High-field states of Dirac-like electrons in graphene and bismuth

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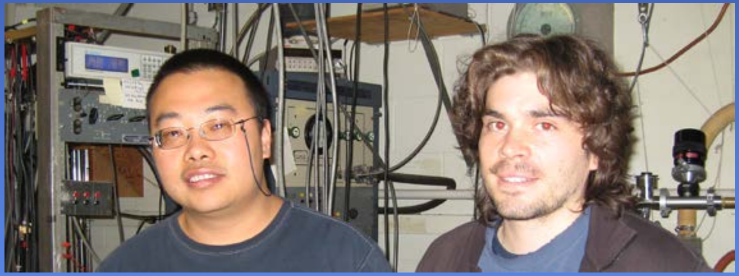
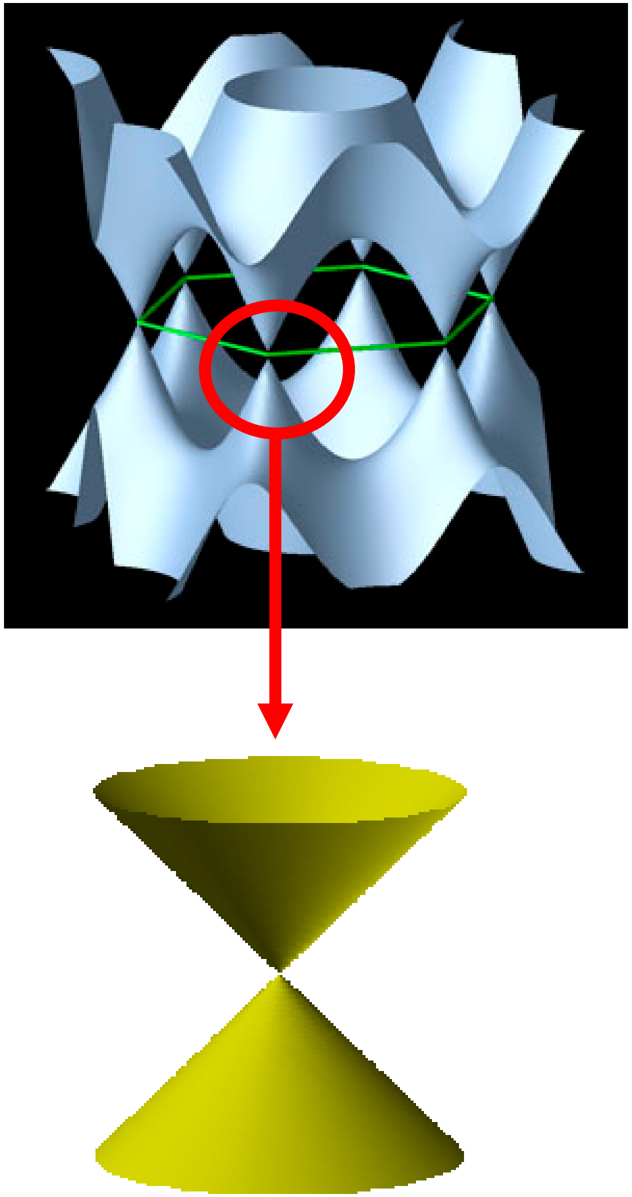
A. Hebard, *U. Florida, Gainsville*

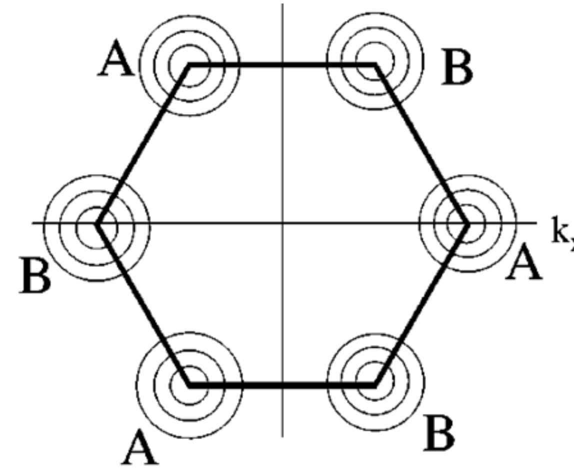


1. Dirac point in graphene in high fields
2. 3D Dirac ellipsoids in Bi
3. Phase transitions in high field

Princeton seminar Mar 2008

Momentum Space





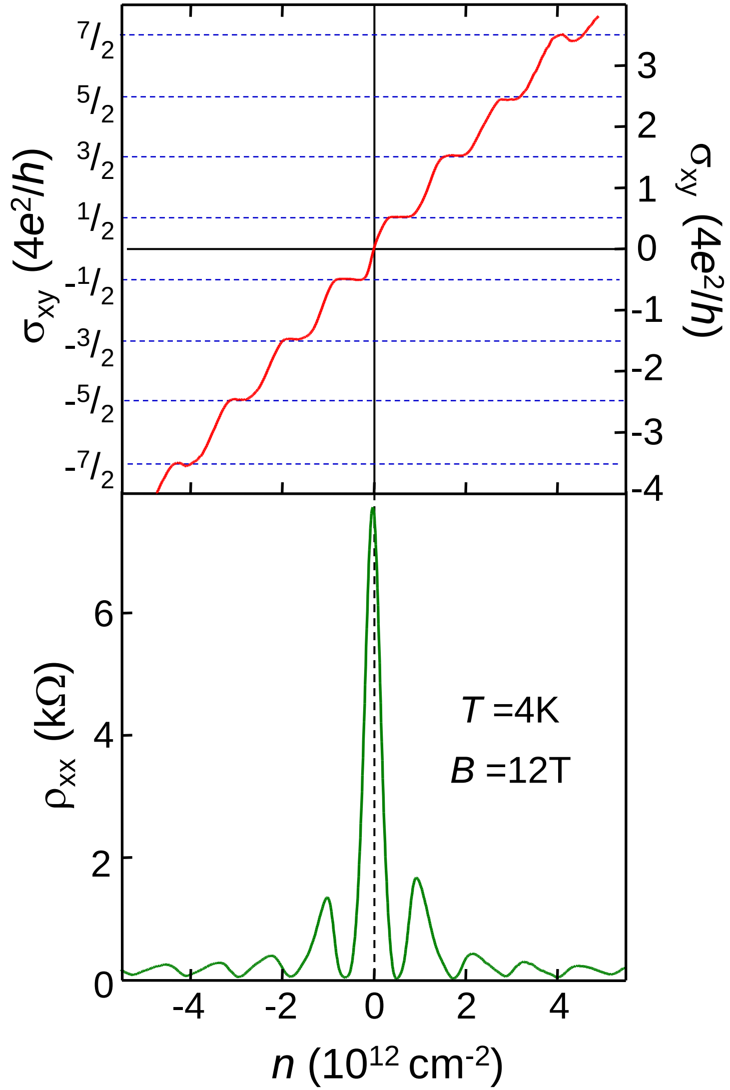
$E = v_0|\,\mathbf{p}\,|$ massless Dirac spectrum

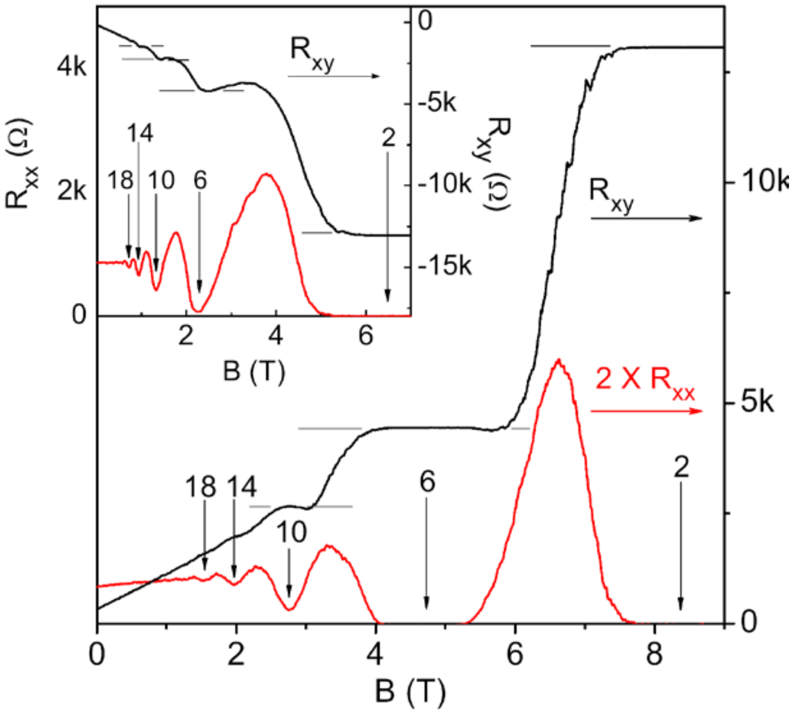
$v_0 = 10^6$ m/s $= c/300$

"Fine-structure" const $\alpha \sim 3$

Orbital quantization in a few Tesla
(need 10^9 T in vacuum)

Quantum Hall Effect in graphene





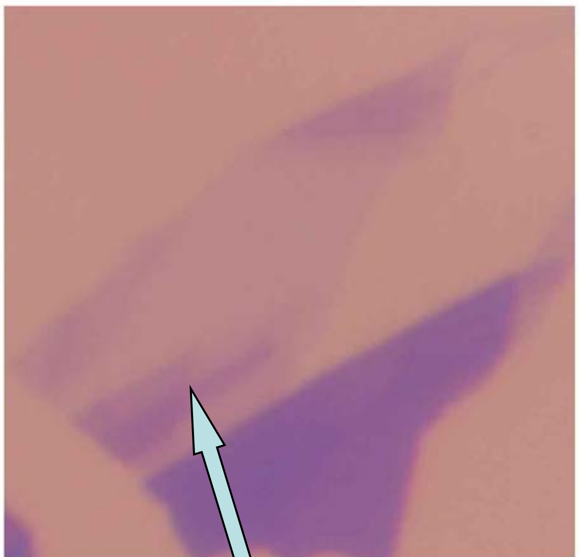
Y.Zhang, Kim et al., Nature **438**, 201 (05)

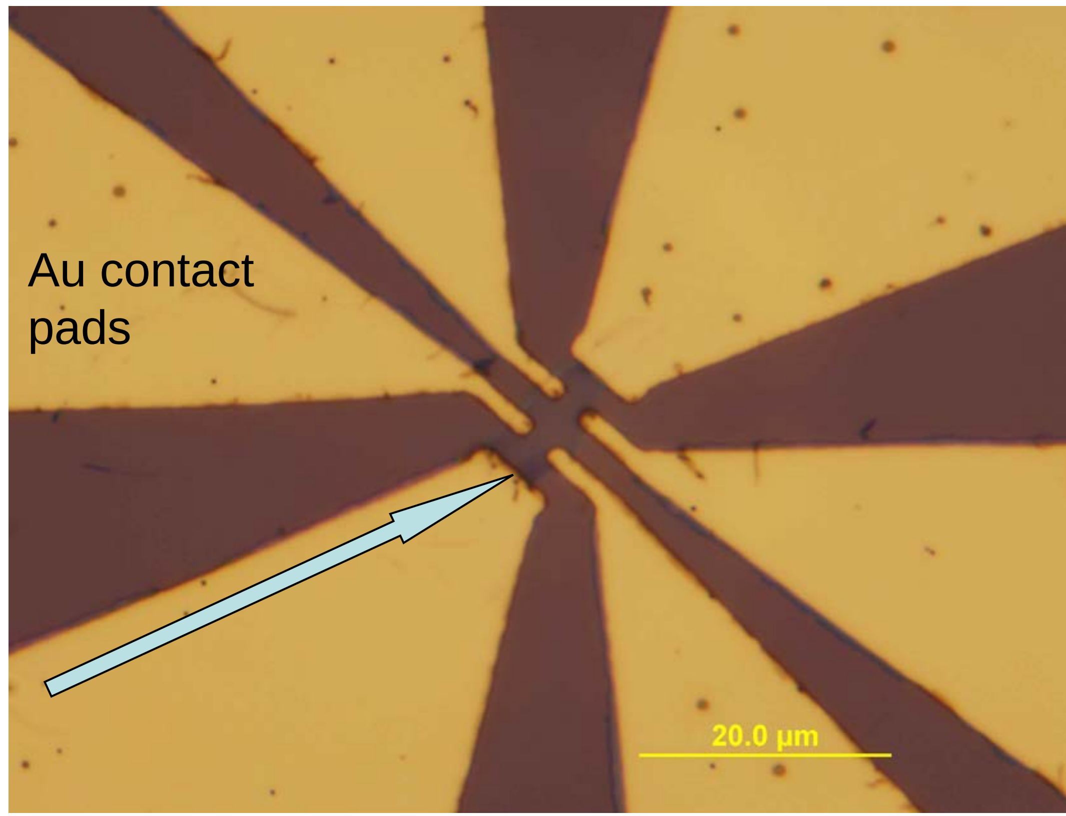
Novoselov, Geim et al., Nature **438**, 197 (05)

Single atomic-layer graphene

- Graphene sheets peeled off onto Si/SiO_2 wafers
- Single atomic-layer samples identified
- Au contacts attached by e-beam lithography

Checkelsky, Li, Ong

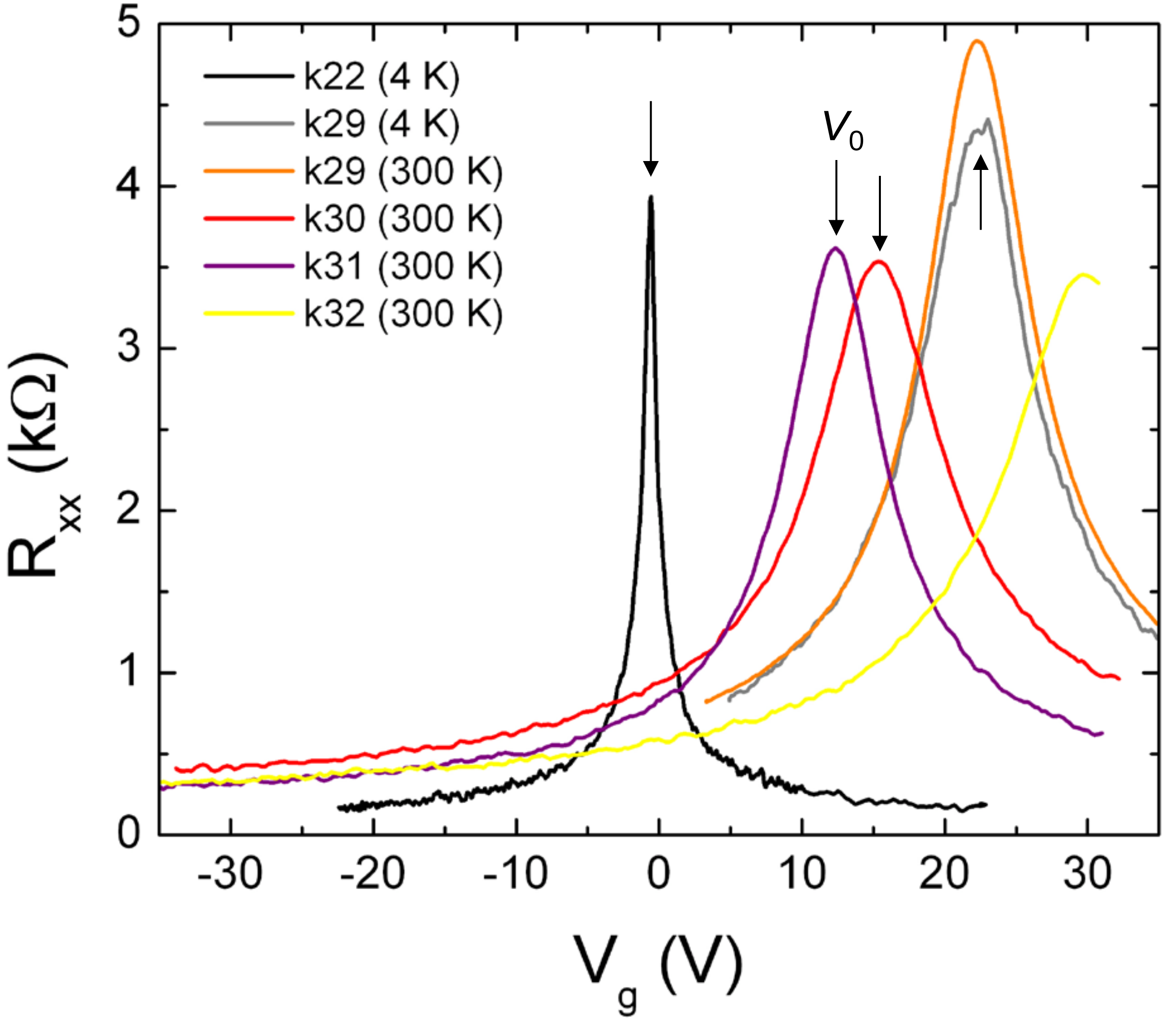




Single atomic-layers


20.0 µm

Offset Gate voltage V_0 -- an important parameter



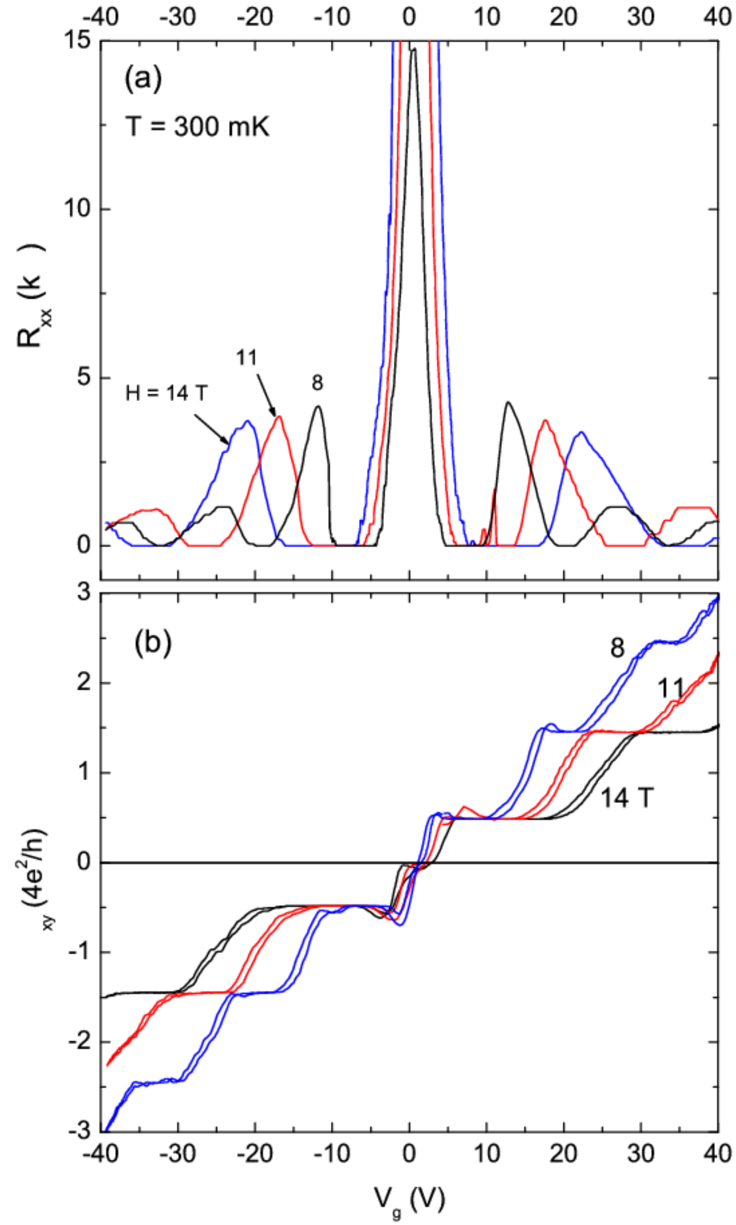
Small offset V_0 correlates with low disorder

The Quantized Hall Effect in graphene

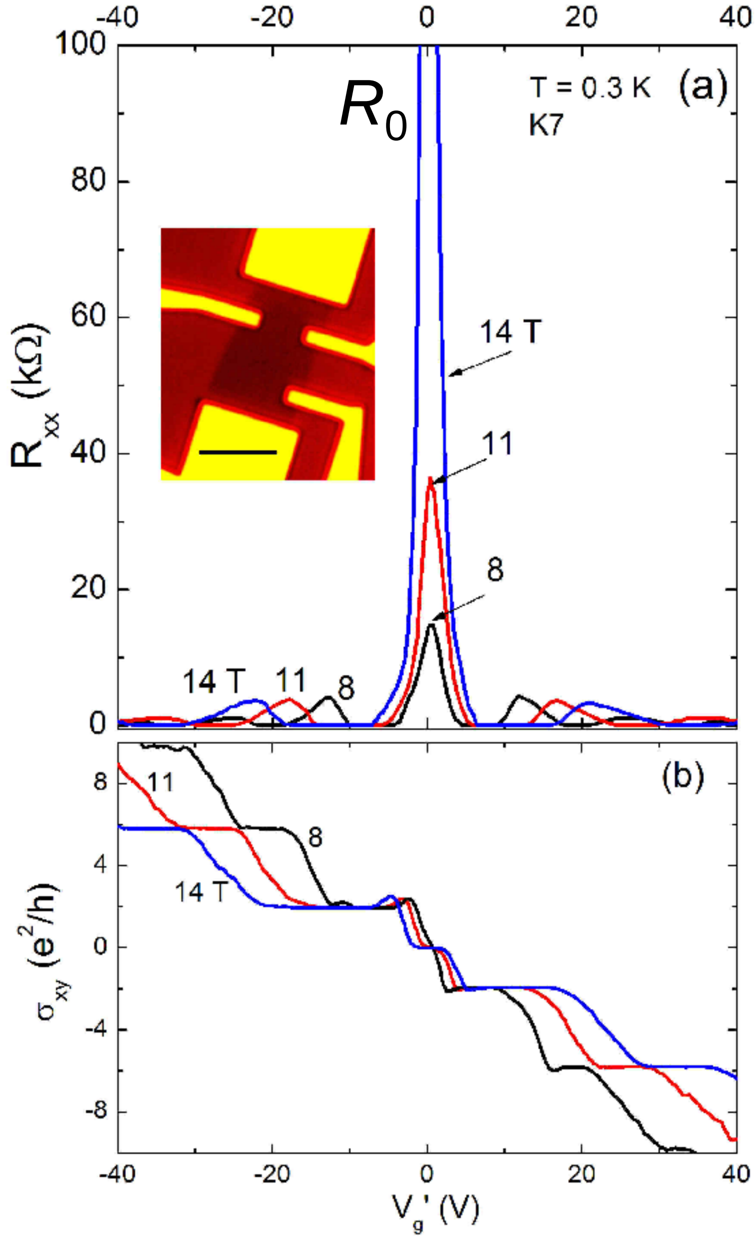
Checkelsky, Li, Ong

Panel (a) Resistivity R_{xx} of graphene vs gate voltage V_g at fields H = 8, 11 and 14 T. R_{xx} peaks at Landau Levels n = 0 and +1 and -1. The peak at n = 0 is singularly large. Temperature fixed at 0.3 K

