankK	Change in ranking
S51	female	84	79	169	264	95
S207	male	82	70	152.5	258	105.5
S146	male	90	64	207.5	250.5	43
S197	female	88	64	194	250.5	56.5
S87	female	78	62	122.5	248.5	126
S36	male	89	59	201	241	40
S56	female	81	58	143.5	238	94.5
S84	male	90	57	207.5	233.5	26
S245	male	73	57	85	233.5	148.5
S192	male	51	57	18.5	233.5	215
S230	female	76	56	108.5	229	120.5
S262	male	76	56	108.5	229	120.5
S72	female	84	55	169	225	56
S44	female	77	55	114.5	225	110.5
S231	male	68	55	64	225	161
S106	male	89	54	201	219.5	18.5
S108	female	89	54	201	219.5	18.5
S257	male	88	54	194	219.5	25.5
S2	male	85	54	175.5	219.5	44
S152	female	84	54	169	219.5	50.5
S143	female	88	53	194	213	19
S252	female	86	53	181.5	213	31.5
S255	male	86	53	181.5	213	31.5
S150	female	82	53	152.5	213	60.5
S19	male	81	53	143.5	213	69.5

Table 15

Students ranked top 5% (white), top 10% (light shaded) and top 20% (dark shaded) in the national test in year 6 and bottom 80% in the mathematical kangaroo, 7 boys and 16 girls.

ID	Gender	Sum6	SumK	Rank6	RankK	Change in ranking
S110	female	100	51	261	197	-64
S172	female	99	38	258.5	118	-140.5
S205	male	99	51	258.5	197	-61.5
S201	female	98	41	256.5	140.5	-116
S94	female	96	42	246	149.5	-96.5
S135	female	96	41	246	140.5	-105.5
S157	female	96	31	246	59	-187
S98	female	95	52	241	205.5	-35.5
S145	male	94	39	235.5	126.5	-109
S162	male	94	51	235.5	197	-38.5
S194	female	94	52	235.5	205.5	-30
S198	female	94	46	235.5	174.5	-61
S99	female	93	44	229.5	161.5	-68
S112	female	93	45	229.5	167.5	-62
S82	female	92	47	223.5	180.5	-43
S121	female	92	52	223.5	205.5	-18
S147	male	92	37	223.5	108.5	-115
S222	female	92	52	223.5	205.5	-18
S7	female	91	29	216	50	-166
S216	male	91	48	216	185	-31
S227	female	91	42	216	149.5	-66.5
S236	male	91	37	216	108.5	-107.5
S240	male	91	42	216	149.5	-66.5

Figure 25 visualises the students identified as high achievers on one test but not on the other and shows their distribution in the two tests used for comparison. In Figure 25 a circle placed at the bottom of the diagram represents a student that was ranked low on the mathematical kangaroo, and a circle placed at the top of the diagram represents a student that is highly ranked on the mathematical kangaroo. In the same way, a circle placed to the left in Figure 25 represents a student that was ranked low on the national test in year 6, and a circle to the right represents a student that was ranked high on the national test in year 6. In Figure 25, students belonging to Group 2 are marked with filled circles on the right side of the diagram, and students belonging to Group 1 are marked with filled circles at the top of the diagram. Students who are among the top 20% on both the national test in year 6 and the mathematical kangaroo in year 7 are represented by non-filled circles in the upper right corner: those students are not the object of study here and are therefore the circles are not filled.

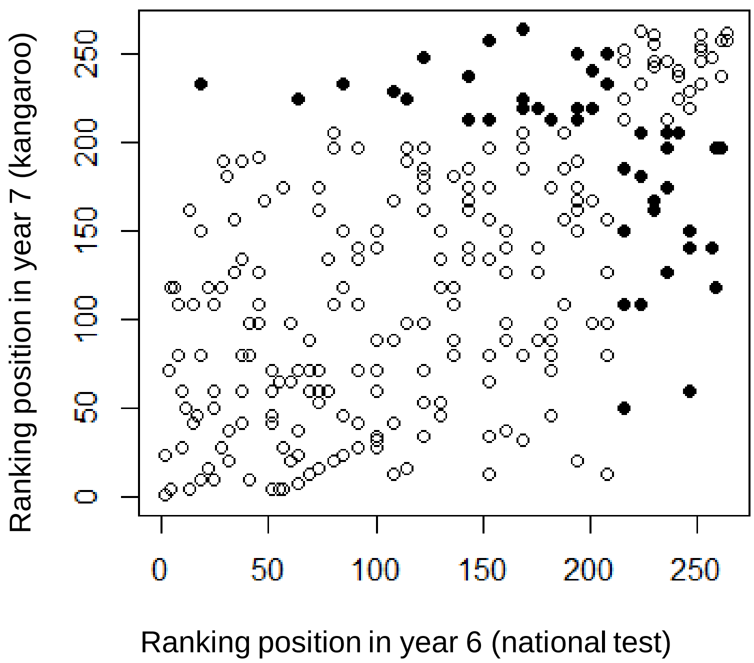


Figure 25. Distribution of students identified to be high achievers in one test but not in the other.

5.3.1 Interpretations of the comparison between the national test in year 6 and the mathematical kangaroo in year 7

Statistical measures used: minimum, lower quartile, median, upper quartile and maximum, see Table 16, for the change in ranking positions in Group 1 and 2 show that students in Group 1 are more widely spread (have a larger variance) in the change of their ranking position as compared to students in Group 2.

For students belonging to the upper half (median to maximum), the distribution of students was broader for Group 1; this is visualised in Figure 25. There is a lack of students in the sample that achieved low in the national test. It is possible that some of those would also have achieved highly on the mathematical kangaroo and thereby be included in the group of students that change their ranking position more than 56.5 places (the median value) between the two tests.

Table 16
Statistical measures of ranking position for the two groups compared.

	Group 1, change in ranking positions.	Group 2, change in ranking positions.
Minimum	18.5	18
Lower quartile	31.5	40.75
Median	56.5	66.5
Upper quartile	110.5	108.25
Maximum	215	187

There is a difference in gender distribution between the groups summarised in Table 14 and Table 15. 13 boys and 12 girls belonged to Group 1. 7 boys and 16 girls belonged to Group 2. This study does not deal with gender differences, but it is still worth noting that in Group 1 the gender distribution is equal, but in Group 2 the majority of the students are girls.

5.4 Summary of results – Study 1

The results of the descriptive analysis of relative achievement in three tests in mathematics shows that the group of students that is among the bottom 10% in both national tests is larger (n=26) than the group of students that is among the top 10% on both national tests (n=15). The analysis of the national tests shows that both tests have a ceiling effect, Figure 10 and Figure 11; this is a possible explanation for why the top 10% group is smaller than the bottom 10% group. There are many students that achieve highly (actual results) on both tests, which leads to a difference of a few points having a large effect on the ranking position. If both tests had a floor effect the opposite result would have been expected. Both tests are mainly aimed to test the passing level, which they do, because they discriminate at the bottom.

The groups of students who either increased or decreased more than 40 percentage points in ranking position were equal in number.

In comparing the national tests given in year 3 and year 6, it can be concluded that the analysis indicates that there are more top students that change their relative achievement than there are bottom students, the ceiling effect of the tests are a plausible explanation. There are approximately the same numbers of students that increase as decrease largely in relative achievement.

Students in Group 1 change in ranking positions between the national test in year 6 to the mathematical kangaroo in year 7 in the range of 18.5 to 215, with a median value of 56.5. The comparative group, Group 2 change in ranking position in a range from 18 to 187, with a median value of 66.5. Students who ranked high in one of the tests therefore show a wider range of achievement in the other test, and there is not a large difference between the two groups.

5.5 Competency profiles in the tests

The results correlated to study 2 are to answer research question 4 and are based on the comparison of two groups, identified in study 1, of how those groups differ in the way they activate mathematical competencies through their result on the mathematical kangaroo. A student is seen to have activated a mathematical competency if she or he has been given points on a task that according to the analyse guide, Table 10, requires the competency to solve the task. The identified groups are;

Group 1 are those students identified to be among the top 20% achievers in the mathematical kangaroo and among the bottom 80% achievers in the national test in year 6.

Group 2 are those students identified to be among the top 20% achievers in the national test in year 6 and among the bottom 80% achievers in the mathematical kangaroo.

5.5.1 The mathematical kangaroo

The analysis of each task in the mathematical kangaroo resulted in the reduced matrixes shown in Table 17. Abbreviations for the competencies are used as follows: App = applying procedures competency, Rea = reasoning competency, Com = communication competency, Rep = representation competency, Con = connection competency, Pro = problem solving competency.

