Chapter 7

Chapter 7 Maintaining Mathematical Proficiency (p. 357)

1. $4(7 - x) = 16$

$$\begin{aligned} \frac{4(7-x)}{4} &= \frac{16}{4} \\ 7 - x &= 4 \\ \underline{-7} \quad &= \underline{-7} \\ -x &= -3 \\ \frac{-x}{-1} &= \frac{-3}{-1} \\ x &= 3 \end{aligned}$$

2. $7(1 - x) + 2 = -19$

$$\begin{aligned} -2 &= -2 \\ 7(1-x) &= -21 \\ \frac{7(1-x)}{7} &= \frac{-21}{7} \\ 1 - x &= -3 \\ \underline{-1} \quad &= \underline{-1} \\ -x &= -4 \\ \frac{-x}{-1} &= \frac{-4}{-1} \\ x &= 4 \end{aligned}$$

3. $3(x - 5) + 8(x - 5) = 22$

$$\begin{aligned} 3x - 15 + 8x - 40 &= 22 \\ 11x - 55 &= 22 \\ \frac{11x}{11} - \frac{55}{11} &= \frac{22}{11} \\ x - 5 &= 2 \\ \underline{+5} &= \underline{+5} \\ x &= 7 \end{aligned}$$

4. Slope of line a: $\dfrac{2 - (-2)}{-2 - 4} = \dfrac{2 + 2}{-6} = \dfrac{4}{-6} = -\dfrac{2}{3}$

Slope of line b: $\dfrac{-4 - (-2)}{0 - (-3)} = \dfrac{-4 + 2}{6} = \dfrac{-4}{6} = -\dfrac{2}{3}$

Slope of line c: $\dfrac{-3 - 0}{3 - (-3)} = \dfrac{-3}{3 + 3} = \dfrac{-3}{6} = -\dfrac{1}{2}$

Slope of line d: $\dfrac{4 - 0}{3 - 1} = \dfrac{4}{2} = 2$

Because the slopes of line a and line b are equal, $a \parallel b$.

Because $\left(-\dfrac{1}{2}\right)(2) = -1$, $c \perp d$.

5. Slope of line a: $\dfrac{1 - (-3)}{3 - 0} = \dfrac{1 + 3}{3} = \dfrac{4}{3}$

Slope of line b: $\dfrac{1 - (-3)}{0 - (-3)} = \dfrac{1 + 3}{3} = \dfrac{4}{3}$

Slope of line c: $\dfrac{4 - 1}{-2 - 2} = \dfrac{3}{-4} = -\dfrac{3}{4}$

Slope of line d: $\dfrac{-4 - 2}{4 - (-4)} = \dfrac{-6}{4 + 4} = -\dfrac{6}{8} = -\dfrac{3}{4}$

Because the slopes of line a and line b are equal, $a \parallel b$.
Because the slopes of line c and line d are equal, $c \parallel d$.
Because $\left(\dfrac{4}{3}\right)\left(-\dfrac{3}{4}\right) = -1$, $a \perp c$, $a \perp d$, $b \perp c$, and $b \perp d$.

6. Slope of line a: $\dfrac{-4 - (-2)}{4 - (-2)} = \dfrac{-4 + 2}{4 + 2} = \dfrac{-2}{6} = -\dfrac{1}{3}$

Slope of line b: $\dfrac{-2 - 2}{-3 - (-2)} = \dfrac{-4}{-3 + 2} = \dfrac{-4}{-1} = 4$

Slope of line c: $\dfrac{-3 - 1}{2 - 3} = \dfrac{-4}{-1} = 4$

Slope of line d: $\dfrac{3 - 4}{0 - (-4)} = \dfrac{-1}{4} = -\dfrac{1}{4}$

Because $4\left(-\dfrac{1}{4}\right) = -1$, $b \perp d$ and $c \perp d$.

Because the slopes of line b and line c are equal, $b \parallel c$.

7. You can follow the order of operations with all of the other operations in the equation and treat the operations in the expression separately.

Chapter 7 Mathematical Practices (p. 358)

1. false; There is no overlap between the set of trapezoids and the set of kites.

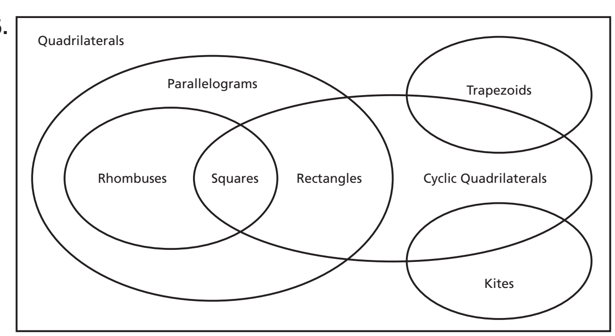
2. true; There is no overlap between the set of kites and the set of parallelograms.

