For official marking scheme, please refer to HKEA publications.

1. **HKDSE 2016 Q.1**

$$\frac{(x^8y^7)^2}{x^5y^{-6}} = \frac{x^{16}y^{14}}{x^5y^{-6}} = x^{11}y^{20}$$

2. **HKDSE 2016 Q.2**

$$\mathrm{A}x = (4x + \mathrm{B})\mathrm{C}$$
$$\mathrm{A}x = 4\mathrm{C}x + \mathrm{BC}$$
$$x(\mathrm{A} - 4\mathrm{C}) = \mathrm{BC}$$
$$x = \frac{\mathrm{BC}}{\mathrm{A} - 4\mathrm{C}}$$

3. **HKDSE 2016 Q.3**

$$\begin{aligned}\frac{2}{4x-5} + \frac{3}{1-6x} &= \frac{2(1-6x) + 3(4x-5)}{(4x-5)(1-6x)} \\ &= \frac{2 - 12x + 12x - 15}{(4x-5)(1-6x)} \\ &= \frac{-13}{(4x-5)(1-6x)}\end{aligned}$$

4. **HKDSE 2016 Q.4**

(a) $5m - 10n = 5(m - 2n)$

(b) $m^2 + mn - 6n^2 = (m + 3n)(m - 2n)$

(c) $$\begin{aligned}m^2 + mn - 6n^2 - 5m + 10n &= (m + 3n)(m - 2n) - 5(m - 2n) \\ &= (m - 2n)(m + 3n - 5)\end{aligned}$$

5. **HKDSE 2016 Q.5**

Let f be the number of female members.

$f(1 + 40\%) + f = 180$

$2.4f = 180$

$f = 75$

$\therefore$ Number of female members $= 75$ and number of male members $= 75(1 + 40\%) = 105$

The required difference $= 105 - 75 = 30$

6. **HKDSE 2016 Q.6**

(a) $x + 6 < 6(x + 11)$

$x + 6 < 6x + 66$

$5x > -60$

$x > -12$

Thus, the solution of the compound ineqaulity is all real solutions.

(b) The required greatest negative integer is -1.

7. **HKDSE 2016 Q.7**

(a) $\angle \text{AOB} = 135° - 75° = 60°$

(b) $\text{AB}^2 = 12^2 + 12^2 - 2(12)(12)\cos 60°$

$\text{AB} = 12$

The required perimeter $= 12 \times 3 = 36$ units

(c) ΔAOB has a rotational symmery of order 3.

8. **HKDSE 2016 Q.8**

(a) Let $f(x) = ax + bx^2$, where a and b are non-zero constants.

So we have $3a + 9b = 48$ and $9a + 81b = 198$.

Solving, we have $a = 13$ and $b = 1$.

$\therefore \quad f(x) = x^2 + 13x$

(b) $f(x) = 90$

$x^2 + 13x = 90$

$x^2 + 13x - 90 = 0$

$(x - 5)(x + 18) = 0$

$x = -18 \quad \text{or} \quad x = 5$

9. **HKDSE 2016 Q.9**

(a) $a = 2,\ a + 4 + b = 13,\ a + 4 + b + c + 15 = 37$

$\therefore \quad a = 2,\ b = 7,\ c = 9$

$x = 2 + 4 = 6$

$y = 13 + 9 = 22$

$z = 37 + 3 = 40$

(b) $\text{P}(0.65\text{ m} \le \text{height} < 1.25\text{ m}) = \dfrac{7+9}{40} = \dfrac{2}{5}$ (or 0.4)

10. **HKDSE 2016 Q.10**

(a) Let (x, y) be the coordinates of P.

$\mathrm{PA} = \mathrm{PB}$

$\sqrt{(x-5)^2 + (y-7)^2} = \sqrt{(x-13)^2 + (y-1)^2}$

$x^2 + y^2 - 10x - 14y + 74 = x^2 + y^2 - 26x - 2y + 170$

$16x - 12y - 96 = 0$

$4x - 3y - 24 = 0$

$\therefore$ The equation of Γ is $4x - 3y - 24 = 0$.

(b) $\mathrm{H} = (6, 0)$ and $\mathrm{K} = (0, -8)$

$\because$ $\angle\mathrm{HOK} = 90^\circ$

$\therefore$ HK is the diameter of the circle C (converse of $\angle$ in semi-circle)

Radius of C $= \sqrt{(6-0)^2 + (0+8)^2} \div 2 = 5$

Circumference of C $= 2\pi(5) \approx 31.41592654 > 30$

Thus, the claim is agreed.

11. **HKDSE 2016 Q.11**

(a) Let $\mathrm{V_F}\ \mathrm{cm}^3$ be the final volume of milk in the vessel.

$\frac{\mathrm{V_F}}{\mathrm{V_F} - 444\pi} = \left(\frac{16}{12}\right)^3$

$27\mathrm{V_F} = 64(\mathrm{V_F} - 444\pi)$

$37\mathrm{V_F} = 28416\pi$

$\mathrm{V_F} = 768\pi$

The final volume of milk in the vessel is $768\pi\ \mathrm{cm}^3$.

