HIGLEY UNIFIED SCHOOL DISTRICT
INSTRUCTIONAL ALIGNMENT

HS Algebra I Semester 1 (Quarter 1)

Unit 1: Relationships Between Quantities and Reasoning with Equations and Their Graphs (40 days)
Topic A: Graphing Stories – Introduction to Functions (Optional)
(The concepts in this topic could be incorporated as you study functions throughout the course.)

By the end of eighth grade, students have learned to solve linear equations in one variable and have applied graphical and algebraic methods to analyze and solve systems of linear equations in two variables. Now, students analyze and explain precisely the process of solving an equation. Students, through reasoning, develop fluency writing, interpreting, and translating between various forms of linear equations and inequalities, and make conjectures about the form that a linear equation might take in a solution to a problem. They reason abstractly and quantitatively by choosing and interpreting units in the context of creating equations in two variables to represent relationships between quantities. They master the solution of linear equations and apply related solution techniques and the properties of exponents to the creation and solution of simple exponential equations.

In this unit, students solidify their previous work with functional relationships as they begin to formalize the concept of a mathematical function. This unit provides an opportunity for students to reinforce their understanding of the various representations of a functional relationship—words, concrete elements, numbers, graphs, and algebraic expressions. Students review the distinction between independent and dependent variables in a functional relationship and connect those to the domain and range of a function. The standards listed here will be revisited multiple times throughout the course, as students encounter new function families.

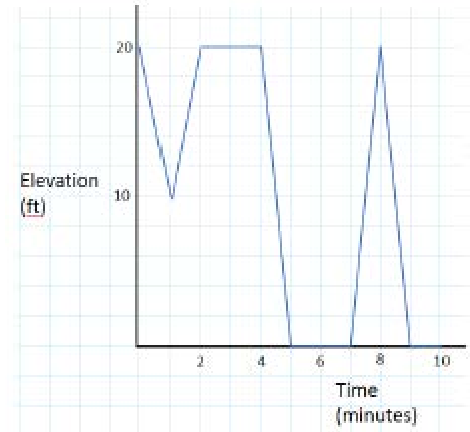
Big Idea:	• Units and quantities define the parameters of a given situation and are used to solve problems. • The different parts of expressions, equations and inequalities can represent certain values in the context of a situation and help determine a solution process. • Relationships between quantities can be represented symbolically, numerically, graphically, and verbally in the exploration of real world situations.
Essential Questions:	• When is it advantageous to represent relationships between quantities symbolically? numerically? graphically? • What is the relationship between physical measurements and representations on a graph? • How are appropriate quantities from a situation (a "graphing story") defined? • How is the scale and origin for a graph chosen and interpreted?
Vocabulary	Piecewise-linear function, intersection point
Assessments	Galileo: Algebra I Module 1 Foundational Skills Assessment; Live Binders/Galileo: Topic A Assessment

Standard	AZ College and Career Readiness Standards	Explanations & Examples	Resources
N.Q.A.1	**A. Reason qualitatively and units to solve problems** Use units as a way to understand problems and to	**Explanation:** Include word problems where quantities are given in different units, which must be converted to make sense of the problem. Graphical representations and data displays include, but are not limited to: line	**Eureka Math**: Module 1 Lesson 1-4 **Other:**

	guide the solution of multi-step problems; choose and interpret units consistently in formulas; choose and interpret the scale and the origin in graphs and data displays. ***This is a modeling standard which means students choose and use appropriate mathematics to analyze situations. Thus, contextual situations that require students to determine the correct mathematical model and use the model to solve problems are essential.***	graphs, circle graphs, histograms, multi-line graphs, scatterplots, and multi-bar graphs. **<u>Use units as a way to understand problems and to guide the solution of multi-step problems</u>** Students use the units of a problem to identify what the problem is asking. They recognize the information units provide about the quantities in context and use units as a tool to help solve multi-step problems. Students analyze units to determine which operations to use when solving a problem. **Examples:** • For example, a problem might have an object moving 12 feet per second and another at 5 miles per hour. To compare speeds, students convert 12 feet per second to miles per hour: $$\frac{12\,ft}{\sec}\bullet\frac{60\sec}{\min}\bullet\frac{60\min}{hr}\bullet\frac{1mi}{5280\,ft}\approx\frac{8.182mi}{hr}$$ which is more than 5 miles per hour. • **Maya and Earl live at opposite ends of the hallway in their apartment building. Their doors are 50 feet apart. They each start at their door and walk at a steady pace towards each other and stop when they meet.** **Suppose that:** ▪ **Maya walks at a constant rate of 3 feet every second.** ▪ **Earl walks at a constant rate of 4 feet every second.** **Graph both people's distance from Maya's door versus time in seconds.** ***Graphs should be scaled and labeled appropriately. Maya's graph should start at $(0,)$ and have a slope of 3, and Earl's graph should start at $(0,)$ and have a slope of -4.*** What do you think the numbers along the horizontal axis represent? What might the numbers along the vertical axis represent? Do we have any indication of the units being used? • Given the speed in *mph* and the time traveled in *hours*, what	MAP – [interpreting distance-time graphs] Gizmos – [Distance-Time Graphs] **This standard is revisited in Topic D.**

		is the distance traveled? o From looking at the units, we can determine that we must multiply *mph* times *hours* to get an answer expressed in miles: $\left(\frac{mi}{hr}\right)(hr) = mi$ **(Note that knowledge of the distance formula is not required to determine the need to multiply in this case.)** **<u>Choose and interpret units consistently in formulas</u>** Students choose the units that accurately describe what is being measured. Students understand the familiar measurements such as length (unit), area (unit squares) and volume (unit cubes). They use the structure of formulas and the context to interpret units less familiar. **Example:** • If $density = \frac{mass\ in\ grams}{volume\ in\ mL}$ then the unit for density is $\frac{grams}{mL}$ **<u>Choose and interpret the scale and the origin in graphs and data displays</u>** When given a graph or data display, students read and interpret the scale and origin. When creating a graph or data display, students choose a scale that is appropriate for viewing the features of a graph or data display. Students understand that using larger values for the tick marks on the scale effectively "zooms out" form the graph and choosing smaller values "zooms in". Students also understand that the viewing window does not necessarily show the x- or y-axis, but the apparent axes are parallel to the x- and y-axes. Hence, the intersection of the apparent axes in the viewing window may not be the origin. They are also aware that apparent intercepts may not correspond to the actual x- or y-intercepts of the graph of a function.	
N.Q.A.2 ●	**A.Reason qualitatively and units to solve problems** Define appropriate quantities for the purpose of	**Explanation:** Determine and interpret appropriate quantities when using descriptive modeling. For example, if you want to describe how dangerous the roads are, you may choose to report the number of accidents per year	**Eureka Math**: Module 1 Lesson 1-4 **Other:**

	descriptive modeling. ***This is a modeling standard which means students choose and use appropriate mathematics to analyze situations. Thus, contextual situations that require students to determine the correct mathematical model and use the model to solve problems are essential.***	on a particular stretch of interstate. Generally speaking, it would not be appropriate to report the number of exits on that stretch of interstate to describe the level of danger. This standard is taught in Algebra I and Algebra II. In Algebra I, the standard will be assessed by ensuring that some modeling tasks (involving Algebra I content or securely held content from grades 6-8) require the student to create a quantity of interest in the situation being described. **Examples:** • What type of measurements would one use to determine their income and expenses for one month? • How could one express the number of accidents in Arizona? • What quantities could you use to describe a safe bungee jump apparatus? If you were to build an in-classroom bungee jump apparatus, what units would be best to use for your measurements? Explain. If you were to build a real bungee jump apparatus, what units would it be best to use for your measurements? Explain. How can you relate the model from the classroom to the real life bungee jump? • ***Darryl lives on the third floor of his apartment building. His bike is locked up outside on the ground floor. At 3:00 p.m., he leaves to go run errands, but as he is walking down the stairs, he realizes he forgot his wallet. He goes back up the stairs to get it and then leaves again. As he tries to unlock his bike, he realizes that he forgot his keys. One last time, he goes back up the stairs to get his keys. He then unlocks his bike, and he is on his way at 3:10 p.m.*** **Sketch a graph that depicts Darryl's change in elevation over time.** ***The graph students produce should appear as follows:***	MAP – interpreting distance-time graphs Gizmos – Distance-Time Graphs **This standard is revisited 2nd semester in Unit 5.**

		 • **A ramp is made in the shape of a right triangle using the dimensions described in the picture below. The ramp length is 10 feet from the top of the ramp to the bottom, and the horizontal width of the ramp is 9.25 feet. A ball is released at the top of the ramp and takes 1.6 seconds to roll from the top of the ramp to the bottom. Find each answer below to the nearest 0.1 feet/sec.** **a. Find the average speed of the ball over the 1.6 seconds.** 10/1.6 ***ft./sec. or*** 6.3 ***ft./sec.*** **b. Find the average rate of horizontal change of the ball over the 1.6 seconds.** 9.251.6 ***ft./sec. or*** 5.8 ***ft./sec.***	
N.Q.A.3	**A.Reason qualitatively and units to solve problems** Choose a level of accuracy appropriate to limitations on measurement when reporting quantities. ***This is a modeling standard which means students choose and use appropriate mathematics to analyze situations. Thus, contextual situations that require students to determine the correct mathematical model and use the model to solve problems are essential.***	**Explanation:** The margin of error and tolerance limit varies according to the measure, tool used, and context. **Students understand the tool used determines the level of accuracy that can be reported for a measurement.** **Examples:** • When using a ruler, one can legitimately report accuracy to the nearest division. If a ruler has centimeter divisions, then when measuring the length of a pencil the reported length must be to the nearest centimeter, or	**Eureka Math**: Module 1 Lesson 1-4 **Other:** MAP – interpreting distance-time graphs Gizmos – Distance-Time Graphs

		• In situations where units constitute a whole value, as the case with people, an answer of 1.5 people would reflect a level of accuracy to the nearest whole base on the fact that the limitation is based on the context. **<u>Students use the measurements provided within a problem to determine the level of accuracy.</u>** Students recognize the effect of rounding calculations throughout the process of solving problems and complete calculation with the highest degree of accuracy possible, <u>reserving rounding until reporting the final quantity.</u> **Examples:** • If lengths of a rectangle are given to the nearest tenth of a centimeter then calculated measurements should be reported to no more than the nearest tenth. • Jason is collecting data on the rate of water usage in the tallest skyscraper in the world during a typical day. The skyscraper contains both apartments and businesses. The electronic water meter for the building displays the total amount of water used in liters. At noon, Jason looks at the water meter and notes that the digit in the **ones** place on the water meter display changes too rapidly to read the digit and that the digit in the **tens** place changes every second or so. a. Estimate the total number of liters used in the building during one 24-hour day. Take into account the time of day when he made his observation. (Hint: Will water be used at the same rate at 2:00 a.m. as at noon?). Explain how you arrived at your estimate. $10 \frac{\text{liters}}{\text{sec}} \times 60 \frac{\text{sec}}{\text{min}} \times 60 \frac{\text{min}}{\text{hr}} \times 18\text{ hr} = 648{,}000$ liters.	**This standard is revisited 2nd semester in Unit 5.**

		Since water is probably only used from about 5:00 am to 11:00 pm, I did not multiply by 24 hours, but by 18 hours instead. • Determining price of gas by estimating to the nearest cent is appropriate because you will not pay in fractions of a cent but the cost of gas is $\frac{\$3.479}{gallon}$.	

A.CED.A.2	**A. Create equations that describe numbers or relationships**
	
	Create equations in two or more variables to represent relationships between quantities; graph equations on coordinate axes with labels and scales.
	
	This is a modeling standard which means students choose and use appropriate mathematics to analyze situations. Thus, contextual situations that require students to determine the correct mathematical model and use the model to solve problems are essential.
		Explanation:
		This standard is taught in Algebra I and Algebra II. In Algebra I, students create equations in two variables for linear, exponential and quadratic contextual situations. Limit exponential situations to only ones involving integer input values.
		
		Linear equations can be written in a multitude of ways; $y=mx+b$ and $ax+by=c$ are commonly used forms (given that x and y are the two variables). Students should be flexible in using multiple forms and recognizing from the context, which is appropriate to use in creating the equation.
		
		Examples:
		• The FFA had a fundraiser by selling hot dogs for \$1.50 and drinks for \$2.00. Their total sales were \$400.
		  o Write an equation to calculate the total of \$400 based on the hot dog and drink sales.
		  o Graph the relationship between hot dog sales and drink sales.
		• A spring with an initial length of 25cm will compress 0.5cm for each pound applied.
		  o Write an equation to model the relationship between the amount of weight applied and the length of the
			Eureka Math:
			Module 1 Lesson 5
			
			Big Ideas:
			Page 120 #52
			Page 124, ex 5
			Page 142 #35
			Page 144 # 50
			
			Other:
			Gizmos – Distance-Time Graphs
			
			This standard is revisited in Topic D and in Unit 2 (Exponential), Unit 3 (Quadratics) and Unit 5.

		spring. o Graph the relationship between pounds and length. o What does the graph reveal about limitation on weight? • ***Duke starts at the base of a ramp and walks up it at a constant rate. His elevation increases by three feet every second. Just as Duke starts walking up the ramp, Shirley starts at the top of the same 25 foot high ramp and begins walking down the ramp at a constant rate. Her elevation decreases two feet every second.*** **Write down the equation of the line that represents Duke's motion as he moves up the ramp and the equation of the line that represents Shirley's motion as she moves down. Show that the coordinates of the point you found in question above satisfy both equations.** If ***y represents elevation in feet and t represents time in seconds, then Duke's elevation satisfies $y=3t$ and Shirley's $y=25-2t$. The lines intersect at $(5,)$, and this point does indeed lie on both lines.*** ***Duke: $15=3(5)$*** ***Shirley: $15=25-2(5)$*** • **Maya and Earl live at opposite ends of the hallway in their apartment building. Their doors are 50 feet apart. They each start at their door and walk at a steady pace towards each other and stop when they meet. Suppose that:** ▪ **Maya walks at a constant rate of 3 feet every second.** ▪ **Earl walks at a constant rate of 4 feet every second.** **Create equations for each person's distance from Maya's door and determine exactly when they meet in the hallway. How far are they from Maya's door at this time?** ***Maya's Equation: $y=3t$.*** ***Earl's Equation: $y=50-4t$.*** ***Solving the equation $3t=50-4t$, gives the solution: $t=7\ 1/7$. The two meet at exactly this time at a distance of $3(71/7)=21\ 3/7$ feet from Maya's door.***	
MP.1	**Make sense of problems and persevere in solving them.**	Students are presented with problems that require them to try special cases and simpler forms of the original problem in order to gain insight into the problem.	**Eureka Math**: Module 1 Lesson 1,2,4,5

MP.2	**Reason abstractly and quantitatively.**	Students analyze graphs of non-constant rate measurements and reason from the shape of the graphs to infer what quantities are being displayed and consider possible units to represent those quantities.	**Eureka Math**: Module 1 Lesson 4
MP.3	**Construct viable arguments and critique the reasoning of others.**	Students reason about water usage at the school; they construct arguments based on finding intersection points graphically and algebraically.	**Eureka Math**: Module 1 Lesson 1,4,5
MP.4	**Model with mathematics.**	Students have numerous opportunities in this module to solve problems arising in everyday life, society, and the workplace from modeling bacteria growth to understanding the federal progressive income tax system.	**Eureka Math**: Module 1 Lesson 2,3
MP.6	**Attend to precision.**	Students formalize descriptions of what they learned before (variables, solution sets, numerical expressions, algebraic expressions, etc.) as they build equivalent expressions and solve equations. Students analyze solution sets of equations to determine processes (like squaring both sides of an equation) that might lead to a solution set that differs from that of the original equation.	**Eureka Math**: Module 1 Lesson 1,3

