# Highly-directive planar leaky-wave antennas: a comparison between metamaterial-based and conventional designs

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Abstract - A comparative study is made of two types of planar leaky-wave antennas. The first type is a "conventional" planar leaky-wave antenna composed of a grounded slab that is covered with a metallic or dielectric partially-reflecting surface, which acts as a leaky parallel-plate waveguide. The second type is a leaky-wave antenna consisting of a grounded metamaterial layer, having either a very low permittivity or permeability. For either type of structure, directive pencil beams at broadside may be produced when the structure is excited with a simple source such as a horizontal electric or magnetic dipole. A high directivity is obtained by the excitation of weakly-attenuated cylindrical leaky waves that propagate radially outward from the source on the planar structure. The comparison is made for the fundamental antenna properties such as broadside directivity, radiated broadside power density, pattern bandwidth, and the attenuation constants of the relevant leaky modes.

*Index terms* – Leaky-wave antennas, metamaterials, planar antennas, high directivity.

## I. Introduction and background

Leaky-wave antennas (LWAs) based on planar layered structures that can produce directive beams have been investigated in recent years due to their advantages in terms of low cost and simplicity [1-3]. Their operating principle is outlined in the following.

As is well known, leaky waves are natural modes supported by open waveguides, displaying a continuous radiation (*leakage*) of power during their propagation. For one-dimensional wave propagation (e.g., along the x axis), leaky modes are characterized by a complex wave number  $k_x = \beta j \alpha$  where  $\beta$  is the phase constant, and  $\alpha$  is the attenuation (or leakage) constant that takes into account the above-mentioned radiation losses; such one-dimensional modal propagation would be excited by a line source inside the structure, with TE/TM modes excited by an electric/magnetic line source. When this source excites an antenna aperture field dominated by a leaky wave with  $\beta > \alpha$ , a far-field pattern with two maxima is produced, whose pointing angles are given by the approximate formula  $\theta \cong arc$ sin  $(\beta / k_0)$  valid when  $\beta >> \alpha$  [4-6]. The frequency dispersion of the phase constant gives rise to an angular scanning of the main beam by varying the operating frequency. On the other hand, when  $\beta < \alpha$ , a beam with a *single* peak at broadside is radiated [6], whose beam

width is directly proportional to the attenuation constant  $\alpha$  when  $\alpha \ll k_0$ . Highly-directive broadside radiation can thus be expected by exciting a weaklyattenuated leaky wave that dominates the antenna aperture field. In the case of *two-dimensional* wave propagation, cylindrical leaky waves have to be considered, with a complex radial wave number  $k_{\rho} = \beta -j\alpha$  [7]. As shown in [7], by properly exciting a pair of TE and TM cylindrical leaky waves with small and equal values of the phase and attenuation constants, a narrow omnidirectional pencil beam pointing at broadside is obtained. This excitation happens naturally when the structure is optimized for broadside radiation.

Planar LWAs analyzed previously consist of a grounded dielectric slab (Fig. 1, where the adopted coordinate system is shown) covered with a "partially-reflecting surface" (PRS), which may assume various practical forms [8-10]. For example, it may consist of a thin high-permittivity dielectric layer, or a stack of alternating high- and low-permittivity layers [1] (Fig. 2(a)), or a periodic array of metallic patches (Fig. 2(b)), or a conducting plate with a periodic array of slots (Fig. 2(c)) [3]. In each case, the PRS creates a leaky parallel-plate waveguide region between the PRS and the bottom ground plane, in which the leaky modes propagate. A simple source, such as a horizontal infinitesimal dipole inside the substrate, can be used to launch the leaky waves. In the limit of very small leakage, when the PRS becomes a short-circuit plane, the two leaky modes launched by the dipole source become the TM<sub>1</sub> and TE<sub>1</sub> parallel-plate waveguide modes, which have one-half cycle of field variation between the PRS and the bottom ground plane.

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**Fig. 1.** Grounded slab excited by an infinitesimal *y*-directed horizontal electric or magnetic dipole, with the adopted coordinate system and the relevant spherical angles shown.

The advent of metamaterials has opened up new possibilities for the creation of planar LWAs. Metamaterials are artificial materials (typically constructed from a periodic structure) that can be described in certain frequency ranges as homogeneous media with frequencydependent constitutive parameters that can assume very small and even negative values. Progress has already been made in studying both surface and leaky modes supported by planar structures that include metamaterial layers, and in using such materials for antenna applications [11-14].

An interesting example of a metamaterial-based planar LWA for broadside radiation consists of a metamaterial layer with positive and small values of the relative permittivity (or permeability) placed on a ground plane [15], with a source (such as an infinitesimal horizontal dipole or line source) inside (Fig. 2(d)). Such an antenna is capable of producing a narrow beam of radiation at broadside, with the beam becoming more directive as the permittivity (permeability) of the layer decreases. Although the fundamental operating principle has been explained in terms of a lensing effect (ray refraction) [16, 17], recent work has demonstrated that this antenna is fundamentally a leaky-wave antenna [18]. Furthermore, a careful analysis of this structure has allowed for the development of simple approximate formulas to predict the main antenna characteristics such as directivity and pattern bandwidth. An alternative configuration is the one proposed in [19] based on grounded metamaterial bi-layers, which can be designed to support weakly-attenuated leaky waves and is capable of overcoming some of the limitations of the single-layer configurations, e.g., in terms of antenna thickness.

