UNIT
2

The Combined Gas Law

How are the Pressure, Volume, and Temperature of a Gas Related to One Another?

All was glorious—a cloudless sky above, a most delicious view around….
How great is our good fortune! I care not what may be the condition of the earth;
it is the sky that is for me now.

Jacques Charles after the first free flight in a manned hydrogen balloon, 1 December 1783

Engage: How Does Temperature Affect the Volume of Gas in a Balloon?

A. What is the mass of the air in a typical bicycle tire?

Before Studying this Unit	After Studying this Unit

Bicycle tires are manufactured in a wide variety of sizes and styles.

B. What is meant when we talk about the *pressure* of the air within a bicycle tire?

Before Studying this Unit	After Studying this Unit

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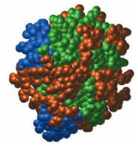

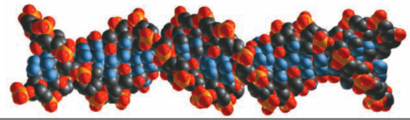

C. What method(s) can you use to change the pressure of the air within a bicycle tire?

Before Studying this Unit	After Studying this Unit

1. Your teacher will demonstrate how temperature affects the volume of a balloon. Describe your observations in the space below.

What affects the behavior of the gaseous air contained within a basketball?

2. Consider familiar items in your life that involve gases: a car or bike tire, a basketball, an aerosol can, a helium balloon, or a propane tank. Describe as many variables as you can that affect the behavior of the gas.

3. Which variables were allowed to change during the demonstration and which variables were held constant? Explain.

4. Provide an explanation for the balloon in a bottle demonstration.

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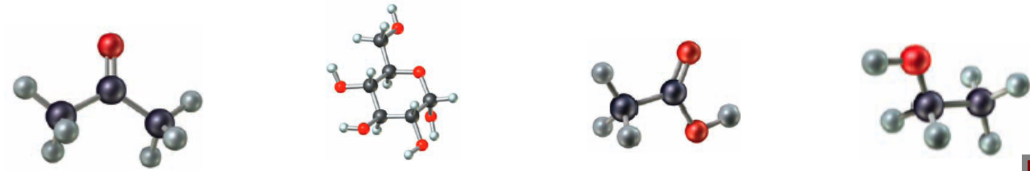

Explore 1: What is the Relationship Between the Volume and the Temperature of a Gas?

Your teacher will tell the class to perform either Option A *or* Option B. If you are using Option B, begin Explore 1 with Item 13.

OPTION A

5. Fasten a capillary tube to the bottom end of a thermometer with two small rubber bands. Orient the tube so that its open end is about 6 mm from the tip of the thermometer bulb.

6. Carefully place the tube and thermometer into the hot oil bath prepared by your teacher. The *entire* capillary tube must be immersed in the oil. (This will allow the air in the tube to reach the same temperature as the oil.) Hold the assembly in the oil until your thermometer is at the same temperature as the thermometer in the oil bath.

CAUTION!
Hot oil will burn skin.
Do not touch the oil.

7. When the temperature of your tube is the same as the temperature of the oil, lift the apparatus up so that only about one quarter of the capillary tube is in the oil. Wait about three seconds. You will see a plug of oil rise in the tube. Remove the apparatus completely and quickly place it on a paper towel. Return to your laboratory bench.

Ask your teacher for help if you do not see an oil plug in your tube.

8. Lay the tube-thermometer apparatus on a clean paper towel on your laboratory bench. Draw a reference line perpendicular to the sealed end of the capillary tube. Draw another line at the inside edge of the oil plug. Next to this line, write the temperature indicated by your thermometer.

9. Mark the length of the air column as the temperature falls about every 10 degrees. Record the temperature at each mark. Continue until the temperature is approximately equal to room temperature.

10. Disassemble and clean the apparatus according to your teacher's instructions. Do not discard your paper towel yet.

11. Measure the length of the air column (cm) between the end of the closed capillary tube and the inside edge of the oil plug, as marked on your paper towel, for each temperature marking. The diameter of the capillary tube is approximately 0.135 cm. Record the length, diameter, and temperature data in the table below.

Length (cm)	Diameter (cm)	Volume (cm^3)	Temperature (°C)

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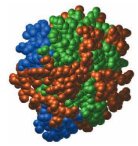

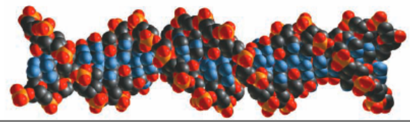

12. Is the length of the trapped gas in your capillary tube proportional to the volume of the gas? Explain. Convert your capillary tube length and diameter data to volume, and enter the results in the table above.

Option A: Skip to Item 19.

OPTION B

A syringe is a plunger within a volume-calibrated tube.

13. Obtain a 60-mL syringe. Remove the stopper from the syringe and move the plunger to a volume reading of about 25 mL. Replace the stopper and do not remove the stopper or plunger from the syringe as you conduct this exercise.

14. You will be observing and recording the volume and temperature of trapped air in the syringe. Record your data in the table below.

Temperature (°C)	Syringe Volume (mL)

15. Obtain a 400-mL or 600-mL beaker. Fill the beaker to about half full with tap water. Add ice until the beaker is about 80 to 90 percent full. Submerse the syringe in the ice water. Use a thermometer to monitor the temperature. When the temperature is below 5°C, record the temperature to the precision allowed by the thermometer and record the volume of trapped air in the syringe.

16. Place your beaker on a hot plate and turn the knob to its highest setting. Monitor the temperature of the system as it rises. Leave the syringe in the water throughout the heating process. When the temperature is between 20°C and 30°C, record the temperature to the precision allowed by the thermometer and record the volume of trapped air in the syringe.

17. When the temperature is between 50°C and 60°C, record the temperature to the precision allowed by the thermometer and record the volume of trapped air in the syringe. *Note:* observe the volume through the side of the beaker. Leave the syringe in the bath while reading the volume.

18. When the water is boiling, record the temperature to the precision allowed by the thermometer and record the volume of trapped air in the syringe. *Note:* observe the volume through the side of the beaker. Leave the syringe in the bath while reading the volume.

