Inspiring Mathematics

How to change the world, and have some fun at the same time

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The problem:

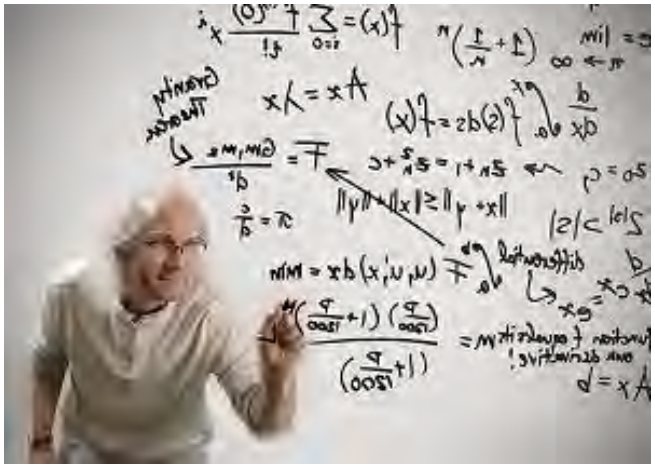
Some very common views on math and mathematicians

Mathematics is completely useless

Mathematics is frightening

Mathematicians are evil souless geeks

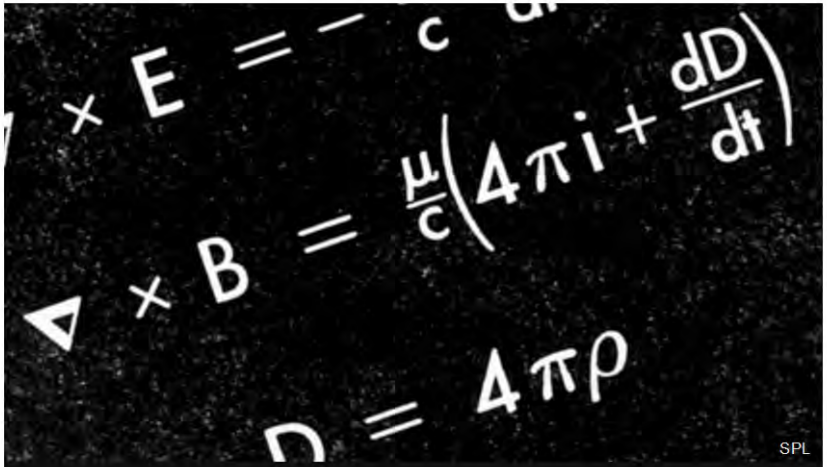
All Mathematicians are mad!



This can cause serious problems

Flight delayed after passenger becomes suspicious of equation

8 May 2016 | US & Canada Share

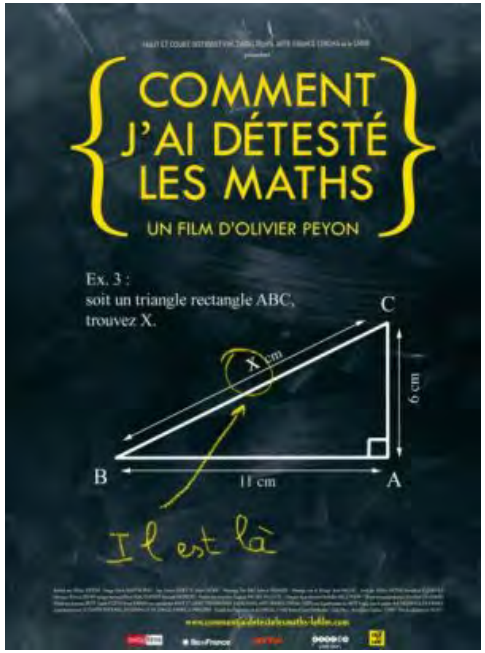


We can't fly on together with suspicious maths: the flight was delayed for two hours

An Italian economist says his flight was delayed after a fellow passenger saw him working on a differential equation and alerted the cabin crew.

Guido Menzio was taken off and questioned by agents who did not identify themselves, after the woman next to him said she felt ill.

And can lead to adults being put off maths for life



Is this attitude reasonable or even justified?

What can we do about it?

Some possible reasons for maths phobia

1. Perceived lack of relevance of mathematics
2. Perceived lack of creativity in maths
3. Perceived lack of ‘humanity’ in maths
4. Perceived impossibility of maths
5. Bias of the media against maths
6. Biased sampling eg. Celebrities

But …. Is maths really worse than other subjects, given that maths is the most popular A level

1. The relevance of mathematics!

The modern world would not exist without maths

Maths lies at the heart of all modern technology

Some statistics

Deloitte report:

Mathematics contributes £20Bn to the UK economy

CBI report

70% of jobs require mathematical skills

Big Data: Mathematical algorithms are increasingly important in the real world

US University Survey

10 BEST JOBS 2015

RANK	OCCUPATION	MEDIAN SALARY
1	ACTUARY	$94,209
2	AUDIOLOGIST	$71,133
3	MATHEMATICIAN	$102,182
4	STATISTICIAN	$79,191
5	BIOMEDICAL ENGINEER	$89,165
6	DATA SCIENTIST	$124,149
7	DENTAL HYGIENIST	$71,102
8	SOFTWARE ENGINEER	$93,113
9	OCCUPATIONAL THERAPIST	$77,114
10	COMPUTER SYSTEMS ANALYST	$81,150

Indeed ….

Much of industry has problems which can potentially be formulated, and solved using mathematics

But .. Too few people recognize that the high technology so celebrated today is essentially a mathematical technology

Edward David, ex-president of Exxon R&D

But …Maths and therefore mathematicians are often invisible or mistaken for something or someone else

Spot the mathematician, and why are they important?

Maxwell and the discovery of electromagnetic waves

$$\nabla \times E = -\frac{\partial B}{\partial t} - M, \quad \nabla \times H = -\frac{\partial D}{\partial t} + J,$$

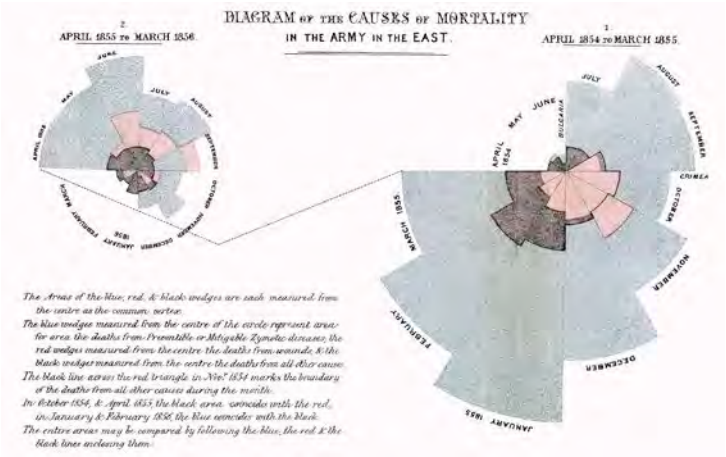
$$\nabla . D = \rho, \qquad \nabla . B = 0.$$

Electromagnetism, radio, WiFi,TV, radar, mobile phones, microwaves all come from the work of Maxwell!

The most famous ever female mathematician?

Florence Nightingale

Medical statistics



Mathematicians really have made the modern world possible

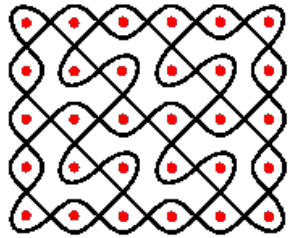
So … who needs maths now?

