Due Monday 10/08/18 at 11:00am Name ________________________________

Homework Chapter 25: Capacitance

25.04 The plates of a spherical capacitor have radii 38.0 mm and 40.0 mm. (a) Calculate the capacitance. (b) What must be the plate area of a parallel-plate capacitor with the same plate separation and capacitance?

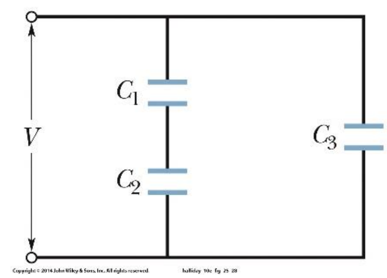
4. (a) We use Eq. 25-17:

$$C = 4\pi\varepsilon_0 \frac{ab}{b-a} = \frac{(40.0\,\text{mm})(38.0\,\text{mm})}{\left(8.99\times10^9\,\frac{\text{N}\cdot\text{m}^2}{\text{C}^2}\right)(40.0\,\text{mm}-38.0\,\text{mm})} = 84.5\ \text{pF}.$$

(b) Let the area required be A. Then $C = \varepsilon_0 A/(b-a)$, or

$$A = \frac{C(b-a)}{\varepsilon_0} = \frac{(84.5\,\text{pF})(40.0\,\text{mm}-38.0\,\text{mm})}{\left(8.85\times10^{-12}\ \text{C}^2/\text{N}\cdot\text{m}^2\right)} = 191\,\text{cm}^2.$$

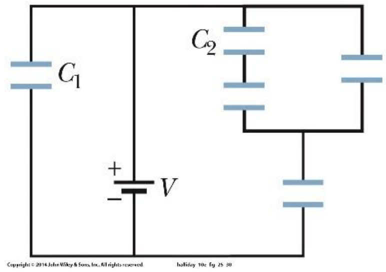
25.10 In Fig. 25-28, find the equivalent capacitance of the combination. Assume that C_1 is 10.0 μF, C_2 is 5.00 μF, and C_3 is 4.00 μF.



10. The equivalent capacitance is

$$C_{\text{eq}} = C_3 + \frac{C_1C_2}{C_1+C_2} = 4.00\,\mu\text{F} + \frac{(10.0\,\mu\text{F})(5.00\,\mu\text{F})}{10.0\,\mu\text{F}+5.00\,\mu\text{F}} = 7.33\,\mu\text{F}.$$

25.14 In Fig. 25-30, the battery has a potential difference of $V = 10.0$ V and the five capacitors each have a capacitance of 10.0 μF. What is the charge on (a) capacitor 1 and (b) capacitor 2?



14. (a) The potential difference across C_1 is $V_1 = 10.0$ V. Thus,

$$q_1 = C_1V_1 = (10.0\ \mu\text{F})(10.0\ \text{V}) = 1.00 \times 10^{-4}\ \text{C}.$$

(b) Let $C = 10.0\ \mu$F. We first consider the three-capacitor combination consisting of C_2 and its two closest neighbors, each of capacitance C. The equivalent capacitance of this combination is

$$C_{\text{eq}} = C + \frac{C_2C}{C+C_2} = 1.50\ C.$$

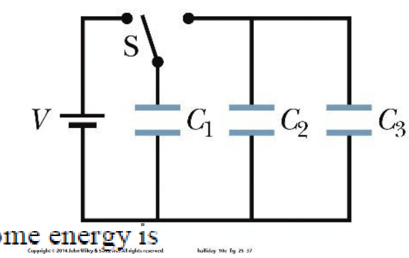
Also, the voltage drop across this combination is

$$V = \frac{CV_1}{C+C_{\text{eq}}} = \frac{CV_1}{C+1.50\ C} = 0.40V_1.$$

Since this voltage difference is divided equally between C_2 and the one connected in series with it, the voltage difference across C_2 satisfies $V_2 = V/2 = V_1/5$. Thus

$$q_2 = C_2V_2 = (10.0\,\mu\text{F})\left(\frac{10.0\text{V}}{5}\right) = 2.00\times10^{-5}\,\text{C}.$$

25.22 In Fig. 25-37, $V = 10$ V, $C_1 = 10\ \mu$F, and $C_2 = C_3 = 20\ \mu$F. Switch S is first thrown to the left side until capacitor 1 reaches equilibrium. Then the switch is thrown to the right. When equilibrium is again reached, how much charge is on capacitor 1?



