How to Conquer Fractions, Decimals & Percentages Vol 1 Fundamentals

#### How to Conquer Fractions, Decimals & Percentages Vol 1 Fundamentals

#### Intelligent Australia Productions

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# Teachers Notes

#### About the Series

The 'How to Conquer Fractions, Decimals & Percentages' series was written in response to a pressing need. Most teachers of upper primary school classes would agree that this area of the Maths syllabus, along with Problem Solving, presents the most difficulties for students.

The series consists of four books:

- How to Conquer Fractions, Decimals & Percentages Vol 1 Fundamentals
- How to Conquer Fractions, Decimals & Percentages Vol 2 Method
- How to Conquer Fractions, Decimals & Percentages Vol 3 Conversions
- How to Conquer Fractions, Decimals & Percentages Vol 4 Problem Solving

The books are sequential, beginning with the most basic concepts in Volume 1, progressing through the steps required to work with and manipulate fractions, decimals and percentages, and concluding with examples based on everyday life where students can apply the skills gained from the earlier volumes.

# How to Conquer Fractions, Decimals & Percentages Vol 1 Fundamentals

The activities in this book have been designed to give students a clear understanding of what fractions, decimals and percentages are.

The best way for teachers to make use of the pages here is to

#### work slowly through the sheets with the students,

encouraging questions as you go. This is especially important in this volume; if worked through slowly and carefully students will come to know and understand these very important but curious abstractions of maths. With new insights and skills comes confidence and a readiness to tackle more challenging examples.

As exercises in this book are very basic it was decided to include answers at the back for only selected questions, those calling on higher processing skills.

# Some of what your students will learn:

- What fractions are

- What decimals are What percentages are  $\frac{1}{2} = 0.5 = 50\%$   $\frac{1}{10} = 0.1 = 10\%$   $\frac{1}{10} = 0.25 = 25\%$
- 1= 1.0= 100%

- 1/2 = 2/4 = 4/8 .....
  1/2 = 0.5=50%

- $\frac{3}{4} = 0.75 = 75\%$

#### **About the Author**

Ron Shaw is a highly experienced classroom teacher.

He is the author of some 30+ educational books, many of which are used in schools in several English-speaking nations.

Ron has teaching qualifications from Edith Cowan University (Perth) and is a graduate of the Australian National University (Canberra).

He is a member of the Australian College of Education, the Australian Teaching Council, the Australian Association of Mathematics Teachers and the Mathematical Association of Western Australia.

# What are Fractions?

A fraction is a part of a whole. One whole is written like this: 1 A half of 1 is written like this: 1/2



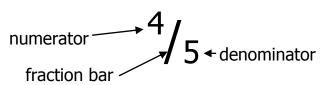
That's because there are two (2...the bottom number) halves in a whole and we are only interested in one of those halves (1....the top number).

All fractions have three parts.

There's the top number, the numerator.

There's the bottom number, the denominator.

And there's the line separating them, the fraction bar.



Every whole (1) can be divided up into equal parts (fractions).

The denominator tells us how many parts the whole has been divided into.

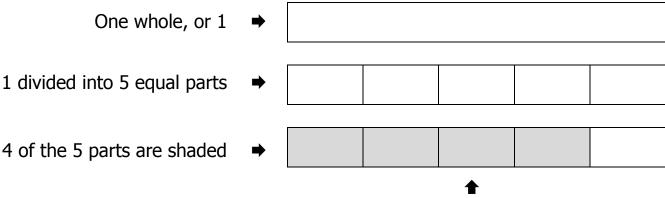
The numerator tells us the number of parts we're interested in.

For example, if we divide a whole up into five equal parts the denominator will be 5.

If we interested in four of those parts the numerator will be 4.

The fraction we will use is then  $\frac{4}{5}$ .

This is shown here:



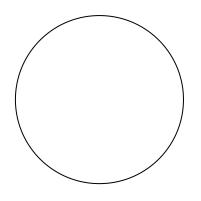
# The fraction that is shaded is $\frac{4}{5}$

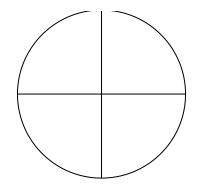
Here is another example...

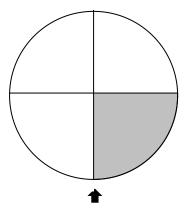
One whole, or 1

1 divided into 4 equal parts

1 of the 4 parts is shaded







The fraction that is shaded is  $\frac{1}{4}$ 

**Challenge:** On the back of this sheet or on another piece of paper use your ruler and pencil to draw a 10cm x 1cm rectangle, an 8cm square and a circle with diameter approx' 8cm.

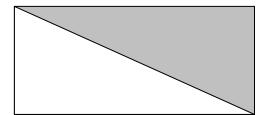
In the same way we worked here, shade

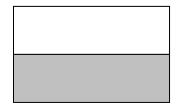
**1.**  $\frac{4}{10}$  of your rectangle **2.**  $\frac{3}{8}$  of your square (you'll need to draw diagonals) **3.**  $\frac{3}{4}$  of your circle

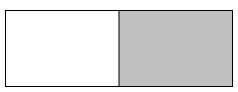






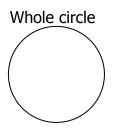


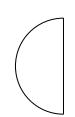




\* Colour the white half of each shape. Then write  $\frac{1}{2}$  in each half.

#### 3 more ways of showing one half:

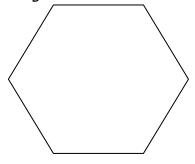




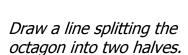


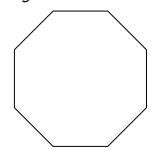


Draw a line splitting the hexagon into two halves.



Colour half of the hexagon red. Write  $^{1}/_{2}$  in each half.

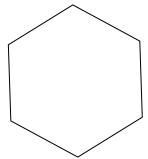




Colour half of the octagon red. Write  $\frac{1}{2}$  in each half.

#### **Hexagons**

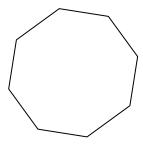
Now draw a line splitting this hexagon into two halves.



Colour half of the hexagon blue. Write  $^{1}/_{2}$  in each half.

