Homework Chapter 21 Solutions

21.7

21.13

21.17

21.19

21.26

21.33

21.45

21.51

21.71

Problem 21.7

A 2 mole sample of oxygen gas is confined to a 5 liter vessel at a pressure of 8 atm. Find the average translational kinetic energy of the oxygen molecules under these conditions.

Solution

The temperature of the gas is

$$PV = nRT \quad \Rightarrow \quad (8 \cdot 101{,}325\ Pa)(5 \times 10^{-3}\ m^3) = (2\ mol)(8.314\ \tfrac{J}{mol \cdot K})T$$

$$T = 243.75\ K$$

At this temperature, the average kinetic energy of a molecules is

$$\bar{K} = \frac{5}{2} k_B T = 8.4130 \times 10^{-21}\ J$$

At this temperature, the total kinetic energy above is distributed to all of the degrees of freedom evenly and there are 5. The translational kinetic energy is only 3/5 of this amount.

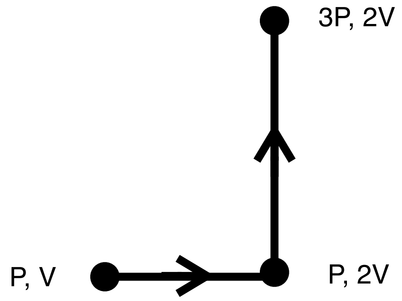
$$\bar{K}_{translational} = \frac{3}{5} 8.4130 \times 10^{-21}\ J = 5.0478 \times 10^{-21}\ J$$

Problem 21.13

A sample of a diatomic ideal gas has pressure P and volume V. When the gas is warmed, its pressure triples and its volume doubles. This warming process has two steps. The first is at constant pressure, and the second is at constant volume. Determine the amount of heat transferred to the gas.

Solution

The P-V diagram looks like this.



Through the first process, the change in the internal energy is

$$\Delta E = \frac{5}{2} nR\Delta T$$

We know this.

$$nR\Delta T = 2PV - PV = PV$$

So

$$\Delta E = \frac{5}{2} PV = 2.5PV$$

The work is just

$$W = -P\Delta V = PV$$

The heat is

$$Q = \Delta E - W = 2.5PV + PV = 3.5PV$$

Through the second process, the work is zero so

$$Q = \Delta E = \frac{5}{2} nR\Delta T = \frac{5}{2}(6PV - 2PV) = 10PV$$

The total heat is 13.5PV.

Problem 21.17

A vertical cylinder with a heavy piston contained air at 300 K. The initial pressure is 2×10^5 Pa and the initial volume is 0.35 m^3. Take the molar mass of air as 28.9 g/mole and assume C_V is (5/2)R.

Find the specific heat of air at constant volume.

Calculate the mass of air in the cylinder.

Supposed the piston is held fixed, how much energy input is required to raise the temperature of the air to 700 K.

What would it be if the pressure were held constant?

Solution

The the specific heat of air at constant volume is

$$C_V = \frac{5}{2}R = \frac{5}{2}(8.314\ \tfrac{J}{mol\cdot K}) = 20.785\ \tfrac{J}{mol\cdot K}\left(\frac{1\ mol}{28.9\times10^{-3}\ kg}\right) = 719.20\ \tfrac{J}{kg\cdot K}$$

The number of moles is

$$PV = nRT \quad \Rightarrow \quad n = \frac{PV}{RT} = \frac{(2\times10^5\ Pa)(0.35\ m^3)}{(8.314\ \tfrac{J}{mol\cdot K})(300\ K)} = 28.065\ mol$$

This has a mass of

$$28.065\ mol\left(\frac{28.9\ g}{1\ mol}\right) = 811.08\ g = 0.81108\ kg$$

To increase the temperature to 700 K, the change in the temperature is 400 K. The volume is held constant, so

$$Q = nC_V\,\Delta T = (0.81108\ mol)(719.20\ \tfrac{J}{kg})(400\ K) = 233.33\ kJ$$

If the process is made isobaric,

$$C_P = \frac{7}{2}R = \frac{7}{2}(8.314\ \tfrac{J}{mol\cdot K}) = 29.099\ \tfrac{J}{mol\cdot K}\left(\frac{1\ mol}{28.9\times10^{-3}\ kg}\right) = 1{,}006.9\ \tfrac{J}{kg}$$

$$Q = nC_P\,\Delta T = (0.81108\ mol)(1{,}006.9\ \tfrac{J}{kg})(400\ K) = 326.67\ kJ$$

Problem 21.19

During the compression stroke of a certain gasoline engine, the pressure increases from 1 atm to 20 atm. If the process is adiabatic in the air-fuel mixture behaves as a diatomic ideal gas, by what factors do the volume and temperature change?

