

# Crosstalk

吳瑞北

Rm. 340, Department of Electrical Engineering

E-mail: [rbwu@ew.ee.ntu.edu.tw](mailto:rbwu@ew.ee.ntu.edu.tw)

url: [cc.ee.ntu.edu.tw/~rbwu](http://cc.ee.ntu.edu.tw/~rbwu)

S. H. Hall et al., *High-Speed Digital Design*, Chap.4

N. N. Rao, *Elements of Engineering Electromagnetics*, Sec. 6.7.

# What will you learn

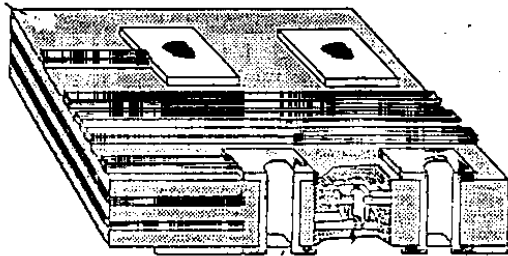
- **Physical** insight to inductive & capacitive coupling
- **Definition** of L and C matrices of multi-coupled tx-lines
- **Electrical parameter extraction** for coupled tx-lines
- Phenomenological description of crosstalk noise by **weakly coupling analysis**
- Analytic derivation for crosstalk noise by **model analysis**
- Construction of equivalent SPICE circuit for transient **simulation** of ideal coupled tx-lines
- Novel **designs** for minimization of crosstalk noise: matched termination, guard trace, spiral delay line.

# Contents

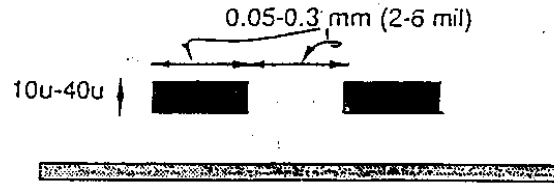
- Two mechanisms that Cause Crosstalk
- Crosstalk-induced Noise
- Even/Odd Mode Decomposition
- Modal Analysis
- Simulation in SPICE
- Design Issues

# MCM Structures

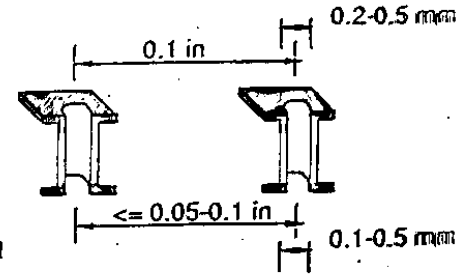
MCM-L



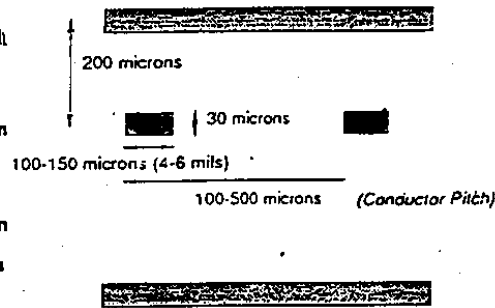
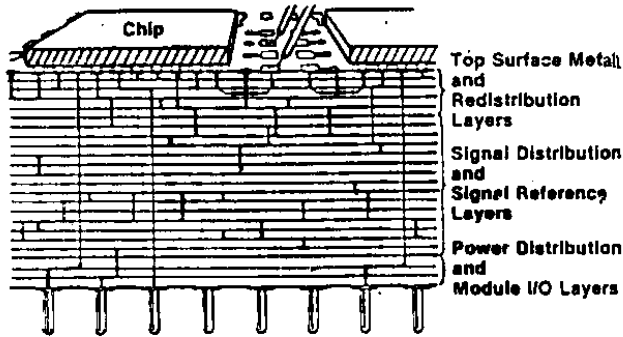
Typical Conductor Cross-section  
MCM-L



Typical via-size and pitch



MCM-C



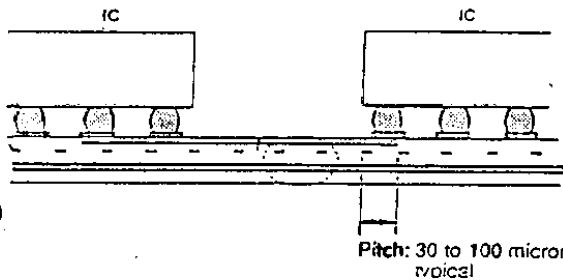
MCM-C



Via Pitch and Size Typical: 100 micron wide 250 micron pitch

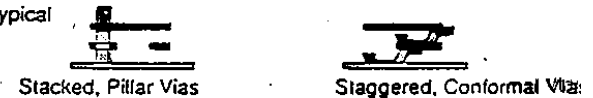
MCM-D

MCM-D

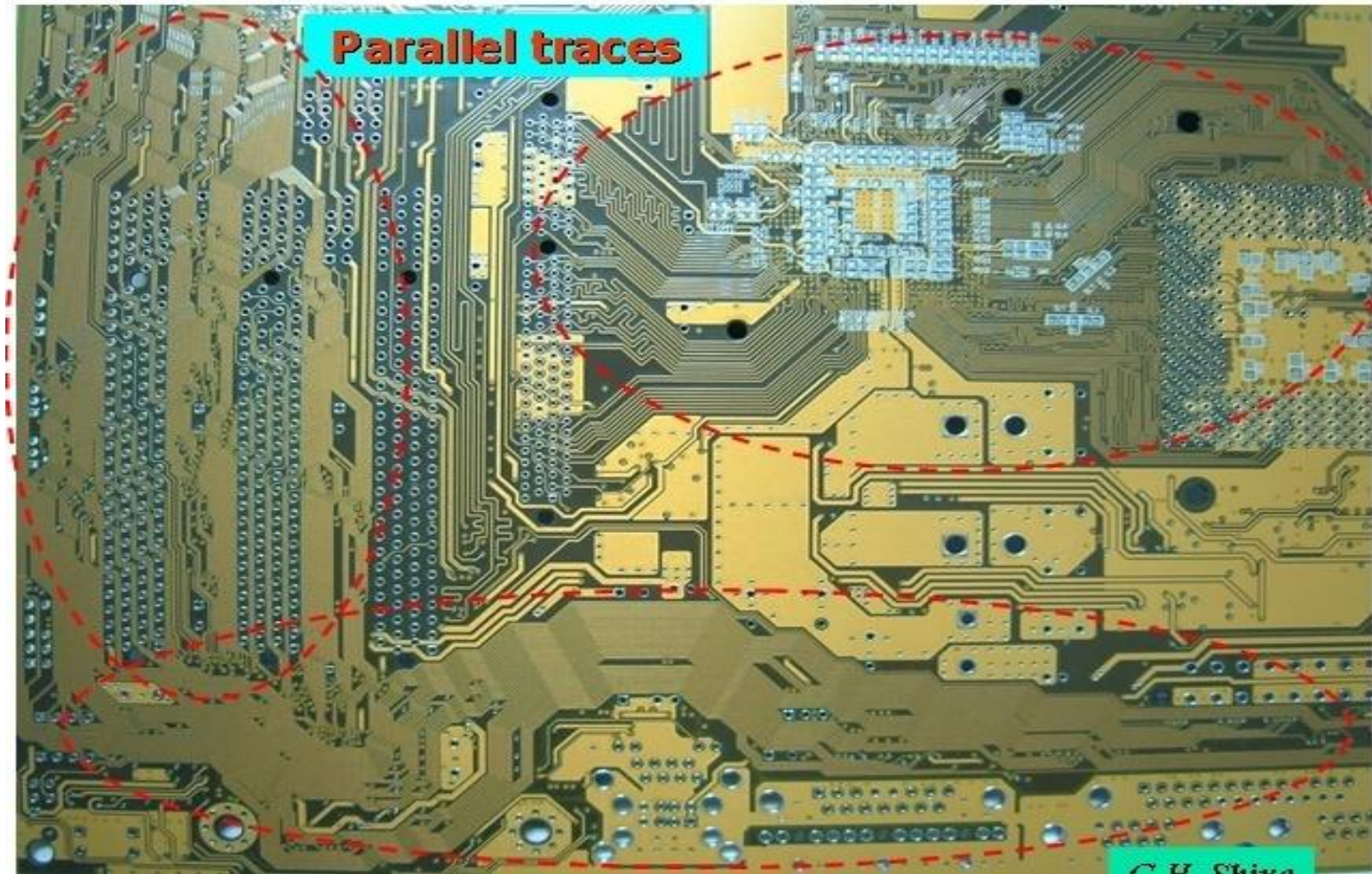


Width: 10 to 30 micron typical  
Height: 2 to 10 micron typical

Via pitch: Usually same or less than wire pitch



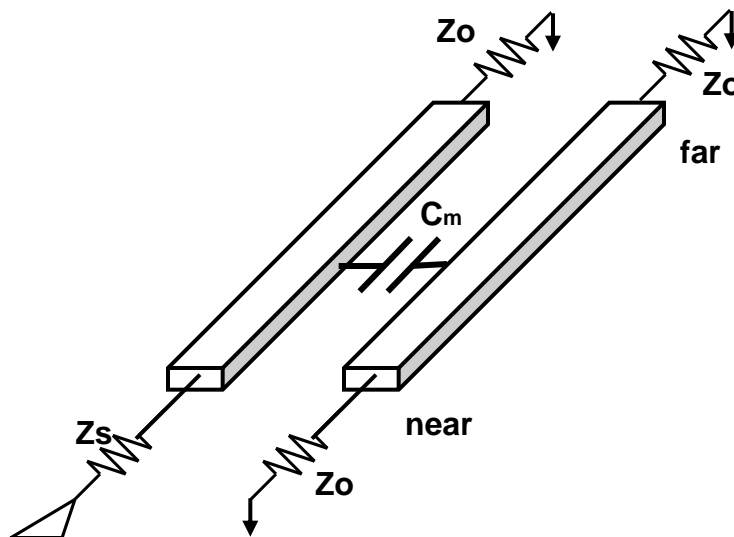
# Example



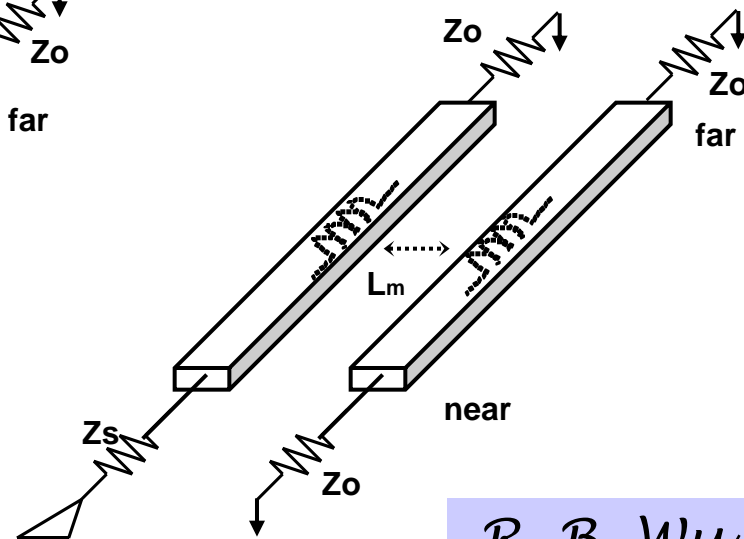
# Mutual Inductance and Capacitance

- Crosstalk is caused by energy coupling from one line to another via:
  - *Mutual capacitance (electric field)*
  - *Mutual inductance (magnetic field)*

Mutual Capacitance,  $C_m$

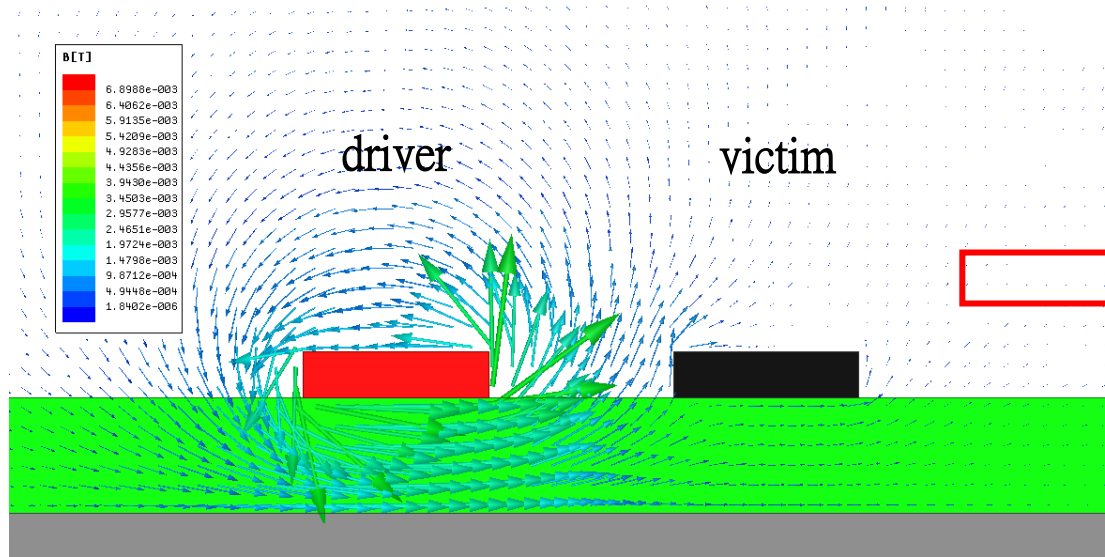


Mutual Inductance,  $L_m$

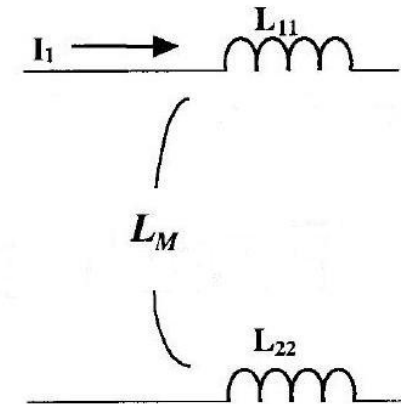


# Mutual Inductance

Mutual L induces current from a driven line onto a quiet line by magnetic field ◦



