Homework Chapter 16 Solutions

16.7

16.15

16.18

16.34

16.39

16.58

16.63

Problem 16.7

A sinusoidal wave is traveling along a rope. The oscillator that generates the wave completes 40 vibrations in 30 s. A given crest of the wave travels 425 cm along the rope in 10 s. What is the wavelength of the wave?

Solution

Completing 40 vibrations in 30 s means a frequency of

$$f = \frac{40\ cycles}{30\ s} = 1.3333\ Hz$$

The crest traveling 425 cm in 10 s means a wave speed of

$$v = \frac{0.425\ m}{10\ s} = 0.0425\ m/s$$

The wavelength is

$$v = \lambda f \quad \Rightarrow \quad \lambda = \frac{v}{f} = \frac{0.0425\ m/s}{1.3333\ 1/s} = 0.031875\ m = 3.1875\ cm$$

Problem 16.15

A transverse wave on a string is described by the wave function

$$y = 0.120\sin\left(\frac{\pi}{8}x + 4\pi t\right)$$

where x and y are in meters and t is in seconds.

(a) Determine the transverse speed at time t = 0.2 s for an element of string located at x = 1.6 m.

(b) Determine the transverse acceleration at time t = 0.2 s for an element of string located at x = 1.6 m.

(c) What is the wavelength of this wave?

(d) What is the period of this wave?

(e) What is the speed of propagation of this wave?

Solution

(a) The transverse speed is

$$\frac{dy}{dt} = (4\pi)(0.120)\cos\left(\frac{\pi}{8}x + 4\pi t\right)$$

At t = 0.2 s and x = 1.6 m, the speed is

$$v_y(1.6, 0.2) = (4\pi)(0.120)\cos\left(\frac{\pi}{8}1.6 + 4\pi(0.2)\right) = 0.48\pi\cos(\pi) = -0.48\pi \ m/s$$

(b) The transverse acceleration is

$$\frac{d^2y}{dt^2} = -(4\pi)^2(0.120)\sin\left(\frac{\pi}{8}x + 4\pi t\right)$$

At t = 0.2 s and x = 1.6 m, the acceleration is

$$a_y(1.6, 0.2) = (4\pi)^2(0.120)\sin\left(\frac{\pi}{8}1.6 + 4\pi(0.2)\right) = 1.92\pi^2\sin(\pi) = 0 \ m/s^2$$

(c) The wavelength is

$$\lambda = \frac{2\pi}{k} = \frac{2\pi}{\pi / 8} = 16 \ m$$

(d) The period is

$$T = \frac{2\pi}{\omega} = \frac{2\pi}{4\pi} = 0.5 \ s$$

(e) The wave speed is

$$v = \lambda f = (16 \ m)(\frac{4\pi \ 1/s}{2\pi}) = 32 \ m/s$$

Problem 16.18

A transverse sinusoidal wave on a string has a period T = 25 ms and travels in the negative direction with a speed of 30 m/s. At t = 0, an element of the string at x = 0 has a transverse position of 2 cm and is traveling downward with a speed of 2 m/s.

(a) What is the amplitude of the wave?

(b) What is the initial phase angle?

(c) What is the maximum transverse speed of an element of the string?

(d) Write the wave function for the wave.

Solution

(a) The motion of the element at x = 0 is

$$y(t) = A\cos(\omega t + \phi) \quad \Rightarrow \quad 0.02\ m = A\cos(\phi)$$

$$v(t) = -\omega A\sin(\omega t + \phi) \quad \Rightarrow \quad -2\ m/s = -\omega A\sin(\phi)$$

The angular frequency is

$$\omega = \frac{2\pi}{T} = \frac{2\pi}{0.025\ s} = 80\pi\ 1/s$$

We have

$$A\cos(\phi) = 0.02$$

$$-2 = -80\pi A\sin(\phi) \quad \Rightarrow \quad A\sin(\phi) = \frac{1}{40\pi}$$

$$\tan(\phi) = \frac{1}{0.02 \cdot 40\pi} = \frac{1}{0.8\pi} = 0.39789 \quad \Rightarrow \quad \phi = 0.37868$$

$$A\cos(0.37868) = 0.02 \quad \Rightarrow \quad A = 0.021525\ m$$

The amplitude is 0.021525 m.

(b) The phase angle is 0.37868 rad.

(c) The maximum speed is

$$v_{\max} = \omega A = (80\pi\ 1/s)(0.021525\ m) = 5.4098\ m/s$$

(d) The wavenumber is

$$v = \lambda f = \frac{2\pi}{k}\frac{\omega}{2\pi} = \frac{\omega}{k} \quad \Rightarrow \quad k = \frac{\omega}{v} = \frac{80\pi\ 1/s}{30\ m/s} = \frac{8\pi}{3}\ 1/m$$

The wave function is

$$y(x,t) = (0.021525\ m)\cos(\frac{8\pi}{3}x + 80\pi t + 0.37868)$$

$$y(x,t) = (0.021525\ m)\cos(8.3776x + 251.33t + 0.37868)$$

Problem 16.34

Sinusoidal waves 5 cm in amplitude are to be transmitted along a string that has a linear mass density of 4×10^{-2} kg/m. The source can deliver a maximum power of 300 W and the string is under a tension of 100 N. What is the highest frequency at which the source can operate?