If there were 0.016 mole of gas at 27 °C, what are the heat, the change in the internal energy, and the work that characterize the process?

Solution

The pressure and volume are related through

$$P_i V_i^{\gamma} = P_f V_f^{\gamma}$$

$$\frac{P_f V_f^{\gamma}}{P_i V_i^{\gamma}} = 1 \quad \Rightarrow \quad \left(\frac{V_f}{V_i}\right)^{\gamma} = \frac{P_i}{P_f} \quad \Rightarrow \quad \frac{V_f}{V_i} = \left(\frac{P_i}{P_f}\right)^{\frac{1}{\gamma}} = \left(\frac{1\ atm}{20\ atm}\right)^{\frac{5}{7}} = 0.11768$$

The temperature and volume are related through

$$T_i V_i^{\gamma-1} = T_f V_f^{\gamma-1}$$

$$\frac{T_f V_f^{\gamma-1}}{T_i V_i^{\gamma-1}} = 1 \quad \Rightarrow \quad \frac{T_f}{T_i} = \frac{V_i^{\gamma-1}}{V_f^{\gamma-1}} = \left(\frac{V_i}{V_f}\right)^{\gamma-1} = \left(\frac{1}{0.11768}\right)^{\frac{7}{5}-1} = 2.3535$$

The heat is obviously zero. The change in the internal energy and the work are the same and they are

$$\Delta E_{\text{int}} = \frac{5}{2} nR\,\Delta T$$

The initial temperature is 300 K and the final temperature is

$$\frac{T_f}{T_i} = 2.3535 \quad \Rightarrow \quad T_f = 2.3535 T_i = 2.3535(300\ K) = 706.06\ K$$

$$\Delta E_{\text{int}} = \frac{5}{2}(0.016\ mol)(8.314\ \tfrac{J}{mol \cdot K})(706.06\ K - 300\ K) = 135.04\ J$$

Problem 21.26

An ideal gas with specific heat ratio gamma confined to a cylinder is put through a closed cycle. Initially, the gas is a P_i, V_i, and T_i. First, its pressure is tripled under constant volume. It then expands adiabatically to its original pressure and finally is compressed isobarically to its original volume.

Draw the P-V diagram diagram.

Determine the volume at the end of the adiabatic expansion.

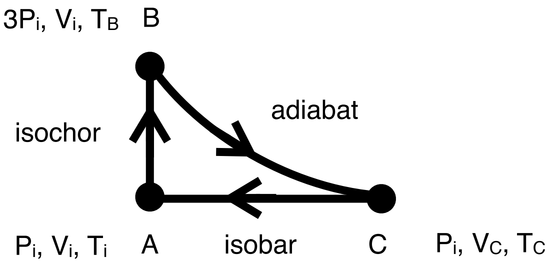
Find the temperature of the gas at the start of the adiabatic expansion.

Find the temperature at the end of the cycle.

What was the net work done on the gas for this cycle?

Solution

Here is the P-V diagram. This includes the obvious relationships between the states.



Use ideal gas equations for the states and the first law for the processes.

$$P_iV_i = nRT_i$$

$$3P_iV_i = nRT_B$$

$$P_iV_C = nRT_C$$

The first and second give

$$3(nRT_i) = nRT_B \quad \Rightarrow \quad T_B = 3T_i$$

The second and third are related by

$$3P_iV_i^\gamma = P_iV_C^\gamma \quad \Rightarrow \quad V_C = (3V_i^\gamma)^{1/\gamma} = 3^{1/\gamma}V_i$$

And

$$P_iV_C = nRT_C \quad \Rightarrow \quad T_C = \frac{P_iV_C}{nR} = 3^{1/\gamma}\frac{P_iV_i}{nR} = 3^{1/\gamma}T_i$$

