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THE SUPPLY OF MONEY AND COMMON STOCK PRICES

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I. INTRODUCTION

ALTHOUGH ECONOMETRIC TECHNIQUES have been applied with a degree of success in studies of the determinants of individual common stock prices, relatively little attention has been given to the use of these techniques in forecasting short run movements in aggregate indices of common stock prices. This lack of attention is surprising since accurate forecasts of the average level of stock prices are of obvious and practical value for determining the timing of stock market investment strategies. Furthermore, econometric forecasting techniques have the important advantage in this context that they yield results that are objective and quantitative and can be consistently replicated.

It remains true, of course, that the application of econometric techniques to stock market forecasting is difficult. The main problem is that any assumed relationship between stock prices and economic variables must depend critically on expectational factors. Thus, given the limited success econometricians have had in explaining the formation of expectations, it is apparent that a complete structural specification of the determinants of stock prices is not possible at this time. On the other hand, this problem need not preclude the derivation of partial relationships between economic variables and a stock index that are sufficiently stable to be useful in forecasting future movements of the index. Consequently, it appears worthwhile at this time to undertake at least an exploratory study of the value of econometric techniques in forecasting the average level of stock prices.

More specifically, the objective of this paper is to develop and estimate a relationship between the supply of money and an index of common stock prices, and then to evaluate the usefulness of this relationship as a forecasting tool in the implementation of investment strategies. The importance of the money supply as a determinant of stock prices may be derived both from the struc-

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tural link of the stock market with monetary conditions and from the role that the money supply plays as a general indicator of economic expectations. The omission of other economic variables from the relationship should not, however, be taken as a presumption that the money supply is the only important determinant of aggregate stock prices. Indeed, it is hoped that the results presented here will provide a basis for the development of more complete and structural specifications of the monetary and expectational variables that determine the average level of stock prices.¹

The plan of the paper is as follows. In Section II the theoretical links between the money supply and the stock market are developed and previous studies are briefly reviewed. In Section III the relationship between the money supply and a stock price index is estimated using the techniques of regression analysis. The estimates are based on quarterly data over the period 1954 to 1969. In addition to the stock market equation, an equation explaining the money supply is also estimated so that "outside of sample" forecasts may be made. In Section IV the stock market forecasts are used to simulate investment strategies over the period of the decade of the 1960s. The results of the paper are summarized and the principal conclusions are drawn in Section V.

II. THE RELATIONSHIP BETWEEN THE MONEY SUPPLY AND THE STOCK MARKET

The nature of the relationship between the money supply and common stock prices can be most easily described if a share of common stock is viewed as an asset that yields its return to the investor over time. The value of the share can then be written in terms of the present discounted value (PDV) of the expected dividends:

$$PDV_0 = \sum_{t=0}^{\infty} \frac{D_0(1 + g_t)^t}{(1 + r_t + \rho_t)^t}, \quad (1)$$

where D_0 is the level of current dividends, g_t is the expected growth rate of dividends at time t , and the expected discount rate consists of the riskless rate r_t and the risk premium ρ_t .² An investor should be willing, by definition, to pay a price equal to the PDV for any share. It is thus apparent that the price of any share of common stock will be determined by three variables: the level and growth rate of dividends, the riskless rate of interest, and the risk premium. It now will be shown that the money supply is positively related to the level and growth rate of dividends, and negatively related to the riskless rate of interest and the risk premium. The average level of stock prices will, consequently, be positively related to the money supply.

The main channel for the influence of the money supply on dividends

1. One attempt at estimating such a relationship for use in simulating monetary effects within a macroeconomic model is described in Robert H. Rasche and Harold T. Shapiro, "The F.R.B.-M.I.T. Econometric Model: Its Special Features," *American Economic Review*, LVIII, 2 (May, 1968), pp. 136-137.

2. A fuller discussion of this common stock valuation formula can be found in Burton G. Malkiel, "Equity Yields, Growth, and the Structure of Share Prices," *American Economic Review*, LIII, 5 (December, 1963), pp. 1004-1031.

operates through the firm's current and expected earnings. Given the demand for money, a decrease in the supply of money will raise interest rates and reduce interest sensitive expenditures such as capital investment. The decrease in expenditures, together with the standard multiplier, will then cause a reduction in the firm's sales and thus a decrease in its earnings. The timing of the effect of the decreased earnings on dividends may depend on the firm's cash flow and liquidity position, but ultimately the full effect must be a decrease in dividends. Although the current price of the common stock share will fall if current dividends are reduced, the main point of leverage for the effect of the money supply is on the expected growth rate of dividends. It is for this reason that the expectational effects of the money supply on dividends are at least as important as any actual short run effects in determining the response of the share price.

The influence of the money supply on the riskless interest rate component of the investor's discount rate is a direct function of the effect of the money supply on market interest rates. The explicit increase in market interest rates caused by increased monetary tightness may, moreover, be reinforced by credit rationing in the loan markets. In this case, monetary tightness will raise the discount rate by an amount greater than would otherwise be indicated by the level of market interest rates alone. As with the effect of the money supply on dividends, it should again be stressed that the major impact of the money supply on the riskless interest rate is in terms of the change in the expected values for future levels of this rate rather than in the actual change in the current value.

The influence of the money supply on the risk premium component of the investor's discount rate is more difficult to quantify. The risk component arises because of the uncertainty associated with future values of the growth rate of dividends and the level of the riskless interest rate. Assuming the investor is a risk-avertter, the risk premium will be positive and it will be positively associated with increased uncertainty. The effect of monetary tightness on the risk premium thus must operate by increasing the uncertainty with which the investor's expectations are held. This suggests that increases in the degree of monetary tightness may be as important as the level of the monetary tightness in influencing the risk premium.