HS Algebra I Semester 1 (Quarter 1)			
Unit 1: Relationships Between Quantities and Reasoning with Equations and Their Graphs (40 days) **Topic B: The Structure of Expressions** Students develop a precise understanding of what it means for expressions to be algebraically equivalent. By exploring geometric representations of the distributive, associative, and commutative properties for positive whole numbers and variable expressions assumed to represent positive whole numbers, students confirm their understanding of these properties and expand them to apply to all real numbers. Students use the properties to generate equivalent expressions and formalize that two algebraic expressions are equivalent if we can convert one expression into the other by repeatedly applying the commutative, associative and distributive properties, and the properties of rational exponents to components of the first expression. Students learn to relate polynomials to integers written in base x, rather than our traditional base of 10. The analogies between the system of integers and the system of polynomials continue as they learn to add, subtract, and multiply polynomials and to find that the polynomials for a system that is closed under those operations (e.g., a polynomial added to, subtracted from, or multiplied by another polynomial) always produces another polynomial.			
Big Idea:	• The different parts of expressions, equations and inequalities can represent certain values in the context of a situation and help determine a solution process. • Relationships between quantities can be represented symbolically, numerically, graphically, and verbally in the exploration of real world situations. • Rules of arithmetic and algebra can be used together with notions of equivalence to transform equations and inequalities. • Equivalent forms of an expression can be found, dependent on how the expression is used. • The Commutative and Associative Properties represent key beliefs about the arithmetic of real numbers. These properties can be applied to algebraic expressions using variables that represent real numbers. • Two algebraic expressions are *equivalent* if we can convert one expression into the other by repeatedly applying the Commutative, Associative, and Distributive Properties and the properties of rational exponents to components of the first expression.		
Essential Questions:	• Why are the commutative, associative, and distributive properties so important in mathematics? • How are polynomials analogous to integers? • If you add two polynomials together, is the result sure to be another polynomial? The difference of two polynomials? • Is the product of two polynomials sure to be another polynomial? • Is a polynomial squared sure to be another polynomial (other integer powers)?		
Vocabulary	numerical symbol, variable symbol, numerical expression, algebraic expression, equivalent numerical expressions, equivalent algebraic expressions, polynomial expression, monomial, degree of a monomial, degree of a polynomial, polynomial, leading term, leading coefficient, constant term, standard form		
Assessments	Live Binders/Galileo: Topic B Assessment		
Standard	**AZ College and Career Readiness Standards**	**Explanations & Examples**	**Resources**

A.SSE.A.2

A. Interpret the structure of expressions

Use the structure of an expression to identify ways to rewrite it. ~~*For example, see*~~ ~~$x^4 - y^4$~~ ~~*as*~~ ~~$(x^2)^2 - (y^2)^2$~~~~*, thus recognizing it as a difference of squares that can be factored as*~~ ~~$(x^2 - y^2)(x^2 + y^2)$~~~~.~~

Explanation:

This standard is taught in Algebra I and Algebra II. In Algebra I tasks are limited to numerical and polynomial expressions in one variable, with a focus on quadratics.

In this unit, only focus on rewriting algebraic expressions in different equivalent forms by combining like terms and using the associative, commutative and distributive properties. In unit 3, you will focus on rewriting algebraic expressions by factoring.

Examples:

- Challenge the students to come up with more than one way to create the number 21:

Value of Expression	Expression (using 1, 2, 3, 4, +, and x)
1	1
2	2
3	3
4	1+3
5	2+3
6	1+2+3
7	2x3 + 1 or 3+4
8	(3+1)x2 or 4x2
9	3(2+1)
10	2x(4+1)
11	4x2 + 3
12	4X3
13	4x3 +1
14	4x3 + 2
15	(4+1)x3

16	(4+3+1)x2
17	3x(4+1)+2
18	(1+3)x4 +2
19	(4+2) x 3 +1
20	(2+3) x 4
21	(3+4)x(1+2)
22	
23	
24	(1+2+3) x4
25	(2+3)(4+1)
30	((4+1)x3)x2
32	4x (3+1) x 2
36	3x(2+1)x4

$$110 = 10 \cdot 10 + 10 = (1+2+3+4) \cdot (1+2+3+4) + (1+2+3+4)$$

- Draw pictures to represent expressions (x + y + 3) X (y + 1).

Eureka Math:
Module 1 Lesson 7,8

Big Ideas:
Pg. 403 (using distributive property only)

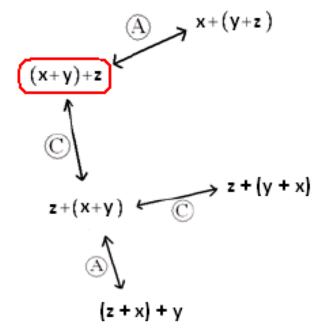
This standard is revisited 2nd semester in Unit 3 (Quadratics).

	x	y	3
y			
1			

Write equivalent expressions by applying the distributive property.

$$(x + y + 3)(y + 1) = xy + x + y^2 + 4y + 3$$

- Rewriting expressions using associative and commutative properties:



- The process of making use of the distributive property "backwards" is factoring.

  Ex. The expression $10x^2+6x^3$ is the result of applying the distributive property to the expression $2x^2(5x^2 + 3x^3)$ or to $x(10x + 6x^2)$; even to $1(10x^2+6x^3)$! (**<u>YOU ARE ARE NOT TEACHING THE FACTORING STEPS</u>**!)

- Replace each of the following expressions with an equivalent expression in which the variable of the expression only appears once with a positive number for its exponent.

		$(16x^2) \div (16x^5)$ $\frac{1}{x^3}$ $(2x)^4(2x)^3$ $128x^7$ $(9z^{-2})(3z^{-1})^{-3}$ $\frac{z}{3}$	
A.APR.A.1	**A. Perform arithmetic operations on polynomials** Understand that polynomials form a system analogous to the integers, namely, they are closed under the operations of addition, subtraction, and ~~multiplication~~; add, subtract, and ~~multiply polynomials~~.	**Explanation:** The primary strategy for this cluster is to make connections between arithmetic of integers and arithmetic of polynomials. In order to understand this standard, students need to work toward both understanding and fluency with polynomial arithmetic. Furthermore, to talk about their work, students will need to use correct vocabulary, such as integer, monomial, polynomial, factor, and term. **Examples:** • Find each sum or difference by combining the parts that are alike. a. $417 + 231$ = _4_ hundreds + _1_ tens + _7_ ones + _2_ hundreds + _3_ tens + _1_ ones = _6_ hundreds + _4_ tens + _8_ ones. b. $(4x^2 + x + 7) + (2x^2 + 3x + 1)$. $6x^2 + 4x + 8$ c. $(3x^3 - x^2 + 8) - (x^3 + 5x^2 + 4x - 7)$. $2x^3 - 6x^2 - 4x + 15$	**Eureka Math**: Module 1 Lesson 8 **Big Ideas:** pp. 357-364, 365-370, 371-376 **This standard is revisited 2nd semester in Unit 3 (Quadratics).**

		d. $3(x^3+8x)-2(x^3+12)$. $x^3+24x-24$ e. $(5-t-t^2)+(9t+t^2)$ $8t+5$ f. $(3p+1)+6(p-8)-(p+2)$ $8p-49$	
MP.7	**Look for and make use of structure.**	Students reason with and about collections of equivalent expressions to see how all the expressions in the collection are linked together through the properties of operations. They discern patterns in sequences of solving equation problems that reveal structures in the equations themselves: $2x+4=10$, $2(x-3)+4=10$, $2(3x-4)+4=10$, etc.	**Eureka Math**: Module 1 Lesson 6
MP.8	**Look for and express regularity in repeated reasoning.**	They have opportunities to pay close attention to calculations involving the properties of operations, properties of equality, and properties of inequality as they find equivalent expressions and solve equations, noting common ways to solve different types of equations.	**Eureka Math**: Module 1 Lesson 7

HS Algebra I Semester 1 (Quarter 1)

Unit 1: Relationships Between Quantities and Reasoning with Equations and Their Graphs (40 days)

Topic C: Solving Equations and Inequalities

Students have written and solved linear equations and inequalities in their previous mathematics courses. The work of this unit should be on bringing students to mastery of this area of their mathematical study. This unit leverages the connection between equations and functions and explores how different representations of a function lead to techniques to solve linear equations, including tables, graphs, concrete models, algebraic operations, and "undoing" (reasoning backwards). This unit provides opportunities for students to continue to practice their ability to create and graph equations in two variables, as described in A-CED.A.2 and A-REI.D.10.

The Common Core Learning Standards rightfully downplay the notion of equivalent equations and instead place a heavy emphasis on students studying the solution sets to equations. Frist, students formalize descriptions of what they learned before (true/false equations, solution sets, identities, properties of equality, etc.) and learn how to explain the steps of solving equations to construct viable arguments to justify their solution methods. They then learn methods for solving inequalities, again by focusing on ways to preserve the (now infinite) solution sets. With these methods now on firm footing, students then investigate in solution sets of equations joined by “and” or “or” and investigate ways to change an equation such as squaring both sides, which changes the solution set in a controlled (and often useful) way. Next, students learn to use these same skills as they rearrange formulas to define one quantity in terms of another. Finally, students apply all of these new skills and understandings as they work through solving equations and inequalities with two variables including systems of such equations and inequalities.

Big Idea:	• An equation is a statement of equality between two expressions. • An equation with variables is viewed as a question asking for the set of values one can assign to the variables of the equation to make the equation a true statement. • Commutative, associate, and distributive properties are identities whose solution sets are the set of all values in the domain of the variables.
Essential Questions:	• What limitations are there to the principle “whatever you do to one side of the equation, you must do to the other side?” • What must be considered when an equation has a variable in the denominator? • How is rearranging formulas the same/different as solving equations that contain a single variable symbol?
Vocabulary	Number sentence, algebraic equation, solution set, set notation, identity, inequality, properties of equality, properties of inequality, zero-product property
Assessments	Live Binders/Galileo: Topic C Assessment

Standard	AZ College and Career Readiness Standards	Explanations & Examples	Resources
A.CED.A.3	**A. Create equations that describe numbers or relationships** Represent constraints by equations or inequalities, and by systems of equations and/or inequalities, and interpret solutions as viable or non-viable options in a modeling context. *For example, represent inequalities*	**Explanation:** Students recognize when a constraint can be modeled with an equation, inequality or system of equations/inequalities. They create, select, and use graphical, tabular and/or algebraic representations to solve the problem. **Examples:**	**Eureka Math**: Module 1 Lesson 10-24

	describing nutritional and cost constraints on combinations of different foods. ***This is a modeling standard which means students choose and use appropriate mathematics to analyze situations. Thus, contextual situations that require students to determine the correct mathematical model and use the model to solve problems are essential.***	**Represent constraints by equations or inequalities:** • The relation between quantity of chicken and quantity of steak if chicken costs \$1.29/lb and steak costs \$3.49/lb, and you have \$100 to spend on a dinner party where chicken and steak will be served. a. Write a constraint b. Justify your reasoning for writing the constraint as either an equation or an inequality. c. Determine two solutions. d. Graph the equation or inequality and identify your solutions. **Represent constraints by a system of equations or inequalities:** • A club is selling hats and jackets as a fundraiser. Their budget is \$1500 and they want to order at least 250 items. They must buy at least as many hats as they buy jackets. Each hat costs \$5 and each jacket costs \$8. a. Write a system of inequalities to represent the situation. b. Graph the inequalities. **Interpret solutions as viable or nonviable options in a modeling context.** • Using the example above: a. If the club buys 150 hats and 100 jackets, will the conditions be satisfied? b. What is the maximum number of jackets that they can buy and still meet the conditions? • Create an expression for the right side of each equation such that the solution set for the equation will be all real numbers.	

<table>
<tr>
<td></td>
<td></td>
<td>
a. $2x - 5 =$ ________

b. $x^2 + x =$ ________

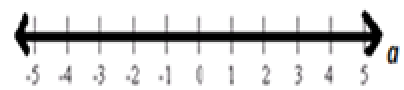
c. $4 \cdot x \cdot y \cdot z =$ ________

d. $(x + 2)^2 =$ ________

• Solve for a: $a + a^2 = a(a+1)$. Describe carefully the reasoning that justifies your solution set in words, in set notation, and graphically.

In Words: By the distributive property we have $a + a^2 = a(1+a)$. This is a true numerical statement no matter what value we assign to a. And by the commutative property of addition, we thus have that $a + a^2 = a(a+1)$ is a true numerical statement no matter what real value we assign to a.

In Set Notation: $\mathbb{R}$ (all real numbers)

In a Graphical Representation:


• Solution set in words, set notation and graphically:
<table>
<tr>
<td>$z - 6 < -2$</td>
<td>The set of real numbers less than 4</td>
<td>$\{z \text{ real} \mid z < 4\}$</td>
<td>
</td>
</tr>
</table>
</td>
<td></td>
</tr>
<tr>
<td>A.CED.A.4</td>
<td>A. Create equations that describe numbers or relationships</td>
<td>Explanation:
Students solve multi-variable formulas or literal equations for a specific variable. Explicitly connect this to the process of solving equations</td>
<td>Eureka Math:
Module 1 Lesson 19</td>
</tr>
</table>

	Rearrange formulas to highlight a quantity of interest, using the same reasoning as in solving equations. *For example, rearrange Ohm's law V = IR to highlight resistance R.*
	
	This is a modeling standard which means students choose and use appropriate mathematics to analyze situations. Thus, contextual situations that require students to determine the correct mathematical model and use the model to solve problems are essential.
		using inverse operations.
		
		Examples:
		• The Pythagorean Theorem expresses the relation between the legs a and b of a right triangle and its hypotenuse c with the equation $a^2 + b^2 = c^2$.
		  o Why might the theorem need to be solved for c?
		  o Solve the equation for c and write a problem situation where this form of the equation might be useful.
		  o Solve $V = \frac{4}{3}\pi r^3$ for radius *r*.
		• Motion can be described by the formula below, where *t* = time elapsed, *u*=initial velocity, *a* = acceleration, and *s* = distance traveled
		$s = ut + ½at^2$
		  o Why might the equation need to be rewritten in terms of *a*?
		  o Rewrite the equation in terms of *a*.
		•
		The area A of a rectangle is 25 in^2. The formula for area is $A = lw$.
		  ▪ **If the width w is 10 inches, what is the length l?**
		  $l = \frac{5}{2}$
		  ▪ **If the width w is 15 inches, what is the length l?**
		  $l = \frac{5}{3}$
		  ▪ Rearrange the area formula to solve for l.
		$A = lw$
		$\frac{A}{w} = \frac{lw}{w}$
		$\frac{A}{w} = l \text{ or } l = \frac{A}{w}$
		•
		Solve each problem two ways. First, substitute the given values and solve for the given variable. Then, solve for the given variable and substitute the given values.
		a. The perimeter formula for a rectangle is $p = 2(l + w)$ where p represents the perimeter; l represents the length, and w represents the width. Calculate l when $p = 70$ and $w = 15$.
		Sample responses:
		Substitute and solve. $70 = 2(l + 15)$ $l = 20$
		Solve for the variable first: $l = \frac{p}{2} - w$
			Big Ideas:
			pp. 35-42

		b. The area formula for a triangle is $A = \frac{1}{2}bh$, where A represents the area; b represents the length of the base, and h represents the height. Calculate b when $A = 100$ and $h = 20$. $b = \frac{2A}{h}, b = 10$ • **Equation Containing More Than One Variable** Solve $ax + b = d - cx$ for x. $ax + cx + b = d$ $ax + cx = d - b$ $x(a + c) = d - b$ $x = \frac{d - b}{a + c}$ **Related Equation** Solve $3x + 4 = 6 - 5x$ for x. $3x + 5x + 4 = 6$ $3x + 5x = 6 - 4$ $x(3 + 5) = 2$ $8x = 2$ $x = \frac{2}{8} = \frac{1}{4}$ • Solve for m. $T = 4\sqrt{m}$ $m = \frac{T^2}{16}$	
A.REI.A.1	**A. Understand solving equations as a process of**	**Explanation:** This standard is taught in Algebra I and Algebra II. In Algebra I tasks are	**Eureka Math**: Module 1 Lesson 12

●	**reasoning and explain the reasoning**
	
	Explain each step in solving a simple equation as following from the equality of numbers asserted at the previous step, starting from the assumption that the original equation has a solution. Construct a viable argument to justify a solution method.
		limited to quadratic equations. **In this unit, the focus is on linear equations only.**
		
		Assuming an equation has a solution, construct a convincing argument that justifies each step in the solution process. Justifications may include the associative, commutative, and division properties, combining like terms, multiplication by 1, etc.
		