Traditional industrial users of maths are

Telecommunications, aerospace, power generation, iron and steel, mining, oil, weather forecasting, security, transport, the tax man, finance, …

But they could equally well be …

Retail, food, zoos, sport, entertainment, graphic design, media, forensic service, hospitals, air-sea-rescue, education, transport, risk, health, biomedical, environmental agencies, art, musicians, …



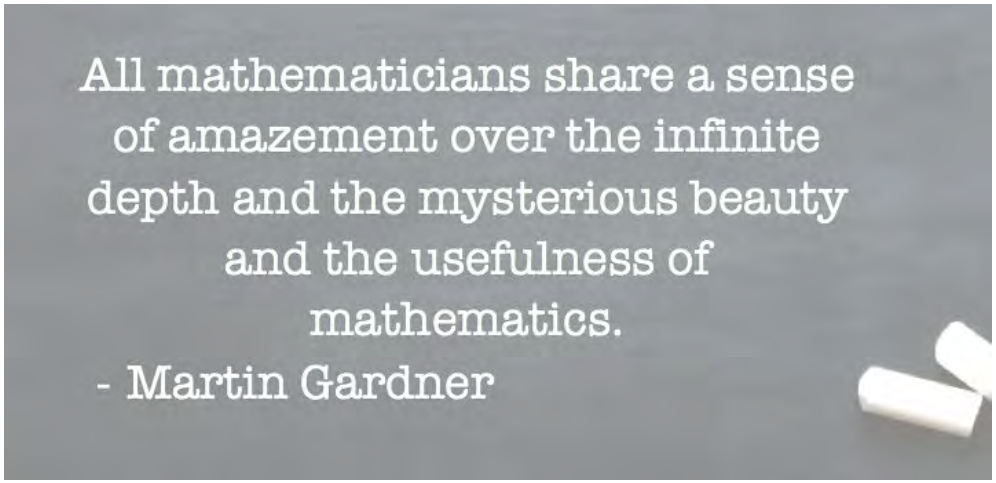
Some examples of uses of maths to inspire you

$$e^{i\pi} = -1$$

Medical imaging

Google

Error correcting codes



Mathematicians save lives

Radon 1917

Studied shadows cast by objects.

Asked the question of: whether you can reconstruct the shape just by knowing line integrals through it?

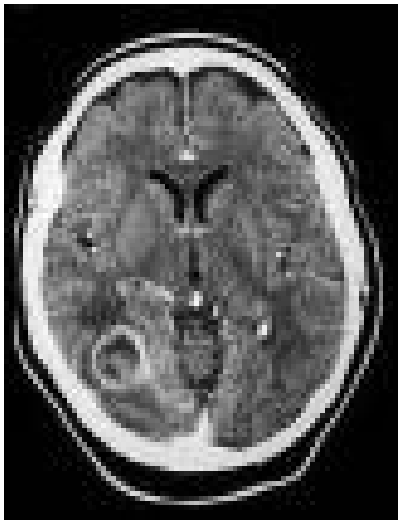
Shadow

$$R(\rho,\theta) = \int f(\rho\cos(\theta) - s\sin(\theta), \rho\sin(\theta) + s\cos(\theta))\, ds$$

Object

$$f(x,y) = \frac{1}{(2\pi)^2}\int_{-\infty}^{\infty}\int_{0}^{\pi}\int_{-\infty}^{\infty} e^{ik(x\cos(\theta)+y\sin(\theta)-\rho)} R(\rho,\theta)|k|\, dk d\theta d\rho$$

Modern CAT (Computerised Axial Tomography) scanner implements this and related formulae to look inside you.

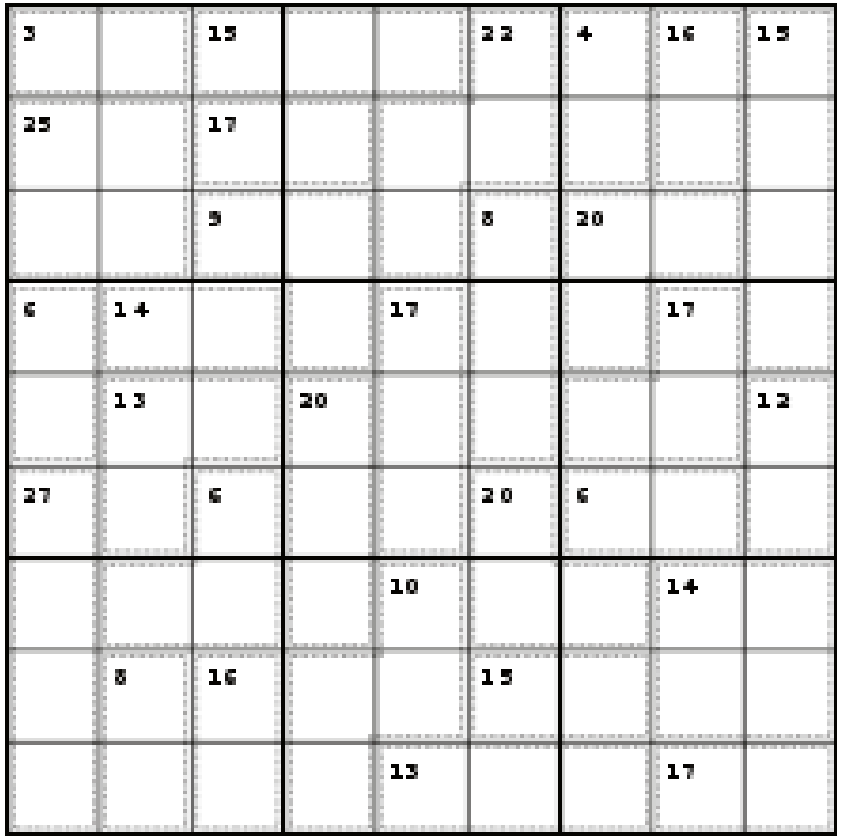


Also used to

X-ray mummies

Detect land mines

Save bees



If you solve Killer Sudoku or Griddler you are using the math of tomography

Example 2:

Google holds somewhere around 10-15 exabytes of data.

An exabyte equals 1 million terabytes

Equates to enough boxes of punch cards to fill up the entire region of New England to a depth of just under 3 miles.

It searches through this in seconds by looking for eigenvectors of large matrices!

Example 3. Math communicates

Error correcting codes.

Used to store and transmit data in such a way that any errors can not only be detected but corrected.

They work by adding extra information to make the code symbols as different as possible so we can still tell the right answer even if it has mistakes in it

They are widely used in

- CDs
- Digital TV and Radio
- Mobile phones
- Satellites

Invented in the 1940s by Hamming in the Bell Labs

Polynomials over finite fields, Galois theory

But isn’t maths still just a geeky non creative subject that only weirdos with super brains can do?

NO!!!!

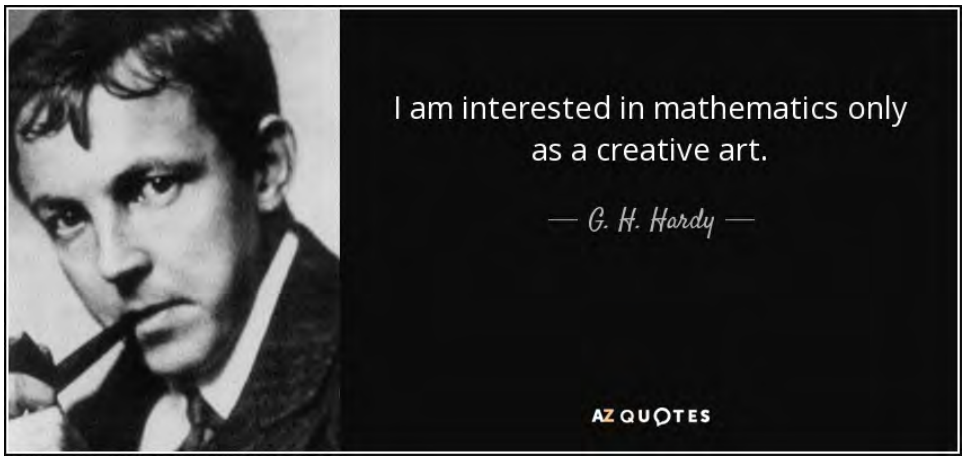
Maths is highly creative and we can all enjoy doing it!