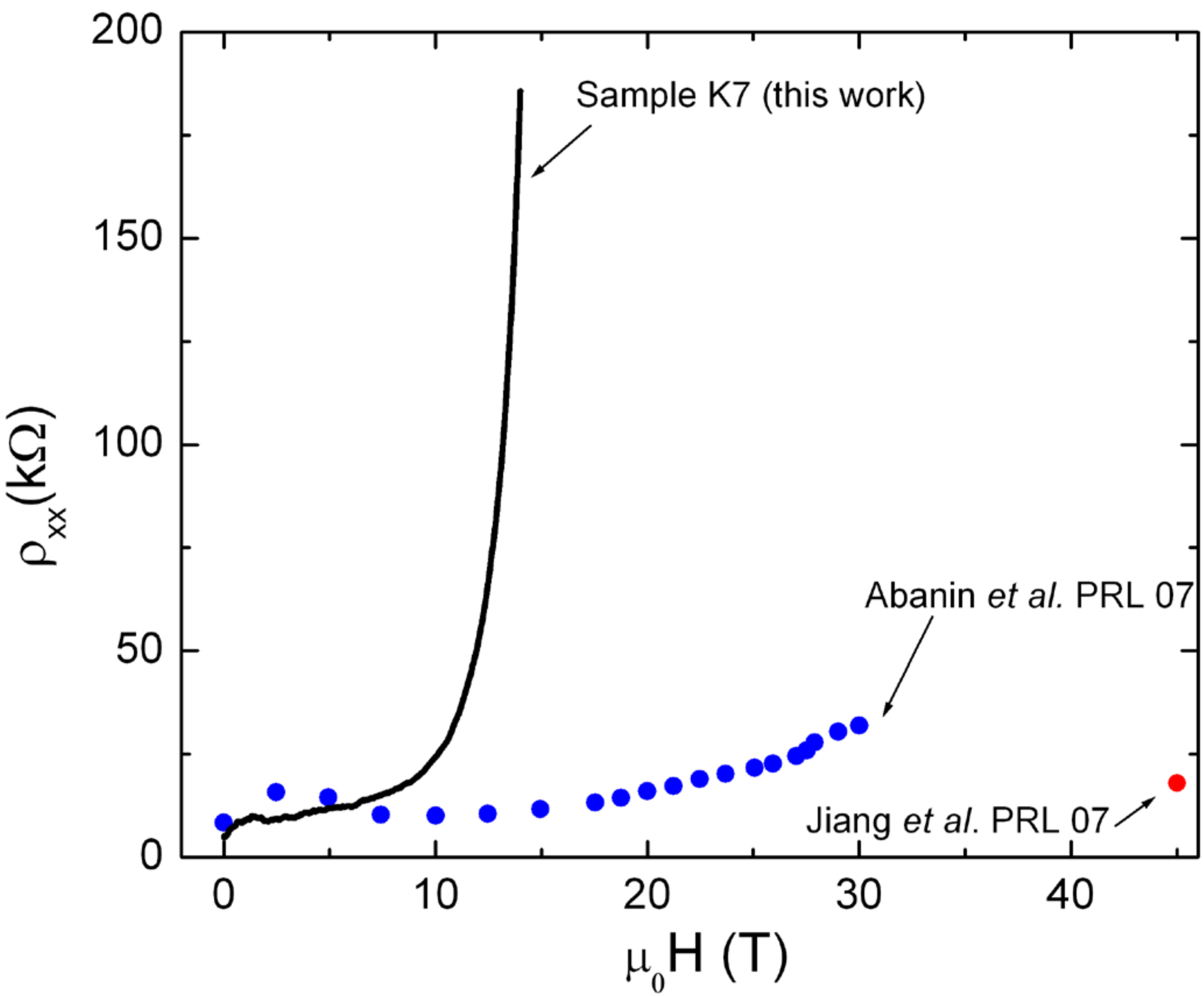
Panel (b) The Hall conductivity σ_{xy} shows step-quantization at universal values 0, $2e^2/h$, $6e^2/h$,…



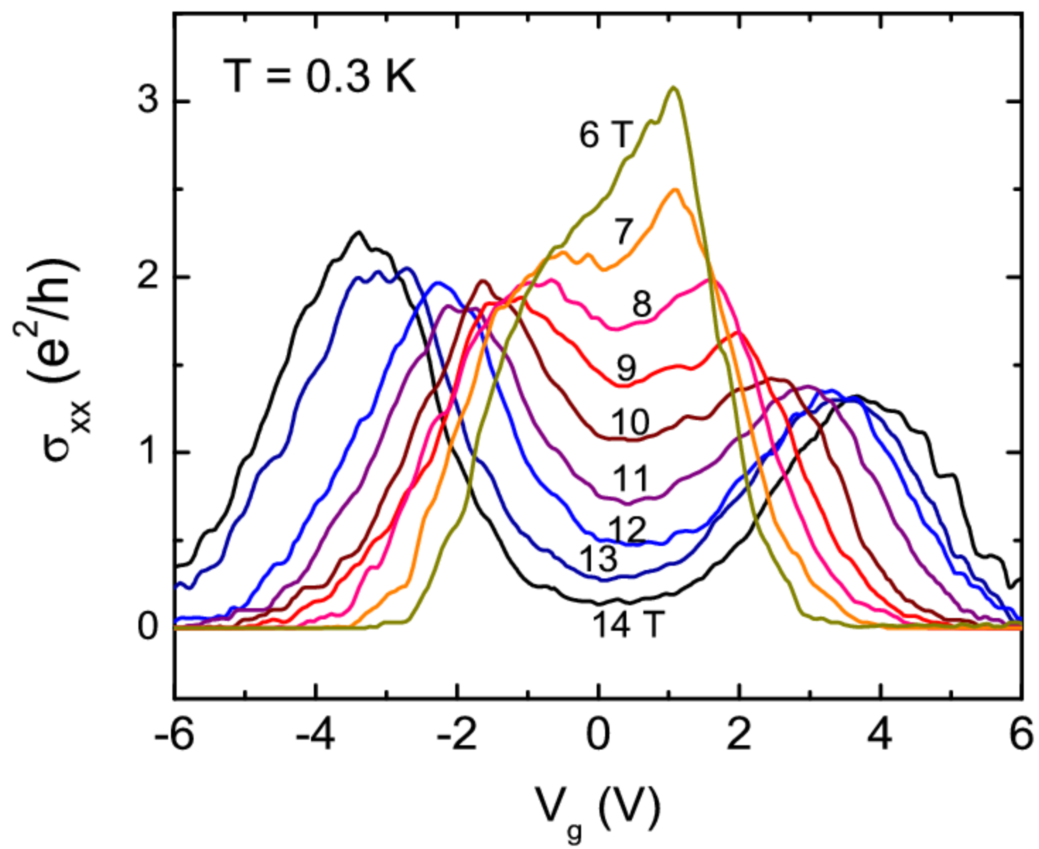
Resistance at Dirac point R_0 diverges with H

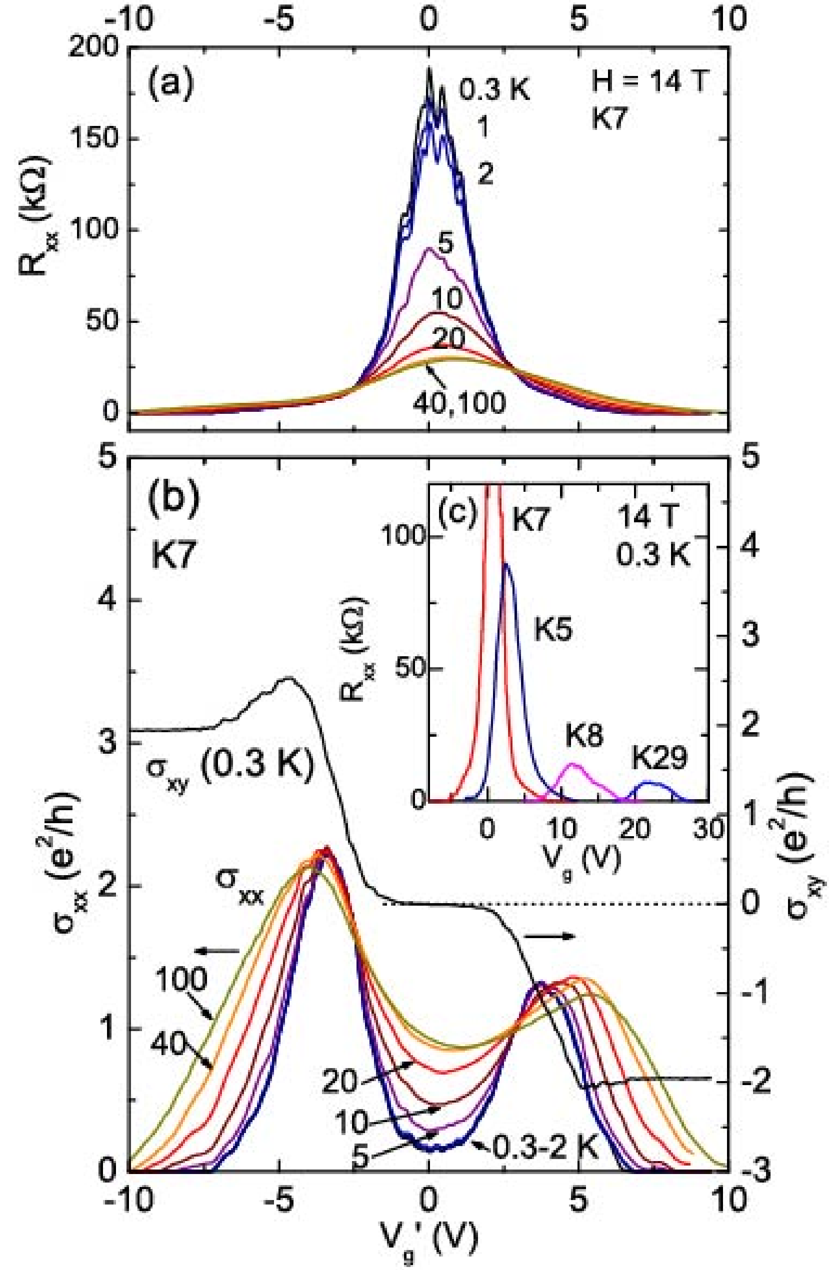


Comparison of divergent R_0 with earlier reports

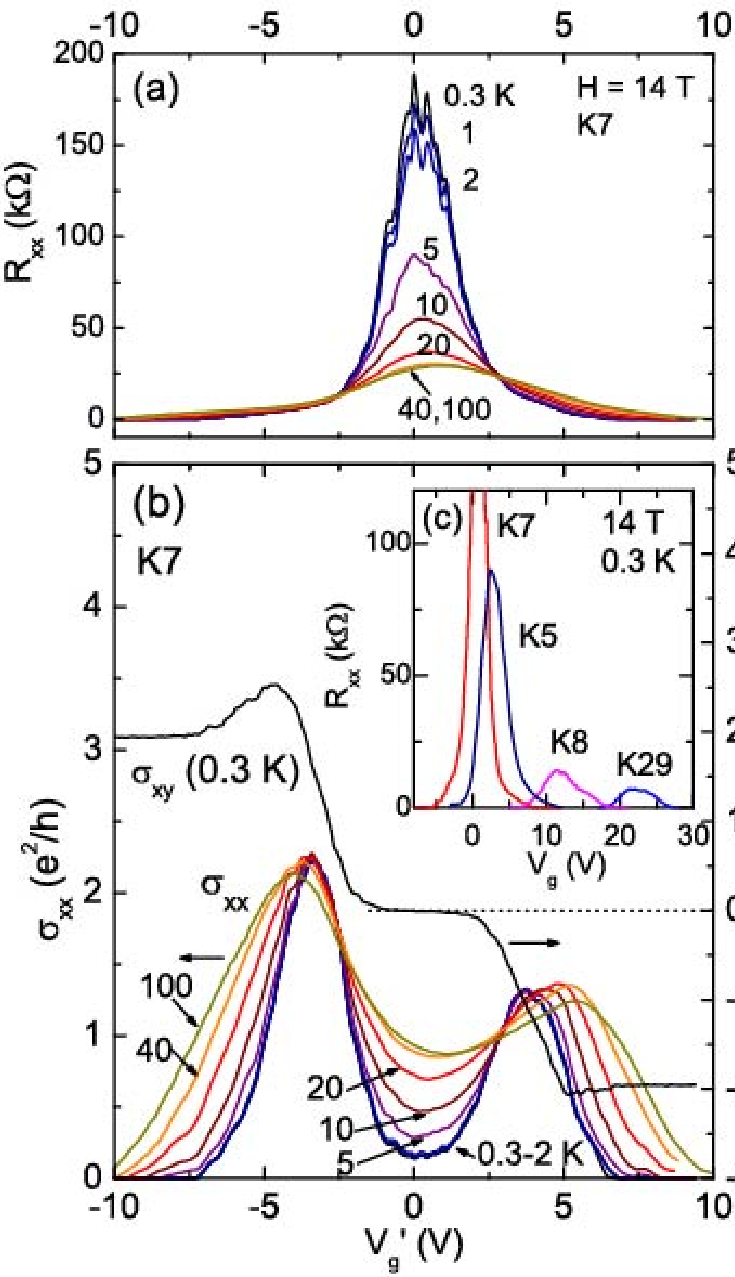


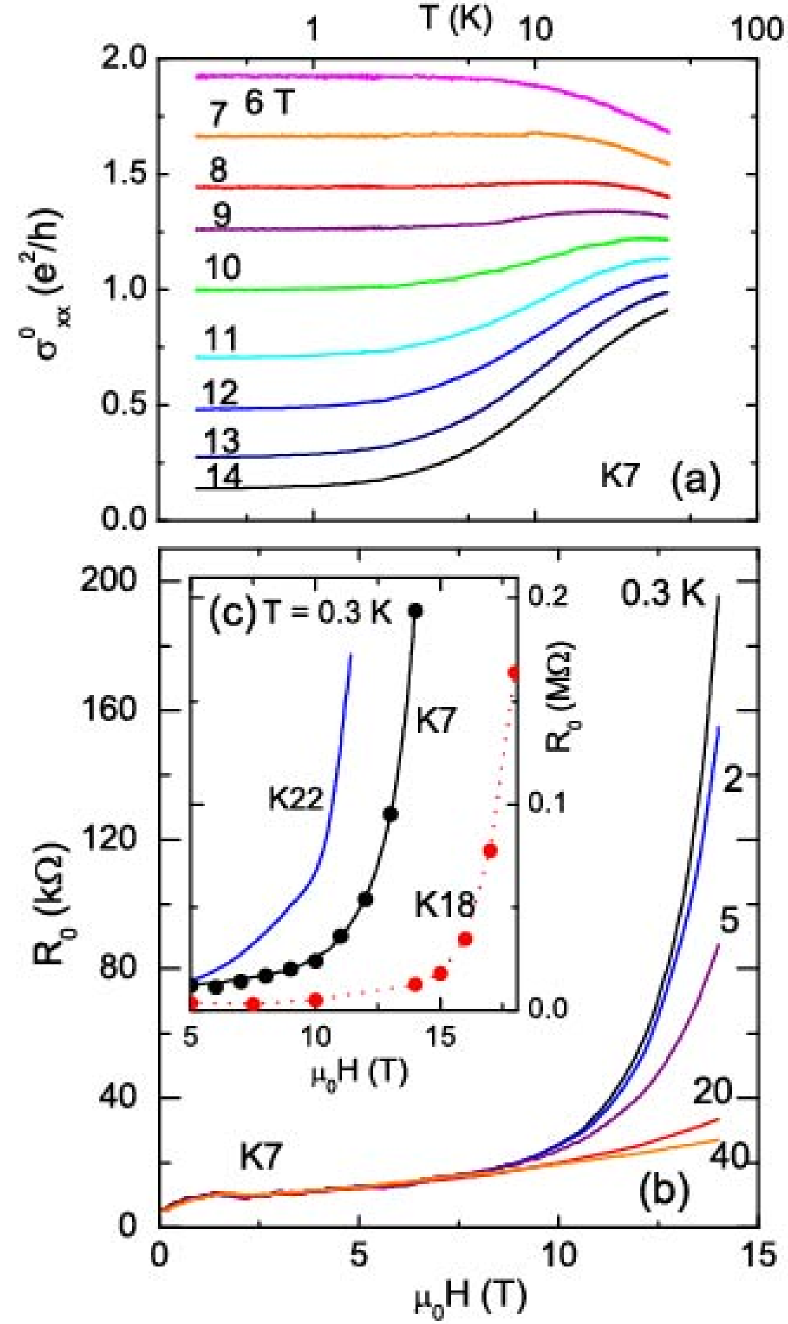
Divergent R_0 → gap "opens" at lower fields



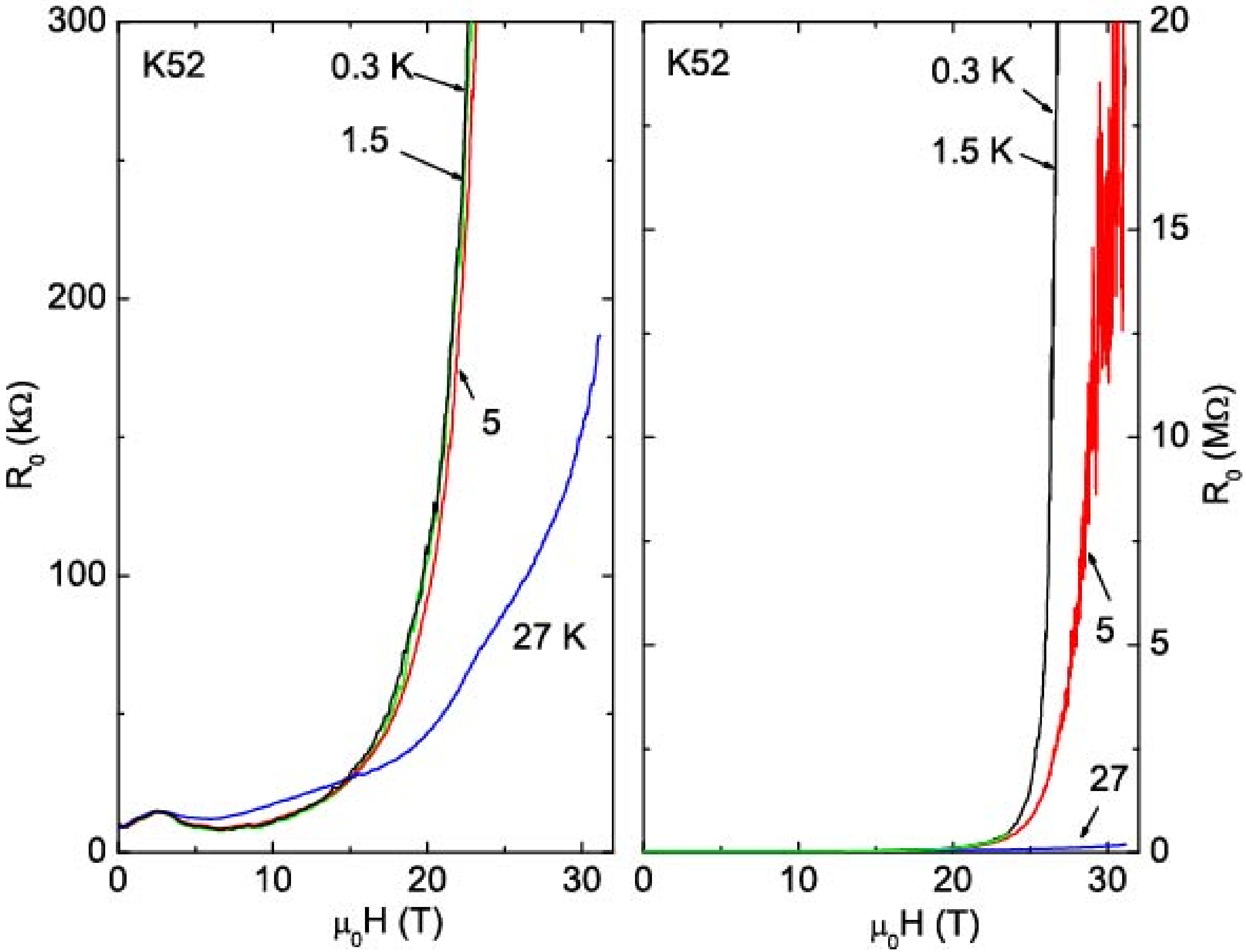


(a)
H = 14 T
K7
0.3 K
1
2
5
10
20
40,100
R_{xx} (kΩ)
(b)
K7
σ_{xy} (0.3 K)
σ_{xx}
100
40
20
10
5
0.3-2 K
σ_{xx} (e^2/h)
σ_{xy} (e^2/h)
V_g' (V)
(c)
K7
14 T
0.3 K
K5
K8
K29
R_{xx} (kΩ)
V_g (V)

