High School Exam Results using Vedic Mathematics compared to Conventional Teaching of Maths, Physics and Chemistry

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ABSTRACT:

Background: Demonstrated Mathematics abilities are required in most professions. Business and engineering in particular regard such skills as essential. High school students and others training for college entrance exams in these subjects require high mathematics grades. Here we present final year exam results in Maths, Physics and Chemistry over the period 2010 to 2015 in an educational institution in Karnataka, India. Whereas the batches graduating in 2010-2013 had only been exposed to conventional teaching methods, the last two batches learned maths using Vedic Mathematics and Geogebra and PowerPoint presentations, visual based computer assisted teaching methods that have proved useful cognitive tools.

Methods: School records of graduates' final exam performance for the six years tabulated for each student, and those for the base period 2010 to 2013 compared to those from 2014 and 2015. Thus results not using Vedic Mathematics or Visual Aids could be analysed against results where they had been employed.

Statistical Analysis: used GraphPad QuickCalcs, and, where necessary SPSS 19.0

Results: Higher scores were obtained for the second period. Statistical differences between the 340 students in the first period and 312 in the second period were both exceptionally significant and exceedingly robust $p < 10^{-6}$ while β was also of this order of magnitude!

Discussion: The extraordinarily conclusive results obtained in this study leave little room for doubt that the methods of Vedic Mathematics combined with computer aided visuals and Geogebra, when well used by enthusiastic and inspiring teachers, can materially improve maths grades received by students graduating from high school. Such results will help students to enter university courses of their choice in more highly rated universities, and give them the confidence to perform well in such courses and contribute more to their professions.

Conclusion: Vedic Mathematics is not merely of cultural interest. This study, the first to quantify its benefits, supplements previous, less formal accounts of its teaching successes.

Introduction

Success in mathematics at high school lays the foundation for success in many careers, particularly information technology, and research. For whole societies it may determine their ability to transform into technology based societies and offer their populations freedom from the drudgery and boredom of jobs in manufacturing. The wide-ranging syllabus of mathematics in high school is a challenge to students, however. The many different methods to be learned in detail present barriers to those not possessing innate maths abilities. Any system that makes acquiring maths skills easier could be of value to societies and nations world-wide. In Chikmaglur in western Karnataka, India, the methods of Vedic Mathematics and related visual based maths teaching were implemented for classes graduating in 2014 and 2015. Here we present a study of the remarkable scores they achieved in their final exams in mathematics.

Vedic Mathematics

For those not familiar with the system its concepts, Vedic Mathematics, abstracted from those methods of guidance and teaching used in ancient times in South Asia, may sound an unlikely addition to modern high school teaching methods. The Vedic system of education, however, focussed more on expanding the container of knowledge, than on stuffing children's brains full of facts. The system of Vedic Mathematics originated in the work of the late Swami Bharati Krsna Tirthaji Maharaja, Shankaracharya of Puri, and a text of the topic (1992) was published in his memory was published in his memory in 1965. The Shankaracharya's book describes a new method of teaching high school maths, based on his own experience as a high school teacher before becoming a monk. Many schools have since started using it, reporting that both students and teachers find its methods new and enjoyable. Several aspects of its system contribute to this. First it empowers students by offering a choice of methods when solving problems. Its focus is on the student, not on the problem. It tacitly incorporates the principle that problem solving in mathematics requires application of patterns of transformation changing questions into answers, and explicitly supplies 32 kinds of transformation that students' minds can utilize to bring about the required transformation for any problem. Learning these patterns and seeing the expanded power that results in the mind is a thrilling process for any student.

The Shankaracharya's book is written in the aphoristic style typical of ancient South Asia, facilitating memorisation of its contents and methods. Its system comprises 16 'sutras' and 16 'upasutras', each with applications to many areas of mathematics. After all, the same mental patterns can be used to solve many different kinds of problem. They offer a number of ways to solve each kind of problem, generalised techniques, inspection methods, and special methods for particular problems. These factors mean that the system promotes strategic thinking about problems, encouraging students to adopt a holistic approach to solving them. In brief, student problem-solving abilities develop. Visualizing patterns to solve problems transforms problem solving into a game making it enjoyable (Ziatdinov & Musa, 2012). For such reasons, Vedic Mathematics is considered most laudable wherever it is utilized (Muehlman, 1998).

