HIGLEY UNIFIED SCHOOL DISTRICT
INSTRUCTIONAL ALIGNMENT

HS Geometry Semester 2 (Quarter 3)

Module 2: Similarity, Proof, and Trigonometry (45 days)
Topic D: Applying Similarity to Right Triangles (4 instructional days)

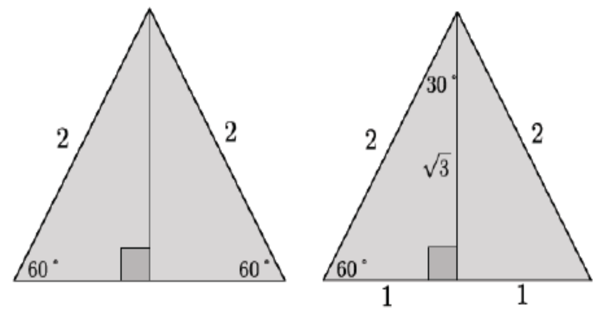
In Topic D, students use their understanding of similarity and focus on right triangles as a lead up to trigonometry. In Lesson 21, students use the AA criterion to show how an altitude drawn from the vertex of the right angle of a right triangle to the hypotenuse creates two right triangles similar to the original right triangle. Students examine how the ratios within the three similar right triangles can be used to find unknown side lengths. Work with lengths in right triangles lends itself to expressions with radicals. In Lessons 22 and 23 students learn to rationalize fractions with radical expressions in the denominator and also to simplify, add, and subtract radical expressions. In the final lesson of Topic D, students use the relationships created by an altitude to the hypotenuse of a right triangle to prove the Pythagorean theorem. (**G.SRT.B.4**)

Big Idea:	• Ratios within figures can be used to find lengths of sides in another triangle when those triangles are known to be similar. • Pythagorean theorem can be proved using similarity.
Essential Questions:	• What is an altitude, and what happens when an altitude is drawn from the right angle of a right triangle? • What is the relationship between the original right triangle and the two similar sub-triangles? • How do you use the ratios of the similar right triangles to determine the unknown lengths of a triangle? • What does it mean to rationalize the denominator of a fractional expression? Why might we want to do it? • How do we use ratios of legs and the hypotenuse to find the lengths of any 30–60–90 triangle. Why does it work? • How do we use ratios of legs and the hypotenuse to find the lengths of any 45–45–90 triangle. Why does it work?
Vocabulary	Pythagorean theorem
Assessments	Galileo: Topic D Assessment

Standard	AZ College and Career Readiness Standards	Explanations & Examples	Resources
G.SRT.B.4	**B. Prove theorems involving similarity** Prove theorems about triangles. *Theorems include: a line parallel to one side of a triangle divides the other two proportionally, and conversely; the Pythagorean Theorem proved using triangle similarity.*		**Eureka Math:** Module 2 Lesson 21-24

An equilateral triangle has sides of length 2 and angle measures of 60, as shown below. The altitude from one vertex to the opposite side divides the triangle into two right triangles.

a. Are those triangles congruent? Explain.



Yes, the two right triangles are congruent by ASA. Since the altitude is perpendicular to the base, then each of the right triangles has angles of measure 90 and 60. By the triangle sum theorem, the third angle has a measure of 30. Then, each of the right triangles has corresponding angle measures of 30 and 60, and the included side length is 2.

b. What is the length of the shorter leg of each of the right triangles? Explain.

Since the total length of the base of the equilateral triangle is 2, and the two right triangles formed are congruent, then the bases of each must be equal in length. Therefore, the length of the base of one right triangle is 1.

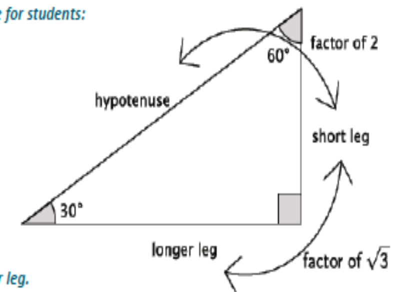
c. Use the Pythagorean theorem to determine the length of the altitude.

Let h represent the length of the altitude.

$$1^2 + h^2 = 2^2$$
$$h^2 = 2^2 - 1^2$$
$$h^2 = 3$$
$$h = \sqrt{3}$$

d. Write the ratio that represents $\textit{shorter leg}:\textit{hypotenuse}$.

$$1:2$$

		e. Write the ratio that represents *longer leg*: *hypotenuse*. $\sqrt{3}:2$ f. Write the ratio that represents *shorter leg*: *longer leg*. $1:\sqrt{3}$ g. By the AA criterion, any triangles with measures 30–60–90 will be similar to this triangle. If a 30–60–90 triangle has a hypotenuse of length 16, what are the lengths of the legs? *Consider providing the following picture for students:*  *Let a represent the length of the shorter leg.* $\frac{a}{16} = \frac{1}{2}$ $a = 8$ *Let b represent the length of the longer leg.* $\frac{b}{16} = \frac{\sqrt{3}}{2}$ $2b = 16\sqrt{3}$ $b = 8\sqrt{3}$ *The length $a = 8$ and the length $b = 8\sqrt{3}$.* *Note: After finding the length of one of the legs, some students may have used the ratio* **shorter leg: longer leg** *to determine the length of the other leg.*	
MP.1	**Make sense of problems and persevere in solving them.**	Mathematically proficient students start by explaining to themselves the meaning of a problem and looking for entry points to its solution. They analyze givens, constraints, relationships, and goals. They make conjectures about the form and meaning of the solution and plan a solution pathway rather than simply jumping into a solution attempt. They consider analogous problems, and try special cases and simpler forms of the original problem in order to gain insight into its solution.	**Eureka Math:** Module 2 Lesson 21

		They monitor and evaluate their progress and change course if necessary.	
MP.2	**Reason abstractly and quantitatively.**	Mathematically proficient students make sense of quantities and their relationships in problem situations. They bring two complementary abilities to bear on problems involving quantitative relationships: the ability to *decontextualize*—to abstract a given situation and represent it symbolically and manipulate the representing symbols as if they have a life of their own, without necessarily attending to their referents—and the ability to *contextualize*, to pause as needed during the manipulation process in order to probe into the referents for the symbols involved. Quantitative reasoning entails habits of creating a coherent representation of the problem at hand; considering the units involved; attending to the meaning of quantities, not just how to compute them; and knowing and flexibly using different properties of operations and objects.	**Eureka Math:** Module 2 Lesson 21
MP.3	**Construct viable arguments and critique the reasoning of others**.	Critical to this module is the need for dilations in order to define similarity. In order to understand dilations fully, the proofs in Lessons 4 and 5 to establish the triangle side splitter and the dilation theorems require students to build arguments based on definitions and previously established results. This is also apparent in Lessons 7, 8, and 9, when the properties of dilations are being proven. Though there are only a handful of facts students must point to in order to create arguments, how students reason with these facts will determine if their arguments actually establish the properties. It will be essential to communicate effectively and purposefully.	**Eureka Math:** Module 2 Lesson 22, 23
MP.6	**Attend to precision.**	Mathematically proficient students try to communicate precisely to others. They try to use clear definitions in discussion with others and in their own reasoning. They state the meaning of the symbols they choose, including using the equal sign consistently and appropriately. They are careful about specifying units of measure, and labeling axes to clarify the correspondence with quantities in a problem. They calculate accurately and efficiently, express numerical answers with a degree of precision appropriate for the problem context. In the elementary grades, students give carefully formulated explanations to each other. By the time they reach high school they have learned to examine claims and make explicit use of definitions.	**Eureka Math:** Module 2 Lesson 24
MP.7	**Look for and make use of structure**.	Much of the reasoning in Module 2 centers around the interaction between figures and dilations. It is unsurprising then that students	**Eureka Math:** Module 2 Lesson

		must pay careful attention to an existing structure and how it changes under a dilation, for example why it is that dilating the key points of a figure by the ratio method results in the dilation of the segments that join them. The math practice also ties into the underlying idea of trigonometry: how to relate the values of corresponding ratio lengths between similar right triangles and how the value of a trigonometric ratio hinges on a given acute angle within a right triangle.	21, 23

HS Geometry Semester 2 (Quarter 3)

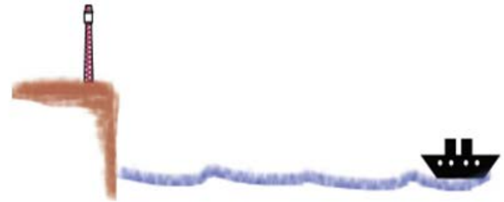
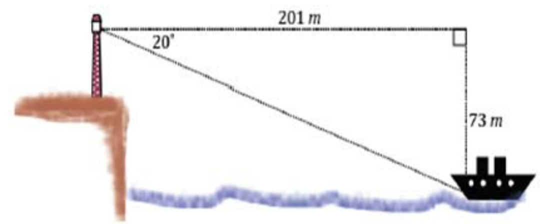
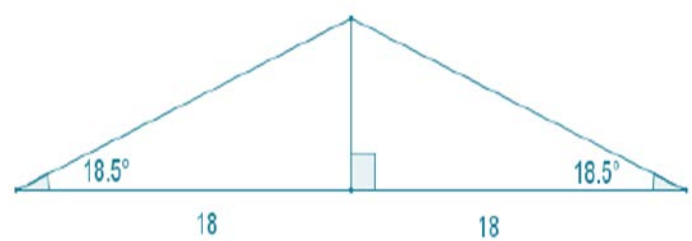
Module 2: Similarity, Proof, and Trigonometry (45 days)

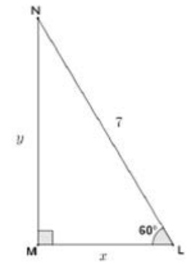
Topic E: Trigonometry (10 instructional days)

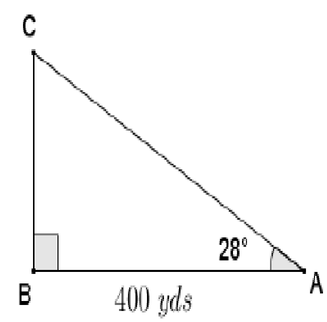
Students begin the study of trigonometry in the final topic of the module. The emphasis in the module on side length relationships within similar triangles (Topic C) and the specific emphasis on right triangles (Topic D) help set the foundation for trigonometry. Lesson 25 is a last highlight of the side length ratios within and between right triangles. Students are guided to the idea that the values of the ratios depend solely on a given acute angle in the right triangle before the basic trigonometric ratios are explicitly defined in Lesson 26 (**G.SRT.C.6**). After practice with ratios labeled as shorter leg:hypotenuse (Lesson 21) and opp:hyp (Lesson 25), students are introduced to the trigonometric ratios sine, cosine, and tangent (**G-SRT.C.6**) in Lesson 26. Students examine the relationship between sine and cosine in Lesson 27, discovering that the sine and cosine of complementary angles are equal (**G-SRT.C.7**). They are also introduced to the common sine and cosine values of angle measures frequently seen in trigonometry. Students apply the trigonometric ratios to solve for unknown lengths in Lessons 28 and 29; students also learn about the relationship between tangent and slope in Lesson 29 (**G-SRT.C.8**). In Lesson 30, students use the Pythagorean theorem to prove the identity $\sin2\theta+\cos2\theta=1$ and also show why $\tan\theta=\sin\theta/\cos\theta$. (**G-SRT.C.6, G-SRT.C.7, G-SRT.C.8**)

