# How to Read Standard Normal Table

This handout will help you to learn how to find probabilities and percentiles when working with the standard normal table. It is not a required reading, but it might help you to acquire necessary skills when solving probability questions.

Look at the standard normal distribution table (I use only the fragment of it below). What does the number 0.3238 represent? It represents the area under the standard normal

| z | 0.00 | 0.01 | 0.02 | 0.03 | 0.04 | 0.05 |
|---|---|---|---|---|---|---|
| 0.8 | 0.2881 | 0.2910 | 0.2939 | 0.2967 | 0.2995 | 0.3023 |
| 0.9 | 0.3159 | 0.3186 | 0.3212 | 0.3238 | 0.3264 | 0.3289 |
| 1.0 | 0.3413 | 0.3438 | 0.3461 | 0.3485 | 0.3508 | 0.8531 |

curve above 0 and to the left of $Z$. Referring to the associated row and column, we'll find that $P(0 < Z < 0.93) = 0.3238$. The number 0.3238 represents the area under the standard normal curve above 0 and to the left of 0.93.

In your homeworks and tests you will encounter two types of questions related to the normal distribution.

## Type 1

First type of questions ask us to find *probability* that a certain event will or will not happen. In statistical notation, it can be written as:

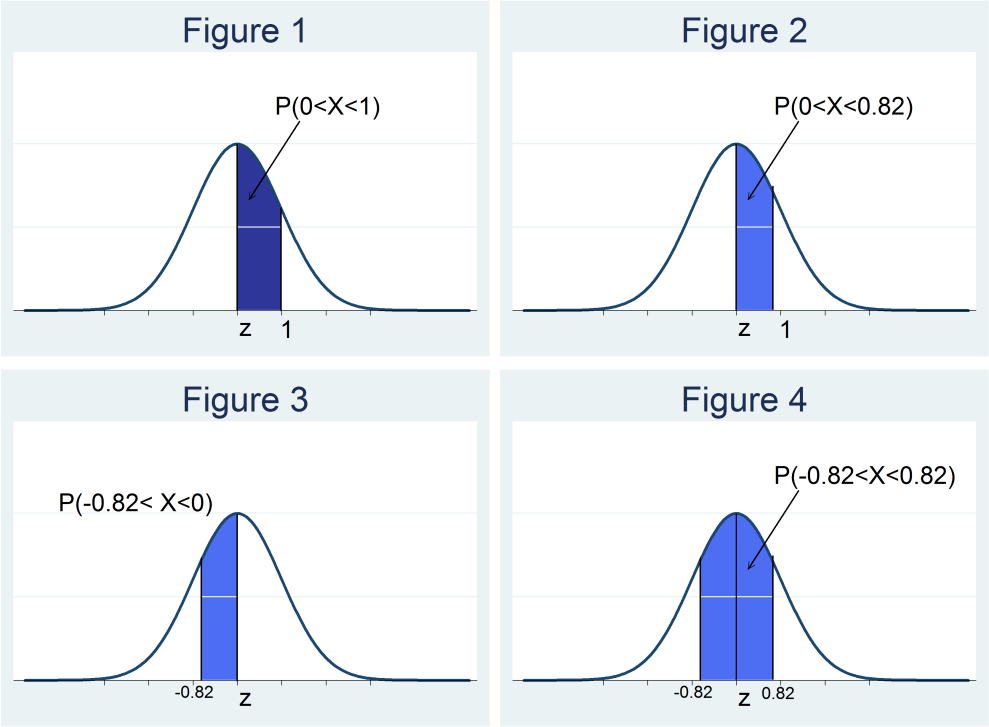
$$P(X \leq x) = ? \tag{1}$$

$$P(X \geq x) = ? \tag{2}$$

$$P(x_1 \leq X \leq x_2) = ? \tag{3}$$

In all three cases the answers can be found *inside* the standard normal table, not in the headings.