Now look at the more creative side of maths

Maths in the movies

Maths and music

Maths and art



Without maths we could not make modern movies!

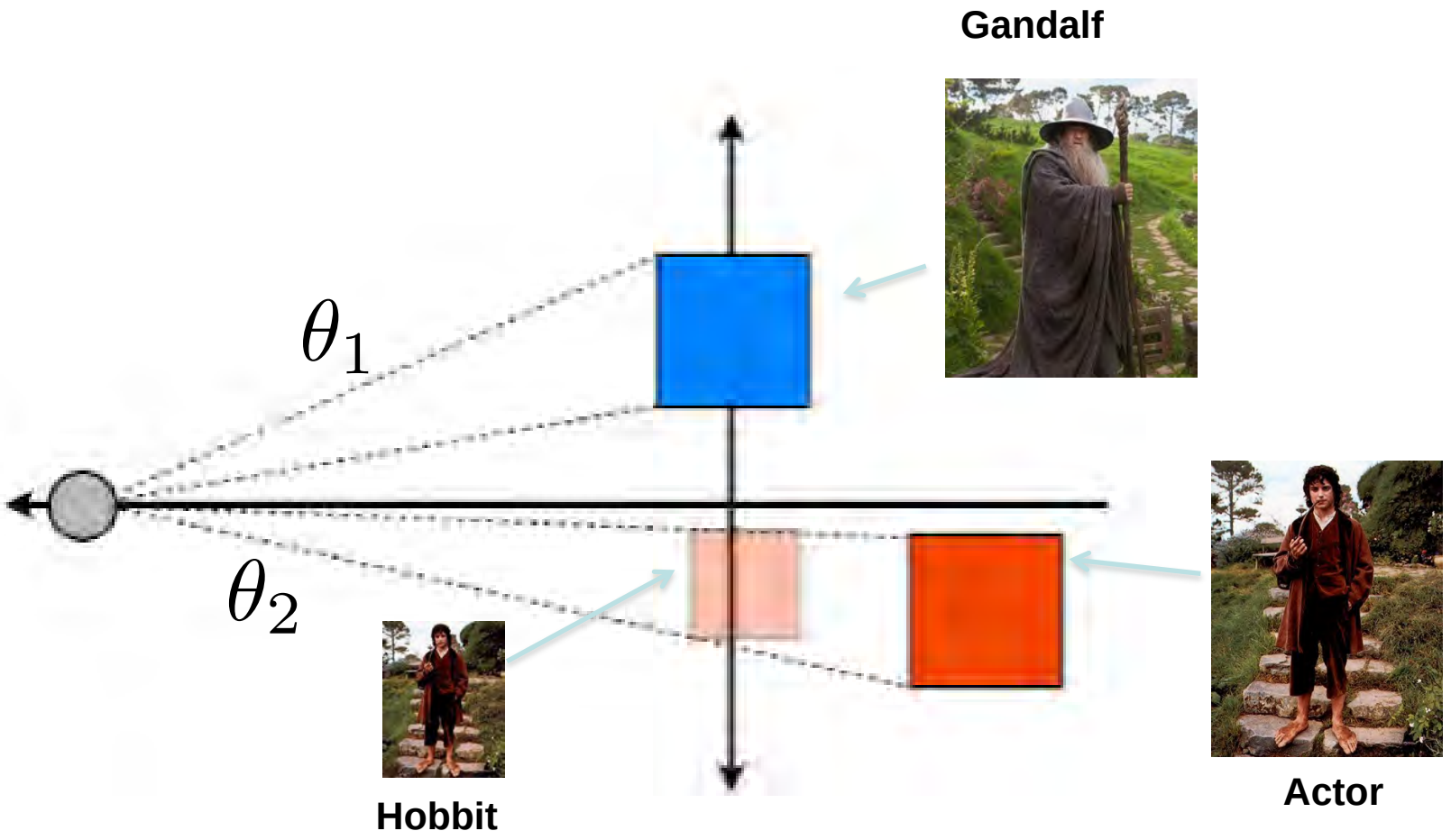
Two hundred and sixty visual effects artists at the Weta Studios worked on the Lord of the Rings trilogy

All of these used maths to do the effects!

The subdivision algorithm invented by Pixar mathematicians is used to create most of the images used in animated films

Differential equations are solved to recreate feathers, lava, water, fire and smoke

Maths can also be used to change the size of actors



Make $\theta_2 < \theta_1$ to make Hobbit appear smaller

Mathematicians can star in films

Film stars can do great maths too: Hedy Lamarr

Inventor of modern communication systems

Interview with Tony DeRose (Pixar)

What advice would you give someone who wanted to use math to make movies?

Learn as much mathematics as you can, particularly applied math. The areas of mathematics we use most heavily today are Euclidean and affine geometry, trigonometry, linear algebra, calculus and numerical analysis. We don't really know what the mathematical tools of tomorrow might be, so we're counting on the next generation of employees to tell us.

Example 2: Mathematicians good for music!

Some musical notes sound better when played together than others

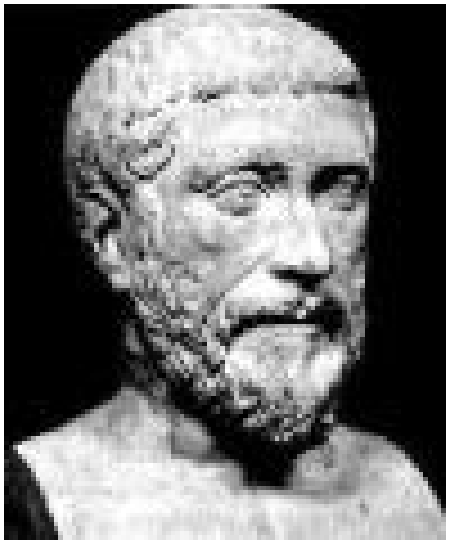
The octave C to C

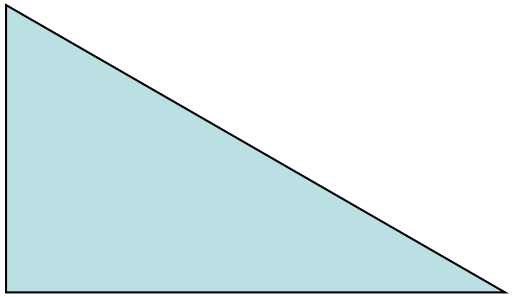
The notes C and G

(a perfect 5th)

