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High School Mathematics Curriculum Study

Prepared for the Washington State Board of Education

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Executive Summary

Scope of Work

At the request of the Washington State Board of Education (SBE), two members of the Strategic Teaching (ST) team, Drs. Guershon Harel and W. Stephen Wilson reviewed four mathematics programs that the Office of the Superintendent of Public Instruction's (OSPI) had identified as matching well to the content in Washington State's standards. The programs, rank ordered according to how well each matched Washington's standards, were:

- *Holt Algebra 1, Geometry, and Algebra 2*
- *Discovering Algebra, Geometry, Advanced Algebra*
- *Glencoe McGraw-Hill Algebra 1, Geometry, and Algebra 2*
- *Core-Plus Mathematics Courses 1, 2, and 3*

ST's review studied the mathematical soundness of these programs in order to inform the OSPI's curriculum recommendations.

For each of the programs, ST examined the following three topics 1) translating between forms of linear functions, equations, and inequalities and using them to solve problems; 2) translating between and applying forms of quadratic equations, and 3) the triangle sum theorem. ST did not consider pedagogy—how the topic is taught—during its review since mathematics can be taught well using a variety of methods.

The following chart summarizes the conclusions drawn by Drs. Harel and Wilson (Curriculum vitae are located in Appendices A and B.) The complete reports are available at www.strategicteaching.com/washington_state_standards_.html.

Findings

Program	Linear functions	Quadratic functions	Triangle Sum Theorem
Holt Algebra 1, Geometry, and Algebra 2	✓	✓	✓
Discovering Algebra, Geometry, Advanced Algebra	—	—	—
Glencoe McGraw-Hill Algebra 1, Geometry, and Algebra 2	✓ and —	—	✓
Core-Plus Mathematics Courses 1, 2, and 3	✓ and —	✓	—

Key	
+	Mathematically sound
✓	Mathematical soundness meets minimum standard
✓ and —	Mathematical soundness received two difference scores
—	Mathematically unsound

As the graphic above indicates, no program earned a "+" in any area. The two reviewers easily agreed on this as well as all of the categories rated "-." While the reviewers also agreed on the mathematical characteristics of all of the programs, they differed on the importance of their specific flaws and therefore on how some programs should be scored. For example, the reviewers agreed that *Glencoe* misuses mathematical language, but one reviewer believed this to be a more profound issue than the other reviewer.

In the end, only *Holt* was found not to be mathematically unsound in any of the topics addressed in this study.

Findings related to linear functions, equations, and inequalities

None of the reviewed programs provided the appropriate justifications for the basis of the graphing of linear functions. First, no program shows that the graph of a linear equation really is a line. Second, no program shows that a line in the plane is the graph of a linear equation. Third, although slope is defined, it is never shown that slope is well-defined, namely, that slope is independent of which two points one chooses to compute it.

That said, *Holt* and *Glencoe* cover the topic moderately well. Both have excellent sets of problems, with Glencoe's collection being outstanding. Glencoe is frequently imprecise in its use of mathematical language. Although both reviewers agreed with this statement, Dr. Harel felt this and other characteristics made the program unacceptable. Dr. Wilson felt that the issues could be overcome and the program should be considered acceptable.

Discovering and *Core-Plus* often side-step the use of formal algebra. Consequently, students do not gain the fluency they will need and they do not learn to appreciate the power of the general form. Moreover, both programs incorrectly imply that calculators, tables, graphs, and equations may be used interchangeably to solve problems and are mathematically equivalent.

Despite this shortcoming, Dr. Harel felt the strength of *Core-Plus'* problems overshadowed other weaknesses and rated the program "✓." Dr. Wilson disagreed and found Core-Plus to be mathematically unsound ("-"). With a split score of "✓" and "-," *Core-Plus* joins *Glencoe* as rating less than minimum expectations.

In *Discovering,* both reviewers found the mathematics related to linear functions as compromised and rated it mathematically unsound ("-"). The problems give students the opportunity to develop a beginning understanding of linear functions, equations, and inequalities but not to consolidate the understanding into the big ideas of mathematics.

Findings related to forms of quadratic functions

None of the four programs do enough to move beyond what is needed to solve a specific problem and move to the general case. Neither do they do enough to provide definitions and justifications. For these reasons, none of the programs was rated as mathematically sound ("+") in its treatment of quadratic functions.

Core-Plus and *Holt* fare the best in their treatment of the mathematics related to forms of quadratic functions.

Holt's coverage of translating between forms of and applying quadratic functions is perfunctory and prescriptive at times, but mathematically legitimate. There is strength in how *Core-Plus* develops quadratic expressions and functions, especially through the lens of max/min problems, but its handling of translating between forms is problematic. For example, translation from the standard form to the factored form is only done in cases where the roots are integers.

Besides the flaws shared by all the programs, *Discovering* and *Glencoe* are mathematically unacceptable for additional reasons. *Discovering* places more emphasis on the study of graphs rather than the study of quadratic functions. *Glencoe* presents many unexplained, unjustified rules and procedures related to algebraic skills. Neither program helps students build an understanding of the structure of algebra.

Findings related to the triangle sum theorem

Holt and Glencoe each present a sound proof of the triangle sum theorem supported by postulates, definitions and other necessary theorems. While each proof is adequate, in both cases there are flaws in the development leading up to the proof. This includes a lack of attention to the distinction between a "postulate" and a "theorem" and an interruption to the development of a deductive structure for synthetic geometry used in the proof by two sections on analytic geometry.

Neither *Discovering* nor *Core-Plus* offers a proof based on sound underpinnings. In *Discovering*, the proof itself looks valid, but it is based on almost 700 prior pages of inductive geometry. In *Core-Plus* the proof depends on empirically derived results to prove theorems that are then used to formally prove those previously empirically derived results. This is an example of circular reasoning.

Suggestions for moving forward

Just as there is no perfect set of standards, there is no perfect set of instructional materials. And less is known than should be about the effectiveness of particular programs.

That said, textbooks do matter. A recent study reveals that students enrolled in either *Math Expressions* or the *Saxon Math Program* score 9 to 12 percentile points higher on one of the federal government exams than their counterparts in classrooms using *Investigations in Number, Data, and Space* or *Scott Foresman-Addison-Wesley Mathematics.*[1] Washington should feel good about its recent recommendation of *Math Expressions.*

The OSPI review and this ST review of high school mathematics texts give the SBE and the OSPI valuable information, but no clear path to recommending more than one program.

The OSPI's and ST's Findings

	OSPI Final Composite Score[2]	ST Mathematical Soundness Rating
Holt	0.838	Meets minimum standard
Discovering	0.835	Unacceptable
Glencoe	0.826	Approaches minimum standard
Prentice Hall	0.820	Unknown
McDougal Littell	0.783	Unknown
Core-Plus	0.780	Approaches minimum standard

[1] Mathematica Policy Research, Inc., *Achievement Effects of Four Early Elementary School Math Curricula*; Retrieved on March 4, 2009 from http://www.mathematica-mpr.com/education/

[2] Composite score is comprised of 1) content/standards alignment, 2) program organization, 3) student learning, 4) assessment, 5) instructional/professional support, and 5) equity/access and is calculated for the series as a whole. The score in this table does not take into account reductions in scores for standards met above or below the expected course level.

Based on the OSPI's and ST's findings, summarized in the chart, ST suggests the following:

1. Recommend *Holt* because it exceeds the threshold for alignment with the content standards and meets the minimum standard for mathematical soundness.

2. Do not recommend *Discovering* because it was found to be mathematically compromised within the scope of this project.

3. Communicate to districts the additional challenges, identified within the scope of this project, that would come from the adoption of *Glencoe* and *Core-Plus.*

4. Communicate the findings of this report and the more detailed reviewer reports to the publishers. Some publishers may be able to make adjustments to make Washington's work easier. Establishing a working relationship between the state and the publishers hopefully may contribute to strengthening the quality of textbooks.

5. Expand the examination of mathematical soundness to other programs with strong matches with respect to content standards. Additionally, the OSPI should explore ways to strengthen the soundness of the programs that are reviewed and meet minimum standards.

6. Have the OSPI consider ways to leverage district work such as by forming a statewide consortium or work groups to share supplements and instructional practices that shore up identified weaknesses in mathematics texts.

7. Track student progress against curriculums adopted by districts. Over time, Washington can make a significant contribution to what is known about program effectiveness.

8. Establish a schedule to conduct a complete review of instructional programs every two years. Consider a policy that recommends all programs provided that they meet minimum thresholds for both standards content and mathematical soundness.

Background on High School Curriculum

Washington State continues to work toward making a world-class education available to all students and to prepare them to be work and college ready, by implementing the steps described in the State Board of Education, Professional Educator Standards Board and the Office of Superintendent of Public Instruction's *Joint Mathematics Action Plan.*[3]

Over the past two years, guided by stakeholder input and legislative mandate, Washington State has established new mathematics standards and recommended instructional programs for the elementary and middle school levels. It must also identify up to three high school mathematics programs that align with the new mathematics standards. The state is working with its partners to create strong professional development and to conduct new assessments of programs to ensure they are aligned to the new mathematics standards.

As required by the legislature, the OSPI must identify not more than three mathematics programs for high school that align to the high school standards adopted on July 30, 2008.

The OSPI's review of instructional programs

After the standards were approved, the OSPI examined thirteen different core mathematics programs, each of which included three years of instructional materials. Some programs are organized into Algebra I, Geometry, and Algebra II and other programs are organized into Integrated Mathematics I, II, and III. As detailed in "*2008 High School Mathematics Core Comprehensive Materials Review & Recommendations Report: Initial Recommendations*" (January 15, 2009), OSPI examined the degree of match between the various programs and Washington's standards. It also looked at program organization and design and it completed a mathematical analysis. This review was conducted with meticulous attention to detail and in ways designed to reduce bias.

As stated in the report on page 10, the process included:

- Rigorous inventory control during publisher check-in, reviewer check-in/out, and publisher check-out
- Training reviewers in how to use the scoring instruments
- Real-time data entry

[3] *Joint Mathematics Action Plan: Building the Proper Foundation*; Washington State Professional Educator Standards Board; November 30, 2006; Retrieved Feb. 7, 2009 from www.pesb.wa.gov/Publications/reports/2006/JointMathematicsActionPlan11-30-06.pdf

- Variance checks and corrective training to reduce variance and increase inter-rater reliability
- Independent reviews of materials
- Five or more reads on all of the material
- Random assignment of materials to reviewers
- Twice-daily progress monitoring

The process was designed to be inclusive, transparent and to make an honest determination about the degree of match between the programs and Washington's standards. It was designed so that each program had a fair chance of being recommended. Thorough statistical analysis ensured that scorer bias was minimal.

The OSPI's review of mathematical soundness
In addition to the content alignment, OSPI asked Drs. George Bright and James King to review the mathematical soundness of the development of specific topics in programs with content that matched best to Washington's state standards.