Below are several examples to help you clarify the concepts:



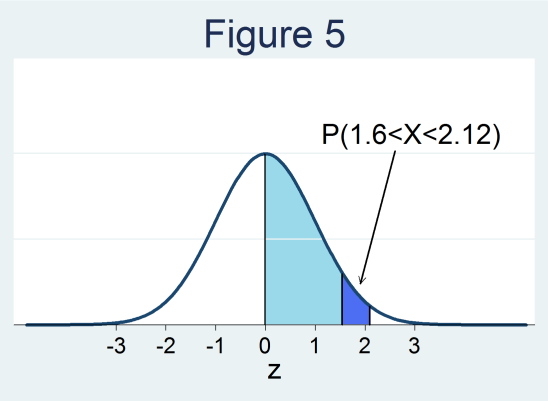

*Refer to Figures 1 - 4*

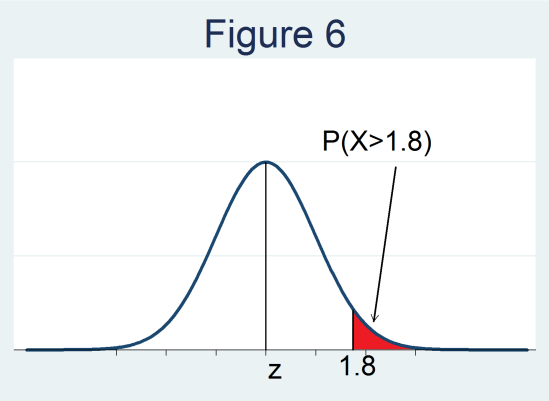
*Example 1: Find probability that $Z$ is between 0 (mean) and 1, i.e. $P(0 < Z < 1)$.* You can read directly the probability from the table. Look at the intersection of row with 1.0 and column with 0.00 decimals. The value at the intersection represents $P(0 < Z < 1) = 0.3413$.

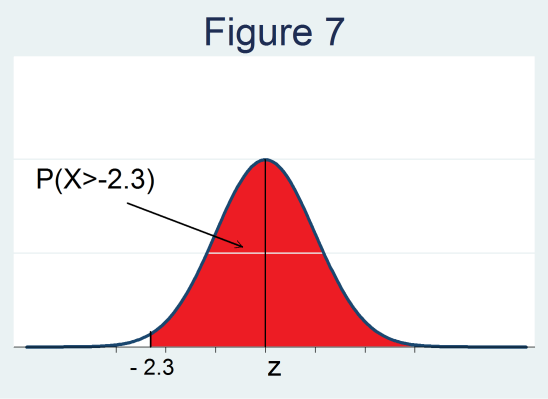
*Example 2: Find probability that $Z$ is between 0 and 0.82 or $P(0 < Z < 0.82)$.* Again, we can read the value directly from the table: look up for the intersection of column 0.02 and row 0.8. The probability at the intersection is 0.2939.

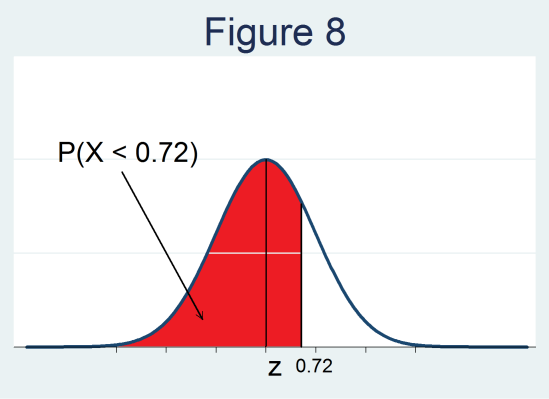
*Example 3: Find probability that $Z$ lies between -0.82 and 0, or $P(-0.82 < Z < 0)$.* We already know that $P(0 < Z < 0.82) = 0.2939$. Since the standard normal distribution is symmetric around its mean, $P(-0.82 < Z < 0) = P(0 < Z < 0.82) = 0.2939$

*Example 4: Find probability that Z is between -0.82 and 0.82.* This is simply two times the area we have found in the previous example (think symmetry!). $P(-0.82 < Z < 0.82) = 2 * P(-0.82 < Z < 0) = 2 * 0.2939 = 0.5878$.