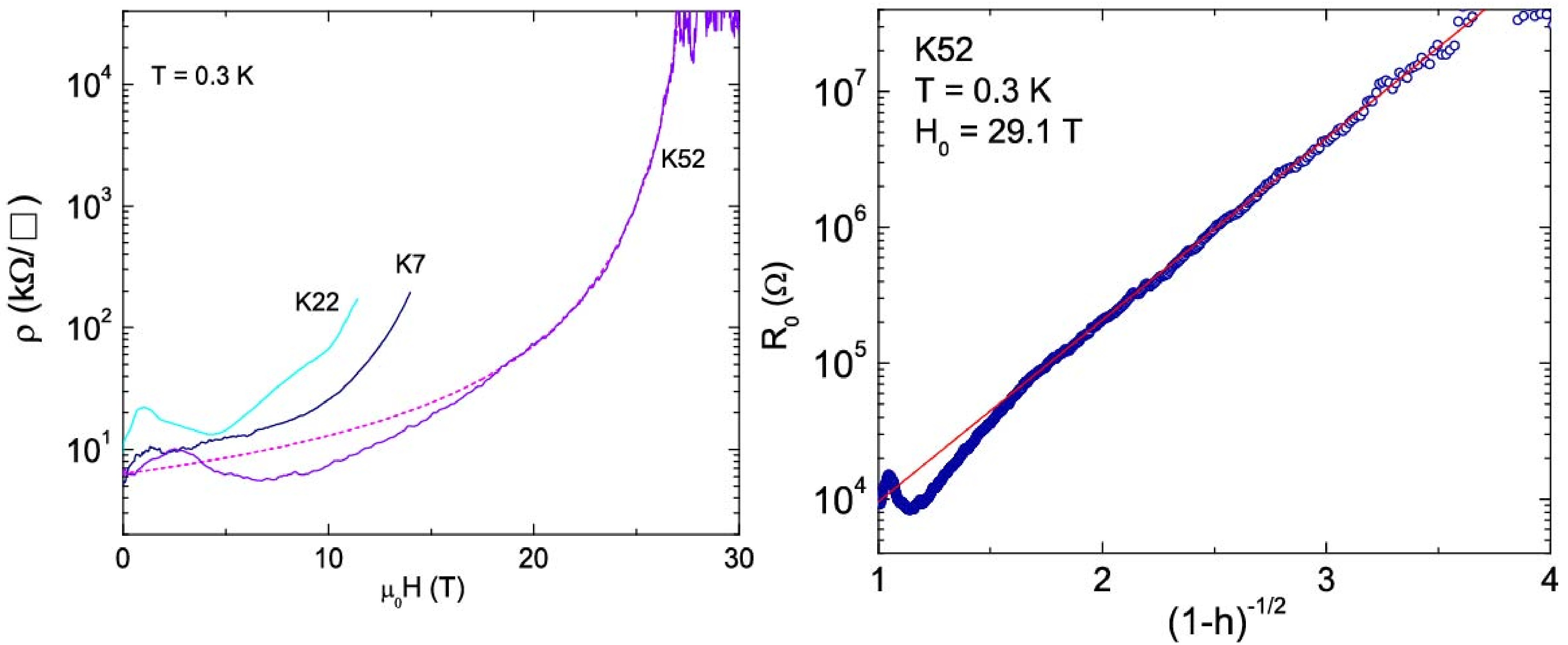

(a)
0.3 K
1
2
5
10
20
40,100
H = 14 T
K7
R_{xx} (kΩ)
(b)
K7
σ_{xy} (0.3 K)
σ_{xx}
100
40
20
10
5
0.3-2 K
σ_{xx} (e^2/h)
σ_{xy} (e^2/h)
V_g' (V)
(c)
K7
14 T
0.3 K
K5
K8
K29
R_{xx} (kΩ)
V_g (V)


T (K)
6 T
7
8
9
10
11
12
13
14
K7
(a)
σ^0_{xx} (e^2/h)
(c) T = 0.3 K
K22
K7
K18
R_0 (MΩ)
μ_0H (T)
0.3 K
2
5
20
40
K7
(b)
R_0 (kΩ)
μ_0H (T)

2D phase transition in graphene in high magnetic field



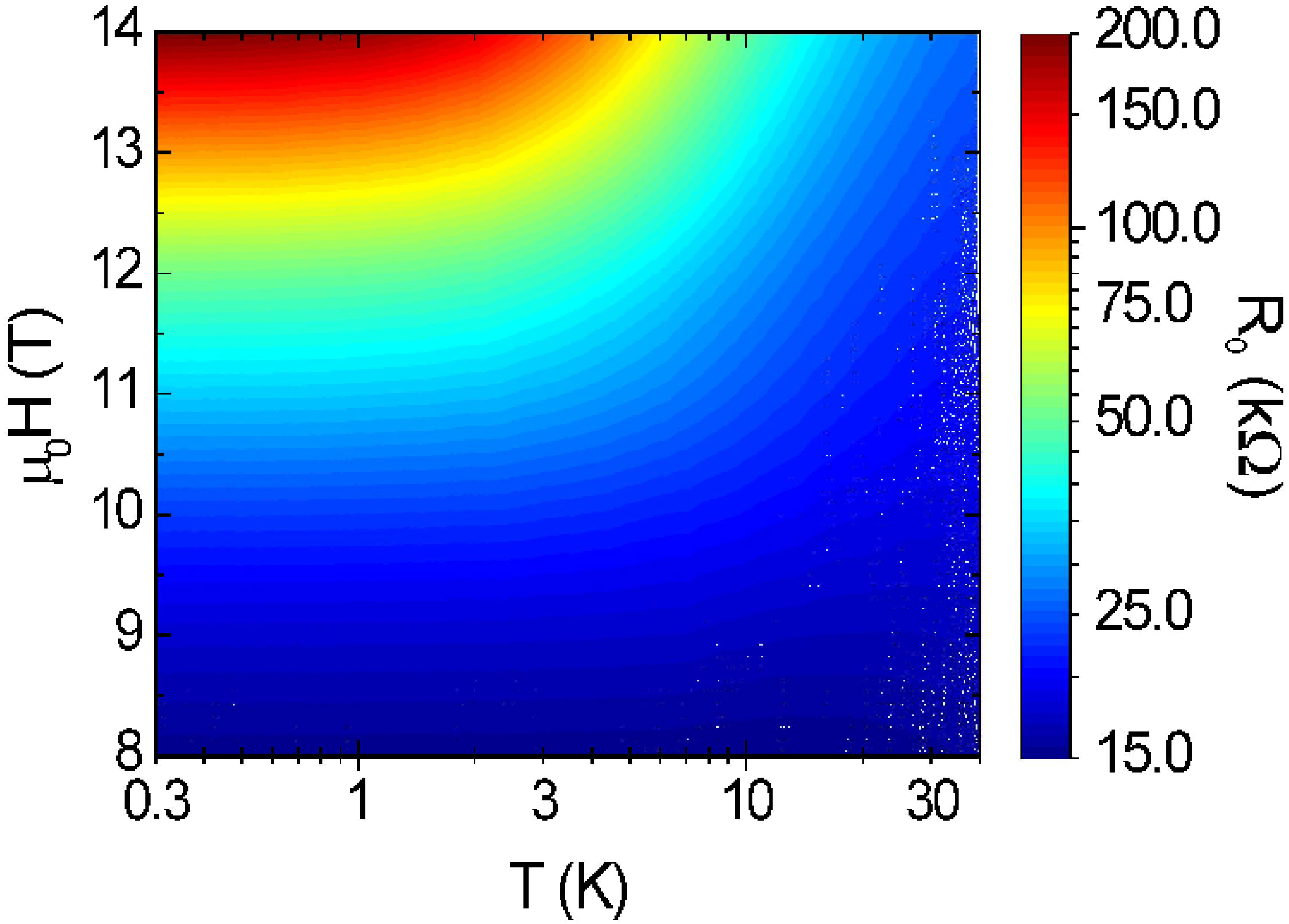
Evidence for a 2D phase transition in intense H field



3-decade fit to
KT correlation

$$R_0 = 440\exp\left[\frac{2b}{\sqrt{1-h}}\right]$$

Contour map of R_0 at Dirac point in *H*-*T* plane

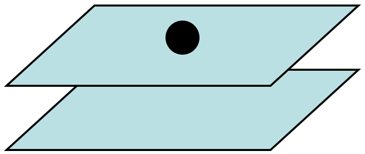


R_0* is exponentially sensitive to *H
but *T*-independent below 2 K

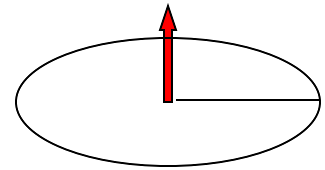
Role of Coulomb Interaction -- quantum Hall ferromagnet

Pseudospin in Bilayer QHE systems

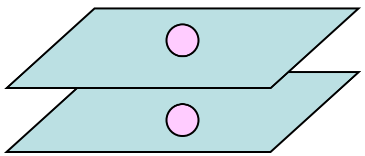
Moon, Yang, Girvin, MacDonald… 1995



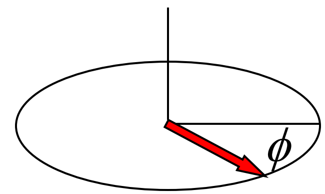
$$\begin{bmatrix}1\\0\end{bmatrix} = 1\left|\uparrow\right\rangle + 0\left|\downarrow\right\rangle$$



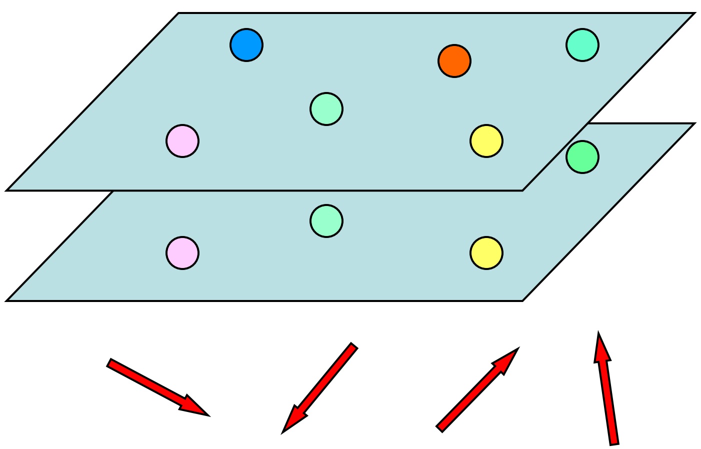
Capacitance → *U(1)* symm.



$$\begin{bmatrix}1\\e^{i\phi}\end{bmatrix} = \left(1\left|\uparrow\right\rangle + e^{i\phi}\left|\downarrow\right\rangle\right)\frac{1}{\sqrt{2}}$$

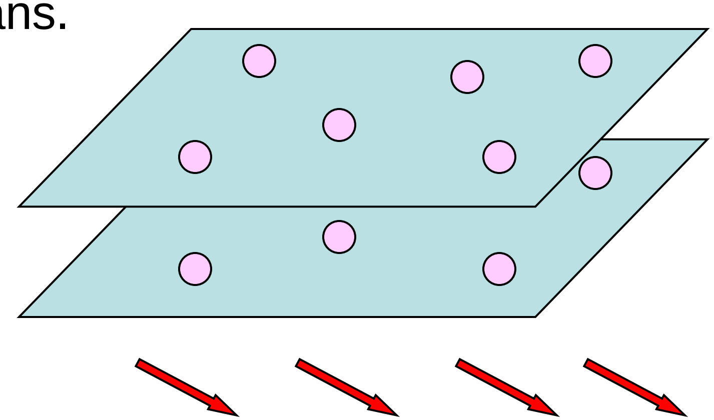


Paramagnet



KT trans.

2DXY Valley Ferromagnet



Coulomb exchange leads to spontaneous alignment of pseudospins (Hund’s rule)

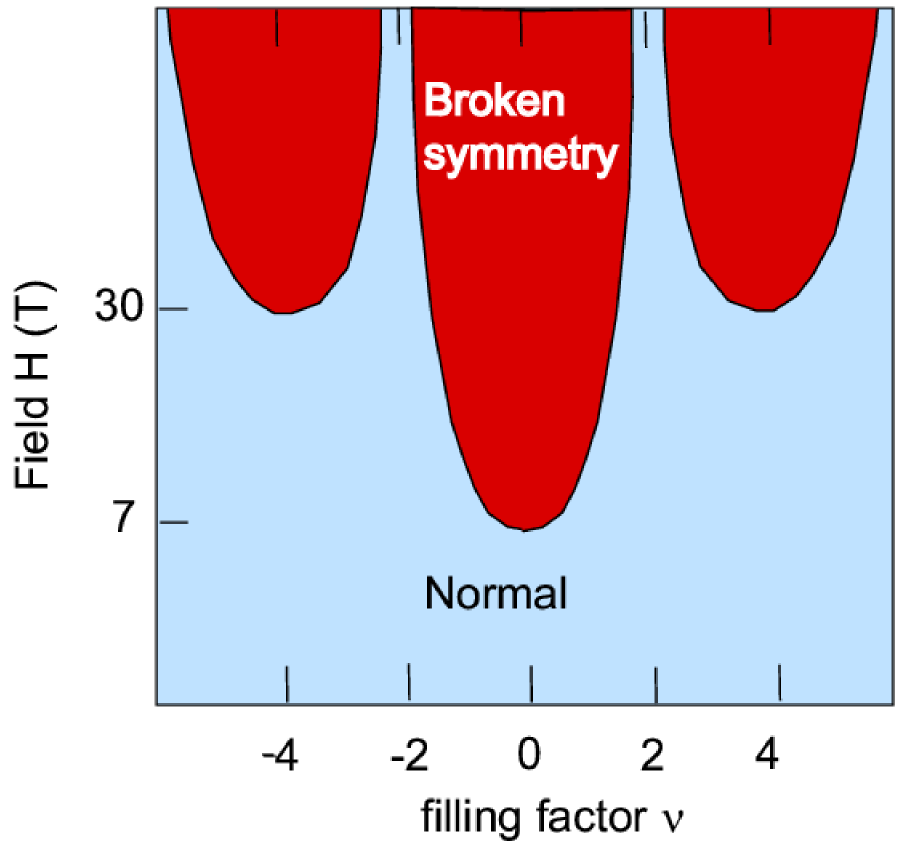
Quantum Hall ferromagnet?

Nomura, MacDonald, PRL06
Goerbig, Moessner, Ducot, PRB06
Alicia, Fisher PRB 06

Layer index → Valley index *K, K'*

1. Coulomb exchange
Splits 4-fold degeneracy
Of n = 0 Landau Level

2. In high fields (and
low disorder), have QHF
state.



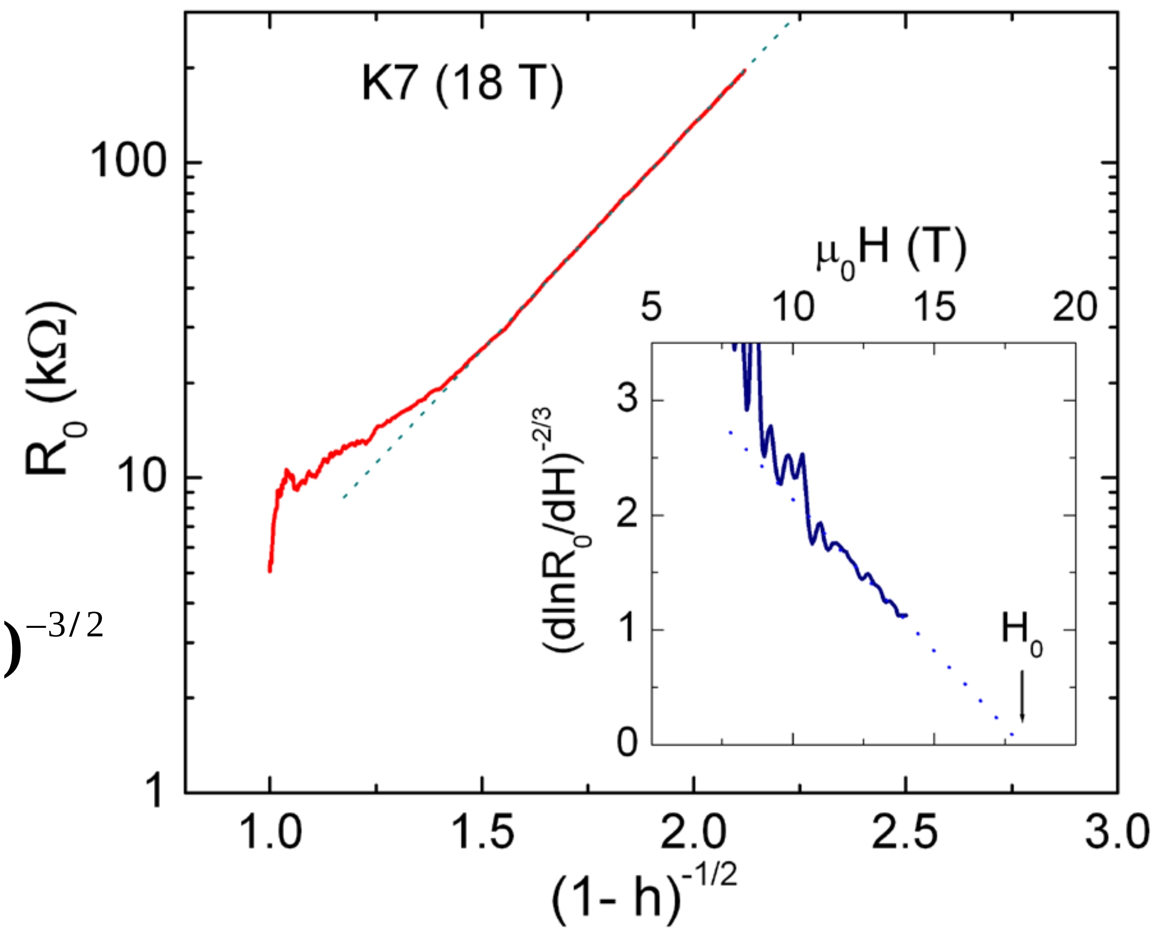
Approaching KT transition?

Correlation length

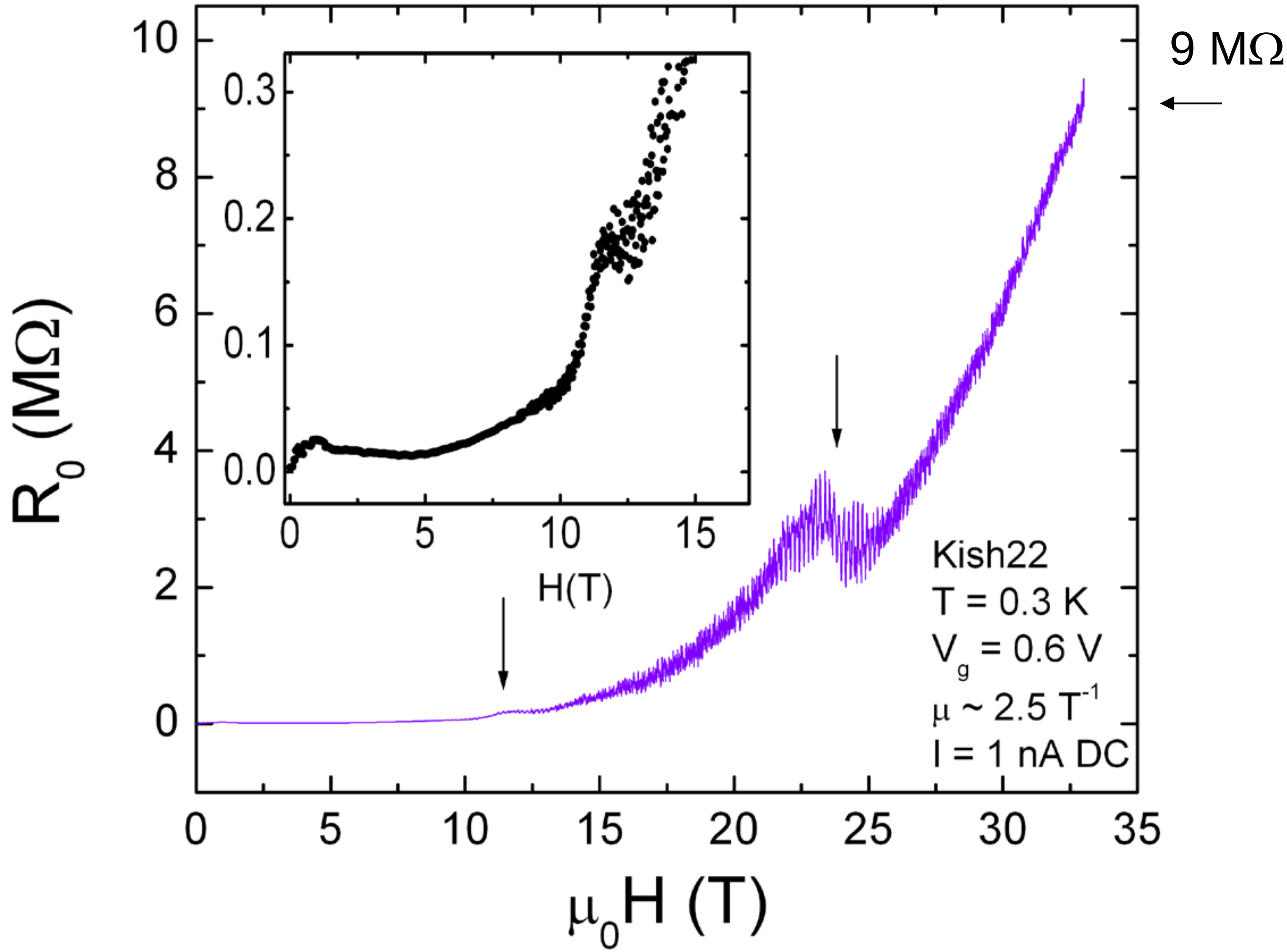
$$\xi = a \exp\left[\frac{b}{\sqrt{h-1}}\right]$$

If $R_0 \sim \xi^2$

$$\frac{d}{dH}\left(\ln R_0\right) \sim (H - H_0)^{-3/2}$$



Difficulties in following divergent R_0 from severe sample heating and non-Ohmicity



Phase transitions in bismuth in high fields

Lu Li, J. G. Checkelsky, Y. S. Hor, R. J. Cava and N. P. Ong,
Princeton
C. Uher, *Univ. Michigan*
A. Hebard, *U. Florida, Gainsville*