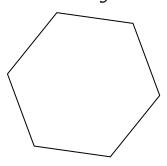
#### **Octagons**

Now draw a line splitting this octagon into two halves.



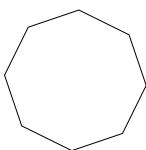
Colour half of the octagon blue. Write  $^{1}/_{2}$  in each half.

Do the same for this hexagon.



Colour half of the hexagon green. Write  $^{1}/_{2}$  in each half.

Do the same for this octagon.



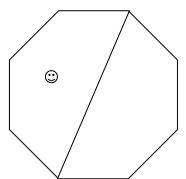
Colour half of the octagon green. Write  $\frac{1}{2}$  in each half.

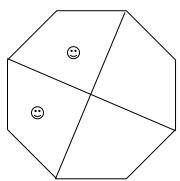
# Other Names for One

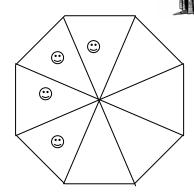
Here are three octagons.

They have been divided up into different numbers of parts.

In each octagon colour all the 'smiley face' parts yellow.







Do you agree that the yellow part of each octagon equals half the octagon?

Alongside every smiley face write the fraction of the octagon its area occupies (ie  $\frac{1}{2}$ ,  $\frac{1}{4}$ ,  $\frac{1}{8}$ )

Can you see that  $\frac{1}{2}$  equals  $\frac{1}{4} + \frac{1}{4}$  equals  $\frac{1}{8} + \frac{1}{8} + \frac{1}{8} + \frac{1}{8}$ ? Another way of saying this is...

 $^{1}/_{2}$  equals  $^{2}/_{4}$  equals  $^{4}/_{8}$ 

That's...

$$^{1}/_{2} = ^{2}/_{4} = ^{4}/_{8}$$

Now, in each of the fractions in the line above, what do you notice about the number on the top (the numerator) compared to the number on the bottom (the denominator)?

That's right, the numerator is exactly half the denominator.

Does this mean that whenever we see a fraction whose numerator is half its denominator we can call it 'one half' ... $^{1}/_{2}$ ? Yes, it does.

We already know that  $^2/_4 = ^1/_2$  We also know that  $^4/_8 = ^1/_2$  Write ten fractions of your own that equal  $^1/_2$ 

5

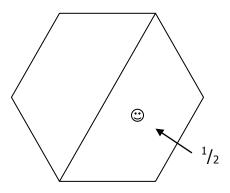
6

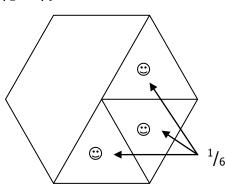
7

10

Now let's look at the two hexagons below.

By colouring the smiley face areas light blue show that  $^{1}/_{2} = ^{3}/_{6}$ 





Write  $\frac{1}{2}$  under the first hexagon and  $\frac{3}{6}$  under the other.

Now write these two fractions again here with an equals sign between them.

# One Quarter One Fourth 1/2



Look at this strip. Its length is one unit, or 1. We may call this <u>one whole</u>. lacktriangle

We shall divide the whole into four parts; the parts are <b>not equal</b> in size.
<ol> <li>Is the length of each part one quarter (¹/₄) the length of the whole strip?</li> <li>Write a thoughtful sentence to explain your answer.</li> </ol>
Now we divide the whole into four <b>equal</b> parts.
3. Is the length of each part one quarter (¹/₄) the length of the whole?
5. Below is one of the four equal parts of the whole. It is one part out of four <sup>1</sup> / <sub>4</sub> Colour it green.  Now use your ruler and pencil to make <b>one whole</b> by carefully drawing three more parts going to the right of the green shape. Make sure each part touches those to its left and right and <u>make each part exactly the same size as the others</u> .
Start drawing here
Now use a red pencil to shade in the three parts you added. You added $^1/_4 + ^1/_4 + ^1/_4$ We can write this in a shorter way $^3/_4$ Does $^1/_4 + ^3/_4$ equal one whole? Does $^1/_4 + ^3/_4 = 1$ ?
6. If you wanted to draw <b>half</b> of a whole $(^{1}/_{2})$ how many $^{1}/_{4}$ parts would you need?
7. If you wanted to draw <b>two wholes</b> (2) how many <sup>1</sup> / <sub>4</sub> parts would you need?
8. Make one half $(^{1}/_{2})$ by drawing another $^{1}/_{4}$ shape.
Draw here
1/ <sub>4</sub>





Look at this strip. Its length is one unit, or 1. We may call this <u>one whole</u>.  $\blacksquare$ 

We shall divide the whole into eight parts; the parts are <b>not equal</b> in size.
1. Is the length of each part one eighth (¹/ <sub>8</sub> ) the length of the whole?
Now we divide the whole into eight <b>equal</b> parts.
3. Is the length of each part one eighth (1/8) the length of the whole?
5. Below is one of the eight equal parts of the whole. It is one part out of eight <sup>1</sup> / <sub>8</sub> Colour it in blue.  Now use your ruler and pencil to make <b>one whole</b> by carefully drawing seven more parts going to the right of the blue shape. Make sure each part touches those to its left and right and <u>make each part exactly the same size as the others</u> .
Start drawing here
Now use a red pencil to shade in the seven parts you added. You added $^1/_8 + ^1/_8 + ^1/_8 + ^1/_8 + ^1/_8 + ^1/_8 + ^1/_8$ We can write this in a shorter way $^7/_8$ Does $^1/_8 + ^7/_8$ equal one whole? Does $^1/_8 + ^7/_8 = 1$ ?
6. If you wanted to draw <b>half</b> of a whole $(^{1}/_{2})$ how many $^{1}/_{8}$ parts would you need?
7. If you wanted to draw <b>two wholes</b> (2) how many <sup>1</sup> / <sub>8</sub> parts would you need?
8. By drawing more $^{1}/_{8}$ shapes make one half $(^{1}/_{2})$
Start drawing here

# **Equivalent Fractions**

1. Use <u>red</u> to colour in the parts with the marked fractions:



- <sup>1</sup>/<sub>2</sub>
- <sup>1</sup>/<sub>4</sub>

  <sup>1</sup>/<sub>4</sub>

1/12	
	1,
	<b>├</b>
	<b>←</b> 1 1
	<b>I</b>
	<del>//</del>
	<b>I</b>
	<b>*</b>

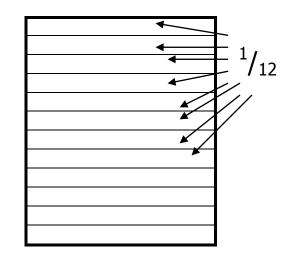
2. Now write in the two missing numerators in the fractions below.