*Refer to Figures 5 - 8*

*Example 5: Find probability that randomly selected variable Z is between 1.6 and 2.12 or* $P(1.6 < Z < 2.12)$. This probability is not given directly by the table. However, the darker shaded area is simple a difference between $P(0 < Z < 2.12)$ (larger area) and $P(0 < Z < 1.6)$ (smaller area). Looking up in the table we find that $P(0 < Z < 1.6) = 0.4452$ and $P(0 < Z < 2.12) = 0.4830$. Therefore, $P(1.6 < Z < 2.12) = P(Z < 2.12) - P(Z < 1.6) = 0.4830 - 0.4452 = 0.0378$.

*Example 6: Find probability that Z is greater than 1.8, or* $P(Z > 1.8)$. Note that now

we are looking for the area to the *right* of 1.8. This is not given directly by our table, but we can figure it out: $P(Z > 1.8) = 1 - P(Z < 1.8) = 0.5 - 0.4641 = 0.0359$.

*Example 7: Find probability that randomly selected Z is greater than -2.3, or* $P(-2.3 < X < \infty)$. Again, such probability is not given by our table directly, but we can figure it out: $P(-2.3 < Z < \infty) = P(-2.3 < Z < 0) + P(0 < Z < \infty)$. By symmetry, $P(-2.3 < Z < 0) = P(0 < Z < 2.3) = 0.4893$ and $P(0 < Z < \infty) = 0.5$ because it is the total area to the right of the mean. Therefore, $P(-2.3 < Z < \infty) = 0.4893 + 0.5 = 0.9893$.

*Example 8: Find probability that Z is below 0.72, or* $P(-\infty < Z < 0.72)$. This is not given directly by our table but we know that $P(-\infty < Z < 0) = 0.5$ and from the table $P(0 < Z < 0.72) = 0.2642$. So, total probability equals to 0.5+0.2642=0.7642.

# Type 2

What I refer to as Type 2 questions are the problems when you are asked to find $z_A$ rather than probability. In other words, you need to find a *number* on the real line rather than the *area* under the curve as in the above examples. In math notation it is:

$$P(Z \leq ?) = (\text{number from 0 to 1}) \tag{4}$$

or

$$P(Z \geq ?) = (\text{number from 0 to 1}) \tag{5}$$

To gain some intuition, ask yourself: what is the 50th percentile of the standard normal distribution? Using math notation: $P(Z \leq ?) = 0.5$. In other words, we need to find the value of $z_A$ below which lies the area of 50%. (*Answer:* $z_A = 0$).

*Example 9: Find the 82nd percentile of the standard normal distribution.* Write it first in

math notation: $P(Z \leq ?) = 0.82$. The ? is what is asked for. First, let's decide whether 82% lies below or above the mean? We already know that 50th percentile (median) is 0 (mean of the standard normal distribution). 82% percentile is therefore above, or to the right of the mean. Our table represents values only above the mean, so we should add 0.5 to each value inside the table to solve for percentiles (make sure you understand why). Looking inside the table, we find that $P(Z < 0.915) = 0.82$. How did I get that? Look at the fragment of the table on the first page. Find the values of 0.3186 and 0.3212 *inside* the table. These represent the areas from 0 to $z$, but since I am looking for a percentile, I understand that these are the same as 0.8186 and 0.8212 (Table does not have value of exactly 0.32, so we have to use the average). Now look at the values of $z$ associated with these probabilities - go left along the row to find first decimal - 0.9, and go up to find second decimal across the top - 0.01 and 0.02 respectively. Use the average of these two values - (0.91+0.92)/2=0.915.

*Example 10: Find the bottom 5th percentile or* $P(Z \leq ?) = 0.05$. Before using the table, let's decide what is the location of 5th percentile - below 0 or above 0? Bottom 5th percentile would be a negative number since we know that 50th percentile or median is equal to 0, and 5th percentile lies below the mean. Our table does not have negative values, so we have to use the symmetry of the distribution and find the top 5th percentile. By symmetry, the top Pth percentile is equal to the bottom Pth percentile with the opposite sign. Top 5th percentile divides the distribution into 95 percent above and 5 percent below. Again, we are dealing with the part of the table which only contains half of the probabilities. The top 5th percentile divide this upper half into 45 percent below and 5 percent above. We should look for the value of 0.45 *inside* the table. We do not have the exact value of 0.45, but we have 0.4495 and 0.4505, so we are going to use the average. Looking for the row number, we find that it is 1.6, and looking for the decimals on top, we find that it is the average of 0.04 and 0.05 which is 0.045. The top 5th percentile of the standard normal distribution is therefore 1.645, and the bottom 5th percentile is -1.645.($z_{0.05} = -1.645$)