Although the basic channels for the effect of the money supply on the price of common stock shares can be distinguished in the preceding manner, an explicit form for the relationship must be specified if the relationship is to be estimated and used for forecasting. One approach to this problem has been developed by Beryl Sprinkel.³ By comparing the turning points in a stock price index with the turning points in the growth rate of money (narrowly defined with a six month moving average), Sprinkel formulated an investment rule that, "a bear market in stock prices was predicted 15 months after each peak in monetary growth, and that a bull market was predicted two months after each monetary growth trough was reached."⁴

3. Beryl Sprinkel, *Money and Stock Prices*, (Homewood, Illinois: Richard D. Irwin, Inc), 1964.

4. Sprinkel, *op. cit.*, p. 120.

This investment rule worked well over most of Sprinkel's sample period from 1918 to 1960, but it has not worked well more recently in the decade of the 1960s. The problem encountered is that variations in the growth rate of the money stock have become less continuous and that the cycles that do exist are less than 15 months. This is illustrated in Table 1 which shows the six

TABLE 1
GROWTH RATES OF THE MONEY SUPPLY: 1960-1 TO 1969-4
(Annual Rates in Percentage Points)^a

1960-1	-3.28	1965-1	3.32
1960-2	-3.00	1965-2 T	2.64
1960-3 T	-.68	1965-3	4.00
1960-4	.80	1965-4	5.88
1961-1	1.12	1966-1 P	6.60
1961-2	2.40	1966-2	5.48
1961-3	2.48	1966-3	1.64
1961-4	3.12	1966-4 T	-.28
1962-1 P	3.20	1967-1	1.96
1962-2	1.84	1967-2	4.84
1962-3 T	.32	1967-3 P	8.00
1962-4	.80	1967-4	7.60
1963-1	3.48	1968-1 T	5.32
1963-2 P	4.04	1968-2	6.20
1963-3 T	3.72	1968-3 P	7.88
1963-4 P	4.04	1968-4	7.12
1964-1	3.36	1969-1	5.80
1964-2 T	2.88	1969-2	5.16
1964-3	4.56	1969-3	2.92
1964-4 P	5.08	1969-4	.92

^a Money defined as seasonally adjusted demand deposits plus currency outstanding, six month moving average at end of quarter.

Source: *Federal Reserve Bulletin*, "Money Supply and Related Data."

month average of the money supply growth rate on an end of quarter basis for the period 1960-1 to 1969-4. Local peaks and troughs in the monetary growth rate series have been marked in the table with the letters P and T respectively. Because Sprinkel's rule indicates a 15 month (5 quarter) lag in the sale of stocks after a peak in the monetary growth rate is reached, peaks and troughs in the growth rate series must be separated by at least this interval if the rule is to be implemented. It can be seen from the table that, in fact, the maximum span between peaks and troughs has been 9 months (3 quarters) with the exception of the most recent peak. The result is that the sell signals (15 months after a peak) are all canceled by buy signals (2 months after a trough) before they can be implemented, and thus an investor using the Sprinkel rule would have simply remained in stocks throughout the full period beginning with the initial buy signal in 1960-3.

In this study the relationship between the money supply and the stock market is estimated using the techniques of regression analysis. In contrast to the turning point method of Sprinkel, regression analysis has several important advantages. First, it allows for more flexibility in the specification of lagged relationships. In particular, specific peaks in the money stock and

monetary growth rate series do not have to be explicitly identified. Second, the forecasts generated by a regression equation are quantitative and the standard error of the forecast can be calculated. This aspect has the useful feature that the uncertainty associated with the forecast can be taken into account when formulating an investment strategy. Finally, regression analysis allows for more continuity in the forecasts since the estimated relationship can be easily revised as more data become available. In the following section the estimation of a stock market equation using regression analysis is described.

III. ESTIMATION OF THE FORECASTING RELATIONSHIPS

The three primary variables to be used in estimating the stock market equation are the level of stock prices, denoted by SP, the level of the money supply, denoted by M, and the rate of growth of the money supply, denoted by G. Stock prices are represented by the Standard and Poor's Corporation combined index of 500 stocks, measured on the last Friday of the quarter. The series is expressed in relatives with the average value for the base period (1941-1943) equal to 10. The money supply series is the quarterly average of the sum of adjusted demand deposits and currency outstanding, seasonally adjusted and measured in billions of current dollars. The money supply growth rate is calculated as $(M - M_{-1})/M_{-1}$, and is measured at a quarterly rate in percentage points. The data were collected for the period from 1954-1 to 1969-4. The beginning of the period was chosen in 1954 to provide the longest possible post-war sample without including the unusual conditions in the periods immediately following World War II and during the Korean War.

The equation was specified with both the level and the growth rate of the money supply, the latter being included because it may be particularly useful in accounting for short run variations in expectations. Because there is little *a priori* information available on the specific form of the relationship, a number of regressions were fitted with alternative lag structures on the money supply and monetary growth rate.⁵ A major problem that arose with these estimates was that the error terms indicated high serial correlation. This was taken into account in further estimates by assuming that the error terms of the equation follow a first order serial correlation pattern. The coefficient of serial correlation was then estimated by including the lagged error term in the regression and using the iterative technique of Cochrane and Orcutt.⁶

The relationship which yielded the best fit was found when the level of the money supply variable was included with its current value and the growth rate of the money supply was specified with both its current value and a one quarter

5. A more detailed discussion of the various specifications that were tested can be found in Kenneth E. Homa, "Money and Stock Prices: An Econometric Study," unpublished senior thesis, Princeton University, April, 1970. See also Joseph S. Mascia, "Monetary Changes and Equity Values," *The Bankers Magazine* (Summer, 1969).

6. The assumption of first order serially correlated errors implies that if the error in estimation in period $t-1$ is u_{t-1} , then the error in period t resulting from the serial correlation will be ρu_{t-1} , where ρ is the coefficient of serial correlation. This effect is taken into account in estimating the relationship by including the lagged residual u_{t-1} in the equation and estimating the serial correlation coefficient ρ . See D. Cochrane and G. H. Orcutt, "Application of Least Squares Regression to Relationships Containing Auto-Correlated Errors," *Journal of American Statistical Association*, March, 1949.

lag. The estimated coefficients for this specification over the sample period from 1954-4 to 1969-4 (taking into account the lagged variables, the sample actually begins in 1954-1) were:⁷

$$SP = -26.77 + .61M + 3.14G + 1.46G_{-1} + .87u_{-1} \quad (2)$$

(1.11) (4.13) (3.16) (1.46)

$$R^2 = .968 \quad S_e = 3.70 \quad D.W. = 2.14$$

The money supply variables all have the expected positive sign, and M and G are statistically significant. Also, over 96 per cent of the variance is explained by the specification. Offsetting this high correlation, however, is the large coefficient of serial correlation (.87), which implies that 87 per cent of the previous period's error would recur if the serial correlation were not taken into account. The effect of this high serial correlation on the forecasting accuracy of the equation can be evaluated in the simulation experiments discussed below.⁸