The work done by the cycle is

$$W_{total} = W_{adiabat} + W_{isobar}$$

Here,

$$W_{adiabat} = \frac{f}{2}nR\Delta T = \frac{f}{2}nR(T_C - T_B) = \frac{f}{2}nR(3^{1/\gamma}T_i - 3T_i) = \frac{1}{\gamma - 1}nRT_i(3^{1/\gamma} - 3)$$

$$W_{adiabat} = \frac{(3^{1/\gamma} - 3)}{\gamma - 1} P_i V_i$$

And

$$W_{isobar} = -P_i \Delta V = -P_i (V_i - V_C) = -P_i (V_i - 3^{1/\gamma} V_i) = (3^{1/\gamma} - 1) P_i V_i$$

So

$$W_{total} = \frac{(3^{1/\gamma} - 3)}{\gamma - 1} P_i V_i + (3^{1/\gamma} - 1) P_i V_i = \left[\frac{(3^{1/\gamma} - 3)}{\gamma - 1} + (3^{1/\gamma} - 1) \right] P_i V_i$$

Problem 21.33

One cubic meter of atomic hydrogen at 0 °C at atmospheric pressure contains approximately 2.70×10^{25} atoms. The first excited state of the hydrogen atom has an energy of 10.2 eV above that of the lowest energy state (ground state). Use the Boltzmann factor to find the number of atoms in the first excited state at 0°C and at 10^4 °C.

Solution

According to the Boltzmann factor,

$$n(E) = n_o e^{-E/k_B T}$$

The number at the energy of 10.2 eV when the temperature is 273 K is

$$n(10.2\ eV) = (2.7\times10^{25})e^{-(10.2\ eV)\Big/(1.38\times10^{-23}\ \frac{J}{K})(273\ K)}$$

We have to convert electron volts to joules first.

$$10.2\ eV\left(\frac{1.60\times10^{-19}\ J}{1\ eV}\right) = 1.632\times10^{-18}\ J$$

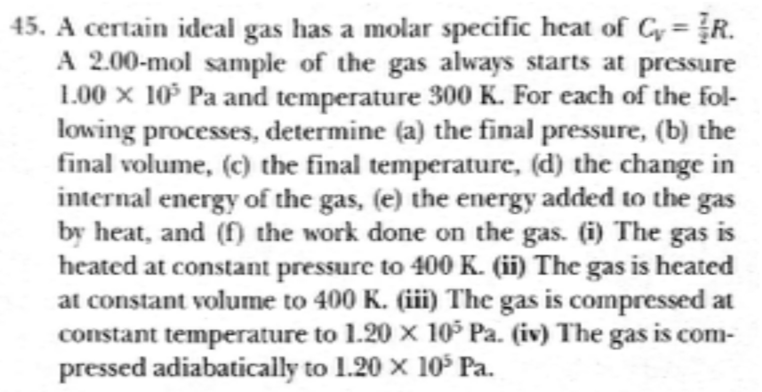
$$n(10.2\ eV) = (2.7\times10^{25})e^{-(1.632\times10^{-18}\ J)\Big/(1.38\times10^{-23}\ \frac{J}{K})(273\ K)} = 2\times10^{-163}$$

We can safely call this zero.

The number at 10273 K is

$$n(10.2\ eV) = (2.7\times10^{25})e^{-(1.632\times10^{-18}\ J)\Big/(1.38\times10^{-23}\ \frac{J}{K})(10273\ K)} = 2.7\times10^{20}$$

Problem 21.45



45. A certain ideal gas has a molar specific heat of $C_V = \frac{7}{2}R$. A 2.00-mol sample of the gas always starts at pressure 1.00×10^5 Pa and temperature 300 K. For each of the following processes, determine (a) the final pressure, (b) the final volume, (c) the final temperature, (d) the change in internal energy of the gas, (e) the energy added to the gas by heat, and (f) the work done on the gas. **(i)** The gas is heated at constant pressure to 400 K. **(ii)** The gas is heated at constant volume to 400 K. **(iii)** The gas is compressed at constant temperature to 1.20×10^5 Pa. **(iv)** The gas is compressed adiabatically to 1.20×10^5 Pa.