3. false; There is area inside the parallelogram circle for rhombuses and other parallelograms that do not fall inside the circle for rectangles.

4. true; All squares are inside the category of quadrilaterals.

5. *Sample answer:* All kites are quadrilaterals. No trapezoids are kites. All squares are parallelograms. All squares are rectangles. Some rectangles are squares. No rhombuses are trapezoids.

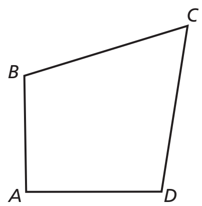
6.



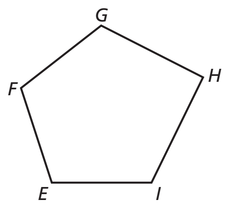
Chapter 7

7.1 Explorations (p. 359)

1. a. The sum of the measures of the interior angles of quadrilateral $ABCD$ is $91° + 106° + 64° + 99° = 360°$.



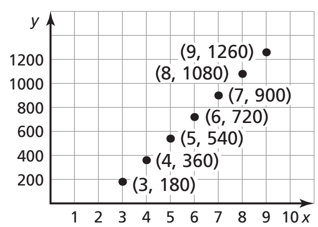
The sum of the measures of the interior angles of pentagon $EFGHI$ is $108° + 110° + 115° + 91° + 116° = 540°$.



b.

Number of sides, n	3	4	5	6	7	8	9
Sum of angle measures, S	180°	360°	540°	720°	900°	1080°	1260°

c.



d. $S = (n - 2) \cdot 180$; Let n represent the number of sides of the polygon. If you subtract 2 and multiply the difference by 180°, then you get the sum of the measures of the interior angles of a polygon.

2. a. An equation used to determine the measure of one angle in a regular polygon is $S = \frac{(n-2) \cdot 180}{n}$

b. $S = \frac{(5-2) \cdot 180}{5}$

$= \frac{3 \cdot 180}{5}$

$= \frac{540}{5}$

$= 108$

The measure of one interior angle of a regular pentagon is 108°.

c.

Number of sides, n	3	4	5	6	7	8	9
Sum of angle measures, S	180°	360°	540°	720°	900°	1080°	1260°
Measure of one interior angle	60°	90°	108°	120°	128.57°	135°	140°

3. The sum S of the measure of the interior angles of a polygon is given by the equation $S = (n - 2)180$, where n is the number of sides of the polygon.

4. $S = \frac{(n-2)180}{n}$

$= \frac{(12-2)\,180}{12}$

$= \frac{(10)(180)}{12} = \frac{1800}{12} = 150°$

The measure of one interior angle in a regular dodecagon is 150°.

7.1 Monitoring Progress (pp. 360–363)

1. $(n - 2) \cdot 180° = (11 - 2) \cdot 180°$

$= 9 \cdot 180°$

$= 1620°$

The sum of the measures of the interior angles of the 11-gon is 1620°.

2. $(n - 2) \cdot 180° = 1440°$

$\frac{(n-2) \cdot 180°}{180°} = \frac{1440°}{180°}$

$n - 2 = 8$

$n = 10$

The polygon has 10 sides, so it is a decagon.

3. $x + 3x + 5x + 7x = 360°$

$16x = 360°$

$\frac{16x}{16} = \frac{360°}{16}$

$x = 22.5°$

$3x = 3 \cdot 22.5° = 67.5°$

$5x = 5 \cdot 22.5° = 112.5°$

$7x = 7 \cdot 22.5° = 157.5°$

The angle measures are 22.5°, 67.5°, 112.5°, and 157.5°.

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4. The total sum of the angle measures of the polygon with 5 sides is 540°.

$93° + 156° + 85° + 2x = 540°$

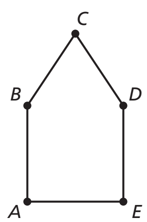
$334 + 2x = 540$

$2x = 206$

$x = 103$

$m\angle S = m\angle T = 103°$

5. Pentagon *ABCDE* is equilateral but not equiangular.



6. The sum of the measures of the exterior angles is 360°.

The sum of the known exterior angle measures is $34° + 49° + 58° + 67° + 75° = 283°$.

So, the measure of the exterior angle at the sixth vertex is $360° - 283° = 77°$.

7. You can find the measure of each exterior angle by subtracting the measure of the interior angle from 180°. In Example 6, the measure of each exterior angle is $180° - 150° = 30°$.

7.1 Exercises (pp. 364-366)

Vocabulary and Core Concept Check

1. A segment connecting consecutive vertices is a side of the polygon, not a diagonal.

2. The one that does not belong is "the sum of the measures of the interior angles of a pentagon". This sum is 540°, but in each of the other three statements the sum is 360°.

Monitoring Progress and Modeling with Mathematics

3. $(n - 2) \cdot 180° = (9 - 2) \cdot 180° = 7 \cdot 180° = 1260°$

The sum of the measures of the interior angles in a nonagon is 1260°.