The development of general ideas related to function—domain, range, and moving among representations of functions—was examined for quadratic functions in *Discovering Algebra and Advanced Algebra*, *Holt Algebra 1 and 2, Glencoe Algebra I and II, and Prentice Hall Algebra 1 and 2.*

The idea of rigorous proof was examined through the lens of parallel/ perpendicular lines and parallelograms for the programs of *Holt Geometry, McDougal-Littell Geometry, Glencoe McGraw-Hill Geometry, and Prentice-Hall Geometry.*

Additionally, the two integrated programs with the highest scores, *Core-Plus Mathematics Courses 1, 2, and 2* and *SIMMS Integrated Mathematics Levels 1, 2 and 3* were analyzed to determine the mathematical soundness of their treatment of both topics identified above.

The OSPI's initial recommendations
Based on the ranking according to the degree of content match with state standards, the results of the mathematical soundness analysis, and taking into account that many districts in Washington currently choose an integrated approach to high school mathematics, the OSPI initially recommended:

- *Holt Algebra 1, Geometry, and Algebra 2*
- *Discovering Algebra, Geometry, and Advanced Algebra*
- *Core-Plus Mathematics, Courses 1, 2, and 3*

ST's review of instructional programs
The SBE also has a responsibility in the process of recommending the curriculum for mathematics. The legislature requires that the SBE examine the results of the

OSPI's work to validate it and offer additional perspective. To this end the SBE contracted with ST to review the mathematical soundness of the programs recommended by the OSPI:[4]

- *Holt Algebra 1, Geometry, and Algebra 2*
- *Discovering Algebra, Geometry, and Advanced Algebra*
- *Core-Plus Mathematics Courses 1, 2* and *3*

The SBE also asked ST to review the program with the next highest match with respect to the state standards on mathematical content:

- *Glencoe McGraw-Hill Algebra I, Geometry, and Algebra II*

Earlier, the SBE had also contracted with ST to do a review of OSPI's analysis of the K-8 mathematics curriculum. During that review process, ST examined four programs in terms of their alignment with the standards for mathematical content as well as mathematical soundness and compared its results to the results of OSPI.[5] The results were similar.

The SBE decided not to request ST to evaluate the high school mathematics programs in terms of the state standards for content because it thought it likely that, with reasonable differences, ST's results would validate the results of the OSPI study. Instead, the SBE asked ST to extend its review of mathematical soundness to the OSPI's third-ranked *Glencoe McGraw-Hill Algebra 1, Geometry,* and *Algebra 2* and to add a second mathematical reviewer. The SBE's request did not require any budget changes; it merely necessitated a reallocation of resources that would provide the SBE and the OSPI with additional, useful information.

There are two notable differences between the OSPI's and ST's mathematical reviews. First, the two groups did not review exactly the same set of programs. Second, they looked at somewhat different topics. While there is significant overlap with both the topics and programs that were reviewed, they are not exactly the same.

The OSPI's final recommendations, as required by legislation, will be made after ST's review of instructional programs.

ST Approach

ST evaluated the four programs with respect to explicit criteria for mathematical soundness provided by its two reviewers. The reviewers employed selected state content standards as a lens to assess the programs. More specifically, ST's review focused on three selected topics that are covered in each program in order to

[4] RCW 28A.305.215(7)(b) of the 2008 legislative session.

[5] *Independent Study of Washington State K-8 Curriculum Review* by Strategic Teaching, November 5, 2008.

determine how well the topics fare with respect to the reviewer's criteria for mathematical soundness.

ST's review of mathematical soundness

Dr. Harel's report lays out the criteria and guiding questions germane to evaluating mathematical soundness of instructional materials.

1. Mathematical justification
   - Are central theorems stated and proved?
   - Are methods for solving problems, conditions, and relations justified?
   - Does the program develop norms for mathematical justification, so that students gradually learn that empirical observations do not constitute justifications, though they can be a source for forming conjectures?

2. Symbolism and structure
   - Does the program develop fluency with algebraic manipulations and reasoning in general terms?
   - Is there an explicit attempt to help students organize what they have learned into a coherent logical structure?
   - Does the text attend to crucial elements of deductive reasoning, such as "existence" and "uniqueness," "necessary condition" and "sufficient condition," and the distinction among "definition," "theorem" and "postulate?"

3. Language
   - Is the language used clear and accurate?

Additionally, both reviewers agree that good problems are more than a pedagogical issue and that a sufficiently large number of good problems are fundamental to the mathematical soundness of texts. The two reviewers describe the guidelines for assessing whether the assigned problems are mathematically sound.

4. Assigned problems
   - Does the text include a sufficiently large number of nontrivial, holistic problems?
   - Does the problem require an equation to be set up and solved?
   - Do mathematical concepts connect to non-contrived problems?

The reviewers describe good problems in slightly different ways and diverge on the value of types and characteristics of problems.

Dr. Harel prefers holistic problems that refer, "to a problem where one must figure out from the problem statement the elements needed for its solution." Dr. Wilson's description for these problems is "word problems that require the setting up of the equation and the solving of the equation." Dr. Wilson observes that, "too many of the problems provide the relevant equation as part of the problem. The problem then frequently reduces to a simple math exercise surrounded by irrelevant words."

Dr. Harel is concerned about the use of non-holistic problems that are, "broken down into small parts, each of which attends to one or two isolated elements of the problem." Dr. Harel, would also prefer to avoid contrived problems—problems that can be solved by tools already available to the student—but which the text solves by applying more sophisticated, new mathematics. For example, the text sets up an equation with an unknown to solve a simple subtraction problem the student can readily solve without an equation. Dr. Harel feels these problems are "alien to mathematical practice."

Dr. Wilson is not opposed to including a limited number of problems of each of these types in order to teach specific skills.

ST's topics

The reviewers applied their criteria for mathematical soundness to three topics: forms of linear functions and equations, forms of quadratic functions, and sum of angles of a triangle. The following discussion of topics contains the specifics that the reviewers looked at and some explanation of why these topics were selected.

Forms of linear functions and equations

The algebraic concepts and skills associated with linear equations are not just useful in ordinary living; they are crucial for the rest of the study of algebra and beyond. Appropriate definitions and justifications for concepts like coefficient and slope provide the basis for understanding linear equations. All forms should be present and each form should be connected to the other forms. Students should have the opportunity to apply their skills and knowledge related to linear equations to solve problems. The standards used as a reference point are:

A1.4.B Write and graph an equation for a line given the slope and the y-intercept, the slope and a point on the line, or two points on the line, and translate between forms of linear equations.

A1.1.B Solve problems that can be represented by linear functions, equations, and inequalities.

Forms of quadratic functions

The ability to put quadratic functions in vertex form allows students to use symmetry and to find the maximum or the minimum of the function. This opens up a new world of problems the student can solve, namely max/min problems. The approach to max/min problems will be analyzed for both the basic algebra and the conceptual development, which includes a coherent definition of a quadratic function and how the line of symmetry is explained and justified. The program will be analyzed to be sure that the connections among strands or topics necessary to meet the standards are explicit and make good sense. The standards used as guideposts are:

A2.3.2. Translate between the standard form of a quadratic function, the vertex form, and the factored form; graph and interpret the meaning of each form.

A1.1.D Solve problems that can be represented by quadratic functions, equations, and inequalities.

Sum of angles of a triangle

The development and application of the theorem that the sum of the angles of a triangle is 180 degrees is analyzed. The analysis begins with the underlying postulates—particularly Euclid's fifth postulate—and includes an examination of how the theorem connects many of the basics in geometry.

For example, the theorem depends on a good understanding of parallel lines, the lines that cross them, and the angles associated with all these lines. The theorem will be looked at carefully, as will the general coherence and logical progression of the geometric material leading up to it. The main concerns will be the foundation for understanding both the geometry of the situation and the logic it depends on. The standard that is guiding evaluation is:

G.3.A Know, explain, and apply basic postulates and theorems about triangles and the special lines, line segments, and rays associated with a triangle.

Program Details

Each program is described in some detail in the following section, with key characteristics highlighted. They are presented in descending order of how well they match with the content in Washington's standards: *Discovering*, *Holt*, *Glencoe*, and *Core-Plus Mathematics*.

Holt Algebra 1, Geometry, and Algebra 2

Program Highlights

Holt Algebra 1, Geometry, and *Algebra 2* is a three-year high school program that organizes content into the courses of *Algebra I, Geometry*, and *Algebra 2*. Most lessons are designed with example problems, including solutions, and numerous practice problems. There are step-by-step examples and online homework help designed to help students become independent learners.

Strengths:

- Students work from graphs, tables and equations to identify linear equations by characteristics.
- All three forms of linear equations are presented and applied.
- Numerous exercises and word problems require application and translation among forms of linear functions and collections of related problems are included.

- Good quadratic problems are provided.
- Proof for triangle sum theorem is legitimate.

Areas of concern:

- Justifications for linear functions are referenced, but not presented.
- Multiplication of inequalities by negative numbers is only shown for a few examples.
- Algebraic facts and procedures overshadow conceptual understanding.
- General forms of linear functions and equations are rarely derived.
- The development is shallow for the three forms of quadratic functions.
- Few quadratic problems require student to produce quadratic functions or equations; max/min problems are not common.
- Triangle sum theorem proof is sound, but could be stronger.

Linear functions, equations, and inequalities in *Holt*

As is true of the other programs examined, *Holt's* study of linear equations and their graphs begins without the necessary mathematical foundation, which is mentioned but not discussed. The topic is first covered in *Holt Algebra 1,* Chapter 5, and then revisited in *Algebra 2,* but not with more care or movement toward more abstract treatments.

Working from graphs, tables, and equations, students learn to identify linear functions by their properties on page 296 of *Holt Algebra 1*. This is typical of *Holt's* approach. For instance, slope is introduced and worked with, in the beginning mostly from tables and reading from graphs, and then moves on to algebraic techniques. The slope intercept form on page 335 is presented and applied. Page 342 introduces the point-slope form. Inequalities in one variable are the focus of Chapter 3.

The material in general is thoroughly developed. There are numerous exercises and word problems, including exercises that require students to translate between the forms of linear equations and quite long collections of related problems.

That said, *Holt* needs to be strengthened in the following ways.

In *Holt Algebra 1,* multiplication of inequalities by negative numbers is only shown for a few examples, which are illustrated on the number line. This is a superficial treatment.

Algebraic facts and procedures overshadow conceptual understanding.

Example #1

"I use the "cover-up" method to find intercepts. … If I have 4x - 3y = 12: First, I cover 4x and solve the equation I can still see -3y = 12; y = -4 [the x-intercept is found in a similar way by covering the -3y.]" (Holt Algebra I, page 304)

This is an insert "student-to-student tip" unrelated to mathematical understanding. On the next page, the intercepts of the line $2x - 4y = 8$ are computed by substituting zero for x to find the y-intercept, and zero for y to find the x-intercept, but no connection is made between this correct method for this specific problem and the student's method.