In this work, the goal is to make a comparative study of the two types of LWAs mentioned above: "conventional" LWAs made from a grounded substrate and a PRS, and those using a grounded metamaterial slab (some preliminary results of this study have been presented in [20]). The comparison will be made for the



**Fig. 2.** Examples of planar LWAs for directive radiation at broadside: (a) Stack of N/2 pairs (N = 4 in the figure) of alternating low- and high-permittivity slabs on a ground plane (dielectric PRS). (b) Grounded slab covered with a periodic array of metal patches (metallic PRS). (c) Grounded slab covered with a metal screen that has a periodic array of slots (metallic PRS). (d) Grounded metamaterial slab with a positive and small value of the relative permittivity.

practical quantities of interest, such as physical size, directivity, broadside-power-density enhancement, and pattern bandwidth.

In our analysis we consider an idealized infinitesimal horizontal dipole source since its radiation properties are similar to those of a more realistic feed (the pattern is mainly determined by the structure, and not the feed). Furthermore, the metamaterial considered here is assumed homogeneous since we have already shown in [18] that this assumption does not affect the physics and the qualitative behaviour of the structure. Furthermore, the homogeneous model gives good quantitative agreement for the radiation behaviour, compared with the exact radiation from realistic periodic metamaterial structures (e.g., those made from a periodic arrangement of thin metallic wires), provided the period is small compared with a wavelength. Other physical quantities such as the input impedance would be more affected by the actual nature of the feed and the interaction between the feed and the surrounding metamaterial, and this will be subject of future investigations.

The paper is organized as follows. In Sec. 2, the radiative properties and frequency bandwidth of the conventional and metamaterial LWAs are studied. On the basis of the expressions thus derived for the antenna parameters, suitable figures of merit describing the antenna performance are introduced and discussed in Sec. 3. In Sec. 4, specific PRS-based and metamaterial-based LWAs are considered, providing numerical results for their radiative properties. Finally, in Sec. 5 conclusions are drawn.

# **II.** Analysis of radiation properties and pattern bandwidth

In this section, the radiation properties and pattern bandwidth of the considered LWAs will be studied. Conditions for achieving a maximum broadside power density will be given in Subsections A and B for conventional and metamaterial-based LWAs, respectively, together with expressions for the phase and attenuation constants of the involved TE and TM leaky modes as a function of the antenna structural parameters, as well as formulas for the relevant antenna bandwidths. In Subsection C the broadside directivity of the antennas will be studied.

### A) PRS-based antennas

As shown in [3] and [21],  $TE_z$  and  $TM_z$  waves propagating on planar LWAs based on a grounded slab covered with a lossless metal (Fig. 2(b), (c)) or dielectric (Fig. 2(a)) PRS can effectively be modeled by means of a simple transverse equivalent network, in which the PRS is represented by a shunt susceptance  $B_s$  placed at the interface between air and slab.

A closed-form design formula can be derived for maximizing the broadside power density radiated by a horizontal electric dipole placed inside the slab as [22]

(1) 
$$\cot(k_{zs}h) = \sqrt{\frac{\mu_{r}}{\varepsilon_{r}}}\overline{B}_{s},$$

where  $k_{zs} = k_0 \sqrt{\mu_r \varepsilon_r}$  is the wavenumber inside the slab and  $\overline{B}_s = B_s / B_0$  where  $\overline{B}_s = B_s / B_0$  is the free-space admittance). This requires the slab thickness to be approximately equal to one half of a wavelength inside the slab when the normalized susceptance  $\overline{B}_s$  is much larger than one, i.e., when the leaky parallel-plate waveguide is almost "closed". The electric dipole source is placed in the middle of the slab (i.e., halfway between the ground plane and the PRS) to get the maximum broadside power density. Condition (1) implies the presence of a pair of dominant TE<sub>z</sub> and TM<sub>z</sub> leaky waves with small and nearly equal values of the phase constants  $\beta^{\text{TE/TM}}$  and the attenuation constants  $\alpha^{\text{TE/TM}}$  [6]. In the absence of material losses, these are given by