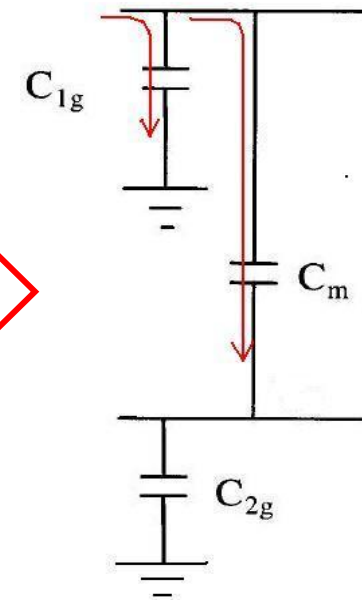
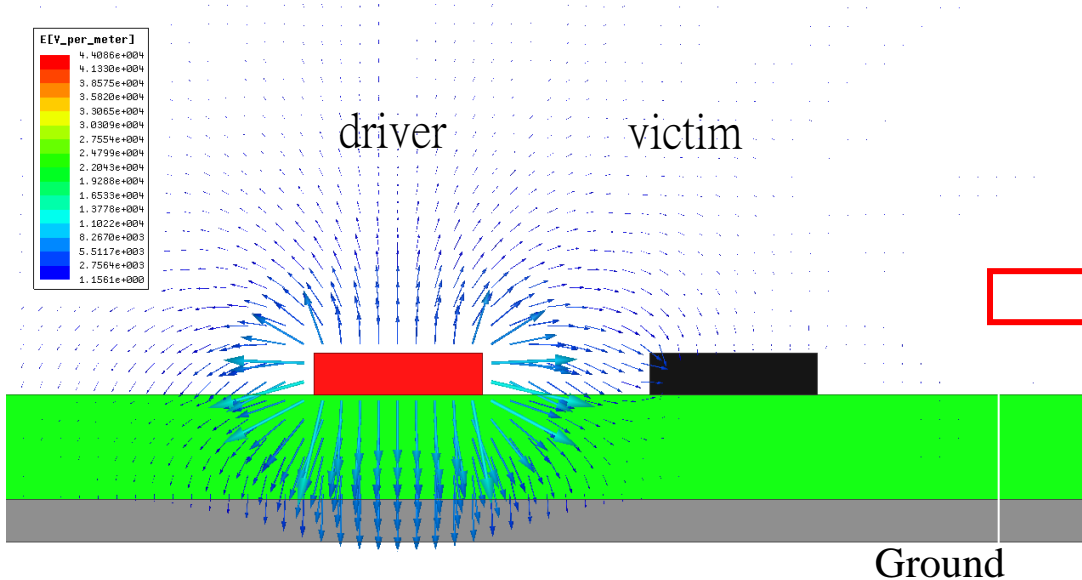
Ground



$$V_{noise} = L_m \frac{dI_{driver}}{dt}$$

# Mutual Capacitance

Mutual C is coupling of two conductors via electric field

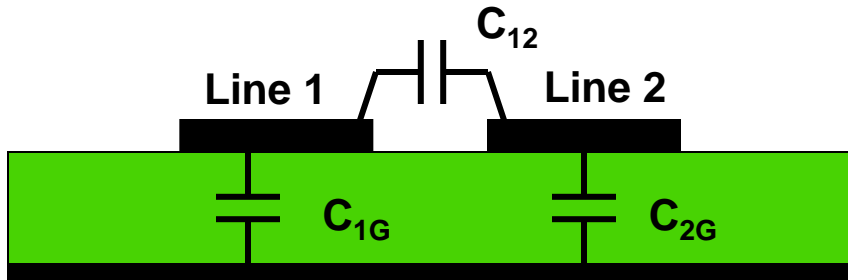


$$I_{noise} = C_m \frac{dV_{driver}}{dt}$$

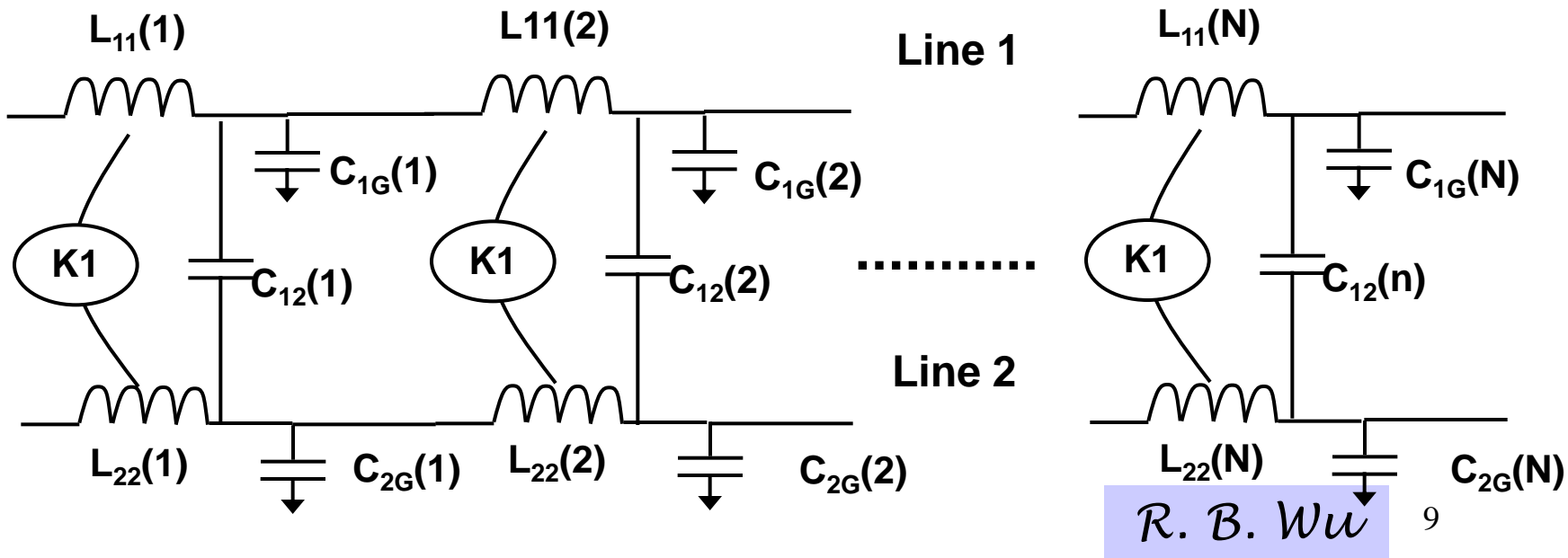


# Crosstalk Model by "Equivalent Circuit"

- Circuit is distributed into N segments



$$K = \frac{L_{12}}{\sqrt{L_{11}L_{22}}}$$



# Inductance Matrix

- Inductance matrix:

- $L_{NN}$  = self inductance of line N per unit length
- $L_{MN}$  = mutual inductance between lines M and N

$$[L] = \begin{bmatrix} L_{11} & L_{12} & \dots & L_{1N} \\ L_{21} & L_{22} & \dots & \vdots \\ \vdots & \dots & \dots & \vdots \\ L_{N1} & \dots & \dots & L_{NN} \end{bmatrix}$$

# Capacitance Matrix

- **Capacitance matrix:**

- $C_{NN}$  = self capacitance of line N per unit length where:

$$C_{NN} = C_{NG} + \sum |C|_{mutuals}$$

- $C_{NG}$  = Capacitance between line N and ground
- $C_{MN}$  = Mutual capacitance between lines M and N

$$[C] = \begin{bmatrix} C_{11} & -|C_{12}| & \dots & -|C_{1N}| \\ -|C_{21}| & C_{22} & & \vdots \\ \vdots & & \ddots & \vdots \\ -|C_{N1}| & \dots & \dots & C_{NN} \end{bmatrix}$$

- **For example, for 2 line circuit:**

$$C_{11} = C_{1G} + |C_{12}|$$

# Numerical Techniques

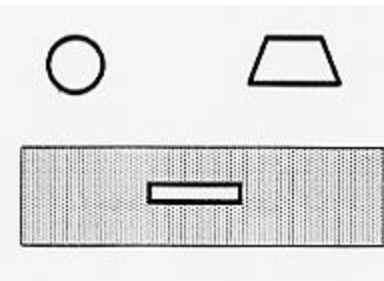
Geometry of lines:  
width, separation,  
thickness, height,  
substrate, etc.

C2D

Electrical parameters:  
capacitance matrix [C]  
Inductance matrix [L]

- Based on IE formulation and MoM
- Exact integration formulae
- Four versions are available

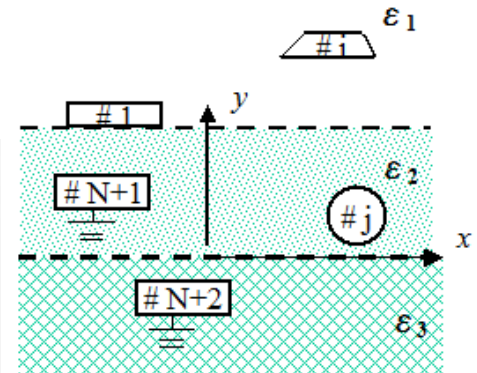
CAP1



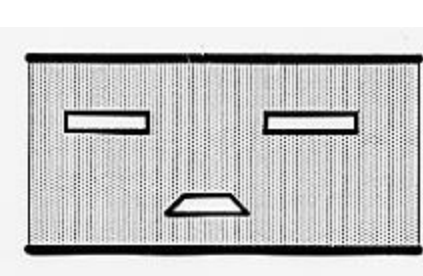
CAP2 - microstrip



CAP4 – e.g. CPW



CAP3 – triplate



W. T. Weeks, "Calculation of coefficients of capacitance of multiconductor transmission lines in the presence of a dielectric interface," *IEEE T-MTT*, Jan. 1970.

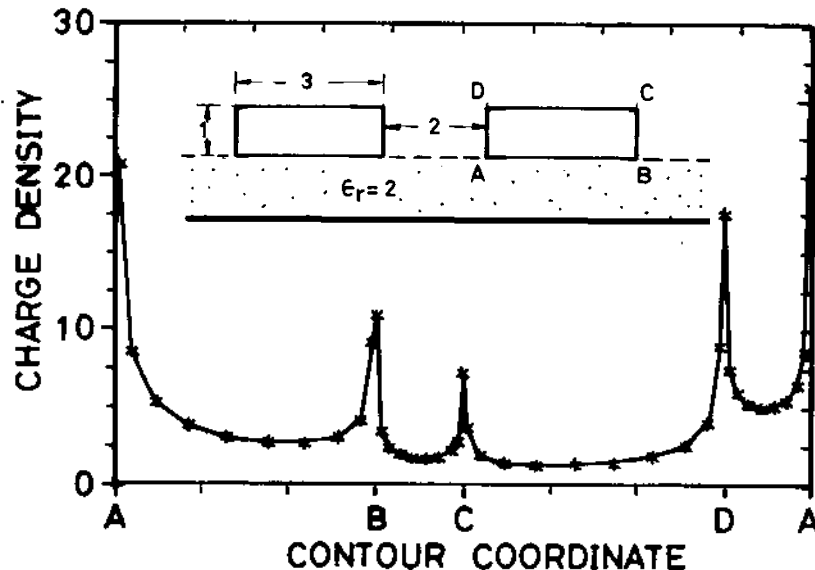
R. B. Wu

# CAP2 Example

輸入檔案 CAP2.DAT

```

1.  2.  1.      # E1, E2, H
2   0           # N, Print/no
1           # Conductor 1 : rectangular
4   3   6       # Polygen K, Len_Div, N_Min_Div
.  -4.  1.  -1.  1.  -1.  2.  -4.  2.  -4.  1.
1           # Conductor 2 ....
4   3   6
1.  1.  4.  1.  4.  2.  1.  2.  1.  1.
    
```



輸出檔案 CAP2.OUT

```

Matrix of Capacitance Coefficients: (pF/m)
  93.4600  -8.5756
 -8.5756   93.4600

Matrix of Inductance Coefficients: (nH/m)
 198.1587  30.1863
 30.1863   198.1588

Approximate Characteristic Impedance: (ohm)
 46.0462   46.0462

Approximate Saturated Near End Noise
 1.0000    0.0610
 0.0610    1.0000
    
```

# Crosstalk Induced Noise

# Mechanism of coupling

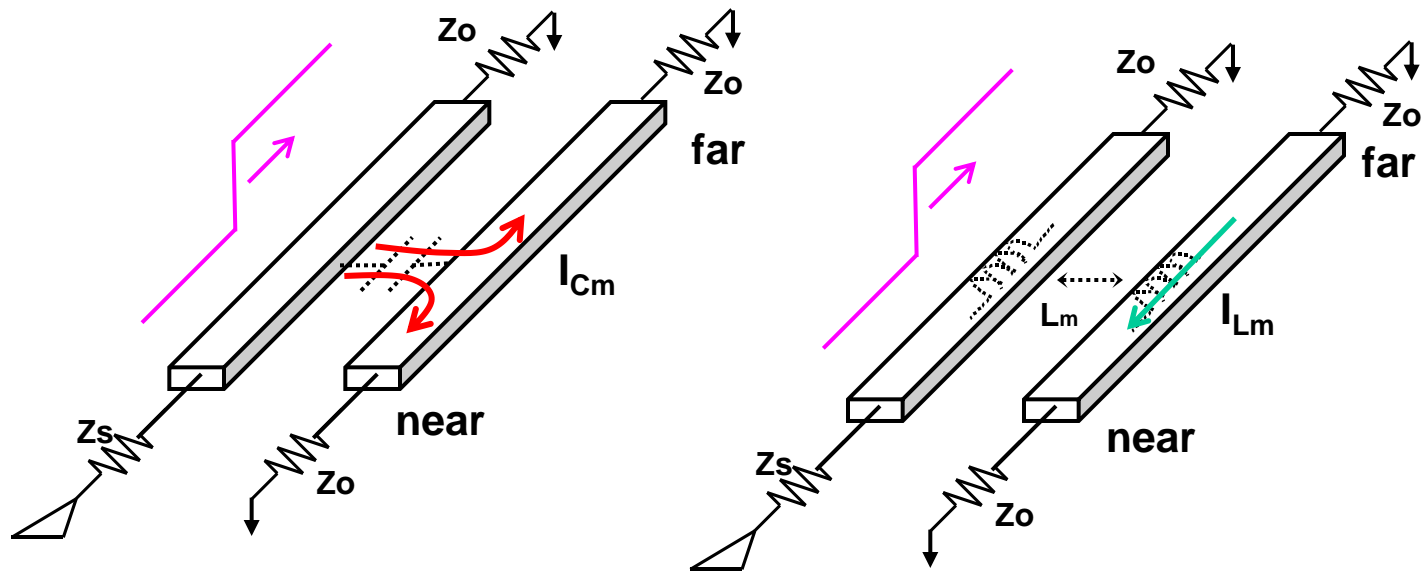
- Circuit element representing transfer of energy

$$V_{Lm} = L_m \frac{dI}{dt} \qquad I_{Cm} = C_m \frac{dV}{dt}$$

- Mutual inductance induces current on victim line opposite of driving current (Lenz's Law)
- Mutual capacitance passes current through mutual capacitance that flows in both directions on the victim line

# Coupled Currents

- Coupled currents on victim line sum to produce near and far end crosstalk noise