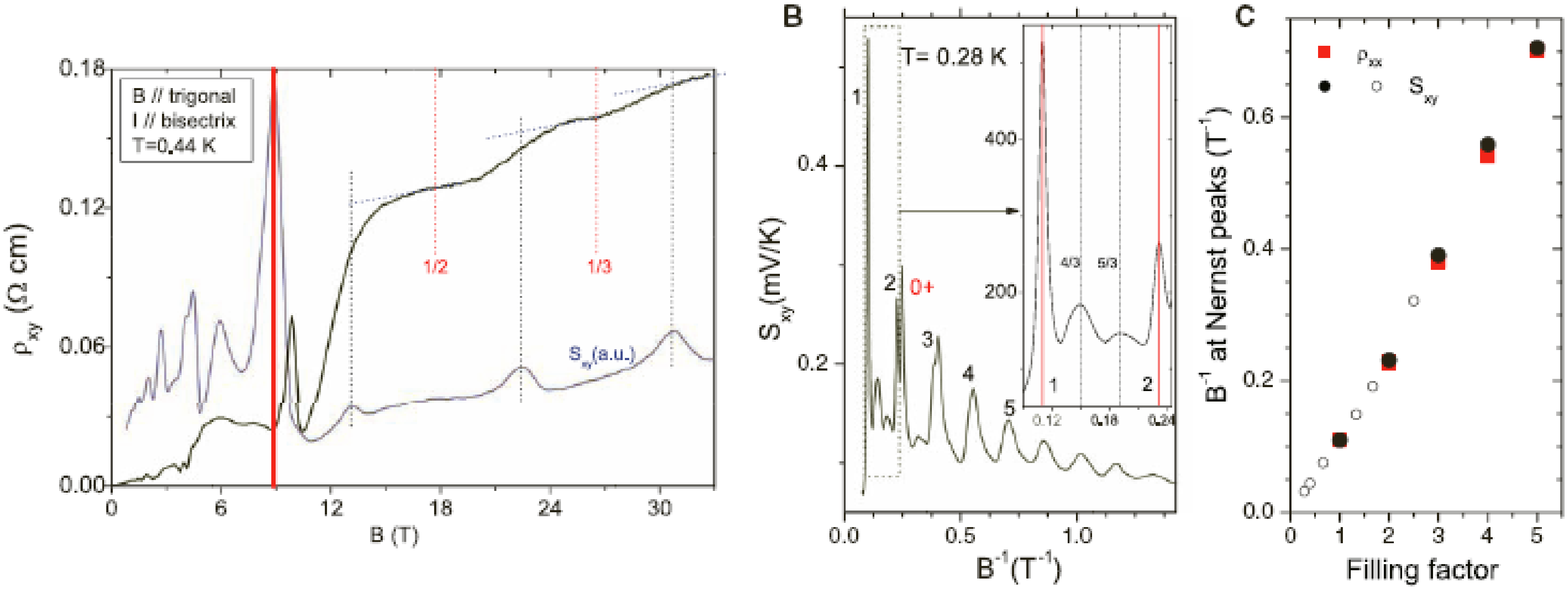
1. **Fractional filling (Behnia, Balicas, Kopelovich)**
2. **High-field Torque and magnetization**
3. **First-order transitions**

Signatures of Electron Fractionalization in Ultraquantum Bismuth

Kamran Behnia,[1*] Luis Balicas,[2] Yakov Kopelevich[3]

Behnia, Balicas, Kopelevich

SCIENCE VOL 317 21 SEPTEMBER 2007

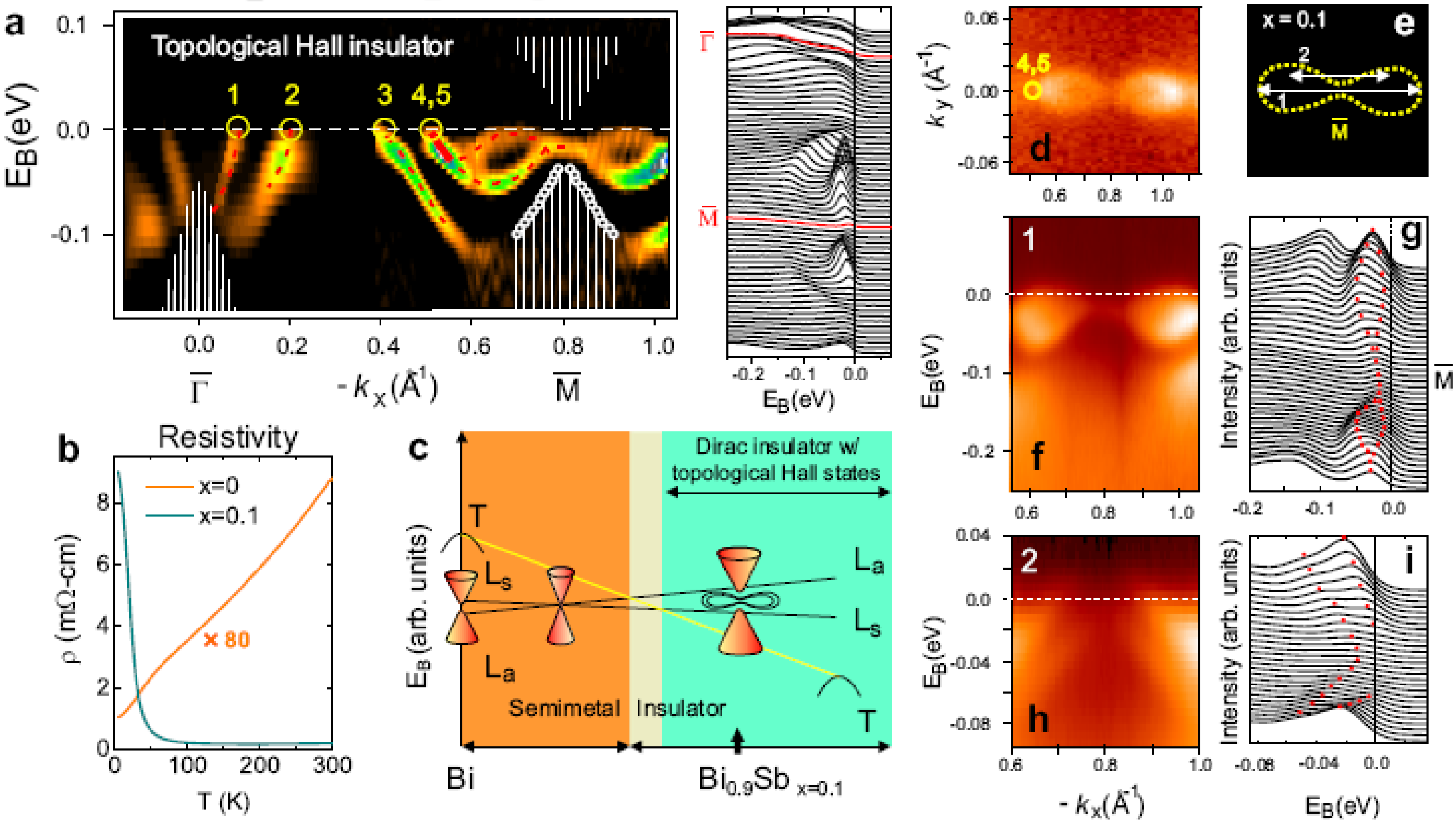


Are they surface states (2D)?
Are electron pockets (vs. hole) involved?

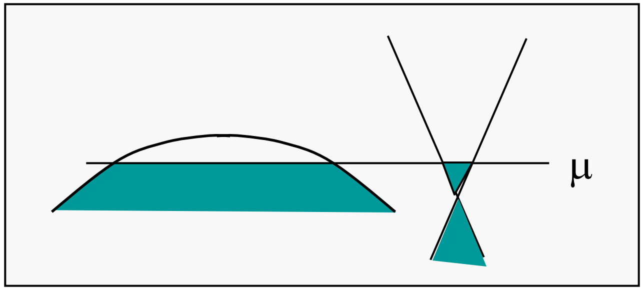
Topological Insulator with surface Hall modes

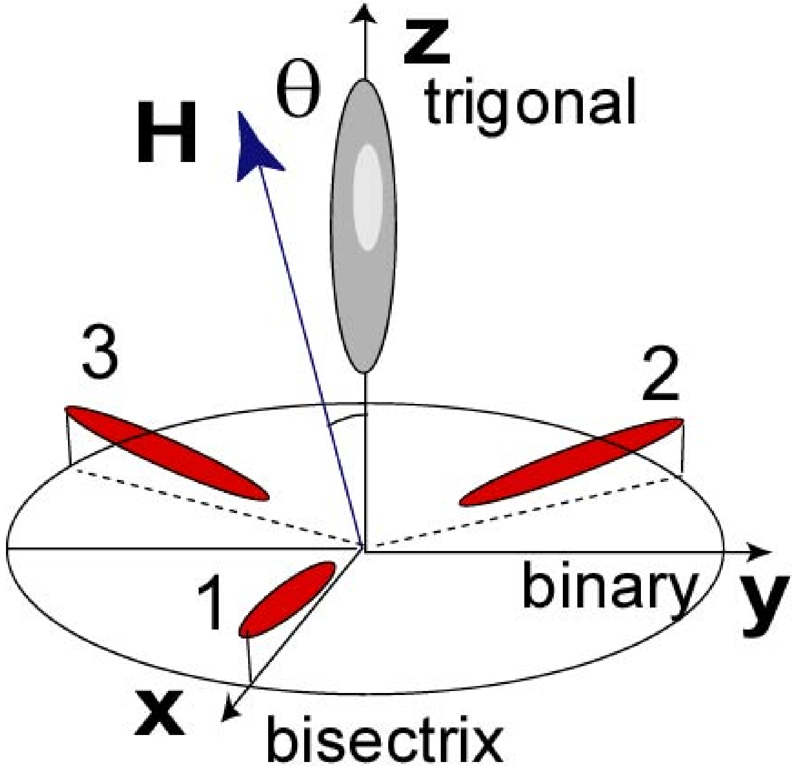
D. Hsieh, M.Z. Hasan et.al., Princeton University (2007)

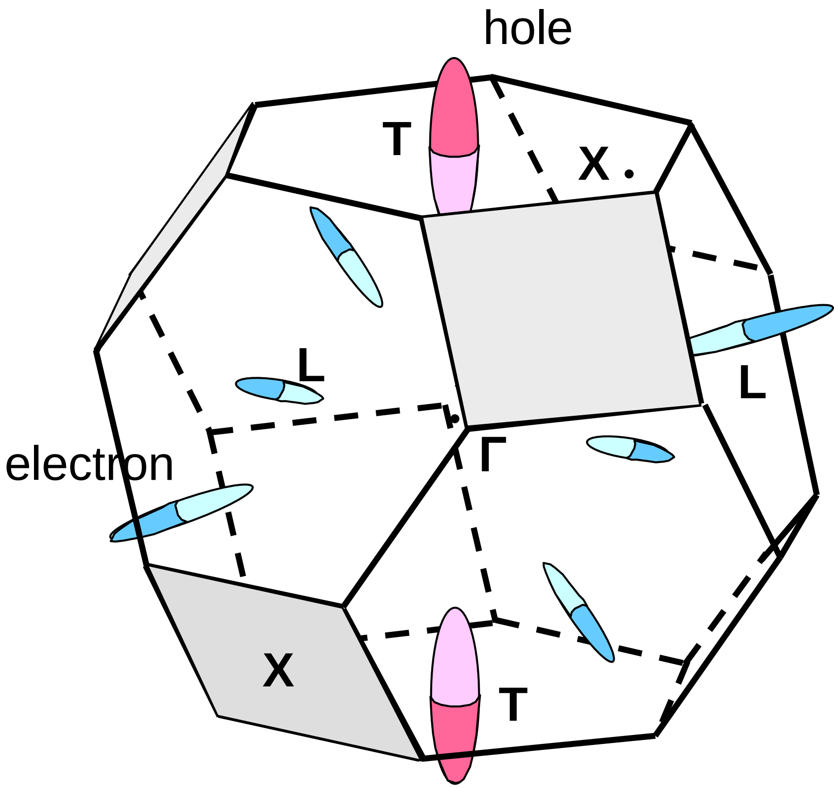
STI: $Z_2 = -1$ topological surface modes

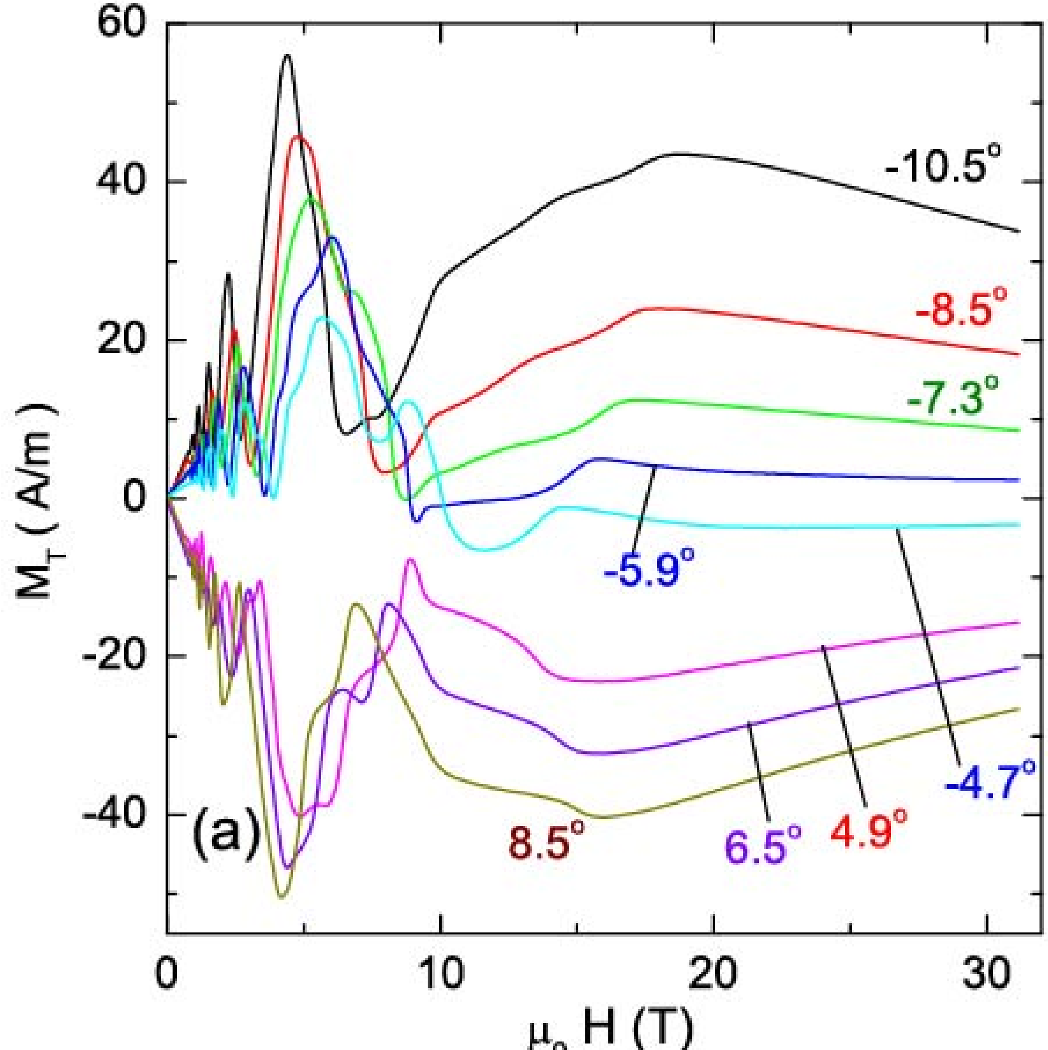


Fermi Surface in Bi: 1 hole ellipsoid + 3 electron ellips.



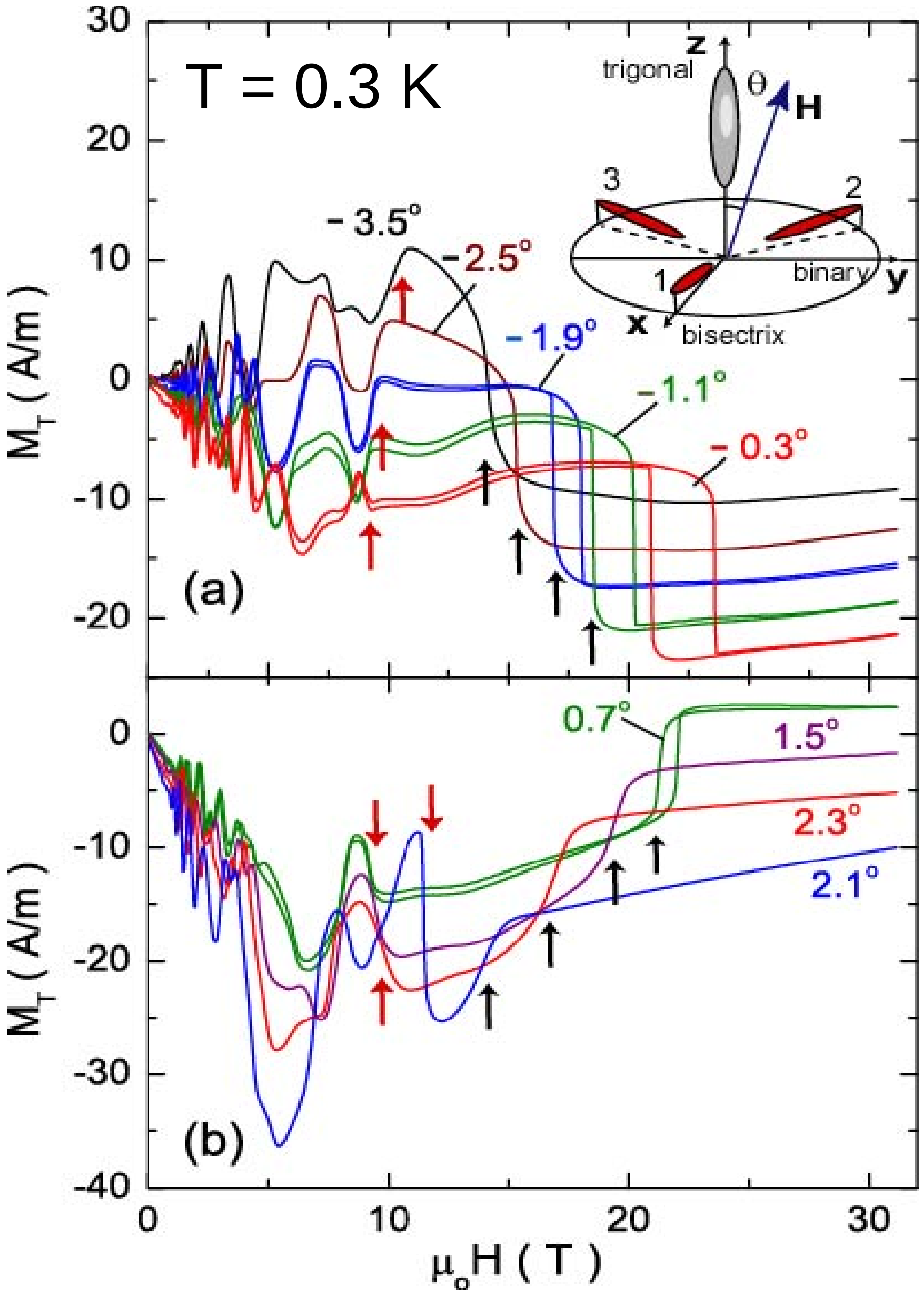






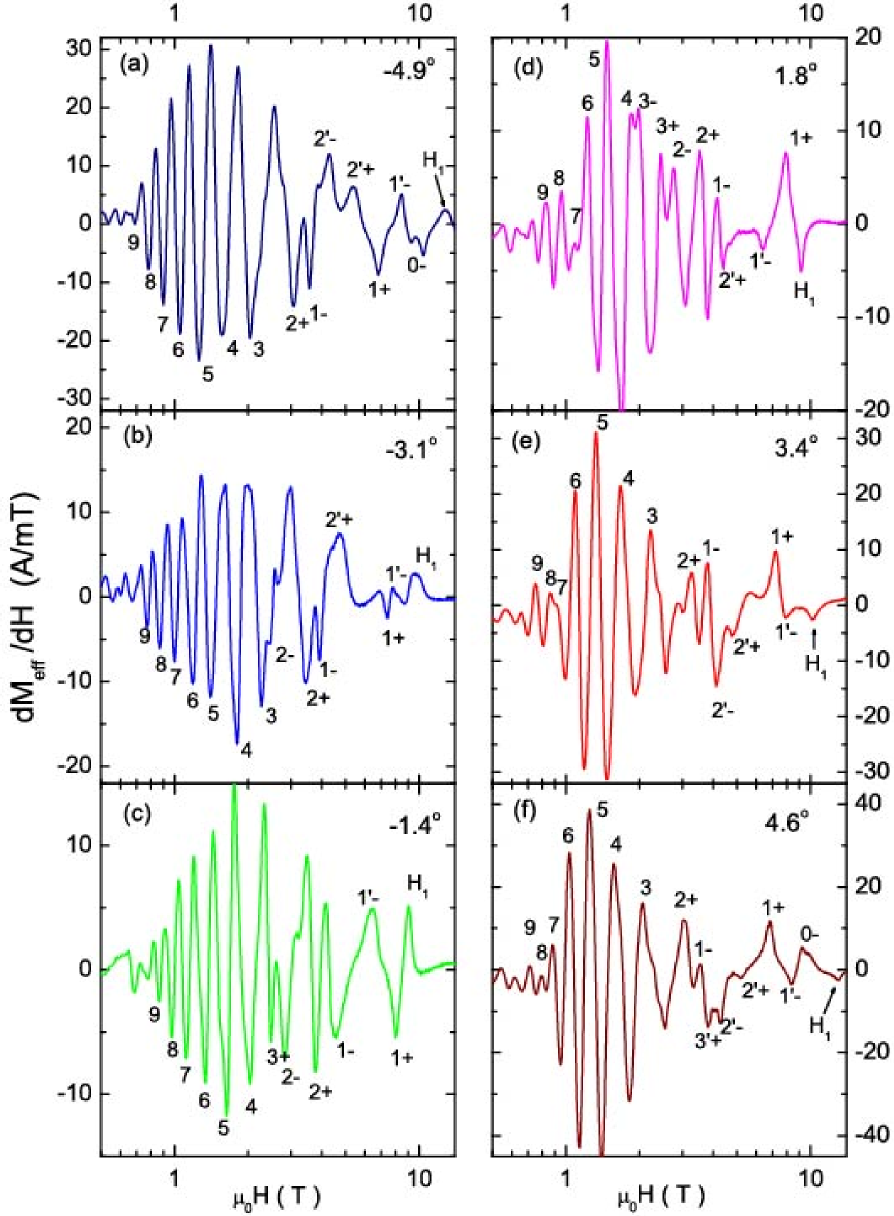
60
40
20
0
-20
-40
0
10
20
30
M_T (A/m)
μ_0 H (T)
-10.5°
-8.5°
-7.3°
-5.9°
-4.7°
8.5°
6.5°
4.9°
(a)

First-order transitions in Bi in high magnetic fields



Transv Magnetiz.
$M_T = \tau/VH$

Transitions at H_1 (red arrows) and H_2 (black)



Landau levels resolved
In derivative curves
dM_T/dH

$(M_T = \tau/VH)$

Dirac model for Bismuth (Cohen Blount '60, Wolff '64)

$$H = \frac{p^2}{2m} + V + \frac{1}{8m^2}\nabla^2 V + \frac{1}{2m^2}\mathbf{p}\,.\,\mathbf{s}\times\nabla V$$