The stock market forecasts generated by equation (2) will be used in Section IV in simulations of investment strategies over a period from 1960-1 to 1969-4. To insure that the forecasts are strictly "outside the sample," it is necessary that two restrictions be considered. First, it is important that the coefficient estimates incorporate only data which would have in fact been available at the time of the forecast. For example, the coefficient estimates used to generate the stock market forecast for 1960-1 should be based only on data for 1959-4 or earlier; the coefficients for the 1960-2 forecast should use only data for 1960-1 or earlier, and so on. To obtain these estimates, 40 additional regressions were fitted using the specification of equation (2) and sample periods each beginning in 1954-4 and ending in the quarters 1959-4 to 1969-3 respectively. These coefficients were then used to generate all the stock market forecasts discussed below.⁹

The second factor that must be considered to insure valid outside of sample forecasts concerns the data that is used with the respective sets of coefficients. The problem arises because the current value of the money supply enters the specification of equation (2) through the M and G variables. Consequently, to use the coefficients to forecast the stock market one period ahead, it is necessary to use one period ahead predictions of the money supply. The accuracy of the stock market forecasts is thus dependent on the ability of the investor to make these money supply predictions. In this study three alternative methods for the prediction of the money supply are used. The first two of

7. The absolute values of T-statistics are shown in parentheses and u_{-1} is the lagged residual used to correct for the serial correlation (see footnote 6). The T-statistics can be only indicative, of course, because of the experimentation used to find the lag structure.

8. An additional problem arises from the potential bias in the estimated coefficients due to simultaneous equations bias. Tests with instrumental estimates of the money supply series, however, did not yield significantly different coefficient estimates, and thus the ordinary least squares fits have been used.

9. It should be noted that the full sample was used to find the appropriate lag structure for equation (2), and thus the forecasts do contain ex-post information to this extent. Since the estimates were not very sensitive to the form of the lag structures, this is unlikely to be an important source of bias.

these methods are: (i) perfect foresight of M and (ii) naive extrapolation of M . For assumption (i) it is assumed that the investor can perfectly predict the money supply one period ahead, and thus the actual values of the money supply are used in generating the forecast values of the stock market. For assumption (ii), it is assumed that the investor predicts the money supply by simply extrapolating the current growth rate of the money supply to the forecast period. The trend extrapolation of assumption (ii) is thus a naive method of prediction that could always be duplicated by an investor.

The stock market forecasts generated by the money supply assumptions (i) and (ii), using the coefficient estimates described above, are shown in columns (2) and (3), respectively, of Table 2. The summary statistics at the bottom of the columns show the correlation (R^2), root-mean-squared-error, and mean error between the forecast value and the actual value of the stock market (shown in column (1)). Because the stock market forecasts are generated by coefficients estimates based on outside of sample procedures, and because in the case of assumption (ii) the current money supply data are not used, the correlation between the forecast stock market series and the actual series could, in principle, be significantly lower than the correlation observed for equation (2) fitted over the entire sample. It is thus quite encouraging that the summary statistics indicate a goodness of fit between the forecast and actual series that is still very high. Furthermore, the correlation for the forecast values generated by the naive extrapolation of M is only slightly lower than the correlation for the forecasts based on perfect foresight of M ; this suggests the stock market forecasting equation may be reasonably robust to errors in predicting the money supply.

In order to further test the stock market equation's ability to provide accurate forecasts, a third method of generating predicted values for the money supply series, regression analysis, was also attempted. The method involved estimating a money supply prediction equation, on the premise that it would improve on the naive predictions of assumption (ii), while using only data actually available at the time of the forecast. The dependent variable in the equation was the growth rate of the money supply, G , and the equation was specified as a reduced form so that both supply variables depicting the factors motivating Federal Reserve behavior and demand variables reflecting the demand for money could be included. In preliminary experiments the coefficients of the demand variables were unstable and not statistically significant, however, so only the supply variables were included in the final specification:

$$G = a_0 + a_1 U^2_{-1} + a_2 P_{-1} + a_3 BOP_{-1} + a_4 G_{-1}, \quad (3)$$

where U is the unemployment rate, P is the rate of inflation, and BOP is a variable positively related to the U.S. international reserve position.¹⁰

The first three of the variables entered in the equation represent the policy

10. The data sources for these variables are as follows:

U: civilian unemployment rate from the BLS Household Survey;

P: private output price deflator from the National Income Accounts;

BOP: the ratio of U.S. reserve assets (gold stock and IMF reserve position) to total U.S. short term liabilities to foreigners, measured as a deviation from a four quarter moving average.

TABLE 2
ACTUAL AND FORECAST STOCK MARKET SERIES: 1960-1 TO 1969-4

Period	Actual SP (1)	Prediction of M based on:		
		Perfect Foresight (assumption (i)) (2)	Naive Extrapolation (assumption (ii)) (3)	Regression Prediction (assumption (iii)) (4)
1960-1	55.98	56.44	56.55	58.04
1960-2	57.68	53.82	53.44	54.18
1960-3	53.52	56.51	54.43	54.93
1960-4	57.44	54.06	54.47	54.36
1961-1	64.42	56.77	56.19	56.66
1961-2	64.64	63.49	63.02	63.63
1961-3	66.73	64.68	65.07	65.69
1961-4	71.55	68.27	67.16	67.77
1962-1	69.55	72.54	73.76	73.08
1962-2	54.75	70.73	71.40	70.52
1962-3	56.27	57.17	58.58	57.82
1962-4	62.96	59.45	57.41	57.32
1963-1	66.57	67.39	65.65	64.06
1963-2	69.37	69.65	70.41	69.96
1963-3	72.13	72.42	72.23	72.09
1963-4	74.44	75.39	75.06	74.96
1964-1	79.19	76.32	77.68	77.89
1964-2	81.46	81.03	80.50	81.02
1964-3	84.21	85.58	83.42	83.78
1964-4	84.15	87.32	88.62	87.68
1965-1	86.20	86.38	87.83	86.99
1965-2	83.06	88.71	88.39	89.19
1965-3	90.02	88.17	86.39	87.51
1965-4	92.43	95.45	94.23	92.21
1966-1	89.54	97.84	97.86	97.33
1966-2	86.58	93.59	95.33	94.11
1966-3	76.56	85.54	90.05	88.38
1966-4	81.47	77.33	75.91	79.24
1967-1	90.20	86.15	82.74	84.00
1967-2	90.64	95.79	92.62	95.74
1967-3	96.71	98.38	94.44	94.49
1967-4	96.47	97.33	102.01	91.22
1968-1	90.20	99.07	99.39	94.44
1968-2	99.58	94.85	92.65	93.00
1968-3	102.31	104.96	103.59	101.32
1968-4	104.74	103.55	106.52	108.19
1969-1	101.51	106.97	106.99	104.85
1969-2	97.33	102.95	104.28	101.49
1969-3	94.16	95.54	99.09	96.01
1969-4	91.18	93.28	94.09	94.11
Correlation (R^2)		.923	.895	.908
Root-Mean-Squared Error		4.727	5.540	4.847
Mean Error		-1.324	-1.289	-.784