$$^{1}/_{2} = /_{4} = /_{12}$$

3. Use <u>blue</u> to colour in the parts with the marked fractions:

1/3
1/3

<sup>1</sup> / <sub>6</sub>	
<sup>1</sup> / <sub>6</sub>	
<sup>1</sup> / <sub>6</sub>	
<sup>1</sup> / <sub>6</sub>	



4. Look at the rectangles above and write in the two missing numerators in the fractions below.

$$^{2}/_{3} = /_{6} = /_{12}$$

- 5. Now write three equivalent fractions for the *non marked* parts in the example above.
- 6. By colouring all marked parts green in the rectangles below show that

$$^{12}/_{20} = ^{6}/_{10} = ^{3}/_{5}$$

1/20	1/20	1/20	1/20	<sup>1</sup> / <sub>20</sub>	<sup>1</sup> / <sub>20</sub>	1/20	1/20	1/20	1/20	1/20	1/20				
1/	10	1/	10	1/	10	1/	10	1/	10	1/	10				
	1/	/ <sub>5</sub>			1/	/ <sub>5</sub>			1	/ <sub>5</sub>					

7. Now write three equivalent fractions for the *non marked* parts in the example above.

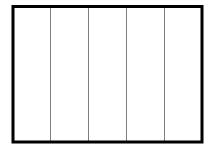
# More Equivalent Fractions



#### The two rectangles below are the same size.

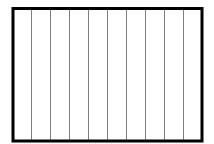
Rectangle 1 Each part is  $\frac{1}{5}$ .

(1) Colour the first  $\frac{1}{5}$  green and the next <sup>4</sup>/<sub>5</sub> red.



Rectangle 2 Each part is  $^{1}/_{10}$ .

(2) Colour the first  $^{2}/_{10}$  green and the next  $^{8}/_{10}$  red.



(3) What do you notice about the green and red parts in the two rectangles?

(4) Write in the missing fractions.

$$\frac{1}{5} + \frac{5}{10} = \frac{5}{5}$$
  
 $\frac{2}{10} + \frac{10}{10} = \frac{10}{10}$ 

(5) Complete the fractions by inserting the correct numerators.

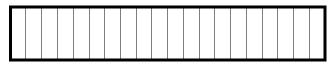
$$/_5 = ^2/_{10}$$

 $l_5 = \frac{2}{10}$   $\frac{4}{5} = \frac{10}{10}$ 

#### The three rectangles below are the same size. $^{12}/_{20} = ^{6}/_{10} = ^{3}/_{5}$

Rectangle 1 Each part is  $^{1}/_{20}$ .

(6) Colour the first  $^{12}/_{20}$  blue and the next  $^{8}/_{20}$  yellow.



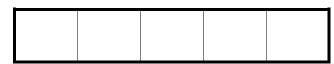
Rectangle 2 Each part is  $^{1}/_{10}$ .

(7) Colour the first  $^{6}/_{10}$  blue and the next  $^{4}/_{10}$  yellow.



Rectangle 3 Each part is  $^{1}/_{5}$ .

(8) Colour the first  $\frac{3}{5}$  blue and the next  $\frac{2}{5}$  yellow.



(9) What do you notice about the blue and yellow parts in the three rectangles?

(10) Write in the missing fractions.

$$^{12}/_{20} + \underline{\phantom{0}} = ^{20}/_{20}$$
 $^{6}/_{10} + \underline{\phantom{0}} = ^{10}/_{10}$ 
 $^{3}/_{5} + \underline{\phantom{0}} = ^{5}/_{5}$ 

(11) Complete the fractions by inserting the correct numerators.

$$\frac{12}{20} = \frac{1}{10} = \frac{1}{5}$$
 $\frac{12}{20} = \frac{1}{10} = \frac{2}{5}$ 

How to Conquer Fractions, Decimals & Percentages Vol 1 Fundamentals

# Same Fraction, Different Name

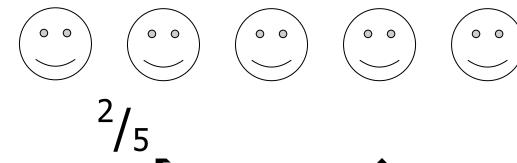
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Each pa											s bli	ue.	Wri	te tl	ne f	rac	ion :	ınde	rne	atl	٦.			
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What do	•														<u>.</u>							 	 	
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Now col	our o you	all re	emaii cice?	ning	, pa	rts	in t	the	ab	ove	e str	ips	yel	low								 	 	
Now col	our you rt is	all re u not <sup>1</sup> / <sub>10</sub>	emaio cice? of th	ning 	pa pa	rts 	in t	the	ab	OV6	e str	rips	yel	low							h.			
Now col What do Each pa	our you rt is	all re u not <sup>1</sup> / <sub>10</sub>	emaio cice? of th	ning 	pa pa	rts 	in t	the	ab	OV6	e str	rips	yel	low							h.	 	 	
Now col What do Each pa	our you rt is	all re u not <sup>1</sup> / <sub>10</sub>	emaio cice? of th	ning 	pa pa	rts 	in t	the	ab	OV6	e str	rips	yel	low							h.	 	 	
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# **Happy Family**

Here is the Smiley family.

There is Mum Smiley, Dad Smiley, two girls (Sue and Sally) and a boy (Stevie). Colour the first two Smileys (Mum and Dad) red and the next three Smileys (the children) blue.





We have written one fraction for you. You write the other fraction here.