Big Idea:	• Triangles occur in nature and everyday life. • Not all mathematical principles are necessary if we solve problems in alternate ways. • Mathematical principles involving triangles are the cornerstone of architectural design.
Essential Questions:	• Is there more than one way to solve a math problem? • What situations in real life require the use of trigonometry? • What are the strengths and weaknesses of the law of sines and the law of cosines? • Can we design and build architecture without math?
Vocabulary	Sides of a right triangle, sine, cosine, tangent, secant, cosecant, cotangent
Assessments	Galileo: Topic E Assessment

Standard	AZ College and Career Readiness Standards	Explanations & Examples	Resources
G.SRT.C.6	**C. Define trigonometric ratios and solve problems involving right triangles.** Understand that by similarity, side ratios in right triangles are properties of the angles in the triangle, leading to definitions of trigonometric ratios for acute angles.		**Eureka Math:** Module 2 Lesson 25,26

		Standing on the gallery of a lighthouse (the deck at the top of a lighthouse), a person spots a ship at an angle of depression of 20°. The lighthouse is 28 m tall and sits on a cliff 45 m tall as measured from sea level. What is the horizontal distance between the lighthouse and the ship? Sketch a diagram to support your answer. *Approximately* $201\ m$.   Tim is designing a roof truss in the shape of an isosceles triangle. The design shows the base angles of the truss to have measures of $18.5°$. If the horizontal base of the roof truss is 36 ft. across, what is the height of the truss?  *Let* h *represent the height of the truss in feet. Using tangent,* $\tan 18.5 = \frac{h}{18}$, *and thus* $h = 18(\tan 18.5)$ $h \approx 6.$ *The height of the truss is approximately* 6 ft.	

G.SRT.C.7	**C. Define trigonometric ratios and solve problems involving right triangles.** Explain and use the relationship between the sine and cosine of complementary angles.	Find the values for θ that make each statement true. a. $\sin\theta = \cos 32$ $\theta = 90 - 32$ $\theta = 58$ b. $\cos\theta = \sin(\theta + 20)$ $\sin(90-\theta) = \sin(\theta+20)$ $90 - \theta = \theta + 20$ $70 = 2\theta$ $35 = \theta$ Triangle LMN is a 30–60–90 right triangle. Find the unknown lengths x and y. $\sin 60 = \frac{\sqrt{3}}{2}$ $\frac{\sqrt{3}}{2} = \frac{y}{7}$ $7\sqrt{3} = 2y$ $y = \frac{7\sqrt{3}}{2}$ $\cos 60 = \frac{1}{2}$ $\frac{1}{2} = \frac{x}{7}$ $7 = 2x$ $\frac{7}{2} = x$  $\sin\theta = \cos(3\theta + 20)$ $\cos(90-\theta) = \cos(3\theta+20)$ $90 - \theta = 3\theta + 20$ $70 = 4\theta$ $17.5 = \theta$	**Eureka Math:** Module 2 Lesson 27, 28, 29

G.SRT.C.8	**C. Define trigonometric ratios and solve problems involving right triangles.** Use trigonometric ratios and the Pythagorean Theorem to solve right triangles in applied problems.	If $\cos\theta = \frac{4}{5}$, find $\sin\theta$ and $\tan\theta$. *Using the identity* $\sin^2\theta + \cos^2\theta = 1$: $\sin^2\theta + \left(\frac{4}{5}\right)^2 = 1$ $\sin^2\theta = 1 - \left(\frac{4}{5}\right)^2$ $\sin^2\theta = 1 - \left(\frac{16}{25}\right)$ $\sin^2\theta = \frac{9}{25}$ $\sin\theta = \sqrt{\frac{9}{25}}$ $\sin\theta = \frac{3}{5}$ *Using the identity* $\tan\theta = \frac{\sin\theta}{\cos\theta}$: $\tan\theta = \frac{\frac{3}{5}}{\frac{4}{5}} = \frac{3}{4}$ A surveying crew has two points A and B marked along a roadside at a distance of 400 yd. A third point C is marked at the back corner of a property along a perpendicular to the road at B. A straight path joining C to A forms a 28° angle with the road. Find the distance from the road to point C at the back of the property and the distance from A to C using sine, cosine, and/or tangent. Round your answer to three decimal places.  $\tan 28 = \frac{BC}{400}$ $BC = 400(\tan 28)$ $BC \approx 212.684$ *The distance from the road to the back of the property is approximately* 212.684 *yds.* $\cos 28 = \frac{400}{AC}$ $AC = \frac{400}{\cos 28}$ $AC \approx 453.028$ *The distance from point C to point A is approximately* 453.028 *yd.*	**Eureka Math:** Module 2 Lesson 29,30

MP.1	**Make sense of problems and persevere in solving them.**	Mathematically proficient students start by explaining to themselves the meaning of a problem and looking for entry points to its solution. They analyze givens, constraints, relationships, and goals. They make conjectures about the form and meaning of the solution and plan a solution pathway rather than simply jumping into a solution attempt. They consider analogous problems, and try special cases and simpler forms of the original problem in order to gain insight into its solution. They monitor and evaluate their progress and change course if necessary.	**Eureka Math:** Module 2 Lesson 28, 29, 31
MP.2	**Reason abstractly and quantitatively.**	Mathematically proficient students make sense of quantities and their relationships in problem situations. They bring two complementary abilities to bear on problems involving quantitative relationships: the ability to *decontextualize*—to abstract a given situation and represent it symbolically and manipulate the representing symbols as if they have a life of their own, without necessarily attending to their referents—and the ability to *contextualize*, to pause as needed during the manipulation process in order to probe into the referents for the symbols involved. Quantitative reasoning entails habits of creating a coherent representation of the problem at hand; considering the units involved; attending to the meaning of quantities, not just how to compute them; and knowing and flexibly using different properties of operations and objects.	**Eureka Math:** Module 2 Lesson 27, 31
MP.3	**Construct viable arguments and critique the reasoning of others**.	Critical to this module is the need for dilations in order to define similarity. In order to understand dilations fully, the proofs in Lessons 4 and 5 to establish the triangle side splitter and the dilation theorems require students to build arguments based on definitions and previously established results. This is also apparent in Lessons 7, 8, and 9, when the properties of dilations are being proven. Though there are only a handful of facts students must point to in order to create arguments, how students reason with these facts will determine if their arguments actually establish the properties. It will be essential to communicate effectively and purposefully.	**Eureka Math:** Module 2 Lesson 26, 30,32,33
MP.7	**Look for and make use of structure**.	Much of the reasoning in Module 2 centers around the interaction between figures and dilations. It is unsurprising then that students	**Eureka Math:** Module 2 Lesson 25,27,30,31

		must pay careful attention to an existing structure and how it changes under a dilation, for example why it is that dilating the key points of a figure by the ratio method results in the dilation of the segments that join them. The math practice also ties into the underlying idea of trigonometry: how to relate the values of corresponding ratio lengths between similar right triangles and how the value of a trigonometric ratio hinges on a given acute angle within a right triangle.	
MP.8	**Look for and express regularity in repeated reasoning.**	Mathematically proficient students notice if calculations are repeated, and look both for general methods and for shortcuts. As they work to solve a problem, mathematically proficient students maintain oversight of the process, while attending to the details. They continually evaluate the reasonableness of their intermediate results.	**Eureka Math:** Module 2 Lesson 26, 29

HS Geometry Semester 2 (Quarter 3)

Module 3: Extending to Three Dimensions (15 days)
Topic A: Area (4 Instructional days)

Module 3 builds on students' understanding of congruence in Module 1 and similarity in Module 2 to prove volume formulas for solids. Topic A studies informal limit arguments to find the area of a rectangle with an irrational side length and of a disk (**G-GMD.A.1**). It also focuses on properties of area that arise from unions, intersections, and scaling. These topics prepare for understanding limit arguments for volumes of solids. (**G-GMD.A.1**)

Big Idea:	•
Essential Questions:	•
Vocabulary	Set, subset, union, intersection, scale factor, scaling principle for triangles, scaling principle for polygons, scaling principle for area, limit, inscribed polygon, circumscribed polygon,
Assessments	Galileo: Geometry Module 3 Foundational Skills Assessment; Galileo: Topic A Assessment

Standard	**AZ College and Career Readiness Standards**	**Explanations & Examples**	**Comments**
G.GMD.A.1	**A. Explain volume formulas and use them to solve problems** Give an informal argument for the formulas for the circumference of a circle, area of a circle, volume of a cylinder, pyramid, and cone. *Use dissection arguments, ~~Cavalieri's principle~~, and informal limit arguments.* *HS.MP.3.* Construct viable arguments and critique the reasoning of others. *HS.MP.4.* Model with mathematics. *HS.MP.5.* Use appropriate tools strategically.	Cavalieri's principle is if two solids have the same height and the same cross-sectional area at every level, then they have the same volume.	**Eureka Math:** Module 3 Lesson 1 - 4

		Approximate the area of a disk of radius 2 using an inscribed regular hexagon. *The interior of a regular hexagon can be divided into 6 equilateral triangles, each of which can be split into two 30-60-90 triangles by drawing an altitude. Using the relationships of the sides in a 30-60-90 triangle, the height of each triangle is $\sqrt{3}$.* $\text{Area} = \frac{1}{2}ph$ $\text{Area} = \frac{1}{2}(2 \cdot 6) \cdot \sqrt{3}$ $\text{Area} = 6\sqrt{3}$ *The area of the inscribed regular hexagon is $6\sqrt{3}$.* Approximate the area of a disk of radius 2 using a circumscribed regular hexagon. *Using the same reasoning for the interior of the hexagon, the height of the equilateral triangles contained in the hexagon is 2, while the lengths of their sides are $\frac{4\sqrt{3}}{3}$.* $\text{Area} = \frac{1}{2}ph$ $\text{Area} = \frac{1}{2}\left(\frac{4\sqrt{3}}{3} \cdot 6\right) \cdot 2$ $\text{Area} = (4\sqrt{3} \cdot 2)$ $\text{Area} = 8\sqrt{3}$ *The area of the circumscribed regular hexagon is $8\sqrt{3}$.*	
MP.1	**Make sense of problems and persevere in solving them.**	Mathematically proficient students start by explaining to themselves the meaning of a problem and looking for entry points to its solution. They analyze givens, constraints, relationships, and goals. They make conjectures about the form and meaning of the solution and plan a solution pathway rather than simply jumping into a solution attempt.	**Eureka Math:** Module 3 Lesson 1,4