Solution

(i) The gas is heated at constant pressure to 400 K.

(a) At the initial state, the pressure is 1.00x10^5 Pa. At the final state, the pressure is the same, 1.00x10^5 Pa.

(b) The final volume is

$$P_f V_f = nRT_f \quad \Rightarrow \quad (1.00\times 10^5\ Pa)V_f = (2.00\ mol)(8.314\ \tfrac{J}{mol\cdot K})(400\ K) \quad \Rightarrow \quad V_f = 0.066512\ m^3$$

(c) The final temperature is 400 K.

(d) Because the constant volume specific heat is 7R/2, there are 7 degrees of freedom. The change in the internal energy is

$$\Delta E_{\text{int}} = \frac{7}{2}nR\Delta T = \frac{7}{2}(2.00\ mol)(8.314\ \tfrac{J}{mol\cdot K})(100\ K) = 5819.8\ J$$

(e) The heat applied to the gas is, for an isobaric process,

$$Q = \frac{9}{2}nR\Delta T = \frac{9}{2}(2.00\ mol)(8.314\ \tfrac{J}{mol\cdot K})(100\ K) = 7482.6\ J$$

(f) The work is done on the gas is

$$W = -nR\Delta T = -(2.00\ mol)(8.314\ \tfrac{J}{mol\cdot K})(100\ K) = -1662.8\ J$$

Or you can use the first law directly.

(ii) The gas is heated at constant volume to 400 K.

(a) At the initial state, the volume is

$$P_i V_i = nRT_i \quad \Rightarrow \quad (1.00\times 10^5\ Pa)V_i = (2.00\ mol)(8.314\ \tfrac{J}{mol\cdot K})(300\ K) \quad \Rightarrow \quad V_i = 0.049884\ m^3$$

At the final state, the volume is the same.

$$P_f V_f = nRT_f \quad \Rightarrow \quad P_f(0.066512\ m^3) = (2.00\ mol)(8.314\ \tfrac{J}{mol\cdot K})(400\ K) \quad \Rightarrow \quad P_f = 133{,}330\ Pa$$

(b) The final volume is 0.049884 m^3.

(c) The final temperature is 400 K.

(d) The change in the internal energy is still

$$\Delta E_{\text{int}} = 5819.8\ J$$

(e) The heat applied to the gas is, for an isochoric process,

$$Q = nC_V\Delta T = \frac{7}{2}nR\Delta T = \frac{7}{2}(2.00\ mol)(8.314\ \tfrac{J}{mol\cdot K})(100\ K) = 5819.8\ J$$

(f) The work is done on the gas is zero.

(iii) The gas is compressed at constant temperature to 1.20x10^5 Pa.

(a) The final pressure is 1.20x10^5 Pa.

(b) At the final state, the volume is, since the final temperature is unchanged,

$$P_fV_f = nRT_f \quad \Rightarrow \quad (1.20\times10^5\ Pa)V_f = (2.00\ mol)(8.314\ \tfrac{J}{mol\cdot K})(300\ K) \quad \Rightarrow \quad V_f = 0.041570\ m^3$$

(c) The final temperature is 300 K.

(d) The change in the internal energy is zero.

(e) The heat applied to the gas is, for an isothermal process,

$$Q = nRT\cdot\ln\left(\frac{V_f}{V_i}\right) = (2.00\ mol)(8.314\ \tfrac{J}{mol\cdot K})\ln\left(\frac{0.049984\ m^3}{0.041570\ m^3}\right) = 3.0649\ J$$

(f) The work is done on the gas is –3.0649 J.

(vi) The gas is compressed adiabatically to 1.20x10^5 Pa.

(a) The final pressure is 1.20x10^5 Pa.

