

Hooke's law in terms of stress and strain is

$$\text{stress} \propto \text{strain}$$

In terms of the definitions

$$\frac{F}{A} = Y \frac{\Delta L}{L}$$

The constant of proportionality is called the **elastic modulus** or **Young's modulus**. It has the same units as stress.  $Y$  is a property of the material used.

Hooke's law holds up to a maximum stress called the **proportional limit**.

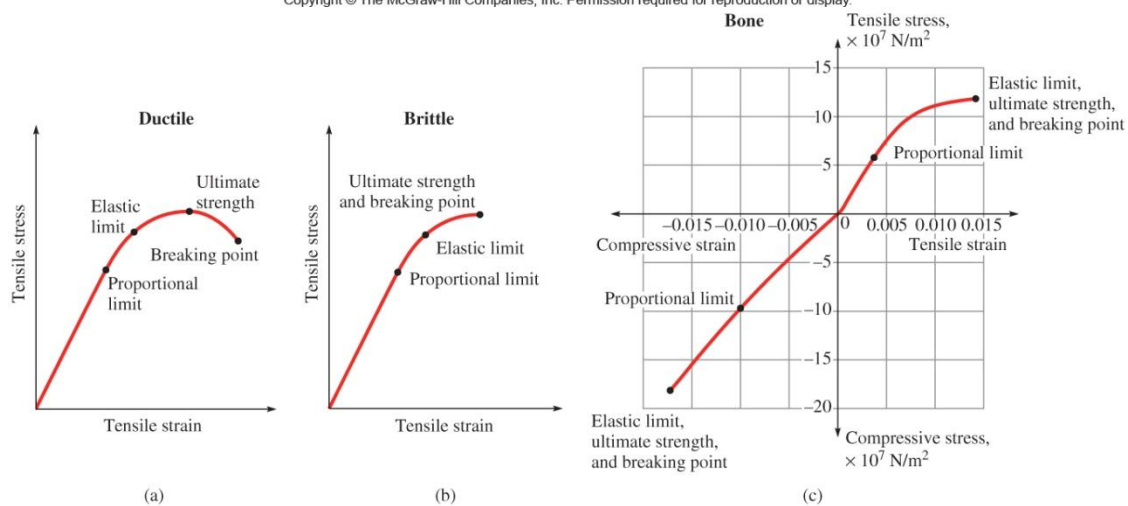
### Beyond the Proportional Limit

If the stress exceeds the proportional limit, the strain is no longer proportional to the stress. The solid will return to its original shape when the stress is removed.

### Some more vocabulary

- **Elastic limit** – When the stress is less than the elastic limit, removing the stress will return the solid to the original shape. If the elastic limit is surpassed, the solid remains permanently deformed.
- **Ultimate strength** – If the ultimate strength is surpassed, the solid fractures. The ultimate strength can be different for tensile and compressive stresses.

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- **Ductile** – A ductile material continues to stretch beyond its ultimate strength without breaking and the stress decreases from the ultimate strength.
- **Brittle** – A brittle material has the ultimate strength and the breaking point close together.

### Other Deformations

There are other ways to deform a solid. Two additional ways are shear deformation and volume deformation.

### Shear Deformation

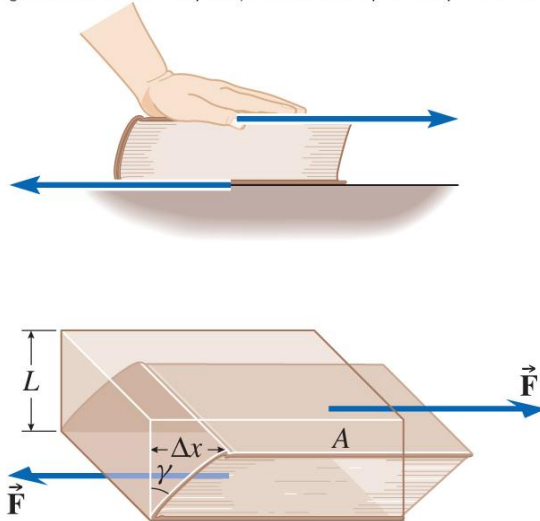
The forces act parallel to the edge of the solid. Tensile and compressive forces act perpendicular to the edges.

