HIGLEY UNIFIED SCHOOL DISTRICT
INSTRUCTIONAL ALIGNMENT

HS Algebra II Semester 1

Module 1: Polynomial, Rational, and Radical Relationships (45 days)

Topic A: Polynomials – From Base Ten to Base X (11 instructional days)

In Topic A, students draw on their foundation of the analogies between polynomial arithmetic and base ten computation, focusing on properties of operations, particularly the distributive property. In Lesson 1, students write polynomial expressions for sequences by examining successive differences. They are engaged in a lively lesson that emphasizes thinking and reasoning about numbers and patterns and equations. In Lesson 2, they use a variation of the area model referred to as the tabular method to represent polynomial multiplication and connect that method back to application of the distributive property.

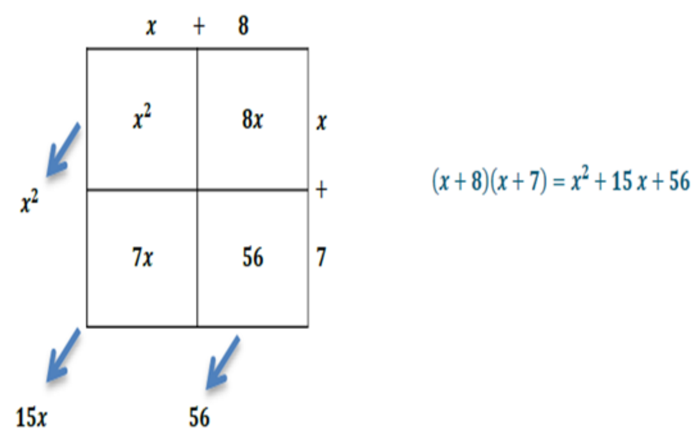
In Lesson 3, students continue using the tabular method and analogies to the system of integers to explore division of polynomials as a missing factor problem. In this lesson, students also take time to reflect on and arrive at generalizations for questions such as how to predict the degree of the resulting sum when adding two polynomials. In Lesson 4, students are ready to ask and answer whether long division can work with polynomials too and how it compares with the tabular method of finding the missing factor. Lesson 5 gives students additional practice on all operations with polynomials and offers an opportunity to examine the structure of expressions such as recognizing that $n(n+1)(2n+1)$ /6
is a 3rd degree polynomial expression with leading coefficient 13 without having to expand it out.

In Lesson 6, students extend their facility with dividing polynomials by exploring a more generic case; rather than dividing by a factor such as $(x+3)$, they divide by the factor $(x+a)$ or $(x-a)$. This gives them the opportunity to discover the structure of special products such as $(x-a)(x^2 + ax + a^2)$ in Lesson 7 and go on to use those products in Lessons 8–10 to employ the power of algebra over the calculator. In Lesson 8, they find they can use special products to uncover mental math strategies and answer questions such as whether or not 2100 − 1 is prime. In Lesson 9, they consider how these properties apply to expressions that contain square roots. Then, in Lesson 10, they use special products to find Pythagorean triples.

The topic culminates with Lesson 11 and the recognition of the benefits of factoring and the special role of zero as a means for solving polynomial equations.

Big Idea:	• Polynomials form a system analogous to the integers. • Polynomials can generalize the structure of our place value system and of radical expressions.
Essential Questions:	• How is polynomial arithmetic similar to integer arithmetic? • What does the degree of a polynomial tell you about its related polynomial function?
Vocabulary	Numerical symbol, variable symbol, algebraic expression, numerical expression, monomial, binomial, polynomial expression, sequence, arithmetic sequence, equivalent polynomial expressions, polynomial identity, coefficient of a monomial, terms of a polynomial, like terms of a polynomial, standard form of a polynomial in one variable, degree of a polynomial in one variable, conjugate, Pythagorean Theorem, converse to the Pythagorean Theorem, Pythagorean Triple, polynomial function, degree of a polynomial function, constant function, linear function, quadratic function, cubic function, zeros or roots of a function
Assessments	Galileo: Geometry Module 1 Foundational Skills Assessment; Galileo: Topic A Assessment

Standard	**Common Core Standards**	**Explanations & Examples**	**Resources**

A.SSE.A.2	**A. Interpret the structure of expressions** Use the structure of an expression to identify ways to rewrite it. For example, see $x^4 - y^4$ as $(x^2)^2 - (y^2)^2$, thus recognizing it as a difference of squares that can be factored as $(x^2 - y^2)(x^2 + y^2)$.	Note: If students have trouble with evaluating or simplifying expressions or solving equations, then you might want to revisit Lessons 6–9 in Grade 9, Module 1, and Lesson 2 in Grade 9, Module 4. Use tabular method to multiply $(x+8)(x+7)$ and combine like terms. 	**Eureka Math:** Module 1 Lesson 2 - 9 Module 1 Lesson 10-11
A.APR.C.4	**C. Use polynomial identities to solve problems** Prove polynomial identities and use them to describe numerical relationships. For example, the polynomial identity $(x^2+y^2)2 = (x^2 - y^2)^2 + (2xy)^2$ can be used to generate Pythagorean triples.		**Eureka Math:** Module 1 Lesson 2 – 7 Module 1 Lesson 10
MP.1	**Make sense of problems and persevere in solving them.**	Students discover the value of equating factored terms of a polynomial to zero as a means of solving equations involving polynomials.	**Eureka Math:** Module 1 Lesson 1 Module 1 Lesson 2 Module 1 Lesson 11
MP.2	**Reason abstractly and quantitatively.**	Students apply polynomial identities to detect prime numbers and discover Pythagorean triples.	**Eureka Math:** Module 1 Lesson 4 Module 1 Lesson 8

MP.3	**Construct viable arguments and critique the reasoning of others.**	Mathematically proficient students understand and use stated assumptions, definitions, and previously established results in constructing arguments. They make conjectures and build a logical progression of statements to explore the truth of their conjectures. They are able to analyze situations by breaking them into cases, and can recognize and use counterexamples. They justify their conclusions, communicate them to others, and respond to the arguments of others.	**Eureka Math:** Module 1 Lesson 5 Module 1 Lesson 7 Module 1 Lesson 8, 9
MP.6	**Attend to precision.**	Mathematically proficient students try to communicate precisely to others. They try to use clear definitions in discussion with others and in their own reasoning. They state the meaning of the symbols they choose, including using the equal sign consistently and appropriately. They calculate accurately and efficiently, express numerical answers with a degree of precision appropriate for the problem context.	**Eureka Math:** Module 1 Lesson 10
MP.7	**Look for and make use of structure.**	Students connect long division of polynomials with the long-division algorithm of arithmetic and perform polynomial division in an abstract setting to derive the standard polynomial identities.	**Eureka Math:** Module 1 Lesson 1 – 6 Module 1 Lesson 8, 9, 10
MP.8	**Look for and express regularity in repeated reasoning.**	Students understand that polynomials form a system analogous to the integers. Students apply polynomial identities to detect prime numbers and discover Pythagorean triples. Students recognize factors of expressions and develop factoring techniques.	**Eureka Math:** Module 1 Lesson 1 - 4 Module 1 Lesson 6 Module 1 Lesson 8 Module 1 Lesson Module 1 Lesson

HS Algebra II Semester 1

Module 1: Polynomial, Rational, and Radical Relationships (45 days)
Topic B: Factoring – Its Use and Its Obstacles (10 instructional days)

Armed with a newfound knowledge of the value of factoring, students develop their facility with factoring and then apply the benefits to graphing polynomial equations in Topic B. In Lessons 12–13, students are presented with the first obstacle to solving equations successfully. While dividing a polynomial by a given factor to find a missing factor is easily accessible, factoring without knowing one of the factors is challenging. Students recall the work with factoring done in Algebra I and expand on it to master factoring polynomials with degree greater than two, emphasizing the technique of factoring by grouping.