The notes C and E

Reason was discovered by Pythagoras



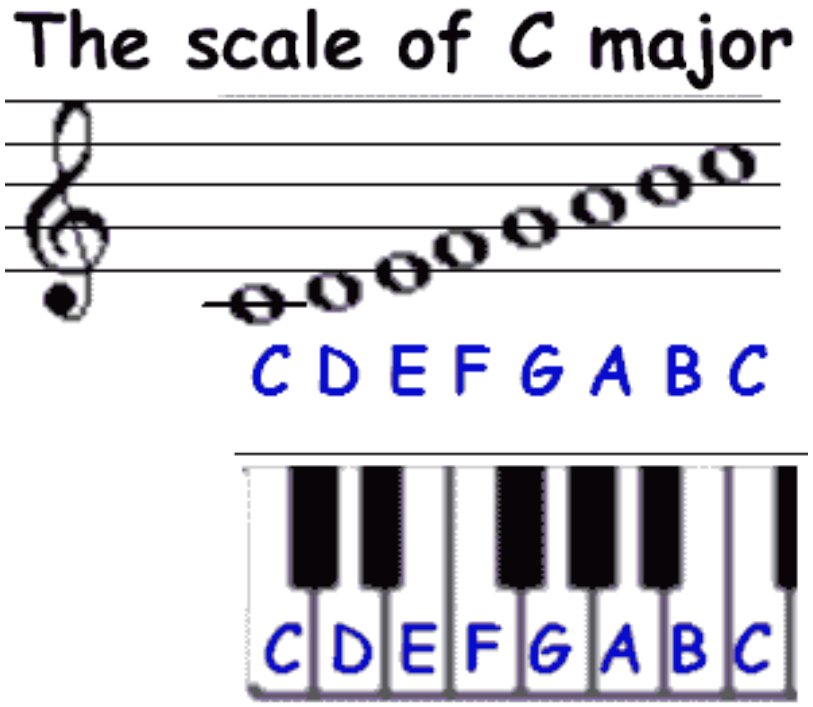


Length of strings giving C and G, and C and E, were in simple fractional proportions

C:C … 2/1 C:G … 3/2 C:E … 5/4

Pythagoras invented the Just Scale .. Sequence of notes with frequencies in simple fractional proportions

1 : 9/8 : 5/4 : 4/3 : 3/2 : 5/3 : 15/8 : 2



Problem: Keyboard instruments could only be tuned for one key

18th Century mathematicians invented a new Well Tempered scale with all notes in the same proportion

a geometric progression of the semi-tone frequencies, ratio

$$1.059.. = 2^{1/12}$$

which works well in all keys

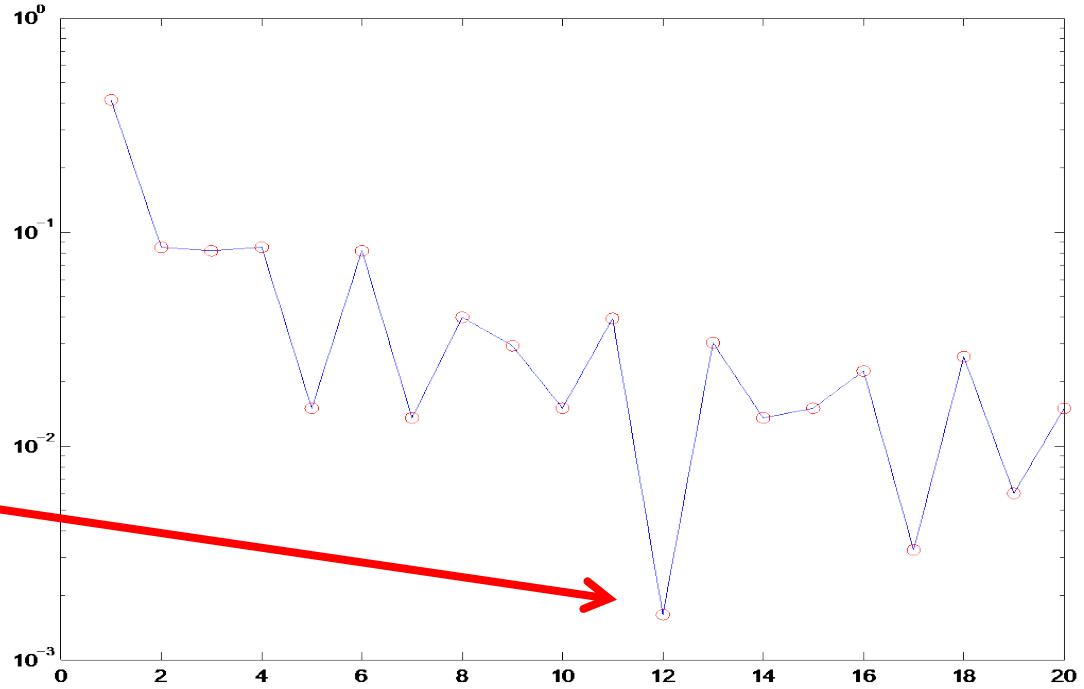
Geometric series, Continued fractions

Why are there 12 semi-tones in the octave?

Let there be n semi-tones, and the 'perfect 5th' is the mth note

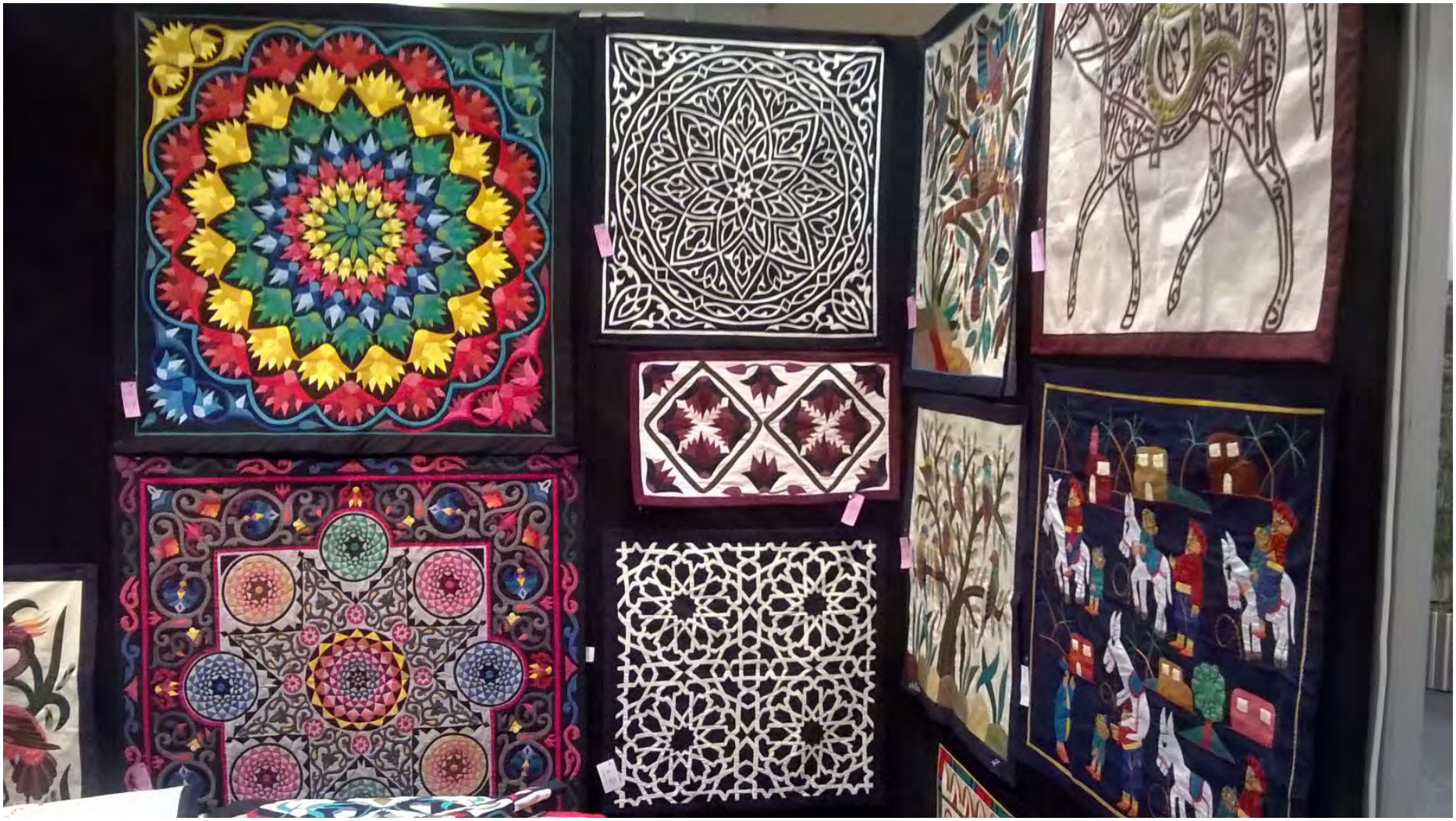
$$2^{m/n} \approx \frac{3}{2} \qquad \frac{m}{n} \approx \log_2\left(\frac{3}{2}\right)$$