4. $(n - 2) \cdot 180° = (14 - 2) \cdot 180° = 12 \cdot 180° = 2160°$

The sum of the measures of the interior angles in a 14-gon is 2160°.

5. $(n - 2) \cdot 180° = (16 - 2) \cdot 180° = 14 \cdot 180° = 2520°$

The sum of the measures of the interior angles in a 16-gon is 2520°.

6. $(n - 2) \cdot 180° = (20 - 2) \cdot 180° = 18 \cdot 180° = 3240°$

The sum of the measures of the interior angles in a 20-gon is 3240°.

7. $(n - 2) \cdot 180° = 720°$

$\frac{(n - 2) \cdot 180°}{180°} = \frac{720°}{180°}$

$n - 2 = 4$

$n = 6$

The polygon has 6 sides, so it is a hexagon.

8. $(n - 2) \cdot 180° = 1080°$

$\frac{(n - 2) \cdot 180°}{180°} = \frac{1080°}{180°}$

$n - 2 = 6$

$n = 8$

The polygon has 8 sides, so it is an octagon.

9. $(n - 2) \cdot 180° = 2520°$

$\frac{(n - 2) \cdot 180°}{180°} = \frac{2520°}{180°}$

$n - 2 = 14$

$n = 16$

The polygon has 16 sides, so it is a 16-gon.

10. $(n - 2) \cdot 180° = 3240°$

$\frac{(n - 2) \cdot 180°}{180°} = \frac{3240°}{180°}$

$n - 2 = 18$

$n = 20$

The polygon has 20 sides, so it is a 20-gon.

11. *XYZW* is a quadrilateral, therefore the sum of the measures of the interior angles is $(4 - 2) \cdot 180° = 360°$.

$100° + 130° + 66° + x° = 360°$

$296 + x = 360$

$x = 64$

12. *HJKG* is a quadrilateral, therefore the sum of the measures of the interior angles is $(4 - 2) \cdot 180° = 360°$.

$103° + 133° + 58° + x° = 360°$

$294 + x = 360$

$x = 66$

13. *KLMN* is a quadrilateral, therefore the sum of the measures of the interior angles is $(4 - 2) \cdot 180° = 360°$.

$88° + 154° + x° + 29° = 360°$

$271 + x = 360°$

$x = 89$

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14. *ABCD* is a quadrilateral, therefore the sum of the measures of the interior angles is $(4 - 2) \cdot 180° = 360°$.

$$x° + 92° + 68° + 101° = 360°$$
$$x + 261 = 360°$$
$$x = 99$$

15. The polygon has 6 sides, therefore the sum of the measures of the interior angles is $(6 - 2) \cdot 180° = 720°$.

$$102° + 146° + 120° + 124° + 158° + x° = 720°$$
$$650 + x = 720$$
$$x = 70$$

16. The polygon has 5 sides, therefore the sum of the measures of the interior angles is $(5 - 2) \cdot 180° = 540°$.

$$86° + 140° + 138° + 59° + x° = 540°$$
$$423 + x = 540$$
$$x = 117$$

17. The polygon has 6 sides, therefore the sum of the measures of the interior angles is $(6 - 2) \cdot 180° = 720°$.

$$121° + 96° + 101° + 162° + 90° + x° = 720°$$
$$570 + x = 720$$
$$x = 150$$

18. The polygon has 8 sides, therefore the sum of the measures of the interior angles is $(8 - 2) \cdot 180° = 1080°$.

$$143° + 2x° + 152° + 116° + 125° + 140° + 139° + x° = 1080°$$
$$815 + 3x = 1080$$
$$3x = 265$$
$$x = 88\frac{1}{3}$$

19. The polygon has 5 sides, therefore the sum of the measures of the interior angles is $(5 - 2) \cdot 180° = 540°$.

$$x° + x° + 164° + 102° + 90° = 540°$$
$$2x + 356 = 540$$
$$2x = 184$$
$$x = 92$$

$m\angle X = m\angle Y = 92°$

20. The polygon has 5 sides, therefore the sum of the measures of the interior angles is $(5 - 2) \cdot 180° = 540°$.

$$x° + 90° + x° + 119° + 47° = 540$$
$$2x + 256 = 540$$
$$2x = 284$$
$$x = 142$$

$m\angle X = m\angle Y = 142°$

21. The polygon has 6 sides, therefore the sum of the measures of the interior angles is $(6 - 2) \cdot 180° = 720°$.

$$90° + 99° + 171° + x° + x° + 159° = 720°$$
$$2x + 519 = 720$$
$$2x = 201$$
$$x = 100.5$$

$m\angle X = m\angle Y = 100.5°$

22. The polygon has 6 sides, therefore the sum of the measures of the interior angles is $(6 - 2) \cdot 180° = 720°$.

$$100° + x° + 110° + 149° + 91° + x° = 720°$$
$$2x + 450 = 720$$
$$2x = 270$$
$$x = 135$$