In Lessons 14–15, students find that another advantage to rewriting polynomial expressions in factored form is how easily a polynomial function written in this form can be graphed. Students read word problems to answer polynomial questions by examining key features of their graphs. They notice the relationship between the number of times a factor is repeated and the behavior of the graph at that zero (i.e., when a factor is repeated an even number of times, the graph of the polynomial will touch the x-axis and "bounce" back off, whereas when a factor occurs only once or an odd number of times, the graph of the polynomial at that zero will "cut through" the x-axis). In these lessons, students will compare hand plots to graphing- calculator plots and zoom in on the graph to examine its features more closely.

In Lessons 16–17, students encounter a series of more serious modeling questions associated with polynomials, developing their fluency in translating between verbal, numeric, algebraic, and graphical thinking. One example of the modeling questions posed in this lesson is how to find the maximum possible volume of a box created from a flat piece of cardboard with fixed dimensions.

In Lessons 18–19, students are presented with their second obstacle: "What if there is a remainder?" They learn the Remainder Theorem and apply it to further understand the connection between the factors and zeros of a polynomial and how this relates to the graph of a polynomial function. Students explore how to determine the smallest possible degree for a depicted polynomial and how information such as the value of the y-intercept will be reflected in the equation of the polynomial.

The topic culminates with two modeling lessons (Lessons 20–21) involving approximating the area of the cross-section of a riverbed to model the volume of flow. The problem description includes a graph of a polynomial equation that could be used to model the situation, and students are challenged to find the polynomial equation itself.

Big Idea:	•
Essential Questions:	• What impact does an even- or odd-degree polynomial function have on its graph? • How do polynomials helps solve real-world problems?
Vocabulary	Difference of squares identity, multiplicities, zeros or roots, relative maximum (maxima), relative minimum (minima), end behavior, even function, odd function, remainder theorem, factor theorem
Assessments	Galileo: Topic B Assessment

Standard	**Common Core Standards**	**Explanations & Examples**	**Resources**

N.Q.A.2	**A. Reason qualitatively and units to solve problems** Define appropriate quantities for the purpose of descriptive modeling.		**Eureka Math:** Module 1 Lesson 15, 16, 17, 20, 21
A.SSE.A.2	**A. Interpret the structure of expressions** Use the structure of an expression to identify ways to rewrite it. *For example, see* $x^4 – y^4$ *as* $(x^2)^2 – (y^2)^2$*, thus recognizing it as a difference of squares that can be factored as* $(x^2 – y^2)(x^2 + y^2)$*.*		**Eureka Math:** Module 1 Lesson 12 – 14 Module 1 Lesson 17 - 21

<table>
<tr>
<td>A.APR.B.2</td>
<td>B. Understand the relationship between zeros and factors of polynomials

Know and apply the Remainder Theorem: For a polynomial $p(x)$ and a number a, the remainder on division by $x - a$ is $p(a)$, so $p(a) = 0$ if and only if $(x - a)$ is a factor of $p(x)$.</td>
<td>Use the Factor Theorem to determine whether $x - 1$ is a factor of $f(x) = 2x^4 + 3x^2 - 5x + 7$

$$\begin{array}{r|rrrrr} 1 & 2 & 0 & 3 & -5 & 7 \\ & & 2 & 2 & 5 & 0 \\ \hline & 2 & 2 & 5 & 0 & 7 \end{array}$$

$x - 1$ is not a factor of $f(x)$

Using the Factor Theorem, verify that $x + 4$ is a factor of $f(x) = 5x^4 + 16x^3 - 15x^2 + 8x + 16$

$$\begin{array}{r|rrrrr} -4 & 5 & 16 & -15 & 8 & 16 \\ & & -20 & 16 & -4 & -16 \\ \hline & 5 & -4 & 1 & 4 & 0 \end{array}$$

$x + 4$ is a factor of $5x^4 + 16x^3 - 15x^2 + 8x + 16$

The Remainder theorem says that if a polynomial $p(x)$ is divided by $x - a$, then the remainder is the constant $p(a)$. That is, $p(x) = q(x)(x - a) + p(a)$. So if p(a) = 0 then p(x) = q(x)(x-a).
Include problems that involve interpreting the Remainder Theorem from graphs and in problems that require long division.</td>
<td>Eureka Math:
Module 1 Lesson 19 - 21</td>
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<tr>
<td>A.APR.B.3</td>
<td>B. Understand the relationship between zeros and factors of polynomials</td>
<td></td>
<td>Eureka Math:
Module 1 Lesson 14, 15,</td>
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	Identify zeros of polynomials when suitable factorizations are available, and use the zeros to construct a rough graph of the function defined by the polynomial.		17, 19, 20, 21
A.APR.D.6	**D. Rewrite rational expressions** Rewrite simple rational expressions in different forms; write $a(x)/b(x)$ in the form $q(x) + r(x)/b(x)$, where $a(x)$, $b(x)$, $q(x)$, and $r(x)$ are polynomials with the degree of $r(x)$ less than the degree of $b(x)$, using inspection, long division, or, for the more complicated examples, a computer algebra system.	The polynomial $q(x)$ is called the quotient and the polynomial $r(x)$ is called the remainder. Expressing a rational expression in this form allows one to see different properties of the graph, such as horizontal asymptotes. Simplify $\frac{x^2+9x+14}{x+7}$ $x + 7 \overline{) x^2 + 9x + 14}$ with quotient $x + 2$ $-x^2 - 7x$ $2x + 14$ $-2x - 14$ 0	**Eureka Math:** Module 1 Lesson 12, 13, 18, 19, 20, 21
F.IF.C.7c	**C. Analyze functions using different representation** Graph functions expressed symbolically and show key features of the graph, by hand in simple cases and using technology for more complicated cases. c. Graph polynomial functions, identifying zeros when suitable factorizations are available, and showing end behavior.		**Eureka Math:** Module 1 Lesson 14, 15, 17, 19, 20, 21