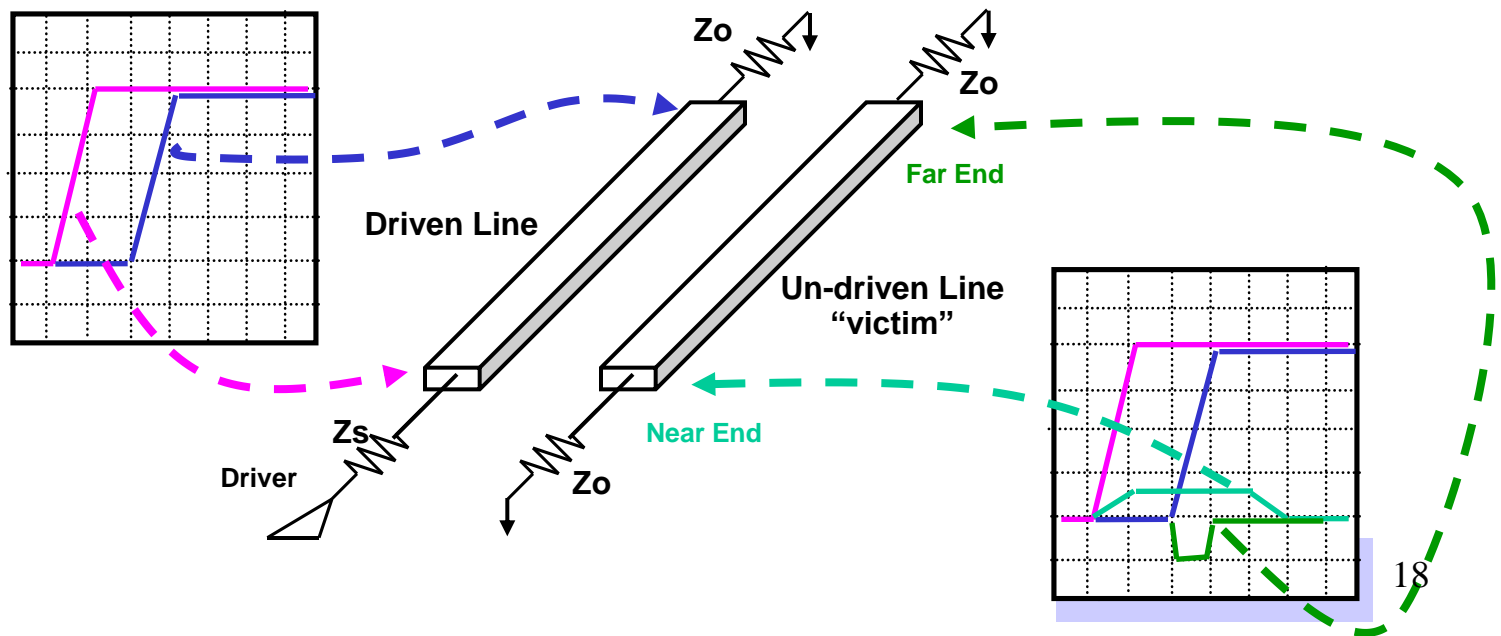


$$I_{near} = I_{Cm} + I_{Lm} \quad I_{far} = I_{Cm} - I_{Lm}$$



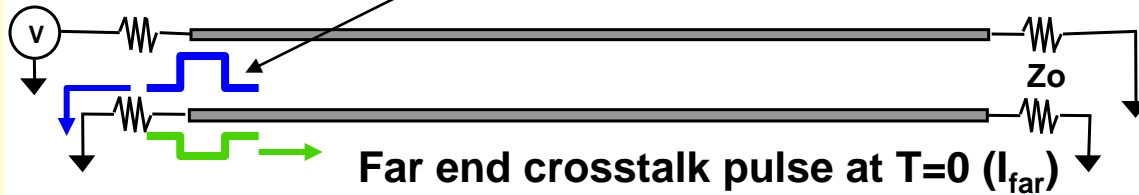
# Voltage Profile of Coupled Noise

- **Near end crosstalk is always positive**
  - Currents from  $L_m$  and  $C_m$  always add & flow into the node.
- **For PCB's, far end crosstalk is "usually" negative**
  - Current due to  $L_m$  larger than current due to  $C_m$
  - Note that far end crosstalk can be positive or nullified.

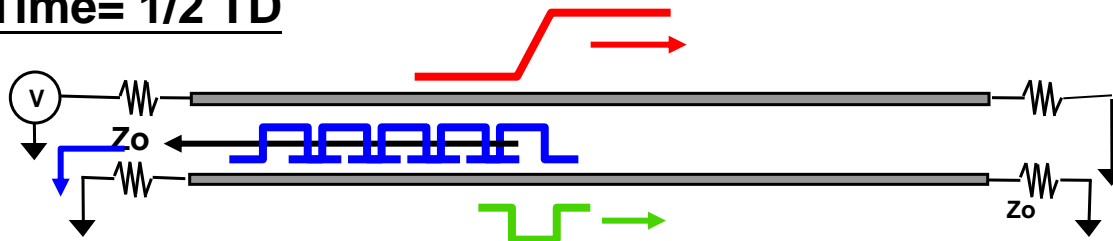


# Graphical Explanation

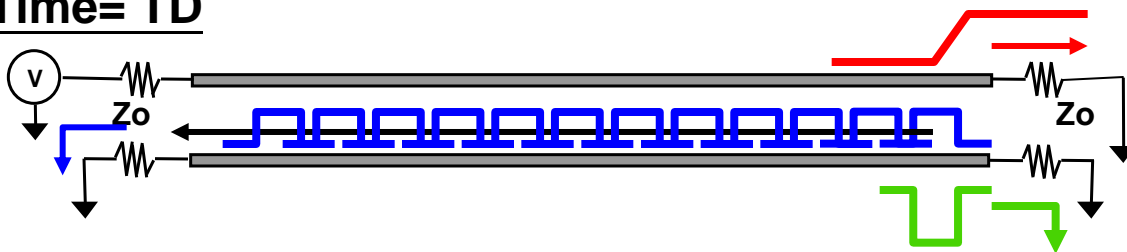
**Time = 0** Near end crosstalk pulse at  $T=0$  ( $I_{near}$ )



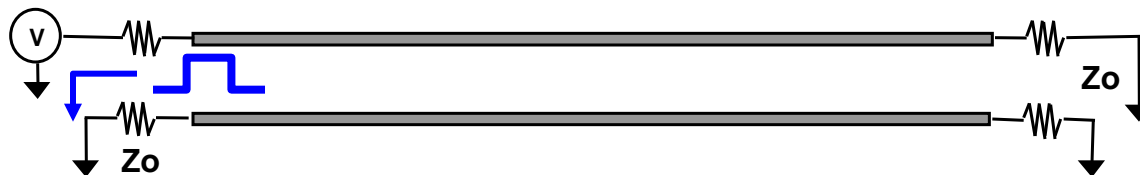
**Time = 1/2 TD**



**Time = TD**

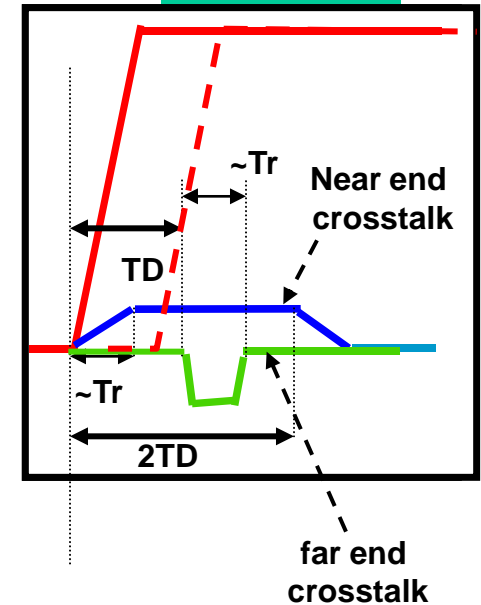


**Time = 2TD**



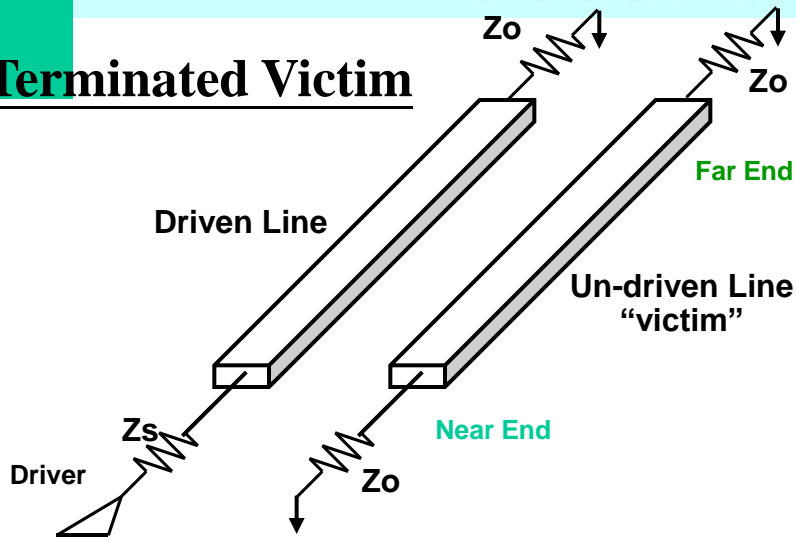
Far end of current terminated at  $T=TD$

Near end current terminated at  $T=2TD$



# Crosstalk Equations

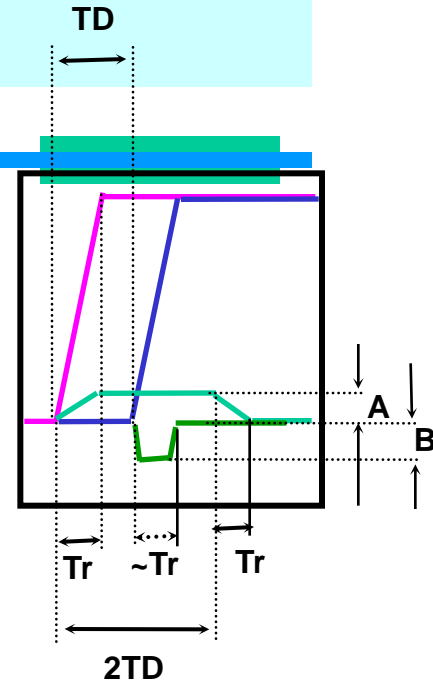
## Terminated Victim



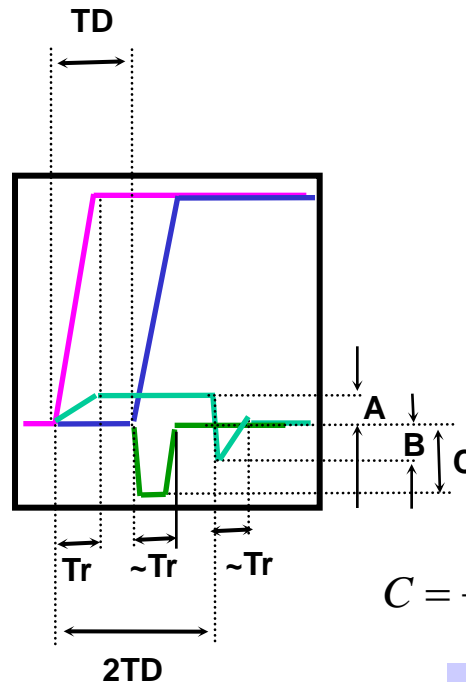
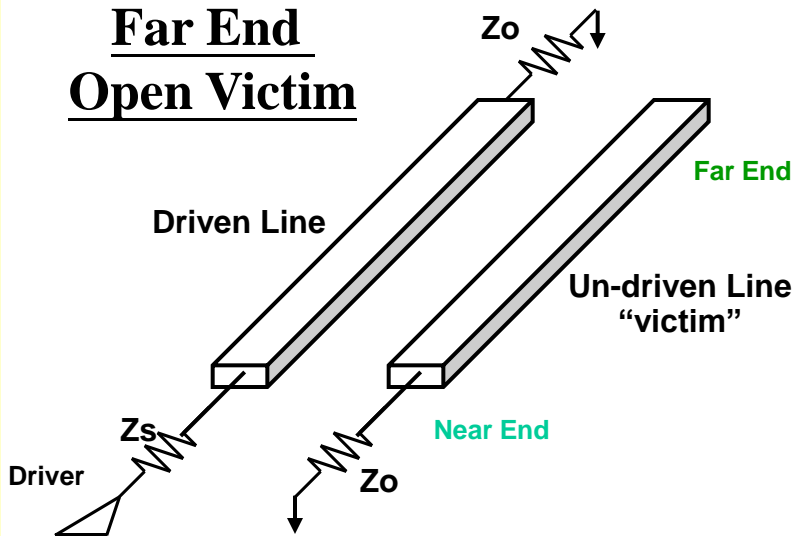
$$A = \frac{V_{input}}{4} \left[ \frac{L_M}{L} + \frac{C_M}{C} \right]$$

$$TD = X \sqrt{LC}$$

$$B = -\frac{V_{input} \cdot TD}{2T_r} \left[ \frac{L_M}{L} - \frac{C_M}{C} \right]$$



## Far End Open Victim



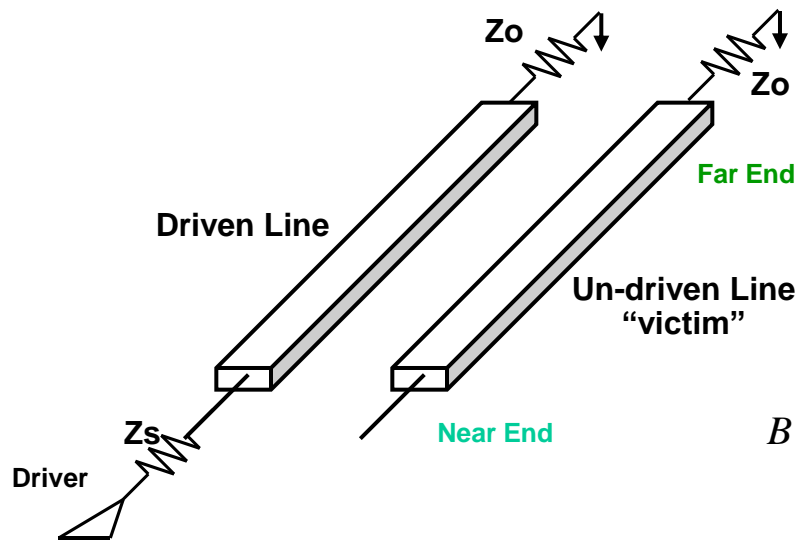
$$A = \frac{V_{input}}{4} \left[ \frac{L_M}{L} + \frac{C_M}{C} \right]$$

$$B = \frac{1}{2} C$$

$$C = -\frac{V_{input} \cdot TD}{T_r} \left[ \frac{L_M}{L} - \frac{C_M}{C} \right]$$

# Crosstalk Equations -2

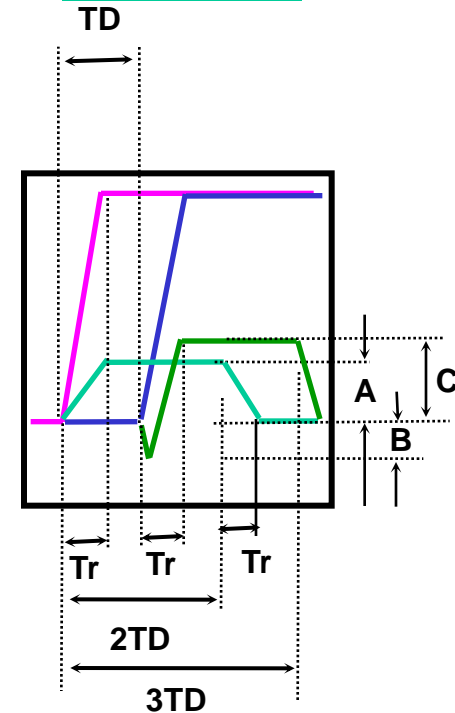
## Near End Open Victim



$$A = \frac{V_{input}}{2} \left[ \frac{L_M}{L} + \frac{C_M}{C} \right]$$

$$C = \frac{V_{input}}{4} \left[ \frac{L_M}{L} - \frac{C_M}{C} \right]$$

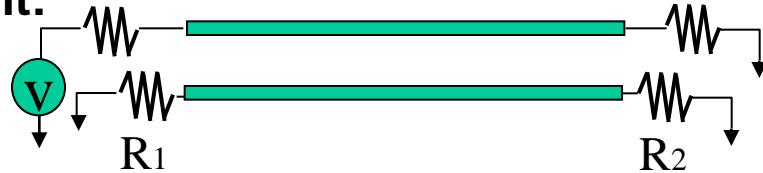
$$B = -\frac{V_{input} \cdot TD}{2T_r} \left[ \frac{L_M}{L} - \frac{C_M}{C} \right]$$



- The Crosstalk noise characteristics are dependent on the termination of the victim line

# Example

Calculate near and far end crosstalk-induced noise magnitudes and sketch the waveforms of circuit:



$V_{source}=2V$ ,  $Trise = 100ps$ .