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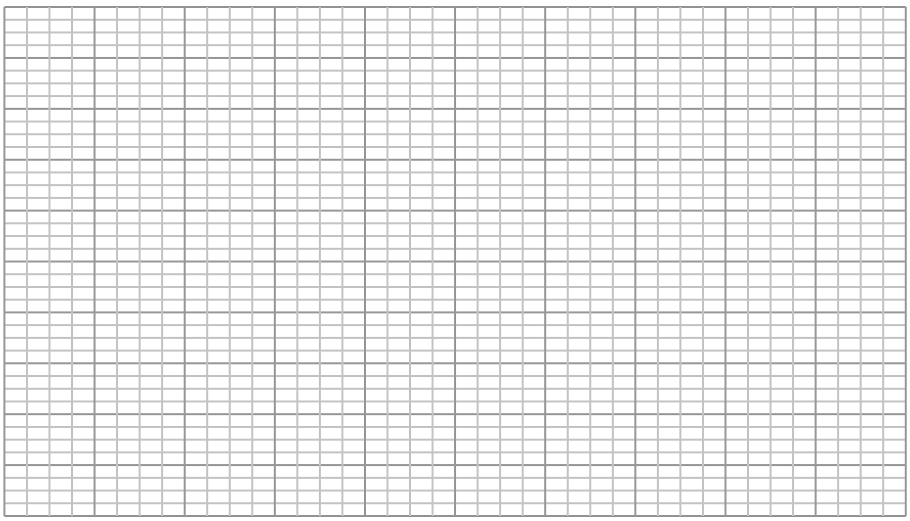
BOTH OPTIONS A AND B CONTINUE HERE

19. Graph the results on the grid below, plotting the volume of the trapped gas on the y-axis and temperature on the x-axis. (Instructions for the second grid will be given later.)



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Explain 1

20. What gas properties were allowed to vary in this experiment?

21. What gas properties were held constant in this experiment?

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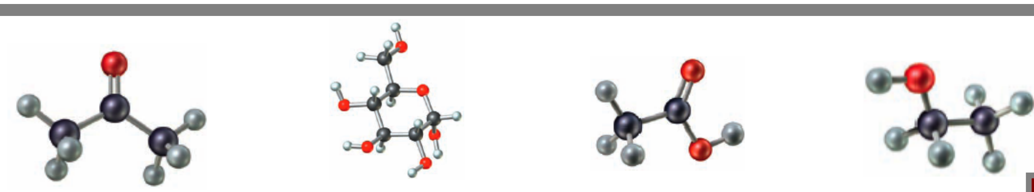

22. Determine the slope of the best-fit straight line to your volume–temperature data.

23. Determine the equation of the best-fit line. Write your final equation in a form that allows you to predict the volume of gas in your capillary tube or syringe at any temperature.

Thinking About Your Thinking
Control of Variables
Proportional Reasoning

24. Use the equation of your line to calculate the temperature necessary to bring the gas volume to zero.

25. On the second grid, scale your axes so that the x-axis extends to –400°C. Plot your data again and draw the best-fit straight line, extrapolating it to y = 0. At what temperature does your trend line intersect the x-axis?

Extrapolation is an extension beyond the range of your data.

26. (a) What is the significance of the trend line intersecting the x-axis?

(b) What would happen to the volume of the gas if the temperature were lower than the point at which it intersects the x-axis?

(c) Describe the physical situation at the point at which the trend line intersects the x-axis.

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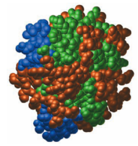

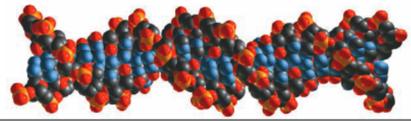

27. The temperature scale on your graph and in your equation can be changed so that the zero volume point corresponds to a zero temperature. What Celsius temperature is this?

28. Your teacher will explain how to pool class data of the Celsius temperature that is the lowest temperature. Write your class average below. Your teacher will tell you the temperature that results from similar experiments with very high quality equipment. What is the percentage error in your class average? Use the value measured by state of the art equipment when answering the questions below.

29. The lowest temperature point is called absolute zero. The temperature scale that sets zero degrees at absolute zero is called the absolute or kelvin temperature scale. The *size* of a kelvin degree is the same as a Celsius degree. Write an equation that allows you to convert between temperature on the Celsius scale and its equivalent on the kelvin scale.

30. Rewrite your volume–Celsius temperature equation from Item 23 so that it becomes a volume–kelvin temperature equation.

31. Complete the following table, using your data collected in Item 11 (Option 1) or Item 14 (Option 2) for the first two columns.

Volume (cm^3 or mL)	**Temperature (°C)**	**Temperature (K)**	**$V \div T_{°C}$ (cm^3 or mL/°C)**	**$V \div T_K$ (cm^3 or mL/K)**

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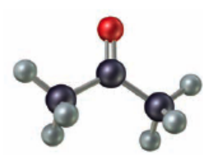
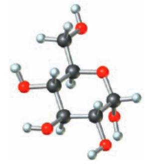
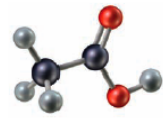
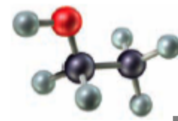

32. Which of the following yields a constant result: $V \div T_{°C}$ or $V \div T_K$?

33. Does your volume–Kelvin temperature equation express a direct proportionality, a linear equation, or an inverse proportionality? Explain.

34. Copy your volume–$T_{°C}$ equation from Item 23 and place it below. Copy your volume–T_K equation from Item 30 and place it below. Classify each equation as a inverse proportion, direct proportion, or linear. Explain your classifications. Which form is most convenient? Explain.

35. Use both words and mathematical symbols to express the proportionality given in your volume–kelvin temperature equation.

36. What is the kelvin temperature at the freezing point of water? At what kelvin temperature does water boil? Show your work.

Elaborate 1: How Can the Volume-Temperature Relationship for a Gas be Used to Make Predictions?

37. The proportionality you discovered in this investigation is called **Charles's Law,** after the French scientist Jacques Charles, whose work with the volume–temperature relationship was stimulated by his interest in hot air ballooning. Write Charles's Law as a proportionality, using the "is proportional to" symbol.

    $V \propto T_K$

38. Write Charles's Law as an equation, using k as the proportionality constant.

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Control of Variables
Proportional Reasoning

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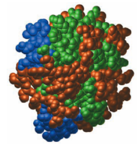

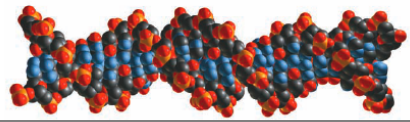

Jacques Charles (1746–1823) (pictured) and his co-pilot Nicolas-Louis Robert were the first people to fly in a basket attached to a floating balloon. From Cracolice, M. S., & Peters, E. I. (2011). *Introductory Chemistry: An Active Learning Approach.* Belmont: CA: Brooks/Cole Cengage Learning.

39. What gas properties were constant in your investigation? Use words to express Charles's Law. Include the restriction of the properties that need to remain constant in order for Charles's Law to be true.