indicators to which the Federal Reserve normally reacts. The coefficients of U and BOP should be positive, assuming the Federal Reserve increases the growth rate of the money supply in periods of high unemployment and a good

international reserve position. The coefficient of P should be negative, assuming the Federal Reserve decreases the growth rate of M in periods of inflation. The unemployment rate is entered as a squared term in the specification since this worked slightly better than the linear form. The lagged value of the growth rate of the money supply is included because inertia on the part of the Federal Reserve would lead to correlation between the present and the lagged growth rate; this variable should thus enter with a positive sign in the equation.¹¹

Equation (3) was initially fitted with the Cochrane and Orcutt method using the same procedures developed for the stock market equation; that is, the initial period for the sample was set in 1954-4 and then 40 regressions were fitted with the end of the sample moving from 1959-4 to 1969-3. Because the explanatory variables all enter the equation with a lag, the problem of predicting them does not arise as it did for the money supply in the stock market equation. The results were not entirely satisfactory, however, because the estimated coefficients varied considerably over the set of regressions. This is perhaps not altogether surprising since the Federal Reserve's priorities, in terms of the variables listed above, may have also changed over the period. In order to at least take this possibility into account, the final set of regressions was estimated using a moving sample of 20 quarterly observations (5 years): the first regression was estimated over the sample 1955-1 to 1959-4, the second was estimated over the sample 1955-2 to 1960-1, and so on to the last regression estimated over the sample 1964-4 to 1969-3. The coefficients for each of these equations were then used to calculate the predicted value of the money supply series for 1 quarter beyond the respective sample period. To illustrate the nature of these results, the estimated coefficients for three sample periods spanning the full sample are shown:

Sample: 1955-1 to 1959-4 (4a)

$$G = -.53 + 2.61 U_{-1}^2 + .20 P_{-1} + .58 BOP_{-1} + .36 G_{-1} + .01 u_{-1}$$

(1.62) (2.65) (1.73) (1.47) (1.31)

$$R^2 = .529 \quad S_e = .413 \quad D.W. = 1.61$$

Sample: 1960-1 to 1964-4 (4b)

$$G = .70 + .60 U_{-1}^2 + .15 P_{-1} + 6.06 BOP_{-1} + .26 G_{-1} + .11 u_{-1}$$

(1.23) (1.40) (1.12) (1.79) (1.26)

$$R^2 = .544 \quad S_e = .385 \quad D.W. = 2.12$$

Sample: 1965-1 to 1969-4 (4c)

$$G = 3.07 - 8.47 U_{-1}^2 - 14.17 P_{-1} + .12 BOP_{-1} + .37 G_{-1} + .58 u_{-1}$$

(1.97) (1.87) (3.43) (1.26) (1.99)

$$R^2 = .692 \quad S_e = .447 \quad D.W. = 2.02$$

11. Several other variables were also tested in the equation, in particular the potential GNP gap and lagged changes in the Federal Reserve's discount rate and required reserve ratio. These variables were significant when included alone, but were not significant when tested with the full specification, presumably because of multicollinearity.

The goodness of fit shown in these results is reasonable, recalling that the dependent variable is the growth rate of the money supply. The coefficient estimates, on the other hand, are disappointing because many are not significant and in three cases the sign is not correct. It should be noted, however, that the unemployment rate is the most significant variable in the 1954 to 1959 sample, the international reserve position variable is the most significant variable in the 1960 to 1964 sample, and the rate of inflation is the most significant variable in the 1965 to 1969 sample. These results are thus broadly consistent with the acknowledged changes in the Federal Reserve System's priorities over the sample period. The insignificant coefficient estimates, moreover, may be attributed to multicollinearity, since the variables were all generally significant and had the correct sign when regressed alone against G . Consequently, the goodness of fit of the equation may still be a valid indicator of the equation's predicting accuracy. In addition, the relatively small coefficient on the lagged growth rate of the money supply is also an encouraging factor, because it suggests that the predicted values for the money supply generated by this equation will be different from, and presumably an improvement on, the values implied by the naive extrapolation method of assumption (ii) discussed above.

The forecast values for the stock market that are generated by the money supply predictions of the regression equation are shown in column (4) of Table 2. The correlation of the forecast stock market series with the actual stock market series is .908. As should be expected, this correlation is less than the correlation of the actual stock market series with the assumption (i) forecast series based on perfect foresight of M , but it is greater than the correlation of the actual series with the assumption (ii) forecast series based on naive extrapolation of the growth rate of M . In the following section, these three series of stock market forecasts are compared in simulations of investment strategies.

IV. SIMULATION OF INVESTMENT STRATEGIES

A. Description of the Simulations

The objective of the simulation experiments presented in this section is to evaluate how well an investor would have fared using the stock market forecasts developed in the preceding section. Under the conditions described below, a hypothetical investor will follow a realistic investment strategy, basing his choice on information that would have been available to a real life investor. The pattern of decisions will thus reflect an investment scheme that could have been actually followed in practice by an investor.