Error

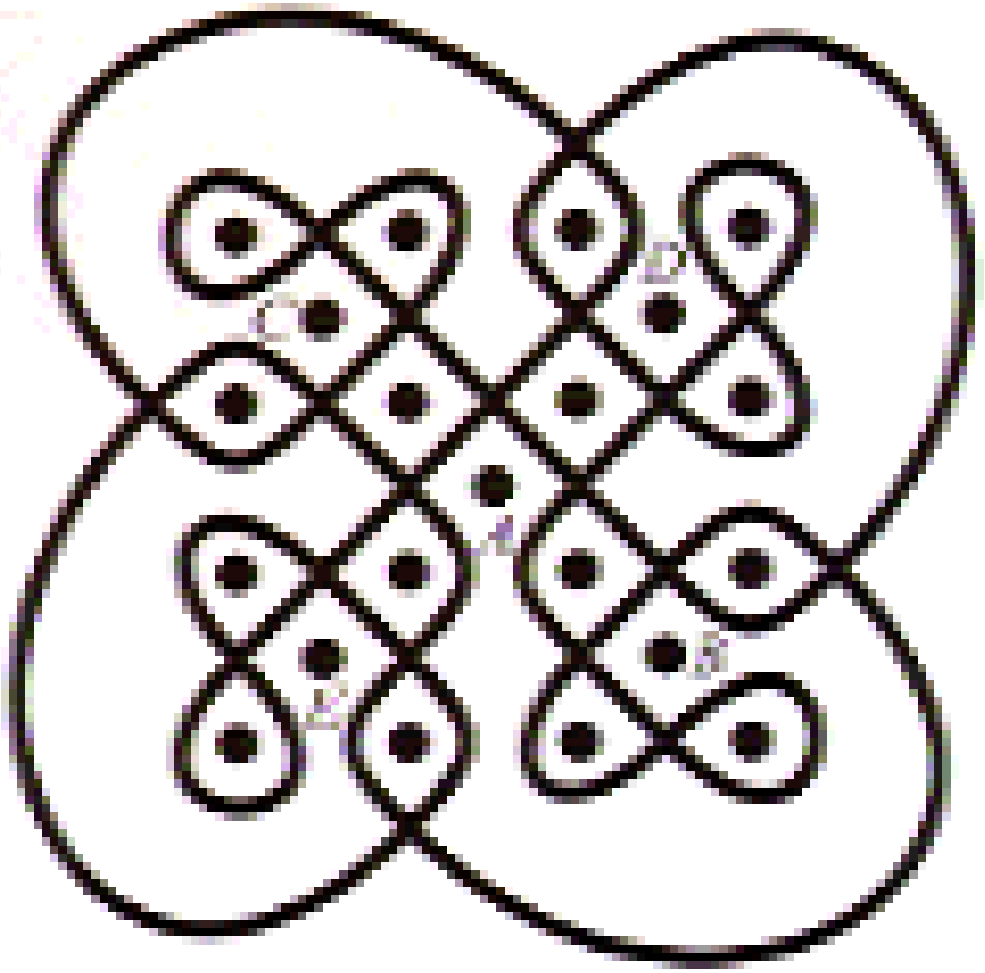


Error minimised when n = 12

Example 3: Maths and art are closely related

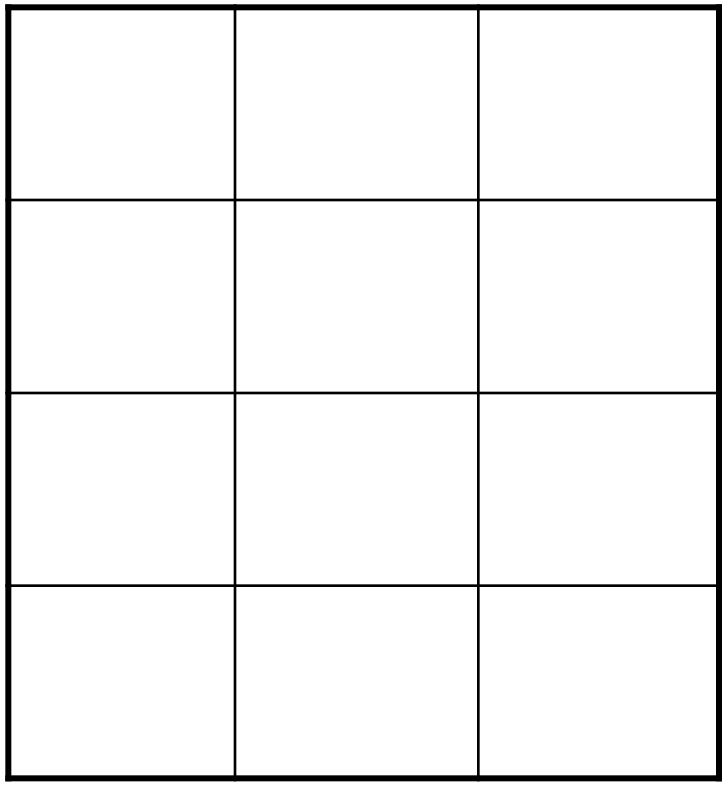


Celtic and African Art

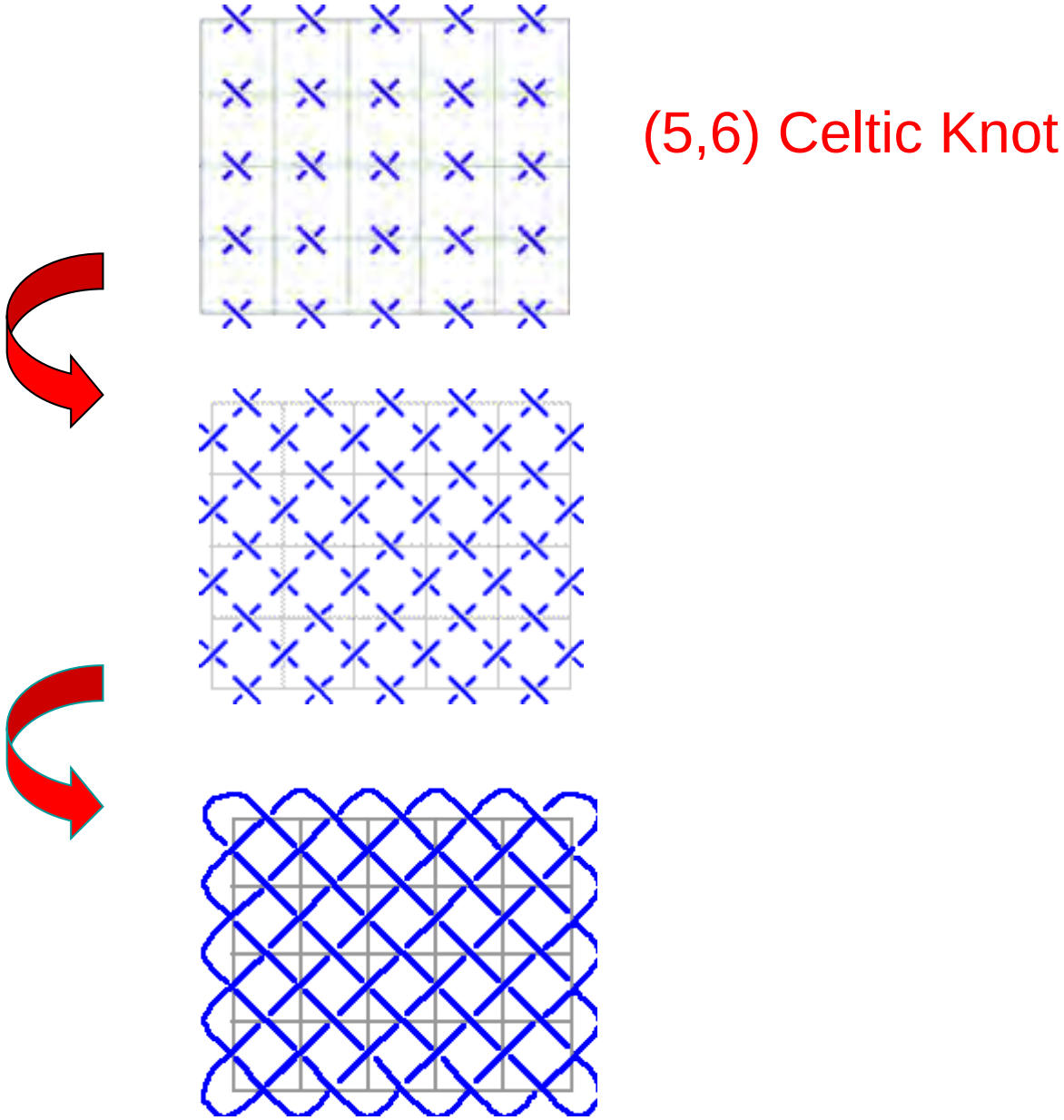


Can use Celtic and African knots to create wonderful art
AND learn some number theory by playful discovery