40. Solve the equation form of Charles's Law for k, the proportionality constant. How does this constant relate the volume of a gas, V_1, at a certain temperature, T_1, to a different volume and temperature of the same quantity of gas at the same pressure, V_2 and T_2?

41. The volume of a gas in a sealed, flexible-walled container is 2.2 L at 22°C. It is then heated to 45°C at constant pressure. What is the new volume?

42. The temperature of a fixed quantity of a gas is 5°C. Its volume is then doubled at constant pressure. What is the new temperature?

43. A fixed quantity of a gas occupies 10.0 m^3 at 15°C. What is the volume of the gas if the kelvin temperature is doubled at constant pressure?

44. How does the volume of a fixed quantity of gas change if the kelvin temperature is reduced to one-fourth of its original value?

45. A helium-filled balloon has a volume of 2.75 L at 22°C. The balloon is placed outside, where its volume decreases to 2.46 L. What is the outside temperature in K and °C?

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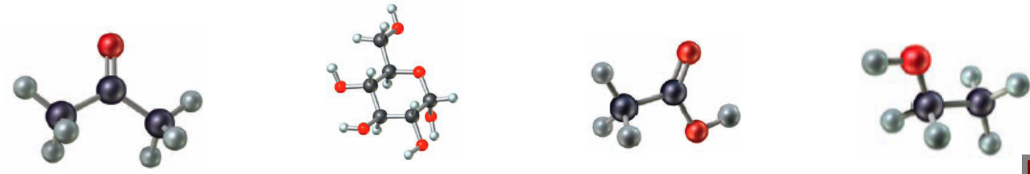

46. A sealed container with flexible walls maintains constant pressure as volume varies with temperature. What is the volume of a gas if a container with 765 mL of gas at –13°C is warmed to 125°C?

Explore 2: What is the Relationship Between the Volume and the Pressure of a Gas?

47. Your teacher will demonstrate an apparatus that shows how the pressure of a fixed quantity of a gas at constant temperature varies with changes in volume. Use the table below to record the data. (The third column will be utilized in a later item.)

Volume (mL)	**Pressure (psi)**	

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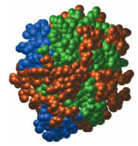

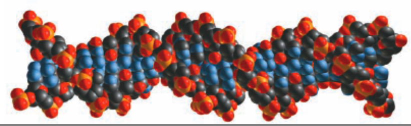

Explain 2

48. Plot pressure (y-axis) versus volume (x-axis) below.

49. Label the third column of the table in Item 47 “1/volume (1/mL).” Calculate and enter the values in the table. Plot the pressure (y-axis) versus the inverse of volume (1/volume) (x-axis) below.

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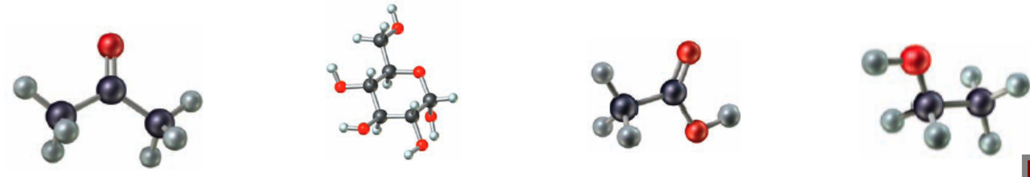

Thinking About Your Thinking
Control of Variables

50. What quantities were allowed to vary in this experiment? What quantities were held constant?

51. Complete the following table, using your data collected in Item 47 for the first two columns.

Volume (mL)	Pressure (psi)	P × V (mL × psi)	P ÷ V (psi/mL)	V ÷ P (mL/psi)

52. Which of the following yields a constant result: P × V, P ÷ V, or V ÷ P?

53. Determine the slope of the best-fit straight line to your pressure vs. inverse of volume data.

54. Determine the equation of the best-fit line. Write your final equation in a form that allows you to predict the pressure of your gas at any volume.

55. Use both words and mathematical symbols to express the proportionality given in your pressure–inverse of volume equation. Is it a direct proportionality or an inverse proportionality? Explain.

Thinking About Your Thinking
Control of Variables
Proportional Reasoning

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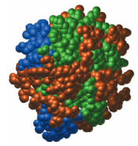
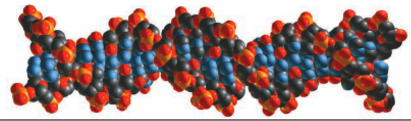
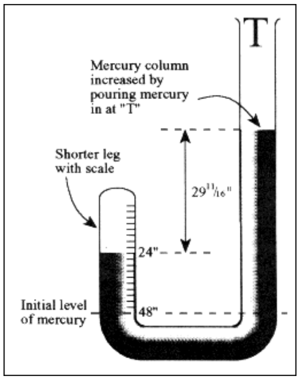




Elaborate 2: How Can the Volume-Pressure Relationship for a Gas be Used to Make Predictions?



A schematic diagram of Boyle's J-tube apparatus. Note, in particular, the size of the device.

56. The proportionality you discovered in this investigation is called **Boyle's Law,** after the Irish-born scientist Robert Boyle, whose work with the volume–pressure relationship was conducted with a giant-sized glass J-tube partially filled with mercury. Write Boyle's Law as a proportionality, using the "is proportional to" symbol.

57. Write Boyle's Law as an equation, using k as the proportionality constant.

58. What gas properties must remain constant for Boyle's Law to be valid?

59. Solve the equation form of Boyle's Law for k, the proportionality constant. How does this constant relate the volume of a gas, V_1, at a certain pressure, P_1, to a different volume and pressure of the same quantity of gas at the same temperature, V_2 and P_2?

60. A fixed quantity of gas at constant temperature has an initial volume of 3.2 ft^3 while at 556 mm Hg pressure. What is the pressure when the volume is reduced to 1.5 ft^3?

61. A teacher demonstrating Boyle's Law has the plunger set at 35 mL when the pressure gauge reads 14.6 psi. To what volume will the plunger need to be depressed to change the pressure to 30.0 psi?

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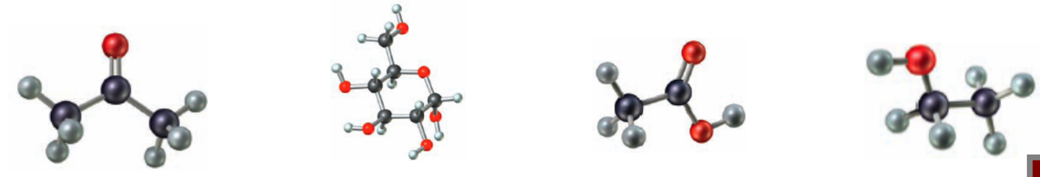

Explore 3: What is the Relationship Between the Temperature and the Pressure of a Gas?