		Properties of operations can be used to change expressions on either side of the equation to equivalent expressions. In addition, adding the same term to both sides of an equation or multiplying both sides by a non-zero constant produces an equation with the same solutions. Other operations, such as squaring both sides, may produce equations that have extraneous solutions.
		
		In Algebra I, students should focus on and master A.REI.1 for linear equations and quadratic equations. They should be able to extend and apply their reasoning to other types of equations in future courses.
		
		Strategy taught here:
		• If we are faced with the task of solving an equation, that is, finding the solution set of the equation:
		  o Use the commutative, associative, distributive properties
		AND
		  o Use the properties of equality (adding, subtracting, multiplying, dividing by non-zeros) to keep rewriting the equation into one whose solution set you easily recognize. (We observed that the solution set will not change under these operations.)
		
		This usually means rewriting the equation so that all the terms with the variable appear on one side of the equation.
			Module 1 Lesson 13
			
			Big Ideas:
			pp. 3-10
			pg 23
			
			This standard is revisited in 2nd semester in Unit 3 (Quadratics).

Examples:

-

Consider the equation $3x^2 + x = (x - 2)(x + 5)x$

a. Use the commutative property to create an equation with the same solution set.

$x + 3x^2 = (x + 5)(x - 2)x$

b. Using the result from (a), use the associative property to create an equation with the same solution set.

$(x + 3x^2) = ((x + 5)(x - 2))x$

c. Using the result from (b), use the distributive property to create an equation with the same solution set.

$x + 3x^2 = x^3 + 3x^2 - 10x$

d. Using the result from (c), add a number to both sides of the equation.

$x + 3x^2 + 5 = x^3 + 3x^3 - 10x + 5$

e. Using the result from (d), subtract a number from both sides of the equation.

$(x + 3x^2 + 5 - 3 = x^3 + 3x^2 - 10x + 5 - 3$

f. Using the result from (e), multiply both sides of the equation by a number.

$4(x + 3x^2 + 2) = 4(x^3 + 3x^2 - 10x + 2)$

g. Using the result from (f), divide both sides of the equation by a number.

$x + 3x^2 + 2 = x^2 + 3x^2 - 10x + 2$

h. What do all 7 equations have in common? Justify your answer.

They will all have the same solution set.

		• Describe the property used to convert the equation from one line to the next: $x(1-x)+2x-4=8x-24-x^2$ $x-x^2+2x-4=8x-24-x^2$ ___distributive property___ $x+2x-4=8x-24$ ___added x^2 to both sides of the equation___ $3x-4=8x-24$ ___collected like terms___ $3x+20=8x$ ___added 24 to both sides of the equation___ $20=5x$ ___subtracted 3x from both sides of the equation___ In each of the steps above, we applied a property of real numbers and/or equations to create a new equation. • Explain why the equation $x/2 + 7/3 = 5$ has the same solutions as the equation $3x + 14 = 30$. Does this mean that $x/2 + 7/3$ is equal to $3x + 14$? • Show that $x = 2$ and $x = -3$ are solutions to the equation $x^2 + x = 6$. Write the equation in a form that shows these are the only solutions, explaining each step in your reasoning.	
A.REI.B.3	**B. Solve equations and inequalities in one variable** Solve linear equations and inequalities in one variable, including equations with coefficients represented by letters.	**Explanation:** Students extend their knowledge of solving equations and inequalities in one variable from 7th grade (7.EE.4) and 8th grade (8.EE.7). In 9th grade, students find solutions of equations that include coefficients represented by letters. They solve inequalities that include variables on both sides of the inequality. Students discover that just like previous work on equations, rewriting an inequality via the commutative, associative, and distributive properties of the real numbers does not change the solution set of that inequality. **Examples:** • Graph the solution set:	**Eureka Math**: Module 1 Lesson 12-14, 19 **Big Ideas:** pp. 3-10, 11-18, 19-24, 27-34, 61-66, 67-72, 73-78 **Tape Diagrams** Intro Solving word problems **Algebra Tiles** Model and solve eq Illuminations **Other:** Learn Zillion

		$7-8x = 7(1+7x)$ $\{0\}$  • Graph the solution set: $2(6b+8) = 4+6b$ $\{-2\}$  • Graph the solution set: $\frac{x+4}{3} = \frac{x+2}{5}$ $\{-7\}$  • Solve for x: ax + 7 = 12 {5/a} • Solve $-q \geq -7$, for q. $-q \geq -7$ $0 \geq -7+q$ *Add q to both sides* $7 \geq q$ *Add 7 to both sides*  	[The Yo-Yo Problem](#) [Reasoning w/Eq and ineq worksheet](#) [Algebra Quiz](#) **Texas Instruments:** [From expressions to Eq](#) [Linear inequalities](#) [Inequalities on number line](#) [Variables on both sides](#) **This standard is revisited again in Topic D.**

		• Use the properties of inequality to show that each of the following are true for any real numbers *p* and *q*. If $p \geq q$, then $-p \leq -q$. $p \geq q$ $p - q \geq q - q$ $p - q \geq 0$ $p - p - q \geq 0 - p$ $-q \geq -p$	
A.REI.C.5	**C. Solve systems of equations** Prove that, given a system of two equations in two variables, replacing one equation by the sum of that equation and a multiple of the other produces a system with the same solutions.	**Explanation:** The focus of this standard is to provide mathematics justification for the addition (elimination) method of solving systems of equations ultimately transforming a given system of two equations into a simpler equivalent system that has the same solutions as the original system. Build on student experiences in graphing and solving systems of linear equations from 8th grade (8.EE.8) to focus on justification of the methods used. Include cases where the two equations describe the same line (yielding infinitely many solutions) and cases where two equations describe parallel lines (yielding no solution); connect to GPE.5, which requires students to prove the slope criteria for parallel lines. **Examples:** • Here is a system of two linear equations. Verify that the solution to this system is $(3, 4)$. Equation A1: $y = x + 1$ Equation A2: $y = -2x + 10$ *Substitute 3 for x and 4 for y into both equations.*	**Eureka Math:** Module 1 Lesson 22, 23 **Big Ideas:** pp. 247-252

		$4 = 3 + 1$ is a true equation. *$4 = -2(3) + 10$ is a true equation.* Equation D1: $y = x + 1$ Equation D2: $3y = -3x + 21$ • What multiple of A2 was added to A1 to create D2? *A2 was multiplied by 2 and then added to A1.* • What is the solution to the system of two linear equations formed by D1 and D2? *The solution is still (3, 4). I checked by substituting (3 ,4) into both equations.* • Is D2 equivalent to the original A1 or A2? Explain your reasoning. *No, the slope of D2 is –1. Neither of the original equations had that slope.* • Start with equation A1. Multiply it by a number of your choice and add the result to equation A2. This creates a new equation E2. Record E2 below to check if the solution is $(3, 4)$. Equation E1: $y = x + 1$ Equation E2: $5y = 2x + 14$ *I multiplied A1 by 4 to get $4y = 4x + 4$. Adding it to A2 gives $5y = 2x + 14$. We already know (3, 4) is a solution to $y = x + 1$. Substituting into E2 give $5(4) = 2(3) + 14$, which is a true equation. Therefore (3,4) is a solution to this new system.*	

<table>
<tr>
<td></td>
<td></td>
<td>

Example 1: Why Does the Elimination Method Work?
Solve this system of linear equations algebraically.
ORIGINAL SYSTEM NEW SYSTEM SOLUTION
$2x + y = 6$
$x - 3y = -11$
ORIGINAL SYSTEM NEW SYSTEM SOLUTION
$2x + y = 6$ $6x + 3y = 18$ $x = 1$
$x - 3y = -11$ $x - 3y = -11$ $7x = 7$ $2(1) + y = 6$ so $y = 4$
Multiply the first equation by 3 and add it to the second. Solve the new system. $(1, 4)$

Example(s): Use the system $\begin{cases} 2x + y = 13 \\ x + y = 10 \end{cases}$ to explore what happens graphically with different combinations of the linear equations.
a. Graph the original system of linear equations. Describe the solution of the system and how it relates to the solutions of each individual equation. (Note: connect to A-REI.10)
b. Add the two linear equations together and graph the resulting equation. Describe the solutions to the new equation and how they relate to the system's solution.
c. Explore what happens with other combinations such as:
i. Multiply the first equation by 2 and add to the second equation
ii. Multiply the second equation by -2 and add to the first equation
iii. Multiply the second equation by -1 and add to the first equation
iv. Multiply the first equation by -1 and add to the second equation

Given that the sum of two numbers is 10 and their difference is 4, what are the numbers? Explain how your answer can be deduced from the fact that they two numbers, x and y, satisfy the equations $x + y = 10$ and $x - y = 4$.

</td>
<td></td>
</tr>
<tr>
<td>A.REI.C.6
▲</td>
<td>C. Solve systems of equations

Solve systems of linear equations exactly and approximately (e.g., with graphs), focusing on pairs of linear equations in two variables.</td>
<td>Explanation:
This standard is taught in Algebra I and Algebra II. In Algebra I tasks are limited to <u>pairs</u> of linear equations in two variables. In Algebra II, systems of three linear equations in three variables are introduced.

Examples:</td>
<td>Eureka Math:
Module 1 Lesson 15-18, 22-24

Big Ideas:
pp. 235-240 (use</td>
</tr>
</table>

		<u>Systems of linear equations in one variable joined by "and" or "or".</u> • Solve each system of equations $x + 8 = 3$ or $x - 6 = 2$ $x = -5$ *or* $x = 8$ $\{-5, 8\}$ $x - 6 = 1$ and $x + 2 = 9$ $x = 7$ *and* $x = 7$ $\{7\}$ • Solve the system and graph the solution set on a number line. $x - 15 = 5$ *or* $2x + 5 = 1.$ $x = 20$ *or* $x = -2$ $\{-2, 20\}$  Equations of the form $(x-a)(x-b)=0$ have the same solution set as two equations joined by "or:" $x-a=0$ or $x-b=0$. • Consider the equation $(x-4)(x+3) = 0.$ a. Rewrite the equation as a compound statement. $x - 4 = 0$ *or* $x + 3 = 0$	problems that replicate the examples that are shown to the left; rather than using the graphing, elimination, substitution approach). Then move to the graphing method. **Texas Instruments:** Balanced Systems Boats in Motion How Many Solutions Solving by Graphing Illuminations: Road Rage

- Find the solutions.

  $(x+1)(x+2) = 0$

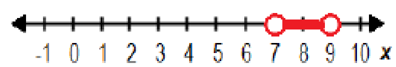
  $\{-2, -1\}$

  $(x+4)(x-6)(x-10) = 0$

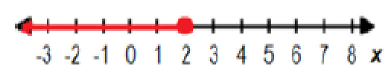
  $\{-4, 6, 10\}$

For a statement separated by "and" to be true BOTH statements must be true. If it is separated by "or," at least one statement must be true.

- $x < 9$ and $x > 7$



$x + 5 < 7$ or $x = 2$



Students interpret equations like $1/x = 3$ as two equations "$1/x = 3$" and "$x \neq 0$" joined by "and."

Note: By stating a restriction for x, we disallow the possibility of ever dividing by zero.

- Consider the equation $\frac{1}{x} = \frac{3}{x-2}$.

  a. Rewrite the equation into a system of equations.

  $\frac{1}{x} = \frac{3}{x-2}$ *and* $x \neq 0$ *and* $x \neq 2$

  b. Solve the equation for x, excluding the value(s) of x that lead to a denominator of zero.

  $x = -1$ *and* $x \neq 0$ *and* $x \neq 2$ *solution set:* $\{-1\}$

This is really a compound statement:

$\frac{1}{x} = \frac{3}{x-2}$ and $x \neq 0$ and $x - 2 \neq 0$

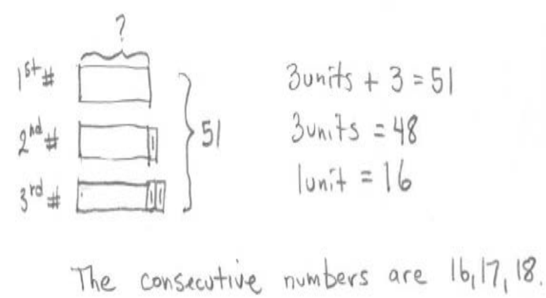
By the properties of equality, we can multiply through by non-zero quantities. Within this compound statement, x and $x-2$ are nonzero, so we may write $x-2=3x$ and $x\neq0$ and $x-2\neq0$, which is equivalent to:

$-2=2x$ and $x\neq0$ and $x\neq2$.

All three declarations in this compound statement are true for $x=-1$. This is the solution set.

- **Find three consecutive integers such that their sum is 51.**

  Tape:

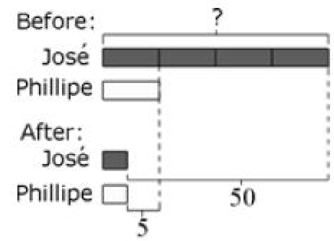


<u>Systems of linear equations in two variables:</u>

- Solve the system of equations:
  x + y = 11 and 3x – y = 5
- Use a second method to check your answer.

<table>
<tr>
<td></td>
<td></td>
<td>

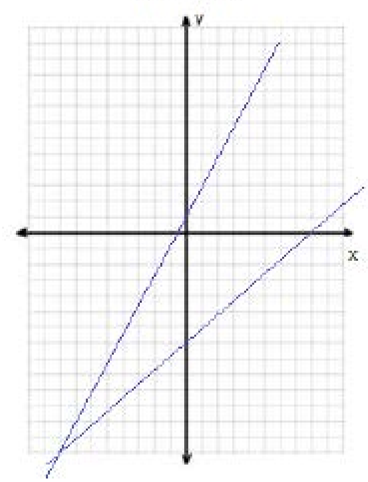
- José had 4 times as many trading cards as Phillipe. After José gave away 50 cards to his little brother and Phillipe gave 5 cards to his friend for this birthday, they each had an equal amount of cards. Write a system to describe the situation and solve the system.



- Solve the following system of equations.

$$\begin{cases} y = 2x + 1 \\ x - y = 7 \end{cases}$$

Graphically:



Algebraically:

$x - (2x + 1) = 7$

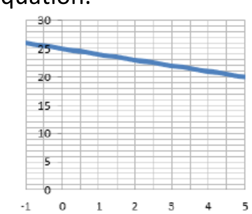
$x = -8$

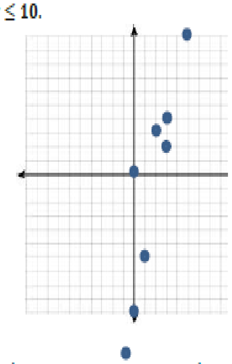
$y = 2(-8) + 1$

$y = -15$

solution: $(-8, -15)$

</td>
<td></td>
</tr>
<tr>
<td>A.REI.D.10</td>
<td>D. Represent and solve equations and inequalities graphically</td>
<td>Explanation:
Students can explain and verify that every point (x,y) on the graph of</td>
<td>Eureka Math:
Module 1 Lesson 20, 22</td>
</tr>
</table>

	Understand that the graph of an equation in two variables is the set of all its solutions plotted in the coordinate plane, often forming a curve (which could be a line).	an equation represents all values for x and y that make the equation true. In Algebra I, students focus on linear, exponential and quadratic equations and are able to adapt and apply that learning to other types of equations in future courses. **Examples:** • Discover as many solutions to the equation 4x – y = 10 as possible. Consider the best way to organize all the solutions you have found. • Create an equation using two variables to represent this situation. o The sum of two numbers is 25. What are the numbers? *Let* x *= one number, and let* y *= another number.* *Equation:* $x + y = 25$ o List at least 6 solutions to the equation. o Create a graph that represents the solution set to the equation.  • Which of the following points are on the graph of the equation -5x + 2y = 20? a. (4, 0) b. (0, 10) c. (-1, 7.5) d. (2.3, 5) How many solutions does the equation have? Justify your answer.	