$$\text{shear stress} = \frac{F}{A}$$

It looks like the previous definition but the picture below shows otherwise.

$$\text{shear strain} = \frac{\Delta x}{L}$$

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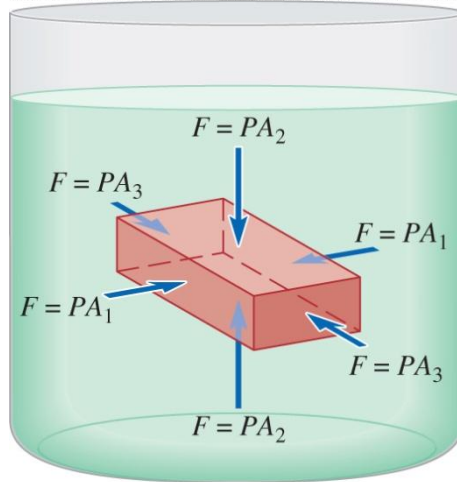
Define the **shear modulus**  $S$  as

$$\frac{F}{A} = S \frac{\Delta x}{L}$$

The shear modulus is also measured in Pa.

### Volume Deformation

As an example, consider a solid immersed in a fluid. The pressure exerted on all sides will change its volume.



The volume stress is created by the pressure.

$$\text{volume stress} = \frac{F}{A} = P$$

The strain will be the change in volume caused by the pressure

$$\text{volume strain} = \frac{\Delta V}{V}$$

The **bulk modulus** is defined in

$$\Delta P = -B \frac{\Delta V}{V}$$

Like the other moduli,  $B$  is measured in Pa.  $\Delta P$  refers to the additional pressure above an atmosphere. Why is there a minus sign?

Unlike the previous stresses and strains, volume stress can be applied to a fluid.

**Problem** A certain man's biceps muscle has a maximum cross-sectional area of  $12 \text{ cm}^2 = 1.2 \times 10^{-3} \text{ m}^2$ . What is the stress in the muscle if it exerts a force of 300 N?

**Solution** From the definition of tensile stress, we have

$$\text{Stress} = \frac{F}{A} = \frac{300 \text{ N}}{1.2 \times 10^{-3} \text{ m}^2} = 2.5 \times 10^5 \text{ Pa}$$

**Problem** A wire 1.5 m long has a cross-sectional area of  $2.4 \text{ mm}^2$ . It is hung vertically and stretches 0.32 mm when a 10-kg block is attached to it. Find (a) the stress, (b) the strain, and (c) Young's modulus for the wire.

**Solution** All measurements must be in SI units.

$$A = 2.4 \text{ mm}^2 \times \left( \frac{1 \text{ m}}{1000 \text{ mm}} \right)^2 = 2.4 \times 10^{-6} \text{ m}^2$$

$$\Delta L = 0.32 \text{ mm} \times \left( \frac{1 \text{ m}}{1000 \text{ mm}} \right) = 3.2 \times 10^{-4} \text{ m}$$

(a) Use the definition of stress

$$\text{Stress} = \frac{F}{A} = \frac{mg}{A} = \frac{(10 \text{ kg})(9.8 \text{ m/s}^2)}{2.4 \times 10^{-6} \text{ m}^2} = 4.08 \times 10^7 \text{ Pa}$$

(b) The definition of strain

$$\text{Strain} = \frac{\Delta L}{L} = \frac{3.2 \times 10^{-4} \text{ m}}{1.5 \text{ m}} = 2.13 \times 10^{-4}$$

(c) Young's modulus

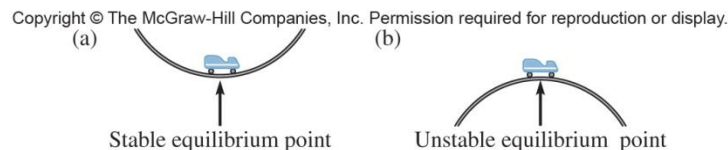
$$\frac{F}{A} = Y \frac{\Delta L}{L}$$

$$\text{Stress} = Y \text{Strain}$$

$$Y = \frac{\text{Stress}}{\text{Strain}} = \frac{4.08 \times 10^7 \text{ N/m}^2}{2.13 \times 10^{-4}} = 1.92 \times 10^7 \text{ Pa}$$