22. We do not employ energy conservation since, in reaching equilibrium, some energy is dissipated either as heat or radio waves. Charge is conserved; therefore, if $Q = C_1V_{\text{bat}} = 100\ \mu$C, and q_1, q_2 and q_3 are the charges on C_1, C_2 and C_3 after the switch is thrown to the right and equilibrium is reached, then

$$Q = q_1 + q_2 + q_3.$$

Since the parallel pair C_2 and C_3 are identical, it is clear that $q_2 = q_3$. They are in parallel with C_1 so that $V_1 = V_3$, or

$$\frac{q_1}{C_1} = \frac{q_3}{C_3}$$

which leads to $q_1 = q_3/2$. Therefore,

$$Q = (q_3/2) + q_3 + q_3 = 5q_3/2$$

which yields $q_3 = 2Q/5 = 2(100\ \mu\text{C})/5 = 40\ \mu\text{C}$ and consequently $q_1 = q_3/2 = 20\ \mu$C.

25.31 A 2.0-μF capacitor and a 4.0-μF capacitor are connected in parallel across a 300-V potential difference. Calculate the total energy stored in the capacitors.

31. **THINK** The total electrical energy is the sum of the energies stored in the individual capacitors.

EXPRESS The energy stored in a charged capacitor is

$$U = \frac{q^2}{2C} = \frac{1}{2}CV^2.$$

Since we have two capacitors that are connected in parallel, the potential difference V across the capacitors is the same and the total energy is

$$U_{\text{tot}} = U_1 + U_2 = \frac{1}{2}(C_1 + C_2)V^2.$$

ANALYZE Substituting the values given, we have

$$U = \frac{1}{2}(C_1 + C_2)V^2 = \frac{1}{2}\left(2.0\times10^{-6}\,\text{F} + 4.0\times10^{-6}\,\text{F}\right)(300\,\text{V})^2 = 0.27\,\text{J}.$$

LEARN The energy stored in a capacitor is equal to the amount of work required to charge the capacitor.

25.32 A parallel-plate air-filled capacitor having area 40 cm^2 and plate spacing 1.0 mm is charged to a potential difference of 600 V. Find (a) the capacitance, (b) the magnitude of the charge on each plate, (c) the stored energy, (d) the electric field between the plates, and (e) the energy density between the plates.

32. (a) The capacitance is

$$C = \frac{\varepsilon_0 A}{d} = \frac{\left(8.85\times10^{-12}\ \text{C}^2/\text{N}\cdot\text{m}^2\right)\left(40\times10^{-4}\ \text{m}^2\right)}{1.0\times10^{-3}\,\text{m}} = 3.5\times10^{-11}\,\text{F} = 35\,\text{pF}.$$

(b) $q = CV = (35\ \text{pF})(600\ \text{V}) = 2.1\times10^{-8}\ \text{C} = 21\ \text{nC}$.

(c) $U = \frac{1}{2}CV^2 = \frac{1}{2}(35\text{pF})(21\text{nC})^2 = 6.3\times10^{-6}\,\text{J} = 6.3\,\mu\text{J}$.

(d) $E = V/d = 600\ \text{V}/1.0\times10^{-3}\ \text{m} = 6.0\times10^{5}\ \text{V/m}$.

(e) The energy density (energy per unit volume) is

$$u = \frac{U}{Ad} = \frac{6.3\times10^{-6}\,\text{J}}{\left(40\times10^{-4}\text{m}^2\right)\left(1.0\times10^{-3}\text{m}\right)} = 1.6\ \text{J/m}^3.$$

25.41. A coaxial cable used in a transmission line has an inner radius of 0.10 mm and an outer radius of 0.60 mm. Calculate the capacitance per meter for the cable. Assume that the space between the conductors is filled with polystyrene.

41. **THINK** Our system, a coaxial cable, is a cylindrical capacitor filled with polystyrene, a dielectric.

EXPRESS Using Eqs. 25-17 and 25-27, the capacitance of a cylindrical capacitor can be written as

$$C = \kappa C_0 = \frac{2\pi\kappa\varepsilon_0 L}{\ln(b/a)},$$

where C_0 is the capacitance without the dielectric, κ is the dielectric constant, L is the length, a is the inner radius, and b is the outer radius.