In fairness, most cases are not this devoid of mathematical sense. But more often the material is presented as a collection of facts about linear functions rather than tied to a conceptual framework.

General forms of functions and equations are rarely derived

Example #1

"*You can sometimes identify a linear function by looking in a table or a list of ordered pairs. In a linear function, a constant change in* x *corresponds to a constant change in y.*" (*Holt Algebra 1*, page 298)

There is no connection between the above example and *Holt's* initial definition of a linear function, $Ax+Bx=C$. Instead, this property is followed by tables of ordered pairs and graphs, whose purpose is to demonstrate that when the change is constant the corresponding order pairs lie on a straight line, and that when the change is not constant the corresponding pairs do not lie on a straight line. This would have been adequate if the text followed this demonstration by justifying that in a linear function (which, according to the text's definition, is a function whose graph is a line), a constant change in x corresponds to a constant change in y.

Forms and graphs of quadratics in *Holt*

The material in *Holt* that is relevant to translating between forms of quadratic functions appears in Chapter 5 of *Holt Algebra 2*. On page 321 the text emphasizes the transformations that take the function x^2 to the vertex form. The symmetry about the y-axis of x^2 is determined and it is asserted that "this shows that parabolas are symmetric curves". This is rather perfunctory. Because of its importance, it would be nice to see symmetry carried through the various transformations between x^2 and the vertex form. With quadratic functions written in the vertex form, max/min problems are direct and easy as the solution is given from knowledge of where the vertex is.

The standard form is introduced, page 324, and the axis of symmetry and vertex are computed. Rather than complete the square on the standard form to get the vertex form, *Holt* derives the formulas for the axis of symmetry and the vertex by comparing the vertex and standard forms algebraically. When problems are solved and exercises worked, these formulas are used instead of the technique of completing the square.

Although there are many good problems, there are few that require the students to produce quadratic functions or equations rather than solve those given in the problem.

There is a tendency in the *Holt* algebra texts to present material without giving students the opportunity to develop a deep understanding of what they are doing. For example, quadratic inequalities appear in Algebra 2 on page 366. The text presents a sample problem on page 368 of how to solve a quadratic inequality by applying three ready-made steps. Specifically, to solve the inequality $x^2 + 4x + 1 > 6$, according to this text, do the following:

> Step 1: Write the related equation $x^2 + 4x + 1 > 6$
> Step 2: Solve the equation by factorization: $(x - 5)(x + 1) = 0$, $x = 5$ or $x = -1$ [and accordingly] divide the number line into three intervals: $x < 1$, $-1 < x < 5$, and $x > 5$.
> Step 3: Test an x-value in each interval. [The values $x = -2$, $x = 0$, and $x = 6$ are tried in the corresponding intervals, from which the solution to the inequality is determined]

Students who can recall these three steps, along with the numerous other mechanical procedures in the text, may solve the assigned problems correctly, but without an adequate understanding of what they are doing.

The mathematical adequacy of a program is compromised when students do not understand *why* something works, *when* it works, and *how* it is connected to other parts of mathematics. These components are sometimes missing in *Holt Algebra 1* and *Algebra 2*.

The triangle sum theorem in *Holt*

Holt Geometry's proof that the sum of the angles in a triangle is 180 degrees can be found on page 223. Interestingly, it is similar to the proof developed in *Glencoe Geometry.*

The triangle sum theorem sits at the top of a pyramid of postulates, definitions, other theorems, and logic, all of which must be in place to make sense of the result. This is a coherent presentation of the geometry but there are concerns.

Although it is mathematically legitimate, a major concern comes from the sequencing of the material. The process of developing a deductive structure for synthetic geometry is interrupted by two sections on analytic geometry (sections 3-5 and 3-6), each of which includes important theorems without proofs. There are at least two problems here. First, it deprives the students of the opportunity to deal with important mathematical ideas. The second concern is that in this presentation the text avoids dealing with the important ideas of "existence" versus "uniqueness" and the distinction between a "postulate" and a "theorem." Another—though minor—issue is that the statement "The Converse of the Corresponding Angles Postulate is used to construct parallel lines" is not completely accurate. The construction of parallel lines involves the duplication of an angle and the Converse of the Corresponding Angles Postulate guarantees that this construction results in parallel lines. A detailed discussion of these issues can be found in Dr. Harel's full report at:
www.strategicteaching.com/washington_state_standards_.html

Still, the proof is mathematically sound. It is developed using the following steps:

- Section 3-1, page 146, starts with definitions and illustrations of basic terms such as parallel lines, parallel planes, and angle pairs formed by a transversal.
- Section 3-2, page 155, starts with the Corresponding Angles Postulate ("If two parallel lines are cut by a transversal, then the pairs of corresponding angles are congruent." This is followed by practice problems on how the postulate is used to compute different angles.
- Section 3-3, page 163, gives the Parallel Postulate ("Through a point P not on line l, there is exactly one line parallel to l.")

The development of the material on parallel lines that leads up to the proof of the triangle sum theorem does not include any apparent circular reasoning but the sequencing of the material, is less than ideal.

Discovering Algebra, Geometry, and Advanced Algebra

Program Highlights

Discovering Algebra, published by Key Curriculum Press is a three-year mathematics program. Most of the time students solve problems based on contexts, often using technology, and in this way do mathematics through discovery. A typical day begins with an investigation that is structured through a series of questions designed to lead students through an opportunity to uncover or apply a mathematical concept. There are open-ended aspects to the investigation as students "practice" along the way with numbers or attributes they select.

Strengths:

- Presents all three forms of linear functions with the point-slope form particularly well developed.
- Includes good problems for linear functions and especially good problems for quadratic functions.
- Includes a proof of the triangle sum theorem.
  - Contains extra, rich content, including that parabolas are conic sections and a definition of parabola in terms of focus and directrix turned into a quadratic equation.

Areas of concern:

- Justifications for linear functions are not present.
- Content is presented in tiny pieces and not consolidated into the big ideas of mathematics.
- Empirical observations are generalized but not consolidated.
- Pictures of graphs rather than quadratic functions are the primary object of study.
- Algebraic skills are not emphasized well enough.
- The idea that one method − graph, table, calculator, or algebraic approach − is superior to another in a specific situation is lost.

- Important math is undefined and assumed.

Linear functions, equations, and inequalities in *Discovering*

Discovering develops some topics well, including the point-slope form for linear equations in *Algebra I* on page 234. All three forms of the linear function—standard, slope-intercept, and point-slope—are included with the minimal symbolic manipulation required so that students translate between the forms. Students also investigate how inequalities function under multiplication and division, although only for specific problems.

However, a major problem of *Discovering* is that the fundamentals of mathematics are not well represented in this program. In addition, it does not integrate and connect topics to the structure of algebra. A common approach is to present the problems and material through small steps in the form of sequences of tasks. This approach is non-mathematical in two respects.

First, it is difficult to discern the underlying ideas of the content taught. The multitude of activities and prescribed steps mask the big ideas underlying linear functions, equations, and inequality. It is difficult to determine the mathematical structure both within a particular lesson, and across an instructional unit. Students learn to solve interesting problems but not to understand algebra.

Second, the text consistently generalizes from empirical observations without attention to mathematical structure and justifications. There is nothing wrong with beginning with particular cases to understand something and make a conjecture about it. In many cases it is advantageous to do so and sometimes even necessary. However, at some point a mathematical text should prove what has been empirically observed.

Additionally, algebraic skills are not emphasized well enough to advance in mathematics. *Algebra 1* is the time for students to begin to learn to fluently manage symbolic manipulations.

The use of technology in *Discovering Algebra* in lieu of algebraic approaches contributes to the lack of opportunity to develop algebraic skills. An example of how this plays out can be found when *Discovering Algebra* introduces solving linear equations in Lesson 3.6, page 199.

> From Example B, you can see that each method has its advantages. The methods of balancing and undoing use the same process of working backward to get an exact solution. The two calculator methods are easy to use but usually give approximate solutions to the equation. You may prefer one method to others, depending on the equation you need to solve.

The four methods presented for solving a simple linear equation raise three issues. First, calculators are highlighted in three out of four solutions, which would not be problematic were it not a reoccurring theme. The implication is that technology replaces mathematics rather than serves as a tool of mathematics. With so much emphasis on calculator use, students do not have the opportunity to build and maintain fluency in symbolic manipulation, the language of mathematics.

The second problem is the idea that one approach is superior to another for a specific problem gets lost. Particular approaches are more efficient in certain cases and it isn't just a "choice."

Third, the precious nature of the general algebraic form is not promoted in this program. A table can be used to solve very few problems. A calculator is a handy tool, but can't pull an equation out of a word problem.

It is the large number of problems of this type that is problematic, not this single example.

Forms and graphs of quadratics in *Discovering*

Discovering Algebra's main strength is in its problems, which are varied and numerous.

Discovering Algebra begins with quadratics in Chapter 3 on page 425 by saying that the graph of $y = x^2$ is a parabola. Symmetry is determined in an investigation from the question, Step 8, "Draw a vertical line through the point (0,0). How is this line like a mirror?" The bulk of the quadratic material in *Discovering Algebra* is in Chapter 9, which begins by explaining that the graph of the height of objects under the influence of gravity is a parabola; that parabolas and their transformations are quadratic functions; and that the parent form of a quadratic function is $f(x) = x^2$. *Advanced Algebra* repeats the same material without going deeper or becoming more abstract.

One way *Discovering Algebra and Discovering Advanced Algebra* compromise mathematics is by leaving important mathematics undefined and assumed.

Parabolas are one case in point. In *Algebra*, on page 447, students are given the image of a transformation of $y = x^2$ and are asked to find the transformation, i.e. find the new function that gives the new graph. Page 466 of *Algebra* requires students to handle a transformation that requires both translation and stretching. In both cases it is assumed that the graph is a parabola. That a parabola has never been defined and that students are working from pictures of graphs—discussed below—projects an imprecision that does not reflect the nature of mathematics.

Several programs fail to define important mathematics. The way *Discovering* handles quadratics is compromised for additional reasons.

There seems to be more emphasis on graphs than on quadratic functions and the algebra of quadratic functions. More precisely, the emphasis is on pictures of graphs in the texts, usually described as parabolas, but not supported with an equation or table. Using information drawn from a picture of a graph, a matching function is determined and then used to study the graph further.

The reliance on pictorial representation versus equations is mathematically unsound and pervasive throughout *Algebra* and *Advanced Algebra*. Unlike an equation, a picture of a graph is not precise mathematical information and students shouldn't be taught to use it as if it were. Although it won't be mentioned again, pictures of graphs are a cornerstone of much of the work related to quadratics.