Table 17
Results of the task analysis in the mathematical kangaroo.

Task 1-10/ Competency	1	2	3	4	5	6	7	8	9	10
App	1	0	0	0	1	1	1	1	0	0
Rea	0	0	1	0	0	0	1	0	1	1
Com	1	0	1	1	1	1	1	0	0	0
Rep	0	1	1	0	1	0	0	1	1	1
Con	0	0	0	0	0	0	0	1	0	1
Pro	0	0	1	1	0	0	1	0	1	1

Task 11-21/ Competency	11	12	13	14	15	16	17	18	19	20	21
App	1	1	1	0	0	1	1	0	0	0	0
Rea	0	1	0	1	1	0	1	1	1	1	1
Com	0	0	1	1	0	1	1	1	1	1	1
Rep	1	1	1	1	1	0	0	0	0	0	0
Con	1	1	1	1	1	0	0	0	0	0	0
Pro	0	1	0	1	1	1	0	1	1	1	1

Pairwise comparison of the competencies shows that the reasoning competency is always activated together with the problem solving competency except on one occasion, task 17, which demands the reasoning competency but not the problem solving competency. It is possible to show the problem solving competency in two tasks, 4 and 16, without activating the reasoning competency. The pairwise comparison also shows that it is not possible to activate the connection competency without also activating the representation competency. It is not a purpose of this study to explore the reason why those competencies are related to each other. In the interpretation of the results, it is important to remember the relation between the reasoning competency and the problem solving competency, as well as the relation between the connection competency and the representation competency.

Summary data from the task analysis results in a competency profile of the whole test as shown in Figure 26. The numbers in Figure 26 show how much each competency is given opportunity to be activated in relation to the total amount of possible competencies on each test.

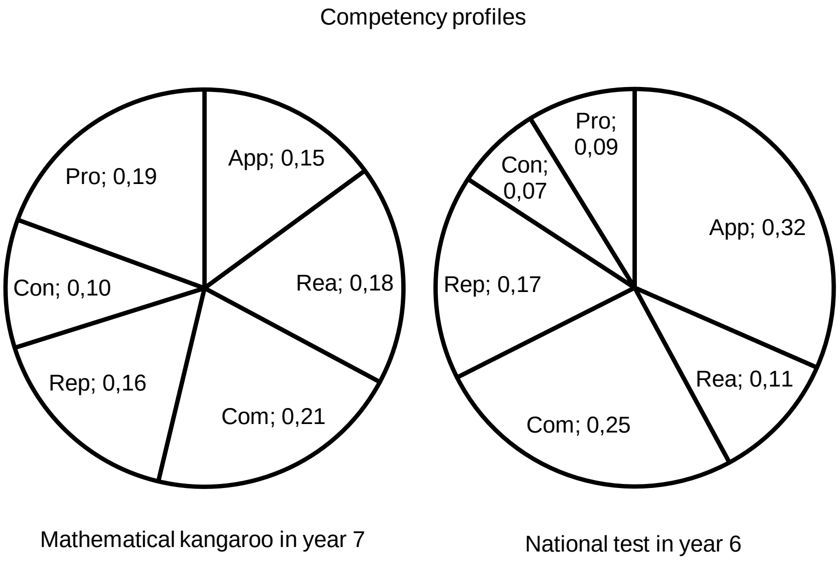


Figure 26. Competency profile for the mathematical kangaroo and the national test in year 6.

The competency profile for the mathematical kangaroo shows that the test gives the greatest opportunity to activate communication competency (21%) and the lowest opportunity to activate the connection competency (10%). The relation between the connection competency and the representation competency shows that taking away one or two tasks that activate the connection competency also lowers the representation competency. The relation between the reasoning competency and the problem solving competency has a similar effect: taking away tasks that activate the reasoning competency lowers the problem solving competency.

5.5.2 The national test in year 6

The national test in year 6 (2012) consists of four parts, A—D. Part A is excluded from the analysis of the competencies. Part A is an oral test that gives 1 or 0 points out of 106 in total. As part A is an oral part and because of its low effect on the results it is not addressed here. The analysis of parts B—D in the national test was made in the same way as the analysis of the mathematical kangaroo. Since the test is classified, results of individual tasks are not shown; the competency profile of the complete test is shown in Figure 26.

5.5.3 Comments on the competency profiles of the tests

The two tests are similar in their competency profiles as regards the communication, representation and connection competencies. The national test gives greater opportunity to activate the applying procedures competency in comparison to the mathematical kangaroo. The mathematical kangaroo gives more opportunity for activating the reasoning and the problem solving competency compared to the national test.

The different ways of analysing problem solving competency can be one explanation for the difference in the problem solving competency. The mathematical kangaroo was analysed by eight experienced year 7 teachers, and the national test is constructed by a group at Stockholm University the "PRIM-gruppen" (Stockholm University, 2013). However the "PRIM-gruppen" also uses experienced teachers as co-constructers and discussion partners (Stockholm University, 2013). There is therefore at least one common factor in the analysis process.

5.6 Mathematical competencies activated by Group 1 and Group 2

Each competency is given a relative activation factor, calculated as the sum of all times that the competency was activated divided by the total sum of activated competencies. Thus the sum of all competencies relative activation factors is 1.

Students who achieved among the top 20% in ranking position in one of the tests, the national test in year 6 (2012) or the mathematical kangaroo in year 7 (2013), and at the same time are among the bottom 80% in the other test are treated as two comparative groups in the following analysis.

In the analysis, one group is called Group 1 that is; “high achievers in the mathematical kangaroo (bottom 80% in the national test)” and the other group are called Group 2 that is, “high achievers in the national test (bottom 80% in the mathematical kangaroo)”.

5.6.1 Favoured competency

To investigate whether one competency is in favour of being activated in relation to the others, in the case of the mathematical kangaroo, a comparison is made of how students activate that competency in relation to the possible outcome of that competency. That a student activate a competency means that the student have been given points on tasks that require that the specific competency is activated according to the analyse guide used.

An example is given in Table 18 and Table 19 of how a competency analysis is done in order to create competency profiles. Table 18 gives the competency profile for the test, and Table 19 gives the competency profile for a specific student. In Table 18, “Sum Comp” is the number of times a specific competency is given the opportunity to be activated through the test; in Table 19, “Sum Comp” is the number of times the specific competency is activated by the student, S71. In both tables, “Rel Comp” tells the activation of each competency relative to the total number of competencies activated.

Table 18

Result of competency analysis in the case of full points.

Competency	Sum Comp	Rel Comp
App	10	0,15
Rea	12	0,18
Com	14	0,21
Rep	11	0,16
Con	7	0,10
Pro	13	0,19
∑	67	1

Table 19

Result of competency analysis of student S71.

S71 Competency	Sum Comp	Rel Comp
App	10	0.17
Rea	9	0.16
Com	14	0.24
Rep	9	0.16
Con	6	0.10
Pro	10	0.17
∑	63	1

To compare how groups of students activate the competencies in a test without actually comparing their test results, a calculation of the relative deviation from the competency profile of the students from the competency profile of the test is introduced.

The relative deviation, denoted f_{cp}, is calculated for each student and each competency using the formula:

$$f_{cp} = \frac{rel.comp_s - rel.comp_t}{rel.comp_t}$$

Where:

$rel.comp_s$ is the number of times a student activate a competency relative to the total number of competencies activated by a student. In the example, it is values from column 3 in Table 19.

$rel.comp_t$ is the opportunities the test gives to activate a competency relative to the total number of competencies the test gives opportunity to. In the example, it is values from column 3 in Table 18.

The values of the relative deviation, f_{cp}, for students belonging to the same identification group, Group 1 or Group 2, are collected and thereafter differences in how groups of students activate each competency can be investigated. In the comparison between the groups each f_{cp} value for each student is represented by a marker in a diagram, one diagram for each group and each competency, see Figure 27 and Figure 28. If a student activates a competency to the same proportion as in the case of getting full points in the mathematical kangaroo, the marker will be placed on the zero line. If the competency proportionally is activated more for a student than in the case of getting full points, the marker is placed above 0; another competency or other competencies will then have to "pay" and will be placed below 0. A point on 0 does not mean that a student has activated the competency as much as possible throughout the test, since it is a proportional measurement.

In the example given in Table 18 and Table 19, the applying procedures competency and the communication competency are activated more relative to the other competencies by the student than they are by getting full points in the test. In the same way the reasoning competency and the problem solving competency are less activated and the representation competency and connection competency are activated to the same extent as in the test profile.

