Homework Chapter 18 Solutions

Problem 18.5

Two pulses traveling on the same string are described by

$$y_1 = \frac{5}{(3x - 4t)^2 + 2} \quad and \quad y_2 = \frac{-5}{(3x + 4t - 6)^2 + 2}$$

(a) In which direction does each pulse travel?

(b) At what instant do the two cancel everywhere?

(c) At what point do the two pulses always cancel?

Solution

(a) The pulse y_1 travels to the right while the pulse y_2 travels to the left.

(b) The time when $y_1 + y_2 = 0$ is when

$$y_1 + y_2 = 0 \quad \Rightarrow \quad \frac{5}{(3x - 4t)^2 + 2} + \frac{-5}{(3x + 4t - 6)^2 + 2} = 0$$

This means

$$3x - 4t = 3x + 4t - 6 \quad \Rightarrow \quad 8t = 6 \quad \Rightarrow \quad t = \frac{2}{3}$$

(c) The position where $y_1 + y_2 = 0$ is where

$$y_1 + y_2 = 0 \quad \Rightarrow \quad \frac{5}{(3x - 4t)^2 + 2} + \frac{-5}{(3x + 4t - 6)^2 + 2} = 0$$

$$3x - 4t = -(3x + 4t - 6) \quad \Rightarrow \quad 6x = 6 \quad \Rightarrow \quad x = 1$$

Problem 18.23

The A string on a cello vibrates in its first normal mode with a frequency of 220 Hz. The vibrating segment is 70.0 cm long and has a mass of 1.20 g.

(a) Find the tension in the string.

(b) Determine the frequency of vibration when the string vibrates in three segments.

Solution

(a) The string is fixed at the ends so its first normal mode represent 0.5 wavelengths so the wavelength is 140 cm. At 220 Hz, the wave speed is

$$v = \lambda f = (1.40\ m)(220\ Hz) = 308\ m/s$$

The means the tension is

$$v = 308\ m/s = \sqrt{\frac{T}{\mu}} \quad \Rightarrow \quad T = (308\ m/s)^2 (1.20 \times 10^{-3}\ kg/0.70\ m) = 162.62\ N$$

(b) The string is fixed at the ends so three segments represent 1.5 wavelengths so the wavelength is

$$1.5\lambda = 70\ cm \quad \Rightarrow \quad \lambda = 46.667\ cm = 0.46667\ m$$

At the wave speed of the string, the frequency is

$$f = \frac{v}{\lambda} = \frac{308\ m/s}{0.46667\ m} = 660\ Hz$$

Problem 18.24

An object is hung from a string that has a mass density of 0.00200 kg/m passes over a light pulley. The string is connected to a vibrator of constant frequency and the length of the string is 2.00 m. When the mass of the object is either 16.0 kg or 25.0 kg, standing waves are observed. No standing waves are observed with any mass between these values.

(a) What is the frequency of the vibrator?

(b) What is the largest object mass for which standing waves could be observed?

Solution

(a) With the given density, the two tensions will generate these two wave speeds.

$$v_1 = \sqrt{\frac{T_1}{\mu}} = \sqrt{\frac{(16\ kg)(9.8\ m/s^2)}{0.002\ kg/m}} = 280\ m/s$$

$$v_2 = \sqrt{\frac{T_2}{\mu}} = \sqrt{\frac{(25\ kg)(9.8\ m/s^2)}{0.002\ kg/m}} = 350\ m/s$$

The standing wave patterns are these.

$$L = n\frac{\lambda}{2} \quad and \quad L = (n+1)\frac{\lambda}{2}$$

The first has a longer wavelength so it goes with the larger wave speed v_2.

$$L = n\frac{\lambda}{2} = n\frac{v_2}{2f} \quad \Rightarrow \quad n = \frac{2fL}{v_2}$$

The other one is

$$L = (n+1)\frac{\lambda}{2} = (n+1)\frac{v_1}{2f} \quad \Rightarrow \quad n = \frac{2fL}{v_1} - 1$$