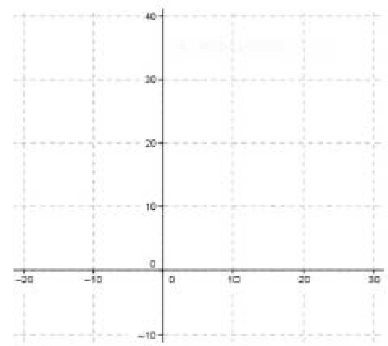
A.REI.D.12	**D. Represent and solve equations and inequalities graphically** Graph the solutions to a linear inequality in two variables as a half-plane (excluding the boundary in the case of a strict inequality), and graph the solution set to a system of linear inequalities in two variables as the intersection of the corresponding half-planes.	**Explanation:** Students graph linear inequalties in two variables, excluding the boundary for non-inclusive inequalities. Students understand that the solution to a system of linear inequalities in two variables are the points that lie in the intersection of the corresponding half-planes. Students may use graphing calculators, programs or applets to model and find solutions for inequaltities of systems of inequalities. **Examples:** • a. Circle each ordered pair (x, y) that is a solution to the equation $4x - y \leq 10$. i. $(3,2)$ $(2,3)$ $(-1,-14)$ $(0,0)$ $(1,-6)$ ii. $(5,10)$ $(0,-10)$ $(3,4)$ $(6,0)$ $(4,-1)$  b. Plot each solution as a point (x, y) in the coordinate plane. c. How would you describe the location of the solutions in the coordinate plane? *(Students may struggle to describe the points. Here is one possible description.) The points do not all fall on any one line, but if you drew a line through any two of the points, the others are not too far away from that line.* To aid in this task have the students complete the following sentence: If an ordered pair is a solution to $4x - y \leq 10$, then it will be located *on the line or above (or on the left side of)* the line $y = 4x - 10$. Explain how you arrived at your conclusion. *I observed that all the points were on one side of the line, and then I tested some points on the other side of the line and found that all the points I tested from that side of the line were not solutions to the inequality.*	**Eureka Math:** Module 1 Lesson 21-22 **Big Ideas:** pp. 267-272 **IXL:** Graph inequalities **Texas Instruments:** Parking Cars Exploring Graphs Linear inequalities Two-variable inequalities

- A publishing company publishes a total of no more than 100 magazines every year. At least 30 of these are women's magazines, but the company always publishes at least as many women's magazines as men's magazines. Find a system of inequalities that describes the possible number of men's and women's magazines that the company can produce each year consistent with these policies. Graph the solution set.

    - Could you express the solution set of a system of inequalities without using a graph?
        - *Yes, using set notation, but a graph makes it easier to visualize and conceptualize which points are in the solution set.*

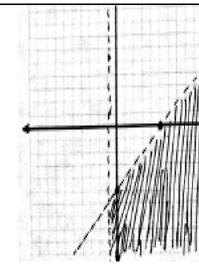
    - How can you check your solution graph?
        - *Test a few points to confirm that the points in the shaded region satisfy all the inequalities.*
    -

Graph the solution to $x + y < 20$



The line should be dashed, and all points below the line should be shaded.

-

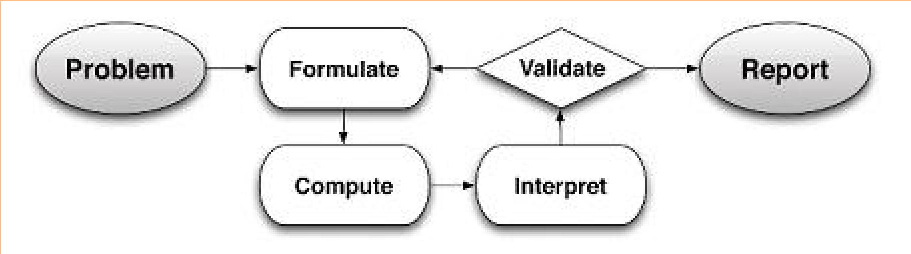
		Graph the solution set to each system of inequalities. a. $\begin{cases} x - y > 5 \\ x > -1 \end{cases}$ 	
MP.1	**Make sense of problems and persevere in solving them.**	Students are presented with problems that require them to try special cases and simpler forms of the original problem in order to gain insight into the problem.	**Eureka Math:** Module 1 Lesson 11, 12, 14, 16, 21, 24
MP.2	**Reason abstractly and quantitatively.**	Students analyze graphs of non-constant rate measurements and reason from the shape of the graphs to infer what quantities are being displayed and consider possible units to represent those quantities.	**Eureka Math:** Module 1 Lesson 11, 12, 14, 15, 20
MP.3	**Construct viable arguments and critique the reasoning of others.**	Students reason about solving equations using "if-then" moves based on equivalent expressions and properties of equality and inequality. They analyze when an "if-then" move is not reversible.	**Eureka Math:** Module 1 Lesson 11-13, 16, 18, 19, 24
MP.6	**Attend to precision.**	Students formalize descriptions of what they learned before (variables, solution sets, numerical expressions, algebraic expressions, etc.) as they build equivalent expressions and solve equations. Students analyze solution sets of equations to determine processes (like squaring both sides of an equation) that might lead to a solution set that differs from that of the original equation.	**Eureka Math:** Module 1 Lesson 15, 17, 20, 24
MP.7	**Look for and make use of structure.**	Students reason with and about collections of equivalent expressions to see how all the expressions in the collection are linked together through the properties of operations. They discern patterns in sequences of solving equation problems that reveal structures in the equations themselves: $2x + 4 = 10$, $2(x - 3) + 4 = 10$, $2(3x - 4) + 4 = 10$, etc.	**Eureka Math:** Module 1 Lesson 17, 22
MP.8	**Look for and express regularity in repeated reasoning.**	After solving many linear equations in one variable (e.g., $3x + 5 = 8x - 17$), students look for general methods for solving a generic linear equation in one variable by replacing the numbers with letters: $ax + b = cx + d$. They have opportunities to pay close attention to calculations involving the properties of operations, properties of equality, and properties of inequality as they find equivalent expressions and solve equations, noting common ways to solve different types of equations.	**Eureka Math:** Module 1 Lesson 17

HS Algebra I Semester 1 (Quarter 1)

Unit 1: Relationships Between Quantities and Reasoning with Equations and Their Graphs (40 days)

Topic D: Creating Equations to Solve Problems

In this topic, students are introduced to the modeling cycle (see page 61 of the Common Core Learning Standards) through problems that can be solved using equations and inequalities in one variable, systems of equations, and graphing. Modeling links classroom mathematics and statistics to everyday life, work, and decision making.



Big Idea:	• Modeling links classroom mathematics and statistics to everyday life, work, and decision-making.
Essential Questions:	• How do I know where to begin when solving a problem? • How does explaining my process help me to understand a problem's solution better? • How do I decide what strategy will work best in a given problem situation? • What do I do when I get stuck?
Vocabulary	Modeling cycle, (formulate, validate, compute, interpret), recursive sequence, sequence
Assessments	Live Binders/Galileo: Topic D Assessment

Standard	AZ College and Career Readiness Standards	Explanations & Examples	Resources
N.Q.A.1	**A. Reason qualitatively and units to solve problems** Use units as a way to understand problems and to guide the solution of multi-step problems; choose and interpret units consistently in formulas; choose and interpret the scale and the origin in graphs and data	**Explanation:** Include word problems where quantities are given in different units, which must be converted to make sense of the problem. Graphical representations and data displays include, but are not limited to: line graphs, circle graphs, histograms, multi-line graphs, scatterplots, and multi-bar graphs.	**Eureka Math:** Module 1 Lesson 25-28

	displays. ***This is a modeling standard which means students choose and use appropriate mathematics to analyze situations. Thus, contextual situations that require students to determine the correct mathematical model and use the model to solve problems are essential.***	**<u>Use units as a way to understand problems and to guide the solution of multi-step problems</u>** Students use the units of a problem to identify what the problem is asking. They recognize the information units provide about the quantities in context and use units as a tool to help solve multi-step problems. Students analyze units to determine which operations to use when solving a problem. **Examples:** • For example, a problem might have an object moving 12 feet per second and another at 5 miles per hour. To compare speeds, students convert 12 feet per second to miles per hour: $\frac{12\,ft}{\sec} \bullet \frac{60\sec}{\min} \bullet \frac{60\min}{hr} \bullet \frac{1mi}{5280\,ft} \approx \frac{8.182mi}{hr}$ which is more than 5 miles per hour. • **Maya and Earl live at opposite ends of the hallway in their apartment building. Their doors are 50 feet apart. They each start at their door and walk at a steady pace towards each other and stop when they meet.** **Suppose that:** ▪ **Maya walks at a constant rate of 3 feet every second.** ▪ **Earl walks at a constant rate of 4 feet every second.** **Graph both people's distance from Maya's door versus time in seconds.** ***Graphs should be scaled and labeled appropriately. Maya's graph should start at $(0,)$ and have a slope of 3, and Earl's graph should start at $(0,)$ and have a slope of -4.*** What do you think the numbers along the horizontal axis represent? What might the numbers along the vertical axis represent? Do we have any indication of the units being used? • Given the speed in *mph* and the time traveled in *hours*, what is the distance traveled? o From looking at the units, we can determine that we	

		must multiply *mph* times *hours* to get an answer expressed in miles: $\left(\frac{mi}{hr}\right)(hr) = mi$ **(Note that knowledge of the distance formula is not required to determine the need to multiply in this case.)** **<u>Choose and interpret units consistently in formulas</u>** Students choose the units that accurately describe what is being measured. Students understand the familiar measurements such as length (unit), area (unit squares) and volume (unit cubes). They use the structure of formulas and the context to interpret units less familiar. **Example:** • If $density = \frac{mass\ in\ grams}{volume\ in\ mL}$ then the unit for density is $\frac{grams}{mL}$ **<u>Choose and interpret the scale and the origin in graphs and data displays</u>** When given a graph or data display, students read and interpret the scale and origin. When creating a graph or data display, students choose a scale that is appropriate for viewing the features of a graph or data display. Students understand that using larger values for the tick marks on the scale effectively "zooms out" form the graph and choosing smaller values "zooms in". Students also understand that the viewing window does not necessarily show the x- or y-axis, but the apparent axes are parallel to the x- and y-axes. Hence, the intersection of the apparent axes in the viewing window may not be the origin. They are also aware that apparent intercepts may not correspond to the actual x- or y-intercepts of the graph of a function.	
A.SSE.A.1 ●	**A. Interpret the structure of expressions** Interpret expressions that represent a quantity in terms of its context. a. Interpret parts of an expression, such as terms,	**Explanation:** This standard is taught in Algebra I and Algebra II. In Algebra I the focus is on linear expressions, exponential expressions with integer exponents and quadratic expressions. Throughout Algebra I, students should:	**Eureka Math:** Module 1 Lesson 25-28 **This standard is revisited later in this unit as well**

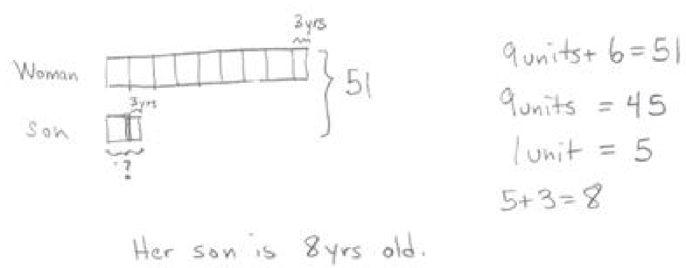
	factors, and coefficients. b. Interpret complicated expressions by viewing one or more of their parts as a single entity. ~~*For example, interpret $P(1+r)^n$ as the product of P and a factor not depending on P.*~~ ***This is a modeling standard which means students choose and use appropriate mathematics to analyze situations. Thus, contextual situations that require students to determine the correct mathematical model and use the model to solve problems are essential.***	• **Explain** the difference between an expression and an equation. • **Use** appropriate vocabulary for the parts that make up the whole expression. • **Identify** the different parts of the expression and explain their meaning within the context of the problem. • **Decompose** expressions and make sense of the multiple factors and terms by explaining the meaning of the individual parts. ***Note:*** Students should understand the vocabulary for the parts that make up the whole expression, be able to identify those parts, and interpret their meaning in terms of a context. **a. Interpret parts of an expression, such as: terms, factors, and coefficients** • Students recognize that the linear expression $mx + b$ has two terms, m is a coefficient, and b is a constant. • Students extend beyond simplifying an expression and address interpretation of the components in an algebraic expression. • Development and proper use of mathematical language is an important building block for future content. Using real-world context examples, the nature of algebraic expressions can be explored. • The "such as" listed are not the only parts of an expression students are expected to know; others include, but are not limited to, degree of a polynomial, leading coefficient, constant term, and the standard form of a polynomial (descending exponents). **Examples:** • A student recognizes that in the expression $2x + 1$, "2" is the coefficient, "2" and "x" are factors, and "1" is a constant, as well as "$2x$" and "1" being terms of the binomial expression. • A student recognizes that in the expression $4(3)^x$, 4 is the coefficient, 3 is the factor, and x is the exponent.	**as 2nd semester in Module 3.**

		• The height (in feet) of a balloon filled with helium can be expressed by $5 + 6.3s$ where s is the number of seconds since the balloon was released. Identify and interpret the terms and coefficients of the expression. • A company uses two different sized trucks to deliver sand. The first truck can transport x cubic yards, and the second y cubic yards. The first truck makes S trips to a job site, while the second makes T trips. What do the following expressions represent in practical terms? a. $S + T$ b. $x + y$ c. $xS + yT$ **b. Interpret complicated expressions by viewing one or more of their parts as a single entity.** • Students view mx in the expression $mx + b$ as a single quantity. **Examples:** • The expression $20(4x) + 500$ represents the cost in dollars of the materials and labor needed to build a square fence with side length x feet around a playground. Interpret the constants and coefficients of the expression in context. • A rectangle has a length that is 2 units longer than the width. If the width is increased by 4 units and the length increased by 3 units, write two equivalent expression for the area of the rectangle. o The area of the rectangle is $(x+5)(x+4) = x^2+9x+20$. Students should recognize $(x+5)$as the length of the modified rectangle and $(x+4)$ as the width. Students can also interpret $x^2+ 9x + 20$ as the sum of the three areas (a square with side length x, a rectangle with side lengths 9 and x, and another rectangle with area 20 that have the same total area as the modified rectangle.	
A.CED.A.1	**A. Create equations that describe numbers or relationships**	**Explanation:** This standard is taught in Algebra I and Algebra II. In Algebra I, tasks	**Eureka Math:** Module 1 Lesson 25-28