This analysis shows whether students in their competency profile activate a competency more or less than the test profile of the mathematical kangaroo shows. Therefore the analysis shows if groups of students favour or disfavour any competencies in relation to each other. It gives a possibility to explore individuals and patterns of how competencies are activated in the two groups.

Results of this analysis of the students identified are shown for Group 1 to the left and for Group 2 to the right in Figure 27 and Figure 28.

Group 1

The f_{cp} *value for each student for the competencies: App, Rea, Com*

Group 2

The f_{cp} *value for each student for the competencies: App, Rea, Com*

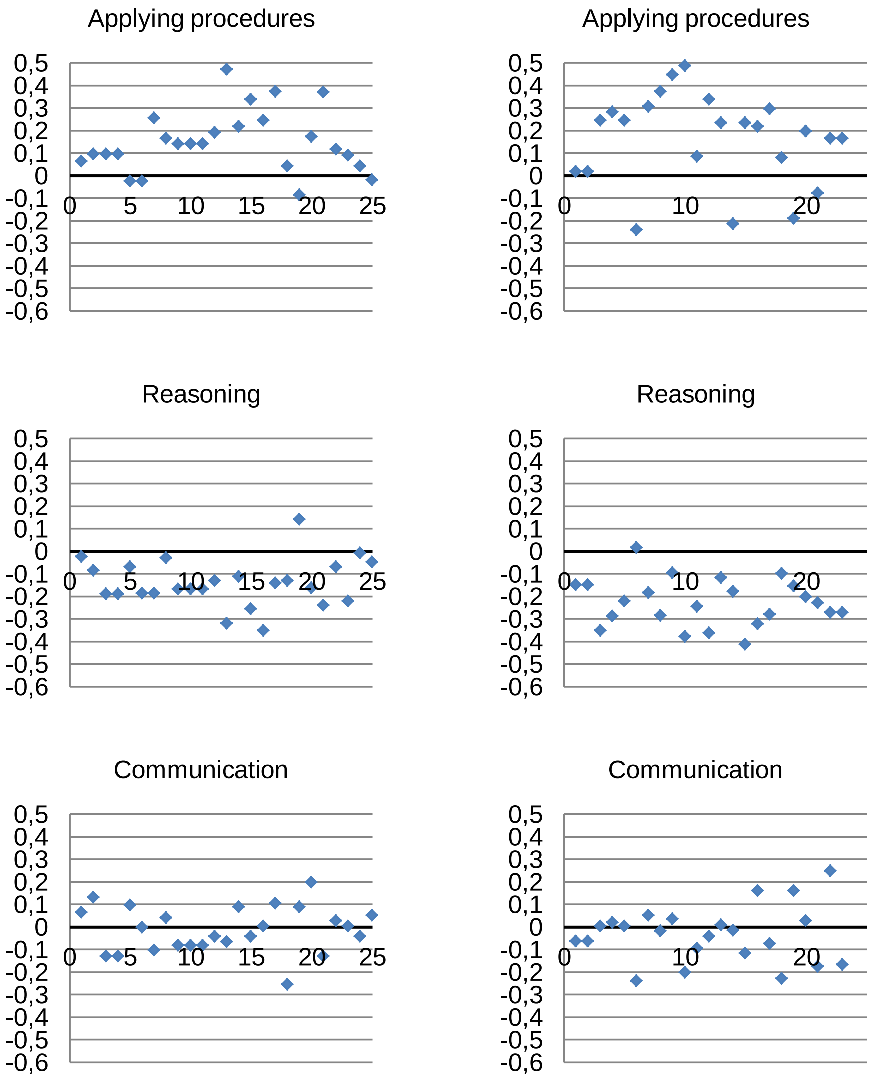


Figure 27. Relative activation of the competencies App, Rea and Com in comparison with the competency profile in the mathematical kangaroo. The x-axis represents students.

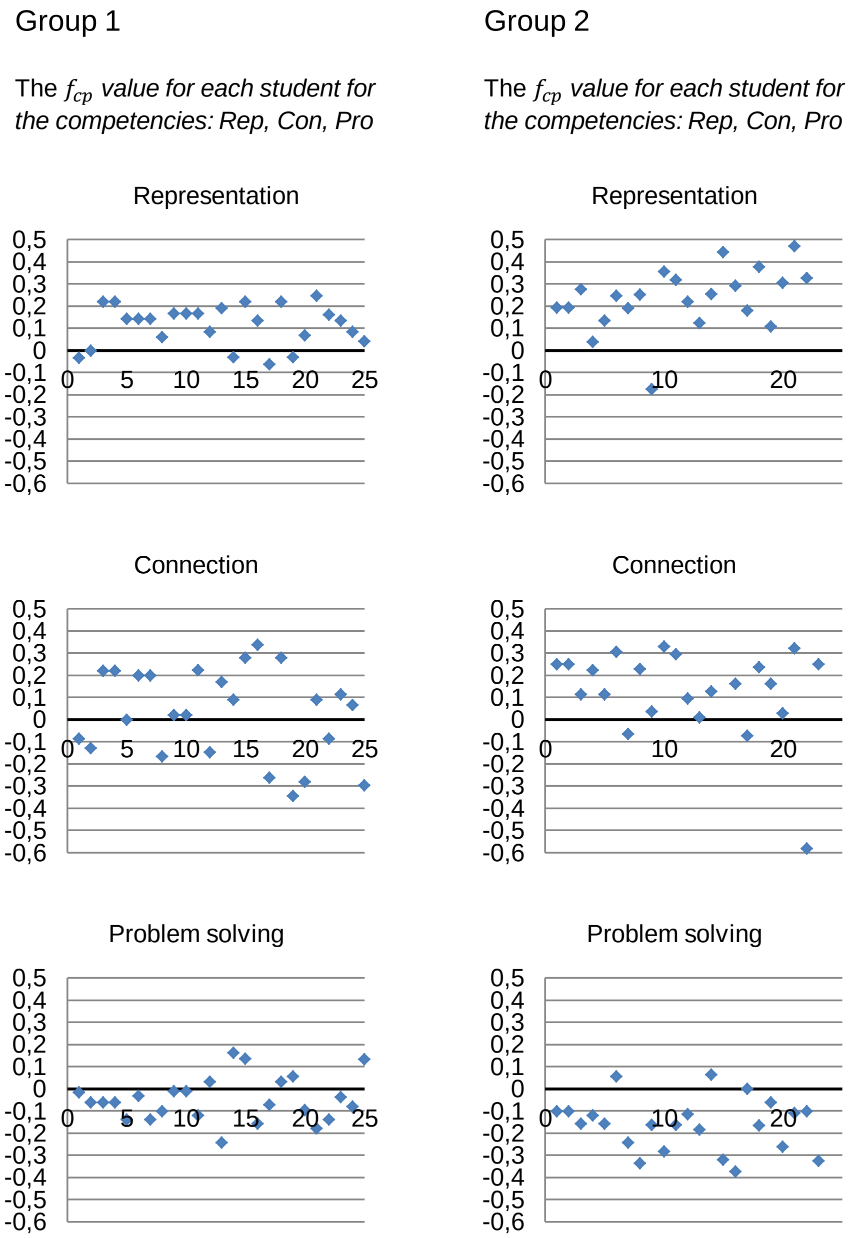


Figure 28. Relative activation of the competencies Rep, Con and Pro in comparison with the competency profile in the mathematical kangaroo. The x-axis represents students.

Figure 27 and Figure 28 visualises the comparison between Group 1 and Group 2 in how they activate the competencies on the mathematical kangaroo. Through the figures it can be suspected that Group 2 activates the applying procedures competency, the representation competency and the connection competency to a greater extent than Group 1, and that Group 1 activate the reasoning competency and the problem solving competency to a greater extent than Group 2. To verify or invalidate the suspicion a Fisher exact test is performed.

5.6.1.1 Fisher exact test in favoured competency analysis

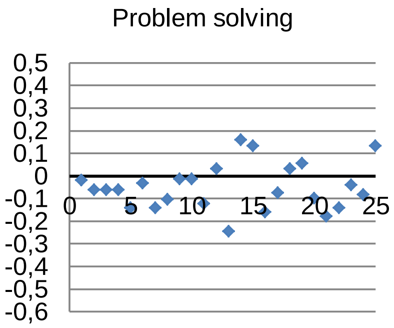
In the favoured competency analysis, competencies activated by individuals shown in their individual kangaroo profiles are compared to the competencies in the competency profile of the test. If a competency is shown more it automatically means that another competency is shown less, because they stand in relation to one another. Because the competency profiles are not related to achievement, the two groups of students identified can be compared to each other.

