Math 130 Solutions – Introduction to Statistics

Homework 4 Solutions

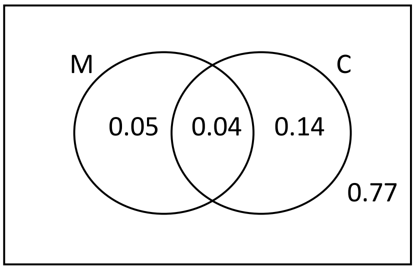
Assignment
Chapter 15: 2, 6, 20, 31, 43
Chapter 16: 9, 20, 24, 28, 47

Hint on 16.47] Think about the mean and standard deviation of a *combination* of six random variables, one for each day the shop is open.

Chapter 15

15.2] Travel. The given information:
Let M = event that a U.S. resident has traveled to Mexico
Let C = event that a U.S. resident has traveled to Canada

P(M) = 0.09   *P*(C) = 0.18   *P*(M and C) = 0.04



A Venn diagram is useful here.

(a) *P*(C but not M) = 0.14. There's a 14% chance that a US resident has traveled to Canada but not Mexico.

(b) *P*(C or M) = *P*(C) + *P*(M) – *P*(C and M) = 0.18 + 0.09 – 0.04 = 0.23.
Or, add the circular sections of the Venn diagram to get 0.05 + 0.04 + 0.14 = 0.23.
There's a 23% chance that a U.S. resident has traveled to Canada or Mexico.

(c) P(neither C or M) = 1 – P(C or M) = 1 – 0.23 = 0.77
There's a 77% chance that a randomly selected resident has traveled to neither Canada nor Mexico.

15.6] Birth Order.

(a) $P(\text{Human Ecology}) = \frac{43}{223} = 0.1928$. The probability that a student is in Human Ecology is 0.1928.

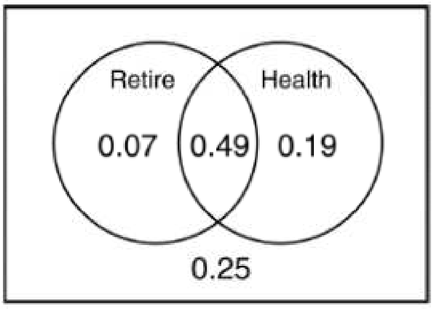
(b) $P(\text{first-born}) = \frac{113}{223} = 0.5067$. The probability that a student is a first born is 0.5067.

(c) $P(\text{first-born and Ecology}) = \frac{15}{223} = 0.0673$. The probability that a student is a first-born and Ecology student is 0.0673.

(d)
$$\begin{aligned} P(\text{first-born or Ecology}) &= P(\text{first-born}) + P(\text{Ecology}) - P(\text{first-born and Ecology}) \\ &= 0.5067 + 0.1928 - 0.0673 \\ &= 0.6322 \end{aligned}$$

The probability that a student is a first-born or Ecology student is 0.6322.

15.20] Benefits. A Venn diagram can help us here.



Let R = Retirement plan
Let H = Health Insurance

(a) P(not R and not H) = 0.25
Using probability rules:

$$\begin{aligned} P(\text{not R and not H}) &= 1 - P(\text{R or H}) \\ &= 1 - [P(R) + P(H) + P(\text{R and H})] \\ &= 1 - [0.56 + 0.68 - 0.49] \\ &= 0.25 \end{aligned}$$

There is a 25% probability that he has neither a retirement plan nor employer sponsored health insurance.

(b) $P(H|R) = \frac{P(\text{H and R})}{P(R)} = \frac{0.49}{0.56} = 0.875$. The probability of health insurance given he has a retirement plan is 0.875.

(c) No, they are not independent, because

$$P(H \text{ and } R) \stackrel{?}{=} P(H)P(R)$$

$$0.49 \stackrel{?}{=} 0.68(0.56)$$

$$0.49 \neq 0.3808$$

(d) They are not mutually exclusive, because there is some overlap between the two events. There are cases with both R and H.