		They consider analogous problems, and try special cases and simpler forms of the original problem in order to gain insight into its solution. They monitor and evaluate their progress and change course if necessary. Mathematically proficient students check their answers to problems using a different method, and they continually ask themselves, "Does this make sense?" They can understand the approaches of others to solving complex problems and identify correspondences between different approaches.	
MP.2	**Reason abstractly and quantitatively.**	Mathematically proficient students make sense of quantities and their relationships in problem situations. They bring two complementary abilities to bear on problems involving quantitative relationships: the ability to *decontextualize*—to abstract a given situation and represent it symbolically and manipulate the representing symbols as if they have a life of their own, without necessarily attending to their referents—and the ability to *contextualize*, to pause as needed during the manipulation process in order to probe into the referents for the symbols involved. Quantitative reasoning entails habits of creating a coherent representation of the problem at hand; considering the units involved; attending to the meaning of quantities, not just how to compute them; and knowing and flexibly using different properties of operations and objects.	**Eureka Math:** Module 3 Lesson 4
MP.3	**Construct viable arguments and critique the reasoning of others.**	Mathematically proficient students understand and use stated assumptions, definitions, and previously established results in constructing arguments. They make conjectures and build a logical progression of statements to explore the truth of their conjectures. They are able to analyze situations by breaking them into cases, and can recognize and use counterexamples. They justify their conclusions, communicate them to others, and respond to the arguments of others. They reason inductively about data, making plausible arguments that take into account the context from which the data arose. Mathematically proficient students are also able to compare the effectiveness of two plausible arguments, distinguish correct logic or reasoning from that which is flawed, and—if there is a flaw in an argument—explain what it is.	**Eureka Math:** Module 3 Lesson 1,3,4
MP.7	**Look for and make use of structure**.	The theme of approximation in Module 3 is an interpretation of structure. Students approximate both area and volume (curved two-dimensional shapes and cylinders and cones with curved bases) polyhedral regions. They must understand how and why it is possible	**Eureka Math:** Module 3 Lesson 2,4

		to create upper and lower approximations of a figure's area or volume. The derivation of the volume formulas for cylinders, cones, and spheres, and the use of Cavalieri's principle is also based entirely on understanding the structure and sub-structures of these figures.	
MP.8	**Look for and express regularity in repeated reasoning.**	Mathematically proficient students notice if calculations are repeated, and look both for general methods and for shortcuts.	**Eureka Math:** Module 3 Lesson 3

HS Geometry Semester 2 (Quarter 3)

Module 3: Extending to Three Dimensions (15 days)
Topic B: Volume (9 Instructional days)

Topic B is introduced by a lesson where students experimentally discover properties of three-dimensional space that are necessary to describe three-dimensional solids such as cylinders and prisms, cones and pyramids, and spheres. Cross-sections of these solids are studied and are classified as similar or congruent (**G-GMD.B.4**). A dissection is used to show the volume formula for a right triangular prism after which limit arguments give the volume formula for a general right cylinder (**G-GMD.A.1**).

In Lesson 10, two-dimensional cross-sections of solids are used to approximate solids by general right cylindrical slices and leads to an understanding of Cavalieri's principle (**G-GMD.A.1**). Congruent cross-sections for a general (skew) cylinder and Cavalieri's principle lead to the volume formula for a general cylinder.
To find the volume formula of a pyramid, a cube is dissected into six congruent pyramids to find the volume of each one. Scaling the given pyramids, according to a scaling principle analogous to the one introduced in Topic A, gives the volume formula for a right rectangular pyramid. The cone cross-section theorem and Cavalieri's principle are then used to find the volume formula of a general cone (**G-GMD.A.1, G-GMD.A.3**).

Cavalieri's principle is used to show that the volume of a right circular cylinder with radius R and height R is the sum of the volume of hemisphere of radius R and the volume of a right circular cone with radius R and height R. This information leads to the volume formula of a sphere (**G-GMD.A.2, G-GMD.A.3**).

(G.GMD.A.1, G-GMD.A.3, G-GMD.B.4, G-MG.A.1, G-MG.A.2, G-MG.A.3) G.GMD.A.2 (Honors Only)

Big Idea:	•
Essential Questions:	•
Vocabulary	Cavalieri's principle
Assessments	Galileo: Topic B Assessment

Standard	AZ College and Career Readiness Standards	Explanations & Examples	Comments
G.GMD.A.1	**A. Explain volume formulas and use them to solve problems** Give an informal argument for the formulas for the	Cavalieri's principle is if two solids have the same height and the same cross-sectional area at every level, then they have the same volume.	**Eureka Math:** Module 3 Lesson 5,8,10-13

	circumference of a circle, area of a circle, volume of a cylinder, pyramid, and cone. *Use dissection arguments, Cavalieri's principle, and informal limit arguments.*		
G.GMD.A.3 ▲	**A. Explain volume formulas and use them to solve problems** Use volume formulas for cylinders, pyramids, cones, and spheres to solve problems.		**Eureka Math:** Module 3 Lesson 5,8,9
G.GMD.B.4 ●	**B. Visualize relationships between two-dimensional and three dimensional objects** Identify the shapes of two-dimensional cross-sections of three-dimensional objects, and identify three-dimensional objects generated by rotations of two-dimensional objects.		**Eureka Math:** Module 3 Lesson 5,6,7,13
G.MG.A.1 ●	**A. Apply geometric concepts in modeling situations** Use geometric shapes, their measures, and their properties to describe objects (e.g., modeling a tree trunk or a human torso as a cylinder).		**Eureka Math:** Module 3 Lesson 5-13
G.MG.A.2 ●	**A. Apply geometric concepts in modeling situations** Apply concepts of density based on area and volume in modeling situations (e.g., persons per square mile, BTUs per cubic foot).		**Eureka Math:** Module 3 Lesson 5-13
G.MG.A.3	**A. Apply geometric concepts in modeling situations**		**Eureka Math:** Module 3 Lesson 5-13

●	Apply geometric methods to solve design problems (e.g., designing an object or structure to satisfy physical constraints or minimize cost; working with typographic grid systems based on ratios).		
MP.1	**Make sense of problems and persevere in solving them.**	Mathematically proficient students start by explaining to themselves the meaning of a problem and looking for entry points to its solution. They analyze givens, constraints, relationships, and goals. They make conjectures about the form and meaning of the solution and plan a solution pathway rather than simply jumping into a solution attempt. They consider analogous problems, and try special cases and simpler forms of the original problem in order to gain insight into its solution. They monitor and evaluate their progress and change course if necessary	**Eureka Math:** Module 3 Lesson 10,12
MP.3	**Construct viable arguments and critique the reasoning of others.**	Mathematically proficient students understand and use stated assumptions, definitions, and previously established results in constructing arguments. They make conjectures and build a logical progression of statements to explore the truth of their conjectures. They are able to analyze situations by breaking them into cases, and can recognize and use counterexamples. They justify their conclusions, communicate them to others, and respond to the arguments of others. They reason inductively about data, making plausible arguments that take into account the context from which the data arose.	**Eureka Math:** Module 3 Lesson 8-11
MP.6	**Attend to precision.**	Students will formalize definitions, using explicit language to define terms such as *right rectangular prism* that have been informal and more descriptive in earlier grade levels.	**Eureka Math:** Module 3 Lesson 6
MP.7	**Look for and make use of structure.**	The theme of approximation in Module 3 is an interpretation of structure. Students approximate both area and volume (curved two-dimensional shapes and cylinders and cones with curved bases) polyhedral regions. They must understand how and why it is possible to create upper and lower approximations of a figure's area or volume. The derivation of the volume formulas for cylinders, cones, and spheres, and the use of Cavalieri's principle is also based entirely on understanding the structure and sub-structures of these figures.	**Eureka Math:** Module 3 Lesson 7,8,11,12
MP.8	**Look for and express regularity in repeated reasoning.**	Mathematically proficient students notice if calculations are repeated, and look both for general methods and for shortcuts. As they work to solve a problem, mathematically proficient students maintain oversight	**Eureka Math:** Module 3 Lesson 9

		of the process, while attending to the details. They continually evaluate the reasonableness of their intermediate results.	

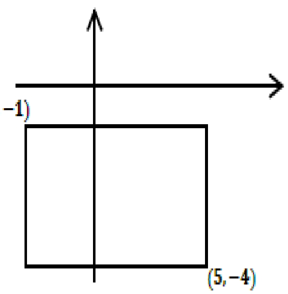
HS Geometry Semester 2 (Quarter 3)

Module 4: Connecting Algebra and Geometry Through Coordinates (25 days)
Topic A: Rectangular and Triangular Regions Defined by Inequalities (4 Instructional days)

In this module, students explore and experience the utility of analyzing algebra and geometry challenges through the framework of coordinates. The module opens with a modeling challenge (G-MG.A.1, G-MG.A.3) that re-occurs throughout the lessons. Students use coordinate geometry to program the motion of a robot bound in a polygonal region (a room) of the plane. MP.4 is highlighted throughout this module as students transition from the verbal tasks to determining how to use coordinate geometry, algebra, and graphical thinking to complete the task. The modeling task varies in each lesson as students define regions, constrain motion along segments, rotate motion, and move through a real-world task of programming a robot. While this robot moves at a constant speed and its motion is very basic, it allows students to see the usefulness of the concepts taught in this module and put them in context. In Lesson 1 students use the distance formula and previous knowledge of angles to program a robot to search a plane. Students impose a coordinate system and describe the movement of the robot in terms of line segments and points. In Lesson 2, students graph inequalities and discover that a rectangular or triangular region (G-GPE.B.7) in the plane can be defined by a system of algebraic inequalities (A-REI.D.12). In Lesson 3, students study lines that cut through these previously described regions. Students are given two points in the plane and a region and determine whether a line through those points meets the region. If it does, they describe the intersection as a segment and name the endpoints. Topic A ends with Lesson 4, where students return to programming the robot while constraining motion along line segments within the region (G-GPE.B.7, A-REI.C.6) and rotating a segment 90° clockwise or counterclockwise about an endpoint (**G-MG.A.1, G-MG.A.3**). Revisiting **A-REI.C.6** (solving systems of linear equations in two variables) and **A-REI.D.12** (graphing linear inequalities in two variables and the solution sets of a system of linear inequalities) shows the coherence between algebra and geometry.