	Create equations and inequalities in one variable and use them to solve problems. *Include equations arising from linear and quadratic functions, and simple rational and exponential functions.* ***This is a modeling standard which means students choose and use appropriate mathematics to analyze situations. Thus, contextual situations that require students to determine the correct mathematical model and use the model to solve problems are essential.***	are limited to **linear**, quadratic or exponential equations with integer exponents. Students recognize when a problem can be modeled with an equation or inequality and are able to write the equation or inequality. Students create, select, and use graphical, tabular and/or algebraic representations to solve the problem. **Examples:** • The Tindell household contains three people of different generations. The total of the ages of the three family members is 85. o Find reasonable ages for the three Tindells. o Find another reasonable set of ages for them. o In solving this problem, one student wrote C + (C+20) + (C + 56) = 85 ▪ What does C represent in this equation? ▪ What do you think the student had in mind when using the numbers 20 and 56? ▪ What set of ages do you think the student came up with? • Mary and Jeff both have jobs at a baseball park selling bags of peanuts. They get paid $12 per game and $1.75 for each bag of peanuts they sell. Create equations, that when solved, would answer the following questions: o How many bags of peanuts does Jeff need to sell to earn $54? o How much will Mary earn if she sells 70 bags of peanuts at a game? o How many bags of peanuts does Jeff need to sell to earn at least $68? • Phil purchases a used truck for $11,500. The value of the truck is expected to decrease by 20% each year. When will the truck first be worth less than $1,000? • A scientist has 100 grams of radioactive substance. Half of it decays every hour. How long until 25 grams remain? Be prepared to share any equations, inequalities, and/or	**Big Ideas:** pp. 6 – 7 ex 3 and 4 pp. 9 -10 #42-44, 47 pg 15 ex 5 pp. 16-17 #29-37 pg. 19 pg 22. Ex 4 pp. 23-24 #27, 28, 33, 34, 39 pg. 53 ex 1 pg. 57 ex 4 pp 58 – 59 #5-14, 25, 26, 46, 47-50, 55-59 pp. 65-66 #26, 29, 32-38 pg 70 ex 3 pp. 71-72 #19-20, 29-32, 35, 38-39 pg. 76 ex 4 pp. 77-78 #31-38 pg 82 ex 1 pg 84 ex 4 pp. 85-86 #3-12, 23-24, 32, 34 Other: Algebra Balance Scales Writing and using inequalities – video Writing and using inequalities **This standard is revisited later in this semester as well as 2nd semester in Units 3 and 5.**

		representations used to solve the problem. • Axel and his brother like to play tennis. About three months ago they decided to keep track of how many games they have each won. As of today, Axel has won 18 out of the 30 games against his brother. How many games would Axel have to win in a row in order to have a 75% winning record? *Solving, $18+n=0.75(30+n)$, results in n=18. He would have to win 18 games.* • A checking account is set up with an initial balance of $9400, and $800 is removed from the account at the end of each month for rent (no other user transactions occur on the account). a. Write an inequality whose solutions are the months, m, in which the account balance is greater than $3000. Write the solution set to your equation by identifying all of the solutions. *For m a non-negative real number, m satisfies the inequality, $9400-800m>3000$. For real numbers m, the solution set is $0\leq m<8$.*	
A.CED.A.2	**A. Create equations that describe numbers or relationships** Create equations in two or more variables to represent relationships between quantities; graph equations on coordinate axes with labels and scales. ***This is a modeling standard which means students choose and use appropriate mathematics to analyze situations. Thus, contextual situations that require students to determine the correct mathematical model and use the model to solve problems are essential.***	**Explanation:** This standard is taught in Algebra I and Algebra II. In Algebra I, students create equations in two variables for linear, exponential and quadratic contextual situations. Limit exponential situations to only ones involving integer input values. **Linear equations** can be written in a multitude of ways; $y=mx+b$ and $ax+by=c$ are commonly used forms (given that x and y are the two variables). Students should be flexible in using multiple forms and recognizing from the context, which is appropriate to use in creating the equation. **Examples:**	**Eureka Math:** Module 1 Lesson 28 **This standard is revisited 2nd semester in Units 3 and 5.**

		• The FFA had a fundraiser by selling hot dogs for \$1.50 and drinks for \$2.00. Their total sales were \$400. o Write an equation to calculate the total of \$400 based on the hot dog and drink sales. o Graph the relationship between hot dog sales and drink sales. • A spring with an initial length of 25cm will compress 0.5cm for each pound applied. o Write an equation to model the relationship between the amount of weight applied and the length of the spring. o Graph the relationship between pounds and length. o What does the graph reveal about limitation on weight? • ***Duke starts at the base of a ramp and walks up it at a constant rate. His elevation increases by three feet every second. Just as Duke starts walking up the ramp, Shirley starts at the top of the same 25 foot high ramp and begins walking down the ramp at a constant rate. Her elevation decreases two feet every second.*** o ***Sketch two graphs on the same set of elevation=versus-time axes to represent Duke's and Shirley's motions.*** o ***Write down the equation of the line that represents Duke's motion as he moves up the ramp and the equation of the line that represents Shirley's motion as she moves down. Show that the coordinates of the point you found in question above satisfy both equations.*** ▪ If y ***represents elevation in feet and*** t ***represents time in seconds, then Duke's elevation satisfies*** $y=3t$ ***and Shirley's*** $y=25-2t$***. The lines intersect at*** $(5,)$***, and this point does indeed lie on both lines.*** ***Duke:*** $15=3(5)$ ***Shirley:*** $15=25-2(5)$ • **Maya and Earl live at opposite ends of the hallway in their apartment building. Their doors are 50 feet apart. They each start at their door and walk at a steady pace towards each other and stop when they meet. Suppose that:** ▪ **Maya walks at a constant rate of 3 feet every second.** ▪ **Earl walks at a constant rate of 4 feet every second.**	

		Create equations for each person's distance from Maya's door and determine exactly when they meet in the hallway. How far are they from Maya's door at this time? *Maya's Equation:* $y=3t$*.* *Earl's Equation:* $y=50-4t$*.* *Solving the equation* $3t=50-4t$*, gives the solution:* $t=7\ 1/7$*. The two meet at exactly this time at a distance of* $3\ (71/7)=21\ 3/7$ *feet from Maya's door.*	
A.REI.B.3	**B. Solve equations and inequalities in one variable** Solve linear equations and inequalities in one variable, including equations with coefficients represented by letters.	**Explanation:** This standard was introduced in Topic C. In this Topic D, the use of tape diagrams and area models are used as a strategy to solve equations and inequalities in real-life contextual situations. The numerical approach is compared to the algebraic approach in investigating real-life situations. **Examples:** • The total age of a woman and her son is 51 years. Three years ago, the woman was eight times as old as her son. How old is her son now? **Tape:** 	**Eureka Math:** Module 1 Lesson 25-28 **Big Ideas:** Pg 9 #42-44, 46, 47 Pg 15 ex 5 Pg 16 #29-37 Pg 65 #25-29 pp 339-346 **Tape Diagrams** Intro Solving word problems **Algebra Tiles** Model and solve eq Illuminations

Equation:

Son's age: S yrs old

Woman's age: $51 - S$

Eqn: $51 - S - 3 = 8(S - 3)$

Solve:
$$48 - S = 8S - 24$$
$$48 - S + S = 8S - 24 + S$$
$$48 = 9S - 24$$
$$48 + 24 = 9S - 24 + 24$$
$$72 = 9S$$
$$8 = S$$

Check: If son is 8 yrs old now, then the woman is 43 yrs old.

Three yrs ago: $\left.\begin{array}{l}\text{woman} - 40 \\ \text{son} - 5\end{array}\right\} \times 8!$

The son is 8 years old.

- Jim tells you he paid a total of $23,078.90 for a car, and you would like to know the price of the car before sales tax so that you can compare the price of that model of car at various dealers. Find the price of the car before sales tax if Jim bought the car in:

  Arizona, where the sales tax is 6.6%.
  Solving $(1+0.066) = 23078.90$ *results in* $x = 21,$. *The car costs* $\$21,$.

- A checking account is set up with an initial balance of $9400, and $800 is removed from the account at the end of each month for rent (no other user transactions occur on the account).

  a. Write an inequality whose solutions are the months, m, in which the account balance is greater than $3000. Write the

		solution set to your equation by identifying all of the solutions. *For m a non-negative real number, m satisfies the inequality, $9400 - 800m > 3000$. For real numbers m, the solution set is $0 \le m < 8$.*	
MP.1	**Make sense of problems and persevere in solving them.**	Students are presented with problems that require them to try special cases and simpler forms of the original problem in order to gain insight into the problem.	**Eureka Math:** Module 1 Lesson 25 Module 1 Lesson 26 Module 1 Lesson 28
MP.2	**Reason abstractly and quantitatively.**	Students analyze graphs of non-constant rate measurements and reason from the shape of the graphs to infer what quantities are being displayed and consider possible units to represent those quantities.	**Eureka Math:** Module 1 Lesson 25 Module 1 Lesson 27 Module 1 Lesson 28
MP.3	**Construct viable arguments and critique the reasoning of others.**	Students reason about solving equations using "if-then" moves based on equivalent expressions and properties of equality and inequality. They analyze when an "if-then" move is not reversible.	**Eureka Math:** Module 1 Lesson 27
MP.4	**Model with mathematics.**	Students have numerous opportunities in this module to solve problems arising in everyday life, society, and the workplace - understanding the federal progressive income tax system.	**Eureka Math:** Module 1 Lesson 26 Module 1 Lesson 27 Module 1 Lesson 28

HS Algebra I Semester 1 (Quarter 2)

Unit 2: Linear and Exponential Functions (35 days)

(S-ID.B.6 and S-ID.C.7 could be incorporated in this unit)

Topic A: Linear and Exponential Sequences (7 days)

In Topic A, students explore arithmetic and geometric sequences as an introduction to the formal notation of functions (**F-IF.A.1**, **F-IF.A.2**). They interpret arithmetic sequences as linear functions with integer domains and geometric sequences as exponential functions with integer domains (**F-IF.A.3**, **F-BF.A.1a**). Students compare and contrast the rates of change of linear and exponential functions, looking for structure in each and distinguishing between additive and multiplicative change (**F-IF.B.6**, **F-LE.A.1**, **F-LE.A.2**, **F-LE.A.3**).

Big Idea:	• Sequences are an ordered list of elements whose pattern is defined by an explicit formula. • Sequences are functions. • Real-life situations can be modeled by linear and exponential functions.
Essential Questions:	• Can one sequence have two different formulas? • Why are there two different types of formula, explicit and recursive, to define a sequence? • What is the difference between an arithmetic sequence and geometric sequence? • Why are arithmetic sequences sometimes called linear sequences? • How are exponential growth and geometric sequences related? • What is the difference between linear growth and exponential growth.
Vocabulary	Sequence, explicit formula, recursive formula, arithmetic sequence, geometric sequence, linear sequence, exponential growth, exponential decay
Assessments	Galileo: Topic A Assessment

Standard	AZ College and Career Readiness Standards	Explanations & Examples	Resources
F.IF.A.1	**A. Understand the concept of a function and use function notation** Understand that a function from one set (called the domain) to another set (called the range) assigns to each element of the domain exactly one element of the range. If *f* is a function and *x* is an element of its domain, then *f*(*x*) denotes the output of *f* corresponding to the input *x*. The graph of *f* is the graph of the equation *y* = *f*(*x*).	**Explanation:** This standard is introduced in this topic via sequences. However, it is not formally taught until Topic B. The function notation, f(n), is introduced without naming it as such and without calling attention to it at this stage. The use of the letter *f* for *formula* seems natural. Watch to make sure that students are using the *f(n)* to stand for *formula for the nth term* and not thinking about it as the product $f \cdot n$.	**Eureka Math:** Module 3 Lesson 1-7

F.IF.A.2	**A. Understand the concept of a function and use function notation** Use function notation, evaluate functions for inputs in their domains, and interpret statements that use function notation in terms of a context.	**Explanation:** This standard is introduced in this topic via sequences. However, it is not formally taught until Topic B. Students are asked to find the n^{th} term (input) of a sequence given a formula *f(n)*; however, the concept of domain and range are not formally taught in this topic. **Example:** • Consider a sequence generated by the formula $\boldsymbol{f(n) = 6n - 4}$ starting with $\boldsymbol{n = 1}$. Generate the terms $\boldsymbol{f(1), f(2), f(3), f(4)}$, and $\boldsymbol{f(5)}$. $\boldsymbol{2, 8, 14, 20, 26}$	**Eureka Math:** Module 3 Lesson 1-7
F.IF.A.3	**A. Understand the concept of a function and use function notation** Recognize that sequences are functions, sometimes defined recursively, whose domain is a subset of the integers. *For example, the Fibonacci sequence is defined recursively by f(0) = f(1) = 1, f(n+1) = f(n) + f(n---1) for n ≥ 1.* *HS.MP.8.* Look for and express regularity in repeated reasoning.	**Explanation:** This standard is taught in Algebra I and Algebra II. In Algebra I, it is part of the major work and will be assessed accordingly. A sequence can be described as a function, with the input numbers consisting of a subset of the integers, and the output numbers being the terms of the sequence. The most common subset for the domain of a sequence is the Natural numbers {1, 2, 3, …}; however, there are instances where it is necessary to include {0} or possibly negative integers. Whereas, some sequences can be expressed explicitly, there are those that are a function of the previous terms. In which case, it is necessary to provide the first few terms to establish the relationship. *Connect to arithmetic and geometric sequences. Emphasize that arithmetic and geometric sequences are examples of linear and exponential functions, respectively.* **Examples:** •	**Eureka Math:** Module 3 Lesson 1-3

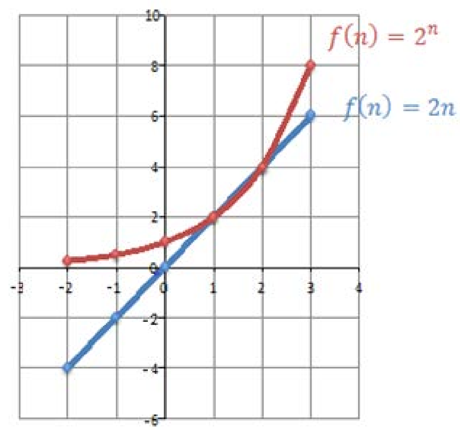
		For each sequence below, an explicit formula is given. Write the first 5 terms of each sequence. Then, write a recursive formula for the sequence.
		a. $a_n = 2n + 10$ for $n \geq 1$
		$12, 14, 16, 18, 20$
		$a_{n+1} = a_n + 2$, *where* $a_1 = 12$ *and* $n \geq 1$
		
		b. $a_n = \left(\frac{1}{2}\right)^{n-1}$ for $n \geq 1$
		$1, \frac{1}{2}, \frac{1}{4}, \frac{1}{8}, \frac{1}{16}$
		$a_n + 1 = a_n \div 2$, *where* $a_1 = 1$ *and* $n \geq$
		
		• A theater has 60 seats in the first row, 68 seats in the second row, 76 seats in the third row, and so on in the same increasing pattern.
		  o If the theater has 20 rows of seats, how many seats are in the twentieth row?
		  o Explain why the sequence is considered a function.
		  o What is the domain of the sequence? Explain what the domain represents in context.	

F.IF.B.6	**B. Interpret functions that arise in applications in terms of the context** Calculate and interpret the average rate of change of a function (presented symbolically or as a table) over a specified interval. Estimate the rate of change from a graph. ***This is a modeling standard which means students choose and use appropriate mathematics to analyze situations. Thus, contextual situations that require students to determine the correct mathematical model and use the model to solve problems are essential.***	**Explanation:** Students were first introduced to the concept of rate of change in grade 6 and continued exploration of the concept throughout grades 7 and 8. In Algebra I, students will extend this knowledge to non-linear functions. This standard will be explored further in topic D. This standard is taught in Algebra I and Algebra II. In Algebra I, tasks have a real-world context and are limited to linear functions, quadratic functions, square root functions, cube root functions, piecewise-defined functions (including step functions and absolute value functions), and exponential functions with domains in the integers. **Examples:** • Let us understand the difference between $f(n) = 2n$ and $f(n) = 2^n$. Complete the tables below, and then graph the points	**Eureka Math:** Module 3 Lesson 4-7 **This standard will be revisited in Units 3 and 5.**

(n,$f(n)$) on a coordinate plane for each of the formulas.

n	$f(n) = 2n$
-2	-4
-1	-2
0	0
1	2
2	4
3	6

n	$f(n) = 2^n$
-2	$\frac{1}{4}$
-1	$\frac{1}{2}$
0	1
1	2
2	4
3	8



Describe the change in each sequence when n increases by 1 unit for each sequence.

For the sequence $f(n) = 2n$, for every increase in n by 1 unit, the $f(n)$ value increases by 2 units. For the sequence $f(n) = 2^n$, for every increase in n by 1 unit, the $f(n)$ value increases by a factor of 2.