The two groups, Group 1 and Group 2, are compared to investigate whether any competency is favoured or disfavoured in the mathematical kangaroo. The Fisher exact test is used to investigate whether one group of students significantly favours or disfavours a competency compared to the other group. The Fisher exact test is a dichotomous test that calculates whether there is a significant difference between two groups around a chosen limit, where data are either above or below the chosen limit. The natural choice of limit is zero in this case, since zero means that the competency is activated to the same proportion to which the test gives an opportunity. However, if there is reason to suspect differences in distribution between the groups although they do not show a significant difference around the zero, a new limit can be chosen.

Example:

Group 1

The f_{cp} value for each student for the competency: Pro



Group 2

The f_{cp} value for each student for the competency: Pro

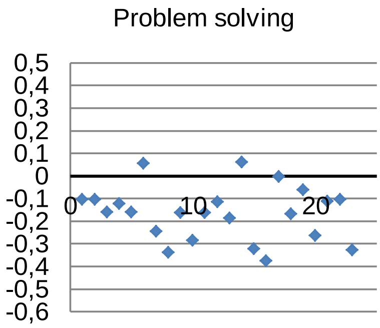


Figure 29. Example of when it can be justified to change the limit of comparison for the Fisher exact test.

It appears in Figure 29 that the f_{cp} values for Group 2 are distributed to a greater extent on the negative side. A comparison of whether there is a significant difference between the two groups around the zero shows that there is not, the hypotheses that Group 1 activates the competency either less or more than Group 2 is not supported (p=0.9678 alternatively p=0.1509). However, when the limit of comparison is changed to -0.1, we see a significant difference, the hypothesis that Group 1 activates the competency more than Group 2 is supported (p=0.0005), indicating that more students in Group 1 are placed above -0.1 compared to Group 2. This means that there are more students in Group 2 that activate the problem solving competency to a lower degree relative to the other competencies as compared to Group 1. If actual achievement is taken into account it is expected that the students that succeed among the best in the mathematical kangaroo also activate the problem solving competency most, since the test offers many opportunities to activate the problem solving competency. Here, the achievement factor is eliminated and the two groups of students are compared to each other with regard to how they activate the competencies relative to each other. It can be concluded that: Group 1 favours the problem solving competency over Group 2 in the mathematical kangaroo despite the achievement factor having been eliminated.

The results of the Fisher exact test are shown in Table 20. In column 2 the p-value tells if the hypothesis that Group 1 activates the competency less than

Group 2 is significant. In column 3 the p-value tells if the hypothesis that Group 1 activates the competency more than Group 2 is significant. In those cases where a difference in distribution between the groups is suspected when looking at Figure 27 and Figure 28 the Fisher exact test is done one more time with a new limit to test around, 0.1 or -0.1. If the new Fisher exact test gives a significant difference between the groups it can be concluded that one of the groups activates that competency more or less than the other group.

Table 20
Summary of Fisher exact test.

Competency	Group 1 activates the competency less than Group 2, p-value	Group 1 activates the competency more than Group 2, p-value
App	0.6971	0.5997
App(0.1 limit)	*0.3611*	*0.8282*
Rea	0.734	0.7757
Rea (-0.1 limit)	*0.9736*	*0.1112*
Com	0.7267	0.4905
Rep	0.114	0.9856
Rep (0.1 limit)	*0.01342*	*0.9985*
Con	0.0365	0.9935
Pro	0.9678	0.1509
Pro (-0.1 limit)	1	0.0004694

5.6.1.2 Comparison between Group 1 and Group 2

The connection competency is activated significantly less among Group 1. No other competency shows any significant difference between the two groups when comparing around the zero. Visually, in Figure 27 and Figure 28, it looks as there are differences in the distribution between the groups in the applying procedures, the reasoning, the representation and the problem solving competency. By raising or lowering the limit in the Fisher exact test when testing those competencies, it is shown that the representation competency is activated less in Group 1 compared to Group 2. This result is expected since the connection competency and the representation competency are related to each other.

Lowering the limit in the Fisher exact test also indicates that the problem solving competency is activated more by Group 1 compared to Group 2. The Fisher exact test with a lower limit on the reasoning competency, which is related to the problem solving competency, does not give a significant difference.

5.6.1.3 Comparison of favoured competencies within Group 1 respective Group 2

In the mathematical kangaroo both groups activate the applying procedures- and the representation competency to a higher degree than other competencies, which means that both groups have to "pay" with other competencies.

Both groups "pay" with the reasoning competency and partly with the problem solving competency, meaning that they activate those competencies to a lower degree than other competencies. However the problem solving competency is more evenly distributed and closer to zero for Group 1.

The communication competency is evenly distributed around the zero for both groups, meaning that both groups activate that competency in the same proportion that the test gives the opportunity to do.

The connection competency is evenly distributed around the zero for Group 1, but activated to a higher degree for Group 2.

The results are summarised in Table 21, where a "+" indicates that the competency is activated more in comparison to the other, a "–" indicates that the competency is less activated and a "0" indicates that the competency is activated to the same degree as expected from the test competency profile.

Table 21

Summary of favoured competency on the mathematical kangaroo.

Competency	Group 1	Group 2
App	+	+
Rea	-	-
Com	0	0
Rep	+	+
Con	0	+
Pro	-	-

5.6.1.4 Interpretations

The analysis of a favoured competency in the mathematical kangaroo shows that Group 1 activate the applying procedures competency and the representation competency more, relative to the other competencies and that they pay with the reasoning competency and the problem solving competency. As a group, they activate the communication competency and the connection competency in the same amount that the test offers. Looking at Group 2, the pattern is the same except for the connection competency, which is activated more in this group, which in turn means that this group must pay to a larger extent with the reasoning competency and the problem solving competency.

The comparison between the two groups shows that Group 1 activate the problem solving competency more than Group 2. It verifies that Group 2 pay more with the problem solving competency. The comparison between the two groups also shows that Group 1 activate the connection competency and the representation competency less than Group 2. This agrees with that Group 2 activating the connection competency more in relation to other competencies, and that the connection competency and the representation competency are closely related.

5.7 Comparison of Group 1 and Group 2 on task level

A comparison of the response rate in percentage for the two groups in the 21 tasks on the mathematical kangaroo is summarised in Table 22. There are 25 students in Group 1, and 23 students in Group 2. The students in Group 1 naturally achieve better on most tasks because of the identification process. Nevertheless, it is interesting to explore whether there are some tasks that show different behaviour than others when comparing Group 1 and 2. To identify tasks that stand out, the difference in response rate between the groups is therefore calculated and is shown in percentage units in Table 22.

Table 22
Comparison of response rate in each task.

Task	Group1 (n=25) response rate (%)	Group 2 (n=23) response rate (%)	%-units between group 1 and group 2
1	100	96	4
2	96	83	13
3	88	83	5
4	88	83	5
5	96	100	-4
6	72	61	11
7	68	30	38
8	96	83	13
9	68	30	38
10	80	83	-3
11	88	91	-3
12	76	65	11
13	76	57	19
14	44	43	1
15	40	17	23
16	72	26	46
17	48	22	26
18	32	17	15
19	96	78	18
20	36	22	14
21	40	13	27

The comparison shows that there are some tasks for which the response rate for group 2 is much lower than for Group 1. Those tasks are number 7, 9, 15, 16, 17 and 21, all with a larger difference in percentage units than 20. The tasks and the competencies they give opportunity to activate are shown in Table 23.

Table 23

Competencies activated by task 7,9, 15, 16, 17 and 21

Task /Competency	7	9	15	16	17	21
App	1	0	0	1	1	0
Rea	1	1	1	0	1	1
Com	1	0	0	1	1	1
Rep	0	1	1	0	0	0
Con	0	0	1	0	0	0
Pro	1	1	1	1	0	1

For those six identified tasks:

- All but one, (no. 17) gave opportunity to activate the problem solving competency,
- All but one, (no. 16) gave opportunity to activate the reasoning competency,
- No one but one, (no. 15) gave opportunity to activate the connection competency.

5.7.1 Interpretations

The six tasks in Table 23 that differ most between the two compared groups, is probably the reason to that Group 1 activate the problem solving competency more than Group 2. A result in this study is that the six tasks are most likely special in some way. One explanation can be that they give students the opportunity to activate the problem solving competency and the reasoning competency, combined with that they do not give opportunity to show the connection competency. However, this could be explained by something other than mathematical competencies offered by the task, although, that question remains for another study to explore.