62. Your teacher will demonstrate an apparatus that shows how the pressure of a fixed quantity of a gas at constant volume varies with changes in temperature. Use the table below to record the data. (The third column will be utilized in a later item.)

Pressure (psi)	Temperature (°C)	

63. Is pressure proportional to Celsius temperature? Explain why or why not.

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Explain 3

64. Plot pressure (y-axis) versus kelvin temperature (x-axis). A column is provided in the table in Item 62 for the Celsius–kelvin conversion.

65. Draw the line of best fit to your data.

66. What gas properties were allowed to vary in this experiment? What properties were held constant?

67. Determine the slope of the best-fit straight line to your pressure vs. temperature data.

68. Determine the equation of the best-fit line. Write your final equation in a form that allows you to predict the pressure of your gas at any temperature.

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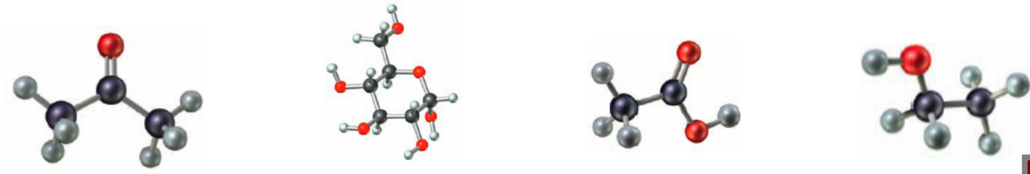

69. Use both words and mathematical symbols to express the proportionality given in your pressure–temperature equation. Is it a direct proportionality or an inverse proportionality? Explain.

Thinking About Your Thinking
Control of Variables
Proportional Reasoning

Elaborate 3: How Can the Pressure-Temperature Relationship for a Gas be Used to Make Predictions?

70. The proportionality you discovered in this investigation is called **Amontons's Law,** after the French scientist Guillaume Amontons, who discovered the relationship in 1699. (This relationship previously was known as Gay-Lussac's Law, after Joseph Louis Gay-Lussac, who was mistakenly given credit for its discovery. Gay-Lussac did, however, in 1808 discover the Law of Combining Volumes of Gases.) Write Amontons's Law as proportionality and as an equation, using k as the proportionality constant.

71. What gas properties must remain constant for Amontons's Law to be valid?

Thinking About Your Thinking
Control of Variables
Proportional Reasoning

72. Solve the equation form of Amontons's Law for k, the proportionality constant. How does this constant relate the pressure of a gas, P_1, at a certain temperature, T_1, to a different pressure and temperature of the same quantity of gas at the same volume, P_2 and T_2?

73. A steel-walled container filled with helium gas has a pressure of 34.5 psi at a temperature of 19°C. What pressure will the gas exert when the temperature falls to –3°C?

74. A sealed glass bulb holds a gas at a pressure of 0.33 atm when the temperature is 22°C. The pressure increases to 0.75 atm. What is the Celsius temperature of the gas?

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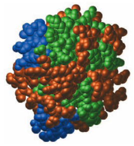

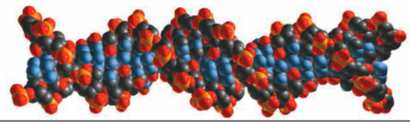

Elaborate 4: How Can We Combine the Three Two-Variable Gas Laws Into One Three-Variable Gas Law?

75. Thus far, you have used experimental data to derive three gas laws:

Charles's Law	$V = kT$
Boyle's Law	$P = k(1/V)$
Amontons's Law	$P = kT$

These laws involve relationships among the volume, pressure, and temperature of a fixed quantity of gas. Each law expresses the relationship between *two* variables. It would be convenient to have a single relationship that accounts for all three variables. To do find this relationship, we can use the fact that when one variable is proportional to two different variables, it is proportional to the product of those variables. In symbols, if $x \propto y$ and $x \propto z$, then $x \propto (y \times z)$. Use this concept and Boyle's and Amontons's Laws to find the relationship among V, T, and P.

Thinking About Your Thinking
Proportional Reasoning

76. Change the proportionality above to an equation by inserting a proportionality constant, k.

77. Solve the equation above for the proportionality constant.

78. Use words to explain the meaning of the equation in Item 77.

79. The relationship expressed in Items 77 and 78 is called the **combined gas law.** It is a combination of the pairwise gas laws. What is the equation that relates the volume, temperature, and pressure of a fixed quantity of gas at one set of conditions, V_1, T_1, and P_1, to the volume, temperature, and pressured at another set of conditions, V_2, T_2, and P_2?

80. Use the form of the combined gas law that relates V_1, T_1, and P_1 to V_2, T_2, and P_2 to derive each of the pairwise gas laws. You can do this by holding constant one variable at a time.

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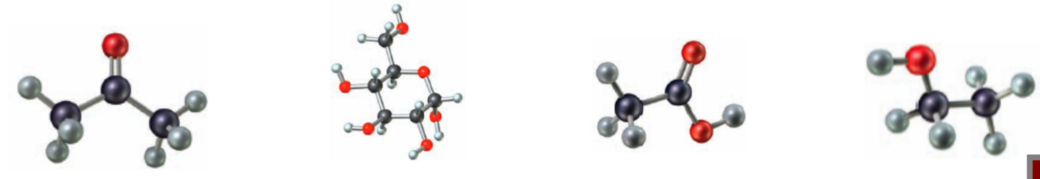

Appendix 1: How is Gas Pressure Measured?

Some of the most important investigations in the history of science were those that led to the discovery that the atmosphere exerts pressure. Evangelista Torricelli (1608–1647), a student of Galileo, while investigating why water could not be pumped from deep mines, designed a mercury **barometer.** Torricelli filled a long glass tube with mercury, placed his finger over the end, inverted the tube, and placed it in a dish of mercury. The height of the column of mercury was about 30 inches, although it varied from day to day. **Figure 2.1** is an illustration of Torricelli's barometer.

Torricelli proposed that the height of the mercury in the column was due to the weight of the atmosphere pushing the liquid up the tube. Torricelli's hypothesis was confirmed, in part, by the experiments of Blaise Pascal (1623–1662). Pascal found that when a barometer was taken up a mountain, where the atmosphere weighs less than at lower altitudes, the height of the mercury column decreased. Thus the barometer is "weighing" the atmosphere.