(2) 
$$\hat{\beta}^{\text{TE/TM}} \simeq \hat{\alpha}^{\text{TE/TM}} \simeq \frac{1}{\overline{B}_{\text{S}}} \sqrt{\frac{\varepsilon_{\text{r}}^{3/2} \mu_{\text{r}}^{1/2}}{\pi}},$$

where the hat indicates normalization with respect to the free-space wavenumber  $k_0$ . Because the TM<sub>z</sub> and TE<sub>z</sub> leaky modes have nearly equal values of complex wavenumber when the condition in Eq. (1) is satisfied, the TE/TM superscript will henceforth be dropped. The maximum broadside power density enhancement E is defined as the ratio of the power density radiated at broadside for the dipole inside the structure, operating at the point of maximum broadside radiation (Eq. (1)), to that radiated by the same dipole in free space. The pattern bandwidth is defined as  $BW_P = (f_H f_L) / f_0$ , where  $f_0$  is the operating frequency (which satisfies Eq. (1)) and the frequency range  $f_L < f < f_H$  is the range over which the broadside power density is within 3 dB of its maximum value, occurring at  $f_0$ . These parameters can be related to the leaky-mode normalized attenuation constant  $\hat{\alpha}$  as follows [6]:

(3) 
$$\mathbf{E} \simeq \frac{4}{\pi} \mu_{\mathbf{r}} \sqrt{\mu_{\mathbf{r}} \varepsilon_{\mathbf{r}}} \frac{1}{\hat{\alpha}^2}, \quad \mathbf{BW}_{\mathbf{P}} \simeq \frac{2}{\mu_{\mathbf{r}} \varepsilon_{\mathbf{r}}} \hat{\alpha}^2.$$

It is to be noted that Eqs. (3) are also valid for a grounded slab covered with a stack of alternating high- and low-permittivity layers (i.e., a "Fabry-Perot Cavity" (FPC) antenna, shown in Figure 2 (a)). This may be seen by using the explicit expression for the normalized attenuation constant  $\hat{\alpha}$  as a function of the antenna parameters given in [1, Eq. 40] for a structure designed for radiation at broadside. The thicknesses *b* (layers with lower permittivity  $\varepsilon_{r1}$ ) and *t* (layers with higher permittivity  $\varepsilon_{r2}$ ) are such that  $k_0 b \sqrt{\varepsilon_{r1}} = \pi$ ,  $k_0 t \sqrt{\varepsilon_{r2}} = \pi/2$ . In this case the attenuation constant is

(4) 
$$\hat{\alpha} \simeq \sqrt{\frac{2}{\eta_0 F}} q^{N/2},$$

where  $q = \sqrt{\varepsilon_{\rm r1} / \varepsilon_{\rm r2}}$  ,  $Y_0 = \sqrt{\varepsilon_0 / \mu_0}$  and

$$\eta_0 F = \frac{\pi}{1-q^2} \left( \frac{1}{\sqrt{\varepsilon_{\rm r1}}} + \frac{q^2}{\sqrt{\varepsilon_{\rm r2}}} \right) + \frac{\pi}{\sqrt{\varepsilon_{\rm r1}}}$$

(assuming non-magnetic media).

From the expression for E in Eq. (3) it follows that the antenna performance in terms of broadside radi $\overline{B}_{s}$ on can be improved by decreasing the value of the leaky-wave attenuation constant. This can be achieved by increasing the value of the equivalent susceptance of the PRS, either by increasing the patch area (or reducing the slot area) in a metal patch (slot) PRS, or by increasing the ratio between the superstrate and substrate permittivities in a dielectric PRS.

#### B) Metamaterial-based antennas

Let us consider now a grounded non-magnetic metamaterial slab as in Figure 1 or Figure 2 (d) with a positive relative permittivity much less than unity, i.e.  $\varepsilon_r$ <<1. One possible way to obtain a low-permittivity medium is by a periodic arrangement of metal wires (*wire medium*) [23, 24]; although such a metamaterial is certainly anisotropic and also spatially dispersive [25], its radiative features in the principal planes when it is excited by a horizontal dipole source parallel to the wires and it has been optimized for broadside radiation can be obtained through a simple homogeneous isotropic constitutive model [18]. For this structure, the maximum power density at broadside is achieved when the slab thickness h is equal to an integer multiple of the half wavelength inside the slab (the integer is assumed to be one here). In fact, this condition implies the existence of a pair of TE<sub>z</sub> and TM<sub>z</sub> leaky modes with nearly equal and small values of the phase and attenuation constants [18] having

(5) 
$$\hat{\beta} \simeq \hat{\alpha} \simeq \sqrt{\frac{\varepsilon_{\rm r}^{3/2}}{\pi}}$$
.