*Example 11: What is the 99th percentile?* We need to find ? in $P(Z \leq ?) = 0.99$. $99^{th}$ will be a positive number and we can find it directly from our table. Find the value 0.49 inside the table (because 0.99=0.5+0.49, but out table gives the values above 0). We can approximate and use 0.4901 instead, which corresponds to z=2.33. $P(Z \leq 2.33) = 0.99$. So, $99^{th}$ percentile is 2.33.

To summarize, below are the steps you should follow to find probabilities/percentiles:

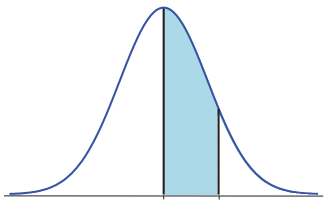
1. Write what is asked for as a mathematical expression (i.e. $P(X \leq x) = ?$ or $P(X \leq ?) = 0.65$)

2. Determine whether the distribution in question is normal (mean is <u>not</u> 0 and variance is <u>not</u> 1) or <u>standard</u> normal (mean 0 and variance 1).

3. If distribution is <u>not</u> standard normal, then standardize it. Do not forget to standardize both parts of your expression (i.e $P(\frac{X-\mu}{\sigma} \leq \frac{x-\mu}{\sigma}) = ?$ or $P(\frac{X-\mu}{\sigma} \leq \frac{?-\mu}{\sigma}) = 0.65$).

4. Find the value of $z = \frac{x-\mu}{\sigma}$ in the table and find corresponding probability (area under the density curve).

5. For type 2 questions, find probability *inside* the table and find corresponding value of $z$. Solve equation $z = \frac{x-\mu}{\sigma}$ for $z$.