(b) At the final state, the following is true.

$$P_iV_i^\gamma = P_fV_f^\gamma \quad \Rightarrow \quad (1.00\times10^5\ Pa)(0.049884\ m^3)^{9/7} = (1.20\times10^5\ Pa)V_f^{9/7} \quad \Rightarrow \quad V_f = 0.043289\ m^3$$

(c) The final temperature is

$$P_fV_f = nRT_f \quad \Rightarrow \quad (1.20\times10^5\ Pa)(0.043289\ m^3) = (2.00\ mol)(8.314\ \tfrac{J}{mol\cdot K})T_f \quad \Rightarrow \quad T_f = 312.40\ K$$

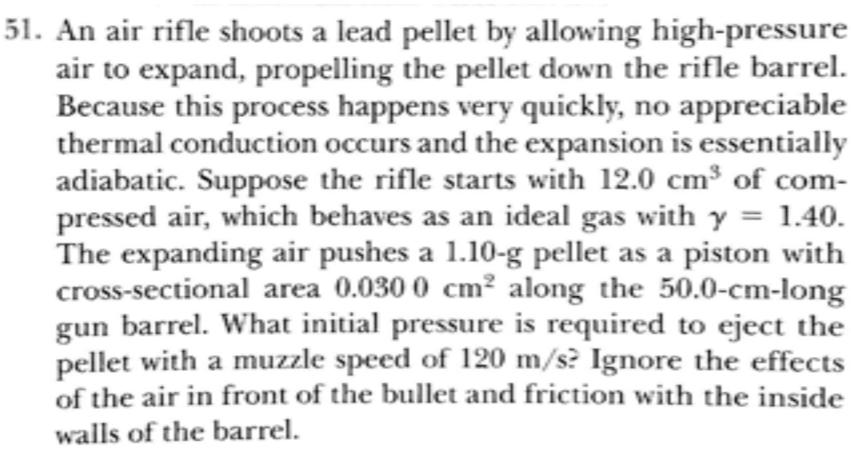
(d) The change in the internal energy is

$$\Delta E_{\text{int}} = \frac{7}{2}nR\Delta T = \frac{7}{2}(2.00\ mol)(8.314\ \tfrac{J}{mol\cdot K})(312.40\ K - 300\ K) = 14.733\ J$$

(e) The heat applied to the gas is zero.

(f) The work is done on the gas is 14.733 J.

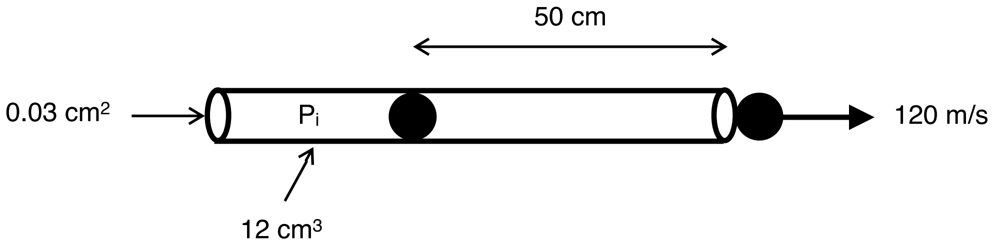
Problem 21.51



51. An air rifle shoots a lead pellet by allowing high-pressure air to expand, propelling the pellet down the rifle barrel. Because this process happens very quickly, no appreciable thermal conduction occurs and the expansion is essentially adiabatic. Suppose the rifle starts with 12.0 cm^3 of compressed air, which behaves as an ideal gas with $\gamma = 1.40$. The expanding air pushes a 1.10-g pellet as a piston with cross-sectional area 0.030 0 cm^2 along the 50.0-cm-long gun barrel. What initial pressure is required to eject the pellet with a muzzle speed of 120 m/s? Ignore the effects of the air in front of the bullet and friction with the inside walls of the barrel.

Solution

Here is what the situation looks like.



We can treat the air as an ideal gas.

$$P_iV_i = nRT_i$$

$$P_fV_f = nRT_f$$

The expansion is adiabatic. The states of the gas are connected adiabatically.

$$P_iV_i^\gamma = P_fV_f^\gamma$$

$$T_iV_i^{\gamma-1} = T_fV_f^{\gamma-1}$$

The total work done on the gas is negative of the work done by the gas expanding against the atmosphere and the kinetic energy of the bullet. The question says to ignore the air so the pressure from the atmosphere is ignored.

$$W = -\frac{1}{2}m\Delta v^2 = -\frac{1}{2}(1.10\times10^{-3}\ kg)(120\ m/s)^2 = -7.92\ J$$