G-GPE.B.7

Big Idea:	• Geometric figures can be represented in the coordinate plane. • The algebraic properties (including those related to the distance between points in the coordinate plane) may be used to prove geometric relationships. • Relationships between geometric objects represented in the coordinate plane may be determined or proven through the use of similarity transformations. • The distance formula may be used to determine measurements related to geometric objects represented in the coordinate plane (e.g., the perimeter or area of a polygon.		
Essential Questions:	• How are dilations used to partition a line segment into two segments whose lengths form a given ratio? • Given a polygon represented in the coordinate plane, what is its perimeter and area? • How can geometric relationships be proven through the application of algebraic properties to geometric figures represented in the coordinate plane?		
Vocabulary	Distance formula		
Assessments	Galileo: Geometry Module 4 Foundational Skills Assessment; Galileo: Topic A Assessment		
Standard	**AZ College and Career Readiness Standards**	**Explanations & Examples**	**Resources**

G.GPE.B.7	**B. Use coordinates to prove simple geometric theorems algebraically** Use coordinates to compute perimeters of polygons and areas of triangles and rectangles, e.g., using the distance formula.	Students may use geometric simulation software to model figures. Consider the rectangular region: a. What boundary points does a line through the origin with a slope of -2 intersect? What is the length of the segment within this region along this line? *It intersects at* $\left(\frac{1}{2}, -1\right)$ *and* $(2, -4)$*; the length is approximately* 3.35*.*  b. What boundary points does a line through the origin with a slope of 3 intersect? What is the length of the segment within this region along this line? *It intersects at* $\left(-\frac{1}{3}, -1\right)$ *and* $\left(-1\frac{1}{3}, -4\right)$*; the length is approximately* 3.2*.*	**Eureka Math:** Module 4 Lesson 1 - 4
MP.1	**Make sense of problems and persevere in solving them.**	Students start the module with the challenge to understand and develop the mathematics for describing of the motion of a robot bound within a certain polygonal region of the plane—the room in which it sits. This a recurring problem throughout the entire module and with it, and through related problems, students discover the slope criteria for perpendicular and parallel lines, the means to find the coordinates of a point dividing a line segment into two lengths in a given ratio, the distance formula of a point from a line, along with a number of geometric results via the tools of coordinate geometry.	**Eureka Math:** Module 4 Lesson 2
MP.3	**Construct viable arguments and critique the reasoning of others.**	Mathematically proficient students understand and use stated assumptions, definitions, and previously established results in constructing arguments. They make conjectures and build a logical progression of statements to explore the truth of their conjectures. They are able to analyze situations by breaking them into cases, and can recognize and use counterexamples. They justify their conclusions, communicate them to others, and respond to the arguments of others. They reason inductively about data, making plausible arguments that take into account the context from which the data arose. Mathematically proficient students are also able to compare the effectiveness of two plausible arguments, distinguish correct logic or reasoning from that which is flawed, and—if there is a flaw in an	**Eureka Math:** Module 4 Lesson 3

		argument—explain what it is.	
MP.4	**Model with mathematics.**	Students model the motion of a robot in the plane in two contexts: determining the extent of motion within the bounds of a polygonal region, and determining and moving to the location of the source of beacon signal in the infinite plane.	**Eureka Math:** Module 4 Lesson 1,4
MP.7	**Look for and make use of structure.**	Students determine slope criteria for perpendicular and parallel lines and use these slope conditions to develop the general equation of a line and the formula for the distance of a point from a line. Students determine the area of polygonal regions using multiple methods including Green's theorem and decomposition. Definitive geometric properties of special quadrilaterals are explored and properties of special lines in triangles are examined.	**Eureka Math:** Module 4 Lesson 2,3,4

HS Geometry Semester 2 (Quarter 3)

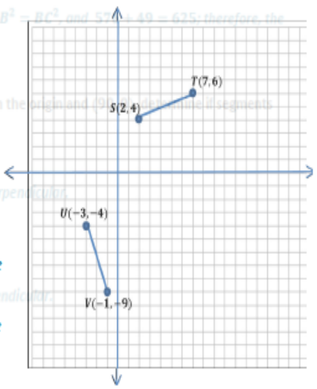
Module 4: Connecting Algebra and Geometry Through Coordinates (25 days)

Topic B: Perpendicular and Parallel Lines in the Cartesian Plane (4 Instructional days)

The challenge of programming robot motion along segments parallel or perpendicular to a given segment leads to an analysis of slopes of parallel and perpendicular lines and the need to prove results about these quantities (**G-GPE.B.5**). MP.3 is highlighted in this topic as students engage in proving criteria and then extending that knowledge to reason about lines and segments. This work highlights the role of the converse of the Pythagorean theorem in the identification of perpendicular directions of motion (**G-GPE.B.4**). In Lesson 5, students explain the connection between the Pythagorean theorem and the criterion for perpendicularity studied in Module 2 (**G-GPE.B.4**). Lesson 6 extends that study by generalizing the criterion for perpendicularity to any two segments and applying this criterion to determine if segments are perpendicular. In Lesson 7, students learn a new format for a line, $a_1x+ a_2y=c$, and recognize the segment from (0,0) to (a_1,a_2) as normal with a slope of $-a_2/a_1$. Lesson 8 concludes Topic B when students recognize parallel and perpendicular lines from their slopes and create equations for parallel and perpendicular lines. The criterion for parallel and perpendicular lines and the work from this topic with the distance formula will be extended in the last two topics of this module as students use these foundations to determine perimeter and area of polygonal regions in the coordinate plane defined by systems of inequalities. Additionally, students will study the proportionality of segments formed by diagonals of polygons. (**G-GPE.B.4, G-GPE.B.5**)

Big Idea:	• Geometric figures can be represented in the coordinate plane. • The algebraic properties (including those related to the distance between points in the coordinate plane) may be used to prove geometric relationships. • The algebraic relationship between the slopes of parallel lines and the slopes of perpendicular lines.
Essential Questions:	• What is the relationship between the slopes of parallel lines and of perpendicular lines? • Given a polygon represented in the coordinate plane, what is its perimeter and area? • How can geometric relationships be proven through the application of algebraic properties to geometric figures represented in the coordinate plane?
Vocabulary	Normal segment to a line, slope
Assessments	Galileo: Topic B Assessment

Standard	AZ College and Career Readiness Standards	Explanations & Examples	Resources
G.GPE.B.4	**B. Use coordinates to prove simple geometric theorems algebraically** Use coordinates to prove simple geometric theorems algebraically. *For example, prove or disprove that a figure defined by four given points in the coordinate plane is a rectangle; prove or disprove that the point (1, √3) lies on the circle centered at the origin and containing the point (0, 2).*	Students may use geometric simulation software to model figures and prove simple geometric theorems.	**Eureka Math:** Module 4 Lesson 5-8

		Prove using the Pythagorean theorem that $\overline{AC}$ is perpendicular to $\overline{AB}$ given $A(-2,-2)$, $B(5,-2)$, and $C(-2,22)$. *$AC = 24$, $BC = 25$, and $AB = 7$. If triangle ABC is right, $AC^2 + AB^2 = BC^2$, and $576 + 49 = 625$; therefore, the segments are perpendicular.* Using the general formula for perpendicularity of segments through the origin and $(90, 0)$, determine if segments $\overline{OA}$ and $\overline{OB}$ are perpendicular. a. $A(-3,-4)$, $B(4,3)$ *$(-3)(4) + (-4)(3) \neq 0$; therefore, the segments are not perpendicular.* b. $A(8,9)$, $B(18,-16)$ *$(8)(18) + (9)(-16) = 0$; therefore, the segments are perpendicular.*	
G.GPE.B.5	**B. Use coordinates to prove simple geometric theorems algebraically** Prove the slope criteria for parallel and perpendicular lines and use them to solve geometric problems (e.g., find the equation of a line parallel or perpendicular to a given line that passes through a given point).	Given points $S(2,4)$, $T(7,6)$, $U(-3,-4)$, and $V(-1,-9)$: a. Translate segments $\overline{ST}$ and $\overline{UV}$ so that the image of each segment has an endpoint at the origin. *Answers can vary slightly. Students will have to choose one of the first two and one of the second two.* *If we translate $\overline{ST}$ so that the image of S is at the origin, we get $S'(0,0)$, $T'(5,2)$.* *If we translate $\overline{ST}$ so that the image of T is at the origin, we get $S'(-5,-2)$, $T'(0,0)$.* *If we translate $\overline{UV}$ so that the image of U is at the origin, we get $U'(0,0)$, $V'(2,-5)$.* *If we translate $\overline{UV}$ so that the image of V is at the origin, we get $V'(0,0)$, $U'(-2,5)$.*  b. Are the segments perpendicular? Explain. *Yes. By choosing any two of the translated segment $\overline{S'T'}$ and $\overline{U'V'}$, we determine whether the equation yields a true statement: $(b_1 - a_1)(d_1 - c_1) + (b_2 - a_2)(d_2 - c_2) = 0$.* *For example, using $S(-5,-2)$, $T'(0,0)$, and $U'(0,0)$, $V'(2,-5)$:* *$-5(2) + (-2)(5) = 0$ is a true statement; therefore, $\overline{S'T'} \perp \overline{U'V'}$ and $\overline{ST} \perp \overline{UV}$.* c. Are the lines $\overleftrightarrow{ST}$ and $\overleftrightarrow{UV}$ perpendicular? Explain. *Yes, lines containing perpendicular segments are also perpendicular.*	**Eureka Math:** Module 4 Lesson 5-8

		Write the equation of the line through $(-5,3)$ and: a. Parallel to $x=-1$. $x=-5$ b. Perpendicular to $x=-1$. $y=3$ c. Parallel to $y=\frac{3}{5}x+2$. $3x-5y=-30$ d. Perpendicular to $y=\frac{3}{5}x+2$. $5x+3y=-16$ Write the equation of the line through $\left(\sqrt{3},\frac{5}{4}\right)$ and: a. Parallel to $y=7$. $y=\frac{5}{4}$ b. Perpendicular to $y=7$. $x=\sqrt{3}$ c. Parallel to $\frac{1}{2}x-\frac{3}{4}y=10$. $8x+12y=15+8\sqrt{3}$	
MP.1	**Make sense of problems and persevere in solving them.**	Students start the module with the challenge to understand and develop the mathematics for describing of the motion of a robot bound within a certain polygonal region of the plane—the room in which it sits. This a recurring problem throughout the entire module and with it, and through related problems, students discover the slope	**Eureka Math:** Module 4 Lesson 6

		criteria for perpendicular and parallel lines, the means to find the coordinates of a point dividing a line segment into two lengths in a given ratio, the distance formula of a point from a line, along with a number of geometric results via the tools of coordinate geometry.	
MP.3	**Construct viable arguments and critique the reasoning of others.**	Mathematically proficient students understand and use stated assumptions, definitions, and previously established results in constructing arguments. They make conjectures and build a logical progression of statements to explore the truth of their conjectures. They are able to analyze situations by breaking them into cases, and can recognize and use counterexamples. They justify their conclusions, communicate them to others, and respond to the arguments of others. They reason inductively about data, making plausible arguments that take into account the context from which the data arose. Mathematically proficient students are also able to compare the effectiveness of two plausible arguments, distinguish correct logic or reasoning from that which is flawed, and—if there is a flaw in an argument—explain what it is.	**Eureka Math:** Module 4 Lesson 5,6,8
MP.4	**Model with mathematics.**	Students model the motion of a robot in the plane in two contexts: determining the extent of motion within the bounds of a polygonal region, and determining and moving to the location of the source of beacon signal in the infinite plane.	**Eureka Math:** Module 4 Lesson 7
MP.7	**Look for and make use of structure.**	Students determine slope criteria for perpendicular and parallel lines and use these slope conditions to develop the general equation of a line and the formula for the distance of a point from a line. Students determine the area of polygonal regions using multiple methods including Green's theorem and decomposition. Definitive geometric properties of special quadrilaterals are explored and properties of special lines in triangles are examined.	**Eureka Math:** Module 4 Lesson 7,8