Commented [AP1]: Muehlman, who published the work in 1998, did not have access to UK works which appeared in 2005, 2009 and 2010! Therefore, the use of 'For such reasons' where it appears is incorrect. A better place to put this statement would be towards the end of previous paragraph after '…student problem solving abilities develop'.

The UK's Ken Williams and James Glover have given significant contribution in popularising Vedic mathematics methods by publishing several books (Nicholas, Williams, & Pickles, 2010; Williams, 2009; Glover, 2005a). However, the Vedic maths students and teachers are not aware of any formal quantitative assessments of the benefits it provides. For that reason, the authors of this paper decided to inspect exam records at the school where the last two authors were teaching near the Western Ghat mountains in the town of Chikmaglur in Karnataka, India. The methods used in Maths teaching also included computer based visual aids such as power points and Geogebra, so these also require description.

Vedic Mathematics Examples: Squaring 0.35 uses sutra, 'By one more than the one before'. Determinants use sutra 'vertically and crosswise'.

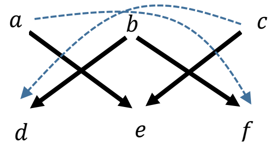
Simultaneous Equations: Find values of x and y if $3x + 5y = 13$ and $4x + 7y = 18$

$3x + 5y = 13$

$4x + 7y = 18$

$$x = \frac{5.18-13.7}{5.4-7.3} = 1, \quad y = \frac{13.4-18.3}{5.4-7.3} = 2,$$

Pattern of Multiplication:



$ax + by = c$

$dx + ey = f$

$$\boldsymbol{x = \frac{bf-ce}{bd-ae}}, \ \boldsymbol{y = \frac{cd-af}{bd-ae}}$$

Integration using Partial Fractions: $\int \frac{\mathbf{1}}{(\boldsymbol{x}+\mathbf{2})(\boldsymbol{x}+\mathbf{3})} = \int \frac{\boldsymbol{A}}{(\boldsymbol{x}+\mathbf{2})} + \int \frac{\boldsymbol{B}}{(\boldsymbol{x}+\mathbf{3})}$

Here put $x = -2$ $(\because \ x + 2 = 0)$ in LHS of the equation, we get $\frac{1}{-2+3} = 1$ which is A.

Put $x = -3$ $(\because \ x + 3 = 0)$ in LHS of the equation, we get $\frac{1}{-3+2} = -1$ which is B.

Then we can integrate to obtain, $I = \log(x + 2) - \log(x + 3) + C$.

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At the school in western Karnataka where the Vedic Maths approach has been implemented, the rules and topics included in the curriculum are listed in Table 1 below.

Table 1: Topics taught using Vedic Mathematics

Topic	**Vedic Math Sutras Used**
1. Quadratic Equations, Integration	Differential Calculus
2. Simultaneous Equations Determinants Multiplication	Vertically and Crosswise Addition and Subtraction Transpose and Apply
3. Partial Fractions, Conics, Straight Line	Paravartya
4. Squaring, Cubing	By One More than the One Before
5. Factorization	Proportionately By Alternate Elimination and Retention The First by the First and the Last by the Last
6. Highest Common Factors	Addition and Subtraction The First by the First and Last by the Last By Alternate Elimination and Retention

Table 1 Caption: Table 1 illustrates various Vedic Maths sutras used to help students solve problems in selected topics of basic high school mathematics. The left column displays some such topics, while the right column names the sutras used in training student to solve problems in those topics.

Learning to implement use the sutras named in Table 1 to execute with ease the geometric and algebraic patterned instructions in Examples 1 and 2 greatly increases student confidence when faced with exam questions. And confidence is more than half the key to performing well in examinations with a strict time-limit designed to distinguish good students from the mediocre.

Computer Based Visual Aids to Teaching:

Information technology tools to visualise dynamics of geometric shapes facilitates student learning (Reis & Ozdemir, 2010). Levels of maths achievement are higher when visualization is used (Arcavi, 2003), and technology based environments enhance conceptual understanding of mathematics (Alagic, 2003). Student motivation to solve problems increases when simulations are used (Santulli, 2006). Geogebra dynamic visuals improve understanding of differential calculus (Diković, 2009), and its simulations improve learning of trigonometry (Zengin, Furkan, & Kutluca, 2012). Sole (2014) found that real-world examples to bridge from Algebra to Calculus increase student confidence. Geogebra visuals can represent both algebraic and geometric aspects of differentiation simultaneously. They create effective ways to examine functions in terms of their graphs (Takači, Stankov, & Milanovic, 2015), they enhance both conceptual and procedural mathematics knowledge (Zulnaidi & Zakaria, 2012). Math classes have been found more enjoyable when teachers use computer-based visual aids. Math anxiety, a problem for high school mathematics studied by the present authors (2016) also improves (Taylor & Fraser, 2013). Subject enjoyment also increases in such environments (Vandecandelaere, Speybroeck, Vanlaar, De Fraine, & Van Damme, 2012).