Dominant
spin-orbit energy

In **k.p** approx., at *L* point

$$H = \frac{E_G}{2}\beta + \frac{k^2}{2m}.1 + \mathbf{k}\,.\,\mathbf{\Gamma}$$

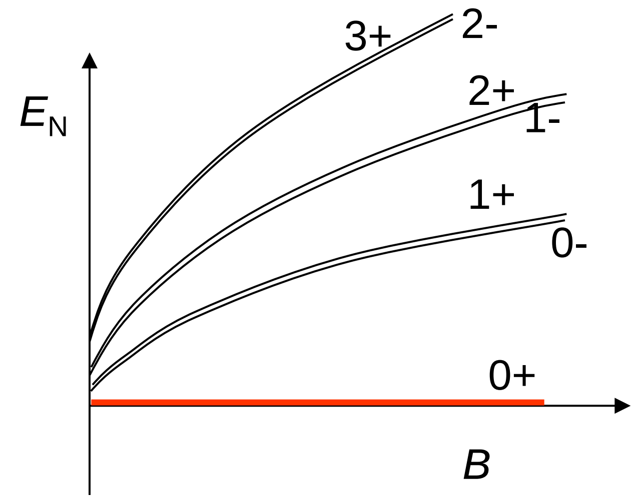
$$\Psi = \begin{pmatrix} \psi_{c\uparrow} \\ \psi_{c\downarrow} \\ \psi_{v\uparrow} \\ \psi_{v\downarrow} \end{pmatrix} \qquad \beta = \begin{bmatrix} 1 & 0 \\ 0 & -1 \end{bmatrix} \qquad \alpha = \begin{bmatrix} 0 & \sigma_\mu \\ \sigma_\mu & 0 \end{bmatrix}$$

$$\mathbf{\Gamma} = i\sum_\mu \mathbf{W}(\mu)\beta\alpha_\mu \qquad (\mathbf{k}\,.\,\mathbf{\Gamma})^2 = E_G\begin{bmatrix} H^* & 0 \\ 0 & H^* \end{bmatrix} \qquad H^* = \frac{\mathbf{k}\,.\,\boldsymbol{\alpha}\,.\,\mathbf{k}}{2} - \boldsymbol{\mu}\,.\,\mathbf{B}$$

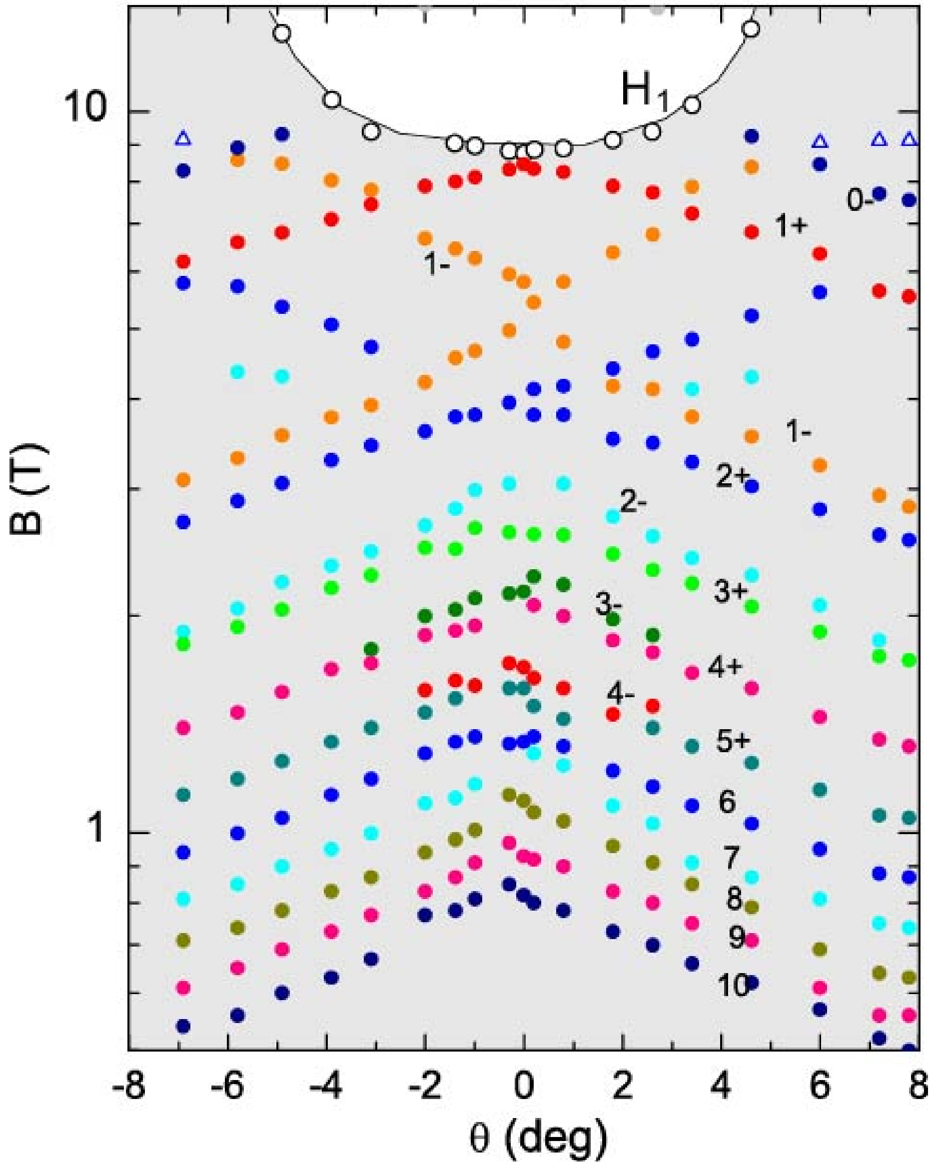
Squared *H* may be block-diagonalized

$$H^2\psi = E^2\psi$$

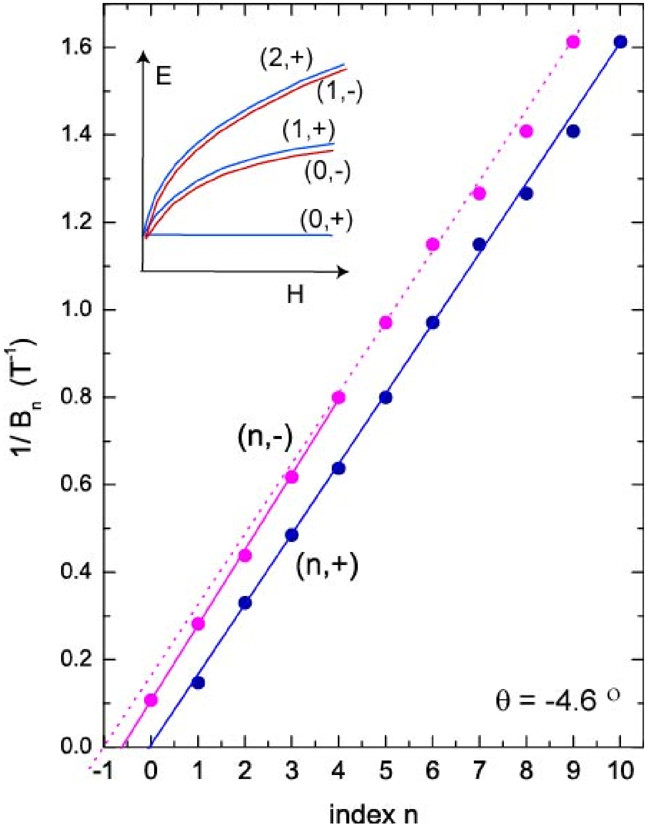
$$E_{Nk} = \pm\left[\left(\frac{E_G}{2}\right)^2 + E_G\left\{\left(N+\frac{1}{2}\right)\omega_c \pm \frac{\omega_c}{2} + \frac{k^2}{2m}\right\}\right]^{\frac{1}{2}}$$



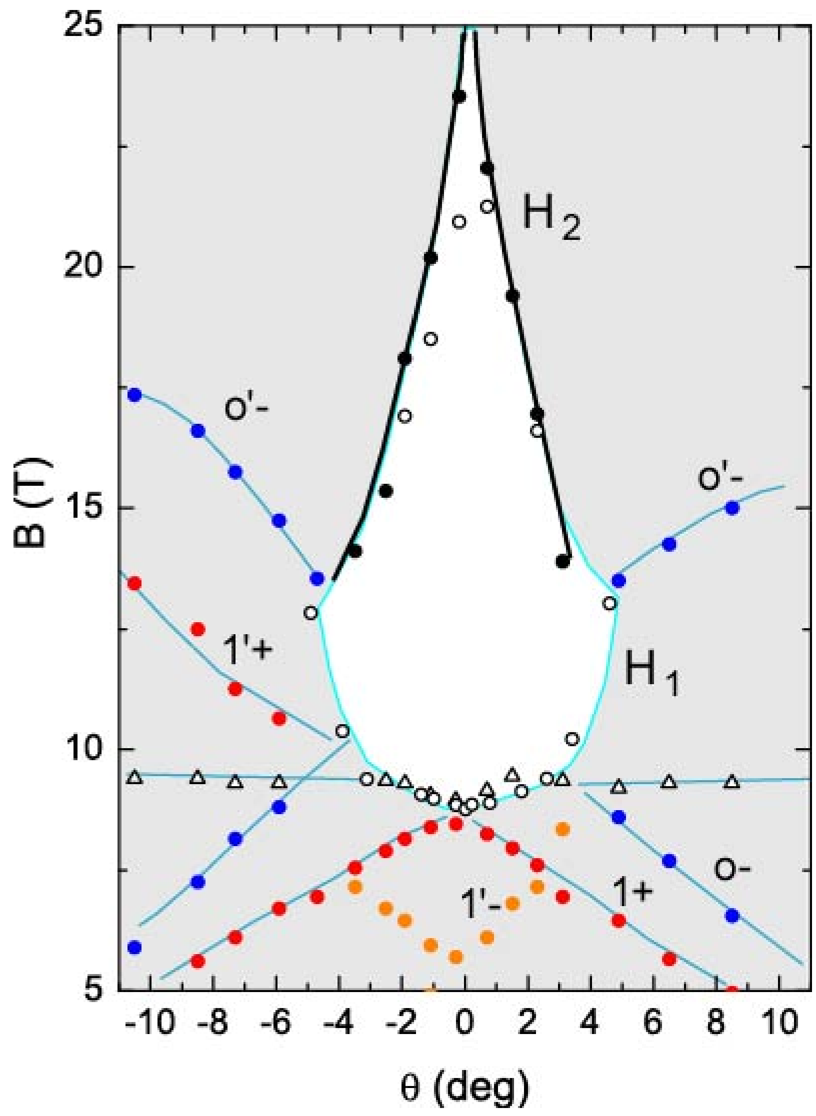
Have identified electron FS sublevels *n* = 0, 1, 2, … 10



Dirac level indexing

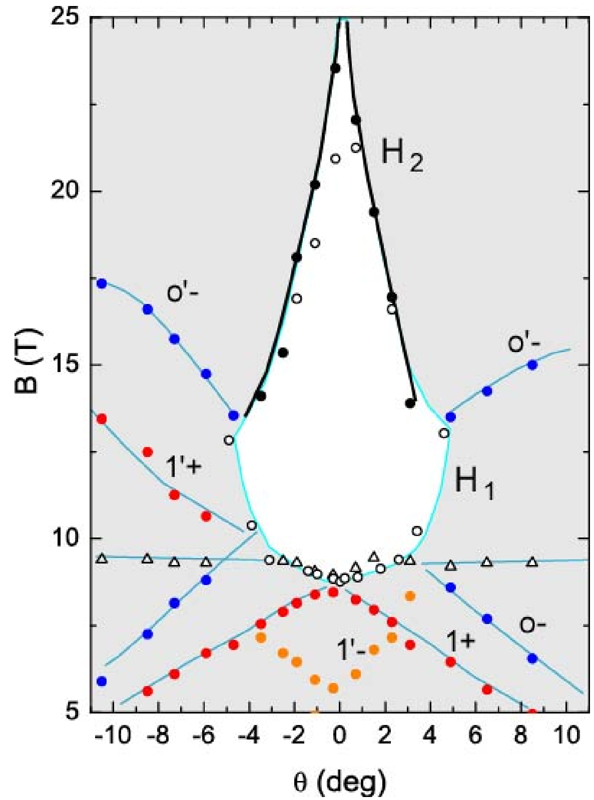


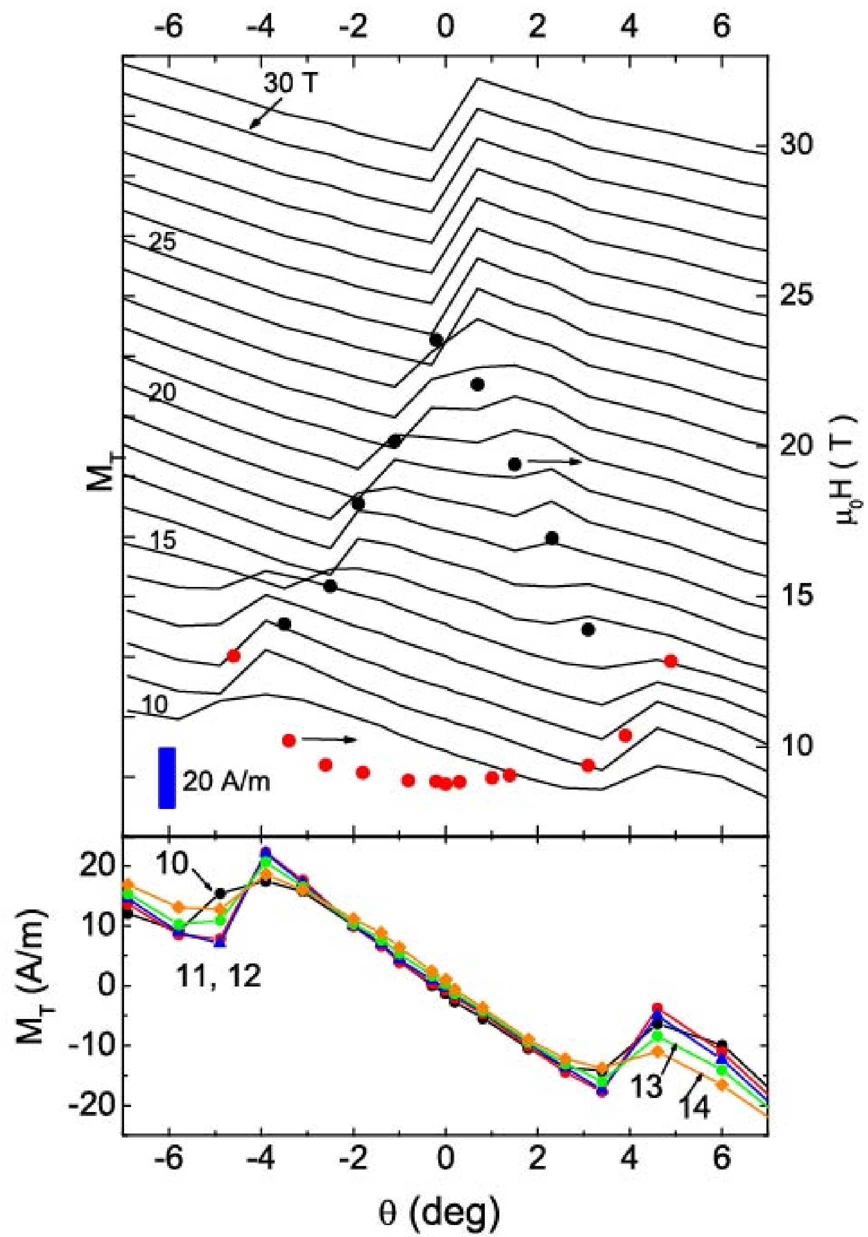
High-field phase diagram of bismuth



Inside cone region

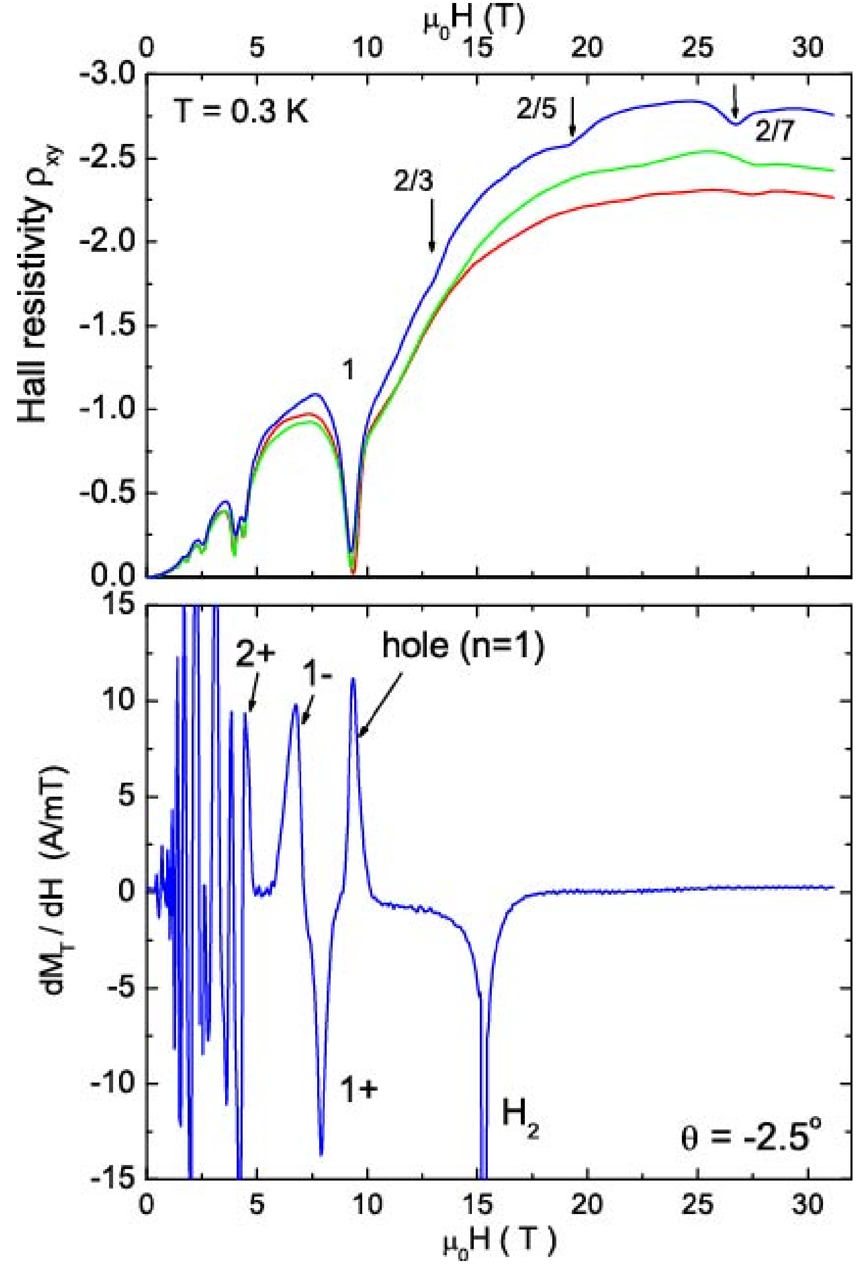
1. No sublevels obs.
2. Zero torque signal
3. First-order jumps at boundaries





Sloped background from weak-field anisotropy (Fukuyama Kubo '80)
Torque signal vanishes inside cone region?
High-field M_T vs. H curve not understood

Fractional-filling states obs. in Hall resistivity but not in magnetization.
Also independent of tilt angle θ (surface states?)