By definition, **pressure is the force exerted on a unit area:**

$$\text{Pressure} \equiv \frac{\text{force}}{\text{area}} \quad \text{or} \quad P \equiv \frac{F}{A} \qquad \textbf{(2.1)}$$

Evangelista Torricelli (1608–1647) invented the mercury barometer while trying to understand why suction pumps could not raise water past a height of about ten meters. From Cracolice, M. S., & Peters, E. I. (2011). *Introductory Chemistry: An Active Learning Approach.* Belmont: CA: Brooks/Cole Cengage Learning.

Units of pressure come from the definition. In the United States Customary System, if force is measured in pounds and area in square inches, the pressure unit is **pounds per square inch (psi).** The SI unit of pressure is the pascal (Pa), which is one newton per square meter. (The *newton* is the SI unit of force.) Although chemists generally follow SI guidelines, one pascal is a very small pressure; the **kilopascal (kPa)** is a more practical unit. Many other pressure units are commonly used.

Weather bureaus generally report *barometric pressure,* the pressure exerted by the atmosphere at a given weather station, in inches of mercury or kilopascals. Atmospheric pressure is measured by a Torricellian barometer or its mechanical equivalent. On a day when the mercury column in a barometer is 752 mm Hg high, we say that atmospheric pressure is 752 mm Hg.

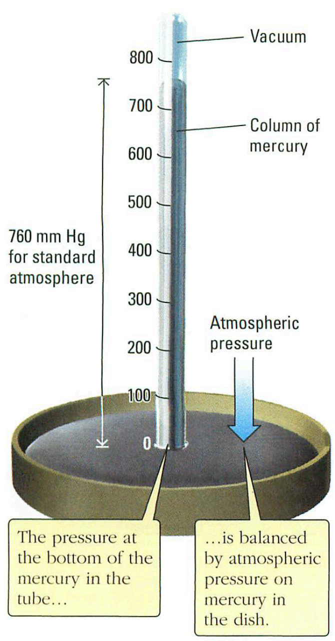

Figure 2.1 Two operational principles govern the mercury barometer. (1) The total pressure at any point in a liquid system is the sum of the pressures of each gas or liquid phase above that point. (2) The total pressures at any two points at the same level in a liquid system are always equal. All points on the liquid surface outside the tube are at the same level as the mercury at the bottom inside the tube. The only thing exerting downward pressure on surface outside the tube is the atmosphere; the downward arrow represents atmospheric pressure. The only thing exerting downward pressure at the bottom of the mercury inside the tube is the mercury above that point. Both points are at the same level, therefore the pressures at these points are equal: $P_{atmosphere} = P_{mercury}$. From Cracolice, M. S., & Peters, E. I. (2011). *Introductory Chemistry: An Active Learning Approach.* Belmont: CA: Brooks/Cole Cengage Learning.

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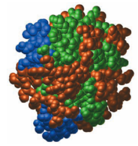

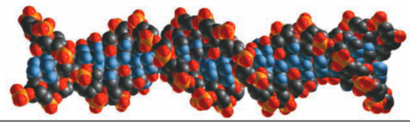

One standard atmosphere of pressure is defined as 760 mm Hg. This is a typical barometric pressure at sea level. Atmospheric pressure on the top of the world's highest mountain is about 270 mm Hg. Normal atmospheric pressure therefore usually varies between 760 and 270 mm Hg, depending on the altitude. The atmosphere unit is particularly useful in referring to very high pressures. Here is a summary of common pressure units and their relationships to one other:

$$1 \text{ atm} \equiv 760 \text{ mm Hg} \quad \textbf{(2.2)}$$

$$1 \text{ atm} = 101.325 \text{ kPa} \quad \textbf{(2.3)}$$

$$1 \text{ atm} = 14.6959 \text{ psi} \quad \textbf{(2.4)}$$

P/Review

Numbers fixed by definition are exact; significant figures do not apply to exact numbers. The relationship 1 atm ≡ 760 mm Hg is such a definition.

An open-end **manometer** is the instrument most commonly used to measure pressure in the laboratory (**Fig. 2.2**).

Outside the laboratory, mechanical gauges are used to measure gas pressure (**Fig. 2.3**). A typical tire gauge is probably the most familiar. Tire gauges show the pressure *above* atmospheric pressure, rather than the absolute pressure measured by a manometer. Even a flat tire contains air that exerts pressure. If it did not, the entire tire would collapse, not just the bottom. The pressure of gas remaining in a flat tire is equal to atmospheric pressure. If a tire gauge shows 25 psi, that is the **gauge pressure** of the gas (air) in the tire. The absolute pressure is nearly 40 psi—the 25 psi shown by the gauge plus about 15 psi (1 atm = 14.7 psi) from the atmosphere.

Figure 2.3 A mechanical gauge used to measure atmospheric pressure. In general, mechanical gauges work via an air-filled tube that changes shape with changing pressure. Levers and/or gears move in response to the tube, and a pointing needle and scale or an electronic display is calibrated to show the corresponding atmospheric pressure. From Cracolice, M. S., & Peters, E. I. (2011). *Introductory Chemistry: An Active Learning Approach.* Belmont: CA: Brooks/Cole Cengage Learning.

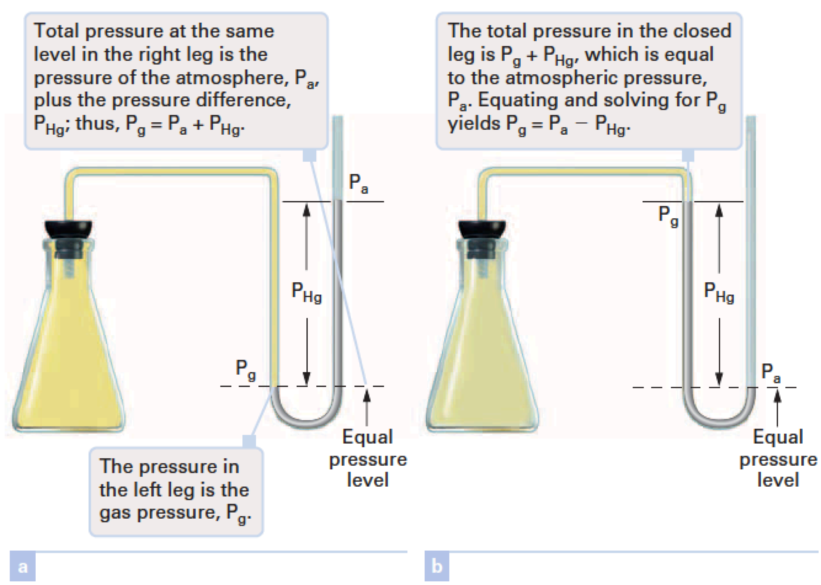

Figure 2.2 Open-end manometers. Open-end manometers are governed by the same principles as mercury barometers. The pressure of the gas, P_g, is exerted on the mercury surface in the closed (left) leg of the manometer. Atmospheric pressure, P_a, is exerted on the mercury surface in the open (right) leg. With a meter stick, the difference between these two pressures, P_{Hg}, may be measured directly in millimeters of mercury (torr). Gas pressure is determined by equating the total pressures at the lower liquid mercury level, indicated with a dashed line.