Now fill in the missing fraction below and then read your equation out loud.

$$^{2}/_{5} + = ^{5}/_{5}$$

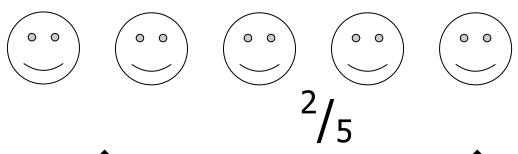
Can you see that  $^5/_5$  is the **whole** Smiley Family? That's right, each person is  $^1/_5$  of the whole family...  $^1/_5 + ^1/_5 + ^1/_5 + ^1/_5 + ^1/_5 = ^5/_5$  And  $^5/_5$  equals 1 (**one whole** family)

Yes, 
$$\frac{5}{5} = \mathbf{1}$$

Now, let's go back to the Smiley Family.

As before, colour the first two Smileys (Mum and Dad) red.

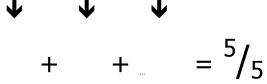
This time, colour the next two (Sue and Sally) yellow and the last one (Stevie) green. We have written the girls' fraction underneath them.



Write the parents' fraction here.

Write Stevie's fraction here.

Now fill in the three missing fractions below and then read your equation out loud.

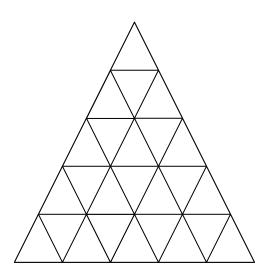


# Building a Gl



Let's build a large triangle out of twenty-five smaller triangles like the one above. We need 15 triangles turned the right way up and 10 turned upside-down.

Here is the result...



The large triangle takes up a large space. Let's give that space a funny name. We'll call it a GLOB.

Each smaller triangle takes up a smaller space. How much space?

When stacked together the small triangles take up a GLOB of space but how much space does one small triangle take up?

Is it  $\frac{1}{2}$  of a **GLOB**? No, it's less isn't it!

One of the fractions below tells how much space a small triangle takes up. Put a tick after the correct one.

- <sup>1</sup>/<sub>4</sub> of a **GLOB**
- 1/10 of a GLOB
   1/25 of a GLOB
- $^{1}/_{100}$  of a **GLOB**

Colour one of the small triangles red.

**1.** What fraction of the large triangle is it? (put a ring around the correct answer below)

$$\frac{1}{2}$$
  $\frac{1}{10}$   $\frac{1}{25}$   $\frac{1}{100}$ 

Colour seven of the small triangles yellow.

**2.** What fraction of the large triangle are they? (put a ring around the correct answer below)

Colour twelve of the small triangles green.

**3.** What fraction of the large triangle are they? (put a ring around the correct answer below)

Colour the remaining five small triangles blue.

**4.** What fraction of the large triangle are they? (put a ring around the correct answer below)

All your small coloured triangles together take up 1 **GLOB** of space. **5.** One of your small coloured triangles takes up  $^1/_2$   $^1/_{10}$   $^1/_{25}$   $^1/_{100}$  **GLOB** of space (circle correct one)

Now look at your answers to questions **1-4** above and fill in the spaces below with fractions. (the first one has been done for you)

6. 
$$\frac{1}{25} + \frac{1}{25} = \frac{25}{25}$$

and...

When our answer to a maths question is a fraction we usually write it in its simplest form. For example  $^{2}/_{4}$  would be written as  $^{1}/_{2}$ 

That's because  $^2/_4$  and  $^1/_2$  are the <u>same amount</u> and its easier to think of and 'see'  $^1/_2$  than  $^2/_4$  Do you think  $^{20}/_{40}$  is also this amount? Yes, it is.

What about  $^{100}/_{200}$  and  $^{500}/_{1000}$ ? Yes, these are the same as  $^{1}/_{2}$  too.

What do you notice in all of the above fractions? ...

The numerator is half the denominator.

So let's say our answer to a maths problem is <sup>24</sup>/<sub>48</sub>

Rather than leave it at  $^{24}/_{48}$  we simplify our answer by cancelling.

In this case both numerator and denominator may be divided by 24 (24 is their highest common factor...HCF).

 $^{124}/_{248} = ^{1}/_{2}$ 

 $^{24}/_{48}$  expressed in its *lowest terms* is  $^{1}/_{2}$ 

This process of cancelling until we bring a fraction down to its lowest terms is called *simplifying*.

Simplify these fractions by dividing the numerator and denominator by 7. You may have to cancel down even further until you reach the fraction's lowest terms.

$$(1)^{14}/_{35} =$$

$$(2)^{7}/_{21} =$$

$$(3)^{28}/_{56} =$$

$$(4)^{14}/_{70} =$$

$$(3)^{28}/_{56} = (4)^{14}/_{70} = (5)^{35}/_{42} =$$

Simplify these fractions by dividing the numerator and denominator by 9. Again, you may have to cancel down even further until you reach the fraction's lowest terms.

$$(6)^{9}/_{36} =$$

$$(7)^{27}/_{81} =$$

$$(8)^{18}/_{54} =$$

$$(9)^{45}/_{90} =$$

$$(10)^{54}/_{72} =$$

By finding a common factor, or better still, a Highest Common Factor, simplify these...

$$(11)^{10}/_{40} =$$

$$(12)^{32}/_{60} =$$

$$(13)^{24}/_{60} =$$

$$(13)^{24}/_{60} = (14)^{6}/_{72} = (15)^{50}/_{80}$$

$$(15)^{50}/_{80}$$

$$(16)^{12}/_{50} =$$

$$(17)^{25}/_{40} =$$

$$(18)^{27}/_{63} =$$

$$(19)^{15}/_{80} =$$

$$(20)^{11}/_{110}$$

Reduce these fractions to their lowest terms:

$$(21)^{12}/_{60} =$$

$$(22)^{13}/_{39} =$$

$$(23)^{5}/_{30} =$$

$$(24)^{14}/_{56} =$$

$$(25)^{11}/_{55}$$

$$(26)^{18}/_{90} =$$

$$(27)^{16}/_{48} =$$

$$(28)^{10}/_{25} =$$

$$(29)^{22}/_{88} =$$

$$(30)^{9}/_{45}$$

$$(31)^{20}/_{55} =$$

$$(32)^{12}/_{72} =$$

$$(33)^{24}/_{80} =$$

$$(34)^{15}/_{50} =$$

$$(35)^{28}/_{60}$$

Simplify these fractions:

$$(36)^{8}/_{60} =$$

$$(36)^{8}/_{60} = (37)^{3}/_{93} =$$

$$(38)^4/_{38} =$$

$$(38)^4/_{38} = (39)^{26}/_{36} =$$

$$(40)^{36}/_{48}$$

$$(41)^{2}/_{54} =$$

$$(42)^{35}/_{45} =$$

$$(43)^{21}/_{36} =$$

$$(44)^{18}/_{60} = (45)^{4}/_{96}$$

$$(45)^{4}/_{96}$$

$$(46)^{25}/_{65} =$$

$$(47)^{13}/_{65} =$$

$$(48)^{40}/_{60} =$$

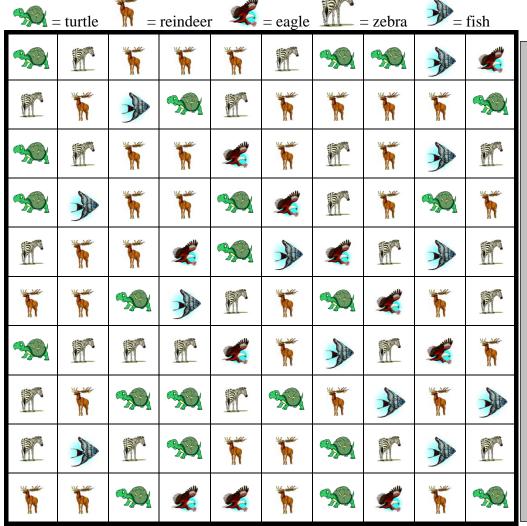
$$(49)^{18}/_{64} =$$

$$(50)^{20}/_{100}$$

# What Percentage?

Including fish and birds there are 100 animals in the grid.





# Why have 100 animals?

We used 100 animals in this example because it's easy to work out percentage if there are 100 objects. If there were only 50 animals we would need to double to work out the percentage of each animal. If there were only 25 animals we would need to multiply the number of each animal by 4 to obtain its percentage score. If there were only 10 animals we would have to multiply each animal's score by 10 to calculate its percentage. Remember: per means 'out of' and cent means 'one hundred'.

Colour the graphs below to show the correct percentages (use the same colour for each graph)

Percentage of these animals that:

can	fly	lay egg	<b>75</b>	have gi	lls	don't hav	e wings	have antlers		
100		100		100		100		100		
90		90		90		90		90		
80		80		80		80		80		
70		70		70		70		70		
60		60		60		60		60		
50		50		50		50		50		
40		40		40		40		40		
30		30		30		30		30		
20		20		20		20		20		
10		10		10		10		10		

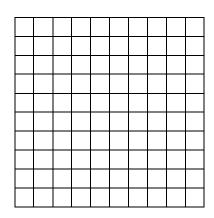
Finished?

On a separate piece of paper or in your maths book draw a graph to show the percentage of these animals that are mammals. Under the graph write this number with a % sign after it.

# Commonly used Percentages



**1.** You know that 10% means 10 parts out of 100 and that it may be written like this:  $^{10}/_{100}$ 

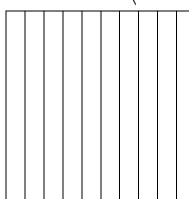


Colour the 1st column (that's 10 parts out of 100) red.

You have now coloured 10% of the large square red.

**2.** Now, 10% may also be written as  $^{1}/_{10}$ (meaning 1 part out of 10). This is easy to see when we cancel:

$$\frac{10}{100} / \frac{100}{100} = \frac{1}{10} / \frac{10}{10}$$



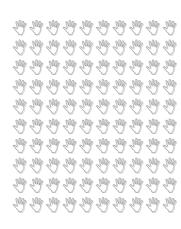
Colour the 1st column (that's 1 part out of 10) red.

You have now coloured 10% of the large square red.

Can you see that 10% is both  ${}^{10}/_{100}$  and  ${}^{1}/_{10}$ ?

**3.** 25% means 25 parts out of 100 and it may be written like this:

$$^{25}/_{100}$$



Colour the 1st two and a half columns (that's 25 hands out of 100) vellow.

You have now coloured 25% of the hands yellow. **4.** 25% may also be written as  $\frac{1}{4}$  (meaning 1 part out of 4).

This is easy to see when we cancel:

$$\frac{25}{100} = \frac{1}{4}$$



Here you can see by the hands' different shadings that you coloured one out of four equal sets.

(your yellow hands are one of four sets, each with 25 hands)

That's  $\frac{1}{4}$ 

Can you see that 25% is both  $^{25}/_{100}$  and  $^{1}/_{4}$ ?

**5.** In question 3 you coloured 25 hands and that was 25% of all the hands.

What percentage of the hands were not coloured?

**6.** In question 4 you saw that  $25\% = \frac{25}{100}$  and that this cancels down to  $\frac{1}{4}$ 

What fraction does 75% cancel down to?

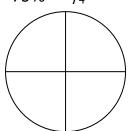
**7.** Put a ring around the correct ones:

 $25\% = \frac{1}{4}$ 

 $75\% = \frac{1}{4}$ 

 $25\% = \frac{3}{4}$   $75\% = \frac{3}{4}$ 

**8.** Shade 25% of this circle blue and 75 % of it yellow.



# Introduction to Decimals

The word decimal comes from the Latin word *decem* which means ten.

Look at these 10 faces.





Colour the happy faces yellow.

There are 5 happy faces.

That's 5 out of 10 and

if we express this as a decimal we say: 0.5 of the faces are happy

Colour the sad faces blue.

There are 2 sad faces.

That's 2 out of 10 and

(1) if we express this as a decimal we say: ..... of the faces are sad

Colour the in-between faces red.

There are 3 in-between faces.

That's 3 out of 10 and

(2) if we express this as a decimal we say: ..... of the faces are in-between



- (3) What decimal of the black shapes are squares? .....
- (4) What decimal of the black shapes have more than 4 sides? .....

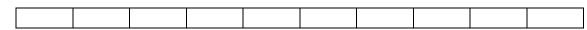
- (5) What decimal of the black shapes have 6 sides? .....
- (6) Quickly sketch 10 small trees on the lines, with 0.4 of them being apple trees.

(7) Sketch 10 simple shapes on the lines, with 0.7 of them being triangles.

(8) Rewrite these decimal numbers, from smallest to largest.

0.5 0.7 0.2 0.9 0.8 0.4 0.6 0.1 0.3

(9) Colour 0.2 of the strip below blue, 0.5 of it red and the other 0.3 yellow. After colouring, write the decimal fraction underneath each colour.