F.BF.A.1a	**A. Build a function that models a relationship between two quantities** Write a function that describes a relationship between	**Explanation:** This standard is explored further in Topic D. In this topic, it is explored via sequences and exponential growth/decay. The students will	**Eureka Math:** Module 3 Lesson 1-7 **This standard will be**

	two quantities. a. Determine an explicit expression, a recursive process, or steps for calculation from a context. ***This is a modeling standard which means students choose and use appropriate mathematics to analyze situations. Thus, contextual situations that require students to determine the correct mathematical model and use the model to solve problems are essential.***	analyze a given problem to determine the function expressed by identifying patterns in the function's rate of change. Students may use graphing calculators or programs, spreadsheets, or computer algebra systems to model functions. This standard is taught in Algebra I and Algebra II. In Algebra I, tasks have a real-world context and are limited to linear functions, quadratic functions, and exponential functions with domains in the integers. **Example:** • If we fold a rectangular piece of paper in half multiple times and count the number of rectangles created, what type of sequence are we creating? Write a function that describes the situation.	**revisited in Unit 5.**
F.LE.A.1	**A. Construct and compare linear, quadratic, and exponential models and solve problems** Distinguish between situations that can be modeled with linear functions and with exponential functions. a. Prove that linear functions grow by equal differences over equal intervals, and that exponential functions grow by equal factors over equal intervals. b. Recognize situations in which one quantity changes at a constant rate per unit interval relative to another. c. Recognize situations in which a quantity grows or decays by a constant percent rate per unit interval relative to another. ***This is a modeling standard which means students choose and use appropriate mathematics to analyze situations. Thus, contextual situations that require students to determine the correct mathematical model and use the model to solve problems are essential.*** *HS.MP.3.* Construct viable arguments and critique the	**Explanation:** Students may use graphing calculators or programs, spreadsheets, or computer algebra systems to model and compare linear and exponential functions. Students distinguish between a constant rate of change and a constant percent rate of change. Students can investigate functions and graphs modeling different situations involving simple and compound interest. Students can compare interest rates with different periods of compounding (monthly, daily) and compare them with the corresponding annual percentage rate. Spreadsheets and applets can be used to explore and model different interest rates and loan terms. **Examples:** • Town A adds 10 people per year to its population, and town B grows by 10% each year. In 2006, each town has 145 residents. For each town, determine whether the population growth is linear or exponential. Explain. • Sketch and analyze the graphs of the following two situations. What information can you conclude about the types of growth	**Eureka Math:** Module 3 Lesson 1-7 **This standard will be revisited in Unit 5.**

	reasoning of others. *HS.MP.4.* Model with mathematics. *HS.MP.5.* Use appropriate tools strategically. *HS.MP.7.* Look for and make use of structure. *HS.MP.8.* Look for and express regularity in repeated reasoning.	each type of interest has? o Lee borrows \$9,000 from his mother to buy a car. His mom charges him 5% interest a year, but she does not compound the interest. o Lee borrows \$9,000 from a bank to buy a car. The bank charges 5% interest compounded annually. • A cell phone company has three plans. Graph the equation for each plan, and analyze the change as the number of minutes used increases. When is it beneficial to enroll in Plan 1? Plan 2? Plan 3? 1. \$59.95/month for 700 minutes and \$0.25 for each additional minute, 2. \$39.95/month for 400 minutes and \$0.15 for each additional minute, and 3. \$89.95/month for 1,400 minutes and \$0.05 for each additional minute. • A computer store sells about 200 computers at the price of \$1,000 per computer. For each \$50 increase in price, about ten fewer computers are sold. How much should the computer store charge per computer in order to maximize their profit?	
F.LE.A.2	**A. Construct and compare linear, quadratic, and exponential models and solve problems** Construct linear and exponential functions, including arithmetic and geometric sequences, given a graph, a description of a relationship, or two input---output pairs (include reading these from a table). ***This is a modeling standard which means students choose and use appropriate mathematics to analyze situations. Thus, contextual situations that require students to determine the correct mathematical model and use the model to solve problems are essential.***	**Explanation:** This standard is taught in Algebra I and Algebra II. In Algebra I, tasks are limited to constructing linear and exponential functions in simple context (not multi-step). While working with arithmetic sequences, make the connection to linear functions, introduced in 8th grade. Geometric sequences are included as contrast to foreshadow work with exponential functions later in the course. Students may use graphing calculators or programs, spreadsheets, or computer algebra systems to construct linear and exponential functions. **Examples:** • Determine an exponential function of the form $f(x) = ab^x$ using data points from the table. Graph the function and identify the	**Eureka Math:** Module 3 Lesson 1-7 **This standard will be revisited in Unit 5.**

<table>
<tr>
<td></td>
<td></td>
<td>key characteristics of the graph.
<table>
<tr><th>x</th><th>f(x)</th></tr>
<tr><td>0</td><td>1</td></tr>
<tr><td>1</td><td>3</td></tr>
<tr><td>3</td><td>27</td></tr>
</table>
Sara’s starting salary is $32,500. Each year she receives a $700 raise. Write a sequence in explicit form to describe the situation.
</td>
<td></td>
</tr>
<tr>
<td>F.LE.A.3</td>
<td>A. Construct and compare linear, quadratic, and exponential models and solve problems

Observe using graphs and tables that a quantity increasing exponentially eventually exceeds a quantity increasing linearly, quadratically, or (more generally) as a polynomial function.

This is a modeling standard which means students choose and use appropriate mathematics to analyze situations. Thus, contextual situations that require students to determine the correct mathematical model and use the model to solve problems are essential.</td>
<td>Explanation:
Students extend their knowledge of linear functions to compare the characteristics of exponential and quadratic functions; focusing specifically on the value of the quantities. Noting that values of exponential functions are eventually greater than the other function types.

Example:

Kevin and Joseph each decide to invest $100. Kevin decides to invest in an account that will earn $5 every month. Joseph decided to invest in an account that will earn 3% interest every month.

Whose account will have more money in it after two years?
After how many months will the accounts have the same amount of money in them?
Describe what happens as the money is left in the accounts for longer periods of time.

Contrast the growth of the $f(x)=x^3$ and $f(x)=3^x$.

</td>
<td>Eureka Math:
Module 3 Lesson 1-7</td>
</tr>
</table>

HS Algebra I Semester 1 (Quarter 2)

Unit 2: Linear and Exponential Relationships (35 days)

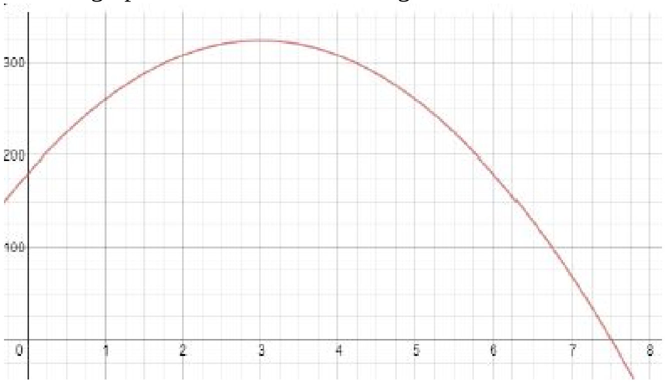
Topic B: Functions and Their Graphs (7 days)

In Topic B, students connect their understanding of functions to their knowledge of graphing from Grade 8. They learn the formal definition of a function and how to recognize, evaluate, and interpret functions in abstract and contextual situations (**F-IF.A.1, F-IF.A.2**). Students examine the graphs of a variety of functions and learn to interpret those graphs using precise terminology to describe such key features as domain and range, intercepts, intervals where the function is increasing or decreasing, and intervals where the function is positive or negative. (**F-IF.A.1, F-IF.B.4, F-IF.B.5, F-IF.C.7a**).

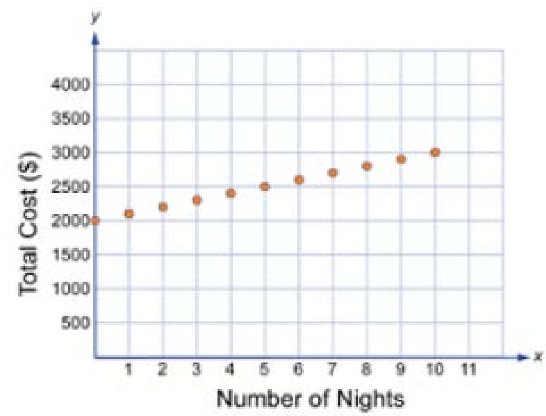
Big Idea:	• A function is a correspondence between two sets, X, and Y, in which each element of X is matched to one and only one element of Y. • The graph of *f* is the same as the graph of the equation y = *f*(*x*). • A function that grows exponentially will eventually exceed a function that grows linearly.
Essential Questions:	• What are the essential parts of a function?
Vocabulary	Function, correspondence between two sets, generic correspondence, range of a function, equivalent functions, identity, notation of *f*, polynomial function, algebraic function, linear function
Assessments	Galileo: Topic B Assessment

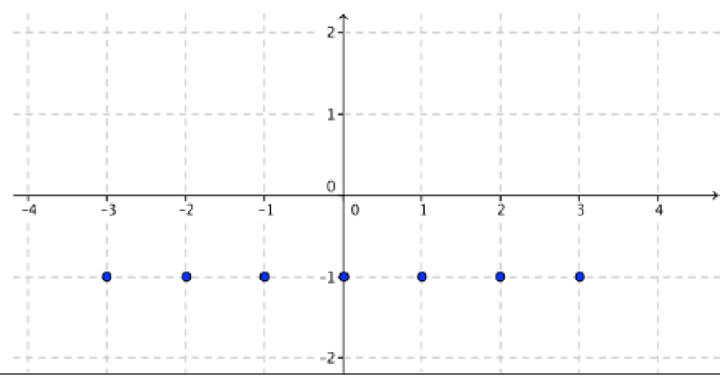
Standard	AZ College and Career Readiness Standards	Explanations & Examples	Resources
F.IF.A.1	**A. Understand the concept of a function and use function notation** Understand that a function from one set (called the domain) to another set (called the range) assigns to each element of the domain exactly one element of the range. If *f* is a function and *x* is an element of its domain, then *f*(*x*) denotes the output of *f* corresponding to the input *x*. The graph of *f* is the graph of the equation *y* = *f*(*x*).	**Explanation:** Students revisit the notion of a function introduced in Grade 8. They are now prepared to use function notation as they write functions, interpret statements about functions and evaluate functions for inputs in their domains. **Examples:** • Is the correspondence described below a function? Explain your reasoning. M:{$women$}→{$people$} Assign each woman their child. ***This is not a function because a woman who is a mother could have more than one child.***	**Eureka Math:** Module 3 Lesson 9-12

F.IF.A.2 ●	**A. Understand the concept of a function and use function notation** Use function notation, evaluate functions for inputs in their domains, and interpret statements that use function notation in terms of a context.	**Explanation:** Students revisit the notion of a function introduced in Grade 8. They are now prepared to use function notation as they write functions, interpret statements about functions and evaluate functions for inputs in their domains. **Examples:** • Create a formula for the area $A(x)$ cm^2 of a square of side length x cm. $A(x) =$ __________ $A(x) = x^2$ Use the formula to determine the area of squares with side lengths of 3 cm, 10.5 cm, and π cm. $A(3) = 9\ cm^2;\ A(10.5) = 110.25\ cm^2;\ A(\pi) = \pi^2\ cm^2$ What does $A(0)$ mean? *In this situation, $A(0)$ has no physical meaning since you cannot have a square whose sides measure 0 cm.* • The function below assigns all people to their biological father. What is the domain and range of the function? o f:{*people*}→{*men*} o *Assign all people to their biological father.* **Domain: all people** **Range: men who are fathers** • ***Let f*:{*positive integers*}→{*perfect squares*}** Assign each term number to the square of that number. o What is (3)? What does it mean? *$f(3)=9$. It is the value of the 3rd square number. 9 dots can be arranged in a 3 by 3 square array.*	**Eureka Math:** Module 3 Lesson 8-10
F.IF.B.4 ▲	**B. Interpret functions that arise in applications in terms of the context**	**Explanation:** This standard is taught in Algebra I and Algebra II. In Algebra I, tasks have a real-world context and they are limited to linear functions,	**Eureka Math:** Module 3 Lesson 8-9, 11-14

	For a function that models a relationship between two quantities, interpret key features of graphs and tables in terms of the quantities, and sketch graphs showing key features given a verbal description of the relationship. *Key features include: intercepts; intervals where the function is increasing, decreasing, positive, or negative; relative maximums and minimums; symmetries; end behavior; and ~~periodicity~~.* ***This is a modeling standard which means students choose and use appropriate mathematics to analyze situations. Thus, contextual situations that require students to determine the correct mathematical model and use the model to solve problems are essential.***	quadratic functions, square-root functions, cube-root functions, piecewise functions (including step functions and absolute-value functions), and exponential functions with domains in the integers. Some functions "tell a story" hence the portion of the standard that has students sketching graphs given a verbal description. Students should have experience with a wide variety of these types of functions and be flexible in thinking about functions and key features using tables and graphs. Examples of these can be found at http://graphingstories.com Students may be given graphs to interpret or produce graphs given an expression or table for the function, by hand or using technology. **Examples:** • The graph represents the height (in feet) of a rocket as a function of the time (in seconds) since it was launched. Use the graph to answer the following:  a. What is the practical domain for t in this context? Why? b. What is the height of the rocket two seconds after it was launched? c. What is the maximum value of the function and what does it mean in context? d. When is the rocket 100 feet above the ground? e. When is the rocket 250 feet above the ground? f. Why are there two answers to part *e* but only one practical answer for part *d*?	**This standard is revisited in Units 3 and 5.**

		g. What are the intercepts of this function? What do they mean in the context of this problem? h. What are the intervals of increase and decrease on the practical domain? What do they mean in the context of the problem? • Marla was at the zoo with her mom. When they stopped to view the lions, Marla ran away from the lion exhibit, stopped, and walked slowly towards the lion exhibit until she was halfway, stood still for a minute then walked away with her mom. Sketch a graph of Marla's distance from the lions' exhibit over the period of time when she arrived until she left. • A *relative minimum* for the function f occurs at the x-coordinate of $\left(\frac{2}{3}\sqrt{3}, -\frac{16}{9}\sqrt{3}\right)$. A similar calculation as you did above shows that this point is also a solution to $y = f(x)$. Plot this point on your graph. *Answer: Students should plot the point* $\left(\frac{2}{3}\sqrt{3}, -\frac{16}{9}\sqrt{3}\right)$ *on their graphs approximately at* $(1.15, -3.08)$. Look at your graph. On what interval(s) is the function f decreasing? *Answer:* $-\frac{2}{3}\sqrt{3} \le x \le \frac{2}{3}\sqrt{3}$ *or* $[-\frac{2}{3}\sqrt{3}, \frac{2}{3}\sqrt{3}]$. Look at your graph. On what interval(s) is the function f increasing? *Answer:* $x \le -\frac{2}{3}\sqrt{3}$ *or* $\frac{2}{3}\sqrt{3} \le x$ *or* $\left(-\infty, -\frac{2}{3}\sqrt{3}\right]$ *or* $\left[\frac{2}{3}\sqrt{3}, \infty\right)$.	
F.IF.B.5 ▲	**B. Interpret functions that arise in applications in terms of the context** Relate the domain of a function to its graph and, where applicable, to the quantitative relationship it describes. *For example, if the function h(n) gives the number of person-hours*	**Explanation:** Students explain the domain of a function from a given context. **Examples:** • Jenna knits scarves and then sells them on Etsy, an online marketplace. Let f(x)=4x+20 represent the cost C in dollars to produce from 1 to 6 scarves. Create a table to show the relationship between the number of scarves x and the cost C.	**Eureka Math:** Module 3 Lesson 8, 11, 12, 14 **This standard is revisited in Units 3 and 5.**

	it takes to assemble n engines in a factory, then the positive integers would be an appropriate domain for the function. ***This is a modeling standard which means students choose and use appropriate mathematics to analyze situations. Thus, contextual situations that require students to determine the correct mathematical model and use the model to solve problems are essential.***	o What are the domain and range of C? o What is the meaning of (3)? o What is the meaning of the solution to the equation f(x)=40? • An all---inclusive resort in Los Cabos, Mexico provides everything for their customers during their stay including food, lodging, and transportation. Use the graph below to describe the domain of the total cost function.  • Oakland Coliseum, home of the Oakland Raiders, is capable of seating 63,026 fans. For each game, the amount of money that the Raiders' organization brings in as revenue is a function of the number of people, n, in attendance. If each ticket costs $30, find the domain of this function. ***Sample Response*: Let *r* represent the revenue that the Raider's organization makes, so that $r=$ (n). Since *n* represents a number of people, it must be a nonnegative whole number. Therefore, since 63,026 is the maximum number of people who can attend a game, we can describe the domain of *f* as follows: Domain = {*n*: $0 \leq n \leq$ 63,026 and *n* is an integer}.	