In effect, you can determine the pressure of a gas, as measured by a manometer, by adding the pressure difference to, or subtracting the pressure difference from, atmospheric pressure: $P_g = P_a \pm P_{Hg}$. From Cracolice, M. S., & Peters, E. I. (2011). *Introductory Chemistry: An Active Learning Approach.* Belmont: CA: Brooks/Cole Cengage Learning.

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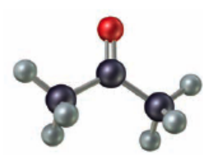 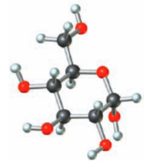 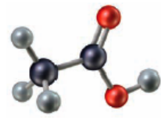 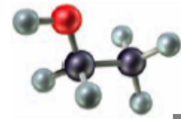

Worked Examples

1. A 6.67 m^3 sample of nitrogen gas at 10°C has its temperature increased to 43°C while the pressure is held constant. What is the adjusted volume after the temperature change?

   Solution
   Identify the unknown: V_2
   Identify the given information: $V_1 = 6.67\ m^3$; $T_1 = 10^0C = 283\ K$; $T_2 = 43^0C = 316\ K$; pressure is constant.
   Make a prediction without calculations first. If the temperature is increasing, what should happen to the volume?
   What gas law and equation would be appropriate?
   Remember that absolute temperatures must be used, not ˚C.

   $$\frac{P_1V_1}{T_1} = \frac{P_2V_2}{T_2} \xrightarrow{P_1 = P_2} \frac{V_1}{T_1} = \frac{V_2}{T_2} \xrightarrow{\text{multiply both sides by } T_2} V_2 = \frac{V_1T_2}{T_1} =$$

   $$\frac{(6.67\ m^3)(316\ K)}{283\ K} = 7.45\ m^3$$

   Did your calculated answer match your larger or smaller prediction?

2. Determine the new volume when 25 L of hydrogen gas at 37°C and 0.95 atm is changed to 25°C and 812 mm Hg.

   Solution
   Identify the unknown: V_2
   Identify the given information: $V_1 = 25\ L$; $T_1 = 37^0C = 310\ K$; $T_2 = 25^0C = 298\ K$; $P_1 = 0.95\ atm$; $P_2 = 812\ mm\ Hg$
   Make a prediction without calculations first.
   If the temperature is decreasing, what should happen to the volume? It should decrease.
   If the pressure is increasing, what should happen to the volume? It should decrease.
   What gas law and equation would be appropriate? Solve the combined gas law equation.

   $$\frac{P_1V_1}{T_1} = \frac{P_2V_2}{T_2} \xrightarrow{\text{cross-multiply}} P_1V_1T_2 = P_2V_2T_1 \xrightarrow{\text{divide both sides by } P_2T_1}$$

   $$V_2 = = 25\ L \times \frac{0.95\ atm}{812\ mm\ Hg} \times \frac{298\ K}{310\ K} \times \frac{760\ mm\ Hg}{1\ atm} = 21\ L$$

   Did your calculated answer match your larger or smaller prediction?

Homework Questions

What is the Relationship between the Volume and the Temperature of a Fixed Amount of Gas at Constant Pressure?

1. A student conducts an investigation similar to the one that you did to explore the volume–temperature relationship of a fixed quantity of gas at constant pressure. She graphs the volume of the gas versus the Celsius temperature and sees that there is a straight-line relationship. She draws the best-fit straight line to the data, and two points that lie on the line are (35 mL, 77°C) and (3.0×10^1 mL, 27°C). Use these data to determine the Celsius temperature value for absolute zero.

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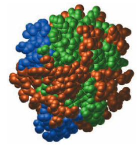
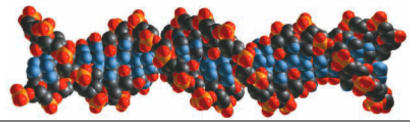




A neon sign is made from electrified glass tubing that contains low-pressure neon.

2. A sample of neon has a volume of 1.83 L at 23°C. At what Celsius temperature would the gas occupy 5.00 L? Pressure is constant.

3. A class conducts an exercise to determine the relationship between the volume and temperature of a gas using the same procedure as your class. Plots of volume versus absolute temperature are drawn, and equations of best-fit straight lines are determined. In all cases, the lines have the form volume equals constant times temperature, but the value of the constant varies. Why?

4. Given that most laboratory thermometers are calibrated in degrees Celsius, why is Charles's Law expressed with temperature in kelvins?

5. A student explores the relationship between the volume and temperature of a gas with a capillary tube that contains an oil plug, as in Option A of Explore 1. She tells her teacher that the same concept could be developed by plotting the length of the trapped gas (instead of volume) versus temperature. Is she correct? Explain why or why not.

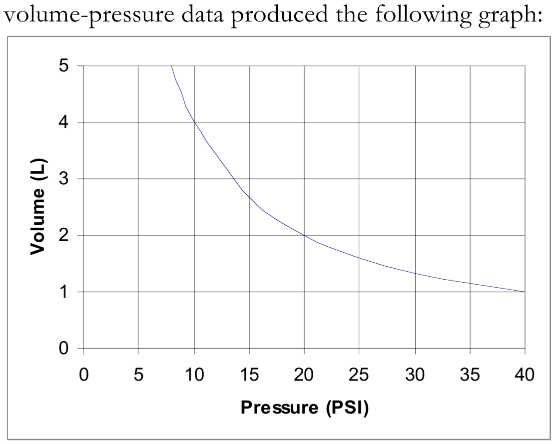
6. After learning about direct proportionalities in Units 1 and 2 in her chemistry class, a student decides to collect data about the time it takes for the bus to travel from her stop to school and the number of students getting on the bus after her stop. Her hypothesis is that the greater the number of students, the longer it takes to get to school because of the loading time. Her prediction is that time will be directly proportional to the number of students. Her data were:

Day of the week	Monday	Tuesday	Wednesday	Thursday	Friday
Time (minutes)	16	8	10	12	18
Number of Students	12	4	6	8	14

Plot the data, determine the equation that expresses the time vs. number of students relationship, and explain if the data support the student's hypothesis. Explain why or why not.