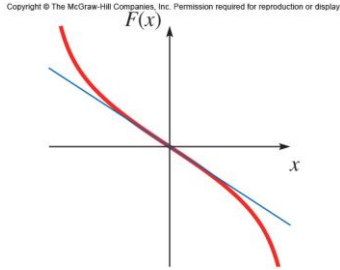
### Simple Harmonic Motion

Vibration is repeated motion back and forth along the same path. Vibrations occur in the vicinity of a point of **stable equilibrium**.



An equilibrium point is stable if the net force on the object when it is displaced a small distance from equilibrium points back towards the equilibrium point.

This type of force is called a restoring force. Simple harmonic motion (SHM) occurs whenever the restoring force is proportional to the displacement from equilibrium.



Simple harmonic motion can be used to approximate small vibrations.

### Energy Analysis

For a spring-mass system, we know that energy is conserved.

$$E = K + U = \text{constant}$$

Recalling the definitions for  $K$  and  $U$

$$E = \frac{1}{2}mv^2 + \frac{1}{2}kx^2$$

We will call the maximum displacement of the body the amplitude  $A$ . At the maximum displacement, the object stops.

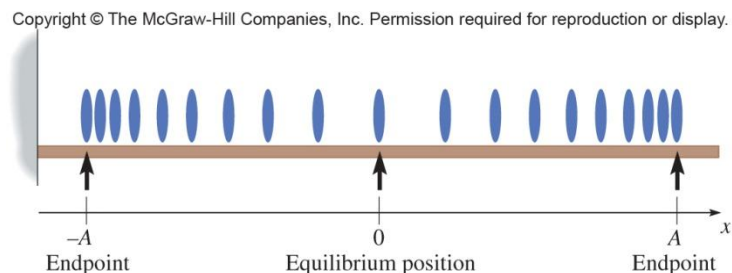
$$E_{total} = \frac{1}{2}m(0)^2 + \frac{1}{2}kA^2 = \frac{1}{2}kA^2$$

The maximum speed occurs at equilibrium ( $x = 0$ ) and  $U = 0$

$$E_{total} = \frac{1}{2}mv_m^2 + \frac{1}{2}k(0)^2 = \frac{1}{2}mv_m^2$$

Equating the two forms of  $E_{total}$  gives

$$v_m = \sqrt{\frac{k}{m}}A$$



### Acceleration in SHM

From Hooke's law for springs

$$F_x = -kx$$

$$ma_x = -kx$$

$$a_x = \frac{-kx}{m}$$

The acceleration (not a constant) is proportional to the displacement and in the opposite direction. The largest acceleration occurs at the largest displacement ( $A$ )

$$a_x = \frac{kA}{m}$$

### Period and Frequency

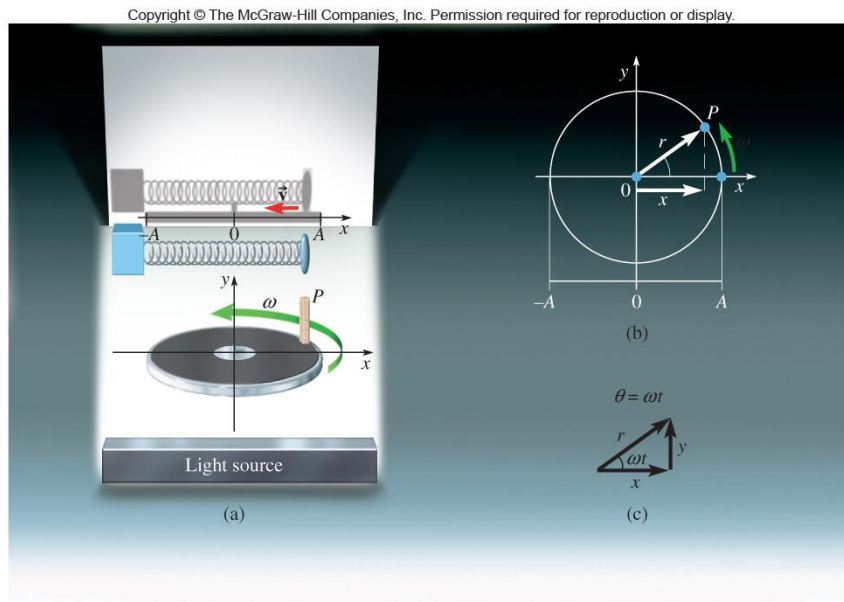
One cycle means that the particle is at the same location and heading the same direction. The period ( $T$ ) is the time to complete one cycle. The frequency ( $f$ ) is the number of cycles per second. Just as before