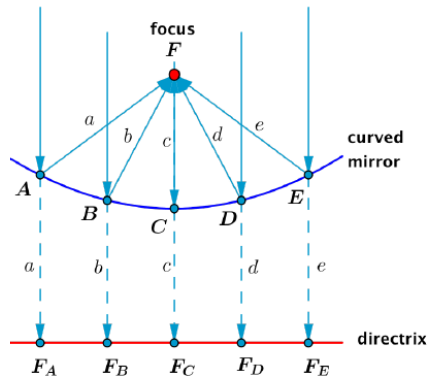
MP.1	**Make sense of problems and persevere in solving them.**	Students discover the value of equating factored terms of a polynomial to zero as a means of solving equations involving polynomials.	**Eureka Math:** Module 1 Lesson 12 Module 1 Lesson 20
MP.2	**Reason abstractly and quantitatively.**	Students apply polynomial identities to detect prime numbers and discover Pythagorean triples. Students also learn to make sense of remainders in polynomial long division problems.	**Eureka Math:** Module 1 Lesson 17
MP.3	**Construct viable arguments and critique the reasoning of others.**	Mathematically proficient students understand and use stated assumptions, definitions, and previously established results in constructing arguments. They make conjectures and build a logical progression of statements to explore the truth of their conjectures. They are able to analyze situations by breaking them into cases, and can recognize and use counterexamples. They justify their conclusions, communicate them to others, and respond to the arguments of others. They reason inductively about data, making plausible arguments that take into account the context from which the data arose. Mathematically proficient students are also able to compare the effectiveness of two plausible arguments, distinguish correct logic or reasoning from that which is flawed, and—if there is a flaw in an argument—explain what it is.	**Eureka Math:** Module 1 Lesson 14, 15, 16, 17
MP.5	**Use appropriate tools strategically.**	Mathematically proficient students consider the available tools when solving a mathematical problem. These tools might include pencil and paper, concrete models, a ruler, a protractor, a calculator, a spreadsheet, a computer algebra system, a statistical package, or dynamic geometry software. Proficient students are sufficiently familiar with tools appropriate for their grade or course to make sound decisions about when each of these tools might be helpful, recognizing both the insight to be gained and their limitations. For example, mathematically proficient high school students analyze graphs of functions and solutions generated using a graphing calculator. They detect possible errors by strategically using estimation and other mathematical knowledge. When making mathematical models, they know that technology can enable them to visualize the results of varying assumptions, explore consequences, and compare predictions with data. They are able to use technological tools to explore and deepen their understanding of concepts.	**Eureka Math:** Module 1 Lesson 14 Module 1 Lesson 21
MP.7	**Look for and make use of structure.**	Students connect long division of polynomials with the long-division algorithm of arithmetic and perform polynomial division in an abstract setting to derive the standard polynomial identities. Students recognize structure in the graphs of polynomials in factored form and develop refined techniques for graphing.	**Eureka Math:** Module 1 Lesson 12 Module 1 Lesson 13 Module 1 Lesson 14 Module 1 Lesson 18

			Module 1 Lesson 20
MP.8	**Look for and express regularity in repeated reasoning.**	Students understand that polynomials form a system analogous to the integers. Students apply polynomial identities to detect prime numbers and discover Pythagorean triples. Students recognize factors of expressions and develop factoring techniques.	**Eureka Math:** Module 1 Lesson 15 Module 1 Lesson 19

HS Algebra II Semester 1			
Module 1: Polynomial, Rational, and Radical Relationships (45 days) **Topic C: Solving and Applying Equations – Polynomial, Rational and Radical (14 instructional days)** In Topic C, students continue to build upon the reasoning used to solve equations and their fluency in factoring polynomial expressions. In Lesson 22, students expand their understanding of the division of polynomial expressions to rewriting simple rational expressions (A-APR.D.6) in equivalent forms. In Lesson 23, students learn techniques for comparing rational expressions numerically, graphically, and algebraically. In Lessons 24–25, students learn to rewrite simple rational expressions by multiplying, dividing, adding, or subtracting two or more expressions. They begin to connect operations with rational numbers to operations on rational expressions. The practice of rewriting rational expressions in equivalent forms in Lessons 22–25 is carried over to solving rational equations in Lessons 26 and 27. Lesson 27 also includes working with word problems that require the use of rational equations. In Lessons 28–29, we turn to radical equations. Students learn to look for extraneous solutions to these equations as they did for rational equations. In Lessons 30–32, students solve and graph systems of equations including systems of one linear equation and one quadratic equation and systems of two quadratic equations. Next, in Lessons 33–35, students study the definition of a parabola as they first learn to derive the equation of a parabola given a focus and a directrix and later to create the equation of the parabola in vertex form from the coordinates of the vertex and the location of either the focus or directrix. Students build upon their understanding of rotations and translations from Geometry as they learn that any given parabola is congruent to the one given by the equation $y = ax^2$ for some value of a and that all parabolas are similar.			
Big Idea:	• Systems of non-linear functions create solutions more complex than those of systems of linear functions. • Mathematicians use the focus and directix of a parabola to derive an equation.		
Essential Questions:	• How do you reduce a rational expression to lowest terms? • How do you compare the values of rational expressions? • Why is it important to check the solutions of a rational or radical equation? • Why are solving systems of nonlinear functions different than systems of linear functions? • What does the focus and directix define a parabola? • What conditions will two parabolas be congruent?		
Vocabulary	Rational expression, complex fraction, equating numerators method, equating fractions method, extraneous solution, linear systems, parabola, axis of symmetry of a parabola, vertex of a parabola, paraboloid, focus, directrix, conic sections, eccentricity, vertical scaling, horizontal scaling, dilation		
Assessments	Galileo: Topic C Assessment		
Standard	**Common Core Standards**	**Explanations & Examples**	**Resources**
A.APR.D.6	**D. Rewrite rational expressions** Rewrite simple rational expressions in different forms; write $a(x)/b(x)$ in the form $q(x) + r(x)/b(x)$, where $a(x)$,		**Eureka Math:** Module 1 Lesson 22 - 27