$m\angle X = m\angle Y = 135°$

23. $65° + x° + 78° + 106° = 360°$

$$x + 249 = 360$$
$$x = 111$$

24. $48° + 59° + x° + x° + 58° + 39° + 50° = 360°$

$$2x + 254 = 360$$
$$2x = 106$$
$$x = 53$$

25. $71° + 85° + 44° + 3x° + 2x° = 360°$

$$5x + 200 = 360$$
$$5x = 160$$
$$x = 32$$

26. $40° + x° + 77° + 2x° + 45° = 360°$

$$3x + 162 = 360$$
$$3x = 198$$
$$x = 66$$

27. The sum of the measures of the interior angles of a pentagon is $(5 - 2) \cdot 180° = 3 \cdot 180° = 540°$.

Each interior angle: $\frac{540°}{5} = 108°$

Each exterior angle: $\frac{360°}{5} = 72°$

The measure of each interior angle of a pentagon is 108° and the measure of each exterior angle is 72°.

28. The sum of the measures of the interior angles of an 18-gon is $(18 - 2) \cdot 180° = 16 \cdot 180° = 2880°$.

Each interior angle: $\frac{2880°}{18} = 160°$

Each exterior angle: $\frac{360°}{18} = 20°$

The measure of each interior angle of an 18-gon is 160° and the measure of each exterior angle is 20°.

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29. The sum of the measures of the interior angles of a 45-gon is $(45 - 2) \cdot 180° = 43 \cdot 180° = 7740°$.

Each interior angle: $\frac{7740°}{45} = 172°$

Each exterior angle: $\frac{360°}{45} = 8°$

The measure of each interior angle of a 45-gon is 172° and the measure of each exterior angle is 8°.

30. The sum of the measures of the interior angles of a 90-gon is $(90 - 2) \cdot 180° = 88 \cdot 180° = 15{,}840°$.

Each interior angle: $\frac{15{,}840°}{90} = 176°$

Each exterior angle: $\frac{360°}{90} = 4°$

The measure of each interior angle of a 90-gon is 176° and the measure of each exterior angle is 4°.

31. The measure of one interior angle of a regular pentagon was found, but the measure of one exterior angle should be found by dividing 360° by the number of angles.

The correct response should be $\frac{360°}{5} = 72°$.

32. The correct response should be $\frac{360°}{5} = 72°$.

33. A regular hexagon has 6 sides. The sum of the measures of the interior angles is $(6 - 2) \cdot 180° = 720°$. The measure of each interior angle is $\frac{720°}{6} = 120°$.

34. A regular decagon has 10 sides. The sum of the measures of the interior angles is $(10 - 2) \cdot 180° = 1440°$. The measure of each interior angle is $\frac{1440°}{10} = 144°$. The measure of each exterior angle is $\frac{360°}{10} = 36°$.

35.

$$\frac{(n - 2) \cdot 180°}{n} = x°$$
$$(n - 2) \cdot 180 = nx$$
$$n \cdot 180 - 2 \cdot 180 = nx$$
$$n \cdot 180 - 360 = nx$$
$$n \cdot 180 = nx + 360$$
$$n \cdot 180 - nx = 360$$
$$n(180 - x) = 360$$
$$\frac{n(180 - x)}{180 - x} = \frac{360}{180 - x}$$
$$n = \frac{360}{180 - x}$$

The formula to find the number of sides n in a regular polygon given the measure of one interior angle $x°$ is: $n = \frac{360}{180 - x}$.

36.

$$x° = \frac{360°}{n}$$
$$n \cdot x = n \cdot \frac{360}{n}$$
$$n \cdot x = 360$$
$$\frac{n \cdot x}{x} = \frac{360}{x}$$
$$n = \frac{360}{x}$$

The formula to find the number of sides n in a regular polygon given the measure of one exterior angle $x°$ is: $n = \frac{360}{x}$.

37. $n = \frac{360°}{180° - 156°} = \frac{360°}{24°} = 15$

The number of sides of a polygon where each interior angle has a measure of 156° is 15.

38. $n = \frac{360°}{180° - 165°} = \frac{360°}{15°} = 24$

The number of sides of a polygon where each interior angle has a measure of 165° is 24.

39. $n = \frac{360°}{9°} = 40$

The number of sides of a polygon where each exterior angle has a measure of 9° is 40.

40. $n = \frac{360°}{6°} = 60$

The number of sides of a polygon where each exterior angle has a measure of 6° is 60.