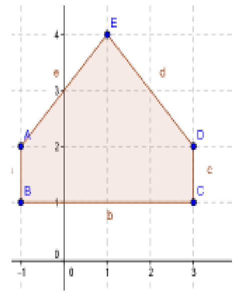
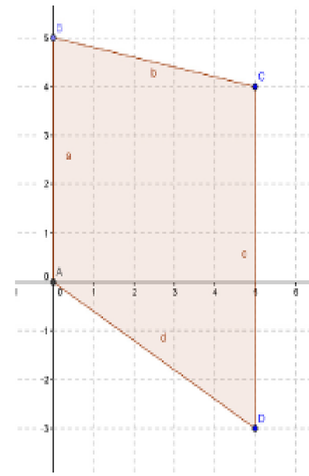
Module 4: Connecting Algebra and Geometry Through Coordinates (20 days)

Topic C: Perimeters and Areas of Polygonal Regions in the Cartesian Plane (3 Instructional days)

Lesson 9 begins Topic C with students finding the perimeter of triangular regions using the distance formula and deriving the formula for the area of a triangle with vertices $(0,0)$,(x_1,y_1), (x_2,y_2) as $A = 1/2|x_1y_2 - x_2y_1|$ (**G-GPE.B.7**). Students are introduced to the "shoelace" formula for area and understand that this formula is useful because only the coordinates of the vertices of a triangle are needed. In Lesson 10, students extend the "shoelace" formula to quadrilaterals, showing that the traditional formulas are verified with general cases of the "shoelace" formula and even extend this work to other polygons (pentagons and hexagons). Students compare the traditional formula for area and area by decomposition of figures and see that the "shoelace" formula is much more efficient in some cases. This work with the "shoelace" formula is the high school Geometry version of Green's theorem and subtly exposes students to elementary ideas of vector and integral calculus. Lesson 11 concludes this work as the regions are described by a system of inequalities. Students sketch the regions, determine points of intersection (vertices), and use the distance formula to calculate perimeter and the "shoelace" formula to determine area of these regions. Students return to the real-world application of programming a robot and extend this work to robots not just confined to straight line motion but also motion bound by regions described by inequalities and defined areas. (**G-GPE.B.7**)

Big Idea:	• Geometric figures can be represented in the coordinate plane. • The algebraic properties (including those related to the distance between points in the coordinate plane) may be used to prove geometric relationships. • The algebraic relationship between the slopes of parallel lines and the slopes of perpendicular lines.
Essential Questions:	• What is the relationship between the slopes of parallel lines and of perpendicular lines? • Given a polygon represented in the coordinate plane, what is its perimeter and area? • How can geometric relationships be proven through the application of algebraic properties to geometric figures represented in the coordinate plane?
Vocabulary	Decompose, shoelace formula (Green's Theorem)
Assessments	Galileo: Topic C Assessment

Standard	AZ College and Career Readiness Standards	Explanations & Examples	Resources
G.GPE.B.7	**B. Use coordinates to prove simple geometric theorems algebraically** Use coordinates to compute perimeters of polygons and areas of triangles and rectangles, e.g., using the distance formula.	Students may use geometric simulation software to model figures.	**Eureka Math:** Module 4 Lesson 9-11

		Given the figures below, find the area by decomposing into rectangles and triangles. **a.**  *Area = 8 square units* **b.**  *Area = 30 square units*	

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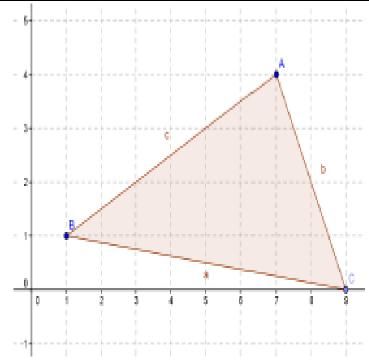
Given triangle ABC with vertices $(7,4)$, $(1,1)$, and $(9,0)$:



a. Calculate the perimeter using the distance formula.

Perimeter: ≈ 19.24 *units*

b. Calculate the area using the traditional area formula.

Area: $\frac{1}{2}\left(\sqrt{45}\right)\left(\sqrt{20}\right) = 15$ *square units*

c. Calculate the area using the shoelace formula.

Area: $\frac{1}{2}\left(7 \cdot 1 + 1 \cdot 0 + 9 \cdot 4 - 4 \cdot 1 - 1 \cdot 9 - 0 \cdot 7\right) = 15$ *square units*

d. Explain why the shoelace formula might be more useful and efficient if you were just asked to find the area.

To use the shoelace formula, all you need are the coordinates of the vertices; you would not have to use the distance formula.

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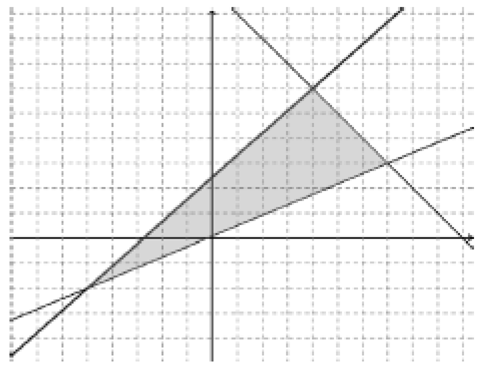
		$8x - 9y \geq -22$ $\quad x + y \leq 10$ $\quad 5x - 12y \leq -1$ *a.*  *b.* $(4, 6), (7, 3), (-5, -2)$ *c.* ≈ 29.28 *units* *d.* 25.5 *square units*	
MP.1	**Make sense of problems and persevere in solving them.**	Students start the module with the challenge to understand and develop the mathematics for describing of the motion of a robot bound within a certain polygonal region of the plane—the room in which it sits. This a recurring problem throughout the entire module and with it, and through related problems, students discover the slope criteria for perpendicular and parallel lines, the means to find the coordinates of a point dividing a line segment into two lengths in a given ratio, the distance formula of a point from a line, along with a number of geometric results via the tools of coordinate geometry.	**Eureka Math:** Module 4 Lesson 11
MP.7	**Look for and make use of structure.**	Students determine slope criteria for perpendicular and parallel lines and use these slope conditions to develop the general equation of a line and the formula for the distance of a point from a line. Students determine the area of polygonal regions using multiple methods including Green's theorem and decomposition. Definitive geometric properties of special quadrilaterals are explored and properties of special lines in triangles are examined.	**Eureka Math:** Module 4 Lesson 9,10

HS Geometry Semester 2 (Quarter 4)

Module 4: Connecting Algebra and Geometry Through Coordinates (20 days)

Topic D: Partitioning and Extending Segments and Parameterization of Lines (4 Instructional days)

Topic D concludes the work of Module 4. In Lesson 12, students find midpoints of segments and points that divide segments into 3 or more equal and proportional parts. Students will also find locations on a directed line segment between two given points that partition the segment in given ratios (**G-GPE.B.6**). Lesson 13 requires students to show that if B' and C' cut AB and AC proportionately, then the intersection of BC' and $C'C$ lies on the median of ABC from vertex A and connects this work to proving classical results in geometry (**G-GPE.B.4**). For instance, the diagonals of a parallelogram bisect one another and the medians of a triangle meet at the point 2/3 of the way from the vertex for each.

Lesson 14 is an optional lesson that allows students to explore parametric equations and compare them with more familiar linear equations (**G-GPE.B.6**, **G-MG.A.1**). Parametric equations make both the x- and y-variables in an equation dependent on a third variable, usually time; for example, $f(t)=(t,2t-1)$ represents a function, f, with both x- and y-coordinates dependent on the independent variable, t (time). In this lesson, parametric equations model the robot's horizontal and vertical motion over a period of time, $P=(20+t/2(100),10+t/2(40))$. Introducing parametric equations in the Geometry course prepares students for higher level courses and also represents an opportunity to show coherence between functions, algebra, and coordinate geometry. Students extend their knowledge of parallel and perpendicular lines to lines given parametrically. Students complete the work of this module in Lesson 15 by deriving and applying the distance formula (**G-GPE.B.4**) and with the challenge of locating the point along a line closest to a given point, again given as a robot challenge. (**G-GPE.B.4, G-GPE.B.6**)

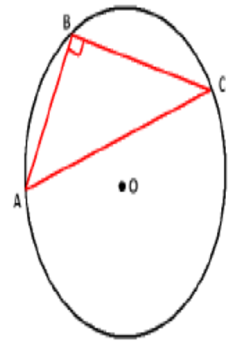
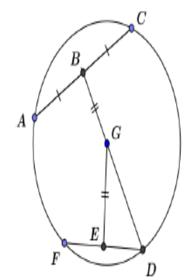
Big Idea:	•
Essential Questions:	•
Vocabulary	Midpoint formula
Assessments	Galileo: Topic D Assessment