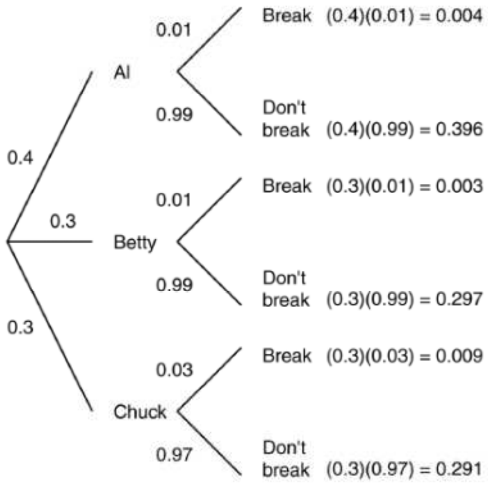
15.31] Montana. Party affiliation is not independent of sex. To see this, let D represent Democrats, and M represent males.

Now $P(D) = \frac{84}{202} = 0.4158$ and $P(D|M) = \frac{P(D \text{ and } M)}{P(M)} = \frac{36}{105} = 0.3429$

Since $P(D|M) \neq P(D)$, these events are not independent.

15.43] Dishwashers. A tree diagram can help us here.

$$P(\text{Chuck}|\text{Break}) = \frac{P(\text{Chuck and Break})}{P(\text{Break})} = \frac{(0.3)(0.01)}{(0.4)(0.01)+(0.3)(0.01)+(0.3)(0.03)} = \frac{0.009}{0.004+0.003+0.009} = \frac{0.009}{0.016} = 0.5625$$



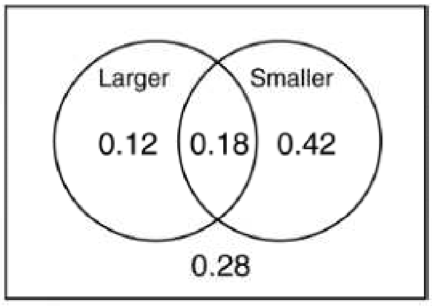
If we hear a dish break, there's a 56.25% chance that Chuck is working.

Chapter 16

16.9] Software. The given information is:

Large Contract: \$50,000 profit 30% chance
Small Contract: \$20,000 profit 60% chance

Both Contracts: \$70,000 profit (0.30)(0.60) = 18% chance



Let *X* be the profit. The distribution of *X* can be tabulated as

X	50,000	20,000	70,000	0
P(X)	0.12	0.42	0.18	0.28

$$E[X] = \sum_{i=1}^{n} X_i P(X_i) = 50000(0.12) + 20000(0.42) + 70000(0.18) + 0(0.28) = \$27{,}000$$

We'd expect an average profit of \$27,000 .

16.20] Insurance. We have the following information:

Costs \$100
Major Injury: Pays \$10,000, Probability of 1/2000
Minor Injury: Pays \$3,000, Probability of 1/500

Let *X* be the profit.

(a) We can make a table for the probability model.

X	100	-9,900	-2900
P(X)	0.9975	0.0005	0.002

(b) The company's expected profit is \$89.

$$E[X] = \sum_{i=1}^{n} X_i P(X_i) = 100(0.9975) - 9900(0.0005) - 2900(0.002) = \$89.$$

(c) The standard deviation is \$260.54.

$$\begin{aligned}\sigma &= \sqrt{\sum_{i=1}^{n} (X_i - \mu)^2 P(X_i)} \\ &= \sqrt{(100 - 89)^2(0.9975) + (-9900 - 89)^2(0.0005) + (-2900 - 89)^2(0.002)} \\ &= \sqrt{120.6975 + 49890.0605 + 17868.242} \\ &= \sqrt{67879} \\ &= 260.536\end{aligned}$$

16.24] Contracts. We have the following information:

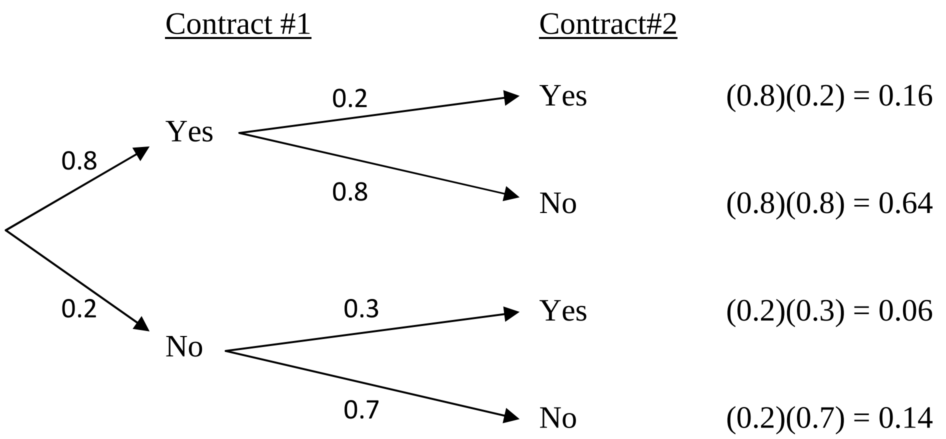
- We bid on two contracts
- *P*(contract #1) = 0.8
- *P*(contract #2) = 0.2, if you get contract #1
- *P*(contract #2) = 0.3, if you don't get contract #1

(a) No, the two contracts are not independent. The probability of the second contract changes depending on whether or not we got the first contract. The two events are *dependent*.

(b) There are several approaches that might work here.
Let A = the event that we get contract #1
Let B = the event that we get contract #2

We want to find P(A and B). We can use the general multiplication rule here.
$P(A \text{ and } B) = P(A)P(B|A) = 0.8(0.2) = 0.16$
There's a 16% probability that we get both contracts.

We could also try a tree diagram:



(c) Let A^C = the event that we don't get contract #1
Let B^C = the event that we don't get contract #2

We want to find $P(A^C$ and $B^C)$. We can use the general multiplication rule again.
$P(A^C \text{ and } B^C) = P(A^C)P(B^C|A^C) = 0.2(0.7) = 0.14$
There's a 6% probability that we get no contracts.

(d) We can make a table for the probability model.

X	0	1	2
$P(X)$	0.14	0.70	0.16

(e) We have:

$$E[X] = \sum_{i=1}^{n} X_i P(X_i) = 0(0.14) + 1(0.70) + 2(0.16) = 1.02$$

$$\begin{aligned}\sigma &= \sqrt{\sum_{i=1}^{n} (X_i - \mu)^2 P(X_i)} \\ &= \sqrt{(0 - 1.02)^2(0.14) + (1 - 1.02)^2(0.70) + (2 - 1.02)^2(0.16)} \\ &= \sqrt{0.1457 + 0.0028 + 0.1537} \\ &= \sqrt{0.3022} \\ &= 0.5497\end{aligned}$$

16.28] Random Variables. Find the mean and standard deviation of each of these variables.

	Mean	SD
X	10	2
Y	20	5
$X - 20$	-10	2
$0.5Y$	10	2.5
$X + Y$	30	5.39
$X - Y$	-10	5.39
$Y_1 + Y_2$	40	7.07

(a) $X - 20$
Mean $= E[X - 20] = E[X] - 20 = 10 - 20 = -10$
$\text{Var}(X - 20) = \text{Var}(X) = 4 \Rightarrow SD = 2$

(b) $0.5Y$
Mean $= E[0.5Y] = 0.5E[Y] = 0.5(20) = 10$
$\text{Var}(0.5Y) = 0.5^2\text{Var}(Y) = 0.25(25) = 6.25 \Rightarrow SD = 2.5$