41. A, B;

A. $n = \frac{360°}{180° - 162°} = \frac{360°}{18°} = 20$ ✓

B. $n = \frac{360°}{180° - 171°} = \frac{360°}{9°} = 40$ ✓

C. $n = \frac{360°}{180° - 75°} = \frac{360°}{105°} = 3.43$ ✗

D. $n = \frac{360°}{180° - 40°} = \frac{360°}{140°} = 2.57$ ✗

Solving the equation found in Exercise 35 for n yields a positive integer greater than or equal to 3 for A and B, but not for C and D.

42. In a pentagon, when all the diagonals from one vertex are drawn, the polygon is divided into three triangles. Because the sum of the measures of the interior angles of each triangle is 180°, the sum of the measures of the interior angles of the pentagon is $(5 - 2) \cdot 180° = 3 \cdot 180° = 540°$.

43. In a quadrilateral, when all the diagonals from one vertex are drawn, the polygon is divided into two triangles. Because the sum of the measures of the interior angles of each triangle is 180°, the sum of the measures of the interior angles of the quadrilateral is $2 \cdot 180° = 360°$.

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44. yes; Because an interior angle and an adjacent exterior angle of a polygon form a linear pair, you can use the Polygon Exterior Angles Theorem (Thm. 7.2) to find the measure of the exterior angles, and then you can subtract this value from 180° to find the interior angle measures of a regular polygon.

45. A hexagon has 6 angles. If 4 of the exterior angles have a measure of $x°$, the other two each have a measure of $2(x + 48)°$, and the total exterior angle sum is 360°, then:

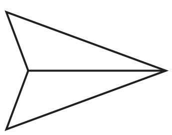
$$4x° + 2[2(x + 48)]° = 360°$$
$$4x + 2[2x + 96] = 360$$
$$4x + 4x + 192 = 360$$
$$8x + 192 = 360$$
$$8x = 168$$
$$x = 21$$

$2(x + 48) = 2(21 + 48) = 2 \cdot 69 = 138$

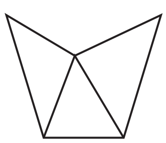
So, the measures of the exterior angles are 21°, 21°, 21°, 21°, 138°, and 138°.

46. yes; The measure of the angle where the polygon caves in is greater than 180° but less than 360°.

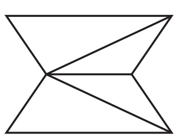
47. Divide the quadrilateral into two triangles. The sum of the measures of the interior angles of each triangle is 180°. Therefore, the total sum of the interior angle measurements for this quadrilateral is $2 \cdot 180° = 360°$.



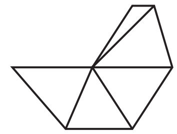
Divide the pentagon into three triangles. The sum of the measures of the interior angles of each triangle is 180°. Therefore, the total sum of the interior angle measurements for this pentagon is $3 \cdot 180° = 540°$.



Divide the hexagon into four triangles. The sum of the measures of the interior angles of each triangle is 180°. Therefore, the total sum of the interior angle measurements for this hexagon is $4 \cdot 180° = 720°$.

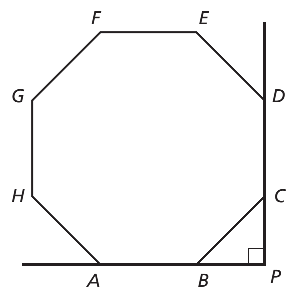


Divide the heptagon into five triangles. The sum of the measures of the interior angles of each triangle is 180°. Therefore, the total sum of the interior angle measurements for this heptagon is $5 \cdot 180° = 900°$.



When diagonals are drawn from the vertex of the concave angle as shown, the polygon is divided into $n - 2$ triangles whose interior angle measures have the same total as the sum of the interior angle measures of the original polygon. So, an expression to find the sum of the measures of the interior angles for a concave polygon is $(n - 2) \cdot 180°$.

48. The base angles of $\triangle BPC$ are congruent exterior angles of the regular octagon, each with a measure of 45°. So, $m\angle BPC = 180° - 2(45°) = 90°$.



49. a. The formula for the number of sides n in a regular polygon, where $h(n)$ is the measure of any interior angle is $h(n) = \frac{(n - 2) \cdot 180°}{n}$.

b. $h(9) = \frac{(9 - 2) \cdot 180°}{9} = \frac{7 \cdot 180°}{9} = \frac{1260°}{9} = 140°$

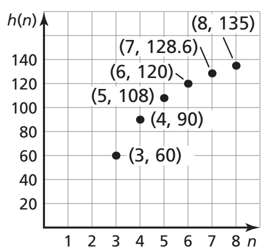
c.
$$150° = \frac{(n - 2) \cdot 180°}{n}$$
$$150n = (n - 2) \cdot 180$$
$$150n = 180n - 360$$
$$-30n = -360$$
$$\frac{-30n}{-30} = \frac{-360}{-30}$$
$$n = 12$$

When $h(n) = 150°$, $n = 12$.

Chapter 7

d.

Number of sides	Measure of each interior angle
3	60°
4	90°
5	108°
6	120°
7	128.6°
8	135°



The value of $h(n)$ increases on a curve that gets less steep as n increases.