Graphene

1. Fate of zero-energy Dirac point in high field is insulating state

2. Energy gap opens -- QHF state?

3. Unusual approach to insulator in *H-T* phase diagram

4. Suggests a KT transition destroys insulating state when $H < H_c$ (17 Tesla).

Bi results

1. Fractional filling confirmed in R_{xx}, R_{xy} and magnetization

2. Additional anomalies in fractional regime
   -- jumps in torque at 18-25 Tesla range

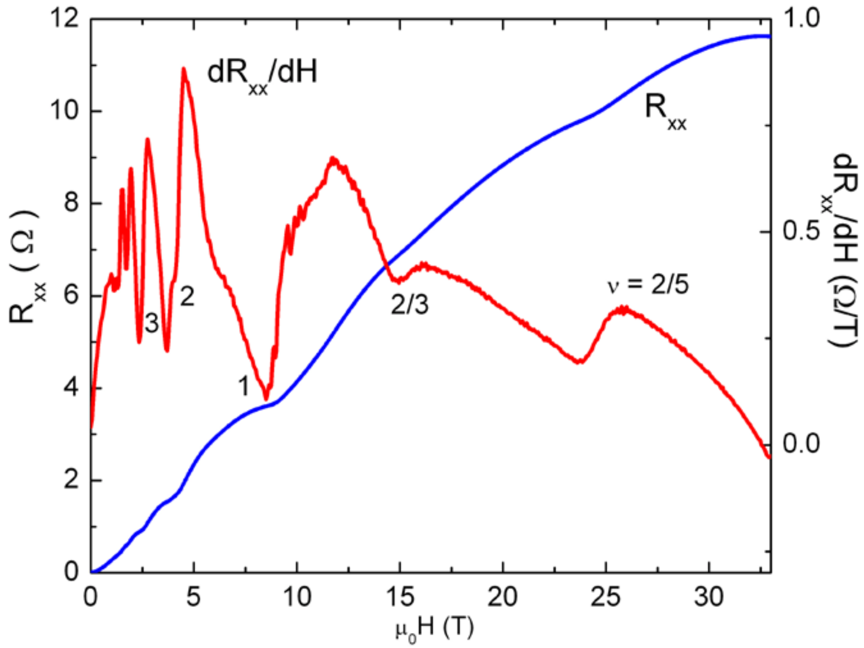
3. Orbital diamagnetic response not understood in quantum limit

End

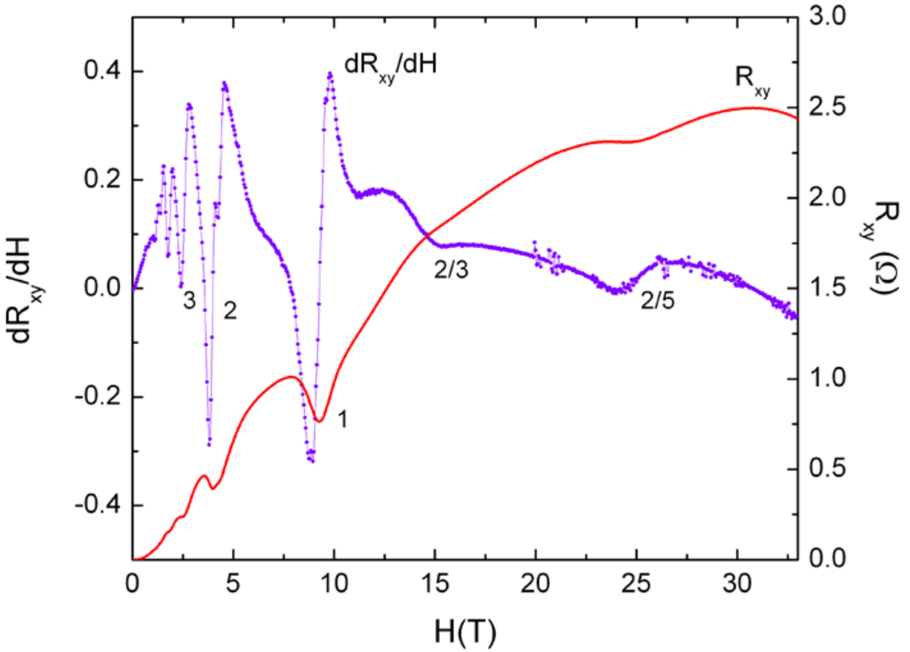
Fractional filling evidence from Rxx and Rxy

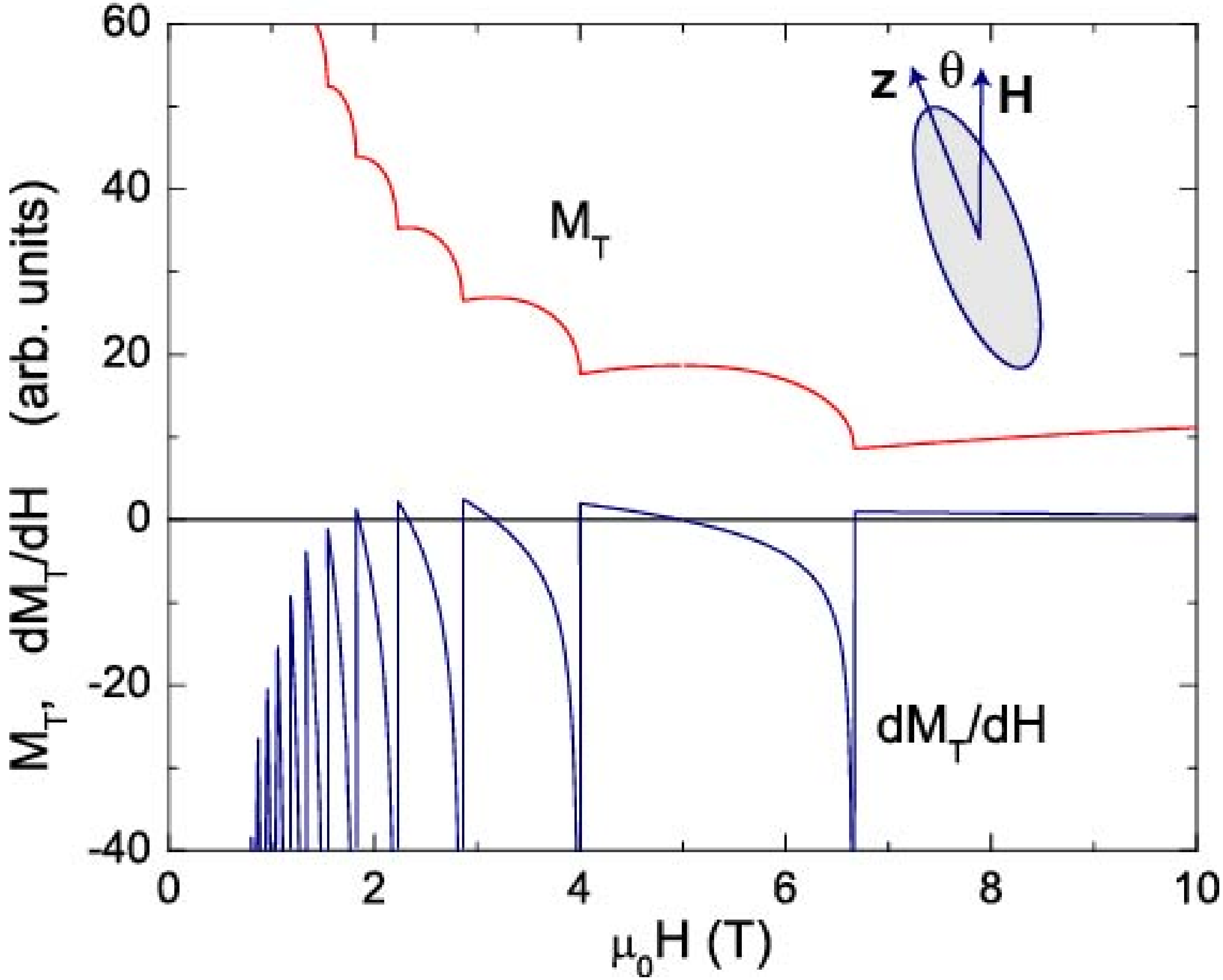
Li, Checkelsky, NPO 2008

R_{xx}

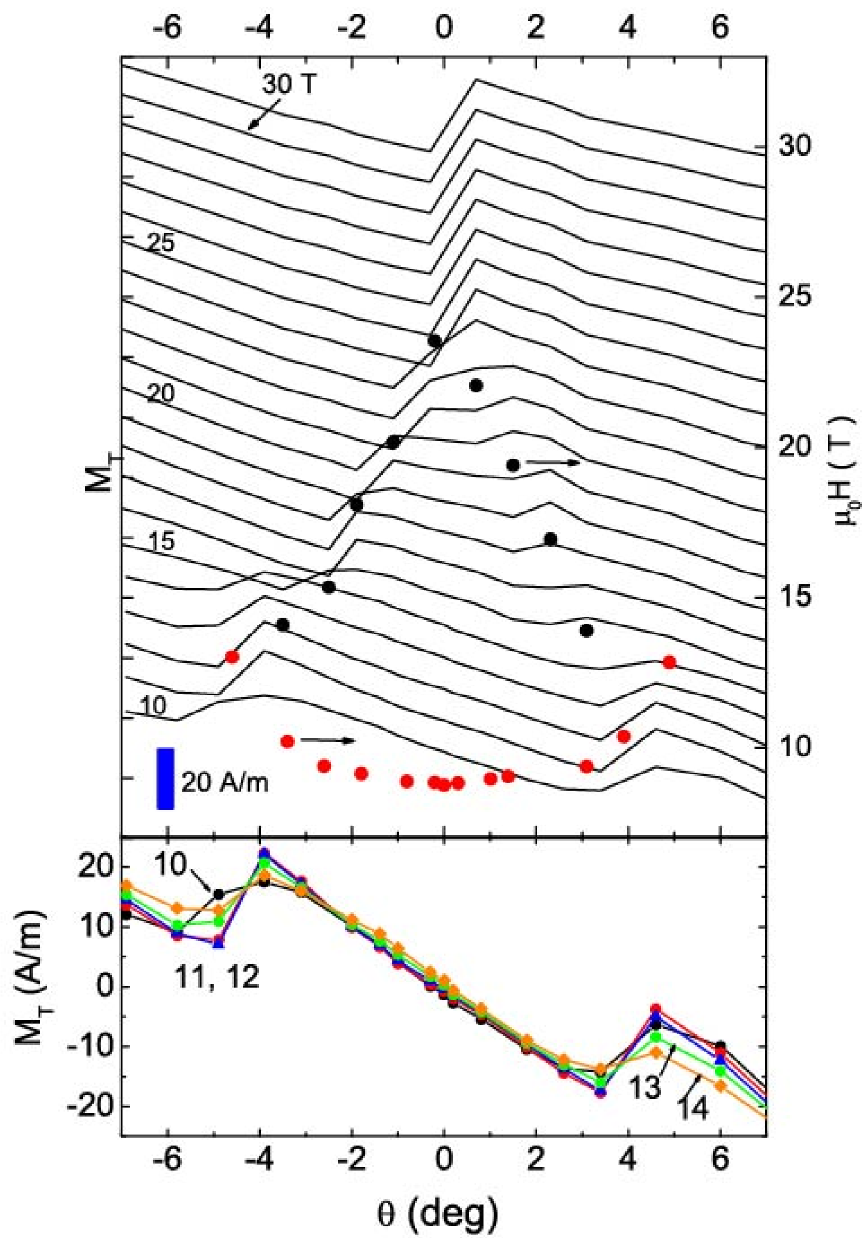


R_{xy}





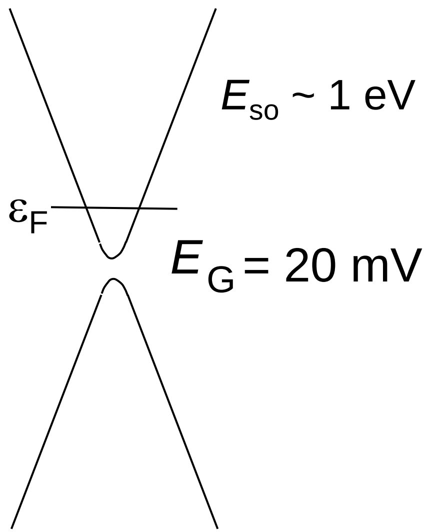
60
40
20
0
-20
-40
0
2
4
6
8
10
M_T, dM_T/dH (arb. units)
μ_0H (T)
M_T
dM_T/dH
z
θ
H



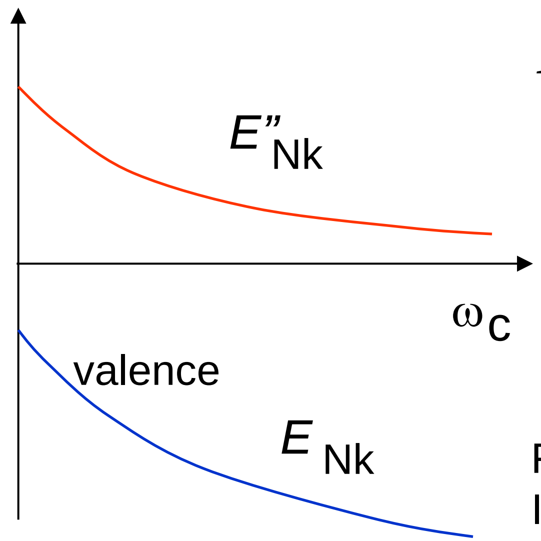
30 T
25
20
15
10
20 A/m
M_T
μ0H (T)
M_T (A/m)
10
11, 12
13
14
θ (deg)

Large Diamagnetism –
From Dirac dispersion and large spin-orbit energy

(Fukuyama Kubo, JPSJ 1970)



$$E_{Nk} = \pm\left[\left(\frac{E_G}{2}\right)^2 + E_G\left\{\left(N+\frac{1}{2}\right)\omega_c \pm \frac{\omega_c}{2} + \frac{k^2}{2m}\right\}\right]^{\frac{1}{2}}$$



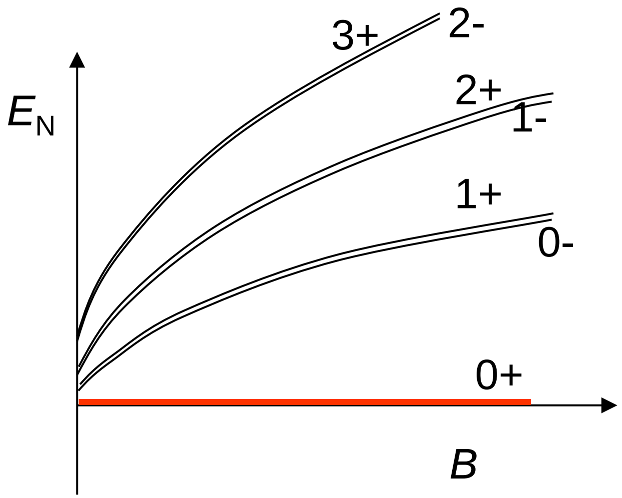
$$F = F_0 - \mathrm{M.B}$$

$$\frac{\partial^2 F}{\partial B^2} = -\chi$$

Filled valence Dirac band
Is diamagnetic

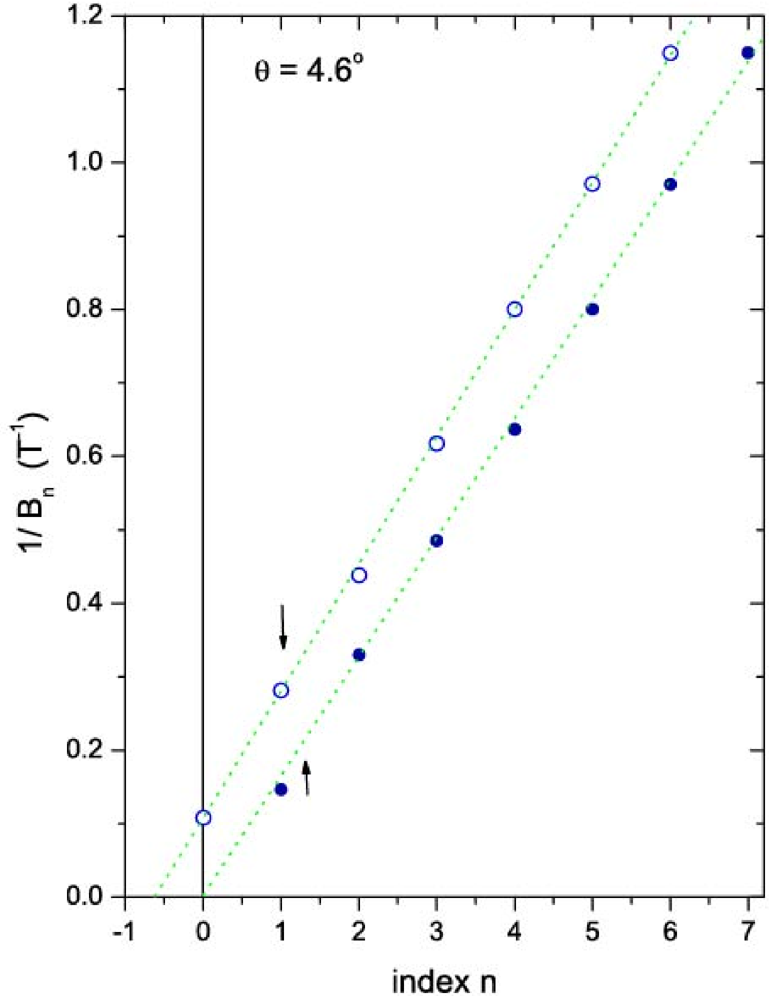
Earlier theories   Peierls, Landau, Jones, Adams, Kohn, Roth, Cohen Blount, Wolff

Landau Levels in 2-band Dirac model (Cohen Blount '60, Wolff '64)



If spin mass $m_s = m_c$

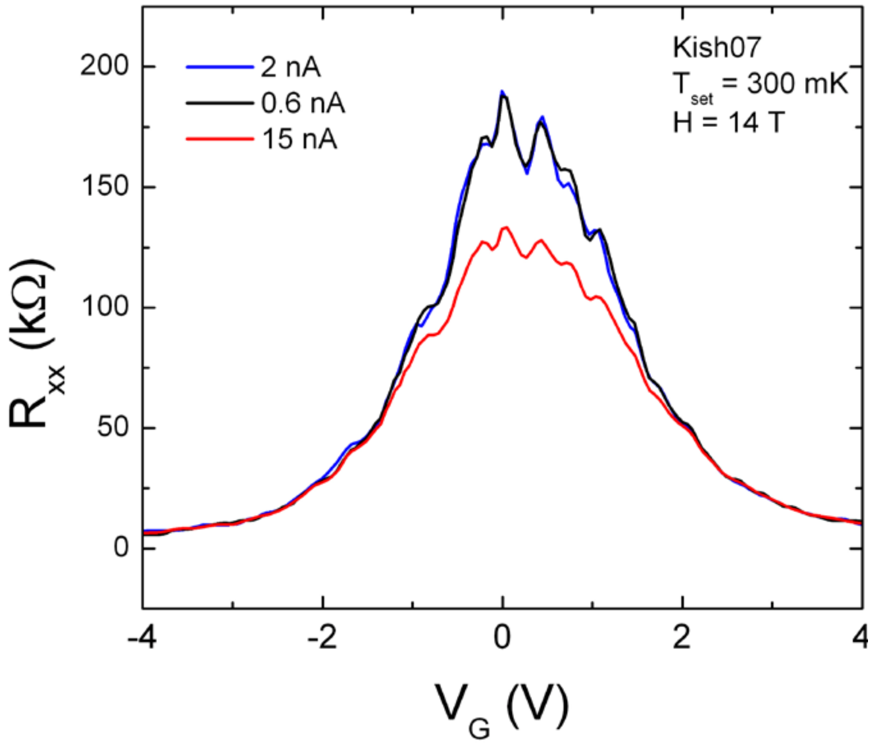
Existence of zero-energy state

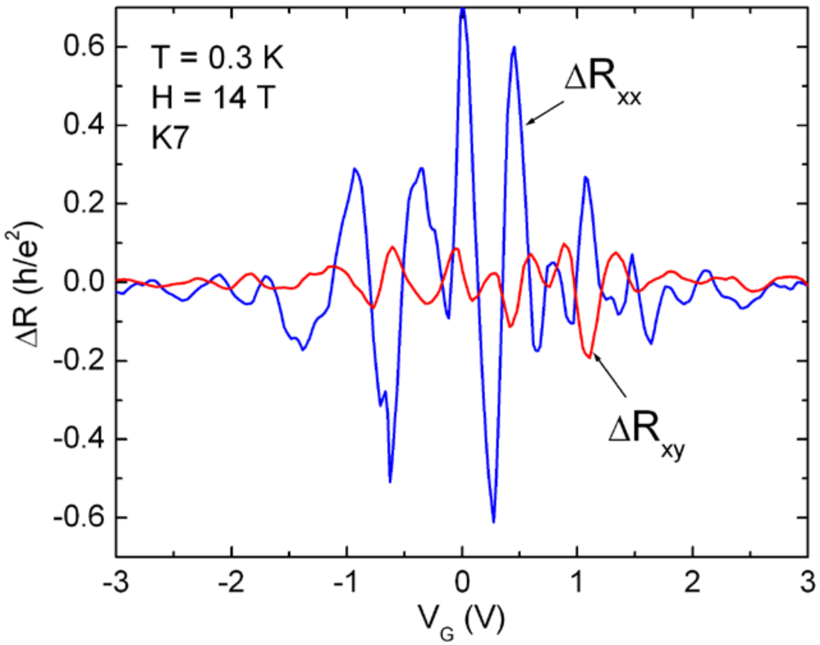


$$E_{Nk} = \pm\left[\left(\frac{E_G}{2}\right)^2 + E_G\left\{\left(N+\frac{1}{2}\right)\omega_c \pm \frac{\omega_c}{2} + \frac{k^2}{2m}\right\}\right.$$

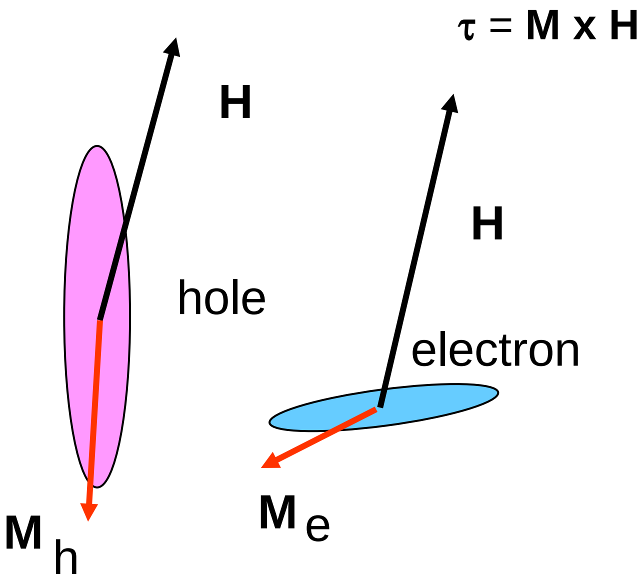
For $N = 0\uparrow$, E is independent of B!