Standard	AZ College and Career Readiness Standards	Explanations & Examples	Resources
G.GPE.B.4	**B. Use coordinates to prove simple geometric theorems algebraically** Use coordinates to prove simple geometric theorems algebraically. *For example, prove or disprove that a figure defined by four given points in the coordinate plane is a rectangle; prove or disprove that the point (1, √3) lies on the circle centered at the origin and*	Students may use geometric simulation software to model figures and prove simple geometric theorems.	**Eureka Math:** Module 4 Lesson 13-15

	containing the point (0, 2).	Given a quadrilateral with vertices $E(0,5)$, $F(6,5)$, $G(4,0)$, and $H(-2,0)$: a. Prove quadrilateral $EFGH$ is a parallelogram. *$\overline{EF}$ and $\overline{GH}$ are horizontal segments, so they are parallel.* *$\overline{HE}$ and $\overline{GF}$ have slopes of $\frac{5}{2}$, so they are parallel.* *Both pairs of opposite sides are parallel, so the quadrilateral is a parallelogram.* b. Prove $(2, 2.5)$ is a point on both diagonals of the quadrilateral. *Since $EFGH$ is a parallelogram, the diagonals intersect at their midpoints. $(2, 2.5)$ is the midpoint of $\overline{HF}$ and $\overline{GE}$, so it is a point on both diagonals.*	
G.GPE.B.6	**B. Use coordinates to prove simple geometric theorems algebraically** Find the point on a directed line segment between two given points that partitions the segment in a given ratio.	Students may use geometric simulation software to model figures or line segments. Find the midpoint of $\overline{ST}$ given $S(-2,8)$ and $T(10,-4)$. $M\left(\frac{1}{2}(-2+10), \frac{1}{2}(8-4)\right) = M(4,2)$ Find the point on the directed segment from $(-2,0)$ to $(5,8)$ that divides it in the ratio of $1:3$. *A ratio of $1:3$ means $\frac{1}{4}$ of the way from $(-2,0)$ to $(5,8)$.* $\left(-2+\frac{1}{4}(5-(-2)), 0+\frac{1}{4}(8-0)\right) = \left(-\frac{1}{4}, 2\right)$ Given $\overline{PQ}$ and point R that lies on $\overline{PQ}$ such that point R lies $\frac{7}{9}$ of the length of $\overline{PQ}$ from point P along $\overline{PQ}$. a. Sketch the situation described. 	**Eureka Math:** Module 4 Lesson 12-15

MP.3	**Construct viable arguments and critique the reasoning of others.**	Mathematically proficient students understand and use stated assumptions, definitions, and previously established results in constructing arguments. They make conjectures and build a logical progression of statements to explore the truth of their conjectures. They are able to analyze situations by breaking them into cases, and can recognize and use counterexamples. They justify their conclusions, communicate them to others, and respond to the arguments of others.	**Eureka Math:** Module 4 Lesson 12,13
MP.4	**Model with mathematics.**	Students model the motion of a robot in the plane in two contexts: determining the extent of motion within the bounds of a polygonal region, and determining and moving to the location of the source of beacon signal in the infinite plane.	**Eureka Math:** Module 4 Lesson 14
MP.7	**Look for and make use of structure.**	Students determine slope criteria for perpendicular and parallel lines and use these slope conditions to develop the general equation of a line and the formula for the distance of a point from a line. Students determine the area of polygonal regions using multiple methods including Green's theorem and decomposition. Definitive geometric properties of special quadrilaterals are explored and properties of special lines in triangles are examined.	**Eureka Math:** Module 4 Lesson 12,15

HS Geometry Semester 2 (Quarter 4)			
Module 5: Circles With and Without Coordinates (25 days) **Topic A: Central and Inscribed Angles (6 Instructional days)** With geometric intuition well established through Modules 1, 2, 3, and 4, students are now ready to explore the rich geometry of circles. This module brings together the ideas of similarity and congruence studied in Modules 1 and 2, the properties of length and area studied in Modules 3 and 4, and the work of geometric construction studied throughout the entire year. It also includes the specific properties of triangles, special quadrilaterals, parallel lines and transversals, and rigid motions established and built upon throughout this mathematical story. This module's focus is on the possible geometric relationships between a pair of intersecting lines and a circle drawn on the page. If the lines are perpendicular and one passes through the center of the circle, then the relationship encompasses the perpendicular bisectors of chords in a circle and the association between a tangent line and a radius drawn to the point of contact. If the lines meet at a point on the circle, then the relationship involves inscribed angles. If the lines meet at the center of the circle, then the relationship involves central angles. If the lines meet at a different point inside the circle or at a point outside the circle, then the relationship includes the secant angle theorems and tangent angle theorems. (**G-C.A.2, G-C.A.3**)			
Big Idea:	•		
Essential Questions:	•		
Vocabulary	Thales' Theorem, diameter, radius, central angle, inscribed angle, inscribed angle theorem, chord, converse of Thales' theorem, equidistant, arc, intercepted arc, minor and major arc, central angle, consequence of inscribed angle theorem		
Assessments	Galileo: Geometry Module 5 Foundational Skills Assessment; Galileo: Topic A Assessment		
Standard	**AZ College and Career Readiness Standards**	**Explanations & Examples**	**Resources**

G.C.A.2	**Understand and apply theorems about circles.** Identify and describe relationships among inscribed angles, radii, and chords. *Include3 the relationship between central, inscribed, and circumscribed angles; inscribed angles on a diameter are right angles; the radius of a circle is perpendicular to the tangent where the radius intersects the circle.*	Show that there is something mathematically wrong with the picture below.  *Draw three radii ($\overline{OA}$, $\overline{OB}$, and $\overline{OC}$). Label $\angle BAC$ as a and $\angle BCA$ as c. Also label $\angle OAC$ as x and $\angle OCA$ as x since $\triangle AOC$ is isosceles (both sides are radii). If $\angle ABC$ is a right angle (as indicated on the drawing), then $a + c = 90°$. Since $\triangle AOB$ is isosceles, $\angle ABO = a + x$. Similarly, $\angle CBO = c + x$. Now adding the angles of $\triangle ABC$ results in $a + a + x + c + x + c = 180°$. Using the distributive property and division, we obtain $a + c + x = 90°$. Substitution takes us to $a + c = a + c + x$, which is a contradiction. Therefore, the figure above is mathematically impossible.* In the figure, $AC = 24$, and $DG = 13$. Find EG. Explain your work. *5, $\triangle ABG$ is a right triangle with $\text{hypotenuse} = \text{radius} = 13$ and $AB = 12$, so $BG = 5$ by Pythagorean theorem. $BG = GE = 5$.*  	**Eureka Math:** Module 5 Lesson 1-6
G.C.A.3	**Understand and apply theorems about circles.** Construct the inscribed and circumscribed circles of a triangle, and prove properties of angles for a quadrilateral inscribed in a circle.		**Eureka Math:** Module 5 Lesson 3

MP.1	**Make sense of problems and persevere in solving them.**	Students solve a number of complex unknown angles and unknown area geometry problems, work to devise the geometric construction of given objects, and adapt established geometric results to new contexts and to new conclusions.	**Eureka Math:** Module 5 Lesson 1,3
MP.3	**Construct viable arguments and critique the reasoning of others.**	Students must provide justification for the steps in geometric constructions and the reasoning in geometric proofs, as well as create their own proofs of results and their extensions.	**Eureka Math:** Module 5 Lesson 1,2,4,6
MP.7	**Look for and make use of structure.**	Students must identify features within complex diagrams (e.g., similar triangles, parallel chords, and cyclic quadrilaterals) which provide insight as to how to move forward with their thinking.	**Eureka Math:** Module 5 Lesson 1,2,4,5
MP.8	**Look for and express regularity in repeated reasoning.**	Mathematically proficient students notice if calculations are repeated, and look both for general methods and for shortcuts. As they work to solve a problem, mathematically proficient students maintain oversight of the process, while attending to the details. They continually evaluate the reasonableness of their intermediate results.	**Eureka Math:** Module 5 Lesson 1,4

HS Geometry Semester 2 (Quarter 4)

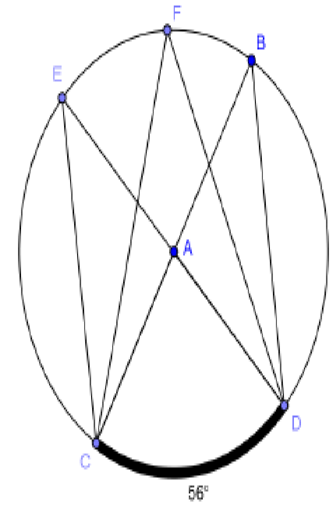
Module 5: Circles With and Without Coordinates (25 days)

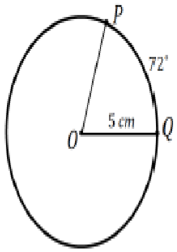
Topic B: Central and Inscribed Angles (4 Instructional days)

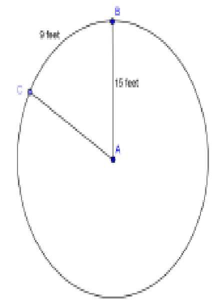
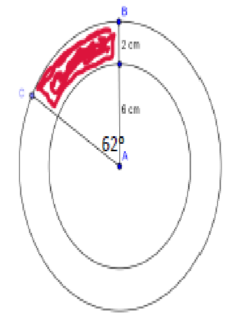
In Topic B, students continue studying the relationships between chords, diameters, and angles and extend that work to arcs, arc length, and areas of sectors. Students use prior knowledge of the structure of inscribed and central angles together with repeated reasoning to develop an understanding of circles, secant lines, and tangent lines (MP.7). In Lesson 7, students revisit the inscribed angle theorem, this time stating it in terms of inscribed arcs (**G-C.A.2**). This concept is extended to studying similar arcs, which leads students to understand that all circles are similar (**G-C.A.1**). Students then look at the relationships between chords and subtended arcs and prove that congruent chords lie in congruent arcs. They also prove that arcs between parallel lines are congruent using transformations (**G-C.A.2**). Lessons 9 and 10 switch the focus from angles to arc length and areas of sectors. Students combine previously learned formulas for area and circumference of circles with concepts learned in this module to determine arc length, areas of sectors, and similar triangles (**G-C.B.5**). In Lesson 9, students are introduced to radians as the ratio of arc length to the radius of a circle. Lesson 10 reinforces these concepts with problems involving unknown length and area. Topic B requires that students use and apply prior knowledge to see the structure in new applications and to see the repeated patterns in these problems in order to arrive at theorems relating chords, arcs, angles, secant lines, and tangent lines to circles (MP.7). For example, students know that an inscribed angle has a measure of half the central angle intercepting the same arc. When they discover that the measure of a central angle is equal to the angle measure of its intersected arc, they conclude that the measure of an inscribed angle is half the angle of its intercepted arc. Students then conclude that congruent arcs have congruent chords and that arcs between parallel chords are congruent. (**G-C.A.1, G-C.A.2, G-C.B.5**)

Big Idea:	•
Essential Questions:	•
Vocabulary	Inscribed angle theorem, arc, minor and major arc, semicircle, inscribed angle, central angle, intercepted arc of an angle, length of an arc, radian, sector, area of a sector
Assessments	Galileo: Topic B Assessment

Standard	AZ College and Career Readiness Standards	Explanations & Examples	Resources
G.C.A.1	**Understand and apply theorems about circles.** Prove that all circles are similar.		**Eureka Math:** Module 5 Lesson 7,8