The hypothetical investor will have only two financial media at his disposal—common stocks and Treasury bills. The former are represented by Standard and Poor's composite index of 500 stocks. The investor will buy and sell shares, or fractions of shares, as the case may be, of the aggregate equity indicator. For example, if the Standard and Poor index is currently at 81.00 and the investor wishes to invest \$40.50, it is assumed that he is able to purchase $1/2$ of a share of the composite indicator. Three month Treasury bills serve

as the proxy for the investor's Treasury bill alternative. For purposes of simplicity, it is assumed that the investor purchases a new issue at the ask yield, holds it until the 90 day maturity is reached, and then redeems the notes at face value. Consequently, there are no explicit transactions costs involved in either the purchase or sale of the bills.

The investor will be permitted only one investment decision per quarter, and it will be made at the very beginning of the period. The first choice will be made at the beginning of 1960-1 and the last choice at the beginning of 1969-4. The stock index and Treasury bill yield are measured on the last Friday of each quarter to coincide with the timing of the investor's decisions. The investment strategy is based on the premise that the investor attempts to maximize his expected profits. At each decision point he will compare the expected return on stocks with the yield on bills, and invest all his capital in the alternative that offers the greater expected profit potential. The expected return on stock is defined to include not only the predicted capital appreciation indicated by the stock market forecast, but also the dividends that accrue to the stock over the holding period. Because dividends are typically announced nearly a quarter ahead of the actual payment date, it is assumed that the investor uses the actual dividend rate for the forecast period in calculating his return from holding stocks.¹²

In one set of simulations, the investor will also be permitted the option to go on margin. In order to simplify the computer programming, it is assumed that a 50 per cent margin rate was available throughout the period and that, if the investor uses the margin option, then he uses it to the full extent. The interest cost of borrowing funds for the margin account is calculated at the same rate as the yield on Treasury bills.¹³ The investor will choose to go on margin for those periods in which he is in stocks, whenever the expected return on stocks exceeds the standard error of estimate of the most recent stock market forecasting equation he has available.¹⁴ By using this method, the investor's willingness to go on margin becomes a function of the expected gain from the stock portfolio relative to the estimated uncertainty attendant with this form of investment.

Transactions costs are also included in the decisions made by the investor. It is assumed that the cost of buying or selling equity claims is one per cent of the total dollar value of the transaction. Thus on any movement in or out of stocks the investor's capital is reduced by one per cent before the calculation of the expected returns for that period. On the other hand, if the investor decides to maintain his current position for another quarter, whether it be in stocks or bonds, then no transactions costs are collected. This will motivate

12. The dividend rate data for the simulations were obtained from the "Bond and Stock Yields" table in the *Federal Reserve Bulletin*.

13. The Treasury bill rate is used here as a representative yield for the opportunity cost of the investor when he is in stocks and as the cost of borrowing when he is on margin. The correct value for the opportunity cost will, of course, vary between investors, depending on the investment alternatives they have available and on the liquidity of their non-stock portfolios.

14. This condition is slightly modified, as explained in footnote 15, to take into account transactions costs when the investor is already on margin and is deciding whether to continue in this position.

the investor to remain in his current position in a manner that is characteristic of actual investment decisions.

The possibility of going on margin also introduces one additional complication in the calculation of the transactions costs. If the investor is moving into stocks with margin from a previous bond position, he then must pay transactions costs both on the investment of his basic capital and on the amount of stock purchased with borrowed funds. Thus if he began the period with capital in the amount C , his portfolio of common stocks will be $2(.99)(C)$, and it is on this portfolio that his rate of return is calculated. He will also have to pay the interest costs of borrowing the amount C for the margin account, and this cost is deducted from his profits at the end of the period. If the investor's initial position is in stocks, but without margin, and he then decides to go on margin, he pays transactions costs only on the purchases necessary for the margin account. Similarly, if his initial position is in stocks with margin and he decides to continue on margin, then no transactions costs are incurred.¹⁵ Finally, in all cases, an investor who sells stock in his margin account must pay the one per cent transactions costs for the sale of the total portfolio.¹⁶

B. *Results of the Simulations*

This section reports the results of 6 simulations using the stock market forecasts developed in Section III under the ground rules described above. The 6 simulations differ in the money supply predictions that are used to generate the stock market forecasts and in whether margin purchases are allowed:¹⁷

Simulation #	Money Supply Predictions Based on:	Margin Allowed:
1	perfect foresight	no
2	naive extrapolation	no
3	regression prediction	no
4	perfect foresight	yes
5	naive extrapolation	yes
6	regression prediction	yes

The investment decisions and the results of simulation 1 are shown in Table 3.¹⁸ The summary statistics for the six simulations are shown in Table 4. The column headings for Table 3 are described as follows:

15. The absence of transactions costs when the investor is considering continuing on margin should motivate him to do so even though he might not invest in a new margin position. This is taken into account in the simulations by adding 1 per cent to the expected return from the stock market that is used in making the decision whether to continue on margin.

16. Two minor elements of transactions costs have been omitted from the analysis for the sake of simplicity. First, the model assumes that dividends are automatically reinvested in stock at the end of each period without the normal transactions costs being charged. Second, in principle the investor would have to pay transactions costs when stock from his margin account is sold in order to pay the interest on the account, but in the model it is assumed that he can sell the stock for this purpose without cost.

17. Simulations allowing for variations in transactions costs and the treatment of dividends are reported in Homa, *op. cit.*, pp. 83-99.

18. Corresponding tables for simulations 2 to 6 have been omitted to save space. They are available upon request to the authors.