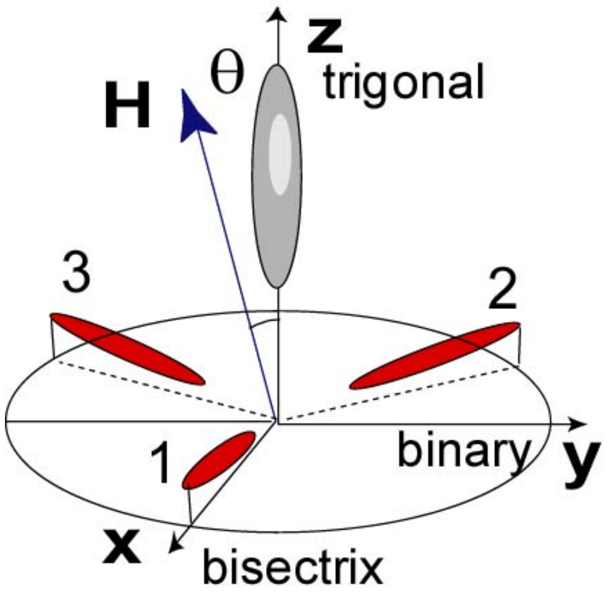
Large universal conductance quantum (UCF) oscillations vs. V_g and H





Torque magnetometry in Bi





Electron FS diamagnetization gives
positive torque signal

$$\tau \parallel \mathbf{z} \times \mathbf{H}$$

Large spin-orbit energy (E_{so}~ 1 eV) and very small gap (E_G ~ 20 meV)
Magnetization dominated by orbital currents

$$\boldsymbol{\mu} = \boldsymbol{\mu}_s + \boldsymbol{\mu}_L$$

Orbital angul. momtm

(Cohen, Blount Phil. Mag. 1960
Laura Roth, PR 1962)

$$\boldsymbol{\mu}_L = -\mu_B \, \mathbf{X} \times m\,\mathbf{v}\,\hbar^{-1}$$

$$\langle n\,\mathbf{k} \,|\, \mathbf{X} \,|\, m\,\mathbf{k} \rangle = \left(u_{n\mathbf{k}}, i\nabla_{\mathbf{k}} u_{m\mathbf{k}} \right)$$

Berry potential

Semiclassical theory (effective mass model)

$E(\mathbf{k}) = \mathbf{k}.\boldsymbol{\alpha}.\mathbf{k}/2m$

1. g-factor ~ 200
2. Spin mass equal to cyclotron mass
3. Susceptibility diamagnetic and very anisotropic
4. Deep connection to Berry curvature

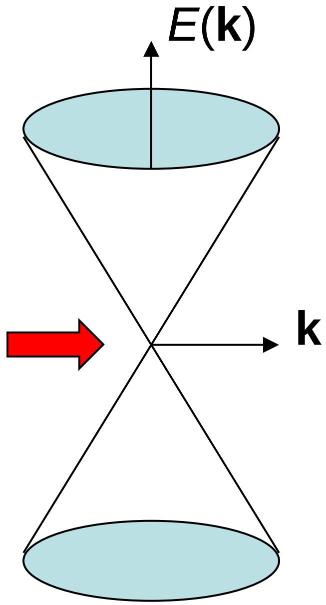
5. What happens in quantum limit?

$$\chi \sim - D_F \mu_B^{\;2} [\alpha_1 \alpha_2 \cos^2\theta + \alpha_2 \alpha_3 \sin^2\theta] \varphi$$

α_i = Inverse mass tensor

(Fukuyama, Kubo JPSJ 1970)

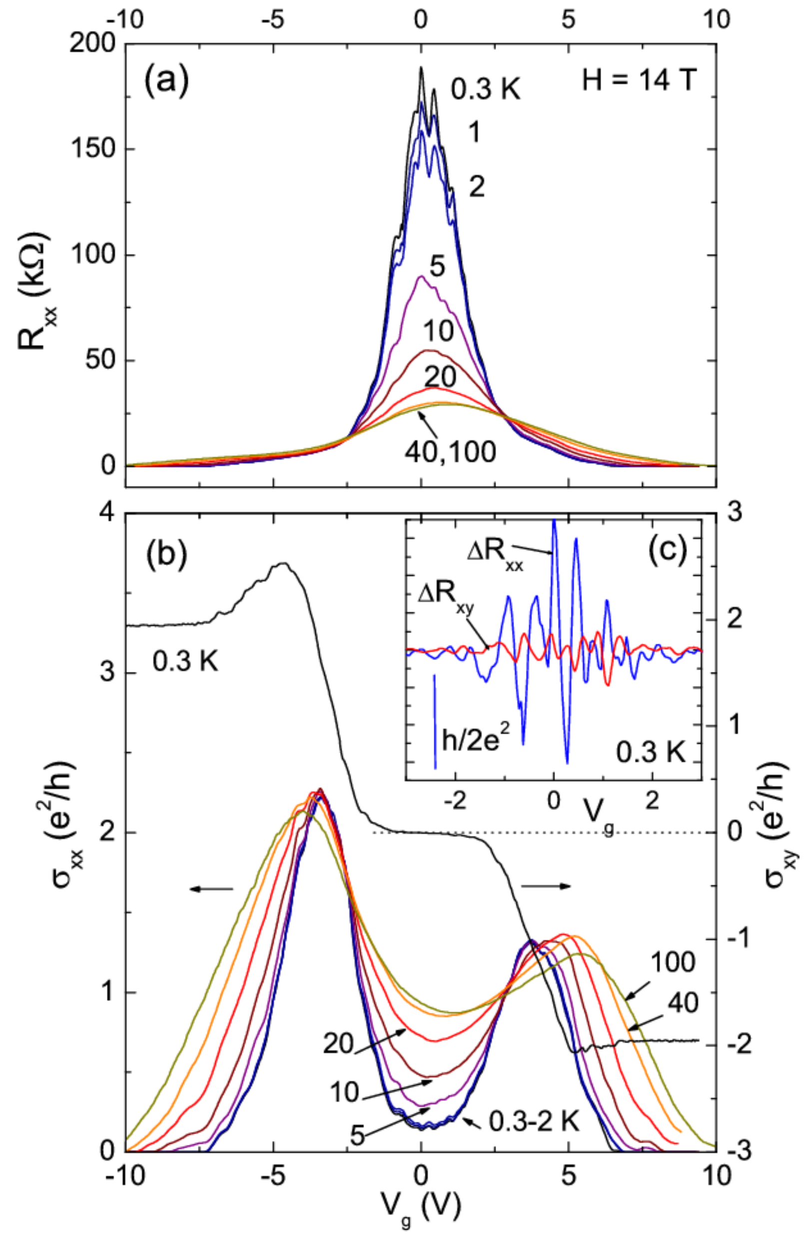
Physics at the Dirac Point (*n* = 0 Landau Level)



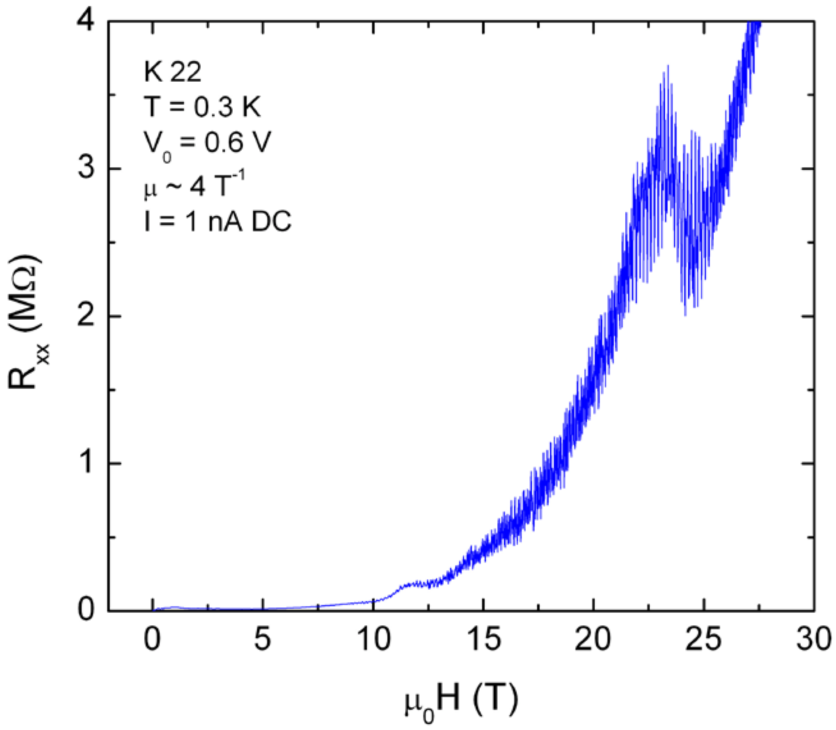
(a) R_{xx} in n = 0 Landau Level increases steeply as $T \rightarrow 0$.

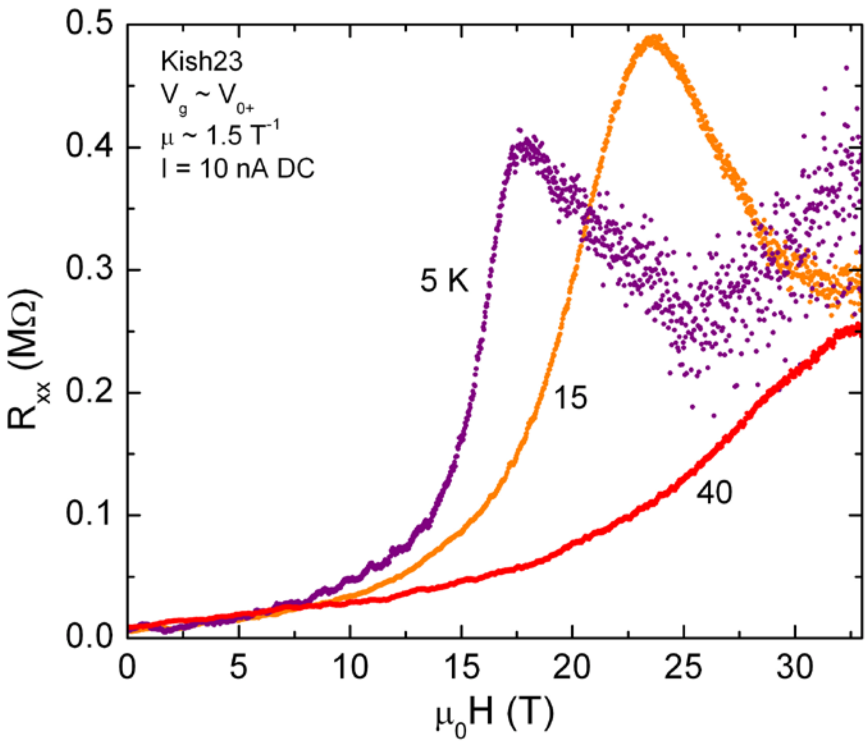
(b) Conductivity shows sublevel split. Hall conductivity displays plateau.

(c) Quantum oscillations in conductance at 0.3 K



Divergent R_0 -- a technical challenge to measure accurately above 1 MΩ





Conductance G_0 at Dirac point $\mu = 0$

Checkelsky, Li, Ong

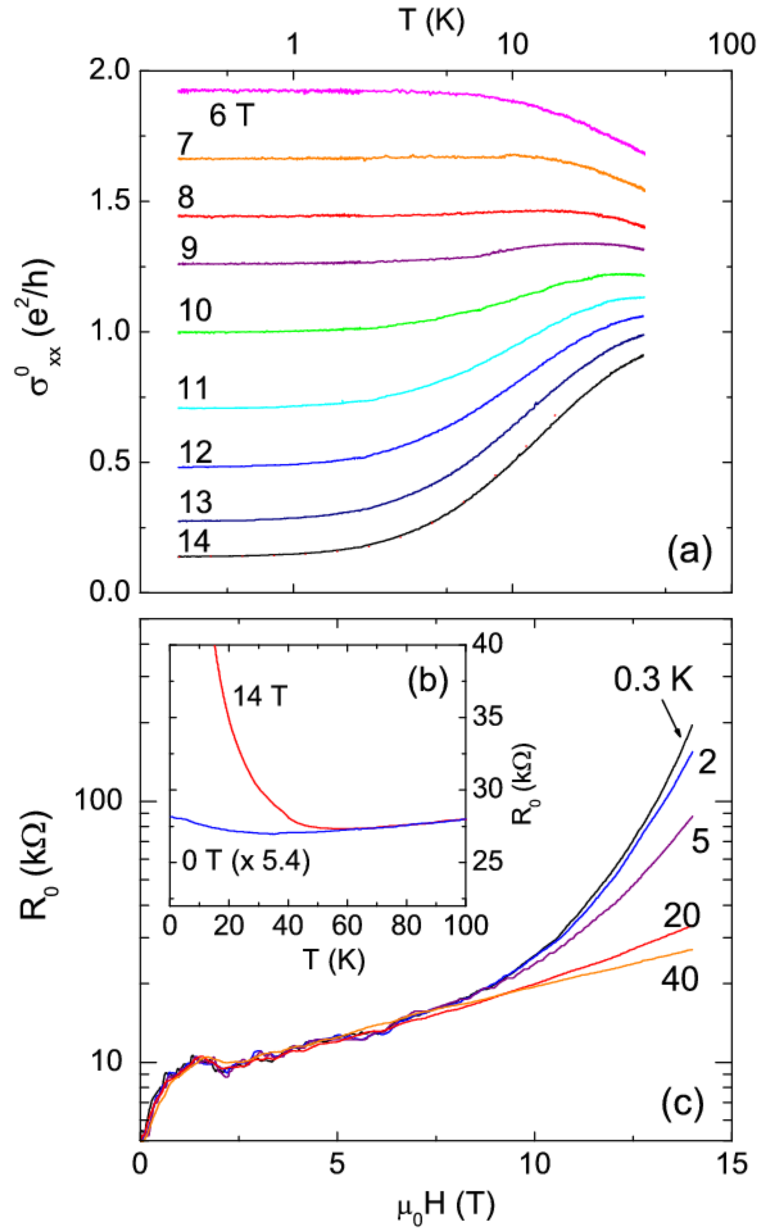
1. At large *H*, G_0 falls as T → 2 K revealing gap

2. G_0 saturates to G_{res} below 2 K *Gapless* excitations

3. G_{res} strongly suppressed by *H* *Faster* than Gaussian *exp(-H²)*

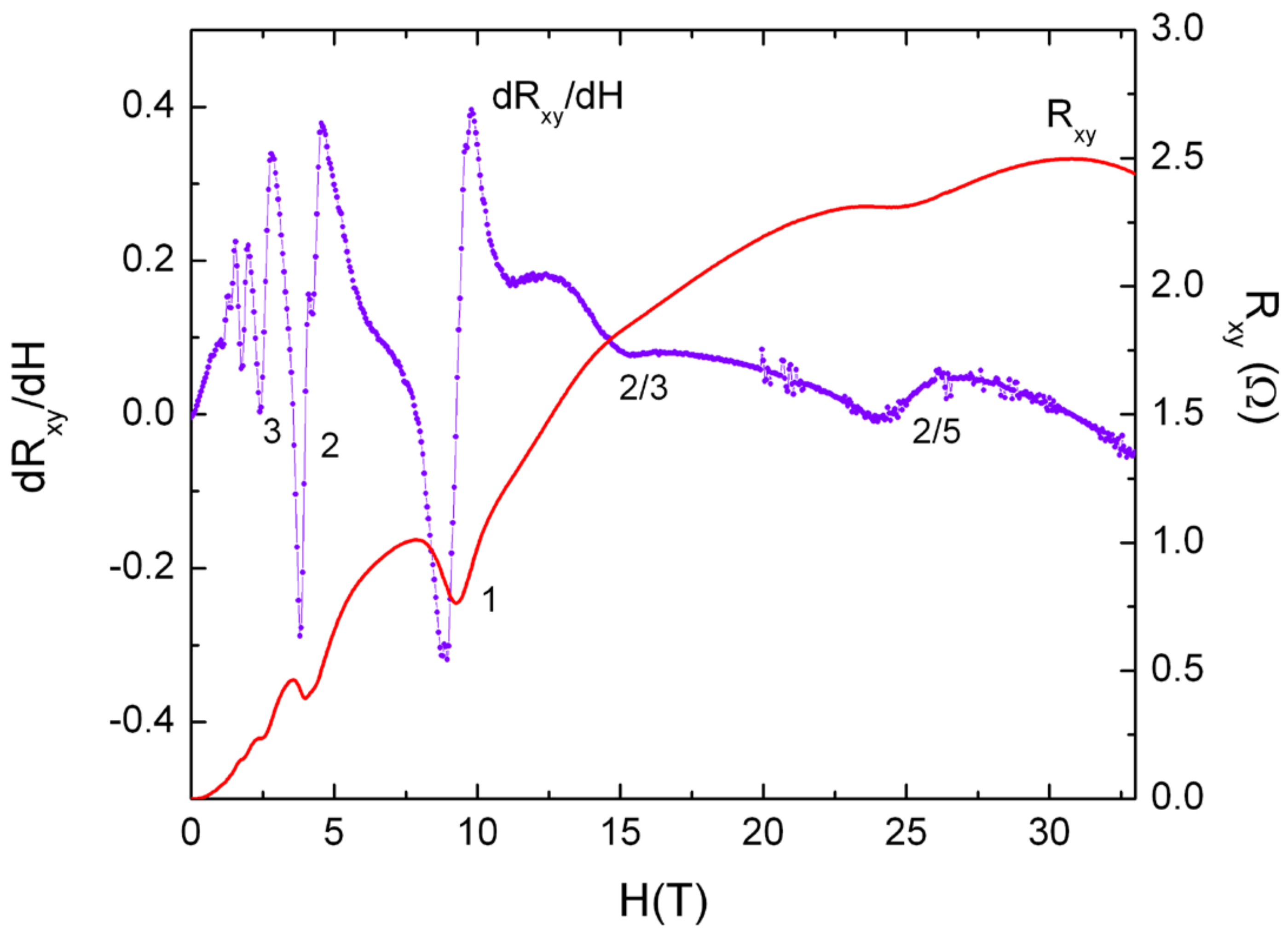
4. Phase diagram reveals unusual approach to insulating state