Calculators are sometimes used in ways that undercut mathematics. While there are certainly disagreements about what kind of technology and how often technology should be used in the classroom, most people would agree that it has a place and that when it is used it should support students' understanding and learning of mathematics. The problem on page 504 of *Discovering Advanced Algebra* shows a misuse of technology. A complete breakdown of the issue can be found in Dr. Wilson's paper at:

www.strategicteaching.com/washington_state_standards_.html.

The main point is that students are led to believe that, "You can see from the graph and the table that the equations $y = x^2 + 3x - 5$ and $y = (x + 1.5)^2 - 7.25$ are equivalent." There are several difficulties, but simply put it is not possible to look at this chart and graph and determine mathematical equivalence. To solve this problem algebraically is trivial. To avoid solving this problem algebraically is misleading to students.

Another concern is that *Discovering* often fails to move from the specific problem to the general case, which is one purpose of algebra. The following two cases illustrate topics that deserve complete development.

- Complete the square for functions and to solve equations, but not for the general case. (*Algebra 1*, page 525)
- Rewrite specific vertex form quadratics in general form, but it is not done for the general case. (*Advanced Algebra*, page 510)

Strengths of the program related to quadratic functions include the excellent problems and the notable sidelines in *Discovering Advanced Algebra.* On page 507 it points out that parabolas are conic sections, which is an important topic. Also, on page 524 of the same book there is a definition of parabolas in terms of a focus and a directrix. Although nothing is done with it at this point, in Chapter 9 this is turned into a definition of quadratic equations – a real treat.

The triangle sum theorem in *Discovering*

As suggested by the title in Discovering Geometry's last chapter, Chapter 13, *Geometry as a Mathematical System*, this is where deductive reasoning is tackled. The first 690 pages of *Geometry* favor an inductive approach to learning about shape and space. As stated on page 693:

> You used informal proofs to explain why a conjecture was true. However, you did not prove every conjecture. In fact, you sometimes made critical assumptions or relied on unproved conjectures in your proofs.
>
> In this chapter you will look at geometry as Euclid did. You will start with premises: definitions, properties, and postulates. From these premises you will systematically prove your earlier conjectures. … You will build a logical

framework using your most important ideas and conjectures from geometry.

From it's beginning on page 691, with the right number of stops at the appropriate theorems and postulates on the way, *Discovering Geometry* arrives at a succinct, coherent, and complete proof of the triangle sum theorem on page 706.

The proof depends on the following theorems and postulates:

- The necessary alternate angles theorem and the vertical angles theorem are both proven on page 704. On pages 696-697 the postulates of geometry are stated, in particular the ones needed for the triangle sum theorem: the corresponding angles postulate, the linear pair postulate, the angle addition postulate, and the parallel postulate (the last is Playfair's postulate, equivalent to Euclid's fifth postulate, that makes this Euclidean geometry).
- On page 694 the properties of arithmetic that are used are given. The discussion and development are very sparse, but 690 pages of discussion precede this chapter.

The theorems and postulates used to prove the triangle sum theorem in turn rest on definitions developed in the first 690 pages of the book.

At this point the proof breaks down.

The informal, inductive approach of the majority of the book means that the definitions and theorems are empirically based and not strong enough to support the weight of a formal proof.

The value of studying geometry is partly to learn to solve geometric problems and partly to learn to work in an axiomatic system and develop the associated logic skills. *Discovering Geometry's* treatment of the axiomatic system is inadequate, despite the proof on page 703.

Glencoe McGraw-Hill Algebra 1, Geometry, and Algebra 2

Glencoe's high school program is a series of three texts organizing conventional mathematical content into the courses of Algebra 1 Geometry, and Algebra 2. The structure of mathematics itself provides the basic structure for the series, rather than realistic problems or a thematic approach.

Most lessons begin with a question posed by the teacher, direct instruction of the new content, and lots of practice problems with an emphasis on well-crafted word problems. Chapters typically start with a pre-test and include both a mid-and post-assessment. They often include one or two hands-on mini-labs designed to build conceptual understanding as well as suggestions for one or two optional group activities in the teachers edition all of which relates to the topic at hand.

The online support offers homework help, extra problems and solutions, and math "challenges."

Program Highlights

Strengths:

- Contains well-crafted word problems for both linear and quadratics
- Meticulous sequencing, building from the simplest case to the more complex
- All aspects of symbolic manipulation related to linear functions, equations and inequalities are developed
- Directly teaches writing equations from word problems

Areas of concern:

- There are errors in content
- Important mathematical ideas are presented as prescribed rules
- Mathematically language is used incorrectly
- Explanations are carelessly formed, confusing and misleading
- Important theorems are not proven
- There is a lack of attention to mathematical accuracy and the difference between a postulate and a theorem in the proof of the triangle sum theorem

Linear functions, equations, and inequalities in *Glencoe*

The content related to linear functions appears in *Glencoe's Algebra 1* in chapters 2 through 5, pages 73 through 330, and is repeated in chapters 1 and 2 in Algebra 2.

One of *Glencoe's* strengths is its meticulous sequencing in content and instruction related to solving linear equations. It is carefully scaffolded from the simplest situation, gradually progressing through those with more and more complexity. All aspects of linear functions, equations and inequalities are present and developed, leading to solid development of algebraic manipulation.

Glencoe also directly tackles word problems, beginning by teaching students to translate the simplest sentences in word problems into mathematical sentences. The skill is critical, but *Glencoe's* approach can be described as formulaic. Students are taught to apply rules rather than think about the mathematical situation being represented – for example, "Look for key words such as *is*, *is as much as*, *is the same as*, or *is identical to* that indicate where you should place the equals sign." (page 75)

That said, *Glencoe's* collection of problems is very strong. The sheer volume of high-quality problems of many different sorts provides abundant opportunities to represent *ad hoc* mathematical situations.

In addition to the superficial treatment of the mathematical foundations, common across all reviewed programs, *Glencoe* has other issues related to content. Mathematical language is used incorrectly and content is presented in misleading or confusing ways.

Mathematical language is used incorrectly

Example #1:

*"The **standard form** of a linear equation is $Ax+By=C$, where $A \geq 0$, A and B are not both zero, and A, B, and C are integers with a greatest common factor of 1."* (*Algebra 1*, page 153.)

The statement is incorrect since it would rule out any equation that had Pi or the square root of 2 in it. This example, although more blatant than most, represents a major problem with *Glencoe:* the abuse of mathematical language.

Example #2:
"*A mathematical statement that contains two algebraic expressions and a symbol to compare them is an open sentence. A sentence that contains an equal sign, =, is an equation." (Algebra 1,* page 31*.)*

An equation is typically thought to be an open sentence, which Glencoe must understand when it states, *"To solve an equation means to find the value of the variable that makes the equation true."* (*Algebra 1,* page 83.)

Is an equation an open sentence? If so, it doesn't make sense to say "an equation is true," as is stated on page 83:

> To solve an equation means to find the value of the variable that makes the equation true.

The term "sentence" in the second sentence in the quote from page 31 is meant to be "open sentence," for otherwise, the purpose of the first sentence in the quote from page 31 is not clear. Based on this, then, 13 is an expression. It is so because $3x$-7=13 is an equation, and an equation is an open sentence that contains two algebraic expressions ($3x$ + 7) and (13) and symbol (=) that compares them. Since 13 is an expression, any number is an expression. So, from here one can conclude that statements such as, 13=13, 3>4, 17<25 are all open sentences, as are the sentences, ($4x$–6 ≥ $5x$+7), ($4x$–6 = $5x$+7). Obviously this is not true: 13=13, 3>4, and 17<25 are not open sentences.

<u>Careless or wrong formulations</u>
Example #1:
"Expressions with absolute values define an upper and lower range in which a value must lie." (Algebra 1, page 103.)

This statement is confusing partly because it is seems to imply that expressions with absolute values follow different sorts of rules than other expressions. What is meant by upper or lower range? Doesn't any algebraic expression define a range? If this is what is meant, what is the purpose of this statement? The sentence that follows, "*Expressions involving absolute value can be evaluated using the given value for the variable,"* adds to the muddle. All expressions, not only those involving absolute value, can be evaluated using a given value for the variable in the expression.

Forms and graphs of quadratics *Glencoe*

Material that is relevant to quadratics appears in Chapters 8 and 9 in *Algebra 1* and Chapter 5 in *Algebra 2*.

The word problems are abundant and, generally, stellar. For instance, "*A square has an area of* $9x^2 + 30xy + 25y^2$ *square inches. What is the perimeter of the square? Explain.*" This is a rich problem that takes more than simple factoring to unlock.

However, there are 1) mathematical errors, 2) concepts presented as rules, and 3) confusing, misleading presentations of content in *Algebra II.* Examples of each are offered below.

<u>Mathematical errors</u>

Example #1:

"*Quadratic equations can be used to model the shape of architectural structures such as the tallest memorial in the United States, the Gateway Arch in St. Louis, Missouri.*" (Algebra I, page 468)

Unfortunately the graph of a quadratic equation is a parabola and the Gateway Arch is not a parabola, but a catenary[6]. Intermingled with *Glencoe's* brilliant word problems are some that are not just ordinary, but reflect sloppy mathematics.

Example #2

"Eddie is organizing a charity tournament. He plans to charge a $20 entry fee for each of the 80 players. He recently decided to raise the entry fee by $5, and 5 fewer players entered with the increase. He used this information to determine how many fee increases will maximize the money raised." (*Algebra 1*, page 252)

Glencoe suggests this situation is represented by a quadratic function, a suggestion not warranted by the information provided. From the solution provided on page 253, it can be seen that *Glencoe* assumes that a linear increase in entry fee results in a linear decrease in players. Teaching students to take such leaps as part of determining a precise answer is not defensible mathematics.

These two problems are especially troubling because they are the introductory, motivational problems for a chapter or unit and so, presumably, receive focused attention.

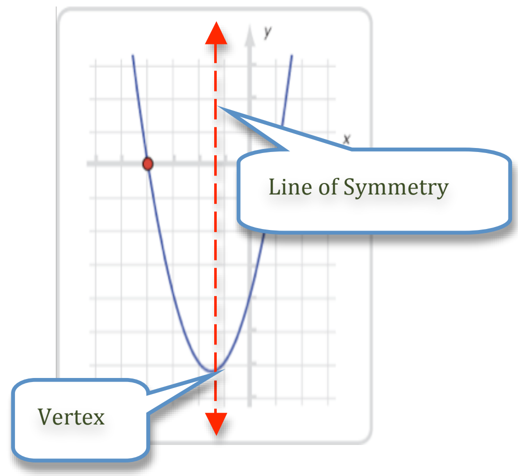
[6] A catenary is the graph of the curve of a hanging chain

<u>Content is presented as rules rather than explained</u>

Example #1

"Parent Function	$f(x)=x^2$
Standard Form	$f(x)=ax^2+bx+c$
Type of Graph	*Parabola*
Axis of Symmetry	$x=-b/2a$
y-intercept	c

When $a>0$*, the graph of* ax^2+bx+c *opens upward; the lowest point on the graph is the minimum. When* $a<0$*, the graph of* ax^2+bx+c *opens downward; the highest point on the graph is the maximum."* (*Algebra 1*, page 525.)



