For official marking scheme, please refer to HKEA publications.

1. **HKDSE 2014 Q.1**

$$\frac{(xy^{-2})^3}{y^4} = \frac{x^3y^{-6}}{y^4} = \frac{x^3}{y^{10}}$$

2. **HKDSE 2014 Q.2**

(a) $a^2 - 2a - 3 = (a-3)(a+1)$

(b) $ab^2 + b^2 + a^2 - 2a - 3 = b^2(a+1) + (a-3)(a+1)$

$= (a+1)(a+b^2-3)$

3. **HKDSE 2014 Q.3**

(a) 200

(b) 123

(c) 123.4

4. **HKDSE 2014 Q.4**

Median $= 1$

Mode $= 2$

Standard deviation ≈ 0.889

5. **HKDSE 2014 Q.5**

(a) $2(3m+n) = m+7$

$6m + 2n = m + 7$

$2n = 7 - 5m$

$$n = \frac{7-5m}{2}$$

(b) Change in $n = \dfrac{7-5(m+2)}{2} - \dfrac{7-5m}{2} = -5$

The decrease in the value of n is 5.

6. **HKDSE 2014 Q.6**

(a) Selling price $= 255 \times (1 - 40\%) = \$153$

(b) Cost $= 153 \div (1 + 2\%) = \$150$

7. **HKDSE 2014 Q.7**

(a) $f(2) = -33$

$4(2)^3 - 5(2)^2 - 18(2) + c = -33$

$c = -9$

$f(-1) = 4(-1)^3 - 5(-1)^2 - 18(-1) - 9 = 0$

$\therefore$ $x + 1$ is a factor of $f(x)$

(b) $f(x) = 0$

$(x + 1)(4x^2 - 9x - 9) = 0$

$(x + 1)(x - 3)(4x + 3) = 0$

$x = -1$ or $x = 3$ or $x = -\frac{3}{4}$

All the roots are rational numbers.

Thus, the claim is agreed.

8. **HKDSE 2014 Q.8**

(a) $\mathrm{P}' = (5, 3)$

$\mathrm{Q}' = (-19, -7)$

(b) Slope of PQ $= \frac{5 + 7}{-3 - 2} = -\frac{12}{5}$

Slope of P′Q′ $= \frac{3 + 7}{5 + 19} = \frac{5}{12}$

Slope of PQ $\times$ slope of P′Q′ $= -\frac{12}{5} \times \frac{5}{12} = -1$

Therefore, PQ is perpendicular to P′Q′.

9. **HKDSE 2014 Q.9**

(a) In $\Delta\mathrm{ABC}$ and $\Delta\mathrm{BDC}$,

$\angle\mathrm{ACB} = \angle\mathrm{BCD}$	(common $\angle$)
$\angle\mathrm{BAC} = \angle\mathrm{DBC}$	(given)
$\angle\mathrm{ABC} = \angle\mathrm{BDC}$	($\angle$ sum of Δ)
$\therefore$ $\Delta\mathrm{ABC} \sim \Delta\mathrm{BDC}$	(A.A.A.)

(b) $\frac{\mathrm{CD}}{\mathrm{BC}} = \frac{\mathrm{BC}}{\mathrm{AC}}$

$\frac{\mathrm{CD}}{20} = \frac{20}{25}$

$\mathrm{CD} = 16$ cm

$\mathrm{BD}^2 + \mathrm{CD}^2 = 12^2 + 16^2 = 400 = 20^2 = \mathrm{BC}^2$

Thus, $\Delta\mathrm{BCD}$ is a right-angled triangle.

10. **HKDSE 2014 Q.10**

(a) Let d km be the required distance.

$$\frac{d}{80}=\frac{0.75}{2}$$

$$d=30$$

Thus, the required distance is 30 km.

(b) Speed of car A $=\dfrac{80}{2}=40$ km/h

Time taken for car A to travel 44 km $=\dfrac{44}{40}=1.1$ hours $=1$ hour 6 minutes

So, the two cars first meet at 8:36.

(c) Average speed of car B during the period 8:15 to 9:30 $=\dfrac{80-44}{1.25}=28.8$ km/h

That is, the average speed of car B is less than that of car A (40 km/h).