Length of line is 2 inches. Assume all terminations are 70 Ohms.

Assume the following capacitance and inductance matrix:

$$\mathbf{L} = \begin{bmatrix} 9.869 & 2.103 \\ 2.103 & 9.869 \end{bmatrix} \text{ nH / inch}$$

$$\mathbf{C} = \begin{bmatrix} 2.051 & -0.239 \\ -0.239 & 2.051 \end{bmatrix} \text{ pF / inch}$$

Characteristic impedance is:

$$Z_o = \sqrt{\frac{L_{11}}{C_{11}}} = \sqrt{\frac{9.869 \text{ nH}}{2.051 \text{ pF}}} = 69.4\Omega$$

Therefore the system has matched termination. ( $V_{input} = 1.0V$ ), crosstalk noise magnitudes can be calculated as follows:

# Example (cont.)

Near end crosstalk voltage amplitude:

$$V_{near} = \frac{V_{input}}{4} \left[ \frac{L_{12}}{L_{11}} + \frac{|C_{12}|}{C_{11}} \right] = \frac{1V}{4} \left[ \frac{2.103}{9.869} + \frac{0.239}{2.051} \right] = 0.082V$$

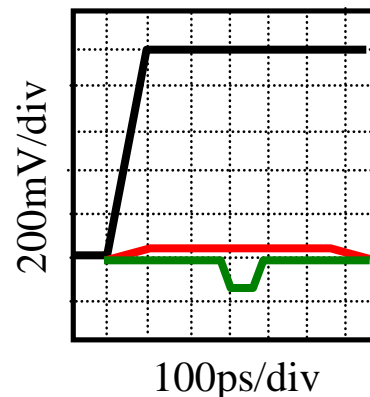
Propagation delay of the 2 inch line is:

$$TD = X\sqrt{LC} = 2inch * \sqrt{(9.869nH * 2.051pF)} = 0.28ns = 280ps$$

Far end crosstalk voltage amplitude:

$$V_{far} = \frac{V_{input}}{2} \cdot \frac{TD}{T_{rise}} \left( \frac{L_{12}}{L_{11}} - \frac{|C_{12}|}{C_{11}} \right) = \frac{1V * 280ps}{2 * 100ps} \left( \frac{2.103}{9.869} - \frac{0.239}{2.051} \right) = -0.137V$$

Thus,



# Even/Odd Mode Decomposition

# *Coupling Effects on Tx-line Parameters*

## *Key Topics:*

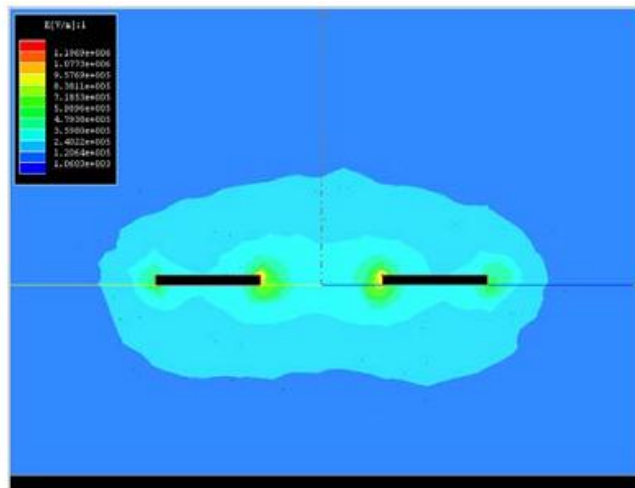
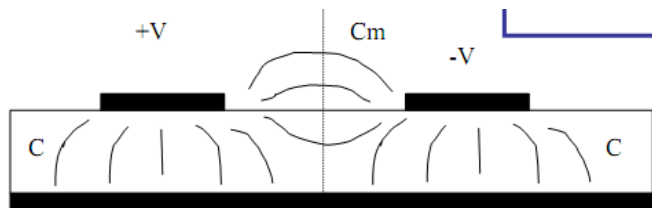
- ✓ **Odd and Even Mode Characteristics**
- ✓ **Microstrip vs. Stripline**
- ✓ **Modal Termination Techniques**
- ✓ **Modal Impedance's for more than 2 lines**
- ✓ **Effect of Switching Patterns**
- ✓ **Single Line Equivalent Model (SLEM)**



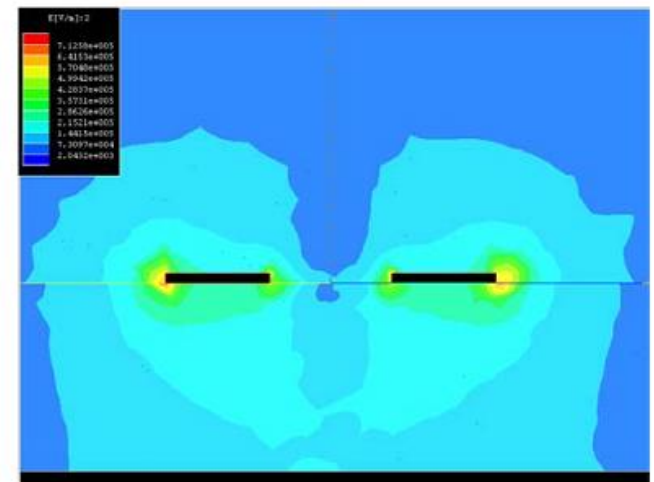
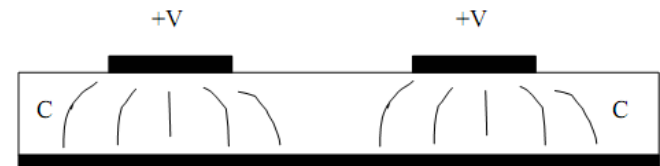
# Field Distribution

- Differential and common mode signals have different field distribution, impedance and propagation velocity.

## Differential mode

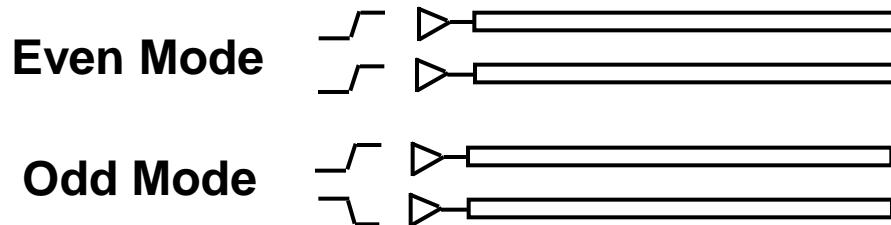


## common mode



# Odd/Even Transmission Modes

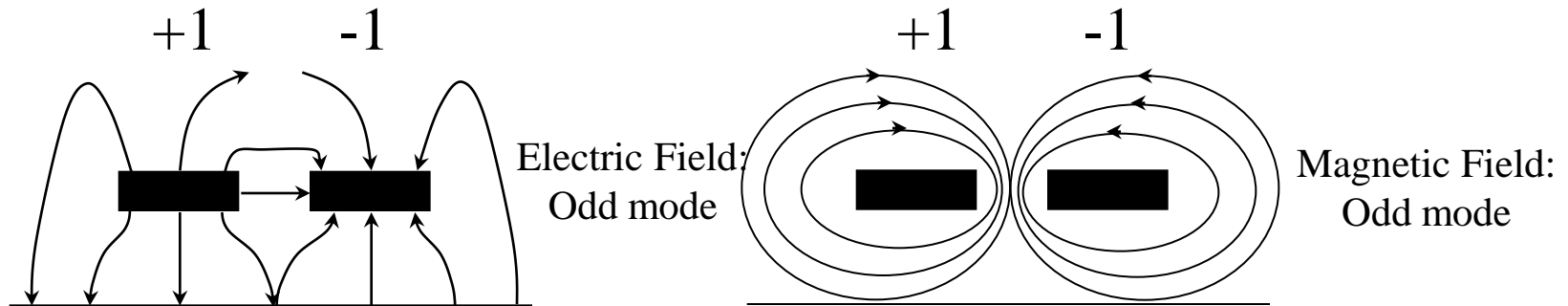
- EM Fields btw two coupled lines interacts with each other. These interactions affect the impedance and delay of tx-line
- A 2-conductor system has 2 propagation modes
  - ↳ Even Mode (Both lines driven in phase)
  - ↳ Odd Mode (Lines driven 180° out of phase)



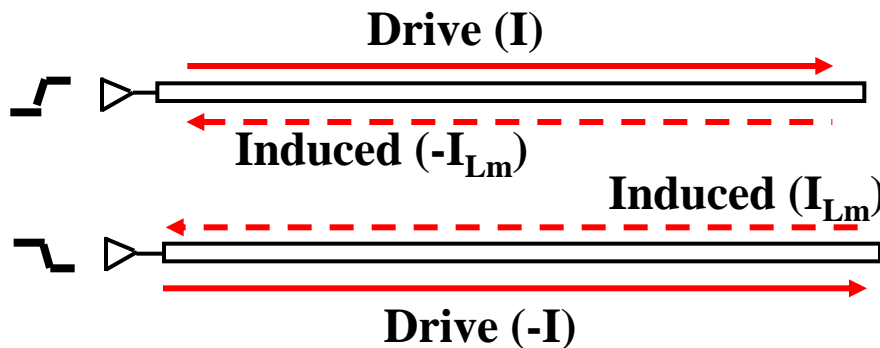
- Field interaction causes the system electrical characteristics to be dependent on the patterns.

# Odd Mode Transmission

- Potential difference btw conductors lead to *increase* of effective  $C$  equal to  $C_m$



- Because currents flow in opposite directions, total  $L$  is *reduced* by  $L_m$



The equivalent circuit consists of two inductors in series. The top inductor has inductance  $L$  and current  $I$  flowing to the right. The bottom inductor has inductance  $L_m$  and current  $-I$  flowing to the right. The total voltage across the series combination is  $V$ .

$$V = L \frac{dI}{dt} + L_m \frac{d(-I)}{dt}$$

$$= (L - L_m) \frac{dI}{dt}$$

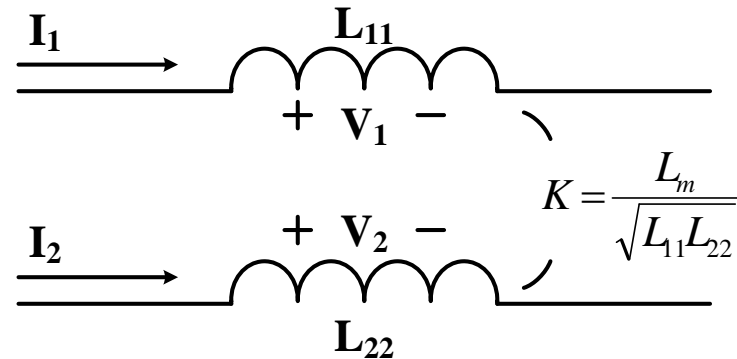
# Derivation of Odd Mode Inductance

Mutual Inductance:

Consider the circuit:

$$V_1 = L_o \frac{dI_1}{dt} + L_m \frac{dI_2}{dt}$$

$$V_2 = L_o \frac{dI_2}{dt} + L_m \frac{dI_1}{dt}$$



Since the signals for odd-mode switching are always opposite,  $I_1 = -I_2$  and  $V_1 = -V_2$ , so that:

$$V_1 = L_o \frac{dI_1}{dt} + L_m \frac{d(-I_1)}{dt} = (L_o - L_m) \frac{dI_1}{dt}$$

$$V_2 = L_o \frac{dI_2}{dt} + L_m \frac{d(-I_2)}{dt} = (L_o - L_m) \frac{dI_2}{dt}$$

Thus, since  $L_o = L_{11} = L_{22}$ ,

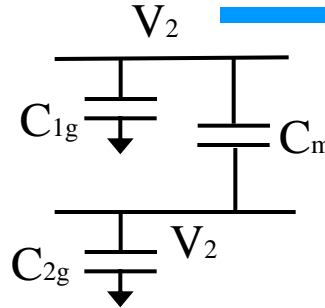
$$L_{odd} = L_{11} - L_m = L_{11} - L_{12}$$

equivalent inductance seen in an odd-mode environment is reduced by mutual inductance.

# Derivation of Odd Mode Capacitance

## Mutual Capacitance:

Consider the circuit:



$$C_{1g} = C_{2g} = C_o = C_{11} - |C_{12}|$$

$$\text{So, } I_1 = C_o \frac{dV_1}{dt} + C_m \frac{d(V_1 - V_2)}{dt} = (C_o + C_m) \frac{dV_1}{dt} - C_m \frac{dV_2}{dt}$$

$$I_2 = C_o \frac{dV_2}{dt} + C_m \frac{d(V_2 - V_1)}{dt} = (C_o + C_m) \frac{dV_2}{dt} - C_m \frac{dV_1}{dt}$$

And again,  $I_1 = -I_2$  and  $V_1 = -V_2$ , so that:

$$I_1 = C_o \frac{dV_1}{dt} + C_m \frac{d(V_1 - (-V_1))}{dt} = (C_{1g} + 2C_m) \frac{dV_1}{dt}$$

$$I_2 = C_o \frac{dV_2}{dt} + C_m \frac{d(V_2 - (-V_2))}{dt} = (C_o + 2C_m) \frac{dV_2}{dt}$$

Thus,

$$C_{odd} = C_{1g} + 2C_m = C_{11} + C_m$$

eq. capacitance for odd mode switching increases.