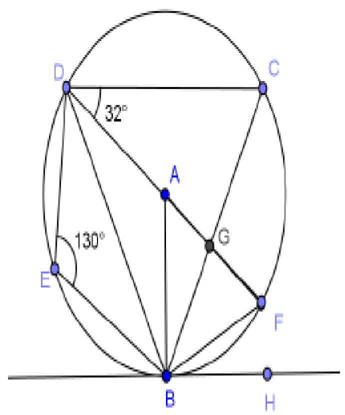
G.C.A.2 ▲	**Understand and apply theorems about circles.** Identify and describe relationships among inscribed angles, radii, and chords. *Include the relationship between central, inscribed, and circumscribed angles; inscribed angles on a diameter are right angles; the radius of a circle is perpendicular to the tangent where the radius intersects the circle.*	Given circle A with diameters $\overline{BC}$ and $\overline{DE}$ and $m\widehat{CD} = 56°$  a. Name a central angle. $\angle CAD$ b. Name an inscribed angle. *Answers will vary.* $\angle CED$. c. Name a chord that is not a diameter. *Answers will vary.* $\overline{CE}$. d. What is the measure of $\angle CAD$? 56° e. What is the measure of $\angle CBD$? 28° f. Name 3 angles of equal measure. $m\angle CED = m\angle CFD = m\angle CBD$ g. What is the degree measure of $\widehat{CDB}$? 180°	**Eureka Math:** Module 5 Lesson 7,8

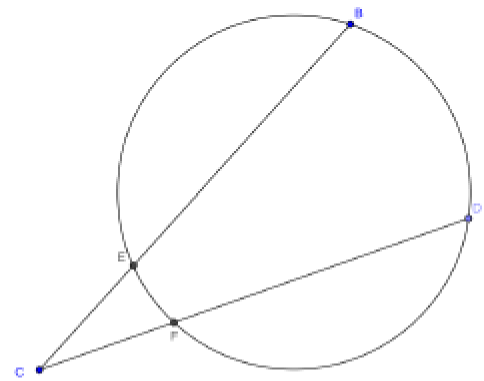
G.C.B.5 ▲	**Find arc lengths and areas of sectors of circles.** Derive using similarity the fact that the length of the arc intercepted by an angle is proportional to the radius, and define the radian measure of the angle as the constant of proportionality; derive the formula for the area of a sector.	P and Q are points on the circle of radius 5 cm and the measure of arc $\widehat{PQ}$ is 72°. Find, to one decimal place each of the following:  a. The length of arc $\widehat{PQ}$ $Arc\ length(\widehat{PQ}) = \frac{72}{360}(2\pi \times 5)$ $Arc\ length(\widehat{PQ}) = 2\pi$ *The arc length of $\widehat{PQ}$ is 2π cm or approximately 6.3 cm.* b. Find the ratio of the arc length to the radius of the circle. $\frac{\pi}{180} \cdot 72 = \frac{2\pi}{5}$ *radians* **What is the radius of a circle if the length of a 45° arc is 9π?** $9\pi = \frac{45}{360}(2\pi r)$ $r = 36$ *The radius of the circle is 36 units.*	**Eureka Math:** Module 5 Lesson 9-10

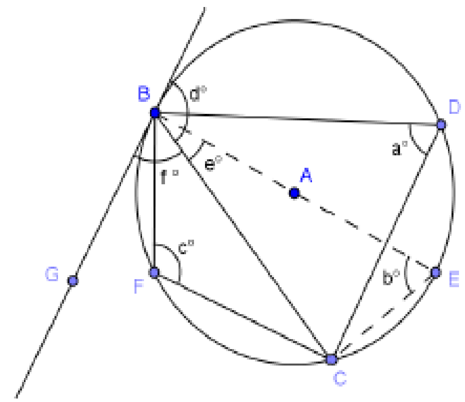
		Given circle A, find the following (round to the nearest hundredth): a. The $m\widehat{BC}$ in degrees. $68.75°$ b. The area of sector $\widehat{BC}$. $135\ \text{ft}^2$  Find the shaded area. $15.15\ \text{cm}^2$  	
MP.1	**Make sense of problems and persevere in solving them.**	Students solve a number of complex unknown angles and unknown area geometry problems, work to devise the geometric construction of given objects, and adapt established geometric results to new contexts and to new conclusions.	**Eureka Math:** Module 5 Lesson 8,9
MP.2	**Reason abstractly and quantitatively.**	Mathematically proficient students make sense of quantities and their relationships in problem situations. They bring two complementary abilities to bear on problems involving quantitative relationships: the ability to *decontextualize*—to abstract a given situation and represent it symbolically and manipulate the representing symbols as if they have a life of their own, without necessarily attending to their referents—and the ability to *contextualize*, to pause as needed during the manipulation process in order to probe into the referents for the symbols involved. Quantitative reasoning entails habits of creating a coherent representation of the problem at hand; considering the units involved; attending to the meaning of quantities, not just how to	**Eureka Math:** Module 5 Lesson 10

		compute them; and knowing and flexibly using different properties of operations and objects.	
MP.3	**Construct viable arguments and critique the reasoning of others.**	Students must provide justification for the steps in geometric constructions and the reasoning in geometric proofs, as well as create their own proofs of results and their extensions.	**Eureka Math:** Module 5 Lesson 7
MP.4	**Model with mathematics.**	Mathematically proficient students can apply the mathematics they know to solve problems arising in everyday life, society, and the workplace. Mathematically proficient students who can apply what they know are comfortable making assumptions and approximations to simplify a complicated situation, realizing that these may need revision later. They are able to identify important quantities in a practical situation and map their relationships using such tools as diagrams, two-way tables, graphs, flowcharts and formulas. They can analyze those relationships mathematically to draw conclusions. They routinely interpret their mathematical results in the context of the situation and reflect on whether the results make sense, possibly improving the model if it has not served its purpose.	**Eureka Math:** Module 5 Lesson 10
MP.7	**Look for and make use of structure.**	Students must identify features within complex diagrams (e.g., similar triangles, parallel chords, and cyclic quadrilaterals) which provide insight as to how to move forward with their thinking.	**Eureka Math:** Module 5 Lesson 8,9,10
MP.8	**Look for and express regularity in repeated reasoning.**	Mathematically proficient students notice if calculations are repeated, and look both for general methods and for shortcuts. As they work to solve a problem, mathematically proficient students maintain oversight of the process, while attending to the details. They continually evaluate the reasonableness of their intermediate results.	**Eureka Math:** Module 5 Lesson 9

<table>
<tr><th colspan="4">HS Geometry Semester 2 (Quarter 4)</th></tr>
<tr><td colspan="4">Module 5: Circles With and Without Coordinates (25 days)
Topic C: Secants and Tangents (6 Instructional days)
Topic C focuses on secant and tangent lines intersecting circles, the relationships of angles formed, and segment lengths. In Lesson 11, students study properties of tangent lines and construct tangents to a circle through a point outside the circle and through points on the circle (G-C.A.4). Students prove that at the point of tangency, the tangent line and radius meet at a right angle. Lesson 12 continues the study of tangent lines proving segments tangent to a circle from a point outside the circle are congruent. In Lesson 13, students inscribe a circle in an angle and a circle in a triangle with constructions (G-C.A.3) leading to the study of inscribed angles with one ray being part of the tangent line (G-C.A.2). Students solve a variety of missing angle problems using theorems introduced in Lessons 11–13 (MP.1). The study of secant lines begins in Lesson 14 as students study two secant lines that intersect inside a circle. Students prove that an angle whose vertex is inside a circle is equal in measure to half the sum of arcs intercepted by it and its vertical angle. Lesson 15 extends this study to secant lines that intersect outside of a circle. Students understand that an angle whose vertex is outside of a circle is equal in measure to half the difference of the degree measure of its larger and smaller intercepted arcs. This concept is extended as the secant rays rotate to form tangent rays, and that relationship is developed. Topic C and the study of secant lines concludes in Lesson 16 as students discover the relationships between segment lengths of secant lines intersecting inside and outside of a circle. Students find similar triangles and use proportional sides to develop this relationship (G-SRT.B.5). Topic C highlights MP.1 as students persevere in solving missing angle and missing length problems; it also highlights MP.6 as students extend known relationships to limiting cases.</td></tr>
<tr><td>Big Idea:</td><td colspan="3">•</td></tr>
<tr><td>Essential Questions:</td><td colspan="3">•</td></tr>
<tr><td>Vocabulary</td><td colspan="3">Interior of a circle, exterior of a circle, secant line, tangent line, tangent segment, tangent to a circle, secant to a circle, polygon inscribed in a circle, circle inscribed in a polygon, tangent-secant theorem, conjecture</td></tr>
<tr><td>Assessments</td><td colspan="3">Galileo: Topic C Assessment</td></tr>
<tr><td>Standard</td><td>AZ College and Career Readiness Standards</td><td>Explanations & Examples</td><td>Resources</td></tr>
</table>

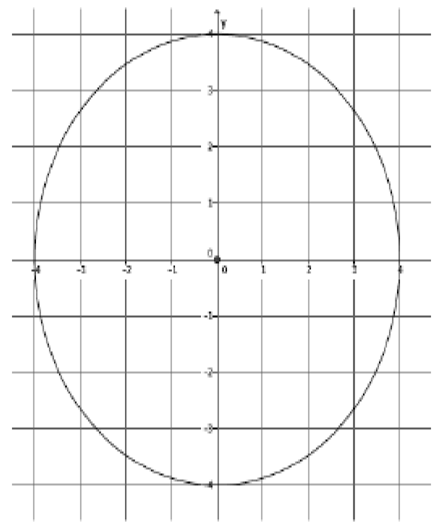
G.C.A.2 ▲	**Understand and apply theorems about circles.** Identify and describe relationships among inscribed angles, radii, and chords. *Include the relationship between central, inscribed, and circumscribed angles; inscribed angles on a diameter are right angles; the radius of a circle is perpendicular to the tangent where the radius intersects the circle.*	$\overleftrightarrow{BH}$ is tangent to circle A. $\overline{DF}$ is a diameter. Find a. $m\angle BCD$ 50° b. $m\angle BAF$ 80° c. $m\angle BDA$ 40° d. $m\angle FBH$ 40° e. $m\angle BGF$ 98°  	**Eureka Math:** Module 5 Lesson 13-16

		$CE = 6, CB = 9, CD = 18.$ Show $CF = 3.$  *$6 \cdot 9 = 54$ and $18 \cdot CF = 54.$ This means $CF = 3.$*	

G.C.A.3	**Understand and apply theorems about circles.** Construct the inscribed and circumscribed circles of a triangle, and prove properties of angles for a quadrilateral inscribed in a circle.	$\overline{BG}$ is tangent to circle A. $\overline{BE}$ is a diameter. Prove: (i) $f = a$ and (ii) $d = c$  $m\angle EBG = 90^0$ *tangent perpendicular to radius* $f = 90 - e$ *sum of angles* $m\angle ECB = 90^0$ *angle inscribed in semi-circle* $In\ \Delta ECB,$ $b + 90 + e = 180$ *sum of angles of a triangle* $b = 90 - e$ $a = b$ *angles inscribed in same arc congruent* $a = f$ *substitution* $a + c = 180$ *inscribed in opposite arcs* $a = f$ *inscribed in same arc* $f + d = 180$ *linear pairs form supplementary angles* $c + f = f + d$ *substitution* $c = d$	**Eureka Math:** Module 5 Lesson 13-16