Solution

The power is

$$P = \frac{1}{2}\mu\omega^2 A^2 v$$

$$300\ W = \frac{1}{2}(4 \times 10^{-2}\ kg\ /\ m)\omega^2(0.05\ m)^2 v$$

The wave speed is

$$v = \sqrt{\frac{T}{\mu}} = \sqrt{\frac{100\ N}{4 \times 10^{-2}\ kg\ /\ m}} = 50\ m/s$$

$$300\ W = \frac{1}{2}(4 \times 10^{-2}\ kg\ /\ m)\omega^2(0.05\ m)^2(50\ m/s) \quad \Rightarrow \quad \omega = 346.41\ 1/s$$

$$\omega = 346.41\ 1/s = 2\pi f \quad \Rightarrow \quad f = 55.133\ Hz$$

Problem 16.39

The wave function for a wave on a taut string is

$$y(x,t) = 0.350 \sin\left(10\pi t - 3\pi x + \frac{\pi}{4}\right)$$

where x and y are in meters and t is in seconds. The linear mass density of the string is 75 g/m.

(a) What is the average rate at which energy is transmitted along the string?

(b) What is the energy contained in each cycle of the wave?

Solution

(a) The power of this wave is

$$P = \frac{1}{2}\mu\omega^2 A^2 v = \frac{1}{2}(0.075\ kg/m)(10\pi\ 1/s)^2(0.35\ m)^2 \frac{10\pi\ 1/s}{3\pi\ 1/m} = 15.113\ W$$

(b) The energy in each cycle is

$$E_\lambda = \frac{1}{2}\mu\omega^2 A^2 \lambda = \frac{1}{2}(0.075\ kg/m)(10\pi\ 1/s)^2(0.35\ m)^2 \frac{2\pi}{3\pi\ 1/m} = 3.0226\ J$$

Problem 16.58

A rope of total m and length L is suspended vertically. Analysis shows that for short transverse pulses, the waves above a short distance from the free end of the rope can be represented to a good approximation by the linear wave equation. Show that a transverse pulse travels the length of the rope in a time interval that is given approximately by

$$\Delta t = 2\sqrt{L / g}$$

Solution

Since this system can be modeled by the linear wave equation, the velocity of a pulse is given by

$$v = \sqrt{\frac{T}{\mu}}$$

In the current model, we have a tension that is dependent on x, the distance from the free end of the rope. This dependence is linear.

$$T(x) = m(x)g = \mu x g$$

The time of travel is just

$$\frac{dx}{dt} = \sqrt{\frac{\mu g x}{\mu}} = \sqrt{gx} \quad \Rightarrow \quad dt = \frac{1}{\sqrt{g}}\frac{dx}{\sqrt{x}}$$

The total time of travel is

$$\int_0^{t_f} dt = \frac{1}{\sqrt{g}}\int_0^L \frac{dx}{\sqrt{x}} = \frac{1}{\sqrt{g}}(2\sqrt{x})\Big|_0^L = 2\sqrt{\frac{L}{g}} \quad \Rightarrow \quad t_f = 2\sqrt{\frac{L}{g}}$$

Problem 16.63

Following problem 16.58,

(a) Over what time interval does a pulse travel halfway up the rope? Give your answer as a fraction of the quantity 2√(L/g).

(b) A pulse starts traveling up the rope. How far has it traveled after a time interval √(L/g)?

Solution

(a) This is just the integration from x = 0 to x = L/2.

$$\int_0^{t_f} dt = \frac{1}{\sqrt{g}}\int_0^{L/2}\frac{dx}{\sqrt{x}} = \frac{1}{\sqrt{g}}(2\sqrt{x})\Big|_0^{L/2} = 2\sqrt{\frac{L\,/\,2}{g}} \quad\Rightarrow\quad t_f = \frac{1}{\sqrt{2}}2\sqrt{\frac{L}{g}} = 0.70711\cdot 2\sqrt{\frac{L}{g}}$$

(b) The mathematical question is what is x_f when t_f = √(L/g)?

$$\int_0^{\sqrt{L/g}} dt = \frac{1}{\sqrt{g}}\int_0^{x_f}\frac{dx}{\sqrt{x}} = \frac{1}{\sqrt{g}}(2\sqrt{x})\Big|_0^{x_f} = 2\sqrt{\frac{x_f}{g}} \quad\Rightarrow\quad \sqrt{\frac{L}{g}} = 2\sqrt{\frac{x_f}{g}} \quad\Rightarrow\quad x_f = L\,/\,4$$