# *Odd Mode Transmission Characteristics*

Impedance:

$$Z_{odd} = \sqrt{\frac{L_{odd}}{C_{odd}}} = \sqrt{\frac{L_{11} - L_{12}}{C_{11} + |C_{12}|}}$$

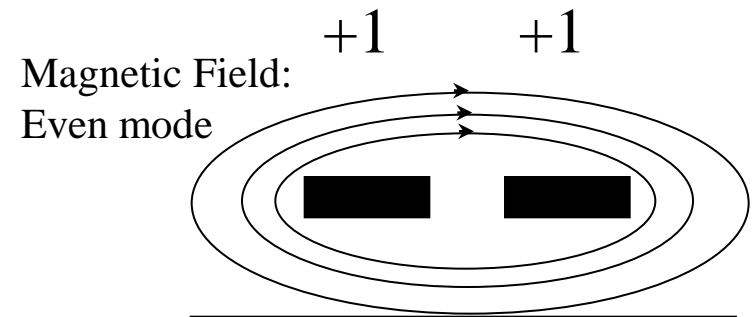
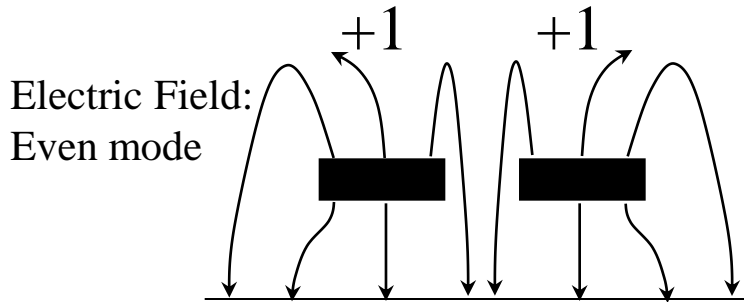
Note:  $Z_{differential} = 2Z_{odd}$  Explain why.

Propagation Delay:

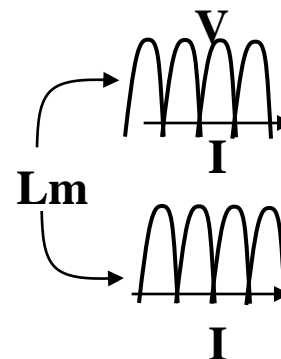
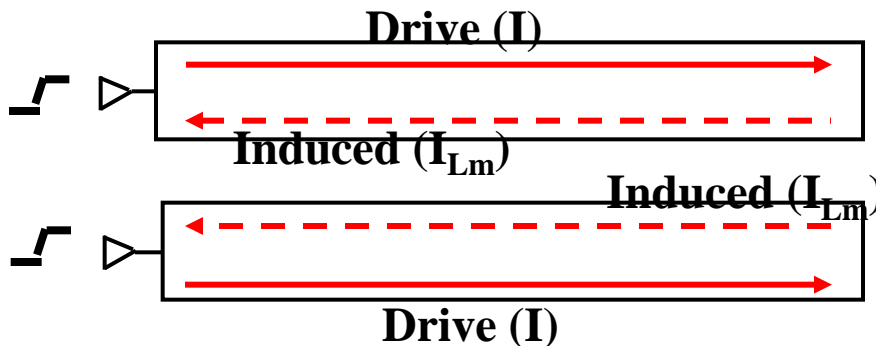
$$TD_{odd} = \sqrt{L_{odd} C_{odd}} = \sqrt{(L_{11} - L_{12})(C_{11} + |C_{12}|)}$$

# Even Mode Transmission

- Since conductors are at equal potential, effective  $C$  is *reduced* by  $C_m$



- Because currents flow in same direction, total  $L$  is *increased* by  $L_m$



$$V = L \frac{dI}{dt} + L_m \frac{d(I)}{dt}$$

$$= (L + L_m) \frac{dI}{dt}$$

# Even Mode Transmission Characteristics

Impedance:

$$Z_{even} = \sqrt{\frac{L_{even}}{C_{even}}} = \sqrt{\frac{L_{11} + L_{12}}{C_{11} - |C_{12}|}}$$

Note:  $Z_{even} > Z_{odd}$

Propagation Delay:

$$TD_{even} = \sqrt{L_{even} C_{even}} = \sqrt{(L_{11} + L_{12})(C_{11} - |C_{12}|)}$$

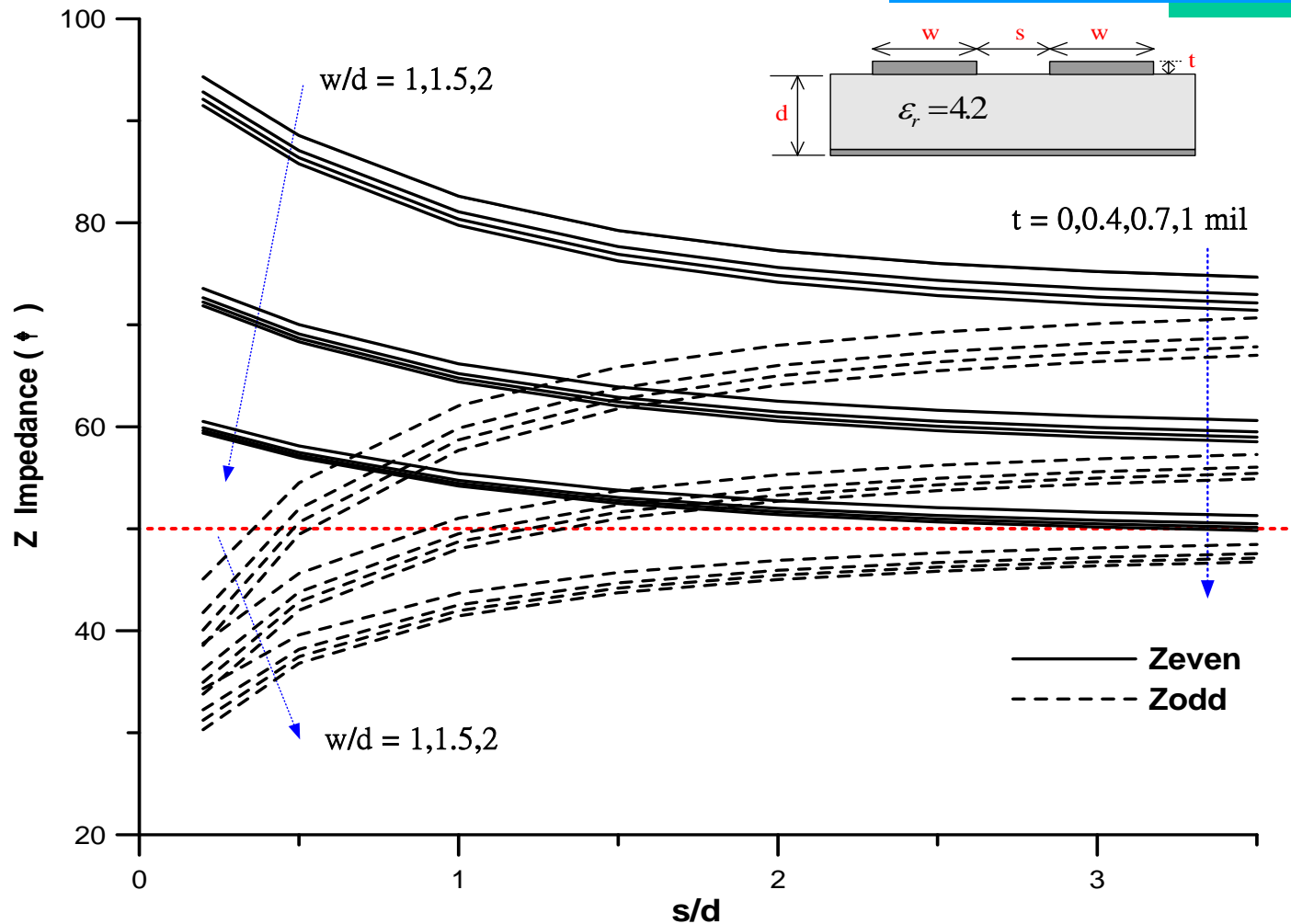
Note:

$TD_{even} > TD_{odd}$  for microstrip lines

$TD_{even} = TD_{odd}$  for strip lines



# Variations in Impedance - microstip

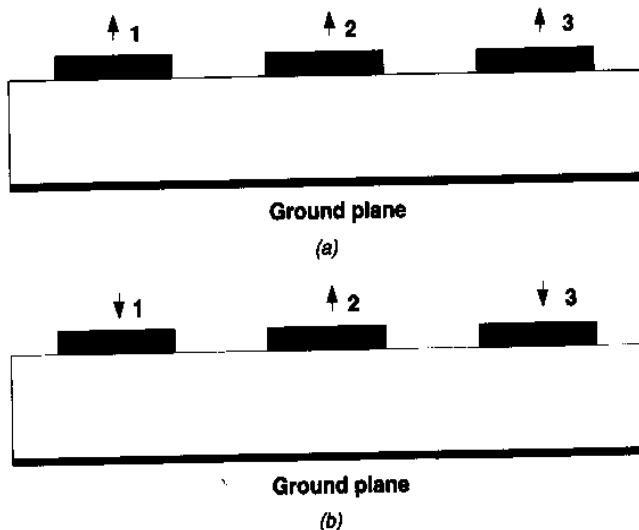


Ref: 吳瑞北、薛光華等，磁碟容錯陣列控制電路板之訊號整合度分析，  
普安案研究計畫報告，民92年2月，第2章。

# Single-Line Eq. Model (SLEM)

- Goal:
  - Determine effective crosstalk-induced impedance & delay variation for a multi-conductor system.
  - Estimate worst-case crosstalk effects during a bus design prior to actual layout

- SLEM



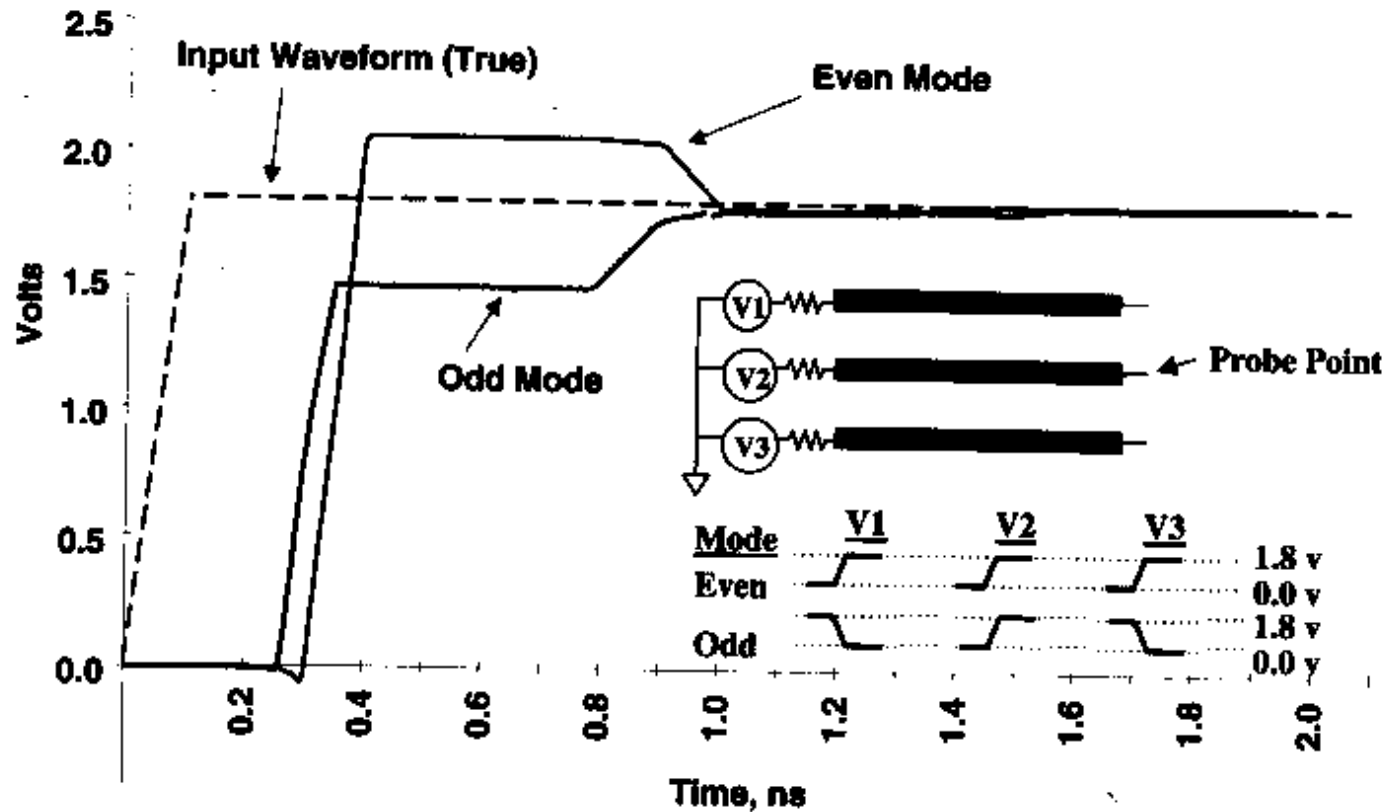
$$Z_{2,\text{eff.}} = \sqrt{\frac{L_{22} + L_{21} + L_{23}}{C_{22} - |C_{12}| - |C_{23}|}};$$

$$TD_{2,\text{eff.}} = \sqrt{(L_{22} + L_{21} + L_{23}) \cdot (C_{22} - |C_{12}| - |C_{23}|)} \cdot \ell$$

$$Z_{2,\text{eff.}} = \sqrt{\frac{L_{22} - L_{21} - L_{23}}{C_{22} + |C_{12}| + |C_{23}|}};$$

$$TD_{2,\text{eff.}} = \sqrt{(L_{22} - L_{21} - L_{23}) \cdot (C_{22} + |C_{12}| + |C_{23}|)} \cdot \ell$$

# Effects on SI & Velocity – on microstrip



**Rem:** No velocity variations due to crosstalk in striplines.

# Modal Analysis

# Model Decomposition Theory

- For coupled lines, 
$$\frac{\partial}{\partial z} \mathbf{v} = -\mathbf{L} \frac{\partial}{\partial t} \mathbf{i}; \quad \frac{\partial}{\partial z} \mathbf{i} = -\mathbf{C} \frac{\partial}{\partial t} \mathbf{v}$$

Transformation:  $\mathbf{v} = \mathbf{M}_V \mathbf{v}_m, \quad \mathbf{i} = \mathbf{M}_I \mathbf{i}_m;$

$$\frac{\partial}{\partial z} \mathbf{M}_V \mathbf{v}_m = -\mathbf{L} \frac{\partial}{\partial t} \mathbf{M}_I \mathbf{i}_m \Rightarrow \frac{\partial}{\partial z} \mathbf{v}_m = -\mathbf{L}_m \frac{\partial}{\partial t} \mathbf{i}_m; \quad \mathbf{L}_m = \mathbf{M}_V^{-1} \mathbf{L} \mathbf{M}_I$$

$$\frac{\partial}{\partial z} \mathbf{M}_I \mathbf{i}_m = -\mathbf{C} \frac{\partial}{\partial t} \mathbf{M}_V \mathbf{v}_m \Rightarrow \frac{\partial}{\partial z} \mathbf{i}_m = -\mathbf{C}_m \frac{\partial}{\partial t} \mathbf{v}_m; \quad \mathbf{C}_m = \mathbf{M}_I^{-1} \mathbf{C} \mathbf{M}_V$$

$\mathbf{L}_m, \mathbf{C}_m$  both diagonal;