TABLE 3
SIMULATION 1—PERFECT FORECAST OF M; NO MARGIN

Time	Begin. Capital	ER (SP)	ER (SP) -Trans	Bills	Bills -Trans	Investment Decision	R (SP)	Gross Yield	Net Act Profit
60-1	100.00	-0.0354	-0.0451	0.0108	0.0007	BUY BILLS	-0.0432	0.0108	1.08
60-2	101.08	-0.0304	-0.0401	0.0073	-0.0027	BUY BILLS	0.0391	0.0073	0.74
60-3	101.83	-0.0118	-0.0217	0.0054	-0.0047	BUY BILLS	-0.0640	0.0054	0.55
60-4	102.38	0.0189	0.0087	0.0061	-0.0039	BUY STOCKS	0.0827	0.0827	7.35
61-1	109.73	-0.0039	-0.0138	0.0054	-0.0047	HOLD STOCKS	0.1303	0.1303	14.30
61-2	124.03	-0.0072	-0.0171	0.0060	-0.0040	BUY BILLS	0.0108	0.0060	-0.50
61-3	123.33	0.0080	-0.0021	0.0058	-0.0043	BUY BILLS	0.0399	0.0058	0.71
61-4	124.24	0.0304	0.0201	0.0056	-0.0044	BUY STOCKS	0.0799	0.0799	8.59
62-1	132.83	0.0214	0.0112	0.0067	-0.0034	HOLD STOCKS	-0.0208	-0.0208	-2.76
62-2	130.07	0.0256	0.0153	0.0069	-0.0032	HOLD STOCKS	-0.2061	-0.2061	-26.81
62-3	103.26	0.0537	0.0431	0.0071	-0.0030	HOLD STOCKS	0.0371	0.0371	3.83
62-4	107.08	0.0659	0.0552	0.0068	-0.0033	HOLD STOCKS	0.1288	0.1288	13.79
63-1	120.87	0.0792	0.0684	0.0072	-0.0028	HOLD STOCKS	0.0660	0.0660	7.98
63-2	128.85	0.0546	0.0440	0.0072	-0.0029	HOLD STOCKS	0.0503	0.0503	6.48
63-3	135.33	0.0521	0.0416	0.0074	-0.0027	HOLD STOCKS	0.0479	0.0479	6.49
63-4	141.81	0.0533	0.0427	0.0084	-0.0017	HOLD STOCKS	0.0400	0.0400	5.68
64-1	147.49	0.0331	0.0228	0.0087	-0.0013	HOLD STOCKS	0.0719	0.0719	10.61
64-2	158.10	0.0309	0.0206	0.0088	-0.0013	HOLD STOCKS	0.0364	0.0364	5.76
64-3	163.86	0.0584	0.0478	0.0085	-0.0016	HOLD STOCKS	0.0415	0.0415	6.80
64-4	170.66	0.0446	0.0342	0.0088	-0.0013	HOLD STOCKS	0.0068	0.0068	1.15

TABLE 4
SUMMARY STATISTICS FOR SIMULATIONS

Simulation	Total Return on Investment	Compounded Annual Yield (Full Period)	Standard Deviation (Full Period)	# of Periods in Stocks	Compounded Annual Yield (In Stocks)	Standard Deviation (In Stocks)
(1) Perfect Foresight of M. No Margin	1.456	.094	.235	31	.114	.262
(2) Naive Extrapolation of M. No Margin	.655	.052	.235	32	.060	.262
(3) Regression Prediction of M. No Margin	1.282	.086	.231	31	.104	.258
(4) Perfect Foresight of M. With Margin	3.061	.150	.396	31	.189	.437
(5) Naive Extrapolation of M. With Margin	.665	.052	.368	31	.060	.409
(6) Regression Prediction of M. With Margin	1.761	.107	.302	31	.131	.336
(7) Buy and Hold Stocks	1.081	.076	.268	40	.076	.268

TIME	identification for the beginning of the quarter in which the decision is made;
BEGIN. CAPITAL	the capital that is available to the investor at the beginning of the quarter;
ER(SP)	the expected quarterly rate of return from holding stocks; the rate of return includes both dividend payments and expected capital appreciation; ¹⁹
ER(SP)-TRANS	the expected return on stocks less the transactions cost incurred in the purchase of stocks;
BILLS	the quarterly rate of return from Treasury bills;
BILLS-TRANS	the return on bills less the transactions cost incurred in the purchase of stocks when the investor moves from bills to stocks; ²⁰
INVESTMENT DECISION	the investor's portfolio decision, which must be one of three alternatives: (i) buy bills, (ii) buy stocks, or (iii) hold stocks;
R(SP)	the actual realized return on stocks (dividends plus capital appreciation);
GROSS YIELD	the gross yield earned by the investor during the period; this must equal either <i>BILLS</i> or <i>R(SP)</i> , depending on whether the investor held bills or stock during the period;
NET ACT PROFIT	the net profits earned by the investor during the period; this must equal the product of the gross yield and the beginning capital, less any transactions costs and, in the case of margin, less the costs of borrowing.

Simulation 1: Perfect Foresight of M; No Margin

The results for this simulation, shown in Table 3, are worth tracing out in detail, since the format for the other simulations is essentially the same. The investor starts at the beginning of 1960-1 with \$100 in capital, which is assumed to be in the form of cash. To make the decision whether to buy bills or to buy stock, the investor compares the expected return on stocks net of the transactions cost ($-.0451$) with the yield on bills (.0108), and, since the bill yield is higher, his decision is to buy bills. The gross yield on the portfolio for the period is thus 0.108 and the profits are 1.08. It can be noted that his decision for this quarter was correct since stocks actually fell by $-.0432$ per cent.

Continuing into 1960-2, the investor's initial capital is now updated by the profits he earned during the first period. He again makes the portfolio decision

19. The expected rate of capital appreciation is calculated as the percentage growth between the actual value of SP at the end of the preceding period (see column (1) of Table 2) and the forecast value for SP generated by the stock market equation (see columns (2) and (4) of Table 2).

20. It is to be stressed that a transactions cost is not incurred in the model when Treasury bills are purchased. The information in this column simply facilitates the description of the investor's decisions as will be apparent below.

by comparing the expected return on stocks net of the transactions cost with the bill yield, and his decision is again to buy bills. The same decision is also made in 1960-3. In 1960-2 this was not the correct decision because the stock market actually rose by almost 4 per cent, but the bills purchase was correct in 1960-3 when the stock market fell by over 6 per cent. In fact, given the transactions costs of buying stocks at the beginning of 1960-2 and then selling them at the end of the quarter, the investor's decision to buy bills in 1960-2 was not a costly error.

In 1960-4 the investor chose to buy stocks for the first time. This decision was made because the expected return on stocks net of the transactions cost (.0087) exceeded the yield on bills (.0061). The gross yield in this quarter thus equals the return on stocks, and the net profits reflect this return after deducting the transactions costs involved in the stock purchase.²¹ The decision was very timely because in this period the stock price index appreciated by over 8 per cent.