Thus, students are handed most of the core formulas and facts for the important properties related to quadratics but without context or any foundation that would provide the meaning needed and with no pretense of caring about justification. Much of the rest of quadratics is just a matter of plugging numbers into the unjustified formulas.

This is the major problem with Glencoe. The major goals of the study of quadratics are presented to students without justification or explanation.

To be clear, this concern is not related to teaching methodology. Students might be given the opportunity to "discover" these properties, direct instruction might be used to teach how the standard form of the quadratic formula is derived, or calculators could be employed to determine what happens to a graph when the coefficients are varied. There are many ways to avoid presenting content devoid of meaning.

Example #2:
"When a quadratic function is in the form $y = ax^2 + bx + c$*, you can complete the square to write the function in vertex form. If the coefficient of the quadratic term is not 1, then factor the coefficient from the quadratic and linear terms* before *completing the square."* (*Algebra 2*, page 305)

This is as close as *Glencoe* gets to showing equivalencies between the standard and vertex form of quadratic functions. Although it does walk through example and problems, nowhere are equivalencies among the three forms of quadratic functions—the standard form, the vertex form, and the factored form—proved.

<u>Some information is incomplete or confusing.</u>

Example #1:
On page 268 of *Algebra 2* the factored form of a quadratic equation is introduced. In this section, FOIL is used is an "acrostic" taking the first letters from the verse,

"First, Inside, Outside, Last," as a mnemonic for multiplying two binomials to help one remember the steps in the process. This common strategy might be helpful to learn the steps in the procedure, but not in lieu of understanding distributivity. FOIL is a very limited tool--it doesn't help with problems like $(2x+4)(x^2+3x+10)$—but more importantly, it is not a mathematical tool.

The triangle sum theorem in *Glencoe*

The proof of the triangle sum theorem in *Glencoe Geometry* is almost identical to the approach used in *Holt Geometry* and so the descriptions are very similar. The development that leads up to the proof the Triangle Sum Theorem (Section 4-2) does not include any apparent circular reason.

The concern about *Glencoe's* presentation is the lack of mathematical accuracy. There is a lack of attention to the distinction between a "postulate" and a "theorem" and the process of developing a deductive structure for synthetic geometry is interrupted by two sections on analytic geometry (Sections 3-3 and 3-4) that does not belong to the development of this structure.

Glencoe Geometry covers the triangle sum theorem in Chapters 3 and 4 and develops it in the following way:

– Section 3-1 includes the definitions and illustrations of basic terms such as parallel lines, parallel planes, and angle pairs formed by a transversal.
– Section 3-2 starts with the Corresponding Angles Postulate: "If two parallel lines are cut by a transversal, then the pairs of corresponding angles are congruent." This is followed by practice problems on how the postulate is used to compute different angles.
– Sections 3-3 and 3-4 digress to analytic geometry about lines and slopes. They included the following two postulates (yes, they are called postulates in this text):

> "Two nonvertical lines have the same slope if and only if they are parallel. All vertical lines are parallel
> Two nonvertical lines are perpendicular if and only if the product of their slopes is -1. Vertical and horizontal lines are perpendicular."

This is a misuse of the concept of "postulate." Both of these assertions are theorems. Dr. Harel includes possible proofs of these in his report at www.strategicteaching.com/washington_state_standards_.html.

– Section 3-5 (page 205) starts with the Converse of the Corresponding Angles Postulate: "If two lines are cut by a transversal so that corresponding angles are congruent, then the two lines are parallel". The important condition that lines are coplanar is missing. Following this, the construction of a parallel line to given line through a point is not shown. This construction involves the duplication of angle.

– Section 3-5 then produces the Parallel Postulate: "If given a line and a point not on the line, then there exists exactly one line through the point that is parallel to the given line." This postulate is preceded by the following statement:

> The construction establishes that there is *at least* one line through C that is parallel to $\overleftrightarrow{AB}$. The following postulate [the above Parallel Postulate] guarantees that this line is the *only* one.

So the construction of the parallel lines is established by the duplication of an angle, which, in turned is established by congruence. Congruence, however, does not appear until later in Chapter 4. The end result is that the parallel postulate is not a postulate but a theorem derivable (through a simple proof by contradiction) from the Corresponding Angles Postulate.

Core-Plus Mathematics, Course I, II, and III

The *Core-Plus Mathematics Project* is a four-year comprehensive high school mathematics textbook series published by Glencoe/McGraw Hill. It is based on the NCTM standards and was developed with funding from the National Science Foundation. The first three Courses in the program, those reviewed by the OSPI, are designed for all students in heterogeneous classrooms. *Course 4* is designed to prepare students for college.

– Mathematical content is integrated into four strands: algebra and functions, geometry and trigonometry, statistics and probability, and discrete mathematics. It is structured around student-centered investigations of problems that are set in contexts designed to allow students to uncover mathematics. Technology tools are favored to enhance learning and problem solving.

Program Highlights

- <u>Strengths:</u>
  - Provides multiple, extended opportunities to solve problems related to linear functions
  - Develops a working understanding that a line in the plane is represented by a linear equation and that the graph of a linear equation is a line.
  - Includes all three forms of the quadratic function.
  - Presents the complete proof of quadratic formula twice.
  - Includes many good problems for quadratic functions.

- Areas of Concern
  - Symbolic manipulation is downplayed; tables, graphs, and calculators are emphasized.
  - Sends the message that any tool – table, graph, calculator, equation – is as good as another.
  - Loses the advantages of the general algebraic approach.
  - Translates from the standard form to the factored form of the quadratic equation only for cases when the roots are integers.

- The theorem that any quadratic function with roots can be expressed in factored form is not stated nor proved explicitly.
- Proof of the triangle sum theorem depends on informally established postulates and definitions

Linear functions, equations, and inequalities in *Core-Plus*

Students have multiple and extended opportunities to solve problems related to linear functions, equations, and inequalities almost always within the context of some situation. There is enough material in the text to convince the students that a line in the plane is represented by a linear equation, and that the graph of a linear equation is a line. Students have the opportunity to develop a good sense of what linear functions, equations, and inequalities are and how they work.

The text excels in its mission to contextualize the mathematics taught but falls short on conveying the abstract nature of mathematics and the holistic nature of the mathematical problems students are likely to encounter in the future. "Holistic" problems—problems that are not broken down into different parts—where one needs to figure out from the problem statement what elements are needed to solve the problem are rare. Also missing are "context free" problems and this type of problem can be used as needed to get to the general form of a function.

By design or happenstance, limiting the mathematics to that which can be taught within context results in downplaying the importance of symbolic manipulation and failing to consolidate the mathematics.

The lack of focus on algebraic methods means that the important form of the linear function Ax+By=C is not included. Also, moving between forms of linear equations is done only for the simple examples and never for the general case. Simply put, paper and pencil manipulation is minimal throughout—there are two pages devoted to it—with most problems solved by completing a table and graphing the results or by using a calculator.

While those methods have their place, they do not replace algebraic fluency. The overall conclusion from these experiences is that any one tool is as good as the others. The advantage of a general algebraic approach, part of the essence of algebra, over the other approaches is lost.

Forms and graphs of quadratics in *Core-Plus*

There is strength in how *Core-Plus* develops quadratic expressions and functions, especially through the lens of max/min problems. A complete proof of the quadratic formula appears twice, with one of those times being in the Homework Section in Course 2. There are many problems on the use of the quadratic formula to solve quadratic equations.

Many problems model physical situations using quadratic functions and attend to both the physical and graphical meanings of the different parts of the modeling functions. As was true of linear functions, most of the problems are about physical situations or particular functions.

However, the concerns noted in the preceding section persist throughout quadratics. There are lapses in foundational mathematics, usually related to the specific form, and a general minimization of algebraic skills. More specifically:

- The symmetry of the graph of $y = ax^2 + bx + c$ and the relationship between the shape of the graph and the coefficients of the function are justified in the teacher's edition for the cases $y = x^2$, $y = ax^2$, and $y = ax^2+c$, but not for the general case $y = ax^2 + bx + c$. Information about the shape of the graph of functions of the form $y =ax^2 + bx + c$ and its intersection with the x-axis is used to solve quadratic inequalities.
- There are many problems with the handling of translating between the standard form, vertex form, and factored form. The translation from the standard form to the factored form is only done in cases where the roots are integers. The factorization approach is demonstrated with the case $(x + m)(x + n) = x^2 + 5x + 6$. The theorem that any quadratic function with roots can be expressed in factored form is not proven or stated explicitly.

The triangle sum theorem in *Core-Plus*

Core-Plus guides students through a proof of the triangle sum theorem in *Course 3* on page 45 and provides a completed proof as supported in the teacher's edition. It is a standard proof and would be quite satisfactory except that it depends, in part, on postulates and terms that have not been properly established.

Course 1 contains a great deal of "informal," deductive geometry, which is fine until *Core-Plus* tries to establish formal geometry in *Course 3* by building on the informal results.

As one example, in order to prove the triangle sum theorem, *Core-Plus* must construct a perpendicular. In the process, it uses theorems about congruent triangles. Congruent triangles have only been "proven" informally in *Course 1.* This creates circular reasoning: congruency of triangles is used to prove theorems that allow the proofs of the congruency of triangles. It might be possible to unravel this and make it sound, but this is the way it is presented.

The program provides experience that leads to empirical understanding with the various concepts, but fails to adequately separate inductive and deductive reasoning.

Conclusion

From the perspective of mathematical soundness, none of the reviewed programs were completely satisfactory.

Holt was the strongest of the four, meaning the mathematics is not compromised in any of the three topics examined. *Discovering* was the weakest with all three areas considered inadequate. *Core-Plus* and *Glencoe* were about the same and between the other two.

The good news is that there are other programs that match well to Washington's standards. Although resources are scarce now, there are other programs available for Washington to investigate with respect to their mathematical soundness in the future.

Appendix A: Curriculum Vitae for Harel

General Information

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Education

BS	1978	Mathematics, Ben-Gurion University, Israel
MS	1980	Mathematics, Ben-Gurion University, Israel
PhD	1985	Mathematics, Ben-Gurion University, Israel

Academic Appointments

Professor	2000-	University of California at San Diego
Professor	1993-2000	Purdue University
Associate Professor	1989-1993	Purdue University
Assistant Professor	1986-1989	Northern Illinois University

Memberships in Academic Organizations

- International Linear Algebra Society (ILAS)
- American Education Research Association (AERA)
- Mathematics Association of America (MAA)
- American Mathematical Society (AMS)
- National Council of Teachers of Mathematics (NCTM)
- Psychology of Mathematics Education (PME)
- Psychology of Mathematics Education-North America Chapter (PME-NA)
- Research in Undergraduate Education (RUME)

Research Interest

Cognition and epistemology of mathematics and their implications to mathematics curricula and teacher education.