$\therefore \mathbf{L}_m \mathbf{C}_m = \mathbf{M}_V^{-1} (\mathbf{L} \mathbf{C}) \mathbf{M}_V \Rightarrow \mathbf{M}_V$  is eigenmatrix of  $\mathbf{L} \mathbf{C}$

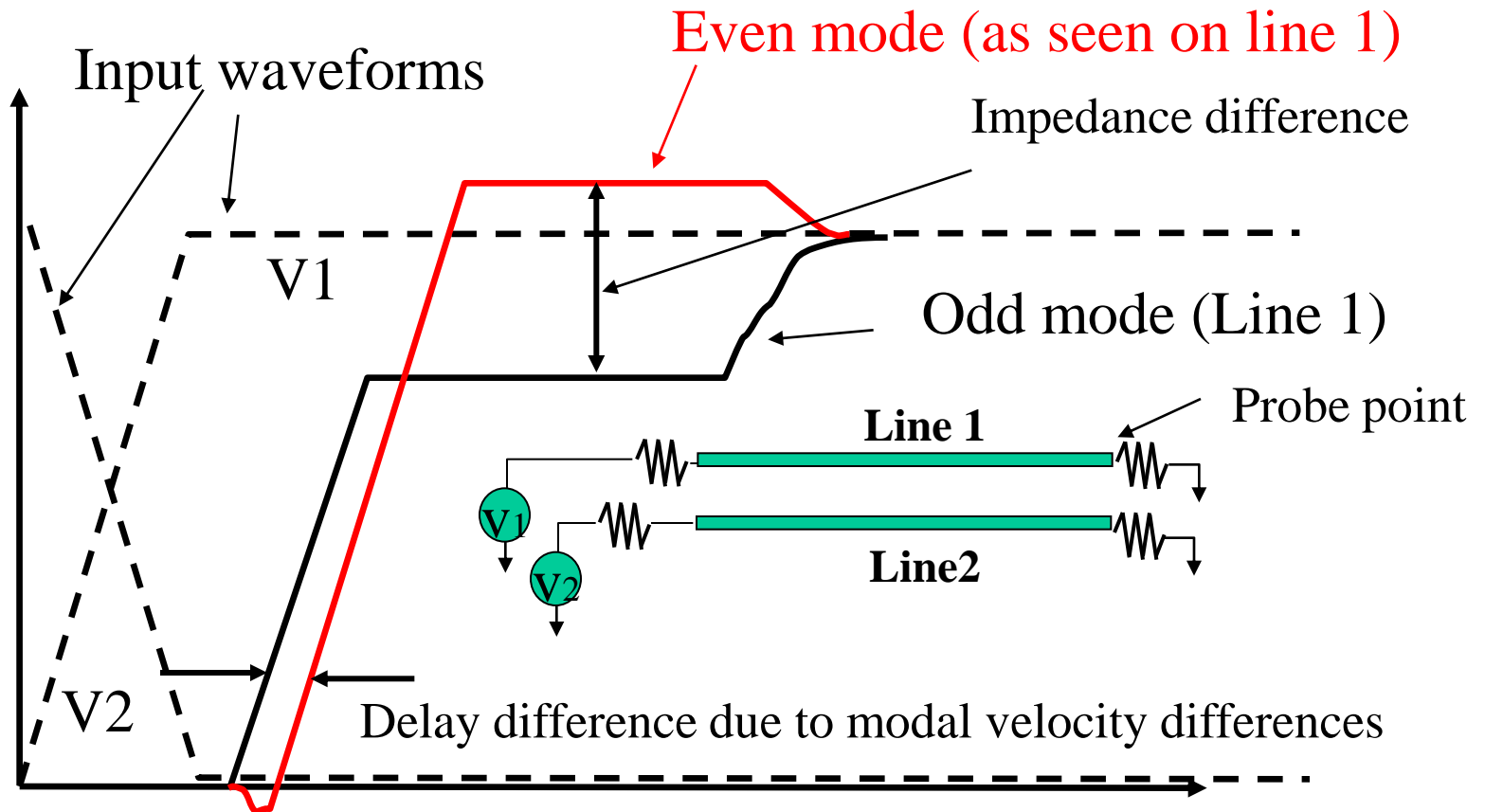
modal impedance  $Z_{m,i} = \sqrt{L_{m,ii} / C_{m,ii}};$

velocity  $v_{pm,i} = 1 / \sqrt{L_{m,ii} \cdot C_{m,ii}}$

# Modal Analysis Procedure

- Find  $\mathbf{M}_V$ , eigenvectors of LC (fails if LC is diagonal)
- Find  $\mathbf{M}_I$ , eigenvectors of CL,
  - usually take  $\mathbf{M}_I = (\mathbf{M}_V^{-1})^T$  s.t.  $\mathbf{v}^T \mathbf{i} = \mathbf{v}_m^T \mathbf{M}_V^T \mathbf{M}_I \mathbf{i}_m = \mathbf{v}_m^T \mathbf{i}_m$
- Use  $\mathbf{M}_V$  and  $\mathbf{M}_I$  to calculate modal inductance, capacitance, voltages, and currents
- Calculate modal impedances and velocities
- Carry out traditional tx-line analysis for each mode
- Convert modal quantities back into line quantities.

# Odd/Even Mode Comparison for Coupled Microstrips



# Symmetric Coupled Lines

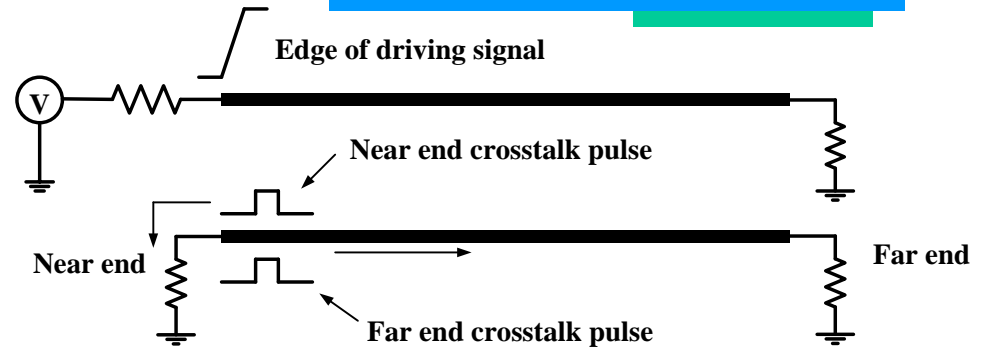
$$[L] = L_{11} \begin{bmatrix} 1 & k_L \\ k_L & 1 \end{bmatrix}$$

$$[C] = C_{11} \begin{bmatrix} 1 & -k_C \\ -k_C & 1 \end{bmatrix}$$

$$\Downarrow \mathbf{M}_V = \frac{1}{\sqrt{2}} \begin{bmatrix} 1 & 1 \\ -1 & 1 \end{bmatrix}; \quad \mathbf{M}_I = \frac{1}{\sqrt{2}} \begin{bmatrix} 1 & 1 \\ -1 & 1 \end{bmatrix}$$

$$\begin{aligned} \mathbf{L}_m &= \mathbf{M}_V^{-1} \mathbf{L} \mathbf{M}_I \\ &= L_{11} \begin{bmatrix} 1-k_L & 0 \\ 0 & 1+k_L \end{bmatrix}; \end{aligned}$$

$$\begin{aligned} \mathbf{C}_m &= \mathbf{M}_I^{-1} \mathbf{C} \mathbf{M}_V \\ &= C_{11} \begin{bmatrix} 1+k_C & 0 \\ 0 & 1-k_C \end{bmatrix} \end{aligned}$$



modal characteristics: (1: odd; 2: even)

$$Z_{m,1} = Z_0 \sqrt{\frac{1-k_L}{1+k_C}}; \quad v_{m,1} = v_0 / \sqrt{(1-k_L)(1+k_C)}$$

$$Z_{m,2} = Z_0 \sqrt{\frac{1+k_L}{1-k_C}}; \quad v_{m,2} = v_0 / \sqrt{(1+k_L)(1-k_C)};$$

$$\text{where } Z_0 = \sqrt{\frac{L_{11}}{C_{11}}}; \quad v_0 = 1 / \sqrt{L_{11} C_{11}}$$

note:  $Z_{m,1} < Z_0 < Z_{m,2}$  R. B. Wu 43

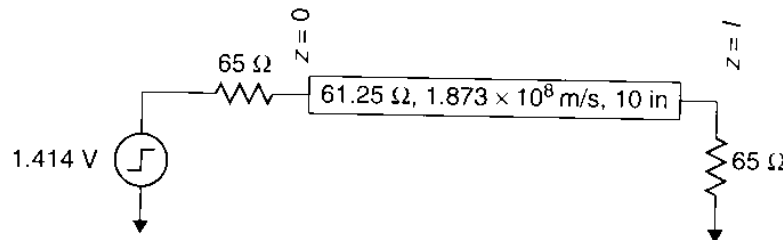


# Equivalent Tx-Lines for Modes

- Modal decomposition

$$\mathbf{v}_s = \begin{bmatrix} 2 \\ 0 \end{bmatrix} u(t) = \left( \begin{bmatrix} 1 \\ -1 \end{bmatrix} + \begin{bmatrix} 1 \\ 1 \end{bmatrix} \right) u(t) \Leftrightarrow \mathbf{v}_{m,s} = \mathbf{M}_V^{-1} \mathbf{v}_s = \left( \begin{bmatrix} \sqrt{2} \\ 0 \end{bmatrix} + \begin{bmatrix} 0 \\ \sqrt{2} \end{bmatrix} \right) u(t)$$

- Equivalent tx-line



$$\Gamma_{L,i} \equiv \frac{Z_L - Z_{m,i}}{Z_L + Z_{m,i}}; \quad \Gamma_{s,i} \equiv \frac{Z_s - Z_{m,i}}{Z_s + Z_{m,i}}$$

- Modal voltages at both ends:

$$\text{near end: } v_{m,i}(0, t) = \frac{Z_{m,i}}{Z_{m,i} + Z_s} \cdot \left[ v_{m,in}(t) + \Gamma_{L,i} \cdot (1 + \Gamma_{s,i}) v_{m,in}\left(t - \frac{2\ell}{v_{m,i}}\right) \right];$$

$$\text{far end: } v_{m,i}(\ell, t) = \frac{Z_{m,i}}{Z_{m,i} + Z_s} \cdot (1 + \Gamma_{L,i}) \cdot v_{m,in}\left(t - \frac{\ell}{v_{m,i}}\right);$$

# Line Voltages at Both Ends

- Line voltages at both ends

$$\mathbf{v} = \mathbf{M}_V \mathbf{v}_m \text{ at } z=0 \text{ or } \ell \Rightarrow \begin{bmatrix} v_1(t) \\ v_2(t) \end{bmatrix} (t) = \frac{1}{\sqrt{2}} \begin{bmatrix} 1 & 1 \\ -1 & 1 \end{bmatrix} \begin{bmatrix} v_{m,1}(t) \\ v_{m,2}(t) \end{bmatrix}$$

- Near end crosstalk

$$v_2(0, t) = \left( \frac{Z_{m,2}}{Z_{m,2} + Z_s} - \frac{Z_{m,1}}{Z_{m,1} + Z_s} \right) u(t) \quad \text{for } 0 < t < 2T$$

$$+ \frac{Z_{m,2} \Gamma_{L,2} (1 + \Gamma_{s,2})}{Z_{m,2} + Z_s} \cdot u\left(t - \frac{2\ell}{v_{m,2}}\right) - \frac{Z_{m,1} \Gamma_{L,1} (1 + \Gamma_{s,1})}{Z_{m,1} + Z_s} \cdot u\left(t - \frac{2\ell}{v_{m,1}}\right)$$

- Far end crosstalk

$$v_2(\ell, t) = \frac{2Z_L Z_{m,2} \cdot u\left(t - \frac{\ell}{v_{m,2}}\right)}{(Z_{m,2} + Z_s)(Z_{m,2} + Z_L)} - \frac{2Z_L Z_{m,1} \cdot u\left(t - \frac{\ell}{v_{m,1}}\right)}{(Z_{m,1} + Z_s)(Z_{m,1} + Z_L)} \quad \text{for } T < t < 3T$$

# Exact Crosstalk at Matched Load

- Zero far end noise  $\implies$

(i)  $v_{m,1} = v_{m,2} \iff k_C = k_L$

$$Z_{eq} = \sqrt{Z_{m,1}Z_{m,2}} \cong Z_0$$

(ii)  $Z_{m,1}Z_{m,2} = Z_s Z_L \implies$  choose  $Z_s = Z_L = Z_{eq}$

- Crosstalk noise (with matched load)

$$v_2(\ell, t) = \frac{2Z_{eq}}{\left(\sqrt{Z_{m,2}} + \sqrt{Z_{m,1}}\right)^2} \cdot \left[ u\left(t - \frac{\ell}{v_{m,2}}\right) - u\left(t - \frac{\ell}{v_{m,1}}\right) \right]$$

$$\approx \frac{1}{2} \left[ u\left(t - \frac{\ell}{v_{m,2}}\right) - u\left(t - \frac{\ell}{v_{m,1}}\right) \right]$$

$$v_2(0, t) = \frac{\sqrt{Z_{m,2}} - \sqrt{Z_{m,1}}}{\sqrt{Z_{m,2}} + \sqrt{Z_{m,1}}} \cdot \left[ u(t) - \frac{2Z_{eq}}{\left(\sqrt{Z_{m,2}} + \sqrt{Z_{m,1}}\right)^2} \cdot \left\{ u\left(t - \frac{2\ell}{v_{m,2}}\right) + u\left(t - \frac{2\ell}{v_{m,1}}\right) \right\} \right]$$

$$\cong \frac{k_L + k_C}{4} \cdot \left[ u(t) - u\left(t - \frac{2\ell}{v_0}\right) \right]$$

# Far End Crosstalk Noise – Short Line

$$v_2(\ell, t) \approx \frac{1}{2} \left[ u\left(t - \frac{\ell}{v_{m,2}}\right) - u\left(t - \frac{\ell}{v_{m,1}}\right) \right]$$

$$\begin{aligned} \Delta\tau &\equiv \tau_{\text{even}} - \tau_{\text{odd}} = \frac{\ell}{v_{m,2}} - \frac{\ell}{v_{m,1}} \\ &= \frac{\ell}{v_0} \left( \sqrt{(1+k_L)(1-k_C)} - \sqrt{(1-k_L)(1+k_C)} \right) \\ &\cong \frac{\ell}{v_0} \cdot (k_L - k_C) \end{aligned}$$

- Case 1:  $|\Delta\tau| \ll t_r$ ;

$$\begin{aligned} v_2(\ell, t) &\approx \frac{1}{2} u'\left(t - \frac{\ell}{v_0}\right) \left[ \left(t - \frac{\ell}{v_{m,2}}\right) - \left(t - \frac{\ell}{v_{m,1}}\right) \right] \\ &\cong -\frac{\Delta\tau}{2} u'\left(t - \frac{\ell}{v_0}\right) = -\frac{\ell(k_L - k_C)}{2v_0} u'\left(t - \frac{\ell}{v_0}\right) \end{aligned}$$

a pulse of value  $-\frac{k_L - k_C}{2} \cdot \frac{T}{t_r} \cdot u_s$ , width  $t_r$