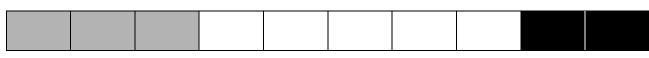
(10) Does 0.2 + 0.5 + 0.3 = 1? .....

# More About Decimals



Did you know that  $^{1}/_{10} = 0.1$ ?



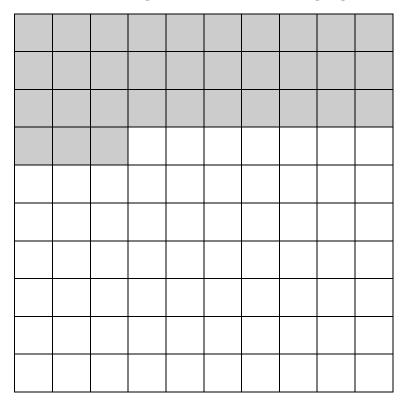


- 1. What decimal of the strip is shaded black? .....
- 2. What decimal of the strip is unshaded? .....

Sometimes with decimals we work with hundredths.

Did you know that  $^{1}/_{100} = 0.01$  ?

#### 0.33 of the grid below is shaded grey.

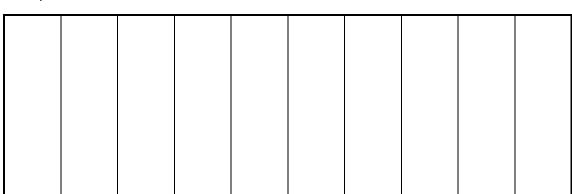


- **3.** Shade 0.48 of the grid red.
- 4. Shade the remainder yellow. What decimal of the grid is yellow? .....
- **5.** Do you think 0.30 = 0.3 ? .....
- **6.** Do you think 0.3 + 0.05 = 0.35?
- **7.** How well can you explain your answer to the previous question?

# Decimal Equivalence

The rectangle below has been divided into 10 parts. Each part is  $\frac{1}{10}$  of the rectangle.

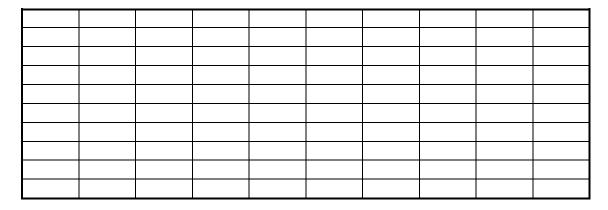
Colour the first 3 parts blue.



- **1.** What fraction of the rectangle did you colour? .....
- 2. What decimal of the rectangle did you colour?

The rectangle below has been divided into 100 parts. Each part is  $\frac{1}{100}$  of the rectangle.

Colour the first 3 columns (30 parts) blue.



- 3. What fraction of the rectangle did you colour?
- 4. What decimal of the rectangle did you colour?

Now look at the areas you coloured blue in the two rectangles. Write in the missing numerator and denominator:

5. 
$$/_{10} = {}^{30}/$$

Fill in the missing numerators and denominators:

**6.** 
$$/_{10} = {}^{70}/_{100}$$

7. 
$$^{2}/_{10} = /_{100}$$

$$/_{10} = {}^{70}/_{100}$$
 7.  ${}^{2}/_{10} = /_{100}$  8.  $/_{10} = {}^{90}/_{100}$  9.  ${}^{1}/_{10} = /_{100}$ 

**9.** 
$$^{1}/_{10} = /_{100}$$

**10.** 
$$\frac{5}{10} = \frac{100}{100}$$

**11.** 
$$\frac{8}{100} = \frac{80}{100} = \frac{100}{100}$$

**12.** 
$$^{10}/_{10} = ^{100}/$$

**10.** 
$$\frac{5}{10} = \frac{100}{100}$$
 **11.**  $\frac{8}{100} = \frac{80}{100}$  **12.**  $\frac{10}{10} = \frac{100}{100}$  **13.**  $\frac{1}{100} = \frac{40}{100}$ 

# Decimal Points and Places

Let's say that each large square below equals **one** or **1**.

You will agree that there are three (3) large squares altogether but not all are fully shaded.

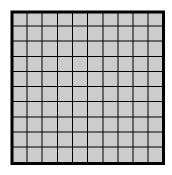
How may we show the amount that is shaded?

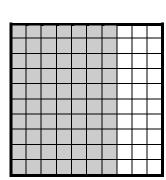
one whole square

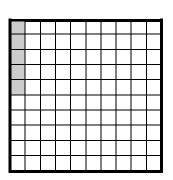
0.7

 $^{7}/_{10}$  of a whole square  $^{5}/_{100}$  of a whole square

0.05



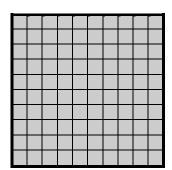


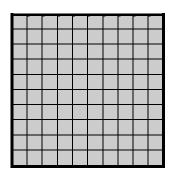


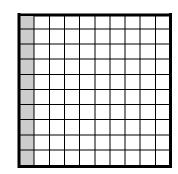


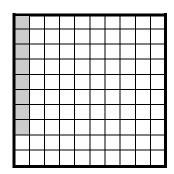
The amount that is shaded is 1 + 0.7 + 0.05 = 1.75

1) What is the total shaded area shown below? (write your answer underneath)



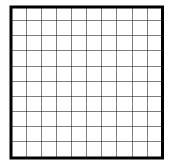


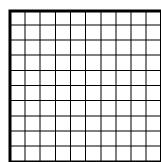


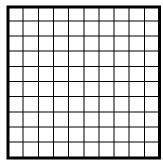


The amount that is shaded is .....