As with the PRS-based antenna, the electric dipole source is placed in the middle of the slab to get the maximum broadside power density. By assuming a hypothetical *non-dispersive* metamaterial medium, the maximum broadside power enhancement E (with respect to the same dipole in free space) and the pattern bandwidth  $BW_P$  are given by

(6) 
$$\mathbf{E} \simeq \frac{4}{\pi^{2/3}} \frac{1}{\hat{\alpha}^{4/3}}, \quad \mathbf{BW}_{\mathbf{P}} \simeq \frac{2}{\pi^{2/3}} \hat{\alpha}^{2/3}.$$

Assuming a *dispersive* metamaterial with a frequencydependent plasma-like effective permittivity  $\varepsilon_{\rm r}(f) = 1 - f_{\rm p}^2 / f^2$  (where  $f_{\rm p}$  is the plasma frequency), the result from Eq. (5) may be further written as [17]

(7) 
$$\hat{\beta}^{\text{TE/TM}} \simeq \hat{\alpha}^{\text{TE/TM}} = \hat{\alpha} \simeq \sqrt{\frac{c^3}{8\pi f_{\rm p}^3 h^3}},$$

where *c* is the speed of light in free space. In this case, the enhancement E is still given by Eq. (6), since the radiation characteristics of the hypothetical and plasma-like LWAs must be the same at a given frequency, provided the permittivities of the slab are the same (and hence the two structures have the same values of the propagation wavenumber). However, the fractional bandwidth  $BW_P$  is different, and is now given by

(8) 
$$BW_P \simeq 2\hat{\alpha}^2$$
,

which is much smaller than the one in Eq. (6), in the limit of small  $\hat{\alpha}$ .

#### C) Antenna directivity

The maximum broadside directivity  $D_{\text{max}}$  of the antenna is considered now. For structures excited by a line source (one-dimensional wave propagation), under the assumption that a leaky mode dominates the aperture field of the antenna, the shape of the radiation pattern does not depend on the specific structure supporting the leaky mode [17]. Therefore, the dependence of maximum directivity on the normalized attenuation constant is common to all the considered structures.

For structures excited by a horizontal dipole source (two-dimensional wave propagation) that are optimized for maximum broadside radiation, a pair of  $TE_z$  and

TM<sub>z</sub> cylindrical leaky waves are excited with nearly the same values of the phase and attenuation constants  $\hat{\beta}^{\text{TE/TM}} \simeq \hat{\alpha}^{\text{TE/TM}} = \hat{\alpha}$ . For high-directivity structures these values are very small and the excitation coefficients of the TE<sub>z</sub> and TM<sub>z</sub> leaky modes are approximately equal; therefore, the pencil beam radiated at broadside is very directive and has a nearly circular cross section [1, 7]. The beamwidths  $\Delta \theta_{3dB}$  in the principal E and H planes can be evaluated by considering the radiation in the xz plane from a structure excited by a line source. This follows from the fact that the normalized patterns in this plane are the same for the dipole and line-source excitations, and also from the fact that the E- and Hplane beamwidths are nearly the same for the optimized structure excited by a dipole source. From the leakywave radiation pattern of the structure excited by a line source, which is available in closed form, we have, in the limit of small beamwidths,

(9) 
$$\Delta \theta_{3dB} \Big|_E \simeq \Delta \theta_{3dB} \Big|_H \simeq 2\sqrt{2}\hat{\alpha}$$

The maximum directivity of the structure excited by a dipole is inversely proportional to the product of the beamwidths  $\Delta \theta_{3dB}$  in the principal E and H planes, according to the approximate formula in [26, Eq. 2.27a]; thus we obtain

(10) 
$$D_{\text{max}} \simeq \frac{9.9}{\Delta \theta_{3\text{dB}}|_E \Delta \theta_{3\text{dB}}|_H} \simeq \frac{1.24}{\hat{\alpha}^2}$$

In order to provide quantitative information on the directivity performance of the considered LWAs excited by a horizontal electric dipole placed in the middle of the slab, the maximum broadside directivity  $D_{\text{max}}$  (in dB) is reported in Figures 3 and 4 as a function of the relevant structural parameters. In Figure 3,  $D_{\text{max}}$  is reported as a function of the adimensional parameter



**Fig. 3.** Broadside directivity (in dB) for a metamaterial LWA made of a grounded low-permittivity slab excited by a horizontal electric dipole placed in the middle of the slab at the optimum frequency, as a function of the normalized slab thickness  $h/\lambda_0$ .



**Fig. 4.** Broadside directivity (in dB) for a LWA consisting of a grounded dielectric slab with  $\mu_r = 1$  covered with a PRS and excited by a horizontal electric dipole placed in the middle of the slab, as a function of the normalized susceptance  $\overline{B}_s$  of the PRS, for different values of the relative permittivity  $\varepsilon_r$  of the slab.

 $h/\lambda_0$  for a low-permittivity metamaterial LWA. As the slab thickness increases, the slab permittivity correspondingly decreases, so that  $k_0 h \sqrt{\varepsilon_r} = \pi$ , in order to maintain the optimal electrical thickness as shown in [18].

In Figure 4,  $D_{\text{max}}$  is reported as a function of the adimensional parameter  $\overline{B}_{\text{s}} = B_{\text{s}} / Y_0$  for a PRS-based LWA as that in Figures 2(b) and 2(c), with slab permeability  $\mu_{\text{r}} = 1$ , for different values of the slab permittivity  $\varepsilon_{\text{r}}$ .