# *Far End Crosstalk Noise – Long Line*

$$v_2(\ell, t) \approx \frac{1}{2} \left[ u\left(t - \frac{\ell}{v_{m,2}}\right) - u\left(t - \frac{\ell}{v_{m,1}}\right) \right]$$

$$\Delta\tau \equiv \tau_{\text{even}} - \tau_{\text{odd}} = \frac{\ell}{v_{m,2}} - \frac{\ell}{v_{m,1}} \approx \frac{\ell}{v_0} \cdot (k_L - k_C)$$

- Case 2:  $|\Delta\tau| > t_r$   
(long line or short risetime)

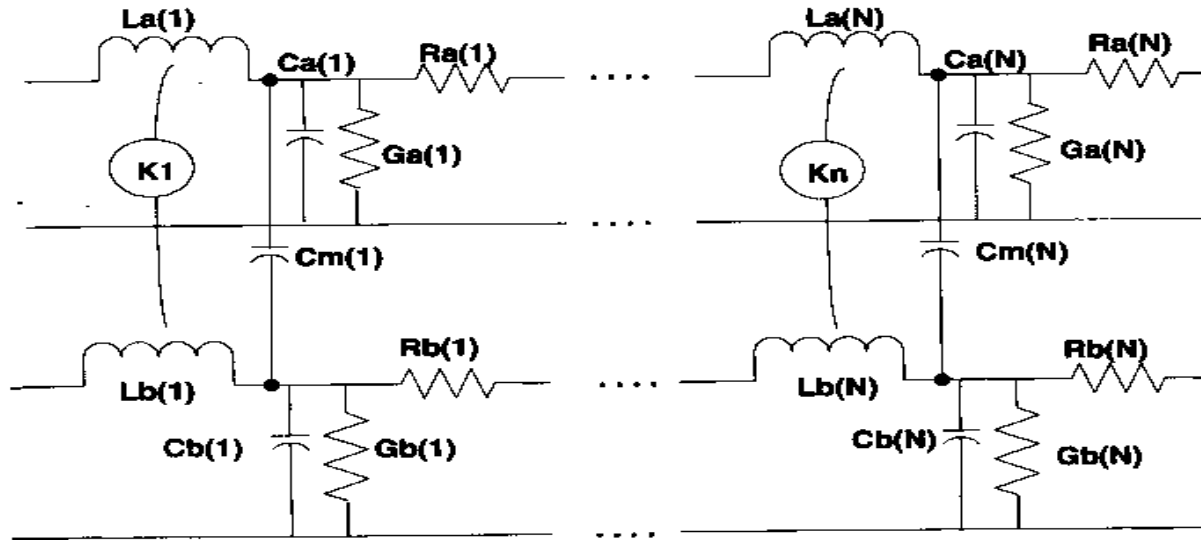
$v_2(\ell, t)$ : a pulse of value  $\pm \frac{1}{2} u_s$   
and width  $|\Delta\tau|$

# Remarks

- NEN occurs because odd (even) mode has smaller (larger) launched voltage and positive (negative) reflected wave.
- FEN occurs because even and odd modes arrives at a different time.
- Under matched load, modal analysis yields same results with weakly coupling analysis at near end.
- At far end, weakly coupling analysis fails for long line or shorter risetime, while modal analysis is still applicable and yield correct results.
- Modal analysis can apply to more complicated cases, i.e., asymmetric, multiple, lossy coupled lines, etc.

# Simulation in SPICE

# *Eq-ckt Model in SPICE (1)*



Ex. : length = 5in,  $t_r = 100\text{ps}$

$$C = \begin{bmatrix} 2.1 & -0.1 \\ -0.1 & 2.1 \end{bmatrix} (\text{pF/in}); L = \begin{bmatrix} 9 & 0.7 \\ 0.7 & 9 \end{bmatrix} (\text{nH/in})$$

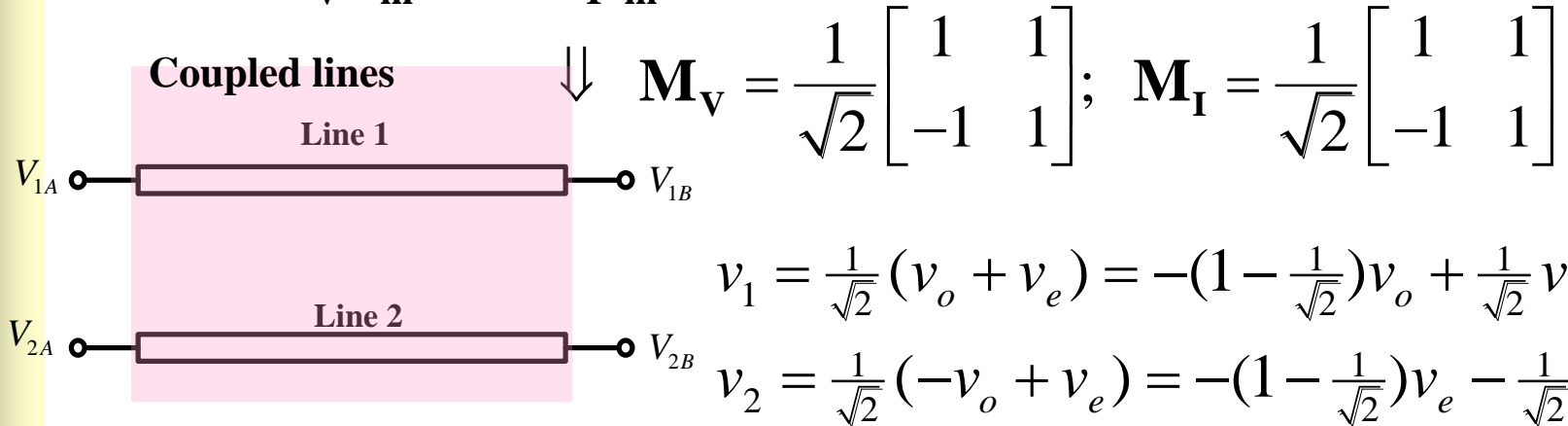
$$\text{TD} = \sqrt{L_{11}C_{11}} = 134\text{ps/in} \rightarrow 670\text{ps}$$

$$\# \text{ segments} \geq 10 \left( \frac{\text{TD}}{t_r} \right) = 67 \rightarrow \# \text{ SPICE elements} \geq 67 * 2 * 3$$

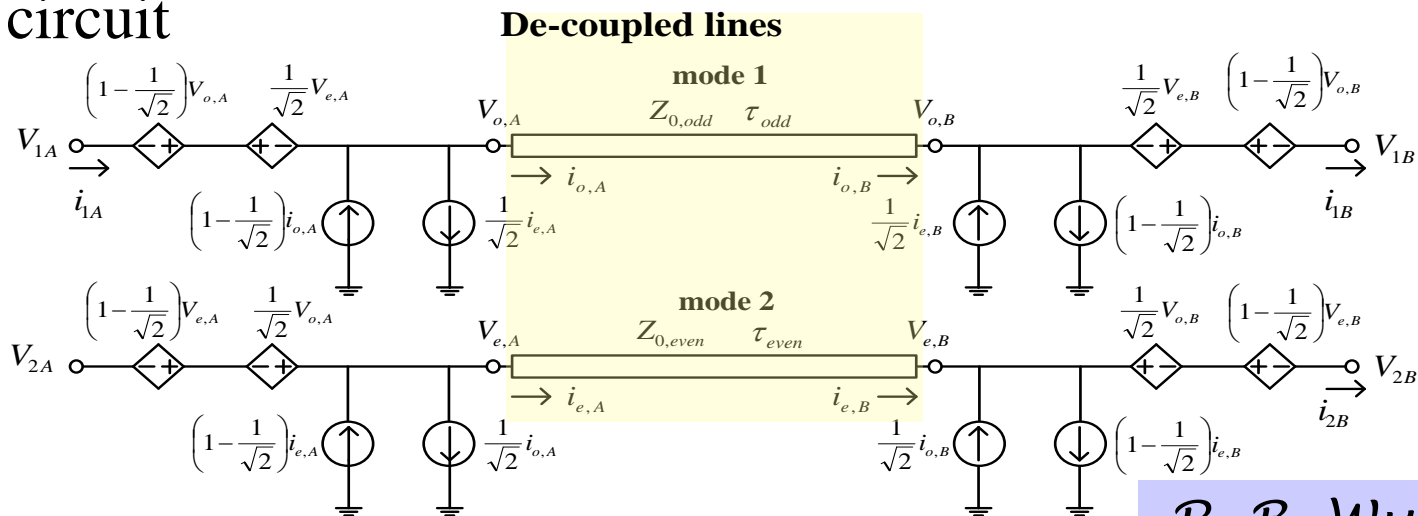


# Eq. ckt by Even/Odd Decomposition

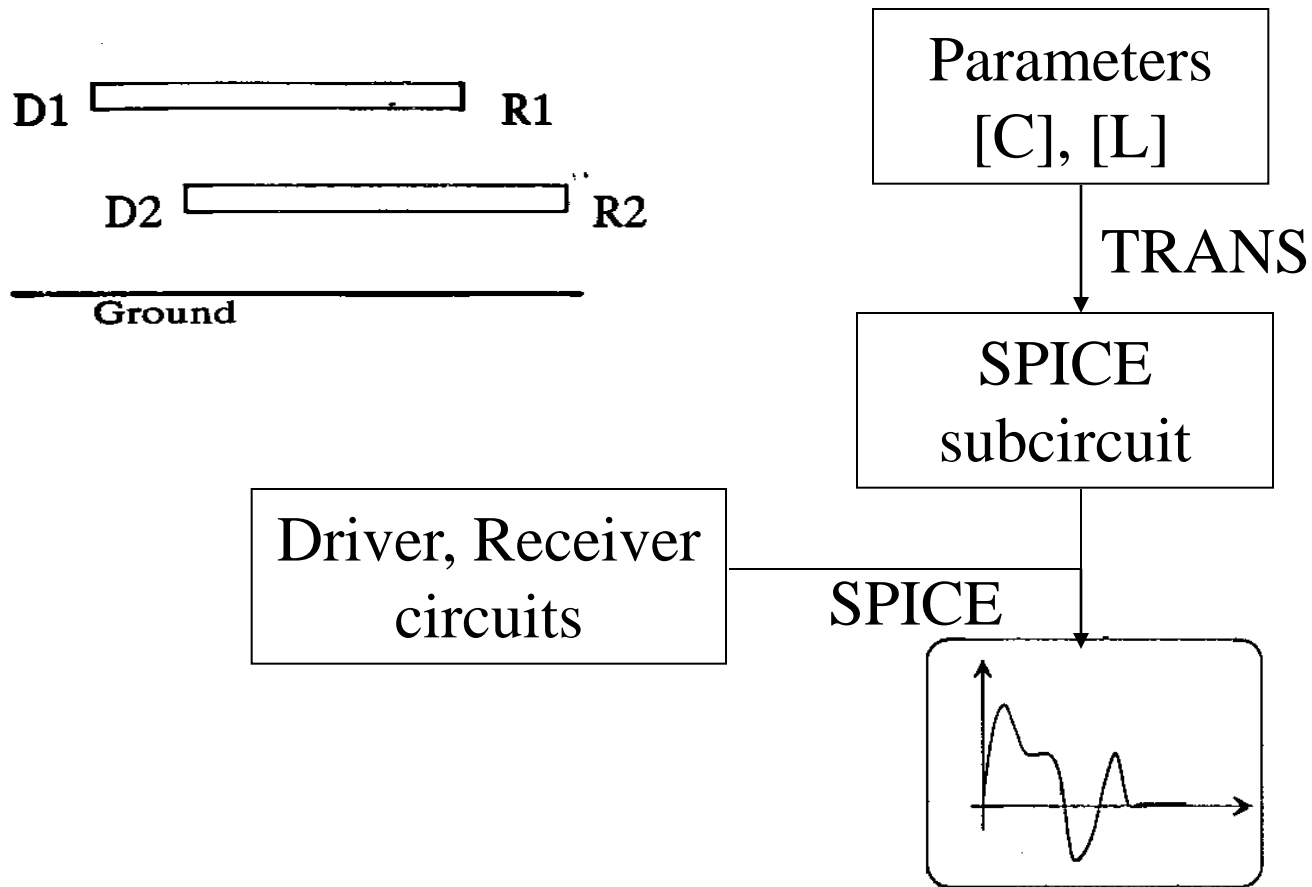
$$\mathbf{v} = \mathbf{M}_V \mathbf{v}_m, \quad \mathbf{i} = \mathbf{M}_I \mathbf{i}_m;$$



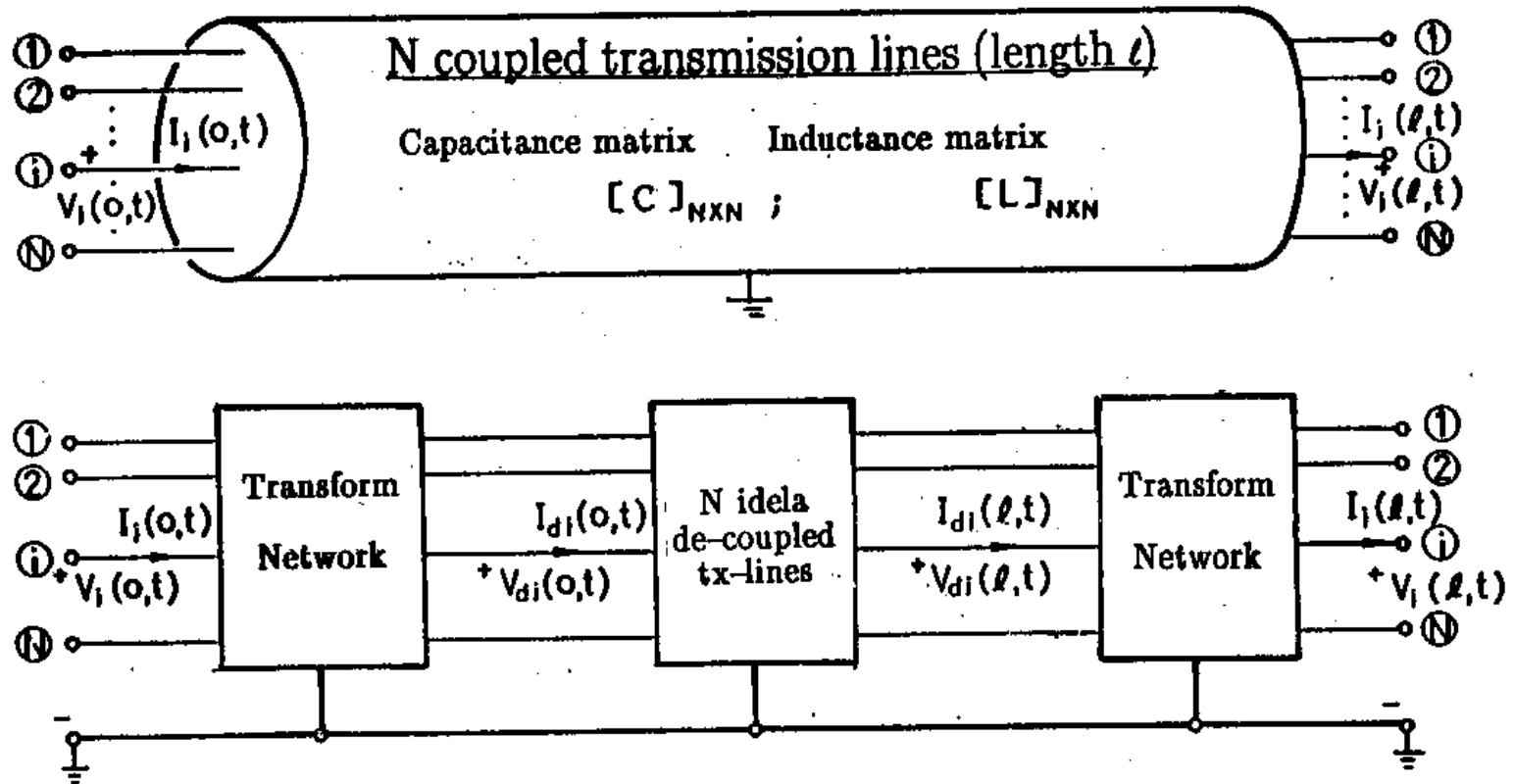
Eq. circuit



# Crosstalk Simulation in SPICE



# Equivalent SPICE Subcircuit (2)

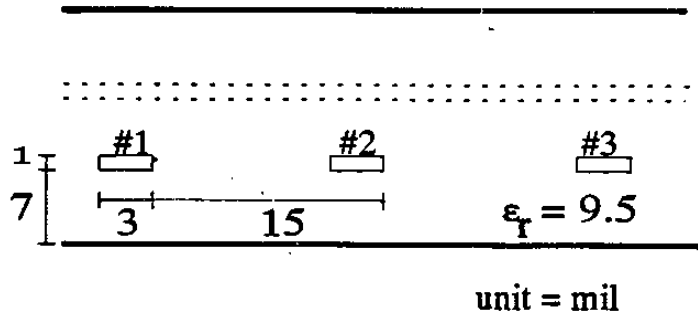


F. Romeo and M. Santomauro, "Time-domain simulation of n coupled transmission lines,"

*IEEE Trans. Microwave Theory Tech.*, vol. 35, pp. 131-136, Feb. 1987.