MP.1	**Make sense of problems and persevere in solving them.**	Students solve a number of complex unknown angles and unknown area geometry problems, work to devise the geometric construction of given objects, and adapt established geometric results to new contexts and to new conclusions.	**Eureka Math:** Module 5 Lesson 13
MP.3	**Construct viable arguments and critique the reasoning of others.**	Students must provide justification for the steps in geometric constructions and the reasoning in geometric proofs, as well as create their own proofs of results and their extensions.	**Eureka Math:** Module 5 Lesson 12,15
MP.6	**Attend to precision.**	Mathematically proficient students try to communicate precisely to others. They try to use clear definitions in discussion with others and in their own reasoning. They state the meaning of the symbols they choose, including using the equal sign consistently and appropriately. They are careful about specifying units of measure, and labeling axes to clarify the correspondence with quantities in a problem. They calculate accurately and efficiently, express numerical answers with a degree of precision appropriate for the problem context.	**Eureka Math:** Module 5 Lesson 11
MP.7	**Look for and make use of structure.**	Students must identify features within complex diagrams (e.g., similar triangles, parallel chords, and cyclic quadrilaterals) which provide insight as to how to move forward with their thinking.	**Eureka Math:** Module 5 Lesson 11,14
MP.8	**Look for and express regularity in repeated reasoning.**	Mathematically proficient students notice if calculations are repeated, and look both for general methods and for shortcuts. As they work to solve a problem, mathematically proficient students maintain oversight of the process, while attending to the details. They continually evaluate the reasonableness of their intermediate results.	**Eureka Math:** Module 5 Lesson 11,16

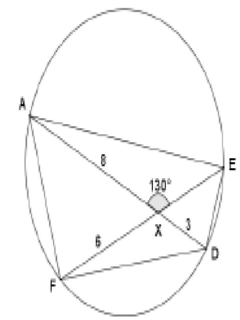
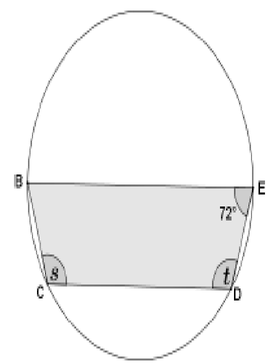
HS Geometry Semester 2 (Quarter 4)			
Module 5: Circles With and Without Coordinates (25 days) **Topic D: Equations for Circles and Their Tangents (3 Instructional days)** Topic D consists of three lessons focusing on MP.7. Students see the structure in the different forms of equations of a circle and lines tangent to circles. In Lesson 17, students deduce the equation for a circle in center-radius form using what they know about the Pythagorean theorem and the distance between two points on the coordinate plane (**G-GPE.A.1**). Students first understand that a circle whose center is at the origin of the coordinate plane is given by $x^2+y^2=r^2$, where r is the radius. Using their knowledge of translation, students derive the general formula for a circle as $(x-a)^2+(y-b)^2=r^2$, where r is the radius of the circle, and (a,b) is the center of the circle. In Lesson 18, students use their algebraic skills of factoring and completing the square to transform equations into center-radius. Students prove that $x^2+y^2+Ax+By+C=0$ is the equation of a circle and find the formula for the center and radius of this circle (**G-GPE.A.4**). Students know how to recognize the equation of a circle once the equation format is in center-radius. In Lesson 19, students again use algebraic skills to write the equations of lines, specifically lines tangent to a circle, using information about slope and/or points on the line. Recalling students' understanding of tangent from Lesson 11 and combining that with the equations of circles from Lessons 17 and 18, students determine the equation of tangent lines to a circle from points outside of the circle.			
Big Idea:	•		
Essential Questions:	•		
Vocabulary	Tangent to a circle		
Assessments	Galileo: Topic D Assessment		
Standard	**AZ College and Career Readiness Standards**	**Explanations & Examples**	**Resources**

G.GPE.A.1	**Translate between the geometric description and the equation for a conic section.** Derive the equation of a circle of given center and radius using the Pythagorean Theorem; complete the square to find the center and radius of a circle given by an equation.	Describe the circle given by the equation $(x-7)^2+(y-8)^2=9$. *The circle has a center at* $(7,8)$ *and a radius of* 3. Write the equation for a circle with center $(0,-4)$ and radius 8. $x^2+(y+4)^2=64$ Write the equation for the circle shown below.  $x^2+y^2=16$ A circle has a diameter with endpoints at $(6,5)$ and $(8,5)$. Write the equation for the circle. $(x-7)^2+(y-5)^2=1$	**Eureka Math:** Module 5 Lesson 17,19

G.GPE.B.4	**Use coordinates to prove simple geometric theorems algebraically.** Use coordinates to prove simple geometric theorems algebraically. *For example, prove or disprove that a figure defined by four given points in the coordinate plane is a rectangle; prove or disprove that the point* (1,√3) *lies on the circle centered at the origin and containing the point* (0,2).	Identify the center and radii of the following circles. a. $(x+25)+y^2=1$ *The center is* $(-25, 0)$*, and the radius is* 1. b. $x^2+2x+y^2-8y=8$ $(x+1)^2+(y-4)^2=25$ *The center is* $(-1, 4)$*, and the radius is* 5. c. $x^2-20x+y^2-10y+25=0$ $(x-10)^2+(y-5)^2=100$ *The center is* $(10, 5)$*, and the radius is* 10. d. $x^2+y^2=19$ *The center is* $(0, 0)$*, and the radius is* $\sqrt{19}$.	**Eureka Math:** Module 5 Lesson 18,19

		Show that a circle with equation $(x-2)^2+(y+3)^2=160$ has two tangent lines with equations $y+15=\frac{1}{3}(x-6)$ and $y-9=\frac{1}{3}(x+2)$. *Assume that the circle has the tangent lines given by the equations above. Then, the tangent lines have slope $\frac{1}{3}$, and the slope of the radius to those lines must be -3. If we can show that the points $(6,-15)$ and $(-2,9)$ are on the circle, and that the slope of the radius to the tangent lines is -3, then we will have shown that the given circle has the two tangent lines given.* $(6-2)^2+(-15+3)^2$ $=4^2+(-12)^2$ $=160$ $(-2-2)^2+(9+3)^2$ $=(-4)^2+12^2$ $=160$ *Since both points satisfy the equation, then the points $(6,-15)$ and $(-2,9)$ are on the circle.* $m=\frac{-15-(-3)}{6-2}=-\frac{12}{4}=-3$ $m=\frac{9-(-3)}{-2-2}=-\frac{12}{4}=-3$ *The slope of the radius is -3.*	
MP.1	**Make sense of problems and persevere in solving them.**	Students solve a number of complex unknown angles and unknown area geometry problems, work to devise the geometric construction of given objects, and adapt established geometric results to new contexts and to new conclusions.	**Eureka Math:** Module 5 Lesson 18-19
MP.3	**Construct viable arguments and critique the reasoning of others.**	Students must provide justification for the steps in geometric constructions and the reasoning in geometric proofs, as well as create their own proofs of results and their extensions.	**Eureka Math:** Module 5 Lesson 17,19

HS Geometry Semester 2 (Quarter 4)			
Module 5: Circles With and Without Coordinates (25 days) **Topic E: Cyclic Quadrilaterals and Ptolemy's Theorem (2 Instructional days)** Topic E is a two lesson topic recalling several concepts from the year, e.g., Pythagorean theorem, similarity, and trigonometry, as well as concepts from Module 5 related to arcs and angles. In Lesson 20, students are introduced to the term *cyclic quadrilaterals* and define the term informally as a quadrilateral whose vertices lie on a circle. Students then prove that a quadrilateral is cyclic if and only if the opposite angles of the quadrilateral are supplementary. They use this reasoning and the properties of quadrilaterals inscribed in circles (**G-C.A.3**) to develop the area formula for a cyclic quadrilateral in terms of side length. Lesson 21 continues the study of cyclic quadrilaterals as students prove Ptolemy's theorem and understand that the area of a cyclic quadrilateral is a function of its side lengths and an acute angle formed by its diagonals. Students must identify features within complex diagrams to inform their thinking, highlighting MP.7. For example, students use the structure of an inscribed triangle in a half-plane separated by the diagonal of a cyclic quadrilateral to conclude that a reflection of the triangle along the diagonal produces a different cyclic quadrilateral with an area equal to the original cyclic quadrilateral. Students use this reasoning to make sense of Ptolemy's theorem and its origin.			
Big Idea:	•		
Essential Questions:	•		
Vocabulary	cyclic quadrilateral		
Assessments	Galileo: Topic E Assessment		
Standard	**AZ College and Career Readiness Standards**	**Explanations & Examples**	**Resources**

G.C.A.3 ▲	**Understand and apply theorems about circles.** Construct the inscribed and circumscribed circles of a triangle, and prove properties of angles for a quadrilateral inscribed in a circle.	Quadrilateral $FAED$ is cyclic, $AX = 8$, $FX = 6$, $XD = 3$, and $m\angle AXE = 130°$. Find the area of quadrilateral $FAED$. *Using the two-chord power rule,* $(AX)(XD) = (FX)(XE)$. $8(3) = 6(XE)$, *thus* $XE = 4$. *The area of a cyclic quadrilateral is equal to the product of the lengths of the diagonals and the sine of the acute angle formed by them. The acute angle formed by the diagonals is* $50°$. $\text{Area} = (8+3)(6+4)\cdot \sin(50)$ $\text{Area} = (11)(10)\cdot \sin(50)$ $\text{Area} \approx 84.3$  In the diagram below, $\overline{BE} \parallel \overline{CD}$, and $m\angle BED = 72°$. Find the value of s and t. *Quadrilateral* $BCDE$ *is cyclic, so opposite angles are supplementary.* $s + 72° = 180°$ $s = 108°$ *Parallel chords* BE *and* CD *intercept congruent arcs* $\widehat{CB}$ *and* $\widehat{ED}$. *By angle addition,* $m\widehat{CBE} = m\widehat{BED}$, *so it follows by the inscribed angle theorem that* $s = t = 108°$.  	**Eureka Math:** Module 5 Lesson 20
MP.7	**Look for and make use of structure.**	Students must identify features within complex diagrams (e.g., similar triangles, parallel chords, and cyclic quadrilaterals) which provide insight as to how to move forward with their thinking.	**Eureka Math:** Module 5 Lesson 20,21