# III. Comparison of performances: figures of merit

# A) Enhancement-fractional bandwidth product (EBW)

A first figure of merit for the considered antennas can be introduced in order to describe their capability of producing high levels of broadside radiation. Equation (3) implies that the product EBW of the maximum broadside power enhancement E and the antenna pattern bandwidth  $BW_P$  for the conventional PRS-based LWA is a constant that is independent of the equivalent susceptance  $\overline{B}_s$  that models the PRS, and is given by

(11) EBW 
$$\simeq \frac{8}{\pi} \sqrt{\frac{\mu_{\rm r}}{\varepsilon_{\rm r}}}$$
.

Therefore, for this class of LWAs a reduction in the antenna pattern bandwidth that is equivalent to the increase in broadside radiation has to be accepted.

With reference to a *non-dispersive* low-permittivity metamaterial LWA, from Eq. (6) it is found that the EBW figure of merit is instead directly proportional to a *negative* power of  $\hat{\alpha}$ :

(12) EBW 
$$\simeq \frac{8}{\pi^{4/3}} \frac{1}{\hat{\alpha}^{2/3}},$$

whereas for a *dispersive plasma-like* metamaterial LWA, from Eqs. (6) and (8) the EBW figure of merit is directly proportional to a *positive* power of  $\hat{\alpha}$ :

(13) EBW 
$$\simeq \frac{8}{\pi^{2/3}} \hat{\alpha}^{2/3}$$
.

By comparing Eqs. (11), (12), and (13) it can be observed that the best performance in terms of EBW product would be provided by a non-dispersive metamaterial-based LWA, followed by the PRS-based and the dispersive metamaterial-based LWAs. The first is a very idealized configuration, since, as is well known, any metamaterial medium is inherently frequency-dependent. Equation (13) shows that, with a plasma-like dispersive model, the EBW figure of merit becomes infinitesimal with  $\hat{\alpha}$ , meaning that the antenna performance in terms of enhancement-bandwidth product deteriorates as the power density at broadside is increased.

# **B)** Directivity-fractional bandwidth product (DBW)

A further figure of merit may be related to the maximum broadside directivity of the antennas, i.e., the capability to produce narrow pencil beams. From Eqs. (3), (6), (8), and (10), it can be concluded that the directivity-pattern bandwidth product DBW<sub>P</sub> is a constant independent of  $\hat{\alpha}$  for the PRS-based and the dispersive metamaterial-based LWAs, whereas it is directly proportional to a *negative* power of  $\hat{\alpha}$  for the non-dispersive metamaterial-based LWA. In particular, for the PRS-based LWA

(14) 
$$\text{DBW}|_{\text{PRS}} = \frac{4}{\varepsilon_{\text{r}}},$$

while for the metamaterial-based LWA we have

(15) 
$$\text{DBW}|_{\text{non-disp}} = \frac{4}{\pi^{2/3}} \frac{1}{\hat{\alpha}^{4/3}}$$
,  $\text{DBW}|_{\text{disp}} = 4$ ,

for both the non-dispersive and dispersive cases, respectively.

For the DBW figure of merit, the conclusion is a bit different than for the EBW figure of merit. The DBW figure of merit for the PRS-based and the dispersive metamaterial LWAs behave similarly, with the figure of merit for both antennas equal to a constant in the limit of narrow beams (for which the above approximate expressions are accurate). The DBW figure of merit for the non-dispersive metamaterial LWA is larger than for the other two antennas, and becomes very large in the limit of narrow beams, similar to the behaviour of the EBW figure of merit. The DBW figure of merit is larger than the EBW figure of merit for the non-dispersive metamaterial LWA, however.

### **IV. Numerical results**

In this section, numerical results are presented for the radiative features of a PRS-based LWA and a metamaterial-based LWA. The former consists of an electric dipole source embedded within N (always even) dielectric layers of relative permittivity  $\varepsilon_{r1}$  and  $\varepsilon_{r2}$  ( $\varepsilon_{r2} > \varepsilon_{r1}$ ) stacked in an alternating arrangement with appropriate thicknesses above a ground plane (Fig. 2(a)). The latter consists of an infinitesimal horizontal electric dipole source embedded in a grounded low-epsilon metamaterial slab with a relative permittivity that is either independent of frequency or has a plasma-like dispersive behaviour. In both cases the ideal dipole source is placed at a height  $h_s$  above the ground plane, equal to one-half of the height of the slab forming the cavity (*b* and *h*, for the two respective antennas). The parameters of



**Fig. 5.** Normalized phase and attenuation constants of the relevant  $TE_z$  (*gray lines*) and  $TM_z$  (*black lines*) leaky modes, and broadside power enhancement factor as a function of frequency, for (a) a PRS-based LWA, and (b) a metamaterial-based LWA (non-dispersive medium). *Parameters* are given in the first and the second columns of Table 1, respectively.

the chosen structures are given in Table 1.