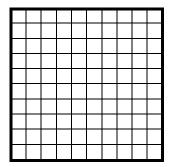
2) Using just one colour shade the rectangles below to show 1.42

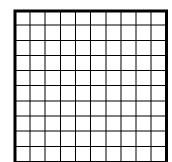


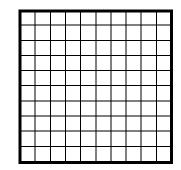


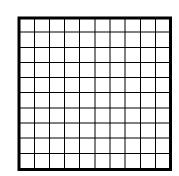


3) Using just one colour shade the rectangles below to show 2.76









# Conversions



1. Complete the table and colour the correct number of boxes.

Fraction	Decimal	Percentage		Visual representation (use green or red)
$\frac{1}{10}$	0.1	10%	$\Rightarrow$	
$^{2}/_{10}$		20%	$\Rightarrow$	
	0.3	30%	$\Rightarrow$	
4/10			$\Rightarrow$	
	0.5		$\Rightarrow$	
		60%	$\Rightarrow$	
7/10			$\Rightarrow$	
	8.0		$\Rightarrow$	
		90%	$\Rightarrow$	
1	1.0		$\Rightarrow$	

2. So....ready for something harder?

Look at each visual representation and then complete the table by writing in the fraction, the decimal and the percentage that it represents.

#### Hints:

- Fractions: cancel –or simplify- when possible.....that is, bring down to lowest terms
- Decimals and Percentages: count the shaded rectangles and multiply by 5

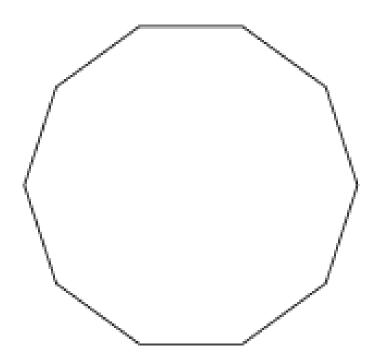
Fraction	Decimal	Percentage	Visual representation
			\$\tag{\tau}\$
			\$\phi\$
			\$\phi\$
			\$\tag{\tau}\$

# **Equalities**

With your ruler and pencil carefully draw in the diagonals in the decagon. (From each corner draw the longest line)

Now you have 10 segments.

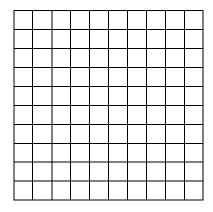
Colour  $\frac{5}{10}$  of the decagon yellow. Colour 0.5 of the decagon blue.



- **1.** Does  $\frac{5}{10}$  equal 0.5?
- **2.** Do  $\frac{5}{10}$  and 0.5 both equal a half?
- **3.** Does 50% equal a half?
- **4.** Do  $^5/_{10}$  and 0.5 and 50% all have the same value (a half) ?



Colour 50% of the square red and  $^{5}/_{10}$  of the square green.



- **5.** Does  $\frac{5}{10}$  equal 50% ? .....
- **6.** Do  $^{5}/_{10}$  and 50% both equal a half?

Colour 0.4 of the rectangle red, 30% of it yellow and  $^2/_{10}$  of it green.



- **7.** What % of the rectangle is coloured? .....
- **8.** What decimal of the rectangle is coloured?
- **9.** What fraction of the rectangle is coloured?
- **10.** What % of the rectangle is <u>not</u> coloured? .....
- **11.** What decimal of the rectangle is <u>not</u> coloured? .....
- **12.** What fraction of the rectangle is not coloured?

# Ways of Saying the Same Thing

# 新新新新新新士士士

 $^{6}/_{10}$  of the pets are dogs. Fraction:

**Decimal:** 0.6 of the pets are dogs.

60% of the pets are dogs. **Percentage:** 

1. Complete these...



Fraction:	$^4/_{10}$ of the pets are cats.
Decimal:	
Percentage:	

2. Draw sketches to match what's below and write in the two missing quantities.

Fraction:	$^{3}/_{10}$ of the people are children.
Decimal:	of the people are children
Percentage:	of the people are children
w sketches to matc	h what's below and write the two

3. Dra missing statements.

Fraction:	
Decimal:	0.6 of the cars are black.
Percentage:	

4. Draw sketches to match what's below and write the two missing statements.

Fraction:	
Decimal:	
Percentage:	90% of the circles are red.

5. Challenge: Think up your own idea, do the sketches, and fill in the numbers.

Fraction:	
Decimal:	
Percentage:	
	17,

6. Extra Challenge: Change  $^{1}/_{100}$  to

(a) a decimal

(b) a percentage

# Ordering

So, you know that fractions, decimals and percentages are all to do with <u>parts of a whole</u>. And maybe you also understand that

- any fraction may be changed to a decimal
- any fraction may be changed to a percentage
- any decimal may be changed to a fraction
- any decimal may be changed to a percentage
- any percentage may be changed to a fraction
- any percentage may be changed to a decimal

Knowing and understanding the six points above can be very useful. For example, if we wish to know which is the greater quantity out of  $^3/_{10}$ , 20% or 0.4 we simply make them

all fractions:  $\frac{3}{10}$ ,  $\frac{2}{10}$ ,  $\frac{4}{10}$  and we see that 0.4 is greater or all decimals: 0.3, 0.2, 0.4 and we see that 0.4 is greater all percentages: 30%, 20%, 40% and we see that 0.4 is greater

Test out your knowledge of this handy trick by completing the examples below:

#### working-out space

# Arrange these quantities in ascending order:

**1.** 80%, 0.3, 
$$\frac{1}{10}$$
, 70%, 0.5

**3.** 
$$0.7$$
,  $\frac{90}{100}$ , 7%, 10%, 0.03,  $\frac{9}{100}$ , 60%, 0.4

**4.** 12%, 14%, 0.09, 
$$\frac{1}{10}$$
, 16%, 0.17,  $\frac{15}{100}$ , 13%, 0.11

**5.** 48%, 0.5, 0.49, 
$$\frac{1}{50}$$
, 54%, 0.1,  $\frac{51}{100}$ , 0.6, 0.57

#### Arrange these quantities in descending order:

**6.** 65%, 0.07, 
$$^{7}/_{10}$$
, 9%, 0.8

**7.** 
$$^{2}/_{10}$$
, 30%, 19%, 0.4,  $^{7}/_{10}$ , 10%, 0.01

**8.** 
$$0.75$$
,  $^{73}/_{100}$ , 8%, 80%,  $0.06$ ,  $^{9}/_{100}$ , 50%,  $0.3$ 

**9.** 
$$0.2$$
,  $\frac{30}{100}$ , 2%, 40%,  $0.03$ ,  $\frac{33}{100}$ , 25%, 1.0

**10.** 72%, 59%, 0.9, 
$$\frac{8}{10}$$
, 89%, 0.48,  $\frac{78}{100}$ , 68%, 8%, 0.6

# Mixed Questions



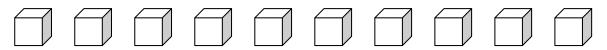




2. What % of the clocks do not get coloured	)
---	---

**3.** Put a ring around the fraction that's more than 0.2 but less than 30%.  $\frac{1}{4}$ 

**4.** Colour  $^{3}/_{10}$  of the cubes red and 40% of them yellow. **5.** What decimal of the cubes are <u>not</u> coloured? .....