What is the Relationship Between the Volume and Pressure of a Fixed Amount of Gas at Constant Temperature?

7. The pressure of a sample of gas at constant temperature is changed and the volume is measured. The volume-pressure data produced the following graph:



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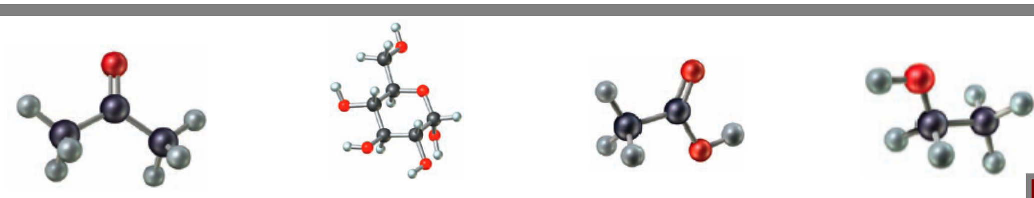
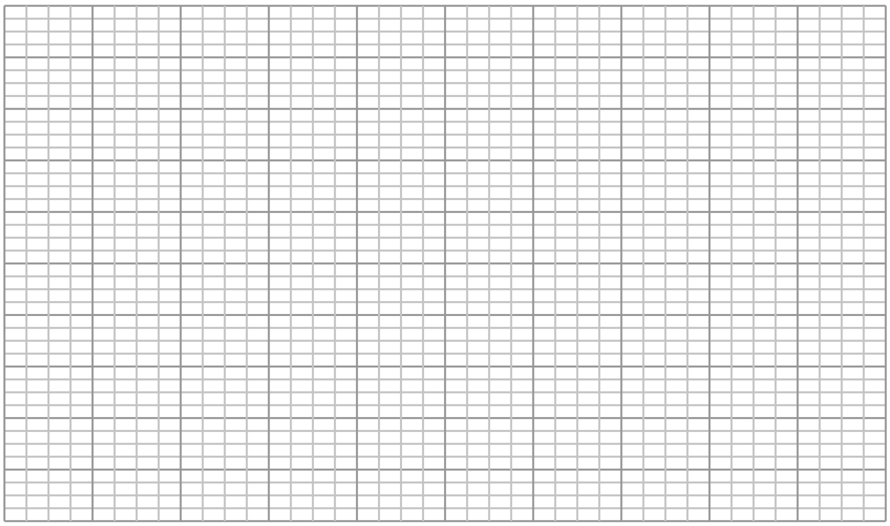


(a) Construct a graph that results in a linear relationship using manipulations of the data above, as necessary. Label the axes appropriately (including units).



(b) What is the value of the slope of the line in this linear graph?

(c) What are the units of the associated with the slope of the line?

(d) Determine the volume of the gas when the pressure is 45 psi.

8. What is Boyle's Law, and what variables must be held constant for this relationship to be valid?

9. Use Boyle's Law to determine the missing value in each of the following:
   a) $P_1 = 3.50 \times 10^2$ mm Hg, $V_1 = 2.00 \times 10^2$ mL, $P_2 = 7.00 \times 10^2$ mm Hg, $V_2 = ?$
   b) $P_1 = 14.4$ psi, $V_1 = 2.00 \times 10^2$ mL, $V_2 = 3.50 \times 10^2$ mL, $P_2 = ?$

10. The plunger of a syringe is pulled out until the volume of air in the syringe is 30.0 mL at a pressure of 694 mm Hg. A stopper is then placed on the syringe so that no air can escape. What will be the air pressure inside the syringe if the plunger is depressed until the volume in the syringe is 26.0 mL?

11. A balloon is filled with enough helium to occupy 2.11 L at sea level (760 mm Hg) when the air temperature is 25°C. If the balloon is taken to a higher elevation where the atmospheric pressure is 670 mm Hg and the temperature remains constant, what will be the volume of the balloon?

12. A steel cylinder contains no matter within its walls. What happens to the gas pressure within the cylinder when it is heated?

What is the pressure within a steel cylinder containing no matter?

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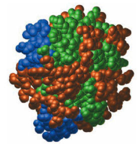

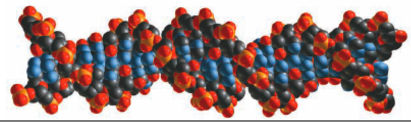

What is the Relationship Between the Temperature and Pressure of a Fixed Amount of Gas at Constant Volume?

13. The pressure of a gas at –73°C is doubled, but its volume is held constant. What is the final temperature in degrees Celsius?

Aerosol cans contain both the product you use such as hair spray or room freshener and a propellant. The propellant is pumped into the can at high pressure.

14. The gas in an aerosol can is at a pressure of 45 psi at 25.0°C. Directions on the can caution that the can should not be warmed above 50.0°C. What is the gas pressure in the can at 50.0°C?

15. The definition of gas pressure is the ratio of the force exerted by the gas to the area over which the force is exerted. Use this definition to explain the cause of Amontons's Law.

16. A teacher demonstrates how the pressure of a fixed quantity of a gas at constant volume varies with temperature. A student in her class records the pressure versus temperature data and constructs a plot, finding that the data fit a straight line. Two points on the best-fit straight line are (14.7 psi, 296 K) and (13.6 psi, 274 K). Write an equation that allows you to predict the pressure of the gas at any given temperature.

17. Two students meet after school to do the data analysis of a lab exercise in which the temperature of a fixed amount of a gas at constant volume was varied and the resulting pressure was measured. The student in the Period 1 class graphed pressure versus absolute temperature and found that the equation of the best-fit straight line to the data was Pressure (psi) = 0.044 psi/K × Temperature (K). The student in the Period 2 class of the same teacher determined that the equation of her best-fit line to the same variables while following the same experimental procedure was Pressure (psi) = 0.054 psi/K × Temperature (K). They check one another's work and conclude that neither made an error. How can this be? Explain.

18. In 1702, Guillaume Amontons reported on his invention of an air thermometer. He heated a fixed quantity of air from the melting temperature of water to the boiling temperature of water. While doing so, he kept the volume of gas constant by adding liquid mercury to a tube that was designed like a manometer, which also allowed him to measure the pressure of the gas. When he started with gas at room pressure, he found that the pressure of the gas increased by 10 inches of mercury as it increased in temperature from water's melting point to its boiling point. He repeated the experiment starting with the gas at twice room pressure at the temperature of water's melting point. How many inches of mercury did the pressure increase by when the gas was at water's boiling temperature? Explain.