Presentation slides communicate mathematical concepts effectively, particularly when pictures and photos, charts and graphics, use low word density (Brock & Joglekar, 2011). Together with good explanations, slides improve visual memory further improving math achievement (Strauss, Corrigan, & Hofacker, 2011).

In the Karnataka high school, Geogebra and presentation slides were used for the 2014 and 2015 batches. Concepts taught and representative visuals employed are listed in Table 1.

TABLE 1: Concepts taught using visual aids

Relations and Functions
Inverse Trigonometric Functions
Continuity and Differentiability
Applications of Derivatives
Application of Integrals
Vector Algebra and Three Dimensional Geometry

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Traditional Mathematics Teaching:

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From 2010 to 2013, usual chalk and board methods without any of the special teaching methods described in the previous two sections were used. Mathematical concepts were explained on the board, following which students wrote problem details and calculated answers if they could. All graphs and pictures were drawn in chalk.

Methods: Results in Maths, Physics and Chemistry achieved by final year students between 2010 and 2015 were obtained from College records, and tabulated by % obtained.

Results:

Means and standard deviations of Physics, Chemistry, and Mathematics marks for each year from 2010 to 2015 are presented in Tables 2a and 2b, the latter showing improvements seen in Maths results for 2014 and 2015, compared to 2010 to 2013.

TABLE 2

Means and Standard Deviations of Physics, Chemistry and Maths Scores 2010-13

Subject → Year (No) ↓	Physics	Chemistry	Maths	Physics Theory Only	Chemistry Theory Only	Maths
2010 (31)	77.71 ± 16.67	68.45 ± 16.15	72.26 ± 19.20	0.75± 0.18	0.65± 0.18	0.72± 0.18
2011 (69)	73.72 ± 16.01	68.77 ± 17.30	61.09 ± 22.97	0.71± 0.18	0.65± 0.19	0.61± 0.23
2012 (108)	77.93 ± 16.73	69.56 ± 17.52	68.98 ± 22.25	0.76± 0.18	0.66± 0.19	0.69± 0.22
2013 (132)	82.36 ± 15.40	81.58 ± 15.24	67.26 ± 21.45	0.81± 0.17	0.80± 0.17	0.67± 0.21
2014 (143)	84.44 ± 12.31	87.06 ± 11.03	86.23 ± 15.89	0.80 ± 0.16	0.84± 0.14	0.86± 0.16
2015 (169)	81.83 ± 12.90	82.49 ± 10.780	82.22 ± 17.22	0.75± 0.18	0.76± 0.15	0.82± 0.17

Table 2a Caption: Table 2a Columns display Means and Standard Deviations of student marks in Physics, Chemistry and Mathematics for each year's batch. Fractional scores for theory alone without practical examination marks are displayed in the last columns to highlight improvement in Maths.

Note particularly that mean scores were above 80 for all subjects in 2014 and 2015. These results are graphically displayed in Figure 1

Figure 1: Average ~~Marks~~ marks for the ~~Six~~ six ~~Batches~~ batches from 2010 to 2015

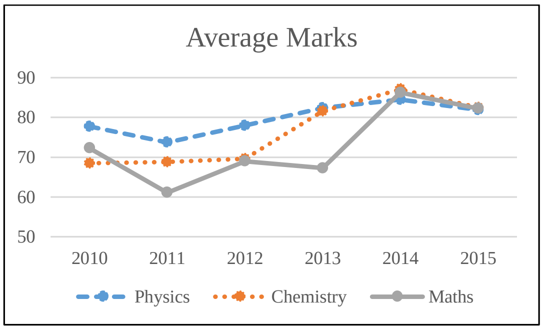

Figure 1 Caption: Figure 1 displays average marks in Physics, Chemistry and Mathematics for all batches from 2010 to 2015.