$$f = \frac{1}{T}$$

Not all vibrations are simple harmonic.

### Circular Motion and Simple Harmonic Motion

The projection of uniform circular motion along any axis (the  $x$ -axis here) is the same as simple harmonic motion. We use our understanding of uniform circular motion to arrive at the equations of simple harmonic motion.



The projection of the position on the  $x$ -axis gives

$$x(t) = A \cos \omega t$$

The acceleration is inward and

$$a(t) = -\omega^2 A \cos \omega t$$

The velocity can be shown to be

$$v(t) = -\omega A \sin \omega t$$

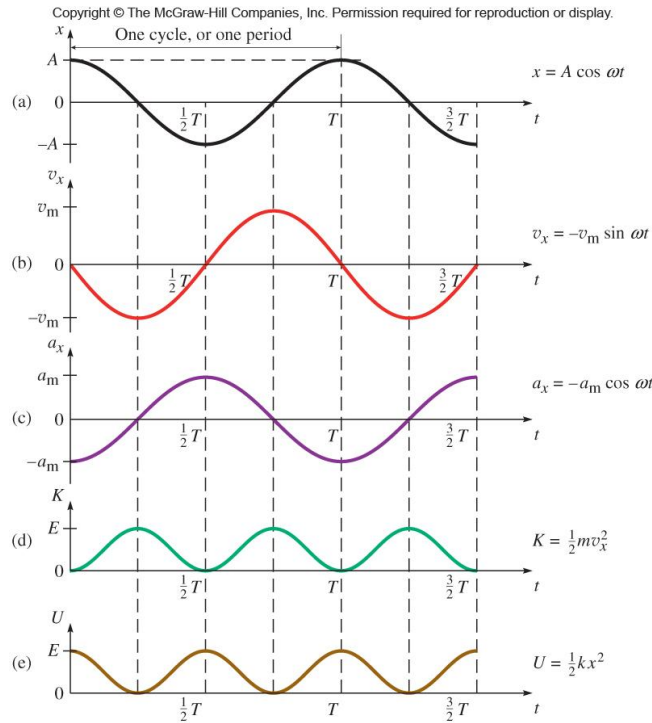
The angular frequency is

$$\omega = \sqrt{\frac{k}{m}}$$

For the ideal spring mass system

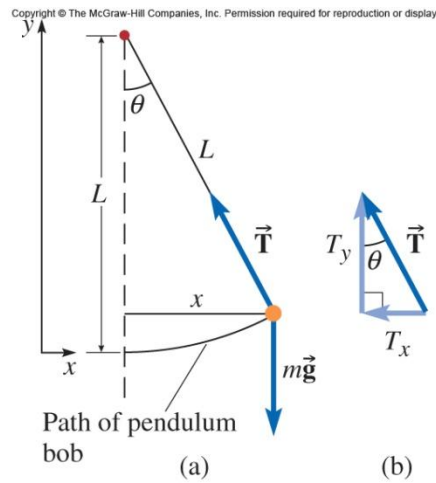
$$f = \frac{\omega}{2\pi} = \frac{1}{2\pi} \sqrt{\frac{k}{m}} \quad T = \frac{1}{f} = 2\pi \sqrt{\frac{m}{k}}$$

How do you keep these straight? Memorize one and derive the other.



## The Pendulum

For small oscillations, the simple pendulum executes simple harmonic motion. A simple pendulum is point mass suspended from a string.