The problem solving competency and the reasoning competency are closely related, as discussed in chapter 3.3.2. The close relationship explains the results in the pairwise comparison of the competencies made in section 5.5.1, which in turn explains that most of the tasks identified in Table 23 demand both the problem solving competency and the reasoning competency. Reasoning is seen to be the "juridical counterpart" to problem solving (Lithner et al., 2010), and this should make it almost impossible to activate the problem solving competency without activating the reasoning competency. Task number 16 is one of the two tasks on the mathematical kangaroo in which this is possible according to the analysis used in this study, although, following the analysis guide, Table 10, no reasoning competency is considered to be needed in task number 16.

5.8 Summary of results – Study 2

Two groups of students were identified in study 1 as being high achievers, top 20%, in one of the tests, the mathematical kangaroo, or the national test in year 6, but among the bottom 80% in the other test, Group 1 respective Group 2.

How those two groups activated mathematical competencies in the mathematical kangaroo was investigated and compared to explore the differences in achievement. The method used is based on an existing competency framework, MCRF (Lithner et al., 2010), discussed and developed both with in-service teachers and research colleagues. One of the aims of this study was to investigate the method, which has been done relatively thoroughly. When using the method, it can be said that the group of students in Group 1 activates the problem solving competency more in the mathematical kangaroo than students in Group 2. Students in Group 2 activate the connection competency and the representation competency more in the mathematical kangaroo compared to students in Group 1.

The competency profiles of the mathematical kangaroo and of the national test in year 6, Figure 26, show that the mathematical kangaroo gives greater opportunity to activate the problem solving competency, although, it must be remembered that the problem solving competency is not analysed in the same way for the two tests. The reasoning competency that is closely related to the problem solving competency is also given more opportunity for activation in the mathematical kangaroo then in the national test. The connection competency and the representation competency are given similar opportunity for activation in the two tests.

There were six tasks in the mathematical kangaroo out of a total 21 where there was a large discrepancy in the response rate between the two groups. Those tasks have in common that, to come to a solution, the solver needs to activate both the problem solving competency and the reasoning competency or one of those competencies. Both are needed for four of the tasks and one of the competencies is needed for two of the tasks.

The investigated method can compare how two different groups of students activate mathematical competencies through a test. The method can also give a competency profile of a test that says which competencies the test gives an opportunity to activate, and how those competencies are given opportunity relative to each other. The method can be used to analyse individual students and indicate how each individual activates the competencies relative to other students in a test, showing both the strength and the weakness for that individual. Further, the method gives the opportunity to explore tasks of special interest according to which mathematical competencies the solver needs to activate.

6 Conclusions and discussion

This chapter summarises the conclusions of the study in point form. Thereafter follow sections that discuss the study in relation to the ceiling effect of the national tests, movements in achievement, assessment and the problem solving competency. Then comes a section that discusses the method chosen and some alternatives that could have been used. The chapter ends with a section of how this study can influence both practice and research, and suggestions for further research.

6.1 Conclusions

- The national tests given in year 3, 2009, and in year 6, 2012, both had a ceiling effect, which means, when measuring over time, that it is more likely for a student to remain among the bottom achievers than to remain among the top achievers.
- The national tests given in year 3, 2009, and in year 6, 2012, did not guide teachers to discover students that might have needed greater challenge according to their high achievement, because the tests did not discriminate at the top.
- There are students who do not achieve among the top in the national test, some of whom are ranked very low, that, when given a non-curriculum test, achieve among the top students. This group is not negligible in size: in this study the group consists of 9% of the population (25 of 264).
- The problem solving competency is of special interest in the explanation of why some students succeed in a non-curriculum bounded test but not in a curriculum bounded one.
- The method explored that compares tests and groups of students according to activation of mathematical competencies can be used to:

- Analyse mathematical tests to show what opportunity a test gives students to activate mathematical competencies on a task level and to give a summary of the test in a competency test profile.
- On the individual level, show how a student activates mathematical competencies, which can be used to identify a student's weak competencies and strong competencies.
- Compare two groups of students to investigate how they differ in activating the mathematical competencies relative to each other in a test, without taking the achievement factor into account.
- Identify tasks that differ more than others in response rate between the compared groups and to discover what mathematical competencies these tasks give the opportunity to activate.

6.2 Discussion

The main aim of this study was to investigate students that have good mathematical competencies although they are not able to show them in conventional school mathematics. A further aim was to describe the phenomenon in a quantitative approach and to investigate a method that can explain differences in achievement between groups according to mathematical competencies.

6.2.1 Ceiling effect and tests

None of the national tests used in this study discriminates at the top which makes it difficult or perhaps impossible to use those tests to identify students who might need greater mathematical challenge in school mathematics to be able to develop further. The aim of the two national tests involved here are mainly to test the passing level (Skolverket, 2010; 2012). Although, the national tests in mathematics should show the mathematical qualities a student has in the subject (Skolverket, 2014) and, according to Swedish law, students have the right to be challenged further when they meet the requirements of the curricula (SFS 2010:800).

Teachers therefore need some sort of assessment system that also discriminates at the top to be able to identify those students that might need greater challenge. The national test influences the assessment of the students (Korp, 2006) and it is therefore important that it also discriminates at the top. Assessment in education ought to be assessment *for* learning, to stimulate students' learning (A. Pettersson, 2007; Wiliam, 2007).

6.2.2 Movements in relative achievement

With the empirical data and definitions used in this study, it has been shown that it is more certain for a student to stay among the bottom achievers as compared to staying among the top achievers in the national tests. This conclusion is a contradiction to both Gagné (2005) and Pettersson (2010), who both in separate research fields concluded that most students who have once been high achievers continue to be high achievers. Pettersson (2010) draws the same conclusion for low achievers. On the other hand, Häggblom (2000) shows that it is very difficult to predict achievement several years in advance according to how a student achieves in the present on the basis of large movement in achievement, both upwards and downwards. None of us make measurements in the same way, which is most likely the main reason for the different conclusions.

In the present study, there is a ceiling effect in the tests that can explain why there are fewer students who continue to be top achievers compared to those who continue to be bottom achievers. Most of the tests included in the work of Häggblom (2000) do not discriminate at the top, which is a similar situation as in the case that a test has a ceiling effect. Pettersson (2010) uses both specially designed tests and standardised tests in her comparison, and she uses actual achievement; I use relative achievement. Gagné drew his conclusions from six other studies[6] (Chen, Lee, & Stevenson, 1996; Dumay, Coe, & Anumendem, 2014; Marques, Pais-Ribeiro, & Lopez, 2011; Marsh, 1990; Muijs, 1997) that

[6] Verified trough an email conversation with Professor Gagné in Spring 2014.

measured a wide range of factors including cognitive and environmental factors.

My conclusion is that, when discussing achievement, it is very important to take the context, the method and the analysis into consideration. What sort of test is used, is it a normally distributed test or does it have a ceiling or a roof effect? Does the test measure one thing in depth or is it broad? This is a current issue in Swedish schools; some schools are discussing measuring students' knowledge from the age of six to be able to help each child to develop according to his or her potential. If this mapping will be implemented, schools need a variety of tests that are normally distributed to be able not only to find students who need more help but also those who need greater challenges in school.

In Sweden, in terms of support to students in schools, the focus is mostly on students with special needs. If a student is at risk of failing a subject, it is the headmaster's responsibility to produce an action plan for that student in order to help him or her to pass the subject (SFS 2010:800). However, there is one group of students that needs more attention and perhaps also needs an action plan, these being the students who easily reach the goals in school subjects and are most often not challenged at a level that benefits them most. Not being challenged on a suitable level can make it harder for a student to concentrate and to achieve.

6.2.3 Assessment and challenges

Teachers have to analyse and collect evidence concerning their students' knowledge (Wiliam, 2007). However, it is important to use different kinds of assessment tools or tests in a multiple context (Boaler, 2006; Jönsson & Svingby, 2008; Moltzen, 2009). This study shows that there are students that achieve among the top in one kind of test but not necessarily do in another.

The present study describes a group of students that achieves high in a non-curriculum bounded test but not in a curriculum bounded test. Those students might have developed a personal mathematical knowledge (A. Pettersson, 2007) on their own, perhaps through different out-of-school activities. However, the school serves to develop students' official mathematical knowledge (A.

Pettersson, 2007), i.e. knowledge defined by the curriculum that curriculum bounded tests such as the national test measure.

To challenge and motivate a student, the task or the problem has to be on the right level – not too easy, not too difficult - which of course is individual. Especially gifted students are less motivated when they work with tasks or problems that are too easy for them (Nolte, 2012a). To achieve in mathematics, it is therefore important that each student, gifted or not, is given tasks and problems that are challenging for him or her. If the tasks and problems are too easy, this can actually lead to student achieving lower than if they were given more challenging tasks or problems. The student can naturally not be passive if she/he should succeed and achieve highly; the student must be motivated and have endurance and self-discipline (Lucas & Claxton, 2010; Nolte, 2012a). Time limits in the present study did not give the opportunity to explore a student's motivation, endurance or self-discipline. The author is aware that the achievement of a student also depends on the student, not only on the teacher or other environmental aspects.