Thus, the claim is disagreed.

11. **HKDSE 2014 Q.11**

(a) Range $=91-18=73$ thousand dollars

Inter-quartile range $=63-42=21$ thousand dollars

(b) New mean $=\dfrac{53\times 33-32-34-58-59}{33-4}=54$ thousand dollars

The original median is 55 thousand dollars.

Note that 32 and 34 are less than 55, while 58 and 59 are larger than 55.

So, the median remains unchanged.

Therefore, new median $=55$ thousand dollars.

12. **HKDSE 2014 Q.12**

(a) $\mathrm{AG}=\sqrt{(6-0)^2+(11-3)^2}=10$

Equation of C : $x^2+(y-3)^2=10^2$

$$x^2+y^2-6y-91=0$$

(b) (i) Let $\mathrm{P}=(x,y)$.

Equation of Γ : $\sqrt{(x-6)^2+(y-11)^2}=\sqrt{x^2+(y-3)^2}$

$$x^2+y^2-12x-22y+157=x^2+y^2-6y+9$$

$$12x+16y-148=0$$

$$3x+4y-37=0$$

(ii) Γ is the perpendicular bisector of the line segment AG.

(iii) The requried perimeter $=4\times 10=40$

13. **HKDSE 2014 Q.13**

(a) Let $f(x) = a + bx^2$, where a and b are non-zero constants.

So, we have $a + b(2)^2 = 59$ and $a + b(7)^2 = -121$.

Solving, we have $a = 75$ and $b = -4$.

$\therefore \quad f(x) = 75 - 4x^2$

$f(6) = 75 - 4(6)^2 = -69$

(b) Note that $a = -69$.

$f(-6) = 75 - 4(-6)^2 = -69$

i.e. $b = -69$

Area of $\Delta\text{ABC} = \frac{1}{2} \times 12 \times 69 = 414$ sq. units

14. **HKDSE 2014 Q.14**

(a) Let r cm be the base radius of the vessel and x cm be the radius of the water surface.

$\frac{r}{72} = \frac{96-60}{96}$ and $\frac{x}{72} = \frac{96-60+28}{96}$

$\therefore \quad r = 27$ and $x = 48$

Area of the wet curved surface of the vessel $= \pi(48)\sqrt{48^2 + 64^2} - \pi(27)\sqrt{27^2 + 36^2}$

$= 2625\pi \text{ cm}^2$

(b) Volume of water in the vessel $= \left[\frac{1}{3}\pi(48)^2(64) - \frac{1}{3}\pi(27)^2(36)\right] \div 100^3$

$\approx 0.126932909 \text{ m}^3$

$> 0.1 \text{ m}^3$

Thus, the claim is agreed.

15. **HKDSE 2014 Q.15**

$$\log_8 y = -\frac{1}{3}(\log_4 x - 3)$$

$$\log_8 y = -\frac{1}{3}\log_4 x + 1$$

$$\log_8 y = \log_4 x^{-\frac{1}{3}} + \log_4 4$$

$$\log_8 y = \log_4 4x^{-\frac{1}{3}}$$

$$\log_8 y = \frac{\log_8 4x^{-\frac{1}{3}}}{\log_8 4}$$

$$\log_8 y = \frac{3}{2}\log_8 4x^{-\frac{1}{3}}$$

$$\log_8 y = \log_8 \left(4x^{-\frac{1}{3}}\right)^{\frac{3}{2}}$$

$$\log_8 y = \log_8 8x^{-\frac{1}{2}}$$

$$\therefore \quad y = 8x^{-\frac{1}{2}}$$

16. **HKDSE 2014 Q.16**

$S(m) > 6888$

$\frac{m}{2}[2(3) + (m-1)(2)] > 6888$

$m(m+2) > 6888$

$m^2 + 2m - 6888 > 0$

$(m+84)(m-82) > 0$

$m < -84$ or $m > 82$

Thus, the least value of m is 83.