ANALYZE With $\kappa = 2.6$ for polystyrene, the capacitance per unit length of the cable is

$$\frac{C}{L} = \frac{2\pi\kappa\varepsilon_0}{\ln(b/a)} = \frac{2\pi(2.6)(8.85\times10^{-12}\ \text{F/m})}{\ln[(0.60\ \text{mm})/(0.10\ \text{mm})]} = 8.1\times10^{-11}\,\text{F/m} = 81\ \text{pF/m}.$$

LEARN When the space between the plates of a capacitor is completely filled with a dielectric material, the capacitor increases by a factor κ, the dielectric constant characteristic of the material.

25.45. A certain parallel-plate capacitor is filled with a dielectric for which $\kappa = 5.5$. The area of each plate is 0.034 m^2, and the plates are separated by 2.0 mm. The capacitor will fail (short out and burn up) if the electric field between the plates exceeds 200 kN/C. What is the maximum energy that can be stored in the capacitor?

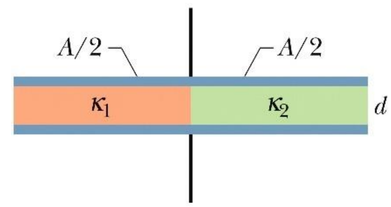
45. Using Eq. 25-29, with $\sigma = q/A$, we have

$$\left|\vec{E}\right| = \frac{q}{\kappa\varepsilon_0 A} = 200\times10^3\ \text{N/C}$$

which yields $q = 3.3\times10^{-7}$ C. Eq. 25-21 and Eq. 25-27 therefore lead to

$$U = \frac{q^2}{2C} = \frac{q^2 d}{2\kappa\varepsilon_0 A} = 6.6\times10^{-5}\ \text{J}\ .$$

25.48. Figure 25-47 shows a parallel-plate capacitor with a plate area $A = 5.56\ \text{cm}^2$ and separation $d = 5.56$ mm. The left half of the gap is filled with material of dielectric constant $\kappa_1 = 7.00$; the right half is filled with material of dielectric constant $\kappa_2 = 12.0$. What is the capacitance?



48. The capacitor can be viewed as two capacitors C_1 and C_2 in parallel, each with surface area $A/2$ and plate separation d, filled with dielectric materials with dielectric constants κ_1 and κ_2, respectively. Thus, (in SI units),

$$C = C_1 + C_2 = \frac{\varepsilon_0 (A/2)\kappa_1}{d} + \frac{\varepsilon_0 (A/2)\kappa_2}{d} = \frac{\varepsilon_0 A}{d}\left(\frac{\kappa_1 + \kappa_2}{2}\right)$$

$$= \frac{(8.85\times10^{-12}\ \text{C}^2/\text{N}\cdot\text{m}^2)(5.56\times10^{-4}\ \text{m}^2)}{5.56\times10^{-3}\ \text{m}}\left(\frac{7.00+12.00}{2}\right) = 8.41\times10^{-12}\ \text{F}.$$

25.60. *The chocolate crumb mystery.* This story begins with Problem 60 in Chapter 23. As part of the investigation of the biscuit factory explosion, the electric potentials of the workers were measured as they emptied sacks of chocolate crumb powder into the loading bin, stirring up a cloud of the powder around themselves. Each worker had an electric potential of about 7.0 kV relative to the ground, which was taken as zero potential. (a) Assuming that each worker was effectively a capacitor with a typical capacitance of 200 pF, find the energy stored in that effective capacitor. If a single spark between the worker and any conducting object connected to the ground neutralized the worker, that energy would be transferred to the spark. According to measurements, a spark that could ignite a cloud of chocolate crumb powder, and thus set off an explosion, had to have an energy of at least 150 mJ. (b) Could a spark from a worker have set off an explosion in the cloud of powder in the loading bin? (The story continues with Problem 60 in Chapter 26.)

60. (a) Equation 25-22 yields

$$U = \frac{1}{2}CV^2 = \frac{1}{2}\left(200\times10^{-12}\ \text{F}\right)\left(7.0\times10^{3}\ \text{V}\right)^2 = 4.9\times10^{-3}\ \text{J}.$$

(b) Our result from part (a) is much less than the required 150 mJ, so such a spark should not have set off an explosion.