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17. Harel, G., & Sowder, L (2007). Toward a comprehensive perspective on proof, In F. Lester (Ed.), *Second Handbook of Research on Mathematics Teaching and Learning*, National Council of Teachers of Mathematics, 805-842.

18. Harel, G. (2007). Students' proof schemes revisited. In P. Boero (Ed.), *From History, Epistemology and Cognition to Classroom Practice*, Sense Publishing.
19. Koichu, B. & Harel, G. (2007). Triadic interaction in clinical task-based interviews with mathematics teachers. *Educational Studies in Mathematics, 65(3),* 349-365.
20. Harel, G. (2008). What is Mathematics? A Pedagogical Answer to a Philosophical Question. In R. B. Gold & R. Simons (Eds.), *Current Issues in the Philosophy of Mathematics from the Perspective of Mathematicians,* Mathematical American Association.
21. Harel, G., & Brown, S. (2008). *Mathematical Induction: Cognitive and Instructional* Considerations. In M. Carlson, & C. Rasmussen (Eds.), *Making the Connection: Research and Practice in Undergraduate Mathematics*, Mathematical American Association.
22. Harel, G. (2008). *DNR Perspective on Mathematics Curriculum and Instruction,* Part I. *Zentralblatt fuer Didaktik der Mathematik.*
23. Harel, G. (2008). DNR Perspective on Mathematics Curriculum and Instruction, Part II. *Zentralblatt fuer Didaktik der Mathematik.*
24. Harel, G. & Sowder, L. (2009). College Instructors' Views of Students vis a vis Proof. *Teaching and Learning Proof across the Grades: A K-16 Perspective*. Routledge/Taylor & Francis.
25. Harel, G. & Fuller, E. (2009). Current Contributions toward Comprehensive Perspectives on the Learning and Teaching of Proof. *Teaching and Learning Proof Across the Grades: A K-16 Perspective.* Routledge/Taylor & Francis.

Recent Refereed Proceedings Articles

1. Heid, K., Harel, G., Ferrini-Mundy, J., & Graham, K. (1998). The role of advanced mathematical thinking in mathematics education reform, The Proceeding of the 20th Annual Conference of the PME-NA, Raleigh, North Carolina, pp. 53-58.
2. Cramer, K, Harel, G., Kieren, T. & Lesh, R. (1998). Research on rational number, ratio and proportionality, The Proceeding of the 20th Annual Conference of the PME-NA, Raleigh, North Carolina, pp. 89-93.
3. Harel, G., & Lim, K. (2004). Mathematics Teachers' Knowledge Base: Preliminary Results. The Proceeding of the Psychology of Mathematics Education, Bergen, Sweden.
4. Zaslavsky, O., & Harel, G. (1996). Teachers' use of examples as a pedagogical tool. The proceeding of the Psychology of Mathematics Education, Prague, Check Republic.
5. Harel, G. (2008). Topic Study Group 19: Reasoning, Proof and Proving in Mathematics Education, In Emborg, E. and Niss, M. (Eds.), Proceedings of the 10th International Congress on Mathematical Education 2004. IMFUFA, Department of Science, Systems and Models, Roskilde University, Denmark.
6. Ignatova, O., Mezentsev, R., Kazachkov, A., & Harel, G. (2008). DNR-based Instruction in Physics: Sliding a Stick towards its Center of Gravity. Proceedings of The 8th Student's Regional Conference on Modern Problems of Physics and Their Computer Support. National Technical University and Kharkov Polytechnic Institute.

Keynote and Plenary Addresses

1. Pedagogical principle in teaching mathematics, with particular reference to the teaching of linear algebra; The International Conference of the International Linear Algebra Society (ILAS); Athens, Georgia; August 1995.
2. A fundamental principle of learning and its application in modifying students' conception of proof; The Annual Joint Meeting of the MAA-MAS; San Diego, California; January 1997.
3. A developmental model of students' conception of mathematics: cognitive, epistemological, and historical considerations; The International Conference of the International Linear Algebra Society (ILAS); University of Wisconsin; June 1998.

4. Students’ conception of mathematical proof; Research in Undergraduate Mathematics Education (RUME); Chicago, Illinois; September 2000.
5. The role of mathematical knowledge in mathematics education, European Society for Research in Mathematics Education (ERME), Summer School for Graduate Study, Pod•brady, Czech Republic; August 2004.
6. Disequilibria in transitioning between proof schemes, Conference on Understanding Linkages Between Social And Cognitive Aspects Of Students’ Transition to Mathematical Proof, Providence, RI; September 2004.
7. What mathematics do mathematics teachers need to know to be effective? Annual Conference of Mathematics Diagnostic Testing Project, University of California, Los Angles; March 2005.
8. DNR-based instruction in mathematics; focus on diagnostic teaching, Annual Conference of Mathematics Diagnostic Testing Project, University of California, San Diego, March 2005.
9. A Research-based framework for teaching mathematics effectively, 46th Annual CMC-South Fall Conference; Palm Spring, California; November 2005.
10. What is mathematics? A pedagogical answer to a philosophical question; European Society for Research in Mathematics Education (ERME), Summer School for Graduate Studies; University of Jyväskylä; Jyväskylä, Finland; August 06.
11. DNR’s definition of mathematics: Some Pedagogical Consequences; The Mathematical Association of America, New Jersey Section; Seton Hall University, South Orange, New Jersey; October 06.
12. Transitions between proof schemes; Annual Conference of Research in Undergraduate Mathematics Education (RUME); San Diego, California; February 07.
13. Thinking in terms of ways of thinking; Annual Conference of Mathematics Diagnostic Testing Project, University of California, San Diego; San Diego, California; March 07.
14. What Is Mathematics? A Pedagogical Answer with a Particular Reference to Proving; Asian Pacific Economic Cooperation (APEC)-Tsukuba International Conference III: Innovation of Mathematics Teaching through Lesson Study; Tokyo, Japan; December 07.
15. DNR-Based Instruction in Mathematics: Focus on Teacher’s Knowledge Base; The 1st Conference on Preparing the Next Generation of Secondary Mathematics Teachers: How Pedagogy Emerges from Learning Mathematics; University of California, San Diego; San Diego, California; May 08.
16. Intellectual Need and Its Role in Mathematics Instruction; The American Mathematical Association, MathFest; Madison, Wisconsin; August 08.

Recent, Selected Invited Seminars/Colloquia/Conferences Talks

1. Principles of Learning and Teaching: Application to School Algebra; Great San Diego Mathematics Conference; San Diego, California; February, 2002.
2. Didactique of Mathematics and Mathematics Education; Las Vegas, Nevada, Annual Meeting of the National Council of Teachers of Mathematics Research Presession; April 2002.
3. DNR-Based Instruction in Mathematics, with Particular Reference to the Concept of Mathematical Proof; First Joint International Meeting; Pisa, Italy; June 2002.
4. Promoting Ways of Thinking Through Ways of Understanding, and Vice Versa, Center for Research in Mathematics and Science Education, San Diego State University; San Diego, California; March 2004.
5. On the Learning and Teaching of Proof; Department of Mathematics, University of Michigan, Ann Arbor, Michigan; April, 04.
6. The Causality of Proof Scheme, Conference on the History and Pedagogy of Mathematics;
7. Uppsala, Sweden; July 04.
8. Disequilibria in Transitioning Between Proof Schemes; Department of Cognitive Science, University of California, San Diego; San Diego, California; October 2004.

9. Disequilibria in Transitioning Between Proof Schemes, Conference on Understanding
10. Linkages Between Social And Cognitive Aspects Of Students' Transition to Mathematical Proof; Providence, Road Island; September 2004.
11. On the Development of Students' Proof Schemes, Department of Mathematics, University of Oregon; Eugene, Oregon; December 2004.
12. What is Mathematics? Pedagogical and Philosophical Considerations, ILAS (International Linear Algebra Society); Regina, Canada; June 2005.
13. On the development of students' conceptions of proof; University of Delaware; Newark, Delaware; November 05.
14. What is Mathematics? A Pedagogical Answer to a Philosophical Question; Leibniz Laboratory; Grenoble, France; January 06.
15. DNR-based instruction in mathematics; Indiana University Purdue University at Indianapolis (IUPUI); Indianapolis, Indiana; March 06.
16. The role of proof in mathematics curricula; Marquette University; Milwaukee, Michigan; March 06.
17. Students' mathematical experience; International Linear Algebra Society; Drexel University; Philadelphia, Pennsylvania; March 06.
18. Workshop on students' intellectual needs; The Mathematical Association of America, New Jersey Section; Seton Hall University; South Orange, New Jersey; October 06.
19. DNR's definition of mathematics: Some Pedagogical Consequences; Kharkiv National V.N.Karazin University; Kharkov, Ukraine; January 07.
20. On the transition between proof schemes; Tel-Aviv University, Tel Aviv, Israel; January 07.
21. DNR as a conceptual framework for curriculum development and instruction in Mathematics; Technion—Israel Institute of Technology; Haifa, Israel; January 07.
22. Students' ways of understanding and ways of thinking; Ben-Gurion University of the Negev; Beer-Sheva, Israel; January 07.
23. Thinking in terms of ways of thinking, California State University at San Marcus; San Diego, California; February 07.
24. A definition of mathematics and its pedagogical consequences; Eastern Carolina University, Greenvile, North Carolina; March 07.
25. Transitions between proof schemes; University of Georgia; Athens, Georgia; April 07.
26. DNR-based instruction in mathematics; University of London; London, England; April 07.
27. What is mathematics? A DNR perspective; University of Essen; Essen, Germany; April 07.
28. Ways of understanding versus ways of thinking in mathematical practice; Institute for Curriculum and Instruction; Glagenfurt, Austria; April 07.
29. Analyzing different modeling perspectives in undergraduate mathematics education; A DNR's view; The Bi-annual Meeting of The International Community of Teachers of Mathematical Modeling and Applications (ICTMA); Indiana University; Bloomington, Indiana; July 07.
30. Thinking of the learning and teaching of fractions in terms of ways of thinking; A Workshop on the Learning and Teaching of Fractions; Preparing Mathematicians to Educate Teachers (PMET), a Project Sponsored by the MAA and Funded by NSF; University of Michigan; Ann Arbor, Michigan; July 07.
31. What is mathematics? A DNR perspective; Arizona State University; Phoenix, Arizona; October 07.
32. Intellectual Need and Its Role in Mathematics Instruction; Arizona State University; Phoenix, Arizona; October 07.
33. Development of mathematics teachers' knowledge base through DNR-based instruction; National Science Foundation; Washington DC; August 07.
34. The necessity principle and its implementation in mathematics instruction; University of Arizona; Tucson, Arizona; August 07.
35. Setting instructional objectives in terms of mathematical ways of thinking; The Annual Meeting of the California Mathematics Council South; Palm Springs, California; November 07.