	$b(x)$, $q(x)$, and $r(x)$ are polynomials with the degree of $r(x)$ less than the degree of $b(x)$, using inspection, long division, or, for the more complicated examples, a computer algebra system.		
++A.APR.D.7	**D. Rewrite rational expressions** Understand that rational expressions form a system analogous to the rational numbers, closed under addition, subtraction, multiplication, and division by a nonzero rational expression; add, subtract, multiply, and divide rational expressions.	**HONORS ONLY** A major theme of the module is A-APR.7. Teachers should continually remind students of the connections between rational expressions and rational numbers as students add, subtract, multiply and divide rational expressions. **Examples:** • Use the formula for the sum of two fractions to explain why the sum of two rational expressions is another rational expression. • Express $\frac{1}{x^2+1} - \frac{1}{x^2-1}$ in the form $a(x)/b(x)$, where $a(x)$ and $b(x)$ are polynomials.	
A.REI.A.1	**A. Understand solving equations as a process of reasoning and explain the reasoning** Explain each step in solving a simple equation as following from the equality of numbers asserted at the previous step, starting from the assumption that the original equation has a solution. Construct a viable argument to justify a solution method.	In Algebra II, tasks are limited to simple rational or radical equations. Properties of operations can be used to change expressions on either side of the equation to equivalent expressions. In addition, adding the same term to both sides of an equation or multiplying both sides by a non-zero constant produces an equation with the same solutions. Other operations, such as squaring both sides, may produce equations that have extraneous solutions. **Examples:** • Explain why the equation $x/2 + 7/3 = 5$ has the same solutions as the equation $3x + 14 = 30$. Does this mean that $x/2 + 7/3$ is equal to $3x + 14$? • Show that $x = 2$ and $x = -3$ are solutions to the equation $x^2 + x = 6$. Write the equation in a form that shows these are the only solutions, explaining each step in your reasoning.	**Eureka Math:** Module 1 Lesson 24-25 Module 1 Lesson 28-29
A.REI.A.2	**A. Understand solving equations as a process of reasoning and explain the reasoning** Solve simple rational and radical equations in one variable, and give examples showing how extraneous solutions may arise.	**Examples:** • $\frac{x+2}{x+3} = 2$	**Eureka Math:** Module 1 Lesson 24-29

A.REI.B.4b	**B. Solve equations and inequalities in one variable** Solve quadratic equations in one variable. b. Solve quadratic equations by inspection (e.g., for x2 = 49), taking square roots, completing the square, the quadratic formula and factoring, as appropriate to the initial form of the equation. Recognize when the quadratic formula gives complex solutions and write them as a ± bi for real numbers a and b.	In Algebra II, in the case of equations having roots with nonzero imaginary parts, students write the solutions as a+/-bi where a and b are real numbers.	**Eureka Math:** Module 1 Lesson 31
A.REI.C.6	**C. Solve systems of equations** Solve systems of linear equations exactly and approximately (e.g., with graphs), focusing on pairs of linear equations in two variables.	In Algebra II, tasks are limited to 3x3 systems.	**Eureka Math:** Module 1 Lesson 30-31
A.REI.C.7	**C. Solve systems of equations** Solve a simple system consisting of a linear equation and a quadratic equation in two variables algebraically and graphically. For example, find the points of intersection between the line $y = -3x$ and the circle $x^2 + y^2 = 3$.	Example:	**Eureka Math:** Module 1 Lesson 31

G.GPE.A.2	**A. Translate between the geometric description and the equation for a conic section** Derive the equation of a parabola given a focus and directrix.	 Given a focus and a directrix, create an equation for a parabola. Focus: $\boldsymbol{F}=(\mathbf{0},\mathbf{2})$ Directrix: $\boldsymbol{x}$-axis Parabola: $\boldsymbol{P}=\{(\boldsymbol{x},\boldsymbol{y}) \mid (\boldsymbol{x},\boldsymbol{y})$ is equidistant to *F* and to the $\boldsymbol{x}$-axis.} Let $\boldsymbol{A}$ be any point $(\boldsymbol{x},)$ on the parabola $\boldsymbol{P}$. Let $\boldsymbol{F}'$ be a point on the directrix with the same $\boldsymbol{x}$-coordinate as point $\boldsymbol{A}$. What is the length of $\boldsymbol{A}$F′? $\boldsymbol{AF}'=\boldsymbol{y}$ Use the distance formula to create an expression that represents the length of AF. $\boldsymbol{A}$F= sqtrt$(\boldsymbol{x}-\mathbf{0})^2+(\boldsymbol{y}-\mathbf{2})^2$ Identify the focus and directrix of the parabola given by $y^2=-4x$ Identify the focus and directrix of the parabola given by $x^2=12y$ Write the standard form of the equation of the parabola with its Vertex at (0,0) and focus at (0, −4) Write the standard form of the equation of the parabola with its vertex at (0, 0) and directrix y = 5 Write the standard form of the equation of the parabola with its vertex at (0, 0) and directrix $x = 2$	**Eureka Math:** Module 1 Lesson 33-34

		Create an equation that relates the two lengths and solve it for y. *Therefore,* $P = \left\{(x,y) \mid \sqrt{(x-0)^2+(y-2)^2} = y\right\}$. *The two segments have equal lengths.* $AF' = AF$ *The length of each segment.* $y = \sqrt{(x-0)^2+(y-2)^2}$ *Square both sides of the equation.* $y^2 = x^2 + (y-2)^2$ *Expand the binomial.* $y^2 = x^2 + y^2 - 4y + 4$ *Solve for y.* $4y = x^2 + 4$ $y = \frac{1}{4}x^2 + 1$ *Replacing this equation in the definition of* $P = \{(x,y) \mid (x,y)$ *is equidistant to* F *and to the* x*-axis*$\}$ *gives the statement* $P = \left\{(x,y) \mid y = \frac{1}{4}x^2 + 1\right\}$. *Thus, the parabola* P *is the graph of the equation* $y = \frac{1}{4}x^2 + 1$. Verify that this equation appears to match the graph shown.	
MP.1	**Make sense of problems and persevere in solving them.**	Students solve systems of linear equations and linear and quadratic pairs in two variables. Further, students come to understand that the complex number system provides solutions to the equation $x^2 + 1 = 0$ and higher-degree equations.	**Eureka Math:** Module 1 Lesson 26-27 Module 1 Lesson 29-31 Module 1 Lesson 33
MP.2	**Reason abstractly and quantitatively.**	Students apply polynomial identities to detect prime numbers and discover Pythagorean triples. Students also learn to make sense of remainders in polynomial long division problems.	**Eureka Math:** Module 1 Lesson 23 Module 1 Lesson 27 Module 1 Lesson 34
MP.3	**Construct viable arguments and critique the reasoning of others.**	Mathematically proficient students understand and use stated assumptions, definitions, and previously established results in constructing arguments. They make conjectures and build a logical progression of statements to explore the truth of their conjectures. They are able to analyze situations by breaking them into cases, and can recognize and use counterexamples. They justify their conclusions, communicate them to others, and respond to the arguments of others.	**Eureka Math:** Module 1 Lesson 23 Module 1 Lesson 28-29 Module 1 Lesson 34-35