**6.** 60% of the webs have a spider hanging from them. Draw in the spiders (one has been done for you).



**7.** 40% of the books below are children's books and the remainder are for adults. Colour the children's books blue and the adults' books red. **8.** What **fraction** of the books are for adults?

**9.** What **decimal** of the books are for children?



**10.** What **percentage** of the bicycles are middle-size ones?



- **11.** What **decimal** of the bicycles are small ones?
- **12.** What **fraction** of the bicycles are big ones?

**13.** In 2006 the weightlifter won 0.8 of the events he entered, the skier won 85% of her events, the golfer won  $^{7}/_{10}$  of his and the motorcyclist won  $^{3}/_{4}$  of his events.

Under each sportsperson put either 1, 2, 3 or 4 to show their success rate compared to the others.









**14.** There are 100 stars below. Colour 42% of them blue and 38% of them red.



#### Pages 4-6 teacher to check

#### Page 7

- 1 no 2 the parts are not equal to 1/4 because 1/4 means 1 part out of 4 equal parts
- g yes 4 the parts are all equal to  $^1/_4$  because each is 1 of 4 equal parts 5 teacher to check
- 5 2 7 8 8 teacher to check

# Page 8

- 1 no 2 the parts are not equal to 1/8 because 1/8 means 1 part out of 8 equal parts
- $\blacksquare$  yes  $\blacksquare$  the parts are all equal to 1/8 because each is 1 of 8 equal parts  $\blacksquare$  teacher to check
- 5 4 7 16 S teacher to check

#### Page 9

**1-4** teacher to check **5**  $\frac{1}{3} = \frac{2}{6} = \frac{4}{12}$  **6** teacher to check **7**  $\frac{8}{20} = \frac{4}{10} = \frac{2}{5}$ 

# Page 10

- 3 the green areas are the same size as each other; the red areas are the same size as each other
- $4^{4}/_{5}$ ,  $8/_{10}$  5 1, 8
- 2 the blue areas are the same size as each other; the yellow areas are the same size as each other
- **10**  $^{8}/_{20}$ ,  $^{4}/_{10}$ ,  $^{2}/_{5}$  **11** 6, 3 8,4

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- $2^{10}/_{15}$   $3^{4}/_{6}$   $4^{1}/_{3}$  5 the blue areas are the same size as each other areas are the same size as each other
- $8^{8}/_{10}$   $9^{4}/_{5}$  10 the red areas are the same size as each other 12 the green areas are the same size as each other

# Page 13

**1**  $\frac{1}{25}$  **2**  $\frac{7}{25}$  **3**  $\frac{12}{25}$  **4**  $\frac{5}{25}$  **5**  $\frac{1}{25}$  **6**  $\frac{1}{25} + \frac{7}{25} + \frac{12}{25} + \frac{5}{25} = \frac{25}{25}$  **7**  $\frac{25}{25}$ 

continued....

#### Page 14

Page 16 5 75% 6 
$$\frac{3}{4}$$
 7 25% =  $\frac{1}{4}$  75% =  $\frac{3}{4}$ 

Page 19 1 
$$\frac{3}{10}$$
 2 0.3 3  $\frac{30}{100}$  or  $\frac{3}{10}$  4 0.3 5 3, 100 6 7 7 20 8 9 9 10 10 50

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Fraction	Decimal	Percentage
<sup>13</sup> / <sub>20</sub>	0.65	65%
<sup>9</sup> / <sub>10</sub>	0.9	90%
$^{3}/_{20}$	0.15	15%
<sup>9</sup> / <sub>20</sub>	0.45	45%
<sup>3</sup> / <sub>4</sub>	0.75	75%
<sup>7</sup> / <sub>20</sub>	0.35	35%
<sup>1</sup> / <sub>20</sub>	0.05	5%
<sup>19</sup> / <sub>20</sub>	0.95	95%
1/.	0.25	25%

1.0

100%

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**1** yes **2** yes **3** yes **4** yes **5** yes **6** yes **7** 90% **8** 0.9 **9** 
$$\frac{9}{10}$$
 **10** 10% **11** 0.1 **12**  $\frac{1}{10}$ 

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#### Page 24

**1** 
$$^{1}/_{10}$$
, 0.3, 0.5, 70%, 80% **2** 0.2, 30%, 40%,  $^{6}/_{10}$ , 0.8,  $^{9}/_{10}$ , 100%

2

**3** 0.03, 7%, 
$$\frac{9}{100}$$
, 10%, 0.4, 60%, 0.7,  $\frac{90}{100}$  **4** 0.09,  $\frac{1}{10}$ , 0.11, 12%, 13%, 14%,  $\frac{15}{100}$ , 16%, 0.17

$$\mathbf{5}^{1}/_{50}$$
, 0.1, 48%, 0.49, 0.5,  $\mathbf{5}^{1}/_{100}$ , 54%, 0.57, 0.6  $\mathbf{6}^{1}$  0.8,  $\mathbf{7}/_{10}$ , 65%, 9%, 0.07

$$\mathbf{Z}^{7}/_{10}$$
, 0.4, 30%,  $^{2}/_{10}$ , 19%, 10%, 0.01  $\mathbf{E}$  80%, 0.75,  $^{73}/_{100}$ , 50%, 0.3,  $^{9}/_{100}$ , 8%, 0.06

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**2** 20% **3** 
$$\frac{1}{4}$$
 **5** 0.3 **8**  $\frac{6}{10}$  or  $\frac{3}{5}$  **9** 0.4 **10** 40% **11** 0.4 **12**  $\frac{2}{10}$  or  $\frac{1}{5}$  **13** 2-1-4-3 **14** 0.8 **15**  $\frac{1}{5}$