The third alternative open to the investor is illustrated in the next quarter, 1961-1, when the decision is made to continue holding stocks. This decision was based on a comparison of the expected yield on stocks, ER(SP), with the bill yield net of the transactions cost of selling stocks, BILLS-TRANS. The transactions costs are not subtracted from the expected stock yield because the costs are not incurred if the investor maintains his stock position. The transactions costs are subtracted from the bill yield, on the other hand, since the investor would incur a transactions cost if he sells stocks in order to purchase the bills. The decision in 1961-1 was again correct; stocks rose by over 13 per cent during this period. Again, the investor's profits are calculated as the product of his beginning of period capital and the gross yield.

The final alternative open to the investor is illustrated in 1961-2 when the investor moved from stocks to bills. In this period ER(SP), $-.0072$, was less than BILLS-TRANS, $-.0040$. The investor also continued to hold bills in 1961-3. The purchases of bills in 1961-2 and 1961-3 were wrong since the stock market rose in both periods and the investor unnecessarily incurred transactions costs.

The investor returned to stocks in 1961-4 and continued in this position through the end of 1966-2. During this period the decision to hold stocks was generally correct, even though the investor suffered a significant loss during the market decline of 1962-2. In 1966-3 the decision was a return to holding bills. The decision to buy bills at this time was correct, since the market actually fell by over 10 per cent in the quarter. The purchase of bills in 1966-4, on the other hand, was wrong, but the investor recovered his loss by purchasing stocks in 1967-1. He then continued to hold stocks through the end of 1969-2, during which time the market generally rose. In 1969-3 and 1969-4 the investor moved back to bills and both of these decisions were correct. The investor's

21. The arithmetic involved in this calculation is somewhat complicated. Letting C be the original capital, t the rate of transactions costs (1 per cent in fact), and r the gross yield on the stock portfolio, the amount of profit is then $(1 - t)(r)(C) - (t)(C)$, where the first term represents the return on the portfolio after payment of the transactions cost and the second term is the deduction of the transactions cost from the capital.

capital at the end of the simulation, \$245.60, is shown as the last entry in the table.²²

The summary statistics for the investor's earnings following the simulation (1) strategy are shown in the first row of Table 4. The first column in the table shows the total return on the investment, which is calculated as the ratio of his profits over the period (\$145.60) to his initial investment (\$100.00). The second column shows this return converted to a compounded annual yield (C.A.Y.) basis. The third column shows the standard deviation of the quarterly returns converted to an annual basis for comparison with the C.A.Y. The fourth column shows the number of periods in which the investor was holding stocks, and the fifth and sixth columns show the C.A.Y. and the standard deviation for only those periods in which the investor was holding stocks. The information in these last two columns will allow the reader to calculate the total C.A.Y. and standard deviation for alternative assumptions about the investor's opportunity cost of holding stocks.

The summary statistics for simulation (1) can be compared with the return and the standard deviation that would have been the result of following a "buy and hold" strategy; that is, purchasing stocks in 1960-1 and holding them through the end of 1969-4. The summary statistics for the buy and hold strategy are shown in the last row of Table 4. In terms of overall performance, the C.A.Y. of .094 earned by simulation (1) compares favorably with the C.A.Y. of .076 resulting from a buy and hold strategy. The simulation (1) results also dominate the buy and hold strategy in terms of risk, since the standard deviation of the returns from simulation (1), .235, is less than the standard deviation of the returns from buy and hold, .268. This indicates that the greater return of simulation (1) could even have been obtained with smaller quarterly fluctuations in the return.

Simulation 2: Naive Extrapolation of M; No Margin

The conditions for this simulation differ from simulation (1) in that the naive trend extrapolations for the money supply are used in generating the forecast stock market series. Only the distinguishing features of the simulation are discussed here (see footnote 18). The investment decisions of simulation (2) differ from simulation (1) in that bills were purchased in 1961-1 and 1961-4, and stocks were purchased in 1966-3, 1969-3 and 1969-4. The simulated investment decision made in each of these quarters was wrong. That is, the stock market forecasts generated by the naive extrapolation of the money supply failed to predict the magnitude of the rise in the stock market that occurred in 1961-1 and 1961-4, and it failed to forecast the decline in the stock market that actually occurred in 1966-3, 1969-3, and 1969-4.

Turning to the summary statistics for simulation (2) in Table 4, it is not surprising that the results for this simulation are inferior to both the simulation (1) and the buy and hold strategy. The C.A.Y. for simulation (2) over the entire period is .052, which is more than two percentage points less than the

22. It is assumed that the investor always returns to a cash portfolio at the end of the simulation. Thus, if the decision for 1969-4 is to hold stocks, a 1 per cent transactions cost is deducted in calculating the final value of the portfolio.

buy and hold strategy and more than four percentage points less than the simulation (1) results. The standard deviation of the returns for simulation (2) equals the standard deviation for simulation (1) and is less than the standard deviation for the buy and hold strategy. It seems clear, however, that if the money supply can be predicted only by using naive extrapolation of its previous growth rate, then the stock market forecasting equation will be inferior to a simple buy and hold strategy.

Simulation 3: Regression Prediction of M; No Margin

Compared with simulation (1), the investment decisions for strategy (3) are distinguished by the holding of bills in 1961-1, 1967-4, and 1968-1, and by the holding of stocks in 1961-3, 1966-3 and 1966-4. Of these decisions, only the holding of bills in 1961-1 and the holding of stocks in 1966-3 were incorrect. The mistakes in these periods were costly, however, since the stock market actually rose by over 13 per cent in 1961-1 and it fell by over 10 per cent in 1966-3.

These factors are reasonably reflected in the summary statistics for simulation (3) shown in Table 4. The C.A.Y. for simulation (3) is .086. This is one percentage point better than a buy and hold strategy, but it is almost one percentage poorer than the results of simulation (1) which was based on perfect foresight of the money supply. In terms of the standard deviation of the returns, on the other hand, the quarterly returns of simulation (3) indicate a less volatile pattern than was generated by the other strategies tested. One can thus conclude from these results that the stock market forecasts generated by regression predictions of the money supply allow an investment strategy that is superior to a buy and hold strategy both in terms of return and risk, although the return is not as high as could be obtained with perfect foresight of the money supply.