The work done on the gas under an adiabatic expansion process is also equal to the change in the internal energy is

$$W = \Delta E_{int} = \frac{5}{2}nR\Delta T = -7.92\ J$$

Here, gamma is 1.40 = 7/5, which means f = 5. This can be written in terms of the pressures and the volumes.

$$nR\Delta T = P_fV_f - P_iV_i$$

So

$$W = \frac{5}{2}nR\Delta T = \frac{5}{2}(P_f V_f - P_i V_i) = -7.92\ J$$

The two pressures are unknown. But we do know this also about the two pressures.

$$P_i V_i^\gamma = P_f V_f^\gamma$$

Solve for the final pressure to get the initial pressure. The final volume is 12 cm^3 + (50 cm)(0.03 cm^2) = 13.5 cm^3.

$$W = -7.92\ J = \frac{5}{2}(P_f V_f - P_i V_i)$$

$$P_f = \frac{\frac{2}{5}W + P_i V_i}{V_f} = \frac{\frac{2}{5}(-7.92\ J) + P_i(12\times 10^{-6}\ m^3)}{13.5\times 10^{-6}\ m^3} = -2.3467\times 10^5\ Pa + 0.88888P_i$$

The initial pressure is

$$P_i V_i^\gamma = P_f V_f^\gamma \quad \Rightarrow \quad P_i = P_f\left(\frac{V_f}{V_i}\right)^\gamma = (-2.3467\times 10^5\ Pa + 0.88888P_i)\left(\frac{13.5\times 10^{-6}\ m^3}{12\times 10^{-6}\ m^3}\right)^{1.4}$$

$$P_i = -2.7674\times 10^5\ Pa + 1.0482P_i \quad \Rightarrow \quad P_i = 5.7379\times 10^6\ Pa = 56.629\ atm$$

Problem 21.71

A cylinder is closed at both ends and has insulating walls. It is divided into two compartments by an insulating piston that is perpendicular to the axis of the cylinder as shown in Figure P21.71a. Each compartment contains 1.00 mol of oxygen that behaves as an ideal gas with $\gamma = 1.40$. Initially, the two compartments have equal volumes and their temperatures are 550 K and 250 K. The piston is then allowed to move slowly parallel to the axis of the cylinder until it comes to rest at an equilibrium position (Fig. P21.71b). Find the final temperatures in the two compartments.

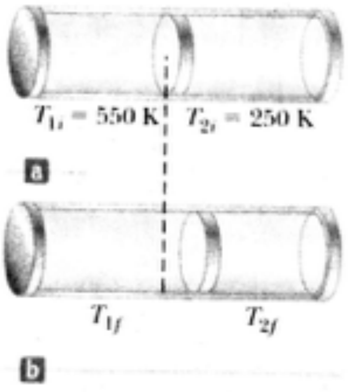


Figure P21.71

Solution

There are two states here and each state have two gases. Let the initial volumes be V.

$$P_{1i}V_{1i} = nRT_{1i} \quad \Rightarrow \quad P_{1i}V = nR(550)$$

$$P_{2i}V_{2i} = nRT_{2i} \quad \Rightarrow \quad P_{2i}V = nR(250)$$

Let the final pressures be P.

$$P_{1f}V_{1f} = nRT_{1f} \quad \Rightarrow \quad PV_{1f} = nRT_{1f}$$

$$P_{2f}V_{2f} = nRT_{2f} \quad \Rightarrow \quad PV_{2f} = nRT_{2f}$$

No heat flows between these gases and the outside so the initial state and the final state for both sides are on their own adiabats. This means that the total internal energy for both sides is conserved. This also means that the total work done on the two gases add up to zero.

$$\Delta E_1 + \Delta E_2 = 0$$

$$W_1 + W_2 = 0$$

Here is the algebra. The ratio of the initial state equations is this.

$$\frac{P_{1i}V}{P_{2i}V} = \frac{nR(550)}{nR(250)} \quad \Rightarrow \quad \frac{P_{1i}}{P_{2i}} = \frac{550}{250} = \frac{11}{5}$$

The states on each side are connected adiabatically. The adiabatic index is 1.4.