Challenging a student at the correct individual level is important but is not an easy mission. The fact that in a heterogeneous classroom at year 9 the 15% lowest achieving students possess mathematical knowledge corresponding to expected knowledge in year 4 (Engström & Magne, 2006) does not make it easier. This wide distribution of knowledge in a heterogeneous classroom, together with the fact that the "challenge factor" is important, tells us that it might be appropriate to let classmates work with different tasks and problems on different levels of challenge, at least if all students should get the opportunity to develop as far as possible, which they have the legal right to do (SFS 2010:800).

Students are different: some need one sort of challenge, perhaps in problem solving, while other students need another sort of challenge, perhaps in communicating mathematics. Some students may have good mathematical competencies but are not able to show them in conventional teacher made tests or in the national test. By giving those students opportunities to show their skills in other, different tests for example the mathematical kangaroo, is one way to lift students that are traditionally not noticed by the teacher. It might give the teacher the possibility to, through positive feedback, help the student to gain better self confidence in mathematics and perhaps also in the end help the student to succeed in school mathematics, also improving their official knowledge.

It is important to see the group of students who need to be challenged as fluent and not fixed (Moltzen, 2009). In the present study, the fluidity of students' achievement is seen in the fact that the groups of students who made large movements up or down in relative achievement are large: approximately 10% of those who had the possibility to make large movements did so. It is reasonable to conclude that, if students' achievement is fluent, then the need of challenges and support is also fluent.

6.2.4 The problem solving competency

This study compares two groups with each other, one being of special interest. The group of the greatest interest is Group 1 because those students' achievement is most surprising. The mathematical kangaroo did not affect the students' subject grade as it was done by the students on a normal school day. Competing in the mathematical kangaroo is a very relaxed happening in Sweden, although students are told to do their best. The reason that the competition is relaxed and that it does not affect the students' grade might give the effect that students do not do their best. The situation is the same for all students who participate. It can explain why students that are highly ranked in the national test decrease their ranking position in the mathematical kangaroo - why should they make any effort? Nevertheless, it does not explain why some students who are ranked low and have a low achievement in the national test increase their ranking position in the mathematical kangaroo to become among the top 20% — why should they make any effort?

The problem solving competency seems to be an important factor in the explanation. The mathematical kangaroo offers more opportunity to activate the problem solving competency than does the national test in year 6. Group 1 activate the problem solving competency more than Group 2, and the tasks that differ most in response rate between the two groups are also connected to the problem solving competency.

What is special in the problem solving competency? According to the chosen framework MCRF (Lithner et al., 2010) a problem is a problem if the solution strategy is not known in advance by the solver. Tasks in the mathematical kangaroo are discussed in problem solving situations (E. Pettersson, 2011), and mathematical problems are used in research on gifted students i.e. (Krutetskii, 1976; Nolte, 2012a). Using a problem solving approach can also result in students achieving better and that they continue their mathematics education path by choosing more advanced mathematics courses (Boaler, 2006).

The tasks in the mathematical kangaroo are often mentioned as mathematical problems when talking to people in the field of mathematics, both teachers and researchers. The present study confirms that many of the tasks in the mathematical kangaroo give the opportunity to activate the problem solving competency. One aim of the mathematical kangaroo is to offer interesting challenges (Wettbewerbsbedingungen, 2013). It is important to challenge students with tasks at the right level; the task cannot be too easy or too hard if the student is to be motivated. Nolte (2012a) writes that especially gifted students are at risk of losing motivation if the tasks are too easy. I believe it is plausible that all students are at most motivated when working with tasks at the right level for them. The fact that the mathematical kangaroo aims to offer challenging tasks and that the challenge level is important for gifted students makes it interesting to mention that the mathematical kangaroo has inspired part of a model that aims to identify mathematically gifted students (Pitta-Pantazi et al., 2011).

A conclusion could be that, since the tasks in the mathematical kangaroo are interesting and challenging and to relatively high degree mathematical problems, some students find those tasks challenging and become motivated and, for that reason, achieve better than they normally do in the national test.

6.3 Method discussion

One aim of the study is to investigate a method that can be used to explain differences in achievement and connect this to mathematical competencies. The framework used to analyse mathematical competencies is crucial for the results of this study. This chapter discusses some of the methods used in the study:

Were there other choices? In what way could those choices have affected the study?

6.3.1 Mathematical competencies

The framework chosen to analyse mathematical competencies is the MCRF (Lithner et al., 2010). This is chosen partly because it has been used in Sweden in situations involving national tests (Lithner, 2011).

However analysing tasks can be done in many different ways; for example it is possible to analyse tasks according to the work of Krutetskii (1976). Using Krutetskii (1976) would change the focus from competencies mentioned in school mathematics and curricula to mathematical abilities mentioned as being important for students gifted in mathematics. This shift would have been interesting, but the main aim of the study has to do with students who not are visualised through school mathematics despite their probably possessing good mathematical competencies.

With the aim to connect the study with school mathematics, there are still other choices for analysing the tasks that could have been chosen. For example it could have been possible to analyse what opportunities the tasks in the different tests gives for imitative reasoning and creative reasoning (Lithner, 2008). The results would probably have been different and would have been discussed from another perspective, with different kinds of reasoning in focus. The reasoning framework (Lithner, 2008) is partly used in a study that concludes that learning through creative mathematically-founded reasoning (CMR) is more beneficial for students with low cognitive ability (Jonsson, Norqvist, Liljekvist, & Lithner, 2014). The use of creative and imitative reasoning could make it possible to analyse the differences between the groups in a different light, or as a complement to the one used, to view the phenomena from different perspectives.

6.3.2 Relative comparison

The analysis uses relative measurements, which sometimes makes the comparison complex and difficult to follow. With regard to individuals, the purpose is to, within an individual measure how the competencies are distributed. This means that calculating how many times a competency is activated in relation to the total amount of activated competencies for that individual fulfils the purpose. Another option that at first sight seems most natural is to calculate for each individual how much a competency is activated according to how much the test gives the opportunity for that competency. If that method is used, however, the results would be connected to the individual's achievement in the test. In the comparison of activation of mathematical competencies between the two groups identified, the desire is to eliminate the achievement factor. The groups are identified according to achievement on tests, but the achievement factor is not important when comparing how mathematical competencies are activated.

6.3.3 Empirical data

This study uses already existing data on students' results in the national test. As an alternative, the study could have collected data from national tests in "real time", this would have given a richer material with access to results on the task level and students' solutions. A richer material would have given possibility for a deeper analysis.

6.3.4 Participants

Students participating in this study all come from the same municipality, this choice was made to make it possible to carry out the study. Another way to choose participants would be to distribute the participants over a larger geographic area, either randomly or consciously trying to achieve, for example, an even distribution of socio economics and other factors known to influence achievement. It is difficult to speculate how it would have affected the study since, in the method used here, all public schools in the municipality participated, which means that the participants come from a widely spread background. However, no investigation of their socio economic status or background is involved in the data.

6.3.5 Implementation of the mathematical kangaroo

To be able to smoothly collect as much data as possible, the participants' ordinary teachers were asked to implement the mathematical kangaroo. All teachers were given the same instructions, both verbally and in writing. However, the author was not present at the time of the implementation of the mathematical kangaroo in the schools. Another way to implement the kangaroo and secure the equality of the implementation would have been to either let one person take care of the implementation in all schools or to gather the participants in one single place during the implementation.

6.3.6 Representativeness of the sample

The thesis has a strong focus on showing the representativeness of the sample which is important because the study starts from a whole population and continues to deal with a sample: the validity of the results is therefore directly connected to how well the sample represents the population. Several methods are used see: the box plots in Figure 7 and Figure 8, the frequency graphs in Figure 10 and Figure 11, and a chi-square test 4.2.1.4. The combination of methods both reveals the ceiling effect in the national tests and where there is a lack of individuals in the sample compared to the population. There are other methods that come to mind for measuring the representativeness, such as the t-test. However, the t-test is most often used when data follow a normal distribution, which is not the case for the data in this study.

7 Implications

7.1 Practice

Looking only at high achievement can be wrong when we try to identify students who need greater challenge; for example students from low socio economic homes are disadvantaged (Mattsson, 2013; Silverman & Miller, 2009). Support from schools, teachers and parents is important for nurturing achievement (Nolte, 2012a). Schools need to learn how to reward, support and stimulate those that do not necessarily achieve high in a school subject but still have the potential to develop a talent. Some patterns are of extra importance in helping those students such as supportive adults and opportunities to be awarded honours and take advanced classes (Reis, Colbert, & Hébert, 2004).