MP.4	**Model with mathematics.**	Students use primes to model encryption. Students transition between verbal, numerical, algebraic, and graphical thinking in analyzing applied polynomial problems. Students model a cross-section of a riverbed with a polynomial, estimate fluid flow with their algebraic model, and fit polynomials to data. Students model the locus of points at equal distance between a point (focus) and a line (directrix) discovering the parabola.	**Eureka Math:** Module 1 Lesson 27 Module 1 Lesson 33
MP.5	**Use appropriate tools strategically.**	Mathematically proficient students consider the available tools when solving a mathematical problem. These tools might include pencil and paper, concrete models, a ruler, a protractor, a calculator, a spreadsheet, a computer algebra system, a statistical package, or dynamic geometry software. Proficient students are sufficiently familiar with tools appropriate for their grade or course to make sound decisions about when each of these tools might be helpful, recognizing both the insight to be gained and their limitations. For example, mathematically proficient high school students analyze graphs of functions and solutions generated using a graphing calculator. They detect possible errors by strategically using estimation and other mathematical knowledge.	**Eureka Math:** Module 1 Lesson 31
MP.7	**Look for and make use of structure.**	Students connect long division of polynomials with the long-division algorithm of arithmetic and perform polynomial division in an abstract setting to derive the standard polynomial identities. Students recognize structure in the graphs of polynomials in factored form and develop refined techniques for graphing. Students discern the structure of rational expressions by comparing to analogous arithmetic problems. Students perform geometric operations on parabolas to discover congruence and similarity.	**Eureka Math:** Module 1 Lesson 22 Module 1 Lesson 24-26 Module 1 Lesson 28-30 Module 1 Lesson 34
MP.8	**Look for and express regularity in repeated reasoning.**	Students understand that polynomials form a system analogous to the integers. Students apply polynomial identities to detect prime numbers and discover Pythagorean triples. Students recognize factors of expressions and develop factoring techniques. Further, students understand that all quadratics can be written as a product of linear factors in the complex realm.	**Eureka Math:** Module 1 Lesson 22 Module 1 Lesson 31

HS Algebra II Semester 1			
Module 1: Polynomial, Rational, and Radical Relationships (45 days) **Topic D: A Surprise from Geometry-Complex Numbers Overcome All Obstacles (5 instructional days)** In Topic D, students extend their facility with finding zeros of polynomials to include complex zeros. Lesson 36 presents a third obstacle to using factors of polynomials to solve polynomial equations. Students begin by solving systems of linear and non-linear equations to which no real solutions exist, and then relate this to the possibility of quadratic equations with no real solutions. Lesson 37 introduces complex numbers through their relationship to geometric transformations. That is, students observe that scaling all numbers on a number line by a factor of −1 turns the number line out of its one-dimensionality and rotates it 180° through the plane. They then answer the question, "What scale factor could be used to create a rotation of 90°?" In Lesson 38, students discover that complex numbers have real uses; in fact, they can be used in finding real solutions of polynomial equations. In Lesson 39, students develop facility with properties and operations of complex numbers and then apply that facility to factor polynomials with complex zeros. Lesson 40 brings the module to a close with the result that every polynomial can be rewritten as the product of linear factors, which is not possible without complex numbers. Even though standards N-CN.C.8 and N-CN.C.9 are not assessed at the Algebra II level, they are included instructionally to develop further conceptual understanding.			
Big Idea:	• Every polynomial can be rewritten as the product of linear factors.		
Essential Questions:	•		
Vocabulary	Complex numbers, imaginary, discriminant, conjugate pairs, [Fundamental Theorem of Algebra (Honors only)]		
Assessments	Galileo: Topic D Assessment		
Standard	**Common Core Standards**	**Explanations & Examples**	**Comments**
N.CN.A.1	**A. Perform arithmetic operations with complex numbers** Know there is a complex number *i* such that $i^2 = -1$, and every complex number has the form $a + bi$ with a and b real.	Multiplying by I rotates every complex number in the complex plane by $90°$ about the origin. Every complex number is in the form $\boldsymbol{a} + \boldsymbol{b}$i, where $\boldsymbol{a}$ is the real part and $\boldsymbol{b}$ is the imaginary part of the number. Real numbers are also complex numbers; the real number $\boldsymbol{a}$ can be written as the complex number $\boldsymbol{a} + \mathbf{0}$i.	**Eureka Math:** Module 1 Lesson 37

		Express each of the following in $a + bi$ form. a. i^5 $0 + i$ b. i^6 $-1 + 0i$ c. i^7 $0 - i$ d. i^8 $1 + 0i$ e. i^{102} $-1 + 0i$	
N.CN.A.2	**A. Perform arithmetic operations with complex numbers** Use the relation $i^2 = -1$ and the commutative, associative, and distributive properties to add, subtract, and multiply complex numbers.	Addition and subtraction with complex numbers: $(a + bi) + (c + di) = (a + c) + (b + d)i$ Multiplication with complex numbers $(a + bi) \cdot (c + di) = ac + bci + adi + bd\, i^2$ $= (ac - bd) + (bc + ad)i$ **Example 1: Addition with Complex Numbers** Compute $(3 + 4i) + (7 - 20i)$. $(3 + 4i) + (7 - 20i) = 3 + 4i + 7 - 20i = (3 + 7) + (4 - 20)i = 10 - 16i$ **Example 2: Subtraction with Complex Numbers** Compute $(3 + 4i) - (7 - 20i)$. $(3 + 4i) - (7 - 20i) = 3 + 4i - 7 + 20i = (3 - 7) + (4 + 20)i = -4 + 20i$	**Eureka Math:** Module 1 Lesson 37

		Verify that $-1+2i$ and $-1-2i$ are solutions to $x^2+2x+5=0$. $-1+2i$: $$\begin{aligned}(-1+2i)^2+2(-1+2i)+5 &= 1-4i+4i^2-2+4i+5\\ &= 4i^2-4i+4i+1-2+5\\ &= -4+0+4\\ &= 0\end{aligned}$$ $-1-2i$: $$\begin{aligned}(-1-2i)^2+2(-1-2i)+5 &= 1+4i+4i^2-2-4i+5\\ &= 4i^2+4i-4i+1-2+5\\ &= -4+0+4\\ &= 0\end{aligned}$$ **Example:** • Simplify the following expression. Justify each step using the commutative, associative and distributive properties. $(3-2i)(-7+4i)$ Solutions may vary; one solution follows:	

		$(3-2i)(-7+4i)$ $3(-7+4i)-2i(-7+4i)$ Distributive Property $-21+12i+14i-8i^2$ Distributive Property $-21+(12i+14i)-8i^2$ Associative Property $-21+i(12+14)-8i^2$ Distributive Property $-21+26i-8i^2$ Computation $-21+26i-8(-1)$ $i^2=-1$ $-21+26i+8$ Computation $-21+8+26i$ Commutative Property $-13+26i$ Computation	
N.CN.C.7	**C. Use complex numbers in polynomial identities and equations** Solve quadratic equations with real coefficients that have complex solutions.	**Examples:** • Within which number system can $x^2 = -2$ be solved? Explain how you know. • Solve $x^2 + 2x + 2 = 0$ over the complex numbers. • Find all solutions of $2x^2 + 5 = 2x$ and express them in the form $a + bi$. Write a polynomial P with the lowest possible degree that has the given solutions. Explain how you generated each answer. a. $-2, 3, -4i, 4i$ *The polynomial P has two real zeroes and two complex zeros. Since the two complex zeros are members of a conjugate pair, P may have as few as four total factors. Therefore, P has degree at least 4.* $P(x) = (x+2)(x-3)(x+4i)(x-4i)$ $= (x^2-x-6)(x^2-16i^2)$ $= (x^2-x-6)(x^2+16)$ $= x^4-x^3-6x^2+16x^2-16x-96$ $= x^4-x^3+10x^2-16x-96$	**Eureka Math:** Module 1 Lesson 37-39