$$P_{1i}V_{1i}^{\gamma} = P_{1f}V_{1f}^{\gamma} \quad \Rightarrow \quad P_{1i}V^{\gamma} = PV_{1f}^{\gamma}$$

$$P_{2i}V_{2i}^{\gamma} = P_{2f}V_{2f}^{\gamma} \quad \Rightarrow \quad P_{2i}V^{\gamma} = PV_{2f}^{\gamma}$$

Their ratio is this.

$$\frac{P_{1i}}{P_{2i}} = \left(\frac{V_{1f}}{V_{2f}}\right)^{\gamma} = \frac{11}{5} \quad \Rightarrow \quad \frac{V_{1f}}{V_{2f}} = \left(\frac{11}{5}\right)^{1/\gamma} = 1.7563$$

The ratio of the final state equations is this.

$$\frac{PV_{1f}}{PV_{2f}} = \frac{nRT_{1f}}{nRT_{2f}} \quad \Rightarrow \quad \frac{V_{1f}}{V_{2f}} = \frac{T_{1f}}{T_{2f}} = 1.7563 \quad \Rightarrow \quad T_{1f} = 1.7563 T_{2f}$$

The energy equation says this

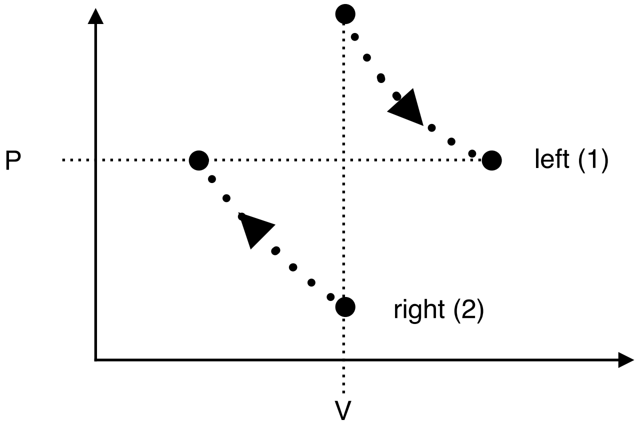
$$\Delta E_1 + \Delta E_2 = 0 \quad \Rightarrow \quad \Delta T_1 + \Delta T_2 = 0 \quad \Rightarrow \quad T_{1f} - T_{1i} + T_{2f} - T_{2i} = 0$$

$$T_{1f} + T_{2f} = 800\ K$$

$$1.7563 T_{2f} + T_{2f} = 2.7563 T_{2f} = 800\ K \quad \Rightarrow \quad T_{2f} = 290.24\ K$$

$$T_{1f} = 1.7563 T_{2f} = 1.7563(290.24\ K) = 509.76\ K$$

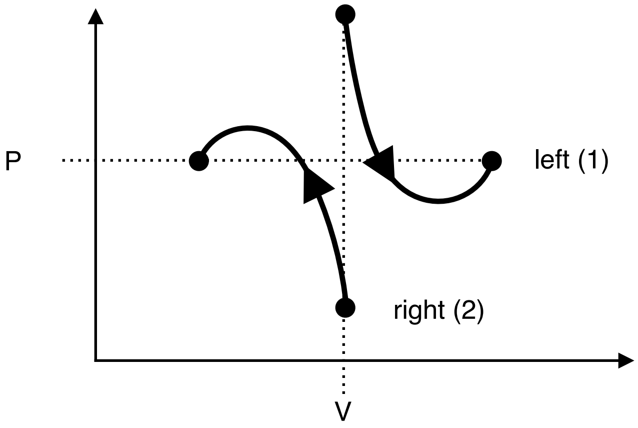
Here is what the states of the processes look like.



Above, I said this.

$$W_1 + W_2 = 0$$

This is clearly not the area under the adiabats. The only way that this can be consistent is if the processes were not adiabatic. The adiabats connect only the end states of the two gases, it is not the path the gas actually took going from the initial to the final states. Here is one possible example of how to connect the end states that would make the work equal in magnitude.



The question states that “the piston is allowed to move slowly”. This means a process that is quasi-static. It implies reversibility. In this case, we can, at most, say that the process is quasi-static and that’s likely untrue. It is definitely not reversible.