How is Gas Pressure Measured?

19. What is the definition of the term *pressure,* as it is used in the sciences?

20. A woman walks across a grassy field in high heels and finds that her shoes tend to fall off because the heels get stuck in the dirt. When she walks across the same field in tennis shoes, she has no problem. Explain why.

21. What type of instrument is used to measure atmospheric pressure?

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22. What does a mercury barometer look like and how does it work?

23. When a mercury barometer is used to measure atmospheric pressure, in what unit is the pressure measured?

24. How does an open-ended manometer measure pressure?

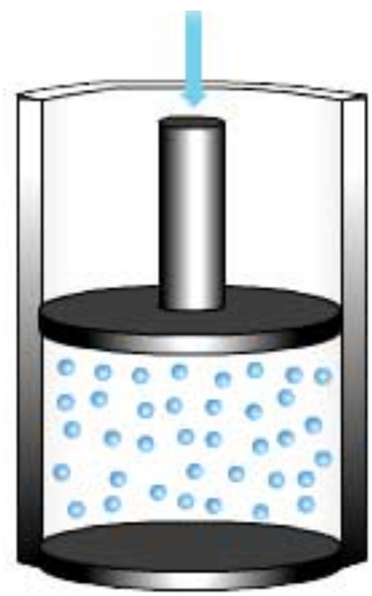
25. What is the average value of atmospheric pressure at sea level?

26. Identify four units used to measure pressure.

27. (a) What is atmospheric pressure?

    (b) What is the value of average atmospheric pressure value at sea level in mm Hg?

Additional Questions



28. In the diagram in the margin, a gas is contained in cylinder is fitted with a piston. The piston locked so that it can not move. The gas in the cylinder is heated from 200 K to 400 K. Describe what will happen to the pressure.

29. In the diagram in the margin, a gas is contained in cylinder is fitted with a movable piston. The piston is pushed down. Describe what will happen to the pressure.

30. A flexible-walled container is filled with argon until it has a volume of 225 cubic inches. The temperature is 22°C and the atmospheric pressure is 749 mm Hg. To what Fahrenheit temperature must the balloon be heated if you want the volume to increase to 3.00×10^2 cubic inches on a day when the pressure is 7.60×10^2 mm Hg?

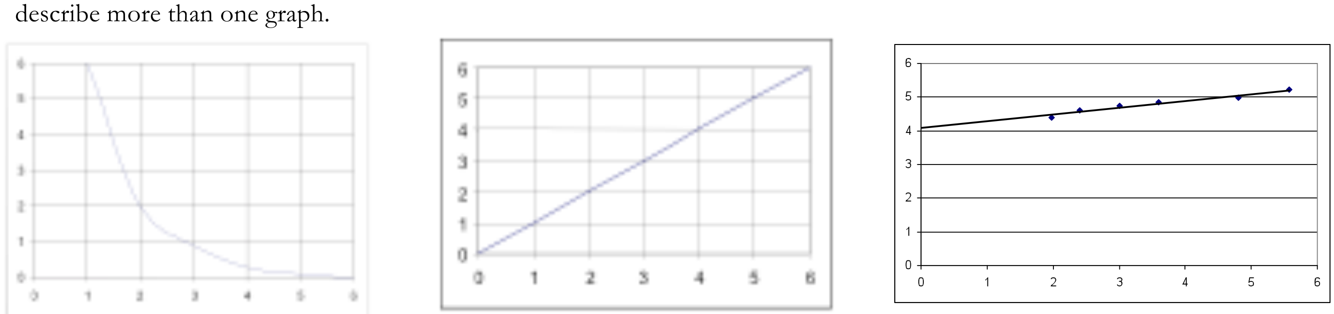
31. People often are intuitively aware of how gases behave in everyday circumstances. Classify each of the following situations as to whether it describes either Boyle's Law, Charles' Law, Amontons's Law, or the Combined Gas Law. Explain each classification in words and with a mathematical relationship.

    (a) Changes to the air in an automobile or bicycle tire as it heats up from friction with the road.

    (b) Changes to the gas in a weather balloon as the balloon rises through the atmosphere.

    (c) Changes to the air in an inflated balloon that is placed in a refrigerator.

32. Match the three graphs below with the appropriate descriptions. Some questions may describe more than one graph.



(1) (2) (3)

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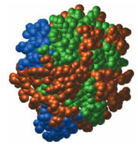

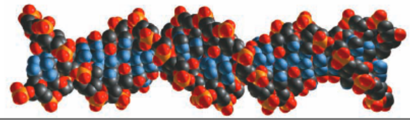

(a) graph(s) showing the relationship of gas volume and Celsius temperature
(b) graph(s) showing the relationship of gas volume and kelvin temperature
(c) graph(s) showing the relationship of kelvin temperature and gas pressure
(d) graph(s) showing the relationship of gas volume and pressure

As part of scuba certification, divers must show that they understand Boyle's Law.

33. The gas behavior described by Boyle's Law is a matter of life and death to scuba divers. On the surface of the water, the diver's lungs and body are at atmospheric pressure. However, under water, a diver's body is under the combined pressure of the atmosphere and water.

    (a) What would happen to the volume of the tank if it were not strong enough to withstand the pressure of the water outside?

    (b) How do the problems of a diver compare with those of a pilot climbing to a higher altitude in an unpressurized plane?

34. A scuba diver on a boat fills a balloon to a volume of 3.0 liters when the atmospheric conditions are 88°F and 14.7 psi. She takes the balloon on a dive to a depth of 28 meters, where the temperature is 63°F. The balloon has shrunk to a volume of 0.75 liter. What is the pressure (in atmospheres) at this depth?

35. Explain why car owners in severely cold climates often add air to their tires in winter and release some air from tires in the summer.

Weather balloons are used to carry instruments to high altitudes.

36. A girl in San Francisco packs a 555-mL bag of potato chips in her luggage when the temperature is 62°F and the atmospheric pressure is 757 mm Hg. What will be the volume of the bag when she arrives in mile-high Denver where the temperature is –3°C and the atmospheric pressure is 83.7 kPa?

37. Consider the operation of a hot air balloon. (a) How do you get it to go up? Why does it go up? (b) How do you get it to come down? Why does it come down?

38. A weather balloon is filled with helium on the ground to a volume of 188 cubic meters when the pressure is 14.5 psi and the temperature is 65°F. The balloon is launched and rises to an altitude where the temperature is –48°C and the pressure is 205 mm Hg. What is the volume of the balloon at this altitude?

39. Why are weather balloons only partially filled when launched?

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