++N.CN.C.8	**C. Use complex numbers in polynomial identities and equations** Extend polynomial identities to the complex numbers. *For example, rewrite $x^2 + 4$ as $(x + 2i)(x - 2i)$.*	******HONORS ONLY******	**Eureka Math:** Module 1 Lesson 39
++N.CN.C.9	**C. Use complex numbers in polynomial identities and equations** Know the Fundamental Theorem of Algebra; show that it is true for quadratic polynomials.	******HONORS ONLY******* **Examples:** • How many zeros does $-2x^2 + 3x - 8$ have? Find all the zeros and explain, orally or in written format, your answer in terms of the Fundamental Theorem of Algebra. • How many complex zeros does the following polynomial have? How do you know? $p(x) = (x^2 - 3)(x^2 + 2)(x - 3)(2x - 1)$	**Eureka Math:** Module 1 Lesson 40
A.APR.D.6	**D. Rewrite rational expressions** Rewrite simple rational expressions in different forms; write $a(x)/b(x)$ in the form $q(x) + r(x)/b(x)$, where $a(x)$, $b(x)$, $q(x)$, and $r(x)$ are polynomials with the degree of $r(x)$ less than the degree of $b(x)$, using inspection, long division, or, for the more complicated examples, a computer algebra system.	The polynomial $q(x)$ is called the quotient and the polynomial $r(x)$ is called the remainder. Expressing a rational expression in this form allows one to see different properties of the graph, such as horizontal asymptotes. **Examples:** • Find the quotient and remainder for the rational expression $\frac{x^3-3x^2+x-6}{x^2+2}$ and use them to write the expression in a different form. • Express $f(x) = \frac{2x+1}{x-1}$ in a form that reveals the horizontal asymptote of its graph.	**Eureka Math:** Module 1 Lesson 36-40
++A.APR.D.7	**D. Rewrite rational expressions** Understand that rational expressions form a system analogous to the rational numbers, closed under addition, subtraction, multiplication, and division by a nonzero rational expression; add, subtract, multiply,	******HONORS ONLY******	

	and divide rational expressions.		
A.REI.A.2	**A. Understand solving equations as a process of reasoning and explain the reasoning** Solve simple rational and radical equations in one variable, and give examples showing how extraneous solutions may arise.		
A.REI.B.4b	**B. Solve equations and inequalities in one variable** Solve quadratic equations in one variable. b. Solve quadratic equations by inspection (e.g., for $x^2 = 49$), taking square roots, completing the square, the quadratic formula and factoring, as appropriate to the initial form of the equation. Recognize when the quadratic formula gives complex solutions and write them as a ± bi for real numbers a and b.	**How does the value of the discriminant for each equation relate the number of solutions you found?** ***If the discriminant is negative, we get complex solutions. If the discriminant is zero, we get one real solution. If the discriminant is positive, we get two real solutions.*** Consider the equation $3x + x^2 = -7$. ▪ What does the value of the discriminant tell us about number of solutions to this equation? ▫ The equation in standard form is $x^2 + 3x + 7 = 0$. ▫ $a = 1, b = 3, c = 7$ ▫ The discriminant is $3^2 - 4(1)(7) = -19$. The negative discriminant indicates that no real solutions exist. There are two complex solutions. ▪ Solve the equation. Does the number of solutions match the information provided by the discriminant? Explain. ▫ *Using the quadratic formula,* $x = \frac{-3+\sqrt{-19}}{2}$ *or* $x = \frac{-3-\sqrt{-19}}{2}$. ▫ *The solutions, in* $a + bi$ *form, are* $-\frac{3}{2} + \frac{\sqrt{19}}{2}i$ *and* $-\frac{3}{2} - \frac{\sqrt{19}}{2}i$. ▫ *The two complex solutions are consistent with the rule for a negative discriminant.*	**Eureka Math:** Module 1 Lesson 38
MP.2	**Reason abstractly and quantitatively.**	Mathematically proficient students make sense of quantities and their relationships in problem situations. They bring two complementary abilities to bear on problems involving quantitative relationships: the	**Eureka Math:** Module 1 Lesson 37-38

		ability to *decontextualize*—to abstract a given situation and represent it symbolically and manipulate the representing symbols as if they have a life of their own, without necessarily attending to their referents—and the ability to *contextualize*, to pause as needed during the manipulation process in order to probe into the referents for the symbols involved. Quantitative reasoning entails habits of creating a coherent representation of the problem at hand; considering the units involved; attending to the meaning of quantities, not just how to compute them; and knowing and flexibly using different properties of operations and objects.	
MP.3	**Construct viable arguments and critique the reasoning of others.**	Mathematically proficient students understand and use stated assumptions, definitions, and previously established results in constructing arguments. They make conjectures and build a logical progression of statements to explore the truth of their conjectures. They are able to analyze situations by breaking them into cases, and can recognize and use counterexamples. They justify their conclusions, communicate them to others, and respond to the arguments of others.	**Eureka Math:** Module 1 Lesson 36 Module 1 Lesson 38-39 Module 1 Lesson 40
MP.7	**Look for and make use of structure.**	Students connect long division of polynomials with the long-division algorithm of arithmetic and perform polynomial division in an abstract setting to derive the standard polynomial identities. Students recognize structure in the graphs of polynomials in factored form and develop refined techniques for graphing. Students discern the structure of rational expressions by comparing to analogous arithmetic problems. Students perform geometric operations on parabolas to discover congruence and similarity.	**Eureka Math:** Module 1 Lesson 37 Module 1 Lesson 39
MP.8	**Look for and express regularity in repeated reasoning.**	Students understand that polynomials form a system analogous to the integers. Students apply polynomial identities to detect prime numbers and discover Pythagorean triples. Students recognize factors of expressions and develop factoring techniques. Further, students understand that all quadratics can be written as a product of linear factors in the complex realm.	**Eureka Math:** Module 1 Lesson 40