36. Setting instructional objectives in terms of mathematical ways of thinking; The Annual Meeting of the California Mathematics Council North; Monterey, California; November 07.
37. Research on the learning and teaching of proof; University of Tsukuba; Tsukuba, Japan; December 07.
38. The Necessity principle and its implementation in mathematics instruction; AMS-MAA Special Session on Scholarship of Teaching and Learning in Mathematics; Joint Mathematics Meeting; San Diego, California, January 08.
39. A definition of mathematics and its pedagogical consequences; AMS-MAA-MER Special Session on Mathematics and Education Reform; Joint Mathematics Meeting; San Diego, California, January 08.
40. Mathematical induction: cognitive and instructional considerations; Special Interest Group of the Mathematical Association of America on Research in Undergraduate Mathematics Education (SIGMAA on RUME); Joint Mathematics Meeting; San Diego, California, January 08.
41. What is mathematics?; Project NExT (New Experiences in Teaching); Joint Mathematics Meeting; San Diego, California, January 08.
42. Advancing teachers' knowledge base through DNR-based instruction in mathematics; Principal Investigators Meeting; US Department of Education; Washington DC; January 08.
43. Mathematics curriculum and instruction: A DNR perspective; Illinois Institute of Technology; February 08.
44. Building a community of mathematicians, teachers, and educators secondary teacher preparation in mathematics: a reaction to Stevens' presentation; University of Arizona; Tucson Arizona; March 08.
45. Categories of intellectual need in mathematical practice, University of California, Los Angeles Mathematics Department's 2nd annual Mathematics and Teaching Conference; Los Angeles, California; March 08.
46. Mathematics curriculum and instruction: A DNR perspective; University of Munich; Munich, Germany; April 08.
47. DNR-Based instruction in mathematics and its application in physics education; Kharkov Pedagogical University; Kharkov, Ukraine; April 08.
48. Some essential algebraic ways of thinking for success in (beginning) collegiate mathematics; Critical Issues in Education Workshop: Teaching and Learning Algebra;
49. Mathematical Sciences Research Institute (MSRI); Berkeley, California; May 08.

Recent Presentations in National and International Conferences

1. Mathematics Teachers' Knowledge Base: Preliminary Results, Annual Conference of the International Group of the Psychology of Mathematics Education, Bergen, Sweden; July 2004.
2. Teachers' Reconceptualization of Proof Schemes; Annual Conference on Research in Undergraduate Mathematics Education, Phoenix, Arizona; February 2005.
3. A Dilemma Concerning Semi-Structured Clinical Interviews: Interviewer-Interviewee Interaction Revisited; Annual Conference on Research in Undergraduate Mathematics Education, Phoenix, Arizona; February 2005
4. Effects of DNR-based Instruction on the Knowledge Base of Algebra Teachers; Annual Conference on Research in Undergraduate Mathematics Education, Phoenix, Arizona; February 2005.
5. Teachers' ways of thinking associated with the mental act of problem posing. Annual Conference of the International Group of the Psychology of Mathematics Education, Prague, Check Republic; July 2006.
6. Teachers' use of examples as a pedagogical tool. Annual Conference of the International Group of the Psychology of Mathematics Education, Prague, Check Republic; July 2006.

Recent Grants

1. Developing, Assessing, and Disseminating an Alternative Program for Teacher Preparation in Mathematics at the Secondary School Level

Granting Agency	US Department of Education
Award Amount	$550,000
Time Period	2007-2009
Role	PI

2. DNR-Based Instruction: A Model for Professional Development of math Teachers.

Granting Agency	National Science Foundation
Award Amount	$1,439,192
Time Period	2003-2007
Role	PI

3. Collaborative Research with the Preuss School

Granting Agency	University of California, Office of the President
Award Amount	$113,000
Time Period	2002-2003
Role	PI

4. Algebraic Thinking

Granting Agency	University of California, Office of the President
Award Amount	$546,000
Time Period	2000-2004
Role	Co-PI

PhD Students

1. Dr. James Petty (1996)
2. Dr. Jack Tedeski (1998)
3. Dr. Sonia Hristovitch (1999)
4. Dr. Elisabetta Ferrando (2004)
5. Dr. April Maskiewicz (2005)
6. Dr. Kien Lim (2006)

Appendix B: Curriculum Vitae for Wilson

W. Stephen Wilson

Johns Hopkins University

Department of Mathematics

404 Krieger Hall

3400 N. Charles Street

Baltimore, MD 21218

Education:

S.B., M.I.T. (Math) 1969

S.M., M.I.T. (Math) 1969

Ph.D., M.I.T. (Math) 1972

Fields:

Algebraic Topology: Homotopy Theory: Complex Cobordism: Brown Peterson: Homology: Morava K-theory

Advisor:

F.P. Peterson (1930-2000)

Positions:

1980-present	Professor of Mathematics, The Johns Hopkins University
1993-1996	Chair, Department of Mathematics, The Johns Hopkins University
1977-80	Associate Professor, The Johns Hopkins University
1972-74	Instructor, Princeton University

Visiting Positions:

2006	Senior Advisor for Mathematics: Office of Elementary and Secondary Education, U.S. Department of Education
1998-99	Visiting Professor:

Kyoto University
Centre de Recerca Matematica, Institute d'Estudis Catalans

1983-84 Visiting Professor:
I.M.P.A., Rio de Janeiro
University of Witwatersrand
University of Melbourne
National Taiwan University
R.I.M.S., Kyoto University

1982 (Spring) Visiting Professor, Porto University

1980-81 Visiting Professor:
Hebrew University
Tata Institute of Fundamental Research
Osaka City University

1978 (Spring) Visiting Senior Mathematician, Oxford University

1977-78 Member, Institute for Advanced Study, Princeton

1975 (Spring) Visiting Assistant Professor, U.C.S.D.

1974-75 Member, Institute for Advanced Study, Princeton

Honors:

- Conference and banquet in honor of my, and Douglas C. Ravenel's, 60th birthdays, March 10-13, 2007.
- The Mathematical Society of Japan's Seki-Takakazu Prize for the Japan-U.S.
- Mathematics Institute, March, 2006
- The Johns Hopkins University Homewood Student Council Award for Excellence in Teaching, 2000
- Alfred P. Sloan Research Fellow, 1977-1979
- Invited One Hour Address, AMS Summer meeting, Duluth, Minn., 1979
- A series of 10 lectures, CBMS Regional Conference, 1980, SUNY at Albany

Conferences organized:

- Special session at the AMS annual meeting, Baltimore, January, 1992.
- Special session at the AMS annual meeting in honor of J. Michael Boardman's 60th birthday, Baltimore, January, 1998.
- Special session at the AMS annual meeting, Washington, January, 2000.
- JAMI conference at Johns Hopkins University, March, 2000.
- Special session at the AMS annual meeting, Baltimore, January, 2003.
- Conference in Kinosaki, Japan, in honor of Goro Nishida's 60th birthday, July, 2003.
- Special session at the AMS regional meeting in honor of the 60th birthdays of
- Martin Bendersky and Don Davis, Newark, Deleware, April, 2005.

References:

[1] W. S. Wilson. A new relation on the Stiefel-Whitney classes of spin manifolds. Il linois Journal of Mathematics, 17:115–127, 1973.

[2] W. S. Wilson. The Ω-spectrum for Brown-Peterson cohomology, Part I. Commentarii Mathematici Helvetici, 48:45–55, 1973.

[3] D. C. Johnson and W. S. Wilson. Pro jective dimension and Brown-Peterson homology. Topology, 12:327–353, 1973.

[4] D. C. Ravenel and W. S. Wilson. Bipolynomial Hopf algebras. Journal of Pure and Applied Algebra, 4:41–45, 1974.

[5] D. C. Ravenel and W. S. Wilson. The Hopf ring for complex cobordism. Bul letin of the American Mathematical Society, 80:1185–1189, 1974.

[6] W. S. Wilson. The Ω-spectrum for Brown-Peterson cohomology, Part II. American Journal of Mathematics, 97:101–123, 1975.

[7] W. S. Wilson. The Ω-spectrum for Brown-Peterson cohomology, Part III. Unpublished, 1975.

[8] D. C. Johnson and W. S. Wilson. BP-operations and Morava's extraordinary K-theories. Mathematische Zeitschrift, 144:55–75, 1975.

[9] H. R. Miller and W. S. Wilson. On novikov's Ext_1 modulo an invariant prime ideal. In D. Davis, editor, Reunion Sobre Teoria de homotopia, Universidad de Northwestern, Agosto 1974, number 1 in Serie notas de matem´atica y simposia, pages 159–166, Mexico, D.F., 1975. Sociedad Matematica Mexicana.

[10] D. C. Johnson, H. R. Miller, W. S. Wilson, and R. S. Zahler. Boundary homomorphisms in the generalized Adams spectral sequence and the nontriviality of infinitely many γ_t in stable homotopy. In D. Davis, editor, Reunion Sobre Teoria de homotopia, Universidad de Northwestern, Agosto 1974, number 1 in Serie notas de matem ´atica y simposia, pages 47–63, Mexico, D.F., 1975. Sociedad Matematica Mexicana.

[11] H. R. Miller, D. C. Ravenel, and W. S. Wilson. Novikov's Ext_2 and the non-triviality of the gamma family. Bul letin of the American Mathematical Society, 81:1073–1075, 1975.

[12] H. R. Miller and W. S. Wilson. On Novikov's Ext_1 modulo an invariant prime ideal. Topology, 15:131–141, 1976.

[13] D. C. Ravenel and W. S. Wilson. The Hopf ring for complex cobordism. Journal of Pure and Applied Algebra, 9:241–280, 1977.

[14] H. R. Miller, D. C. Ravenel, and W. S. Wilson. Periodic phenomena in the Adams-Novikov spectral sequence. Annals of Mathematics, 106:469–516, 1977.

[15] D. C. Johnson and W. S. Wilson. The pro jective dimension of the complex bordism of Eilenberg- Mac Lane spaces. Osaka Journal of Mathematics, 14:533–536, 1977.

[16] D. M. Latch, R. W. Thomason, and W. S. Wilson. Simplicial sets from categories. Mathematische Zeitschrift, 164:195–214, 1979.

[17] D. C. Ravenel and W. S. Wilson. The Morava K-theories of Eilenberg-Mac Lane spaces and the Conner-Floyd conjecture. American Journal of Mathematics, 102:691–748, 1980.

[18] R. W. Thomason and W. S. Wilson. Hopf rings in the bar spectral sequence. Quarterly Journal of Mathematics, 31:507–511, 1980.