All three antennas have been designed to support  $\text{TE}_z$ and  $\text{TM}_z$  leaky modes with the same value of the normalized attenuation constant  $\hat{\alpha} \cong 0.06$  at f = 20 GHz, according to the expressions for  $\hat{\alpha}$  given in Eq. (4) for the multiple-layer dielectric LWA (FPC antenna) and in Eqs. (5) and (7) for the metamaterial-based LWAs. The antennas thus have the same directivity, and are all assumed to be of infinite extent (all of the previous formulas that are related to the radiation patterns have assumed this). If truncated, one has to be sure that the leaky waves are sufficiently attenuated when reaching the truncation. Since the attenuation constant is the same for all three structures, they would have the same lateral size.

In Figures 5 (a) and (b), the normalized phase and attenuation constants of the TE<sub>z</sub> and TM<sub>z</sub> leaky modes are reported for the PRS-based and the non-dispersive metamaterial-based LWAs together with the relevant broadside power enhancement factors as a function of frequency. It can be observed that, as expected, the peak in the power-enhancement curve occurs when the phase and the attenuation constants of the TE and TM leaky modes are equal. For the PRS-based LWA the TE and TM curves are almost superimposed and the exact maximum power enhancement is  $E \cong 353$ , while the approximate result provided by Eq. (3) is  $E \cong 324$ . The exact maximum power enhancement is obtained from the exact radiation pattern, which is computed using reciprocity together with a transmission line model for the layered structure [3]. In the non-dispersive metamaterial-based LWA the exact result is  $E \cong 79.05$  while the approximate one provided by Eq. (6) is  $E \cong 79.36$ . The maximum level of broadside power density is lower for the metamaterial-based LWA, as expected from Eqs. (3) and (6). However, the dispersion curves for the frequency-independent metamaterial slab are

PRS-based LWA	Metamaterial-based LWAs
$\mathcal{E}_{rl} = 1$	Non- dispersive case:
$\mathcal{E}_{r2} = 9$	$\varepsilon_{\rm r} = 0.050$
<i>b</i> = 7.5 mm	Dispersive case
t = 1.25  mm	
N = 4	h = 33.342  mm
$h_{\rm s}b = /2$	$h_{\rm s}h=/2$

**Tab. 1.** Parameters for the two types of LWAs, chosen so that they both have the same attenuation and propagation constants  $b\hat{\alpha} \equiv 0.06$  at f = 20 GHz, and thus have the same directivity. In the first column the parameters are for a FPC antenna with three dielectric layers above the bottom cavity layer used to realize a PRS. In the second column the parameters are for two low-permittivity metamaterial antennas, based on either a non-dispersive or a dispersive metamaterial slab.



**Fig. 6.** Same as in Figure 5 for a dispersive metamaterial-based LWA. *Parameters:* see the second column of Table 1.

much flatter near the operating frequency, and the pattern bandwidth for the power enhancement is correspondingly larger: for the PRS-based LWA one has  $BW_P \cong 0.63\%$ , whereas for the metamaterial-based LWA  $BW_P \cong 14.68\%$  (the corresponding approximate values are  $BW_P \cong 0.72\%$ , from Eq. (3), and  $BW_P \cong$ 



**Fig. 7.** E-plane (*solid line*) and H-plane (*dashed line*) radiation patterns for (a) a PRS-based LWA, and (b) a metamaterial-based LWA, at f = 20 GHz. *Parameters* are given in the first and the second columns of Table 1, respectively. The two antennas have the same directivity.

14.29%, from Eq. (6)). The bandwidth for the directivity (based on the frequencies for which the directivity drops by 3 dB from the maximum value), calculated numerically, is very different as well:  $BW_D \cong 1.1\%$ (PRS-based) and  $BW_D \cong 22.65\%$  (metamaterial-based). The remarkable difference in the pattern and directivity bandwidths is due to the idealized, non-dispersive nature of the metamaterial medium. Taking into account the frequency dependence of the metamaterial permittivity by means of a plasma-like dispersive behaviour, the situation becomes very different. In fact, considering a medium with  $f_{\rm P} = 19.4874$  GHz, such that  $\varepsilon_{\rm r} = 0.0506$ and  $\hat{\alpha} \cong 0.06$  at f = 20 GHz, the results reported in Figure 6 are obtained. Now the fractional bandwidth for the power density is  $BW_P \cong 0.73\%$  (the corresponding approximate value is  $BW_P \cong 0.72\%$ , from Eq. (8)), whereas for the directivity  $BW_D \cong 0.72\%$ .

This shows that accurate modeling of the (unavoidable) dispersive behaviour of the metamaterial is crucial in order to obtain accurate predictions of the bandwidth properties of the considered antennas. Also, this suggests that a significant improvement in the antenna bandwidth could possibly be achieved by using a metamaterial design that has reduced dispersion in the frequency range where  $|\varepsilon_r| <<1$ . Such a metamaterial would require a different implementation than those proposed so far, such as the wire medium, which are inherently rather dispersive (see text after Eq. (6), and [24]).

In Figure 7, radiation patterns in the principal (E and H) planes are shown for both the PRS-based and the metamaterial-based LWAs considered here, excited by a unit-amplitude horizontal electric dipole source. Since we operate at the fixed frequency f = 20 GHz, no distinction needs to be made between the dispersive and non-dispersive metamaterial LWAs, since they have been designed to have the same substrate permittivity at this frequency. The radiated power density has been expressed in dB relative to one Watt per unit solid angle.