(b) Let r cm be the base radius of the milk inside vessel.

$\frac{1}{3}\pi r^2(16) = 768\pi$

$r = 12$

The final area of the wet curved surface $= \pi(12)\sqrt{12^2 + 16^2}$

$= 240\pi$

≈ 753.9822369

< 800

Thus, the claim is disagreed.

12. **HKDSE 2016 Q.12**

(a) $11 + a = 11 + b + 4$

$a = b + 4$

$\because \quad 4 < b < 10$

$\therefore \quad 8 < a < 14$

$\because \quad a > 11$

$\therefore \quad a = 12$ or $a = 13$

When $a = 12$, $b = 12 - 4 = 8$.

When $a = 13$, $b = 13 - 4 = 9$.

(b) (i) The median is the greatest when the ages of these four children are 7, 8, 9 and 10.

Therefore, the required greatest median is 8.

(ii) The mean is the least when the ages of these four children are 6, 7, 8 and 9.

Case 1 : $a = 12$, $b = 8$

The least possible mean of the ages of the children in the group is 7.6.

Case 2 : $a = 13$, $b = 9$.

The least possible mean of the ages of the children in the group ≈ 7.615384615.

Therefore, the required least possible mean is 7.6.

13. **HKDSE 2016 Q.13**

(a) $\because \quad \angle ADE = \angle AED$ (given)

$\therefore \quad AD = AE$ (sides opp., equal $\angle$s)

In ΔACD and ΔABE,

$AD = AE$ (proved)

$\angle ADC = \angle AEB$ (given)

$DC = DM + ME + EC$

$= DM + ME + BD$ (given)

$= EB$

$\therefore \quad \Delta ACD \cong \Delta ABE$ (SAS)

(b) (i) Since ΔABE is an isosceles triangle and M is the mid-point of DE, we have $AM \perp DE$.

$AM = \sqrt{15^2 - 9^2} = 12$ cm (Pyth. theorem)

(ii) $AB = \sqrt{AM^2 + MB^2}$ (Pyth. theorem)

$= \sqrt{12^2 + 16^2}$

$= 20$ cm

$AB^2 + AE^2 = 20^2 + 15^2 = 625$

$BE^2 = 25^2 = 625$

$\because \quad AB^2 + AE^2 = BE^2$

$\therefore \quad \angle BAE = 90°$ (converse of Pyth. theorem)

Thus, ΔABE a right-angled triangle.

14. **HKDSE 2016 Q.14**

(a) By comparing the coefficient of x^4, we have $2l = 6$.

$\therefore$ $l = 3$

By comparing the coefficient of x^3, we have $3m + 10 = 7$.

$\therefore$ $m = -1$

$p(2) = 6(2)^4 + 7(2)^3 + a(2)^2 + b(2) + c = 96 + 56 + 4a + 2b + c$

$p(-2) = 6(-2)^4 + 7(-2)^3 + a(-2)^2 + b(-2) + c = 96 - 56 + 4a - 2b + c$

$p(2) - p(-2) = 0$

$112 + 4b = 0$

$b = -28$

By comparing the coefficient of x, we have $5n - 8 = -28$.

$\therefore$ $n = -4$

(b) $p(x) = 0$

$(3x^2 + 5x + 8)(2x^2 - x - 4) = 0$

$3x^2 + 5x + 8 = 0$ or $2x^2 - x - 4 = 0$

Consider $3x^2 + 5x + 8 = 0$.

$\Delta = 5^2 - 4(3)(8) = -71 < 0$

$\therefore$ There is no real root.

Consider $2x^2 - x - 4 = 0$.

$\Delta = (-1)^2 - 4(2)(-4) = 33 > 0$

$\therefore$ There are 2 distinct real roots.

Thus, there are 2 distinct real roots of the equation $p(x) = 0$.

15. **HKDSE 2016 Q.15**

The required probability $= \dfrac{5! \times C_4^6 \times 4!}{9!} = \dfrac{43200}{362880} = \dfrac{5}{42}$ (or 0.119)

16. **HKDSE 2016 Q.16**

Let σ be the standard deviation of the distribution.

$$\frac{22 - 61}{\sigma} = -2.6$$

$\sigma = 15$

Let x be the mark of Mary.

$$\frac{x - 61}{15} = 1.4$$

$x = 82$

$\therefore$ Mary gets 82 marks.

If Albert gets the least mark and Mary gets the highest mark, then the range of the distribution is $82 - 22 = 60$ marks, which is greater than 59 marks.

Thus, the claim is not correct.

17. **HKDSE 2016 Q.17**

(a) Let d be the common difference of the sequence.

$666 + (38 - 1)d = 555$

$d = -3$

Thus, the common difference is -3.

(b) $\frac{n}{2}[2(666) + (n-1)(-3)] > 0$

$1335 - 3n > 0$

$3n < 1335$

$n < 445$

Thus, the greatest value of n is 444.