[19] W. S. Wilson. Unstable cohomology operations. In Topics in homotopy theory and cohomology theory. Proceedings of a Symposium held at the Research Institute for Mathematical Sciences, Kyoto University, Kyoto, October 27-29, 1980., volume 419, pages 18–25, Kyoto University, Kyoto, Japan, March 1981. RIMS Kokyuroku.

[20] W. S. Wilson. Brown-Peterson homology: an introduction and sampler. Number 48 in C.B.M.S. Regional Conference Series in Mathematics. American Mathematical Society, Providence, Rhode Island, 1982.

[21] W. S. Wilson. Towards BP_*X. In S. Gitler, editor, Symposium on Algebraic Topology in Honor of Jos´e Adem, Contemporary Mathematics, pages 345–351, Providence, Rhode Island, 1982. American Mathematical Society.

[22] W. S. Wilson. The complex cobordism of BO_n. Journal of the London Mathematical Society,

29(2):352–366, 1984.

[23] W. S. Wilson. Brown-Peterson metastability and the Bendersky-Davis conjecture. Publications of Research Institute of Mathematical Sciences, Kyoto University, 20:1037–1051, 1984.

[24] W. S. Wilson. The Hopf ring for Morava K-theory. Publications of Research Institute of Mathematical Sciences, Kyoto University, 20:1025–1036, 1984.

[25] D. C. Johnson and W. S. Wilson. The Brown-Peterson homology of elementary p-groups. American Journal of Mathematics, 107:427–454, 1985.

[26] H. S. Song and W. S. Wilson. On the non-immersion of products of real pro jective spaces. Transactions of the American Mathematical Society, 318:327–334, 1990.

[27] D. C. Johnson, W. S. Wilson, and D. Y. Yan. Brown-Peterson homology of elementary p-groups, II. Topology and its Applications, 59:117–136, 1994.

[28] J. M. Boardman, D. C. Johnson, and W. S. Wilson. Unstable operations in generalized cohomology. In I. M. James, editor, The Handbook of Algebraic Topology, chapter 15, pages 687–828. Elsevier, 1995.

[29] D. C. Ravenel and W. S. Wilson. The Hopf ring for P (n). Canadian Journal of Mathematics, 48(5):1044–1063, 1996.

[30] W. S. Wilson. Wei-Liang Chow. Notices of the American Mathematical Society, 43(10):1117–1124, October 1996. Organizer.

[31] P. J. Eccles, P. R. Turner, and W. S. Wilson. On the Hopf ring for the sphere. Mathematische Zeitschrift, 224(2):229–233, 1997.

[32] D. C. Johnson and W. S. Wilson. On a theorem of Ossa. Proceedings of the American Mathematical Society, 125(12):3753–3755, 1997.

[33] W. S. Wilson. Hopf ring. In Encyclopaedia of Mathematics, Supplementary Volume I, pages 299–300. Kluwer Academic Publishers, 1997.

[34] M. J. Hopkins, D. C. Ravenel, and W. S. Wilson. Morava Hopf algebras and spaces K(n) equivalent to finite Postnikov systems. In Paul S. Selick et. al., editor, Stable and Unstable Homotopy, volume 19 of The Fields Institute for Research in Mathematical Sciences Communications Series, pages 137–163, Providence, R.I., 1998. American Mathematical Society.

[35] D. C. Ravenel, W. S. Wilson, and N. Yagita. Brown-Peterson cohomology from Morava K-theory. K-Theory, 15(2):149–199, 1998.

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[38] W. S. Wilson. K(n + 1) equivalence implies K(n) equivalence. In J.-P. Meyer, J. Morava, and W. S. Wilson, editors, Homotopy invariant algebraic structures: a conference in honor of J. Michael Boardman, volume 239 of Contemporary Mathematics, pages 375–376, Providence, Rhode Island, 1999. American Mathematical Society.

[39] J. M. Boardman, R. Kramer, and W. S. Wilson. The periodic Hopf ring of connective Morava K- theory. Forum Mathematicum, 11:761–767, 1999.

[40] J.-P. Meyer, J. Morava, and W. S. Wilson, editors. Homotopy invariant algebraic structures: a conference in honor of J. Michael Boardman, volume 239 of Contemporary Mathematics. American Mathematical Society, Providence, Rhode Island, 1999.

[41] W. S. Wilson. The impossible made easy: Learning to calculate with generalized cohomology. In 46th Annual Japanese Topology Symposium Proceedings, Hokkaido University, pages 20–30, Hokkaido, Japan, July 1999.

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[47] N. Kitchloo, G. Laures, and W.S. Wilson. Splittings of bicommutative Hopf algebras. Journal of Pure and Applied Algebra, 194:159–168, 2004.

[48] W.S. Wilson and D.Q. Naiman. K-12 calculator usage and college grades. Educational Studies in Mathematics, 56:119–122, 2004.

[49] E.C. Scott with a few hundred Steves including W.S. Wilson. The morphology of Steve. Annals of Improbable Research, 10(4):24–29, July/August 2004.

[50] David Klein, with Bastiaan J. Braams, Thomas Parker, William Quirk, Wilfried Schmid, and W. Stephen Wilson. Technical assistance from Ralph A. Raimi and Lawrence Braden. Analysis by Justin Torres. Foreword by Chester E. Finn, Jr. The State of the State MATH Standards. Thomas B. Fordham Foundation, Washington, D.C., January 2005.

[51] W.S. Wilson. Short response to Tunis's letter to the editor on technology in college. Educational Studies in Mathematics, 58:415–420, 2005.

[52] Barbara Reys, Carolyn Baldree, Donna Taylor, and W.S. Wilson. Grade 6: Process and standards. In Johnny W. Lott and Kathleen Nishimura, editors, Standards & Curriculum: A view from the Nation, pages 29–32, Reston, Virginia, 2005. NCTM.

[53] N. Kitchloo and W.S. Wilson. On fibrations related to real spectra. In M. Ando, N. Minami, J. Morava, and W.S. Wilson, editors, Proceedings of the Nishida Fest (Kinosaki 2003), volume 10 of Geometry & Topology Monographs, pages 237–244, 2007.

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[55] N. Kitchloo and W.S. Wilson. On the Hopf ring for ER(n). Topology and its Applications, 154:1608–1640, 2007.

[56] J. M. Boardman and W. S. Wilson. k(n)-torsion-free H -spaces and P (n)-cohomology. Canadian Journal of Mathematics. To appear.

[57] W.S. Wilson. Are our students better now? Preprint, 2007.

[58] W.S. Wilson. Elementary school mathematics priorities. Preprint, 2007.

[59] N. Kitchloo and W.S. Wilson. The second real Johnson-Wilson theory and non-immersions of RP Submitted.

[60] N. Kitchloo and W.S. Wilson. The second real Johnson-Wilson theory and non-immersions of RP Part 2. Submitted.

[61] J. Gonz´alez and W.S. Wilson. The BP-theory of two-fold products of pro jective spaces. Submitted.

Education activities:

Winter-Spring 1999: While visiting Japan for 8 months we enrolled our son in the local public school. This gave us a very personal view of mathematics education in Japan. Through my many visits to Japan and with my many connections to Japanese mathematicians I have kept track of mathematics education issues in Japan.

2000: The Johns Hopkins University Homewood Student Council Award for Excellence in Teaching.

February 2002: Survey of mathematicians about arithmetic and calculator usage: http://www.math.jhu.edu/~wsw/ED/list

March 4, 2002: Attend Lynn Cheney's American Enterprise Institute conference: Does two plus two still equal four? http://www.aei.org/events/eventID.193/transcript.asp

December 2002: Surveyed all students at Johns Hopkins University in big service courses on their calculator usage in K-12.

2004:	Served on a panel for the Fordham Foundation to evaluate the K-12 mathematics standards for all states.
July 21-24, 2004:	A research mathematician representative at the conference: Mathematics Curriculum: A National View, a meeting of the Association of State Supervisors of Mathematics and the Board of the National Council of Teachers of Mathematics and a few research mathematicians run by Johnny Lott.
July 25-28, 2004:	Participated in a conference, the Mathematics Standards Study Group, organized by Roger Howe, of a dozen research mathematicians interested in K-12 mathematics education.
September 2004-Present:	Appointed by Provost Steven Knapp to the Johns Hopkins Council on K-12 Education.
October 15, 2004:	Invited participant of a meeting of the Board of the Adult Numeracy Network.
Winter 2004-05:	The Johns Hopkins University contact person for Teach for America.
January 2005:	David Klein, with Bastiaan J. Braams, Thomas Parker, William Quirk, Wilfried Schmid, and W. Stephen Wilson. Technical assistance from Ralph A. Raimi and Lawrence Braden. Analysis by Justin Torres. Foreword by Chester E. Finn, Jr. The State of the State MATH Standards. Thomas B. Fordham Foundation. (130 pgs.) http://www.math.jhu.edu/~wsw/ED/mathstandards05FINAL.pdf
January 2005:	Testified before the Maryland State Board of Education.
January 2005:	Advised Ralph Fessler, the Dean of Johns Hopkins University's School of Professional Studies in Business and Education, on elementary school mathematics programs for the new elementary school Johns Hopkins is going to run in the near future.
February-June 2005:	Consult with Johns Hopkins University Professor Peggy King- Sears to participate in focus group and support development of a Johns Hopkins University course of study and certificate for mathematics teachers for middle and high school.
May 24, 2005:	Testify before the Governor's Commission on Quality Education. http://www.math.jhu.edu/ ~wsw/ED/steele.pdf
September 21, 2005:	Johns Hopkins University Council on K-12 Education. Planning for STEM initiative.
January-August 2006:	Senior Advisor for Mathematics, Office of Elementary and Secondary Education, United States Department of Education.
May 5, 2006:	Talk to the Conference Board of the Mathematical Sciences about the National Mathematics Advisory Panel.

May 7-10, 2006: Invited participant in a Mathematical Sciences Research Institute conference: Raising the floor: Progress and setbacks in the struggle for quality mathematics education for all, Berkeley, California.

May-June 2006: Advisor to the National Mathematics Advisory Panel.

May 23, 2006: Served on the panel: Thinking Big: Setting the K-12 Math and Science Agenda at Accelerating the K-12 Mathematics and Science Curriculum: Agenda for the 21st Century. The Third Annual Johns Hopkins Education Summit.

March 2007: Reviewed a draft of the revision for the mathematics content standards for Florida, at the request of the Florida K-12 Chancellor.

April - August 2007: Wrote a background overview of Washington state's K-12 mathematics standards for Strategic Teaching's Washington State Mathematics Standards: Review and Recommendations. Also helped with the final touches on the writing of the report.

August 2007: Compared the American Diploma Project with standards from other countries for Strategic Teaching.

February 2008: Reviewed the revised Washington State mathematics standards for Strategic Teaching.

July 2007 - Present: On the American Mathematical Society's Advisory Board for the Working Group on Preparation for Technical Fields.