Together,

$$\frac{2fL}{v_2} = \frac{2fL}{v_1} - 1 \quad \Rightarrow \quad f \cdot 2L\left(\frac{1}{v_2} - \frac{1}{v_1}\right) = -1 \quad \Rightarrow \quad f = -\frac{1}{2L}\left(\frac{1}{v_2} - \frac{1}{v_1}\right)^{-1}$$

$$f = -\frac{1}{2(2\ m)}\left(\frac{1}{350\ m/s} - \frac{1}{280\ m/s}\right)^{-1} = 350\ Hz$$

(b) The largest tension possible is the largest speed possible which provides for the largest wavelength and smallest mode n = 1.

$$v_{max} = 2fL \quad \Rightarrow \quad T_{max} = \mu(2fL)^2 \quad \Rightarrow \quad m_{max} = \frac{\mu(2fL)^2}{g}$$

$$m_{max} = \frac{(0.002\ kg/m)[2(350\ Hz)(2\ m)]^2}{9.8\ m/s^2} = 400\ kg$$

Problem 18.34

The overall length of a piccolo is 32 cm. The resonating air column is open at both ends.

(a) Find the frequency of the lowest note a piccolo can sound.

(b) Opening holes in the side of a piccolo effectively shortens the length of the resonant column. Assume the highest note a piccolo can sound is 4,000 Hz. Find the distance between adjacent anti-nodes for this mode of vibration.

Solution

(a) This is the longest wavelength possible which means n = 1. Opened on both ends means

$$L = \frac{\lambda_{max}}{2} \quad \Rightarrow \quad \lambda_{max} = 2L = 2(0.32\ m) = 0.64\ m$$

$$f_{min} = \frac{v_{air}}{\lambda_{max}} = \frac{343\ m/s}{0.64\ m} = 536\ Hz$$

(b) The maximum frequency means the shortest wavelength. The wavelength is

$$f_{max}\lambda_{min} = v_{air} \quad \Rightarrow \quad \lambda_{min} = \frac{v_{air}}{f_{max}} = \frac{343\ m/s}{4{,}000\ Hz} = 0.085750\ m = 8.575\ cm$$

Problem 18.45

A student uses an audio oscillator of adjustable frequency to measure the depth of a water well. The student reports hearing two successive resonances at 51.87 Hz and 59.85 Hz.

(a) How deep is the well?

(b) How many anti-nodes are in the standing wave at 51.87 Hz?

Solution

(a) The two resonance conditions are

$$L = (2n_1 + 1)\frac{\lambda_1}{4} \quad and \quad L = (2n_2 + 1)\frac{\lambda_2}{4}$$

Since the first frequency is smaller than the second frequency, the first wavelength is larger than the second wavelength. This means

$$n_1 < n_2 \quad or \quad n_2 = n_1 + 1$$

Together,

$$(2n_1 + 1)\frac{v}{f_1} = (2n_1 + 3)\frac{v}{f_2} \quad \Rightarrow \quad (2n_1 + 1)f_2 = (2n_1 + 3)f_1 \quad \Rightarrow \quad (2n_1 + 1) = (2n_1 + 3)\frac{f_1}{f_2}$$

$$(2n_1 + 1) = 2n_1\frac{f_1}{f_2} + 3\frac{f_1}{f_2} \quad \Rightarrow \quad 2n_1(1 - \frac{f_1}{f_2}) = 3\frac{f_1}{f_2} - 1 \quad \Rightarrow \quad n_1(0.26666) = 1.6$$

$$n_1 = 6 \quad and \quad n_2 = 7$$

The depth of the well is

$$L = (2n_1 + 1)\frac{\lambda_1}{4} = \frac{13}{4}\frac{343\ m/s}{51.87\ Hz} = 21.491\ m$$

(b) The number of anti-nodes for n = 6 is 7.

Problem 18.49

In certain ranges of the piano keyboard, more than one string is two into the same note to provide extra loudness. For example, the note at 110 Hz has two strings at this frequency. If one string slips from its normal tension of 600 N to 540 N, what beat frequency is heard when the hammer strength of two strings simultaneously?