The national test aims to mirror the curriculum. Students who succeed in the national test show that they fulfil the curriculum. The national tests are aimed to support teachers in the assessment of students' mathematical knowledge according to the curriculum. Therefore the tests should discriminate both in the bottom and in the top. The national tests used in this study both have a ceiling effect which makes them unsuitable to use as support in the assessment, however, both test aimed to test the passing level.

This study implicate that it is important to take into account if a tests have a ceiling effect if it is used to support assessment. In Sweden the curriculum is goal oriented which means that in theory all students can reach the highest grade. Despite the goal oriented curriculum, in reality there is always a distribution of students' knowledge. To be able to develop their knowledge students need to be challenged in their education, using tests that discriminate both in the bottom and in the top helps a teacher to find the correct level of challenges for each student. According to Nolte (Nolte, 2012a) it is of extra importance to challenge gifted students, why the discrimination at the top can not be omitted.

As shown in this study, there are students who achieve low in the national test but high in the mathematical kangaroo. It might be that they do not have the mathematical knowledge that fulfils the curriculum, but it is still possible that they have some competencies that are important according to the curriculum. According to Krutetskii (1976), one well developed competency can help to outweigh another, although the students probably need some teacher guidance to help one competency compensate for another. Another possibility is that they have learned mathematics outside the curriculum; they have their own informal personal curriculum that does not follow the national official curriculum. This personal curriculum might give them good mathematical competence but still low grades in school mathematics – they do not follow the rules.

The results of this study say that there are students that can achieve in mathematics although they do not do so in traditional school mathematics. The results also indicate that some of those students can be identified by means of the mathematical kangaroo. The study indicates that those students are better in the problem solving competence compared to some of the students who achieve among the best in the national test. This study thus verifies what other studies have held, i.e. (Boaler, 2006; Jönsson & Svingby, 2008): that variation is important not only in the teaching situation but also in assessment situations. This study does not say that using the mathematical kangaroo is the solution and using it does not help teachers to discover all students. The study says that using the mathematical kangaroo as a complement in assessments is <u>one</u> way to find students that possess some mathematical competencies that not are visualised in traditional tests such as the national test.

7.2 Research

Mathematical competencies are important in mathematics education today, partly because of the great influence that international standardised measurements such as PISA (OECD, 2014) have on the curriculum in an international perspective. The mathematical framework of PISA uses the word capability; competency was earlier used (OECD, 2013). It has also become more and more common for countries to use external standardised tests, which has brought equality in education (Jönsson & Svingby, 2008). Equality is good in one perspective because it makes it easier to compare different countries and contexts. However, it is important to remember that education should, according to Swedish law, give opportunities to students to show their competencies and to be challenged (SFS 2010:800). It is therefore not only important to use variation in teaching but also in assessments.

This study shows that one way to "see" students that not are "seen" through national tests is to use a non-curriculum bounded test – for example the mathematical kangaroo. The study also investigates a method that aims to explore differences in achievement according to mathematical competencies. The results say that some students that achieve among the highest in the mathematical kangaroo to a greater extent activate the problem solving competency than do some of the highest achievers in the national test.

The results of the study tell the research society in mathematics education that it is also important to look at non-curriculum bounded assessment activities and how those can be used to lift up and support students that might possess good mathematical competencies.

7.2.1 Further research

The method used in this study can be further developed and explored. In further research, the method can be used in studies that compare different groups of students and how they activate mathematical competencies. Especially the part that analyses tasks according to what mathematical competencies they give the opportunity to activate can be made more reliable, for example by using a larger amount of active teachers to analyse and discuss the tasks or discussing the analysis in greater depth with more research colleagues in mathematics education.

The national tests in the study both had a ceiling effect. It would be interesting to follow movements in achievement over time using tests where the response rates are normally distributed, discriminating both at the bottom and at the top, thereby offering challenges to almost all students by means of that specific test.

The work of investigating what factors there are in tasks that allow one group of students to achieve while another group does not is also important in further investigations. Factors such as mathematical competencies can be investigated in greater depth, but it is also important to investigate other factors such as creativity, challenge level and cognitive aspects such as for example motivation.

However, the most important thing is to develop strategies in teaching and learning situations that aim to challenge each student in the classroom, the non-gifted as well as the gifted. The importance of using tasks that are challenging has already been stated; the next step is to place challenging tasks into the heterogeneous classroom situation and develop strategies that help to create a meaningful school day with opportunities to be a challenge for all students, not least the gifted ones.

8 Words ending the thesis or "What if?"

In a seminar held by Professor Jeppe Skott I was inspired to always ask myself when I have come to a conclusion or make a statement "What if?" to never be satisfied with finding an answer, because there might be another explanation. In the last words of this thesis I want to relate to the "what if?" question, because I am not satisfied; this study has given me more questions than it has answers. At the beginning of the treatment of a "what if?" question, most of the thoughts that come to the mind during the focused work of trying to answer a question exist inside some chosen frames.

The phenomenon that there are students who are high achievers in the mathematical kangaroo but not in the national test is described quantitatively in this study. The phenomenon has been noticed by me and other in-service teachers (Mattsson, 2013). Some teachers have wondered whether students who achieve highly in the mathematical kangaroo but not in the national test are gifted (Mattsson, 2013). This study has not shown in any way that the students identified are gifted or not.

But **what if** they are?

Or…

What if the tasks in the mathematical kangaroo are creative and make students with less cognitive abilities succeed?

Or…

What **if** …?

The questions will never end…

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Appendicies

Appendix 1

Chi-square test comparing the sample and the population through their results in the national test in year 3.

Points	No. of students, actual value, V_A	No. of students in population	Expected value, $V_E = \frac{V_A}{654} \cdot 247$		$\frac{(V_E - V_A)^2}{V_E}$
0-56	1	20	7.553517		5.685905
57-62	4	21	7.931193		1.948544
63-66	3	20	7.553517		2.745015
67-69	7	26	9.819572		0.809606
70-71	6	24	9.06422		1.03588
72-73	8	31	11.70795		1.174322
74-75	8	32	12.08563		1.381173
76	7	20	7.553517		0.040561
77	12	26	9.819572		0.484162
78	8	28	10.57492		0.626977
79	13	31	11.70795		0.142586
80	21	52	19.63914		0.094298
81	11	33	12.4633		0.171805
82	10	26	9.819572		0.003315
83	11	29	10.9526		0.000205
84	19	40	15.10703		1.003187
85	16	29	10.9526		2.326046
86	17	32	12.08563		1.998329
87	11	30	11.33028		0.009627
88	11	25	9.441896		0.257119
89	12	25	9.441896		0.69307
90	13	20	7.553517		3.927201
91	11	20	7.553517		1.572545
92-93	7	14	5.287462		0.554668
Sum	247	654		calc	28.68615
				df	23
				crit (0.05)	35.172

Appendix 2

Chi-square test comparing the sample and the population through their results in the national test in year 6.

Points	No. of students, actual value, V_A	No. of students in population	Expected value, $V_E = \frac{V_A}{611} \cdot 264$		$\frac{(V_E - V_A)^2}{V_E}$
0-32	3	21	9.07365		4.065533
33-40	2	19	8.209493		4.696733
41-45	6	21	9.07365		1.041182
46-51	9	24	10.36989		0.180965
52-55	9	23	9.937807		0.088499
56-58	3	21	9.07365		4.065533
59-60	7	21	9.07365		0.473902
61-62	8	22	9.505728		0.238511
63-65	8	26	11.23404		0.931012
66-67	6	24	10.36989		1.841476
68-69	9	19	8.209493		0.076119
70-71	8	26	11.23404		0.931012
72-73	9	29	12.53028		0.99462
74-75	18	36	15.55483		0.384374
76-78	22	29	12.53028		7.156715
79-80	12	24	10.36989		0.256249
81-82	18	33	14.25859		0.981733
83-84	16	32	13.82651		0.341665
85-86	12	22	9.505728		0.654489
87-88	13	29	12.53028		0.017608
89-90	13	21	9.07365		1.699011
91-92	15	23	9.937807		2.578617
93-94	12	21	9.07365		0.94378
95-97	17	28	12.0982		1.986051
98-106	9	17	7.345336		0.372742
Sum	264	611		calc.	36.99813
				df	24
				crit (0.05)	36.415

Efter underskrift, lämna till matematikläraren på skolan så snart som möjligt.