It can be observed that in both cases the patterns in the E and H planes are almost equal in a neighborhood of broadside, as expected from the (approximate) equality of the TE<sub>z</sub> and TM<sub>z</sub> leaky-wave phase and attenuation constants; the E- and H-plane patterns start to differ at angles greater than 15°, where the level of the radiated power density is more than 20 dB lower than at the maximum. The two antennas are thus seen to radiate a broadside omnidirectional pencil beam with approximately the same beamwidth, and thus the same directivity, since they have been designed to support leaky modes with the same attenuation constants at the operating frequency. (The peak levels of broadside radiation are very different, however, since the enhancement factors E for

the two antennas are quite different). The radiation patterns of the two antennas differ at angles far from broadside, however. In particular, the pattern in the E plane of the metamaterial LWA shows two sharp nulls at angles close to  $\pm 13^{\circ}$ , which may be shown to arise from the destructive interference of the dominant leaky-mode far field with the far field radiated by an additional TM<sub>z</sub> leaky mode excited by the source.

## V. Conclusion

Two types of planar leaky-wave antennas (LWAs) have been compared in this investigation. One is a "conventional" LWA that consists of a grounded dielectric slab with a partially reflecting surface (PRS) on top. This structure acts as a leaky parallel-plate waveguide, in which the parallel-plate modes become leaky by virtue of radiation that takes place through the PRS surface. The other type of structure is a metamaterial LWA, consisting of a grounded slab of low permittivity (or permeability) material. Two classes of metamaterial LWAs were considered: (a) dispersionless, where the permittivity of the slab was independent of frequency (a hypothetical best-case scenario), and (b) those where the permittivity follows a plasma-like dispersion relation (modelling a practical metamaterial LWA). In both types of antennas, a simple horizontal dipole source is used to provide the excitation of the leaky modes. Both structures are capable of producing highly directive omnidirectional pencil beams at broadside when optimised.

The figures of merit used in the comparison are: (1) the enhancement-factor E for the power density radiated at broadside, (2) the pattern bandwidth  $BW_P$ , (3) the enhancement-factor × bandwidth product EBW, and (4) the directivity × bandwidth product DBW. All of these figures of merit have been put in terms of the attenuation constants of the leaky modes on the structure, to facilitate the most direct type of comparison (antennas with the same attenuation constant will have the same beamwidth). By comparing the reported expressions for these figures of merit, the following remarks can be made.

Conventional PRS-based LWAs have a larger enhancement factor E than do the metamaterial LWAs. The PRS-based LWAs have a smaller enhancementbandwidth product EBW than do dispersionless metamaterial LWAs, but they have a larger EBW than do practical plasma-like metamaterial LWAs.

On the other hand, the directivity-bandwidth product DBW for PRS-based LWAs is equal to or less than that for plasma-like metamaterial LWAs, depending on the substrate permittivity used in the PRS-based design. For disperionless metamaterial LWAs, the DBW product is even still larger.

One advantage of the PRS-based LWAs is that the thickness of the slab is much less than for the metamaterial LWAs (especially if they are realized with a PRS consisting of a flat metal-screen). Therefore, they are more easily realizable, and are more cost effective.

One possible additional advantage of metamaterial antennas concerns the efficiency, since grounded slabs with a positive and small epsilon do not support surface waves, whereas PRS-based antennas generally do, although this issue has not been explored here.

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### References

- Jackson, D.R.; Oliner, A.A.; Ip, A.: Leaky-wave propagation and radiation for a narrow-beam multiple-layer dielectric structure. IEEE Trans. Antennas Propagat. 41 (1993), 344-348.
- [2] Ostner, H.; Detlefsen, J.; Jackson, D.R.: Radiation from one-dimensional dielectric leaky-wave antennas. IEEE Trans. Antennas Propagat. 43 (1995), 331-339.
- [3] Zhao, T.; Jackson, D.R.; Williams, J.T.; Oliner, A.A.: 2D periodic leaky-wave antenna using metal patches or slots, Part I and Part II. IEEE Trans. Antennas Propagat. 53 (2005), 3505-3524.
- [4] Tamir, T.; Oliner, A.A.: Guided complex waves. Part I. Fields at an interface. Part II. Relation to radiation patterns. Proc. Inst. Elect. Eng. 110 (1963), 310-334.
- [5] Oliner, A.A.: Leaky-wave antennas. In: Johnson, R.C. (ed.): Antenna Engineering Handbook. New York: McGraw Hill, 1993, Ch. 10.
- [6] Lovat, G.; Burghignoli, P.; Jackson, D.R.: Fundamental properties of broadside radiation from uniform leaky-wave antennas, Digest XXVIII URSI General Assembly, New Delhi, India, October 23-29, 2005.
- [7] Jackson, D.R.; Ip, A.: Radiation from cylindrical leaky waves. IEEE Trans. Antennas Propagat. 38 (1990), 482-488.
- [8] Trentini, G.V.: Partially Reflecting Sheet Arrays. IEEE Trans. Antennas Propagat. 4 (1956), 666-671.
- [9] Feresidis, A.P.; Vardaxoglou, J.C.: High Gain Planar Antenna Using Optimised Partially Reflective Surfaces. IEE Proc. Microw. Antennas Propag. 148 (2001), 345-350.
- [10] Feresidis, A.P.; Goussetis, G.; Wang, S.; Vardaxoglou, J.C.: Artificial magnetic conductor surfaces and their application to low profile high-gain planar antennas. IEEE Transactions on Antennas and Propagation (Special Issue on Artificial Magnetic Conductor, Soft/Hard surfaces, and other Complex Surfaces) 53 (2005), 209-215.
- [11] Shadrivov, I.W.; Sukhorukov, A.A.; Kivshar, Y.S.: Guided