Solution

A string with a tension of 600 N produces a frequency of 110 Hz. The relationship between these quantities is this.

$$110\ Hz = \frac{v}{\lambda_1} = \frac{1}{\lambda_1}\sqrt{\frac{T_1}{\mu}}$$

Since both strings are in resonance, but wavelengths must be the same as the strings have the same lengths.

$$110\ Hz = \frac{1}{\lambda\sqrt{\mu}}\sqrt{T_1}$$

For the second string,

$$f_2 = \frac{1}{\lambda\sqrt{\mu}}\sqrt{T_2}$$

Their ratio is this.

$$\frac{f_2}{110\ Hz} = \sqrt{\frac{T_2}{T_1}} = \sqrt{\frac{540\ N}{600\ N}} \quad \Rightarrow \quad f_2 = 104.36\ Hz$$

The beat frequency is

$$f_b = \left|f_1 - f_2\right| \quad \Rightarrow \quad f_b = 5.64\ Hz$$

Problem 18.63

On a Marimba, the wooden bar that sounds a tone when struck vibrate in a transverse standing wave having three anti-nodes and two notes. The lowest frequency note is 87 Hz, produced by a bar 40 cm long.

(a) Find the speed of transverse waves on the bar.

(b) A resonant pipe suspended vertically below the center of the bar enhances the loudness of the knitted sound. If the pipe is open at the top and only, what length of the pipe is required to resonate with the bar in part (a)?

Solution

(a) The bar with two nodes and three anti-nodes means a single wavelength. The speed of the wave in the wooden bar is

$$v = \lambda f = (0.40\ m)(87\ Hz) = 34.8\ m/s$$

(b) Now the wave is in air. The wavelength is

$$\lambda = \frac{v}{f} = \frac{343\ m/s}{87\ Hz} = 3.9425\ m$$

The shortest pipe is one that contains one-quarter of a wavelength so it is 0.986 m.

Problem 18.73

With no wind, the shown standing wave is maintained. With a wind, the second shown standing wave is maintained.

What is the force required to change the standing wave?

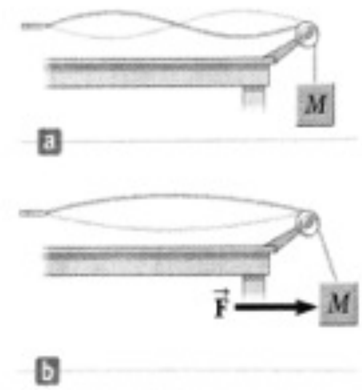


Figure P18.73

Solution

In the initial state, the tension is T = Mg. This tension causes the following state with the wavelength being the length of the string, L.

$$v_1 = \lambda_1 f = \sqrt{\frac{T_1}{\mu}} \quad \Rightarrow \quad \lambda_1 = \frac{1}{f\sqrt{\mu}}\sqrt{T_1} \quad \Rightarrow \quad L = \frac{1}{f\sqrt{\mu}}\sqrt{Mg}$$

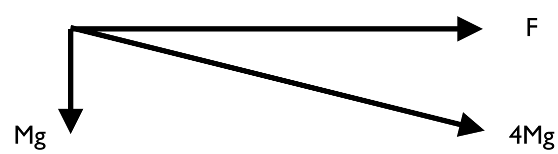
In the second situation, the tension has increased. The result is that the length of the string represents only half of a wavelength.

$$v_2 = \lambda_2 f = \sqrt{\frac{T_2}{\mu}} \quad \Rightarrow \quad \lambda_2 = \frac{1}{f\sqrt{\mu}}\sqrt{T_2} \quad \Rightarrow \quad 2L = \frac{1}{f\sqrt{\mu}}\sqrt{T_2}$$

Together, the second tension is

$$2L = \frac{1}{f\sqrt{\mu}}\sqrt{T_2} \quad \Rightarrow \quad 2\frac{1}{f\sqrt{\mu}}\sqrt{Mg} = \frac{1}{f\sqrt{\mu}}\sqrt{T_2} \quad \Rightarrow \quad T_2 = 4Mg$$

The downward force is Mg but the total force is 4Mg.



The force is

$$F^2 + (Mg)^2 = (4Mg)^2 \quad \Rightarrow \quad F = \sqrt{15}Mg$$