HS Algebra II Semester 1			
Module 2: Trigonometric Functions **Topic A: The Story of Trigonometry and Its Contexts** Topic A starts by asking students to graph the height of a Ferris wheel as a function of time and uses that study to help define the sine, cosine, and tangent functions as functions from all (or most) real numbers to the real numbers. A precise definition of sine and cosine (as well as tangent and the co-functions) is developed using transformational geometry from high school Geometry. This precision leads to a discussion of a mathematically *natural* unit of measurement for angle measures, a radian, and students begin to build fluency with values of sine, cosine, and tangent at $\pi/6$, $\pi/4$, $\pi/3$, $\pi/2$, π, etc. The topic concludes with students graphing the sine and cosine functions and noticing various aspects of the graph, which they write down as simple trigonometric identities.			
Big Idea:	• The unit circle allows all real numbers to work in trigonometric functions. • Key features in graphs and tables shed light on relationships between two quantities. • Trigonometric functions can be represented by a table, graph, verbal description or equation, and each representation can be transferred to another representation		
Essential Questions:	• How do you use/read a unit circle (using radians)? • What do the key features of a trigonometric function represent? • What are the different ways you can represent a trigonometric function?		
Vocabulary	Radian, periodic function, Sine, Cosine, tangent, secant, cosecant cotangent		
Assessment	Galileo: Module 2 Pre-Assessment of Foundational Skills; Topic A assessment		
Standard	**Common Core Standards**	**Explanations & Examples**	**Resources**
F.IF.C.7E	**C. Analyze functions using different representation** Graph functions expressed symbolically and show key features of the graph, by hand in simple cases and using technology for more complicated cases. e. Graph ~~exponential and logarithmic functions, showing intercepts and end behavior, and~~ trigonometric functions, showing period, midline, and amplitude.	Graph the sine function on the interval $[-360, 360]$ showing all key points of the graph (horizontal and vertical intercepts and maximum and minimum points). Then, use the graph to answer each of the following questions. a. On the interval $[-360, 360]$, what are the relative minima of the sine function? Why? *The sine function has relative minima at -90 and 270 because when rotated by $-90°$ or $270°$, the initial ray intersects the unit circle at the bottom of the circle; therefore, the sine is at its smallest possible value.* b. On the interval $[-360, 360]$, what are the relative maxima of the sine function? Why? *The sine function has relative maxima at -270 and 90 because when rotated by $-270°$ or $90°$, the initial ray intersects the unit circle at the top of the circle; therefore, the sine is at its largest possible value.*	**Eureka Math:** Module 2 Lesson 1, 2, 8

F.TF.A.1	**A. Extend the domain of trigonometric functions using the unit circle** Understand radian measure of an angle as the length of the arc on the unit circle subtended by the angle.		**Eureka Math:** Module 2 Lesson 2 - 9
F.TF.A.2	**A. Extend the domain of trigonometric functions using the unit circle** Explain how the unit circle in the coordinate plane enables the extension of trigonometric functions to all real numbers, interpreted as radian measures of angles traversed counterclockwise around the unit circle.		**Eureka Math:** Module 2 Lesson 2 - 9
++F.TF.A.3	**A. Extend the domain of trigonometric functions using the unit circle** Use special triangles to determine geometrically the values of sine, cosine, tangent for $\pi/3$, $\pi/4$ and $\pi/6$and use the unit circle to express the values of sine, cosine, and tangent for $\pi-x$, $\pi+x$, and $2\pi-x$ in terms of their values for x, where x is any real number.	++HONORS ONLY	**Eureka Math:** Module 2 Lesson 8-10

		Demonstrate how to evaluate $\cos\left(\frac{8\pi}{3}\right)$ by using a trigonometric identity. $\cos\left(\frac{8\pi}{3}\right) = \cos\left(\frac{2\pi}{3} + 2\pi\right) = \cos\left(\frac{2\pi}{3}\right) = -\frac{1}{2}$ Determine if the following statement is true or false, without using a calculator. $\sin\left(\frac{8\pi}{7}\right) = \sin\left(\frac{\pi}{7}\right)$ *False.* $\sin\left(\frac{8\pi}{7}\right) = \sin\left(\frac{\pi}{7} + \pi\right) = -\sin\left(\frac{\pi}{7}\right) \neq \sin\left(\frac{\pi}{7}\right)$ If the graph of the cosine function is translated to the right $\frac{\pi}{2}$ units, the resulting graph is that of the sine function, which leads to the identity: For all x, $\sin(x) = \cos\left(x - \frac{\pi}{2}\right)$. Write another identity for $\sin(x)$ using a different horizontal shift. $\sin(x) = \cos\left(x + \frac{3\pi}{2}\right)$	
MP.2	**Reason abstractly and quantitatively.**	Students extend the study of trigonometry to the domain of all (or almost all) real inputs. By focusing only on the linear components of circular motion (the vertical or the horizontal displacement of a point in orbit), students develop the means to analyze periodic phenomena.	**Eureka Math:** Module 2 Lesson 1, 8, 9
MP.3	**Construct viable arguments and critique the reasoning of others.**	The vertical and horizontal displacements of a Ferris wheel passenger car are both periodic. Students conjecture how these functions are related to the trigonometric ratios they studied in geometry, making plausible arguments by modeling the Ferris wheel with a circle in the coordinate plane.	**Eureka Math:** Module 2 Lesson 4, 6, 7
MP.4	**Model with mathematics.**	The main modeling activity of this module is to analyze the vertical and horizontal displacement of a passenger car of a Ferris wheel. As they make assumptions and simplify the situation, they discover the need for sine and cosine functions to model the periodic motion using sinusoidal functions.	**Eureka Math:** Module 2 Lesson 1, 2, 3, 4
MP.6	**Attend to precision.**	Mathematically proficient students try to communicate precisely to others. They try to use clear definitions in discussion with others and in their own reasoning. They state the meaning of the symbols they choose, including using the equal sign consistently and appropriately.	**Eureka Math:** Module 2 Lesson 1

		They are careful about specifying units of measure, and labeling axes to clarify the correspondence with quantities in a problem. They calculate accurately and efficiently, express numerical answers with a degree of precision appropriate for the problem context.	
MP.7	**Look for and make use of structure.**	Mathematically proficient students look closely to discern a pattern or structure.	**Eureka Math:** Module 2 Lesson 1, 3, 6, 8
MP.8	**Look for and express regularity in repeated reasoning.**	Mathematically proficient students notice if calculations are repeated, and look both for general methods and for shortcuts. As they work to solve a problem, mathematically proficient students maintain oversight of the process, while attending to the details. They continually evaluate the reasonableness of their intermediate results.	**Eureka Math:** Module 2 Lesson 6, 9

HS Algebra II Semester 1			
Module 2: Trigonometric Functions **Topic B: Understanding Trigonometric Functions and Putting them to Use** In Topic A, the students developed the ideas behind the six basic trigonometric functions, focusing primarily on the sine function. In Topic B, students will use trigonometric functions to model periodic behavior. We end the module with the study of trigonometric identities and how to prove them.			
Big Idea:	• The unit circle allows all real numbers to work in trigonometric functions. • Key features in graphs and tables shed light on relationships between two quantities. • Trigonometric functions can be represented by a table, graph, verbal description or equation, and each representation can be transferred to another representation		
Essential Questions:	• How do you use/read a unit circle (using radians)? • How does the Pythagorean theorem and the unit circle relate to the identity $\sin^2(\theta) + \cos^2(\theta) = 1$? • What do the key features of a trigonometric function represent? • What are the different ways you can represent a trigonometric function? • What transformations can occur to a trigonometric function/graph and how do you know which one is which?		
Vocabulary	Periodic function, cycle, sinusoidal, amplitude, midline, frequency, period, tangent function		
Assessment	Galileo: Topic B Assessment		
Standard	**Common Core Standards**	**Explanations & Examples**	**Resources**
F.IF.C.7E	**C. Analyze functions using different representation** Graph functions expressed symbolically and show key features of the graph, by hand in simple cases and using technology for more complicated cases. e. Graph ~~exponential and logarithmic functions, showing intercepts and end behavior, and~~ trigonometric functions, showing period, midline, and amplitude.		**Eureka Math:** Module 2 Lesson 11, 12, 13, 14