50. no; The interior angles are supplements of the adjacent exterior angles, and because the exterior angles have different values, the supplements will be different as well.

51. In a convex n-gon, the sum of the measures of the n interior angles is $(n - 2) \cdot 180°$ using the Polygon Interior Angles Theorem (Thm. 7.1). Because each of the n interior angles forms a linear pair with its corresponding exterior angle, you know that the sum of the measures of the n interior and exterior angles is $180n°$. Subtracting the sum of the interior angle measures from the sum of the measures of the linear pairs gives you $180n° - [(n - 2) \cdot 180°] = 360°$.

52. In order to have $\frac{540°}{180°} = 3$ more triangles formed by the diagonals, the new polygon will need 3 more sides.

Maintaining Mathematical Proficiency

53.
$$x° + 79° = 180°$$
$$x = 180 - 79$$
$$x = 101$$

54.
$$x° + 113° = 180°$$
$$x = 180 - 113$$
$$x = 67$$

55.
$$(8x - 16)° + (3x + 20)° = 180°$$
$$11x + 4 = 180$$
$$11x = 176$$
$$x = 16$$

56.
$$(6x - 19)° + (3x + 10)° = 180°$$
$$9x - 9 = 180$$
$$9x = 189$$
$$x = 21$$

7.2 Explorations (p. 367)

1. a. Check students' work; Construct $\overleftrightarrow{AB}$ and a line parallel to $\overleftrightarrow{AB}$ through point C. Construct $\overleftrightarrow{BC}$ and a line parallel to $\overleftrightarrow{BC}$ through point A. Construct a point D at the intersection of the line drawn parallel to $\overleftrightarrow{AB}$ and the line drawn parallel to $\overleftrightarrow{BC}$. Finally, construct $\overline{AB}$, $\overline{BC}$, $\overline{CD}$, and $\overline{DA}$ by removing the rest of the parallel lines drawn.

b. Check students' work. (For sample in text, $m\angle A = m\angle C = 63.43°$ and $m\angle B = m\angle D = 116.57°$.); Opposite angles are congruent, and consecutive angles are supplementary.

c. Check students' work. (For sample in text, $AB = CD = 2.24$ and $BC = AD = 4$.); Opposite sides are congruent.

d. Check students' work; Opposite angles of a parallelogram are congruent. Consecutive angles of a parallelogram are supplementary. Opposite sides of a parallelogram are congruent.

2. a. Check students' work.

b. Check students' work.

c. Check students' work. (For sample in text, $AE = CE = 1.58$ and $BE = DE = 2.55$.) Point E bisects $\overline{AC}$ and $\overline{BD}$.

d. The diagonals of a parallelogram bisect each other.

3. A parallelogram is a quadrilateral where both pairs of opposite sides are congruent and parallel, opposite angles are congruent, consecutive angles are supplementary, and the diagonals bisect each other.

7.2 Monitoring Progress (pp. 369–371)

1. $m\angle G = m\angle E$

$m\angle E = 60°$

$m\angle G = 60°$

$FG = HE$

$HE = 8$

$FG = 8$

In parallelogram $GHEF$, $FG = 8$ and $m\angle G = 60°$.

2. $m\angle J = m\angle L$

$2x° = 50°$

$x = 25$

$JK = ML$

$18 = y + 3$

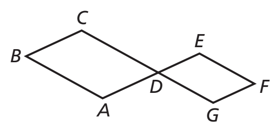
$15 = y$

In parallelogram $JKLM$, $x = 25$ and $y = 15$.

Chapter 7

3. $m\angle BCD + m\angle ADC = 180°$
$m\angle BCD + 2m\angle BCD = 180°$
$3m\angle BCD = 180°$
$m\angle BCD = 60°$

4. **Given** *ABCD* and *GDEF* are parallelograms.
Prove $\angle C$ and $\angle F$ are supplementary angles.



STATEMENTS	REASONS
1. *ABCD* and *GDEF* are parallelograms.	1. Given
2. $\angle C$ and $\angle D$ are supplementary angles.	2. Parallelogram Consecutive Angles Theorem (Thm. 7.5)
3. $m\angle C + m\angle D = 180°$	3. Definition of supplementary angles
4. $\angle D \cong \angle F$	4. Parallelogram Opposite Angles Theorem (Thm. 7.4)
5. $m\angle D = m\angle F$	5. Definition of congruent angles
6. $m\angle C + m\angle F = 180°$	6. Substitution Property of Equality
7. $\angle C$ and $\angle F$ are supplementary angles.	7. Definition of supplementary angles

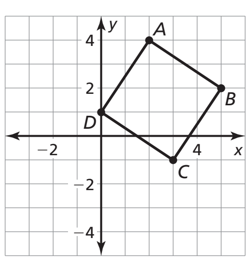
5. By the Parallelogram Diagonals Theorem (Thm. 7.6), the diagonals of a parallelogram bisect each other.