Applying Newton's second law

$$\begin{aligned}\sum F_x &= ma_x \\ -T \sin \theta &= ma_x \\ -T \frac{x}{L} &= ma_x\end{aligned}$$

For small angles  $\sin \theta = \theta$ . The acceleration along the y-axis is negligible

$$\begin{aligned}\sum F_y &= ma_y \\ T \cos \theta - mg &= 0 \\ T &= mg\end{aligned}$$

For small angles  $\cos \theta = 1$ . Substituting for  $T$

$$\begin{aligned}-T \frac{x}{L} &= ma_x \\ -mg \frac{x}{L} &= ma_x \\ a_x &= -\frac{g}{L} x\end{aligned}$$

This is the condition of simple harmonic motion. We can pick off the angular frequency as



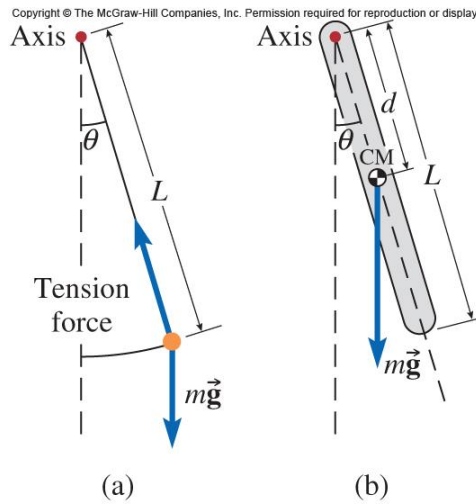
$$\omega = \sqrt{\frac{g}{L}}$$

The period of the pendulum

$$T = 2\pi\sqrt{\frac{L}{g}}$$

### Physical Pendulum.

Instead of a point mass, the vibrating object has size.



The period is

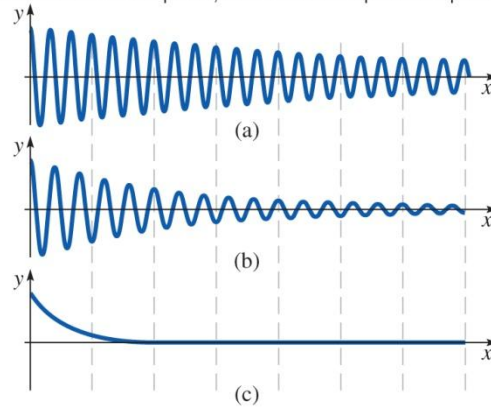
$$T = 2\pi\sqrt{\frac{I}{mgd}}$$

where  $d$  is the distance from the rotation axis to the center of mass of the object and  $I$  is the rotational inertia about the rotation axis (not the center of mass).

### Damped Oscillations

In reality a vibrating system will stop vibrating. Energy is lost to the surroundings. Since it is similar to the drag felts when moving through water, it is called damped motion.

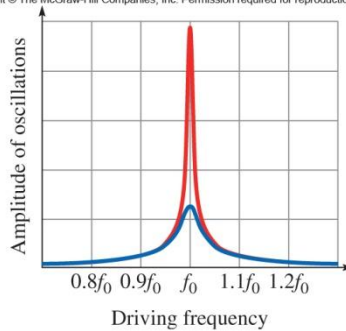
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## Forced Oscillations and Resonance

To compensate for energy lost in damped systems, it is possible to drive the system at its resonant frequency.

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Forced oscillations can occur when the periodic driving force acts on a system that can vibrate. If the system is driven at its resonance frequency, vibrations can increase in amplitude until the system is destroyed.

### The Short Life and Tragic End of Gallopin' Gertie

[http://en.wikipedia.org/wiki/Tacoma\\_Narrows\\_Bridge\\_%281940%29](http://en.wikipedia.org/wiki/Tacoma_Narrows_Bridge_%281940%29)

<http://www.youtube.com/watch?v=j-zczJXSxw>

Best of all. Long but worth it: [http://www.youtube.com/watch?v=tff5Dkk\\_zBE](http://www.youtube.com/watch?v=tff5Dkk_zBE)

You might be interested in other episodes of the *Mechanical Universe* series made in the 1980s at Cal Tech. <http://www.learner.org/resources/series42.html>