(c) $X + Y$
Mean $= E[X + Y] = E[X] + E[Y] = 10 + 20 = 30$
$\text{Var}(X + Y) = \text{Var}(X) + \text{Var}(Y) = 4 + 25 = 29 \Rightarrow SD = 5.39$
(Note: The variances add since X and Y are independent)

(d) $X - Y$
Mean $= E[X - Y] = E[X] - E[Y] = 10 - 20 = -10$
$\text{Var}(X - Y) = \text{Var}(X) + \text{Var}(Y) = 4 + 25 = 29 \Rightarrow SD = 5.39$

(e) $Y_1 + Y_2$
Mean $= E[Y_1 + Y_2] = E[Y_1] + E[Y_2] = 20 + 20 = 40$
$\text{Var}(Y_1 + Y_2) = \text{Var}(Y_1) + \text{Var}(Y_2) = 25 + 25 = 50 \Rightarrow SD = 7.07$

16.47] Coffee and Doughnuts.

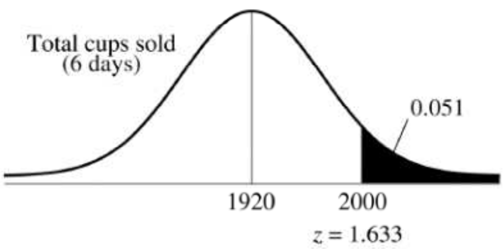
a)

$\mu = E(\text{cups sold in 6 days}) = 6(E(\text{cups sold in 1 day})) = 6(320) = 1920 \text{ cups}$

$\sigma = SD(\text{cups sold in 6 days}) = \sqrt{6(Var(\text{cups sold in 1 day})} = \sqrt{6(20)^2} \approx 48.99 \text{ cups}$

The distribution of total coffee sales for 6 days has distribution $N(1920, 48.99)$.

$$z = \frac{x - \mu}{\sigma}$$
$$z = \frac{2000 - 1920}{48.99}$$
$$z = 1.633$$



According to the Normal model, the probability that he will sell more than 2000 cups of coffee in a week is approximately 0.051.

b) Let C = the number of cups of coffee sold. Let D = the number of doughnuts sold.

$$\mu = E(50C + 40D) = 0.50(E(C)) + 0.40(E(D)) = 0.50(320) + 0.40(150) = \$220$$

$$\sigma = SD(0.50C + 0.40D) = \sqrt{0.50^2(Var(C)) + 0.40^2(Var(D))} = \sqrt{0.50^2(20^2) + 0.40^2(12^2)} \approx \$11.09$$

The day's profit can be modeled by $N(220, 11.09)$. A day's profit of \$300 is over 7 standard deviations above the mean. This is extremely unlikely. It would not be reasonable for the shop owner to expect the day's profit to exceed \$300.

c) Consider the difference $D - 0.5C$. When this difference is greater than zero, the number of doughnuts sold is greater than half the number of cups of coffee sold.

$$\mu = E(D - 0.5C) = (E(D)) - 0.5(E(C)) = 150 + 0.5(320) = -\$10$$

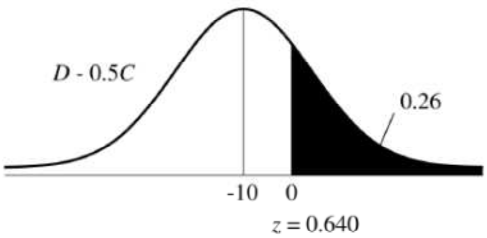
$$\sigma = SD(D - 0.5C) = \sqrt{(Var(D)) + 0.5(Var(C))} = \sqrt{(12^2) + 0.5^2(20^2)} \approx \$15.62$$

The difference $D - 0.5C$ can be modeled by $N(-10, 15.62)$.

$$z = \frac{x - \mu}{\sigma}$$

$$z = \frac{0 - (10)}{15.62}$$

$$z = 0.640$$



According to the Normal model, the probability that the shop owner will sell a doughnut to more than half of the coffee customers is approximately 0.26.