R. B. Wu

# TRANS Example

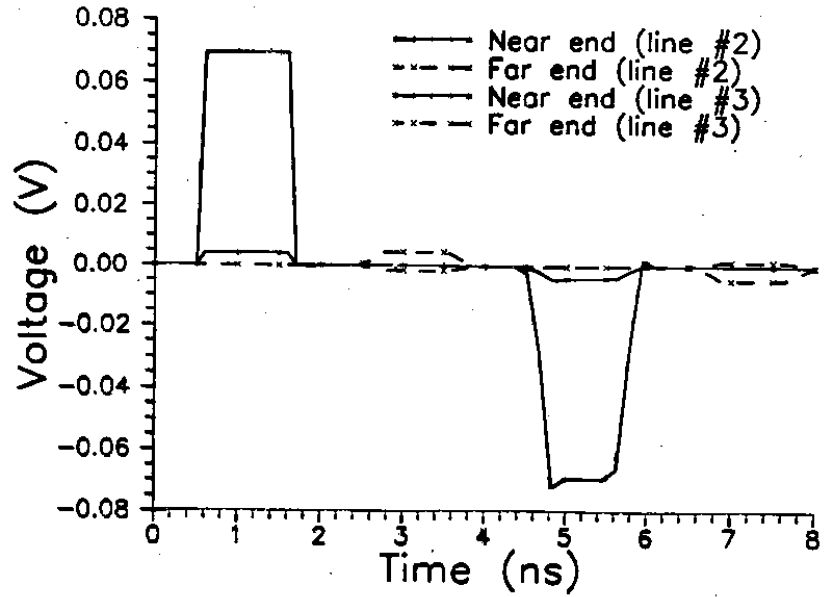
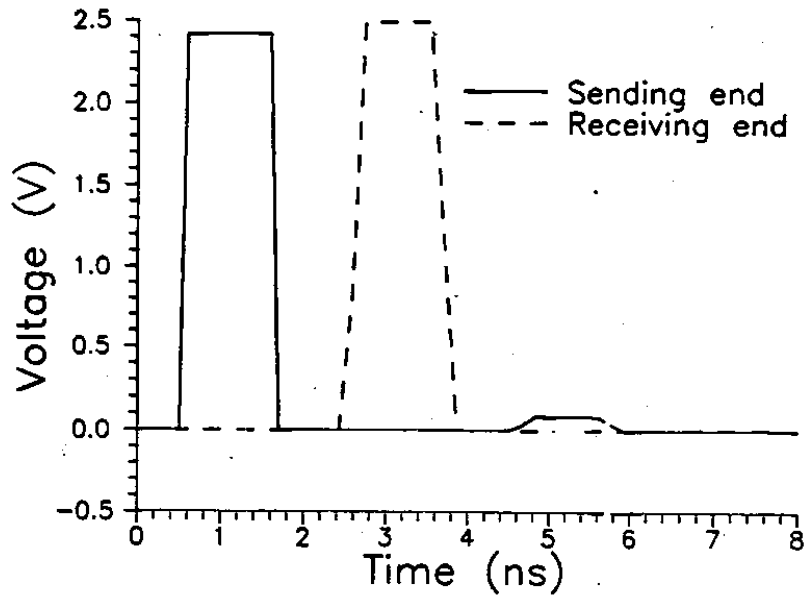
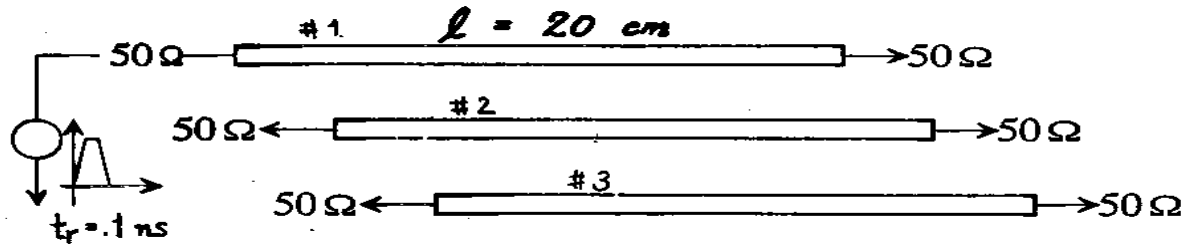


$$[C] = \begin{bmatrix} 219.02 & -12.19 & -0.365 \\ -12.19 & 219.76 & -12.19 \\ -0.365 & -12.19 & 219.02 \end{bmatrix} \text{ pF/m;}$$

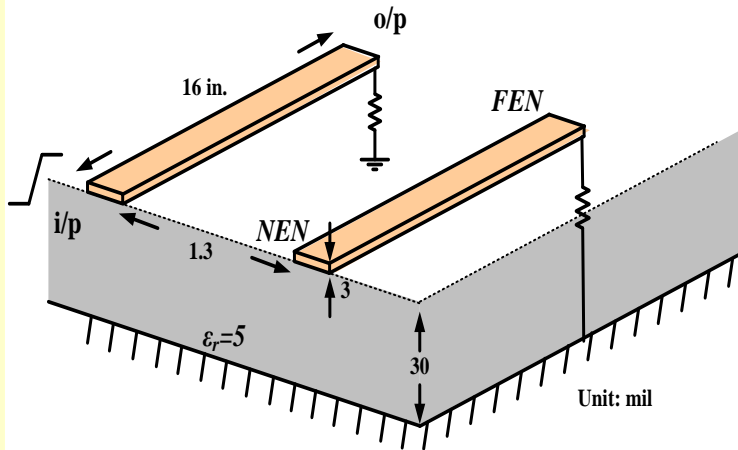
$$[L] = \begin{bmatrix} 479.03 & 26.70 & 2.28 \\ 26.70 & 478.88 & 26.70 \\ 2.28 & 26.70 & 479.03 \end{bmatrix} \text{ nH/m}$$

```
.SUBCKT X33 SF1 SB1 SF2 SB2 SF3 SB3  PARAMS: LENGTH=100
* three ideal de-coupled transmission lines for normal modes
de-coupled tx-line {
T1 TF1 0 TB1 0  ZO=46.62  TD={10.277NS*LENGTH}
T2 TF2 0 TB2 0  ZO=49.01  TD={10.277NS*LENGTH}
T3 TF3 0 TB3 0  ZO=44.88  TD={10.277NS*LENGTH}
}
* three voltage controlled voltage source at sending end
voltage-controlled voltage source {
EF1 SF1 MF1 POLY(3) TF1 0 TF2 0 TF3 0 0.0 -0.2929 -0.6727 -0.1736
EF2 SF2 MF2 POLY(3) TF1 0 TF2 0 TF3 0 0.0 0.0000 -1.3082 0.9694
EF3 SF3 MF3 POLY(3) TF1 0 TF2 0 TF3 0 0.0 -0.7071 -0.6727 -1.1736
}
* three voltage controlled voltage source at receiving end
EB1 SB1 MB1 POLY(3) TB1 0 TB2 0 TB3 0 0.0 -0.2929 -0.6727 -0.1736
EB2 SB2 MB2 POLY(3) TB1 0 TB2 0 TB3 0 0.0 0.0000 -1.3082 0.9694
EB3 SB3 MB3 POLY(3) TB1 0 TB2 0 TB3 0 0.0 -0.7071 -0.6727 -1.1736
}
* three current controlled current source at sending end
current-controlled current source {
FF1 MF1 0 POLY(3) VF1 VF2 VF3 0.0 -0.2929 -0.6869 -0.2184
FF2 MF2 0 POLY(3) VF1 VF2 VF3 0.0 0.0000 -1.2460 0.9534
FF3 MF3 0 POLY(3) VF1 VF2 VF3 0.0 -0.7071 -0.6869 -1.2184
VF1 MF1 TF1 0
VF2 MF2 TF2 0
VF3 MF3 TF3 0
}
* three current controlled current source at receiving end
FB1 MB1 0 POLY(3) VB1 VB2 VB3 0.0 -0.2929 -0.6869 -0.2184
FB2 MB2 0 POLY(3) VB1 VB2 VB3 0.0 0.0000 -1.2460 0.9534
FB3 MB3 0 POLY(3) VB1 VB2 VB3 0.0 -0.7071 -0.6869 -1.2184
VB1 MB1 TB1 0
VB2 MB2 TB2 0
VB3 MB3 TB3 0
.ENDS
```

# Simulation Results

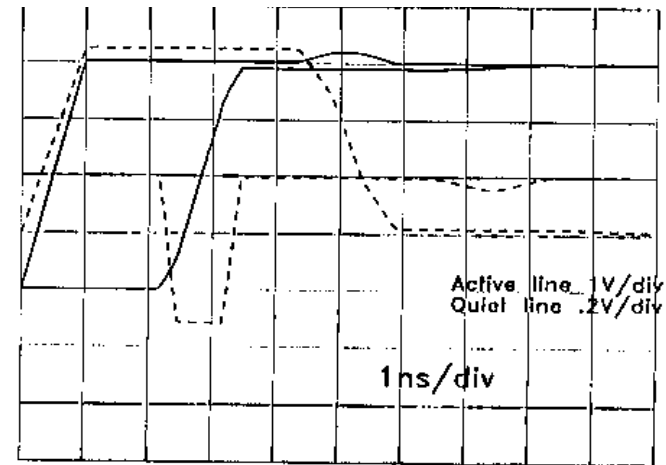
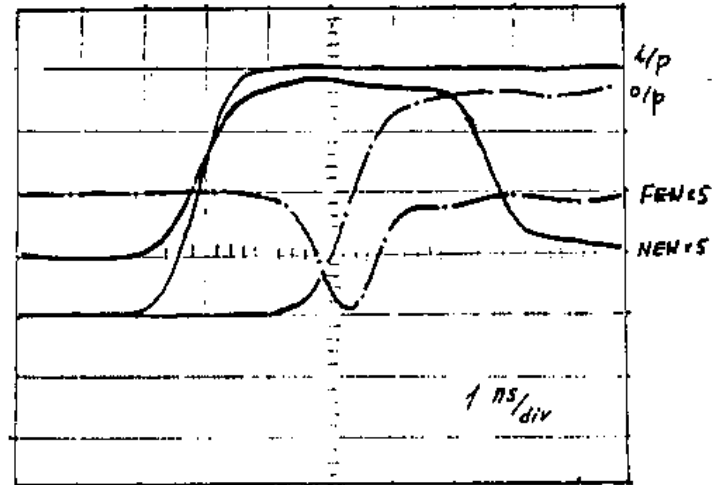


# Measurement and Simulation



$$[C] = \begin{bmatrix} 62.1 & -16.3 \\ -16.3 & 62.1 \end{bmatrix} \text{pF/m}$$

$$[L] = \begin{bmatrix} 593 & 217 \\ 217 & 593 \end{bmatrix} \text{nH/m}$$



# Further Reading -1

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- F. Romeo et al., “Time-domain simulation of n coupled transmission lines,” *IEEE T-MTT*, vol. 35, pp. 131-136, Feb. 1987.
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# Further Reading -2

- J. Bernal, et al., “Crosstalk in coupled microstrip lines with a top cover,” *IEEE T-EMC*, vol. 56, pp. 375-384, Apr. 2014.
- G.-H. Shiue, et al., “Ground bounce noise induced by crosstalk noise for two parallel ground planes with a narrow open-stub line and adjacent signal traces in multilayer package structure,” *IEEE T-CPMT*, vol. 4, pp. 870-881, May 2014.
- M. S. Halligan and D. G. Beetner, “Maximum crosstalk estimation in weakly coupled transmission lines”, *IEEE T-EMC*, vol. 56, pp. 736-744, Jun. 2014.
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- F. Grassi, et al., “On mode conversion in geometrically unbalanced differential lines and its analogy with crosstalk,” *IEEE T-CPMT*, vol. 57, pp. 283-291, April 2015.



# Further Reading -3

- G. Li, et al., “Measurement-based modeling and worst-case estimation of crosstalk inside an aircraft cable connector,” *IEEE T-EMC*, vol.57, pp.827-835, Aug. 2015.
- R. Araneo, et al., “Modal propagation and crosstalk analysis in coupled graphene nano-ribbons,” *IEEE T-EMC*, vol. 57, pp. 726-733, Aug. 2015.
- A. Chada, et al., “Crosstalk impact of periodic coupled routing on eye opening of high speed links in PCBs,” *IEEE T-EMC*, vol. 57, pp. 1676-1689, Oct. 2015.
- V. Ramesh Kumar, et al., “An unconditionally stable FDTD model for crosstalk analysis of VLSI interconnects,” *IEEE T-CPMT*, vol. 5, pp. 1810-1817, Dec. 2015.
- B. R. Huang, et al., “Far-end crosstalk noise reduction using decoupling capacitor,” *IEEE T-EMC*, vol. 58, pp. 836-848, June 2016.
- F. Grassi, et al., "Crosstalk and mode conversion in adjacent differential lines," *IEEE T-EMC*, vol. 58, pp. 877-886, June 2016.