**Normal Probabilities:**

| z | 0.00 | 0.01 | 0.02 | 0.03 | 0.04 | 0.05 | 0.06 | 0.07 | 0.08 | 0.09 |
|---|---|---|---|---|---|---|---|---|---|---|
| 0.0 | 0.0000 | 0.0040 | 0.0080 | 0.0120 | 0.0160 | 0.0199 | 0.0239 | 0.0279 | 0.0319 | 0.0359 |
| 0.1 | 0.0398 | 0.0438 | 0.0478 | 0.0517 | 0.0557 | 0.0596 | 0.0636 | 0.0675 | 0.0714 | 0.0753 |
| 0.2 | 0.0793 | 0.0832 | 0.0871 | 0.0910 | 0.0948 | 0.0987 | 0.1026 | 0.1064 | 0.1103 | 0.1141 |
| 0.3 | 0.1179 | 0.1217 | 0.1255 | 0.1293 | 0.1331 | 0.1368 | 0.1406 | 0.1443 | 0.1480 | 0.1517 |
| 0.4 | 0.1554 | 0.1591 | 0.1628 | 0.1664 | 0.1700 | 0.1736 | 0.1772 | 0.1808 | 0.1844 | 0.1879 |
| 0.5 | 0.1915 | 0.1950 | 0.1985 | 0.2019 | 0.2054 | 0.2088 | 0.2123 | 0.2157 | 0.2190 | 0.2224 |
| 0.6 | 0.2257 | 0.2291 | 0.2324 | 0.2357 | 0.2389 | 0.2422 | 0.2454 | 0.2486 | 0.2517 | 0.2549 |
| 0.7 | 0.2580 | 0.2611 | 0.2642 | 0.2673 | 0.2704 | 0.2734 | 0.2764 | 0.2794 | 0.2823 | 0.2852 |
| 0.8 | 0.2881 | 0.2910 | 0.2939 | 0.2967 | 0.2995 | 0.3023 | 0.3051 | 0.3078 | 0.3106 | 0.3133 |
| 0.9 | 0.3159 | 0.3186 | 0.3212 | 0.3238 | 0.3264 | 0.3289 | 0.3315 | 0.3340 | 0.3365 | 0.3389 |
| 1.0 | 0.3413 | 0.3438 | 0.3461 | 0.3485 | 0.3508 | 0.3531 | 0.3554 | 0.3577 | 0.3599 | 0.3621 |
| 1.1 | 0.3643 | 0.3665 | 0.3686 | 0.3708 | 0.3729 | 0.3749 | 0.3770 | 0.3790 | 0.3810 | 0.3830 |
| 1.2 | 0.3849 | 0.3869 | 0.3888 | 0.3907 | 0.3925 | 0.3944 | 0.3962 | 0.3980 | 0.3997 | 0.4015 |
| 1.3 | 0.4032 | 0.4049 | 0.4066 | 0.4082 | 0.4099 | 0.4115 | 0.4131 | 0.4147 | 0.4162 | 0.4177 |
| 1.4 | 0.4192 | 0.4207 | 0.4222 | 0.4236 | 0.4251 | 0.4265 | 0.4279 | 0.4292 | 0.4306 | 0.4319 |
| 1.5 | 0.4332 | 0.4345 | 0.4357 | 0.4370 | 0.4382 | 0.4394 | 0.4406 | 0.4418 | 0.4429 | 0.4441 |
| 1.6 | 0.4452 | 0.4463 | 0.4474 | 0.4484 | 0.4495 | 0.4505 | 0.4515 | 0.4525 | 0.4535 | 0.4545 |
| 1.7 | 0.4554 | 0.4564 | 0.4573 | 0.4582 | 0.4591 | 0.4599 | 0.4608 | 0.4616 | 0.4625 | 0.4633 |
| 1.8 | 0.4641 | 0.4649 | 0.4656 | 0.4664 | 0.4671 | 0.4678 | 0.4686 | 0.4693 | 0.4699 | 0.4706 |
| 1.9 | 0.4713 | 0.4719 | 0.4726 | 0.4732 | 0.4738 | 0.4744 | 0.4750 | 0.4756 | 0.4761 | 0.4767 |
| 2.0 | 0.4772 | 0.4778 | 0.4783 | 0.4788 | 0.4793 | 0.4798 | 0.4803 | 0.4808 | 0.4812 | 0.4817 |
| 2.1 | 0.4821 | 0.4826 | 0.4830 | 0.4834 | 0.4838 | 0.4842 | 0.4846 | 0.4850 | 0.4854 | 0.4857 |
| 2.2 | 0.4861 | 0.4864 | 0.4868 | 0.4871 | 0.4875 | 0.4878 | 0.4881 | 0.4884 | 0.4887 | 0.4890 |
| 2.3 | 0.4893 | 0.4896 | 0.4898 | 0.4901 | 0.4904 | 0.4906 | 0.4909 | 0.4911 | 0.4913 | 0.4916 |
| 2.4 | 0.4918 | 0.4920 | 0.4922 | 0.4925 | 0.4927 | 0.4929 | 0.4931 | 0.4932 | 0.4934 | 0.4936 |
| 2.5 | 0.4938 | 0.4940 | 0.4941 | 0.4943 | 0.4945 | 0.4946 | 0.4948 | 0.4949 | 0.4951 | 0.4952 |
| 2.6 | 0.4953 | 0.4955 | 0.4956 | 0.4957 | 0.4959 | 0.4960 | 0.4961 | 0.4962 | 0.4963 | 0.4964 |
| 2.7 | 0.4965 | 0.4966 | 0.4967 | 0.4968 | 0.4969 | 0.4970 | 0.4971 | 0.4972 | 0.4973 | 0.4974 |
| 2.8 | 0.4974 | 0.4975 | 0.4976 | 0.4977 | 0.4977 | 0.4978 | 0.4979 | 0.4979 | 0.4980 | 0.4981 |
| 2.9 | 0.4981 | 0.4982 | 0.4982 | 0.4983 | 0.4984 | 0.4984 | 0.4985 | 0.4985 | 0.4986 | 0.4986 |
| 3.0 | 0.4987 | 0.4987 | 0.4987 | 0.4988 | 0.4988 | 0.4989 | 0.4989 | 0.4989 | 0.4990 | 0.4990 |