Simulation 4: Perfect Foresight of M; With Margin

The results for simulation (4) are based on the same assumptions as simulation (1) except that the opportunity of going on margin is now open to the investor under the conditions described above. The decision of buying bills or stocks is made exactly as before so simulations (1) and (4) are identical in this respect. The difference is that the investor chose, given the opportunity, to go on margin in 18 (out of 31) of the periods he was holding stocks. Of the 18 periods the investor was on margin, the stock market rose by over 2 per cent (approximately the average gain) in 13 quarters, rose but by less than 2 per cent in two quarters, and declined in three of the quarters. This would indicate that on net the decisions to move into margin were being made at advantageous times. These advantages are partially offset, of course, by the transactions costs and interest payments made necessary by the margin account.

The summary statistics for simulation (4), shown in Table 4, indicate that the margin strategy out-performed both the no-margin variant of simulation (1) and the buy and hold strategy. The C.A.Y. for simulation (4) for the full period was .150, which is almost double the return of the buy and hold strategy and is over 4 percentage points better than the simulation (1) results. Not

surprisingly, however, the standard deviation of the returns for simulation (4) is significantly higher than that obtained for any of the non-margin experiments. In fact, the increase in the standard deviation is sufficiently large (from .235 for simulation (1) to .396 for simulation (4)) that a risk-averting investor might well prefer the lower but safer returns obtained when the margin option is not used.

Simulation 5: Naive Extrapolation of M; With Margin

The results for simulation (5) are based on the same assumptions as simulation (2) with the addition of the margin option. The investment strategy indicated the investor should hold stocks on margin in 21 of the 32 quarters he was holding stocks. Although 13 of the periods chosen for margin holdings coincide with the periods selected in simulation (4), in general the margin decisions in simulation (5) are inferior.

This evaluation is confirmed in the summary statistics for simulation (5) shown in Table 4. The C.A.Y. for the full period, .052, is unchanged from the results without margin using stock forecasts generated by naive extrapolation of the money supply. In addition, the standard deviation of the quarterly returns increases in simulation (5) to .368 from the no-margin result of .235 in simulation (2). These results are thus consistent with the conclusion for simulation (2), namely that the stock market forecasting equation is inferior to a simple buy and hold strategy if only naive growth rate extrapolations are available for the money supply predictions.

Simulation 6: Regression Prediction of M; With Margin

The results of simulation (6) are based on the same regression predictions of M used in simulation (3), but the margin option is now available. The investment strategy indicated the investor should go on margin in a relatively small number of periods, 14. Only two of these periods, 1964-1 and 1968-4, were not included in the set of 18 quarters in which simulation (4) indicated use of margin, and, in fact, the stock market rose in both of these quarters. On the other hand, simulation (6) failed to go on margin, whereas simulation (4) did, in three quarters, 1962-4, 1967-1 and 1968-2, in which the stock market rose by over 10 per cent.

In line with these results, the summary statistics, shown in Table 4, indicate that simulation (6) performed adequately, but not as well as simulation (4). The C.A.Y. over the entire period is .107, which is over 3 percentage points better than a buy and hold strategy, but is more than 4 percentage points less than the simulation (4) strategy. In terms of the standard deviation of the returns, the values for simulation (6), not surprisingly, exceed the values for all the non-margin simulations, but they are encouragingly smaller than the other margin simulations. The results thus extend the conclusions drawn from simulation (3), which used the regression predictions of M without the margin option, in that the availability of the margin option allows a further increase in the C.A.Y. relative to a buy and hold strategy, although this is achieved only with a corresponding increase in the standard deviation of the returns.

V. CONCLUSIONS

The results of this paper are perhaps best evaluated on two levels. The first level concerns the evidence that a significant and systematic relationship exists between the money supply and the stock market. This relationship has been substantiated in several ways. First, the stock market equation estimated in Section III indicated a good fit and significant correlation between the money supply variables, the level of the money supply and its growth rate, and the index of common stocks. Secondly, the forecasts generated by this relationship, when the actual value of the money supply was assumed known, correlated almost as well with the actual stock market series as did the fitted equation. Finally, the simulation experiments of Section IV, which used the actual value of the money supply, consistently outperformed a buy and hold strategy.

The second level in the evaluation of these results concerns what might be termed "the market test"; that is, would the information generated by the stock market relationship have been of value for an investor. The evaluation here clearly depends on the investor's ability to predict the money supply. An investor who could predict the money supply with perfect foresight would have been significantly more successful than one following a buy and hold strategy. Without the use of margin, the gain in C.A.Y. over the buy and hold strategy was almost 2 percentage points, and this was accompanied by an actual reduction in the standard deviation of the returns. Using margin, the gain in C.A.Y. was over 7 percentage points, although in this case there was an increase in the associated standard deviation.

On the other hand, the results of using only naive growth rate extrapolations to generate the money supply prediction were distinctly inferior to a buy and hold strategy. This implies that at least some current predictive information on the future movements of the money supply is a necessary input for the successful use of the money supply-stock market relationship. The regression equation method for predicting the money supply provides one measure for evaluating how well an informed investor might do, without introducing the extreme assumption of perfect foresight. The results of the simulations using the regression equation predictions of the money supply were generally encouraging although not conclusive. Without margin, the C.A.Y. for the simulation with the regression predicted money supply exceeded the C.A.Y. for the buy and hold strategy by 1 percentage point, and there was also a reduction in the standard deviation of the returns. With the margin option allowed, the C.A.Y. for the regression predicted money supply simulation exceeded the C.A.Y. for the buy and hold strategy by over three percentage points, although in this case there was an increase in the standard deviation.

It thus appears that the results are sufficiently encouraging to indicate the value of further research in this area. In terms of the stock market forecasting equation, significant improvement may be gained by developing a more structural equation that explicitly incorporates the monetary and expectational factors that affect the stock market. In terms of the value of the equation for real investment decision-making, it is clear that any improvements in the

accuracy of the predictions of the money supply will directly improve the usefulness of the forecasting equation. In addition, although it is felt that the simulations shown here provide the basis for a realistic appraisal of the value of the stock market forecasts, it is evident that further realism could be introduced. In particular, short sales of stock, the tax treatment of short-term and long-term capital gains, and mixed portfolios of stocks, bills, and perhaps other assets, all could be introduced into the simulation. Finally, more practical use might be made of the timing implications of the model if forecasts were generated on a monthly or, perhaps, even on a weekly basis.