18. **HKDSE 2016 Q.18**

(a) $f(x) = \frac{-1}{3}x^2 + 12x - 121$

$= \frac{-1}{3}(x^2 - 36x) - 121$

$= \frac{-1}{3}(x^2 - 36x + 18^2 - 18^2) - 121$

$= \frac{-1}{3}(x - 18)^2 + 108 - 121$

$= \frac{-1}{3}(x - 18)^2 - 13$

$\therefore$ Vertex $= (18, -13)$

(b) Since the graph of $y = g(x)$ touches the x-axis, we have $g(x) = f(x) + 13$.

$g(x) = \frac{-1}{3}(x - 18)^2 - 13 + 13$

$g(x) = \frac{-1}{3}(x - 18)^2$

(c) $\frac{-1}{3}x^2 - 12x - 121 = f(-x)$

$y = f(x)$ is reflected about the y-axis.

19. **HKDSE 2016 Q.19**

(a) $\dfrac{\sin \angle ADB}{10} = \dfrac{\sin 86^\circ}{15}$

$\angle ADB \approx 41.68560132^\circ$ or $\angle ADB \approx 138.3143987^\circ$ (rejected)

$\angle ABD \approx 180^\circ - 86^\circ - 41.68560132^\circ \approx 52.31439868^\circ$

$\therefore$ $\angle ABD \approx 52.3^\circ$

$CD^2 = 8^2 + 15^2 - 2(8)(15)\cos 43^\circ$

$CD \approx 10.65246974$

$\therefore$ $CD \approx 10.7$ cm

(b) $AC^2 + BC^2 = 6^2 + 8^2 = 100$

$AB^2 = 10^2 = 100$

$\because$ $AC^2 + BC^2 = AB^2$

$\therefore$ $\angle ACB = 90^\circ$ (converse of Pyth. theorem)

$AD^2 = AB^2 + BD^2 - 2(AB)(BD)\cos \angle ABD$

$AD^2 \approx 10^2 + 15^2 - 2(10)(15)\cos 52.31439868^\circ$

$AD \approx 11.89964475$

$\cos \angle ACD = \dfrac{AC^2 + CD^2 - AD^2}{2(AC)(CD)}$

$\cos \angle ACD \approx \dfrac{6^2 + 10.65246974^2 - 11.89964475^2}{2(6)(10.65246974)}$

$\cos \angle ACD \approx 86.46867599^\circ \neq 90^\circ$

Hence, the angle between AB and the face BCD is not $\angle ABC$.

Thus, the claim is disagreed.

20. **HKDSE 2016 Q.20**

(a) In ΔOPJ and ΔQPJ,

$JO = JQ$	(radii)
$\because$ $JO = JP$	(radii)
$\therefore$ $\angle JOP = \angle JPO$	(base $\angle$s, isos. Δ)
$\because$ $JP = JQ$	(radii)
$\therefore$ $\angle JQP = \angle JPQ$	(base $\angle$s, isos. Δ)
Note that $\angle JPO = \angle JPQ$.	(in-centre of Δ)
$\therefore$ $\angle JOP = \angle JQP$	
$\therefore$ $\Delta OPJ \cong \Delta QPJ$	(A.A.S.)
$\therefore$ $OP = QP$	(corr. sides, $\cong \Delta$s)

(b) (i) Let $P = (p, 19)$.

$OP = PQ$

$\sqrt{p^2 + 19^2} = \sqrt{(p-40)^2 + (19-30)^2}$

$p^2 + 361 = p^2 - 80p + 1721$

$80p = 1360$

$p = 17$

$\therefore \quad P = (17, 19)$

Let the equation of C be $x^2 + y^2 + Dx + Ey + F = 0$.

Since C passes through $O(0, 0)$, we have $F = 0$.

Substitute $P(17, 19)$, we have $17^2 + 19^2 + 17D + 19E = 0$, i.e. $17D + 19E = -650$.

Substitute $Q(40, 30)$, we have $40^2 + 30^2 + 40D + 30E = 0$, i.e. $4D + 3E = -250$.

Solving the above two equations, we have $D = -112$ and $E = 66$.

$\therefore \quad$ Equation of C : $x^2 + y^2 - 112x + 66y = 0$.

(ii) Equation of L_1 : $y - 19 = \frac{3}{4}(x - 17)$

$3x - 4y + 25 = 0$

$\therefore \quad S = (-\frac{25}{3}, 0)$ and $T = (0, \frac{25}{4})$

Let (a, b) be the contact point of C and L_2.

$\frac{a + 17}{2} = 56$ and $\frac{b + 19}{2} = -33$

$a = 95$ $\quad$ $b = -85$

Equation of L_2 : $y + 85 = \frac{3}{4}(x - 95)$

$3x - 4y - 625 = 0$

$\therefore \quad U = (\frac{625}{3}, 0)$ and $V = (0, -\frac{625}{4})$

$TV = \frac{25}{4} - \left(-\frac{625}{4}\right) = \frac{325}{2}$

$$\text{Area of trapezium STUV} = \frac{1}{2}\left(\frac{325}{2}\right)\left(\frac{25}{3}\right) + \frac{1}{2}\left(\frac{325}{2}\right)\left(\frac{625}{3}\right)$$

$$\approx 17604.16667$$

$$> 17000$$

Thus, the claim is correct.