Table 2b displays the means and standard deviations of results in percentages (or fractions after practical examination marks had been omitted) for batches between 2010 and 2013 combined when Vedic Mathematics and computer aided visuals were not in use, and for 2014 and 2015 when such methods were employed. The Table displays improvements, including practical exam marks (3 columns on left), and not including them (3 columns on right). On the right side improvements in mean were: Physics 0.00, Chemistry 0.08, and Maths 0.17.

Table 3: Means and Standard Deviations for batches with and without Vedic Maths

Years (Nos.)	Physics	Chemistry	Maths	Physics Theory Only	Chemistry Theory Only	Maths
2010-13 (340)	78.78 ±16.33	73.97 ± 17.51	67.01 ± 21.99	0.77±0.18	0.71±0.19	0.67± 0.22
2014-15 (312)	83.03 ± 12.68	84.58 ± 11.12	84.06 ± 16.72	0.77±0.17	0.79±0.15	0.84± 0.17
M-W 'Z'	-2.749	-7.970	-10.749	-0.286	-5.389	-10.749
Signif. P	0.006	0.00012	0.000001	0.775	0.00001	0.000001

Table 3 Caption: Table 3 presents mean and standard deviations for marks in Physics, Chemistry and Maths examinations for the 340 students in years 2010-2013 and 312 students in 2014 & 2015. The last three columns on the right display fractional scores for theory alone cutting out scores on practical examinations, further highlighting Maths improvement. Bottom rows display non-parametric test statistics between the first two rows.

Discussion:

Clearly from Table 2 and Figure 1, the two batches in 2014 and 2015 improved greatly over batch scores between 2010 and 2013. Using Vedic Mathematics and computer aided visuals made for highly significant improvements. Academic improvement was also substantial. Student numbers make the power of these results extraordinarily robust. A G*power calculation yields infinitesimal beta all normal values of alpha.

Another paper will report changes in levels of Distinctions (85% or more) and full marks (100%). These had Odds Ratios 4.2 and 9.36, with both p values far below 0.0001.

Improved results in Chemistry are probably due to improved teaching staff quality, rather than higher quality students, which is not consistent with lack of improvement on physics theory exams. High improvement levels in Maths scores (means +17%) are therefore real.

These results show that Vedic Mathematics and visual based teaching methods can greatly improve exam results. Similar improvements have been qualitatively reported following introduction of Vedic maths in other countries (Glover, 2005b). In India, however, such levels of math exam result improvement are new. They are due to methods of teaching described above, Vedic mathematics and visual aids. Learning through Geogebra and computer visuals supports math learning that is problem-oriented and discovery-based; key aspects of math education (Hohenwarter & Preiner, 2007). The high levels of observed improvement reported here favor this. Quantifying this in more detail would be helpful.

Strengths of the study: the robustness of the results and establishment of baseline percentages by the first four years data obtained before introducing Vedic maths or visual aid methods.

Weaknesses of the study: only one teacher, the first author, used Vedic Maths and Visual Aids; the other two did not. However, the first author taught three quarters of the syllabus, so all students received the new approaches to understanding maths and problem solving. A second weakness is that only two years examination result data for students using Vedic mathematics and computer-based visual teaching is available. But consistency and significant statistics in both alpha and beta negate this problem.

Future Outlook: Further studies should follow up these results, particularly including new data when it becomes available at the end of this academic year.

Conclusion: Vedic maths methods and visual aids will greatly improve university entrance results and numbers qualifying to enter college courses in science and engineering. Adding Vedic maths methods to teaching mathematics in high school and university courses, especially in teacher training colleges, will greatly impact India's aspirations to become a technology-based society, and a fully developed country by 2030-2040.

Commented [AP2]: Earlier the paper alludes to the fact that there was significant knowledge in India. For e.g., "…and Vedic Mathematics, abstracted from those methods of guidance and teaching used in ancient times in South Asia …the Vedic system of education, however, focussed more on expanding the container of knowledge". Therefore, the statement "…India's aspirations to become a knowledge-based society…" is mutually contradicting with the earlier assertion in the paper. Combining this with the fact that the introductory remarks in the paper has been about Information technology, suggests that perhaps the intention of the authors is to suggest 'a technology based society' and do not mean to say a 'knowledge-based society'. It is suggested that 'knowledge-based society' be changed to 'technology-based society'.

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