17. **HKDSE 2014 Q.17**

(a) $\dfrac{\sin \angle \mathrm{AVB}}{18} = \dfrac{\sin 110°}{30}$

$\angle \mathrm{AVB} \approx 34.32008291°$

$\angle \mathrm{VBA} \approx 180° - 110° - 34.32008291° \approx 35.67991709° \approx 35.7°$

(b) $\mathrm{MP}^2 \approx 9^2 + 15^2 - 2(9)(15)\cos 35.67991709°$

$\mathrm{MP} \approx 9.310329519$

$\mathrm{NQ} = \mathrm{MP} \approx 9.310329519$

$\mathrm{MN} = 10 \div 2 = 5$

$\mathrm{PQ} = \mathrm{BC} = 10$

Let h be the height of the trapezium PQNM.

$h = \sqrt{9.310329519^2 - \left((10-5) \div 2\right)^2} \approx 8.968402073$

Area of PQNM $\approx \dfrac{(5+10) \times 8.968402073}{2} \approx 67.26301555\ \mathrm{cm}^2 < 70\ \mathrm{cm}^2$

Thus, the claim is agreed.

18. **HKDSE 2014 Q.18**

(a) Equation of L_2 : $y - 0 = \dfrac{90-0}{45-180}(x - 180)$

$y = -\dfrac{2}{3}(x - 180)$

$2x + 3y = 360$

The system of inequalities is $\begin{cases} 6x + 7y \le 900 \\ 2x + 3y \le 360 \\ x \ge 0 \\ y \ge 0 \end{cases}$.

(b) Let x and y be the number of wardrobes X and Y produced that month respectively.

The constraints are $6x + 7y \le 900$ and $2x + 3y \le 360$, where x and y are non-negative integers.

Let \$P be the total profit on the production of wardrobes. So $\mathrm{P} = 440x + 665y$.

The vertices of the shaded region in Figure 7 are (0, 0), (0, 120), (45, 90) and (150, 0).

At (0, 0), $\mathrm{P} = 440(0) + 665(0) = 0$. At (0, 120), $\mathrm{P} = 440(0) + 665(120) = 79\,800$.

At (45, 90), $\mathrm{P} = 440(45) + 665(90) = 79\,650$. At (150, 0), $\mathrm{P} = 440(150) + 665(0) = 66\,000$.

So, the greatest possible profit is \$79 800. Thus, the claim is disagreed.

19. **HKDSE 2014 Q.19**

(a) The required probability $= \frac{1}{6} + \left(\frac{5}{6}\right)^2 \times \frac{1}{6} + \left(\frac{5}{6}\right)^4 \times \frac{1}{6} + \cdots = \frac{\frac{1}{6}}{1 - \left(\frac{5}{6}\right)^2} = \frac{6}{11}$ (or 0.5455)

(b) (i) $P(10 \text{ tokens}) = \frac{8}{8} \times \frac{1}{8} = \frac{1}{8}$

$P(5 \text{ tokens}) = \frac{2}{8} \times \frac{1}{8} + \frac{6}{8} \times \frac{2}{8} = \frac{7}{32}$

$\therefore$ Expected number of tokens $= \frac{1}{8} \times 10 + \frac{7}{32} \times 5 = \frac{75}{32}$ (or 2.34375)

(ii) Consider Option 2.

$P(50 \text{ tokens}) = \frac{8}{8} \times \frac{1}{8} \times \frac{1}{8} = \frac{1}{64}$

$P(10 \text{ tokens}) = \frac{6 \times 3!}{8^3} = \frac{9}{128}$

$P(5 \text{ tokens}) = \frac{7 \times C_2^3 \times 2!}{8^3} = \frac{21}{256}$

Expected number of tokens in Option 2

$= \frac{1}{64} \times 50 + \frac{9}{128} \times 10 + \frac{21}{256} \times 5$

$= \frac{485}{256}$ (or 1.89)

Hence the player should adopt Option 1.

(iii) $P(\text{Ada gets no tokens}) = \left(1 - \frac{6}{11}\right) + \frac{6}{11} \times \left(1 - \frac{1}{8} - \frac{7}{32}\right) = \frac{13}{16} = 0.8125 < 0.9$

$\therefore$ The claim is incorrect.