Midpoint of $\overline{TV}$: $\left(\frac{1+3}{2}, \frac{5+1}{2}\right) = \left(\frac{4}{2}, \frac{6}{2}\right) = (2,3)$

Midpoint of $\overline{SU}$: $\left(\frac{-2+6}{2}, \frac{3+3}{2}\right) = \left(\frac{4}{2}, \frac{6}{2}\right) = (2,3)$

The coordinates of the intersection of the diagonals are (2,3).

6. Slope of $\overline{AB} = \frac{2-4}{5-2} = -\frac{2}{3}$
The rise is 2 units (starting at *C*, go up 2 units and left 3 units). So, the coordinates of *D* are (0,1).



7.2 Exercises (pp. 372-374)

Vocabulary and Core Concept Check

1. In order to be a quadrilateral, a polygon must have 4 sides, and parallelograms always have 4 sides. In order to be a parallelogram, a polygon must have 4 sides with opposite sides parallel. Quadrilaterals always have 4 sides, but do not always have opposite sides parallel.

2. The two angles that are consecutive to the given angle are supplementary to it. So, you can find each of their measures by subtracting the measure of the given angle from 180°. The angle opposite the given angle is congruent and therefore has the same measure.

Monitoring Progress and Modeling with Mathematics

3. The Parallelogram Opposite Sides Theorem (Thm. 7.3) applies here. Therefore, $x = 9$ and $y = 15$.

4. By the Parallelogram Opposite Sides Theorem (Thm. 7.3), $n = 12$ and $m + 1 = 6$. Therefore, $m = 5$.

5. Parallelogram Opposite Sides Theorem (Thm. 7.3):
$z - 8 = 20$
$z = 28$
Parallelogram Opposite Angles Theorem (Thm. 7.4):
$(d - 21)° = 105°$
$d = 126$
Therefore, $z = 28$ and $d = 126$.

6. Parallelogram Opposite Sides Theorem (Thm. 7.3):
$16 - h = 7$
$-h = -9$
$h = 9$
Parallelogram Opposite Angles Theorem (Thm. 7.4):
$(g + 4)° = 65°$
$g = 61$
Therefore, $h = 9$ and $g = 61$.

7. $m\angle A + m\angle B = 180°$
$51° + m\angle B = 180°$
$m\angle B = 129°$

8. $m\angle M + m\angle N = 180°$
$95° + m\angle N = 180°$
$m\angle N = 85°$

9. $LM = 13$; By the Parallelogram Opposite Sides Theorem (Thm. 7.3), $LM = QN$.

10. $LP = 7$; By the Parallelogram Diagonals Theorem (Thm. 7.6), $LP = PN$

11. $LQ = 8$; By the Parallelogram Opposite Sides Theorem (Thm. 7.3), $LQ = MN$.

Chapter 7

12. By the Parallelogram Diagonals Theorem (Thm. 7.6),

$MP = PQ$. $MQ = 2 \cdot MP$

$MQ = 2 \cdot 8.2 = 16.4$

13. Parallelogram Consecutive Angles Theorem (Thm. 7.5)

$m\angle LMN + m\angle MLQ = 180°$

$m\angle LMN + 100° = 180°$

$m\angle LMN = 80°$

14. Parallelogram Opposite Angles Theorem (Thm. 7.4)

$m\angle NQL = m\angle NML$

$m\angle NQL = 80°$

15. Parallelogram Opposite Angles Theorem (Thm. 7.4)

$m\angle MNQ = m\angle MLQ$

$m\angle MNQ = 100°$

16. Alternate Interior Angles Theorem (Thm. 3.2)

$m\angle LMQ = m\angle NQM$

$m\angle LMQ = 29°$

17. $n° + 70° = 180°$

$n = 110$

$2m° = 70°$

$m = 35$

So, $n = 110$ and $m = 35$.

18. $(b - 10)° + (b + 10)° = 180°$

$2b = 180$

$b = 90$

$d° = (b + 10)°$

$d = 90 + 10$

$d = 100$

$c = (b - 10)°$

$c = 90 - 10$

$c = 80$

So, $b = 90$, $c = 80$, and $d = 100$.

19. $k + 4 = 11$

$k = 7$

$m = 8$

So, $k = 7$ and $m = 8$.

20. $2u + 2 = 5u - 10$

$2u = 5u - 12$

$-3u = -12$

$\frac{-3u}{-3} = \frac{-12}{-3}$

$u = 4$

$\frac{v}{3} = 6$

$v = 18$

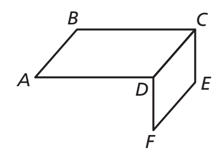
So, $u = 4$ and $v = 18$.