a) Fixed H, gapless conductance
b) Fixed T, insulating limit at large H

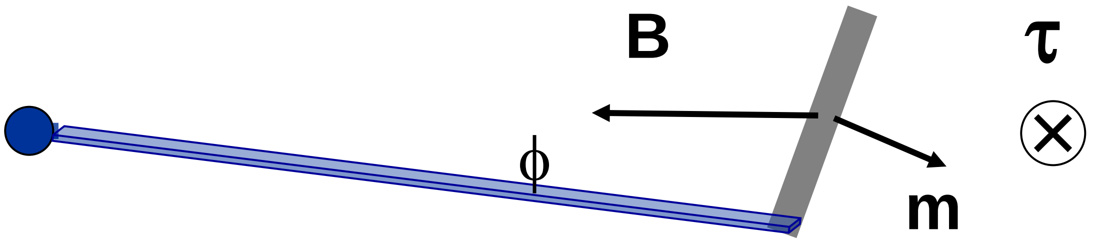


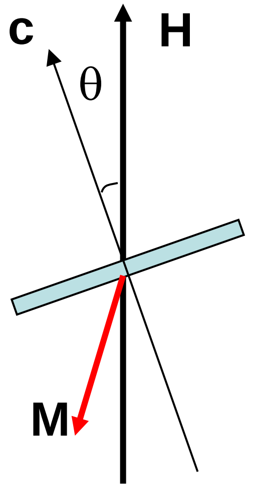
Li, Checkelsky, NPO 2007

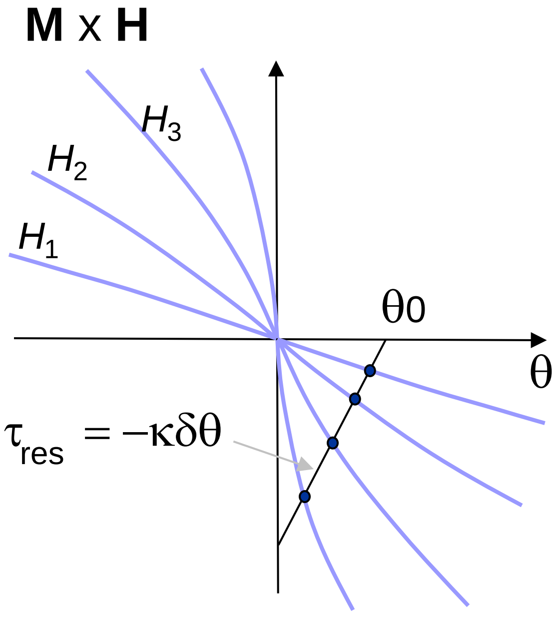
Hall effect



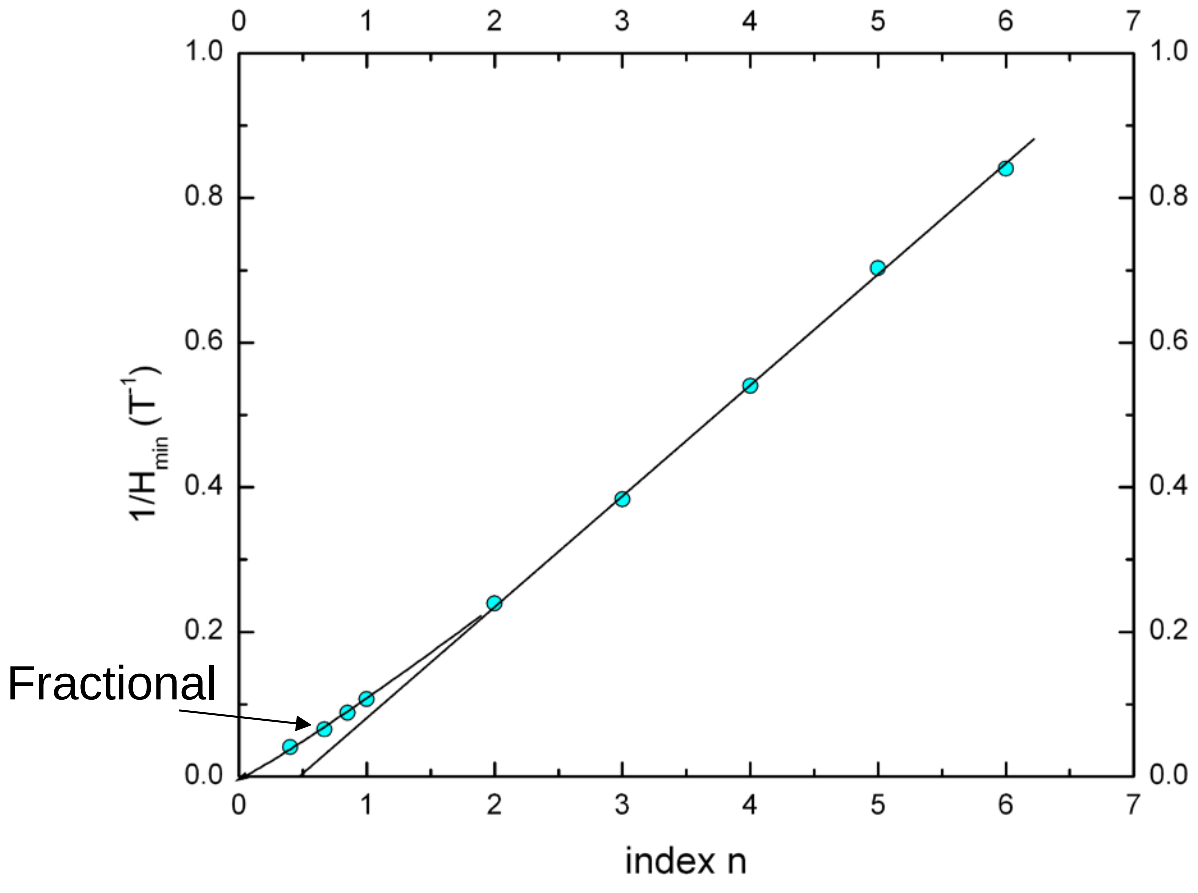
Torque magnetometry with cantilever



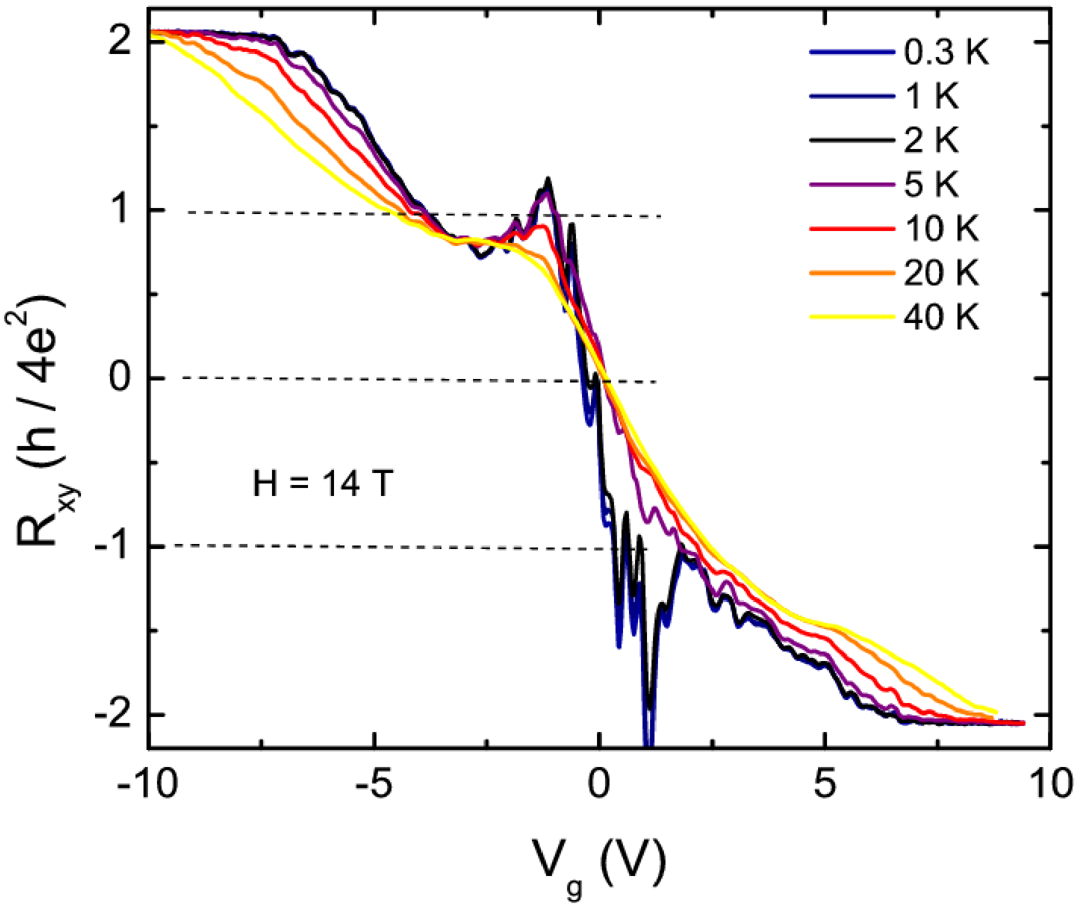




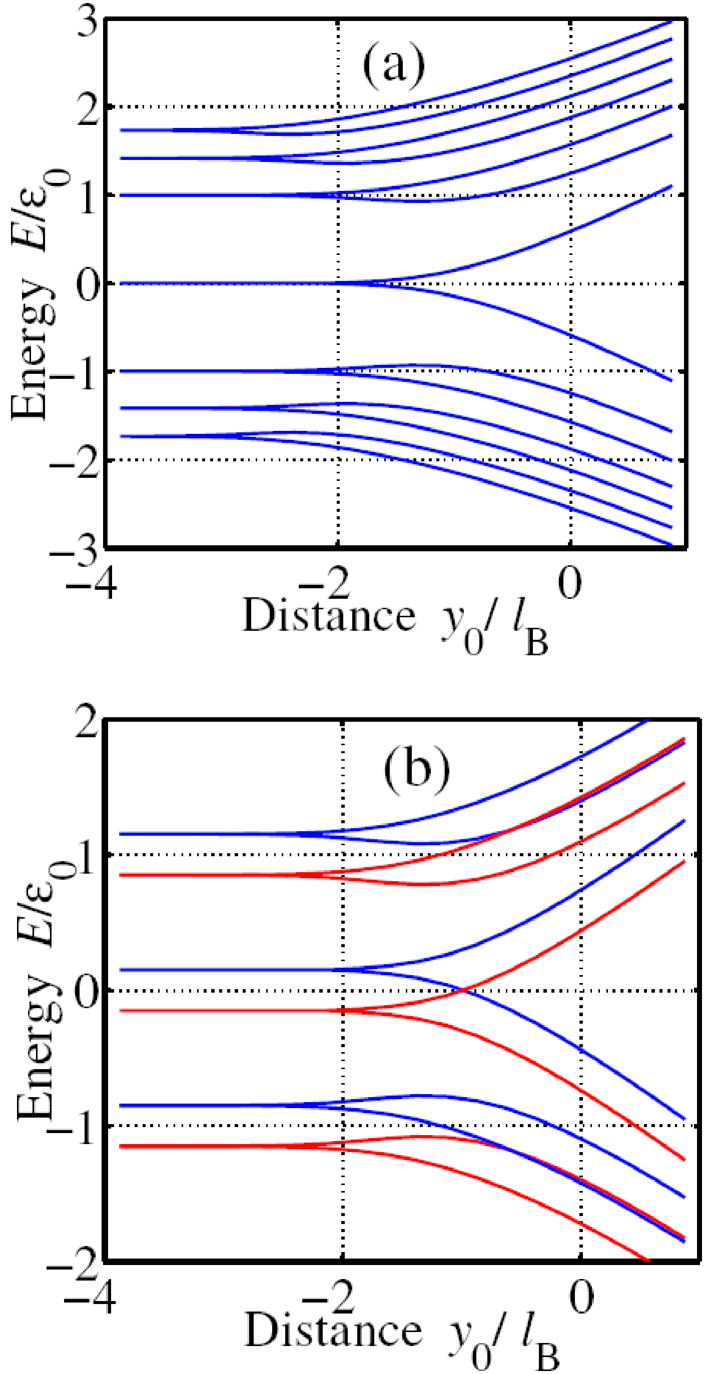
Indexing Landau Levels using Rxx and Rxy



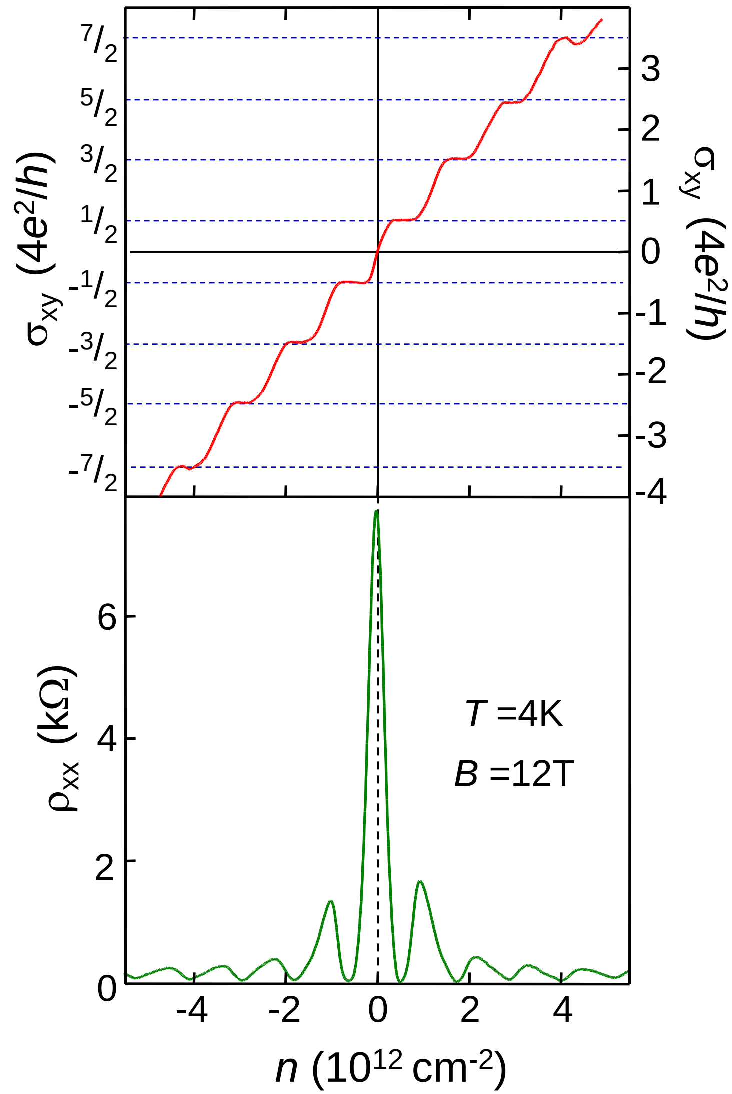
Incipient $\nu = 1$ Hall steps



Spin splitting

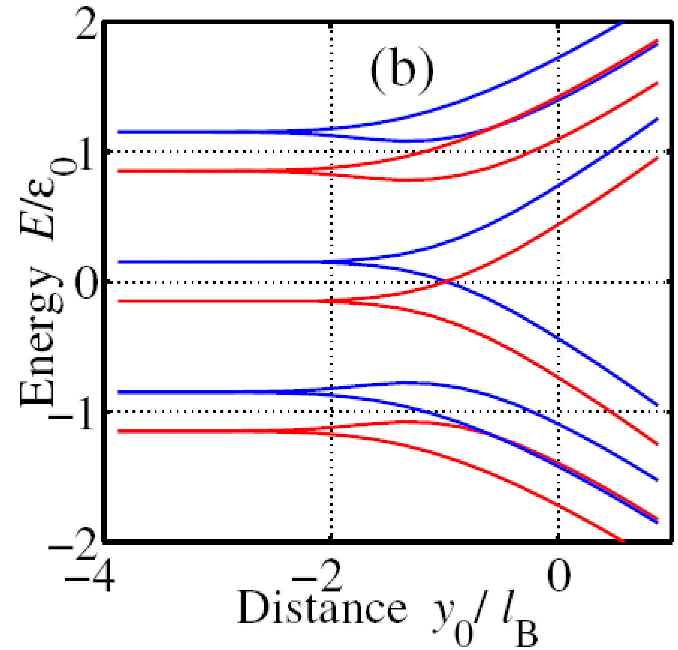


graphene



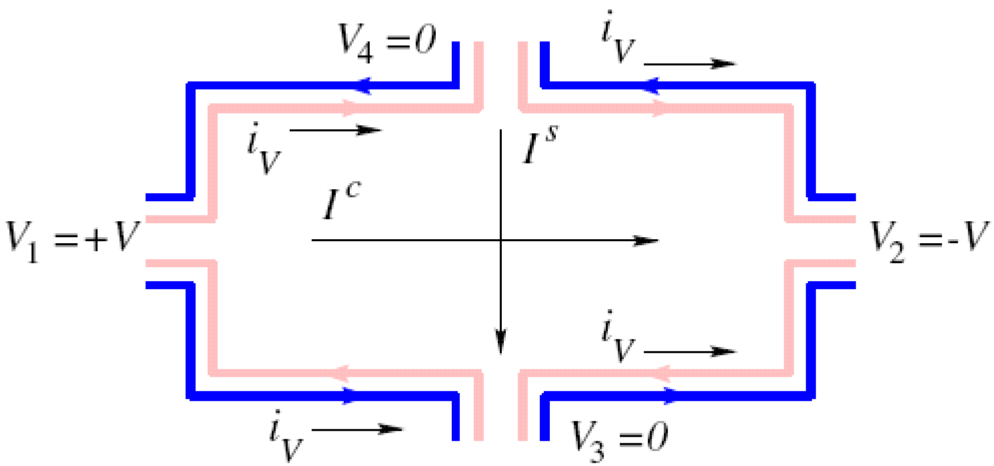
Spin filtered edge states.

Abanin, Lee and Levitov, PRL96,176803(2006)



Spin up moves left

Spin down moves right

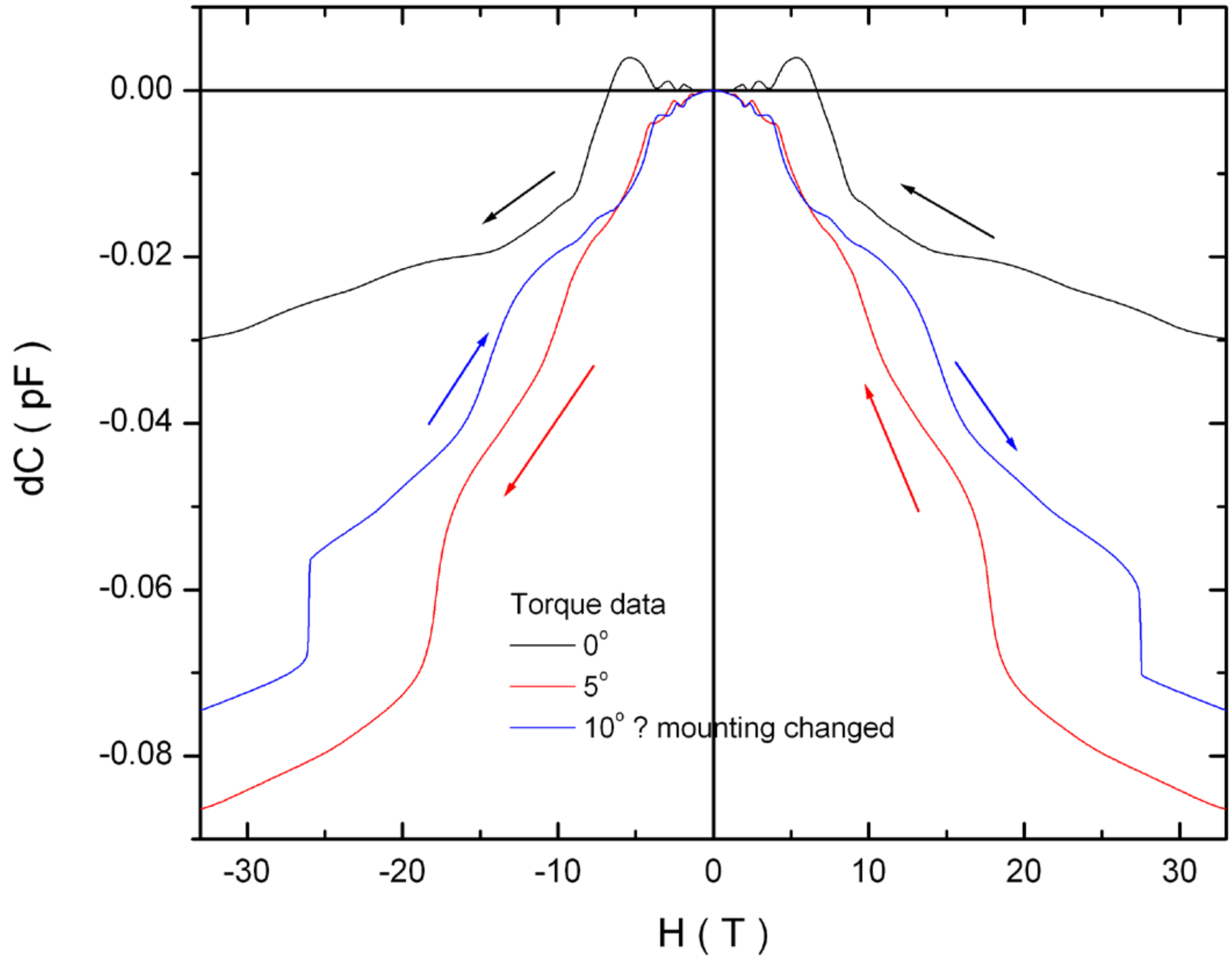


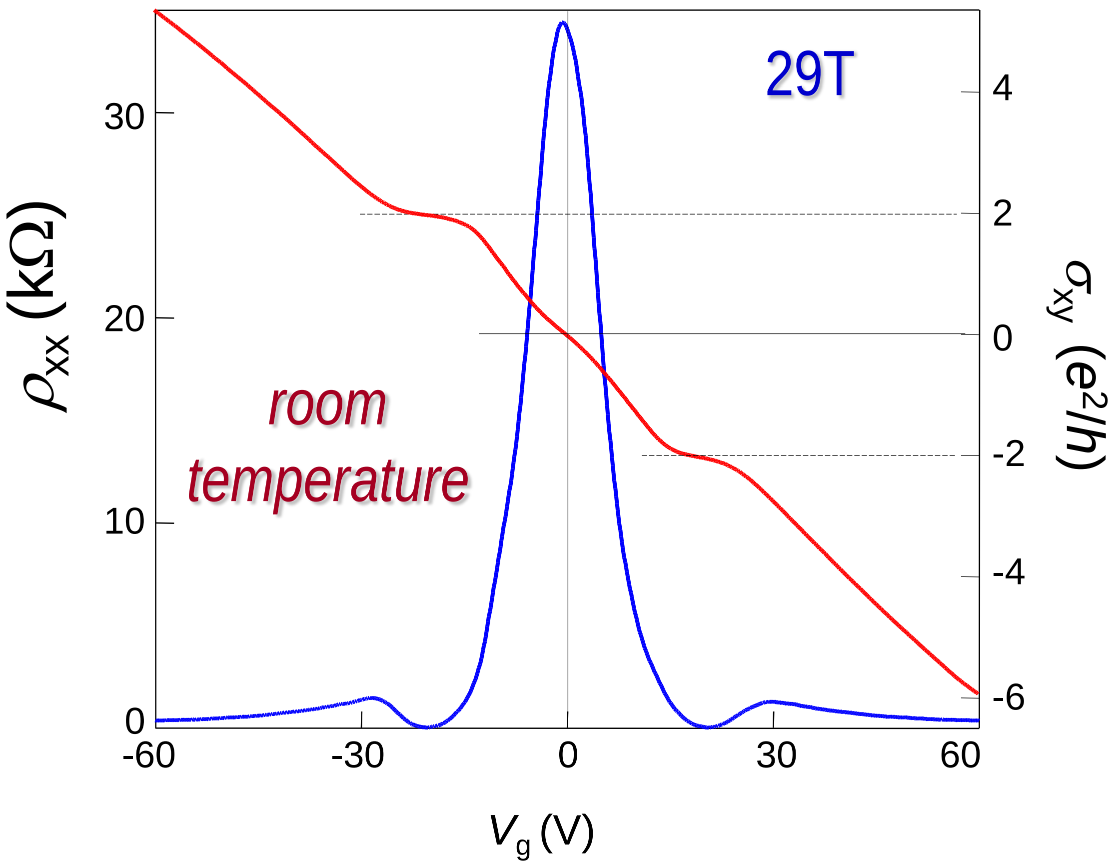
Simple example of a topological Hall insulator. (Kane and Mele,PRL2005) where gap is opened by spin-orbit effect.

No Hall current R_{xy}=0 , but ideal spin current: $I_s=2e^2V/h$.
Also predicts longitudinal charge current, ie
$R_{xx}=h/2e^2$. (13 kOhms)

Torque magnetization of Bi with **H** near trigonal axis

Li, Checkelsky, NPO 2007

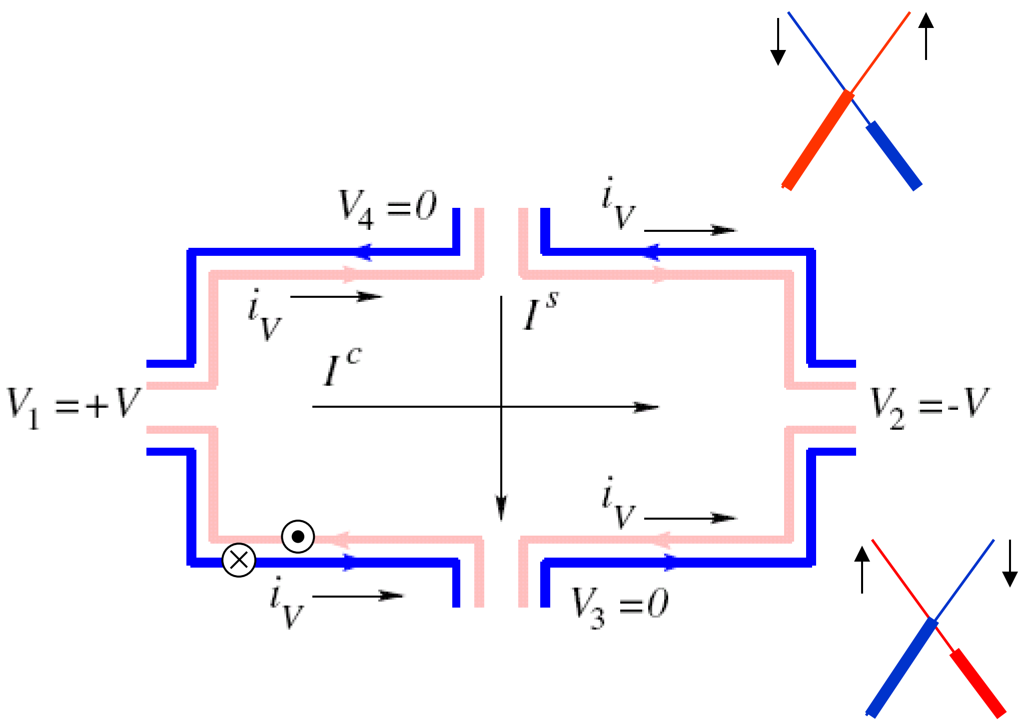


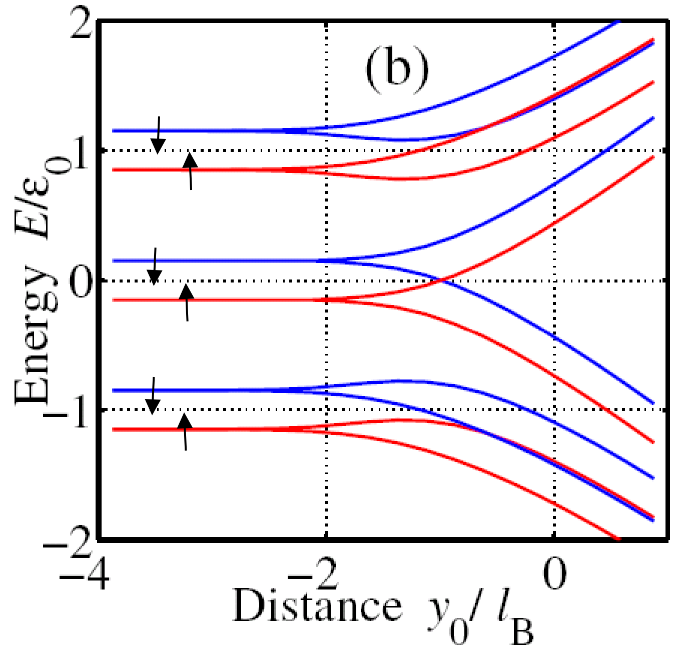

29T
room
temperature
ρxx (kΩ)
σxy (e2/h)
Vg (V)
30
20
10
0
-60
-30
0
30
60
4
2
0
-2
-4
-6

Geim et al

Spin-filtered chiral edge states

Abanin, Lee and Levitov, PRL (2006)





No Hall current R_{xy}=0 , but ideal spin current: $I_s = 2e^2V/h$.
Also predicts longitudinal charge current, i.e.
$R_{xx} = h/2e^2$. (13 kOhms)

Divergent resistance in high field

Approaching KT transition?

Correlation length

$$\xi = a \exp\left[\frac{b}{\sqrt{h-1}}\right]$$

Data suggest H_0 ~ 17 – 18 Tesla