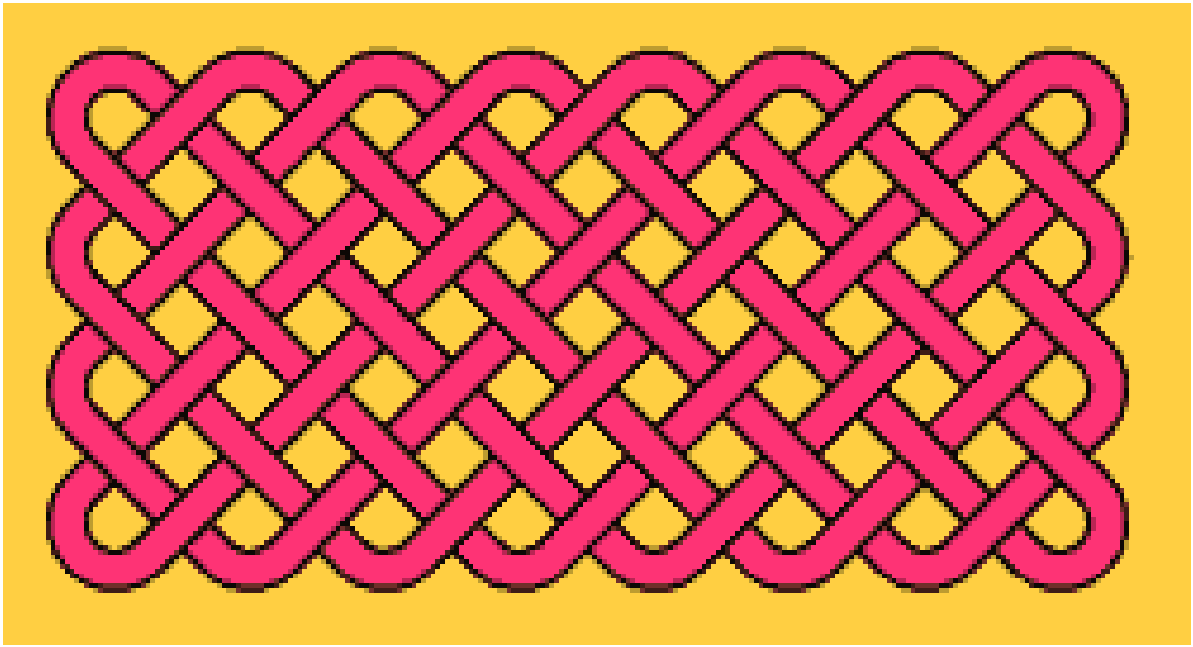
Start with a grid



(5 , 4) grid



(5,6) Celtic Knot

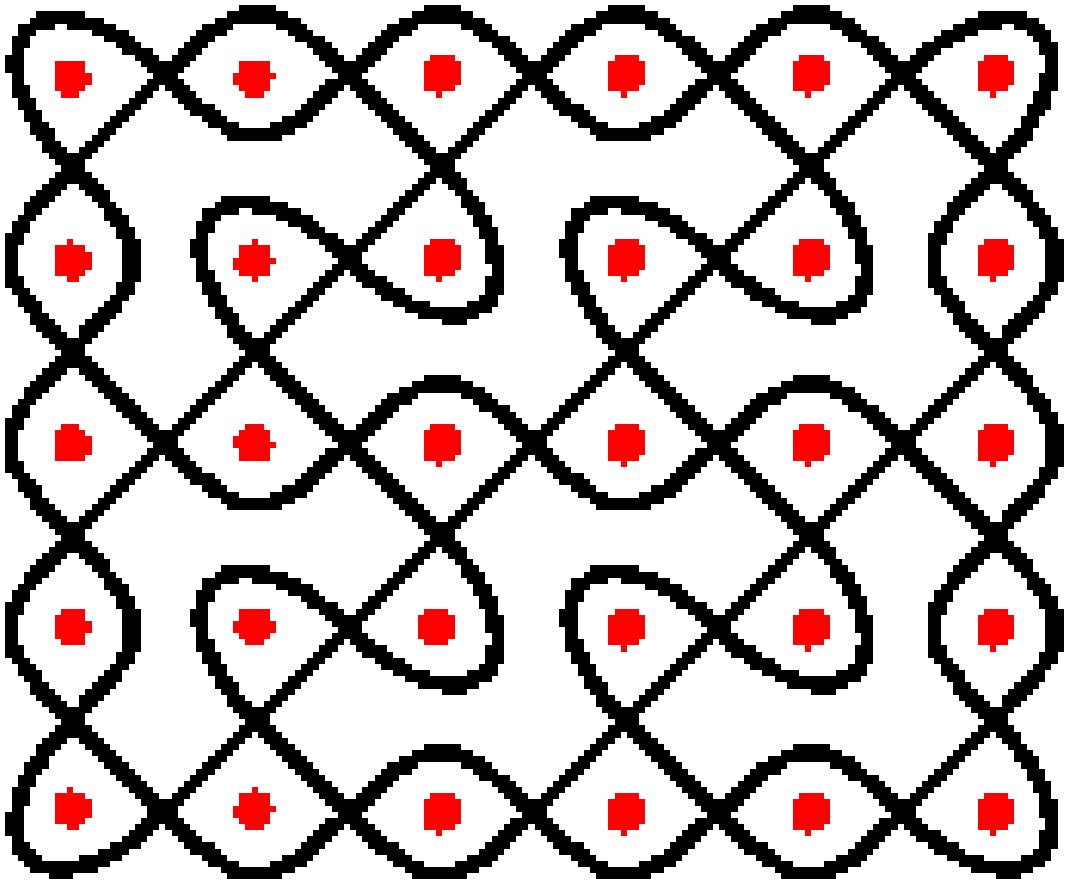


(4,8) Celtic Knot

How many pieces of string are needed?

How many pieces of string are needed?

(2,2)	2
(3,2)	1
(5,3)	1
(4,4)	4



Chase that Chicken!