modes in negative refractive-index waveguides. Phys. Rev. E Stat. Phys. Plasma Fluids Relat. Interdiscip. Top. 67 (2003), 57 602-1 – 57 602-4.

- [12] Wu, B.-I.; Grzegorczyk, T.M.; Zhang, Y.; Kong, J.A.: Guided modes with imaginary transverse wavenumber in a slab waveguide with negative permittivity and permeability. J. Appl. Phys. 93 (2003), 9386-9388.
- [13] Baccarelli, P.; Burghignoli, P.; Frezza, F.; Galli, A.; Lampariello, P.; Lovat, G.; Paulotto, S.: Effects of leakywave propagation in metamaterial grounded slabs excited by a dipole source. IEEE Trans. Microwave Theory Tech. 53 (2005), 32-44.
- [14] Alù, A.; Bilotti, F.; Engheta, N.; Vegni, L.: How Metamaterials May Significantly Affect the Wave Transmission Through a Sub-Wavelength Hole in a Flat Perfectly Conducting Screen. Proc. IEE Seminar on Metamaterials for Microwave and (Sub)millimetre Wave Applications: Photonic Bandgap and Double Negative Designs, Components and Experiments, London, UK, November 24, 2003.
- [15] Gupta, K.C.: Narrow-beam antennas using an artificial dielectric medium with permittivity less than unity. Electronics Lett. 7 (1971), 16-18.
- [16] Poilasne, G.; Pouliguen, P.; Lenormand, J.; Mahdjoubi, K.; Terret, C.; Gelin, P.: Theoretical study of interactions between antennas and metallic photonic bandgap materials. Microwave Opt. Techn. Lett. 15 (1997), 384-389.
- [17] Enoch, S.; Tayeb, G.; Sabouroux, P.; Guérin, N.; Vincent, P.: A metamaterial for directive emission. Phys. Rev. Lett. 89 (2002), 213902.



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- [18] Lovat, G.; Burghignoli, P.; Capolino, F.; Jackson, D.R.; Wilton, D.R. Analysis of directive radiation from a line source in a metamaterial slab with low permittivity. IEEE Trans. Antennas Propagat. 54 (2006), 1017-1030.
- [19] Alù, A.; Bilotti, F.; Engheta, N.; Vegni, L.: Compact leakywave components using metamaterials. Proc. IEEE MTT-S International Microwave Symp. Long Beach, California, June 12-17, 2005.
- [20] Lovat, G.; Burghignoli, P.; Capolino, F.; Jackson, D.R.; Wilton, D.R.: A comparative study of conventional and metamaterial-based planar leaky-wave antennas for directive radiation at broadside, Digest XXVIII URSI General Assembly, New Delhi, India, October 23-29, 2005.
- [21] Zhao, T.; Jackson, D.R.; Williams, J.T.; Oliner, A.A.: Simple CAD model for a dielectric leaky-wave antenna. IEEE Antennas Wireless Propagat. Lett. 3 (2004), 243-245.
- [22] Zhao, T.; Jackson, D.R.; Williams, J.T.; Oliner, A.A.: General formulas for 2D leaky-wave antennas. IEEE Trans. Antennas Propagat. 53 (2005), 3525-3533.
- [23] Brown, J.: Artificial dielectrics having refractive indices less than unity. Proc. IEEE 100 (1953), 51-62.
- [24] Belov, P.A.; Tretyakov, S.A.; Viitanen, A.J.: Dispersion and reflection properties of artificial media formed by regular lattices of ideally conducting wires. J. Electromag. Waves Appl. 16 (2002), 1153-1170.
- [25] Belov, P.A.; Marqués, R.; Maslovski, S.I.; Silveirinha, M.; Simovski, C.R.; Tretyakov, S.A.: Strong spatial dispersion in wire media in the very large wavelength limit. Phys. Rev. B. 67 (2003), 113103-1 – 113103-4.
- [26] Balanis, C.A.: Antenna theory. New York: Wiley, 1997.

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