Samtyckesformulär för elev

Forskare på matematikavdelningen vid Karlstads Universitet vill jämföra ditt resultat på Kängurutävlingen med dina tidigare resultat på nationella prov i matematik.

När jämförelsen görs är ditt namn och personnummer kodat. Ingen, inte ens den som gör jämförelsen, kommer veta vem som hör ihop med resultaten. Ditt personnummer kommer genom ett dataprogram bli en kod som ser ut ungefär så här:

8f3645edc7852a51cd251c9fda56d682

Du tar själv ställning till om du tycker det är okej att använda ditt resultat eller inte. Enligt personuppgiftslagen har du rätt att en gång varje år ta kontakt med oss på universitetet för att ta reda på vilka uppgifter vi har om dig.

☐ **JA**.

Jag samtycker till att Karlstads universitet behandlar personuppgifter om mig i enlighet med det ovanstående.

☐ **NEJ.**

Jag samtycker **inte** till att Karlstads universitet behandlar personuppgifter om mig i enlighet med det ovanstående.

____

Ort och datum

____

Ditt personnummer

____

Underskrift

____

Klass och namnförtydligande

Karlstads universitet, 651 88 Karlstad
Universitetsgatan 2. Telefon 054-700 10 00 , Telefax 054-700 14 60 .
Postgiro 78 81 07-1 Org. nr. 202100-3120. www.kau.se

Appendix 4

Efter underskrift, lämna tillbaka till skolan med underskrift så snart som möjligt, tack.

Samtyckesformulär för vårdnadshavare

Vi bekräftar härmed att vi tagit del av informationen om deltagande i en forskningsstudie där resultatet på Kängurutävlingen kommer att användas. Resultatet jämförs med resultat på ämnesprovet i matematik för årskurs 3 och årskurs 6.

Du har enligt 26 § personuppgiftslagen (1998:204) rätt att, en gång per kalenderår, efter skriftligt undertecknad ansökan ställd till oss, få besked om vilka personuppgifter om dig som vi behandlar och hur vi behandlar dessa. Du har också rätt att enligt 28 § personuppgiftslagen begära rättelse i fråga om personuppgifter som vi behandlar om dig.

☐ **JA**.

Jag samtycker till att Karlstads universitet behandlar personuppgifter om mitt barn i enlighet med det ovanstående.

☐ **NEJ.**

Jag samtycker **inte** till att Karlstads universitet behandlar personuppgifter om mitt barn i enlighet med det ovanstående.

Ort och datum

Elevens namn och personnummer

Elevens klass

Underskrift vårdnadshavare 1

Underskrift vårdnadshavare 2

Karlstads universitet, 651 88 Karlstad
Universitetsgatan 2. Telefon 054-700 10 00 , Telefax 054-700 14 60 .
Postgiro 78 81 07-1 Org. nr. 202100-3120. www.kau.se

Appendix 5

Information till lärare om forskningsstudie om elevers prestationer i matematik

Denna informations riktar sig till dig som undervisar i matematik i årskurs 7 i X kommun och avser en förfrågan om att bistå med utdelning av informationsblad och samtyckesblankett till dina elever samt att bistå med genomförandet av Kängurutävlingen.

Bakgrund och syfte

Denna studie vill jämföra hur elever presterar i matematik när matematikkunskaper mäts på olika sätt. Studien avser att jämföra resultat på de svenska nationella ämnesproven i matematik i årskurs 3 och 6 med resultat på det internationella matematikprovet ”Kängurutävlingen”. Målet är att bidra med kunskap om elevers behov av olika matematikundervisning, en del i vägen att ge alla elever matematikundervisning på den nivå respektive elev behöver.

Deltagande

Förfrågan om deltagande i studien går till samtliga elever som gick i årskurs 3 i skola i X kommun läsåret 2008/2009 och som nu går i årskurs 7.

Kängurutävlingen

Kängurutävlingen genomförs under mars–juni månad 2013 under skoltid av elevernas ordinarie lärare. Resultatet på tävlingen ska inte påverka elevens betyg eller bedömning av elevens skolprestation. De vårdnadshavare eller elever som inte vill att deras resultat ska tas med i studien kommer inte att ingå i studien.

Lärarnas insats

Din insats består i att dela ut informationsblad och samtyckesblanketter till eleverna samt att ta in dessa. Vi hämtar dem när de är insamlade. Du låter sedan dina elever delta i Kängurutävlingen under lektionstid. Vi distribuerar tävlingsmaterial till skolan.

Rättning av provet görs av oss. När de är rättade lämnar vi tillbaka proven till dig. Om du inte har möjlighet att genomföra provet under lektionstid så kan vi medverka. Tag då kontakt med oss.

Hur går studien till?

Proven distribueras och rättas av forskarna på Karlstads universitet. Resultatet jämförs sedan med resultaten på ämnesproven i matematik i årskurs 3 och årskurs 6.

Appendix 5

Hantering av data och sekretess

Alla personuppgifter kommer att kodas av forskargruppen innan databearbetning och analys görs. Ingen obehörig kommer att kunna ta del av uppgifterna. Allt material kommer att förvaras inlåst i dokumentskåp på universitetet.

Studien förväntas vara avslutad under hösten 2014. Materialet och kodnyckeln kommer att förstöras enligt riksarkivets föreskrifter i statliga myndigheters forskningsverksamhet. Enligt Personuppgiftslagen (1998:204) har vårdnadshavare rätt att en gång per kalenderår få besked om vilka uppgifter som finns lagrade om sitt barn.

Hur får jag information om studiens resultat?

Studiens resultat kommer att publiceras i en licentiatavhandling under hösten 2014.

Frivillighet

Allt deltagande i studien är frivilligt och eleven och/eller dess vårdnadshavare har rätt att när som helst och utan särskild anledning välja att avbryta deltagandet i studien. Elevens resultat kommer då att raders från materialet. Kontakta i sådana fall någon av forskarna.

Ansvariga

Karlstads universitet är forskningshuvudman och personuppgiftsansvarig. Ansvarig för genomförandet av studien är docent Arne Engström. Insamling och bearbetning av alla data genomförs av forskarstuderande Elisabet Mellroth. Undrar ni över något är ni välkomna att kontakta oss.

Hälsningar

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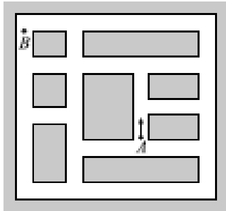
Appendix 6

Competency analysis on Task no. 3 in the mathematical kangaroo Benjamin 2013

3 Nick håller på att lära sig cykla i trafiken. Han får svänga till höger, men han får inte svänga till vänster. Hur många gånger måste han minst svänga för att komma från A till B?

A: 8 B: 4 C: 6

D: 3 E: 10



A reasonable and required solution is that a student tries different tracks and come to a conclusion of which one that is the shortest.

Competency	Question to ask in the analysis of the task. *For all competencies, if the answer is;* *Yes – the classification is '1'* *No – the classification is '0'*	Classifi-cation '0'	Classifi-cation '1'
Applying procedure (App)	It is not necessary to involve a sequence of mathematical actions.	0	
Reasoning (Rea)	It is necessary to justify or argue mentally that the chosen track is the shortest.		1
Communication (Com)	The words in the task are every-day language, but the sentence is mathematically constructed, "*How many times must he at least turn*" It is necessary for the student to correctly interpret the mathematical language to solve the task.		1
Representation (Rep)	The picture is a representation of a map (a map is seen as a mathematically entity), the task cannot be solved without the map.		1
Connection (Con)	No connections between mathematical entities or representations are necessary to do for solving the task.	0	
Problem solving (Pro)	According to the teachers this was a task were the solution process was not known in advanced.		1

The analyse results in a reduced matrix for task 3.

Task 3	Class
App	0
Rea	1
Com	1
Rep	1
Con	0
Pro	1

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High achiever! Always a high achiever?

This thesis describes a study based on teacher observations of students who achieve highly on the international competition 'the mathematical kangaroo' although they do not in the national test. The aim with the study was to investigate students' relative achievement in mathematics over time and how mathematical competencies can be used to explore differences between groups of students on a non-curriculum based test in mathematics. The study was divided in two parts. Study 1 compared students' (n=568) relative achievement in two national tests in mathematics (years 3 and 6), changes in relative achievement between the two tests as well as differences in relative achievement between the national test in year 6 and the mathematical kangaroo in year 7 (age 13) were explored. The study identified two groups of students with high achievements, in only one of the tests, from a sample (n=264) of study 1. Study 2 explored how differences between those students' relative achievement on the mathematical kangaroo could be explained through activation of mathematical competencies.

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