Everyone can enjoy maths!!!

1.It is highly creative and can take you well beyond your imagination

2.It is the subject of puzzles and games

Game of Life, Sudoku, Griddler, number puzzles, topological puzzles, mind reading ….

		3		9	2			
4				3			1	
2	7							
	1		3					8
	5		1	6	7		3	
3					8		6	
							5	3
	3			8				9
			6	2		1		

3. You don’t need a laboratory to do great maths

4. Maths is all about finding and learning about patterns.

To find the patterns you need to play

How Can We Make Maths (More) Playful?

1. Emphasise the discovery and creativity aspects of maths in ALL of our teaching

2. Show the awe and wonder in maths

3. Show that maths is a subject full of surprises and mystery

$$\frac{\pi}{4} = 1 - \frac{1}{3} + \frac{1}{5} - \frac{1}{7} + \dots$$

4. Find the links between maths and art/music/real life/people

5. Demonstrate that above all MATHS IS FUN AND MATHS IS USEFUL

Playing with Pencils and Symmetry
(Maths Masterclass)

Playing with Shapes (at the Big Bang Fair)

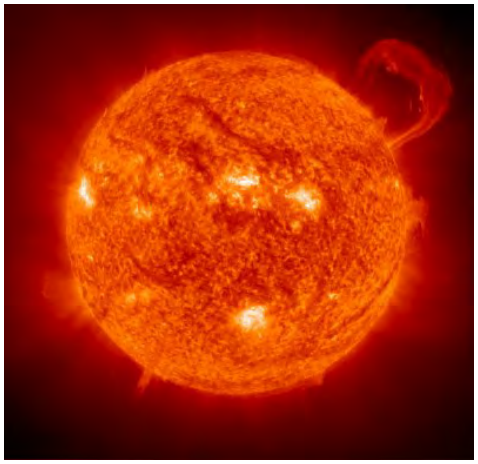
Playing with Bubbles (minimal surfaces)

Playing with Magic (at Bath Taps Into Science)

Where's the Joker? (An exercise in algebra)

10	1	9
11	2	9
12	3	9
13	4	9
14	5	9
15	6	9
16	7	9
17	8	9
18	9	9
19	10	9

Maybe even predict the climate

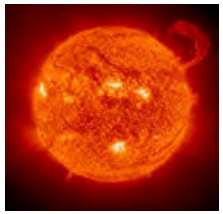


Heat into space

Heat from Sun: S

Earth's mean temperature: T

Heat absorbed $\longrightarrow (1-a)S$



a Albedo: How well the earth reflects the Sun's rays

Heat radiated away $\longrightarrow e\sigma T^4$

e emissivity: How much energy is radiated into space

Balance these to give an estimate of the Earth's temperature

$$e\sigma T^4 = (1-a)S$$

If we know e, σ, a, S we can work out T

Currently

Emissivity e = 0.605,

Boltzmann $\sigma = 5.67 \times 10^{-8}$

Albedo a = 0.31,

Solar heating S = 342 W/metre^2

Work out T from the heat balance equation

$$e\sigma T^4 = (1-a)S \rightarrow T = \left(\frac{(1-a)S}{e\sigma}\right)^{1/4}$$

Play with these values to predict the future climate

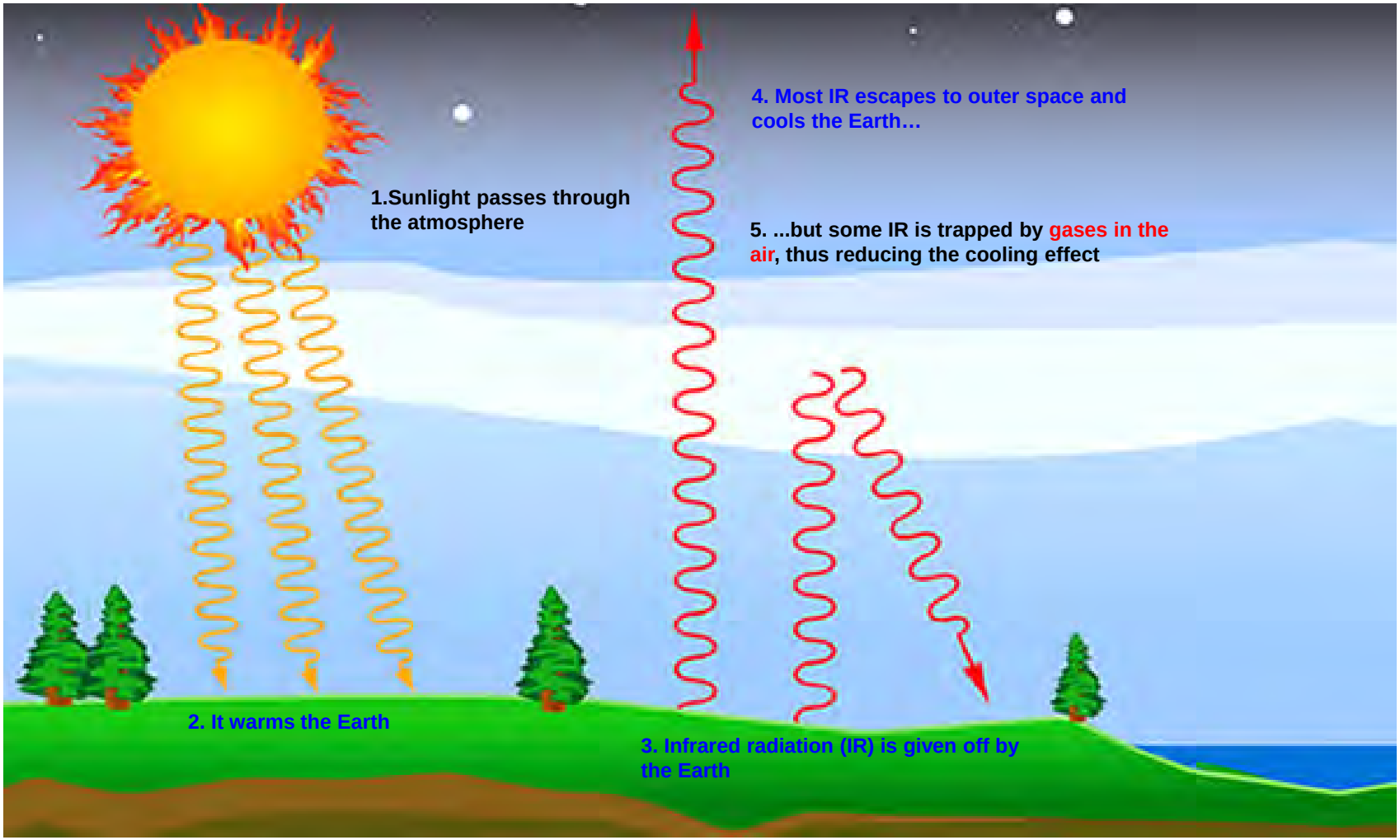
If CO2 increases

Then e decreases

$$T = \left(\frac{(1-a)S}{e\sigma} \right)^{1/4}$$

Formula tells us that T increases.