		The deceptively simple task above asks students to find the domain of a function from a given context. The function is linear and if simply looked at from a formulaic point of view, students might find the formula for the line and say that the domain and range are all real numbers. However, in the context of this problem, this answer does not make sense, as the context requires that all input and output values are non---negative integers, and imposes additional restrictions.	
F.IF.C.7ab	**C. Analyze functions using different representations.** Graph functions expressed symbolically and show key features of the graph, by hand in simple cases and using technology for more complicated cases. a. Graph linear and quadratic functions and show intercepts, maxima, and minima. b. Graph square root, cube root, and piecewise-defined functions, including step functions and absolute value functions. ***This is a modeling standard which means students choose and use appropriate mathematics to analyze situations. Thus, contextual situations that require students to determine the correct mathematical model and use the model to solve problems are essential.***	**Explanation:** Quadratic functions will be formally taught in Module 4. In this module, the focus is on linear functions, piecewise functions (including step functions and absolute-value functions), and exponential functions with domains in the integers. In this topic, the focus is on the use of technology to explore the characteristics of the graphs of functions. **Examples:** Declare x integer Let $f(x) = (x+1)(x-1) - x^2$ Initialize G as {} For all x from -3 to 3 Append $(x, f(x))$ to G Next x Plot G 	**Eureka Math:** Module 3 Lesson 11-14 **This standard is revisited in Unit 3.**

HS Algebra I Semester 1 (Quarter 2)

Unit 2: Linear and Exponential Relationships (35 days)

Topic C: Transformations of Functions (6 days)

In Topic C, students extend their understanding of piecewise functions and their graphs including the absolute value and step functions. They learn a graphical approach to circumventing complex algebraic solutions to equations in one variable, seeing them as $f(x) = g(x)$ and recognizing that the intersection of the graphs of $f(x)$ and $g(x)$ are solutions to the original equation (**A-REI.D.11**). Students use the absolute value function and other piecewise functions to investigate transformations of functions and draw formal conclusions about the effects of a transformation on the function's graph (**F-IF.C.7**, **F-BF.B.3**).

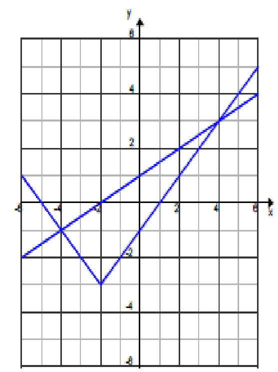
Big Idea:	• Different expressions can be used to define a function over different subsets of the domain. • Absolute value and step functions can be represented as piecewise functions. • The transformation of the function is itself another function (and not a graph).
Essential Questions:	• How do intersection points of the graphs of two functions f and g relate to the solution of an equation in the form $(x)=g(x)$? • What are some benefits of solving equations graphically? What are some limitations?
Vocabulary	Piecewise function, step function, absolute value function, floor function, ceiling function, sawtooth function, vertical scaling, horizontal scaling
Assessments	Galileo: Topic C Assessment

Standard	AZ College and Career Readiness Standards	Explanations & Examples	Resources
A.REI.D.11	**D. Represent and solve equations and inequalities graphically** Explain why the *x*-coordinates of the points where the graphs of the equations $y = f(x)$ and $y = g(x)$ intersect are the solutions of the equation $f(x) = g(x)$; find the solutions approximately, e.g., using technology to graph the functions, make tables of values, or find successive approximations. Include cases where $f(x)$ and/or $g(x)$ are linear, polynomial, rational, absolute value, exponential, ~~and logarithmic~~ functions. ***This is a modeling standard which means students choose and use appropriate mathematics to analyze situations. Thus, contextual situations that require***	**Explanation:** This standard is in Algebra I and Algebra II. In Algebra I, tasks that assess conceptual understanding of the indicated concept may involve any of the function types mentioned except for rational and logarithmic. Finding the solutions approximately is limited to cases where f(x) and g(x) are polynomial functions. Students need to understand that numerical solution methods (data in a table used to approximate an algebraic function) and graphical solution methods may produce approximate solutions, and algebraic solution methods produce precise solutions that can be represented graphically or numerically. Students may use graphing calculators or programs to generate tables of values, graph, or solve a variety of functions.	**Eureka Math:** Module 3 lesson 16 **This standard is revisited in Unit 3.**

<table>
<tr>
<td></td>
<td>students to determine the correct mathematical model and use the model to solve problems are essential.</td>
<td>Examples:

-

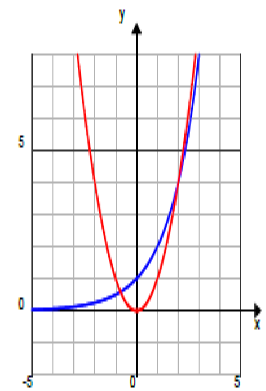
Now let $f(x) = |x+2| - 3$ and $g(x) = 0.5x + 1$. When does $f(x) = g(x)$? To answer this, first graph $y = f(x)$ and $y = g(x)$ on the same set of axes.



When does $f(x) = g(x)$? What is the visual significance of the points where $f(x) = g(x)$?

$f(x) = g(x)$ when $x = 4$ and when $x = -4$; $(4, 3)$ and $(-4, -1)$. The points where $f(x) = g(x)$ are the intersections of the graphs of f and g.

The graphs of the functions f and g are shown.

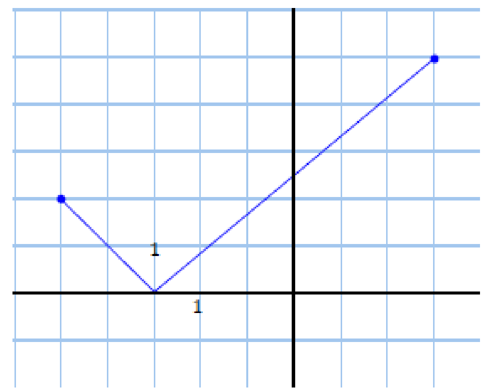
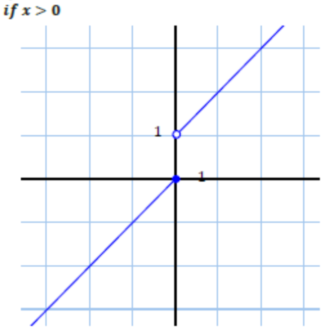


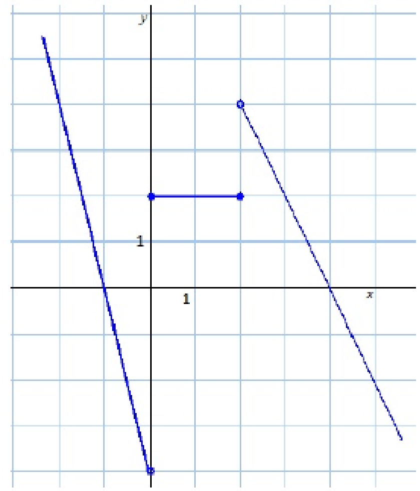
a. Use the graph to *approximate* the solution(s) to the equation $f(x) = g(x)$.

Based on the graphs, the approximate solutions are $\{-0.7, 2\}$.

b. Let $f(x) = x^2$ and let $g(x) = 2^x$. Find <u>*all*</u> solutions to the equation $f(x) = g(x)$. Verify any exact solutions that you determine using the definitions of f and g. Explain how you arrived at your solutions.

By guessing and checking, $x = 4$ is also a solution of the equation because $f(4) = 16$ and $g(4) = 16$. Since the graph of the exponential function is increasing and increases more rapidly than the squaring function, there will only be 3 solutions to this equation. The exact solutions are $x = 2$ and $x = 4$ and an approximate solution is $x = -0.7$.</td>
<td></td>
</tr>
<tr>
<td>F.IF.C.7ab</td>
<td>C. Analyze functions using different representations</td>
<td>Explanation:
Quadratic functions will be formally taught in Module 4. In this module, the focus is on linear functions, piecewise functions (including</td>
<td>Eureka Math:
Module 3 lesson 15, 17, 18</td>
</tr>
</table>

	Graph functions expressed symbolically and show key features of the graph, by hand in simple cases and using technology for more complicated cases. a. Graph linear and quadratic functions and show intercepts, maxima, and minima. b. Graph square root, cube root, and piecewise-defined functions, including step functions and absolute value functions. ***This is a modeling standard which means students choose and use appropriate mathematics to analyze situations. Thus, contextual situations that require students to determine the correct mathematical model and use the model to solve problems are essential.***	step functions and absolute-value functions), and exponential functions with domains in the integers. **Examples:** • Graph. Identify the intercepts, maxima and minima. $f(x) = \|x+3\|$ for $-5 \le x \le 3$  • Graph. Identify the intercepts, maxima and minima. $f(x) = \begin{cases} x & if\ x \le 0 \\ x+1 & if\ x > 0 \end{cases}$  • Write a function that represents the following graph.	**This standard is revisited in Unit 3.**

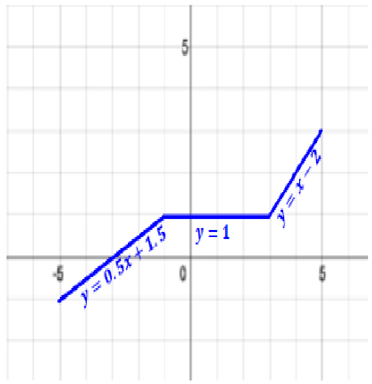
		 $h(x) = \begin{cases} -4x-4 & x<0 \\ 2 & 0 \le x \le 2 \\ -2x+8 & x>2 \end{cases}$	
F.BF.B.3	**B. Build new functions from existing functions** Identify the effect on the graph of replacing *f*(*x*) by *f*(*x*) + *k*, *k f*(*x*), *f*(*kx*), and *f*(*x* + *k*) for specific values of *k* (both positive and negative); find the value of *k* given the graphs. Experiment with cases and illustrate an explanation of the effects on the graph using technology. *Include recognizing even and odd functions from their graphs and algebraic expressions for them.*	**Explanation:** This standard is taught in Algebra I and Algebra II. In Algebra I, focus on vertical and horizontal translations of linear and quadratic functions. Experimenting with cases and illustrating an explanation of the effects on the graph using technology is limited to linear functions, quadratic functions, square root functions, cube root functions, piecewise functions (including step functions and absolute-value functions), and exponential functions with domains in the integers. Tasks in Algebra I do not involve recognizing even and odd functions. **Examples:** • Let $g(x) = \lvert x-5 \rvert$. Graph. Rewrite the function g as a piecewise function. *Solution:*	**Eureka Math:** Module 3 lesson 15, 17, 20 **This standard is revisited in Unit 3.**

		Label the graph of the linear function with negative slope by g_1 and the graph of the linear function with positive slope by g_2 as in the picture above. Function g_1: Slope of g_1 is -1 (why?), and the y-intercept is 5; therefore, $g_1(x) = -x + 5$. Function g_2: Slope of g_2 is 1 (why?), and the y-intercept is -5 (why?); therefore, $g_2(x) = x - 5$. Writing g as a piecewise function is just a matter of collecting all of the different "pieces" and the intervals upon which they are defined: $g(x) = \begin{cases} -x+5 & x < 5 \\ x-5 & x \geq 5 \end{cases}$ How does this graph compare to the graph of the translated absolute value function? ▫ *The graphs are congruent, but the graph of g has been translated to the right 5 units. (Using terms like "congruent" and "translated" reinforces concepts from 8th grade and prepares students for geometry.)* How can you use your knowledge of the graph of $f(x) = \|x\|$ to quickly determine the graph of $g(x) = \|x - 5\|$? ▫ *Watch where the vertex of the graph of f has been translated. In this case, $g(x) = \|x - 5\|$ has translated the vertex point from (0,0) to (5,0). Then, graph a line with a slope of -1 for the piece where $x < 5$ and a line with a slope of 1 for the piece where $x > 5$.* Can we interpret in words what this function does? ▫ *The range values are found by finding the distance between each domain element and the number 5 on the number line.* • **The vertex of the quadratic function $f(x) = x^2$ is at $(0, 0)$, which is the minimum for the graph of f. Based on your work in this lesson, to where do you predict the vertex will be translated for the graphs of $g(x) = (x-2)^2$ and $h(x) = (x+3)^2$?** *The vertex of g will be at $(2, 0)$; The vertex of h will be at $(-3, 0)$.* •	

The graph $y = f(x)$ of a piecewise function f is shown. The domain of f is $-5 \le x \le 5$, and the range is $-1 \le y \le 3$.

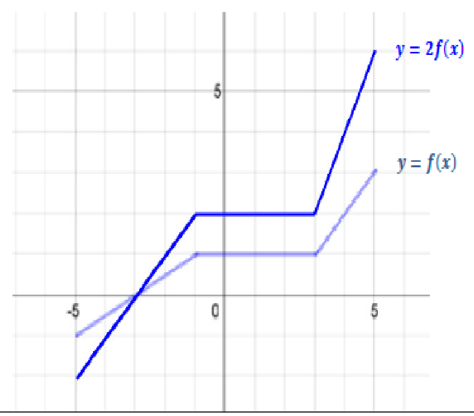
a. Mark and identify four strategic points helpful in sketching the graph of $y = f(x)$.

$(-5,-1)$, $(-1,1)$, $(3,1)$, *and* $(5,3)$



- Sketch the graph of $y = 2f(x)$ and state the domain and range of the transformed function. How can you use part (a) to help sketch the graph of $y = 2f(x)$?

Domain: $-5 \le x \le 5$, range: $-2 \le y \le 6$. For every point (x,y) in the graph of $f(x)$, there is a point $(x,2y)$ on the graph of $y = 2f(x)$. The four strategic points can be used to determine the line segments in the graph of $y = 2f(x)$ by graphing points with the same original x-coordinate and 2 times the original y-coordinate ($(-5,-2)$, $(-1,2)$, $(3,2)$, and $(5,6)$).



MP.3	**Construct viable arguments and critique the reasoning of others.**	They are able to analyze situations by breaking them into cases, and can recognize and use counterexamples. They justify their conclusions, communicate them to others, and respond to the arguments of others	**Eureka Math:** Module 3 lesson 17-19
MP.6	**Attend to precision.**	Mathematically proficient students try to communicate precisely to others. They try to use clear definitions in discussion with others and in their own reasoning. They state the meaning of the symbols they choose, including using the equal sign consistently and appropriately. They are careful about specifying units of measure, and labeling axes to clarify the correspondence with quantities in a problem. They calculate accurately and efficiently, express numerical answers with a degree of precision appropriate for the problem context.	**Eureka Math:** Module 3 lesson 15, 19
MP.8	**Look for and express regularity in repeated reasoning.**	They pay close attention to calculations involving the properties of operations, properties of equality, and properties of inequalities, to find equivalent expressions and solve equations, while recognizing common ways to solve different types of equations.	**Eureka Math:** Module 3 lesson 17,19

HS Algebra I Semester 1 (Quarter 2)

Unit 2: Linear and Exponential Relationships (35 days)

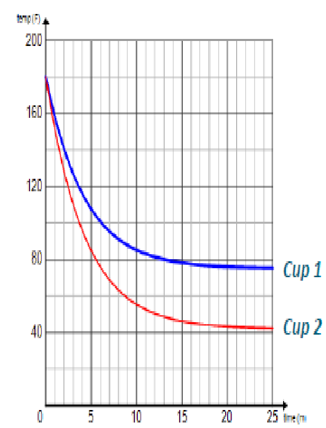
Topic D: Using Functions and Graphs to Solve Problems (4 days)

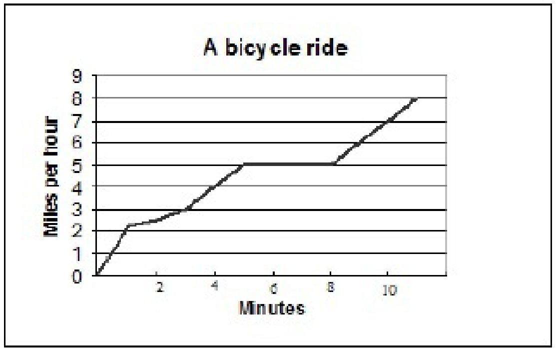
In Topic D, students explore application of functions in real-world contexts and use exponential, linear, and piecewise functions and their associated graphs to model the situations. The contexts include the population of an invasive species, applications of Newton’s Law of Cooling, and long-term parking rates at the Albany International Airport. Students are given tabular data or verbal descriptions of a situation and create equations and scatterplots of the data. They use continuous curves fit to population data to estimate average rate of change and make predictions about future population sizes. They write functions to model temperature over time, graph the functions they have written, and use the graphs to answer questions within the context of the problem. They recognize when one function is a transformation of another within a context involving cooling substances.