F.TF.B.5	**B. Model periodic phenomena with trigonometric functions** Choose trigonometric functions to model periodic phenomena with specified amplitude, frequency, and midline.		**Eureka Math:** Module 2 Lesson 11, 12, 13
F.TF.C.8	**C. Prove and apply trigonometric identities** Prove the Pythagorean identity $\sin^2(\theta) + \cos^2(\theta) = 1$ and use it to find sin(θ), cos(θ), or tan(θ) given sin(θ), cos(θ), or tan(θ) and the quadrant of the angle.		**Eureka Math:** Module 2 Lesson 15
++F.TF.C.9	**C. Prove and apply trigonometric identities** Prove the addition and subtraction formulas for sine, cosine, and tangent and use them to solve problems.	++ HONORS ONLY Does $\sin(x+y)$ equal $\sin(x)+\sin(y)$ for all real numbers x and y? a. Find each of the following: $\sin\left(\frac{\pi}{2}\right)$, $\sin\left(\frac{\pi}{4}\right)$, $\sin\left(\frac{3\pi}{4}\right)$. *$\sin\left(\frac{\pi}{2}\right)=1$, $\sin\left(\frac{\pi}{4}\right)=\frac{\sqrt{2}}{2}$, and $\sin\left(\frac{3\pi}{4}\right)=\frac{\sqrt{2}}{2}$* b. Are $\sin\left(\frac{\pi}{2}+\frac{\pi}{4}\right)$ and $\sin\left(\frac{\pi}{2}\right)+\sin\left(\frac{\pi}{4}\right)$ equal? *No, because $\sin\left(\frac{\pi}{2}+\frac{\pi}{4}\right)=\frac{\sqrt{2}}{2}$, and $\sin\left(\frac{\pi}{2}\right)+\sin\left(\frac{\pi}{4}\right)=1+\frac{\sqrt{2}}{2}$.* c. Are there any values of x and y for which $\sin(x+y)=\sin(x)+\sin(y)$? *Yes. If either x or y is zero, or if both x and y are multiples of π, this is a true statement. In many other cases it is not true, so it is not true in general.*	**Eureka Math:** Module 2 Lesson 16-17

		$\tan(2\pi - x) = -\tan(x)$ *Let x be a real number so that $x \neq \frac{\pi}{2} + \pi k$, for any integer k. Then,* $\tan(2\pi - x) = \frac{\sin(2\pi - x)}{\cos(2\pi - x)} = \frac{-\sin(x)}{\cos(x)}$ $= -\frac{\sin(x)}{\cos(x)} = -\tan(x).$ *Thus,* $\tan(2\pi - x) = -\tan(x)$*, where* $x \neq \frac{\pi}{2} + k\pi$*, for all integers* k*.* Prove that $\cos(2u) = 2\cos^2(u) - 1$ is true for any real number u. *PROOF. Let u be any real number. From Exercise 3 in class, we know that $\cos(2u) = \cos^2(u) - \sin^2(u)$ for any real number u. Using the Pythagorean identity, we know that $\sin^2(u) = 1 - \cos^2(u)$. By substitution,* $\cos(2u) = \cos^2(u) - 1 + \cos^2(u)$. *Thus,* $\cos(2u) = 2\cos^2(u) - 1$ *for any real number u.*	
S.ID.B.6A	**B. Summarize, represent, and interpret data on two categorical and quantitative variables.** Represent data on two quantitative variables on a scatter plot, and describe how the variables are related. a. Fit a function to the data; use functions fitted to data to solve problems in the context of the data. *Use given functions or choose a function suggested by the context.*		**Eureka Math:** Module 2 Lesson 13
MP.2	**Reason abstractly and quantitatively.**	Students extend the study of trigonometry to the domain of all (or almost all) real inputs. By focusing only on the linear components of circular motion (the vertical or the horizontal displacement of a point in orbit), students develop the means to analyze periodic phenomena.	**Eureka Math:** Module 2 Lesson 12, 13

MP.3	**Construct viable arguments and critique the reasoning of others.**	The vertical and horizontal displacements of a Ferris wheel passenger car are both periodic. Students conjecture how these functions are related to the trigonometric ratios they studied in geometry, making plausible arguments by modeling the Ferris wheel with a circle in the coordinate plane. Also, students construct valid arguments to extend trigonometric identities to the full range of inputs.	**Eureka Math:** Module 2 Lesson 11, 12, 13
MP.4	**Model with mathematics.**	The main modeling activity of this module is to analyze the vertical and horizontal displacement of a passenger car of a Ferris wheel. As they make assumptions and simplify the situation, they discover the need for sine and cosine functions to model the periodic motion using sinusoidal functions. Students then model a large number of other periodic phenomena by fitting sinusoidal functions to data given about tides, sound waves, and daylight hours; they then solve problems using those functions in the context of that data.	**Eureka Math:** Module 2 Lesson 12
MP.5	**Use appropriate tools strategically.**	Mathematically proficient students consider the available tools when solving a mathematical problem. These tools might include pencil and paper, concrete models, a ruler, a protractor, a calculator, a spreadsheet, a computer algebra system, a statistical package, or dynamic geometry software. Proficient students are sufficiently familiar with tools appropriate for their grade or course to make sound decisions about when each of these tools might be helpful, recognizing both the insight to be gained and their limitations.	**Eureka Math:** Module 2 Lesson 11, 13
MP.6	**Attend to precision.**	Mathematically proficient students try to communicate precisely to others. They try to use clear definitions in discussion with others and in their own reasoning. They state the meaning of the symbols they choose, including using the equal sign consistently and appropriately. They are careful about specifying units of measure, and labeling axes to clarify the correspondence with quantities in a problem. They calculate accurately and efficiently, express numerical answers with a degree of precision appropriate for the problem context.	**Eureka Math:** Module 2 Lesson 13
MP.7	**Look for and make use of structure.**	Students recognize the periodic nature of a phenomenon and look for suitable values of midline, amplitude, and frequency for it. The periodicity and properties of cyclical motion shown in graphs helps students to recognize different trigonometric identities, and structure in standard proofs (of the Pythagorean theorem, for example) provides the means to extend familiar trigonometric results to a wider range of	**Eureka Math:** Module 2 Lesson 11

		input values.	
MP.8	**Look for and express regularity in repeated reasoning.**	In repeatedly graphing different sinusoidal functions, students identify how parameters within the function give information about the amplitude, midline, and frequency of the function. They express this regularity in terms of a general formula for sinusoidal functions and use the formula to quickly write functions that model periodic data.	**Eureka Math:** Module 2 Lesson 11, 14