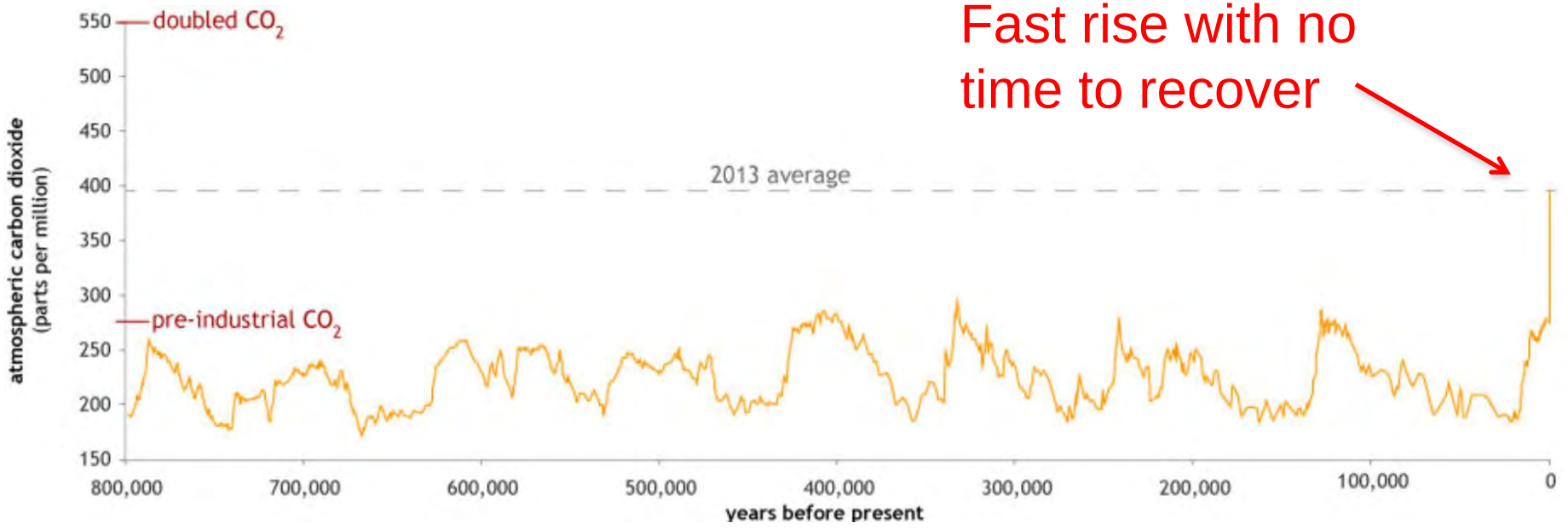
In pictures ….


1.Sunlight passes through the atmosphere
2. It warms the Earth
3. Infrared radiation (IR) is given off by the Earth
4. Most IR escapes to outer space and cools the Earth…
5. ...but some IR is trapped by gases in the air, thus reducing the cooling effect

In numbers …·

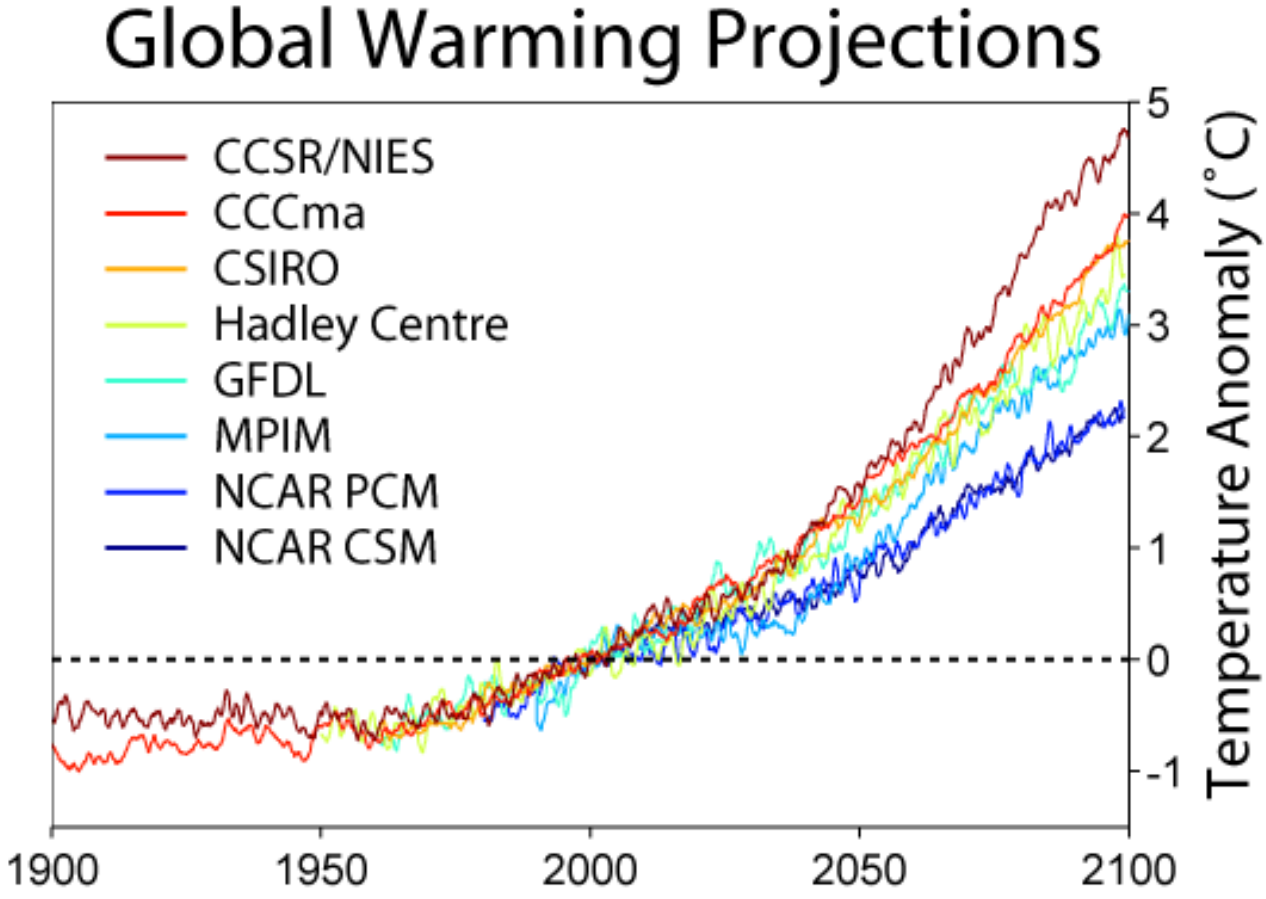
Level of Carbon Dioxide (ppm)	Emissivity e_{CO2}
200	0.194
400	0.14
600	0.108
800	0.085

C02



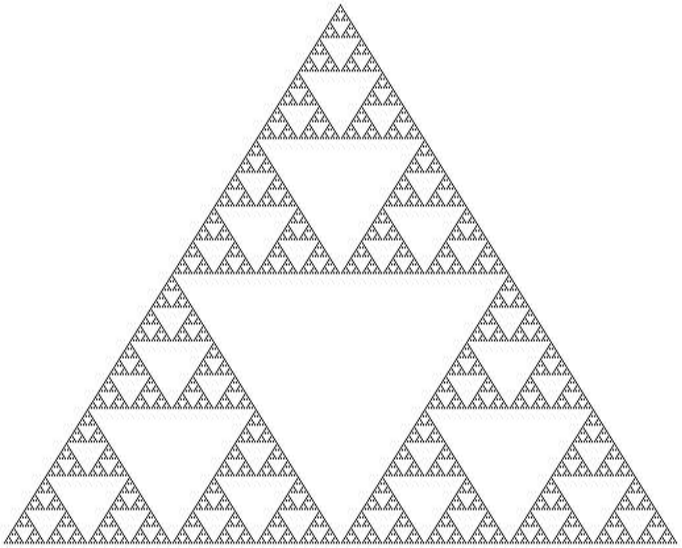
Now used to predict the future:

Gradual rise in temperature



Between a 2 and 5 degree increase by 2100

In Conclusion



Despite rumours to the contrary

- Maths is relevant
- Maths is creative
- Everyone can enjoy maths and get something from doing maths

This is a great message for adults!

Now to impress the celebrities!