Big Idea:	• For every two inputs that are given apart, the difference in their corresponding outputs is constant – dataset could be a linear function. • For every two inputs that are a given difference apart, the quotient if the corresponding outputs is constant-dataset could be an exponential function. • An increasing exponential function will eventually exceed any linear function.
Essential Questions:	• How can you tell whether input-output pairs in a table are describing a linear relationship or an exponential relationship?
Vocabulary	Piecewise function, step function, absolute value function, floor function, ceiling function
Assessment	Galileo: Topic D Assessment

Standard	AZ College and Career Readiness Standards	Explanations & Examples	Resources
A.CED.A.1	**A. Create equations that describe numbers or relationships** Create equations and inequalities in one variable and use them to solve problems. *Include equations arising from linear and quadratic functions, and simple rational and exponential functions.* ***This is a modeling standard which means students choose and use appropriate mathematics to analyze situations. Thus, contextual situations that require students to determine the correct mathematical model and use the model to solve problems are essential.***	**Explanation:** This standard is taught in Algebra I and Algebra II. In Algebra I, tasks are limited to linear, quadratic or **exponential** equations with integer exponents. Students recognize when a problem can be modeled with an equation or inequality and are able to write the equation or inequality. Students create, select, and use graphical, tabular and/or algebraic representations to solve the problem. Equations can represent real world and mathematical problems. Include equations and inequalities that arise when comparing the values of two different functions, such as one describing linear growth and one describing exponential growth. **Examples:**	**Eureka Math:** Module 3 Lesson 21 **This standard is revisited in Units 3 and 5.**

		• Two cups of coffee are poured from the same pot. The initial temperature of the coffee is 180°F and k is 0.2337 (for time in minutes). 1. Suppose both cups are poured at the same time. Cup 1 is left sitting in the room that is 75°F, and cup 2 is taken outside where it is 42°F. a. Use Newton's Law of Cooling to write equations for the temperature of each cup of coffee after t minutes has elapsed. *Cup 1:* $T_1(t) = 75 + (180 - 75) \cdot 2.718^{-0.2337t}$ *Cup 2:* $T_2(t) = 42 + (180 - 42) \cdot 2.718^{-0.2337t}$ • Phil purchases a used truck for \$11,500. The value of the truck is expected to decrease by 20% each year. When will the truck first be worth less than \$1,000? • A scientist has 100 grams of a radioactive substance. Half of it decays every hour. How long until 25 grams remain? Be prepared to share any equations, inequalities, and/or representations used to solve the problem.	
A.SSE.B.3c	**B. Write expressions in equivalent forms to solve problems** Choose and produce an equivalent form of an expression to reveal and explain properties of the quantity represented by the expression. c. Use the properties of exponents to transform expressions for exponential functions. For example the expression 1.15^t can be rewritten as $(1.15^{1/12})^{12t} \approx 1.012^{12t}$ to reveal the approximate equivalent monthly interest rate if the annual rate is 15%. ***This is a modeling standard which means students choose and use appropriate mathematics to analyze situations. Thus, contextual situations that require students to determine the correct mathematical model and use the model to solve problems are essential.***	**Explanation:** Part ***c*** of this standard is taught in Algebra I and Algebra II. In Algebra I, tasks have a real-world context. As described in the standard, there is an interplay between the mathematical structure of the expression and the structure of the situation such that choosing and producing an equivalent form of the expression reveals something about the situation. Tasks are limited to exponential expressions with integer exponents.	**Eureka Math:** Module 3 Lesson 21-24

F.IF.B.4 ▲	**B. Interpret functions that arise in applications in terms of the context** For a function that models a relationship between two quantities, interpret key features of graphs and tables in terms of the quantities, and sketch graphs showing key features given a verbal description of the relationship. *Key features include: intercepts; intervals where the function is increasing, decreasing, positive, or negative; relative maximums and minimums; symmetries; end behavior; and ~~periodicity~~.* ***This is a modeling standard which means students choose and use appropriate mathematics to analyze situations. Thus, contextual situations that require students to determine the correct mathematical model and use the model to solve problems are essential.***	**Explanations:** This standard is taught in Algebra I and Algebra II. In Algebra I, tasks have a real-world context and they are limited to linear functions, quadratic functions, square-root functions, cube-root functions, piecewise functions (including step functions and absolute-value functions), and exponential functions with domains in the integers. Some functions "tell a story" hence the portion of the standard that has students sketching graphs given a verbal description. Students should have experience with a wide variety of these types of functions and be flexible in thinking about functions and key features using tables and graphs. Examples of these can be found at http://graphingstories.com Students may be given graphs to interpret or produce graphs given an expression or table for the function, by hand or using technology. **Examples: (Refer to examples from Topic B in addition to the examples below)** • Graph and label both on the same coordinate plane and compare and contrast the two graphs. 	**Eureka Math:** Module 3 Lesson 21-24 **This standard is revisited in Units 3 and 5.**

<table>
<tr>
<td></td>
<td></td>
<td> Both are decreasing exponentially and have the same y-intercept because they have the same initial temperature. The graph for cup 2 has a larger vertical stretch than cup 1, but cup 1 has a larger vertical translation, which is why they both can have the same initial temperature. The y-values of cup 2 level out lower than the corresponding y-values of cup 1 because of the lower ambient temperature.
The temperature difference (between the cup and the surroundings) drives the cooling. Larger temperature differences lead to faster cooling. This is why the outdoor cup cools much faster. </td>
<td></td>
</tr>
<tr>
<td>F.IF.B.6</td>
<td> B. Interpret functions that arise in applications in terms of the context
Calculate and interpret the average rate of change of a function (presented symbolically or as a table) over a specified interval. Estimate the rate of change from a graph.
This is a modeling standard which means students choose and use appropriate mathematics to analyze situations. Thus, contextual situations that require students to determine the correct mathematical model and use the model to solve problems are essential. </td>
<td> Explanation:
Students were first introduced to the concept of rate of change in grade 6 and continued exploration of the concept throughout grades 7 and 8. In Algebra I, students will extend this knowledge to non-linear functions.
This standard is taught in Algebra I and Algebra II. In Algebra I, tasks have a real-world context and are limited to linear functions, quadratic functions, square root functions, cube root functions, piecewise-defined functions (including step functions and absolute value functions), and exponential functions with domains in the integers.
Examples: (Refer to the examples from Topic A in addition to the ones below)
What is the average rate of change at which this bicycle rider traveled from four to ten minutes of her ride?

</td>
<td> Eureka Math:
Module 3 Lesson 21-22
This standard is revisited in Units 3 and 5. </td>
</tr>
</table>

<table>
<tr>
<td></td>
<td></td>
<td>

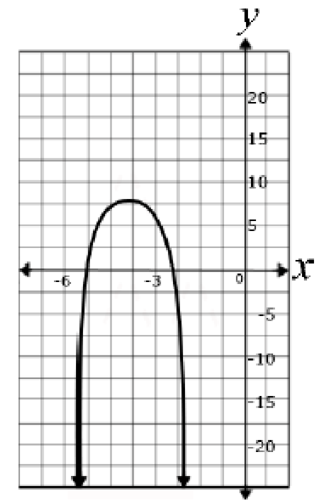
In the table below, assume the function f is deifined for all real numbers. Calculate $\Delta f = f(x+1) - f(x)$ in the last column. What do you notice about Δf? Could the function be linear or exponential? Write a linear or exponential function formula that generates the same input-output pairs as given in the table.

<table>
<tr><th>x</th><th>$f(x)$</th><th>$\Delta f = f(x+1) - f(x)$</th></tr>
<tr><td>0</td><td>2</td><td>$6 - 2 = 4$</td></tr>
<tr><td>1</td><td>6</td><td>$18 - 6 = 12$</td></tr>
<tr><td>2</td><td>18</td><td>$54 - 18 = 36$</td></tr>
<tr><td>3</td><td>54</td><td>$162 - 54 = 108$</td></tr>
<tr><td>4</td><td>162</td><td></td></tr>
</table>
If the entries in the table were considered as a geometric sequence, then the common quotient would be $r = 3$. Since $f(0) = 2$, $a = 2$. Since $f(1) = 6$, we must have $6 = 2 \cdot b$ or $b = 3$. Hence, $f(x) = 2(3)^x$.
In this table, students should see that Δf is not constant for any two inputs that have a difference of 1 unit, which implies that the function cannot be a linear function. However, there is a common quotient between inputs that have a difference of 1 unit: $\frac{6}{2} = \frac{18}{6} = \frac{54}{18} = \frac{162}{54}$. Hence the function f could be exponential.

How do the average rates of change help to support an argument of whether a linear or exponential model is better suited for a set of data?

If the model Δf was growing linearly, then the average rate of change would be constant. However, if it appears to be growing multiplicatively, then it indicates an exponential model.
</td>
<td></td>
</tr>
<tr>
<td>F.IF.C.9</td>
<td>C. Analyze functions using different representation
Compare properties of two functions each represented in a different way (algebraically, graphically, numerically in tables, or by verbal descriptions). For</td>
<td>Explanation:
This standard is taught in Algebra I and Algebra II. In Algebra I, tasks are limited to linear functions, quadratic functions, square root functions, cube root functions, piecewise-defined functions (including step functions and absolute value functions), and exponential functions with domains in the integers.</td>
<td>Eureka Math:
Module 3 Lesson 21-22

This standard is revisited in Unit 3.</td>
</tr>
</table>

	example, given a graph of one quadratic function and an algebraic expression for another, say which has the larger maximum.	**Examples:** • Examine the functions below. Which function has the larger maximum? How do you know? $f(x) = -2x^2 - 8x + 20$  	
F.BF.A.1a	**A. Build a function that models a relationship between two quantities** Write a function that describes a relationship between two quantities. a. Determine an explicit expression, a recursive process, or steps for calculation from a context. ***This is a modeling standard which means students choose and use appropriate mathematics to analyze situations. Thus, contextual situations that require students to determine the correct mathematical model and use the model to solve problems are essential.***	**Explanation:** This standard is taught in Algebra I and Algebra II. In Algebra I, tasks have a real-world context and are limited to linear functions, quadratic functions, and exponential functions with domains in the integers. This standard was introduced in Topic A via sequences. It is explored further in this topic via real-life situations. Students will analyze a given problem to determine the function expressed by identifying patterns in the function's rate of change. They will specify intervals of increase, decrease, constancy, and, if possible, relate them to the function's description in words or graphically. Students may use graphing calculators or programs, spreadsheets, or computer algebra systems to model functions. **Examples:**	**Eureka Math:** Module 3 Lesson 21-24 **This standard is revisited in Unit 5.**

<table>
<tr>
<td></td>
<td></td>
<td>

A cup of coffee is initially at a temperature of 93º F. The difference between its temperature and the room temperature of 68º F decreases by 9% each minute. Write a function describing the temperature of the coffee as a function of time.
The radius of a circular oil slick after t hours is given in feet by $r=10t2-0.5t$, for $0 \le t \le 10$. Find the area of the oil slick as a function of time.

</td>
<td></td>
</tr>
<tr>
<td>F.LE.A.2</td>
<td>
A. Construct and compare linear, quadratic, and exponential models and solve problems

Construct linear and exponential functions, including arithmetic and geometric sequences, given a graph, a description of a relationship, or two input---output pairs (include reading these from a table).

This is a modeling standard which means students choose and use appropriate mathematics to analyze situations. Thus, contextual situations that require students to determine the correct mathematical model and use the model to solve problems are essential.
</td>
<td>
Explanation:
This standard is taught in Algebra I and Algebra II. In Algebra I, tasks are limited to constructing linear and exponential functions in simple context (not multi-step).

While working with arithmetic sequences, make the connection to linear functions, introduced in 8th grade. Geometric sequences are included as contrast to foreshadow work with exponential functions later in the course.

Students may use graphing calculators or programs, spreadsheets, or computer algebra systems to construct linear and exponential functions.

Examples: (Refer to examples from Topic A in addition to the examples below)

Albuquerque boasts one of the longest aerial trams in the world. The tram transports people up to Sandia Peak. The table shows the elevation of the tram at various times during a particular ride.

<table>
<tr><td>Minutes into the ride</td><td>2</td><td>5</td><td>9</td><td>14</td></tr>
<tr><td>Elevation in feet</td><td>7069</td><td>7834</td><td>8854</td><td>10,129</td></tr>
</table>

Write an equation for a function that models the relationship between the elevation of the tram and the number of minutes into the ride.
What was the elevation of the tram at the beginning of the ride?

</td>
<td>
Eureka Math:
Module 3 Lesson 21-24

This standard is revisited in Unit 5.
</td>
</tr>
</table>

<table>
<tr>
<td></td>
<td></td>
<td>
If the ride took 15 minutes, what was the elevation of the tram at the end of the ride?
</td>
<td></td>
</tr>
<tr>
<td>F.LE.B.5</td>
<td>
B. Interpret expressions for functions in terms of the situation they model

Interpret the parameters in a linear or exponential function in terms of a context.

This is a modeling standard which means students choose and use appropriate mathematics to analyze situations. Thus, contextual situations that require students to determine the correct mathematical model and use the model to solve problems are essential.
</td>
<td>
Explanation:
This standard is taught in Algebra I and Algebra II. In Algebra I, tasks have a real-world context. Exponential functions are limited to those with domains in the integers. Use real-world situations to help students understand how the parameters of linear and exponential functions depend on the context.

Students may use graphing calculators or programs, spreadsheets, or computer algebra systems to model and interpret parameters in linear, quadratic or exponential functions.

Examples:

A plumber who charges $50 for a house call and $85 per hour can be expressed as the function $y = 85x + 50$. If the rate were raised to $90 per hour, how would the function change?

Tim deposits money in a Certificate of Deposit account. The balance (in dollars) in his account t years after making the deposit is given by $T(t) = 1{,}000(1.06)^t$ for $t \geq 0$.
a. Explain, in terms of the structure of the expression used to define $T(t)$, why Tim's balance can never be $999.

$(1.06)^0 = 1$ and positive powers of 1.06 are larger than 1, thus the minimum value $T(t)$ attains, if $t \geq 0$, is $1{,}000$. In the context given, a CD account grows in value over time, so with a deposit of $\$1{,}000$ the value will never drop to $999.

Lauren keeps records of the distances she travels in a taxi and what it costs:

<table>
<tr><th>Distance d in miles</th><th>Fare f in dollars</th></tr>
<tr><td>3</td><td>8.25</td></tr>
<tr><td>5</td><td>12.75</td></tr>
<tr><td>11</td><td>26.25</td></tr>
</table>
If you graph the ordered pairs (d, f) from the table,
</td>
<td>
Eureka Math:
Module 3 Lesson 21-24
</td>
</tr>
</table>

		they lie on a line. How can this be determined without graphing them? o Show that the linear function in part a. has equation $F= 2.25d+ 1.5$. o What do the 2.25 and the 1.5 in the equation represent in terms of taxi rides.	
MP.2	**Reason abstractly and quantitatively.**	Students analyze graphs of non-constant rate measurements and apply reason (from the shape of the graphs) to infer the quantities being displayed and consider possible units to represent those quantities.	**Eureka Math:** Module 3 Lesson 22
MP.4	**Model with mathematics.**	Students have numerous opportunities to solve problems that arise in everyday life, society, and the workplace (e.g., modeling bacteria growth and understanding the federal progressive income tax system).	**Eureka Math:** Module 3 Lesson 22 Module 3 Lesson 23
MP.5	**Use appropriate tools strategically.**	Mathematically proficient students consider the available tools when solving a mathematical problem. These tools might include pencil and paper, concrete models, a ruler, a protractor, a calculator, a spreadsheet, a computer algebra system, a statistical package, or dynamic geometry software. They are able to use technological tools to explore and deepen their understanding of concepts.	**Eureka Math:** Module 3 Lesson 24
MP.7	**Look for and make use of structure.**	Students reason with and analyze collections of equivalent expressions to see how they are linked through the properties of operations. They discern patterns in sequences of solving equation problems that reveal structures in the equations themselves. (e.g., $2x$+4=10, 2(x−3)+4=10, 2(3x−4)+4=10)	**Eureka Math:** Module 3 Lesson 21