21. In a parallelogram, consecutive angles are supplementary; Because quadrilateral *STUV* is a parallelogram, $\angle S$ and $\angle V$ are supplementary. So, $m\angle V = 180° - 50° = 130°$.

22. In a parallelogram, the diagonals bisect each other. So the two parts of $\overline{GJ}$ are congruent to each other; Because quadrilateral *GHJK* is a parallelogram, $\overline{GF} \cong \overline{FJ}$.

23. **Given** *ABCD* and *CEFD* are parallelograms.

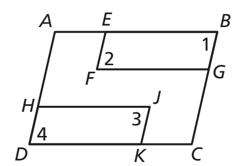
Prove $\overline{AB} \cong \overline{FE}$



STATEMENTS	REASONS
1. *ABCD* and *CEFO* are parallelograms.	**1.** Given
2. $\overline{AB} \cong \overline{DC}, \overline{DC} \cong \overline{FE}$	**2.** Parallelogram Opposite Sides Theorem (Thm. 7.3)
3. $\overline{AB} \cong \overline{FE}$	**3.** Transitive Property of Congruence (Thm. 2.1)

24. **Given** *ABCD, EBGF*, and *HJKD* are parallelograms.

Prove $\angle 2 \cong \angle 3$



STATEMENTS	REASONS
1. *ABCD*, *EBGF*, and *HJKD* are parallelograms.	**1.** Given
2. $\angle 1 \cong \angle 2$, $\angle 3 \cong \angle 4$, $\angle 1 \cong \angle 4$	**2.** Parallelogram Opposite Angles Theorem (Thm. 7.4)
3. $\angle 2 \cong \angle 4$	**3.** Transitive Property of Congruence (Thm. 2.1)
4. $\angle 2 \cong \angle 3$	**4.** Transitive Property of Congruence (Thm. 2.1)

25. By the Parallelogram Diagonals Theorem (Thm. 7.6), the diagonals of a parallelogram bisect each other.

Midpoint of $\overline{WY}$: $\left(\frac{-2 + 4}{2}, \frac{5 + 0}{2}\right) = \left(\frac{2}{2}, \frac{5}{2}\right) = (1, 2.5)$

Midpoint of $\overline{ZX}$: $\left(\frac{2 + 0}{2}, \frac{5 + 0}{2}\right) = \left(\frac{2}{2}, \frac{5}{2}\right) = (1, 2.5)$

The coordinates of the intersection of the diagonals are (1, 2.5).

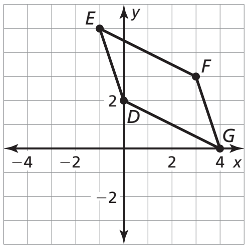
26. By the Parallelogram Diagonals Theorem (Thm. 7.6), the diagonals of a parallelogram bisect each other.

Midpoint of $\overline{QS}$: $\left(\frac{-1 + 1}{2}, \frac{3 + (-2)}{2}\right) = \left(\frac{0}{2}, \frac{1}{2}\right) = (0, 0.5)$

Midpoint of $\overline{TR}$: $\left(\frac{5 + (-5)}{2}, \frac{2 + (-1)}{2}\right) = \left(\frac{0}{2}, \frac{1}{2}\right) = (0, 0.5)$

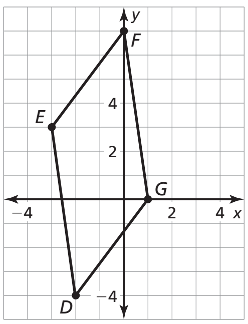
The coordinates of the intersection of the diagonals are (0, 0.5).

Chapter 7

27. 

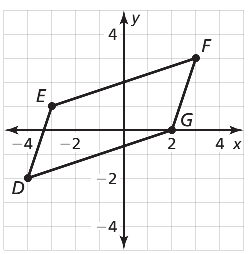
Slope of $\overline{ED} = \frac{5-2}{-1-0} = \frac{3}{-1} = -3$

Starting at G, go up 3 units and left 1 unit. So, the coordinates of F are (3, 3).

28. 

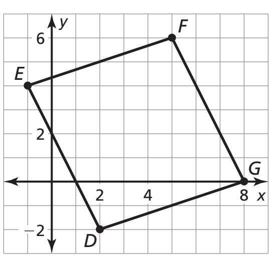
Slope of $\overline{FG} = \frac{7-0}{0-1} = \frac{7}{-1} = -7$

Starting at D, go up 7 units and left 1 unit. So, the coordinates of E are (−3, 3).

29. 

Slope of $\overline{ED} = \frac{1-(-2)}{-3-(-4)} = \frac{3}{-3+4} = \frac{3}{1} = 3$

Starting at F, go down 3 units and left 1 unit. So, the coordinates of G are (2, 0).

30. 

Slope of $\overline{FG} = \frac{6-0}{5-8} = \frac{6}{-3} = -2$

Starting at E, go down 6 units and right 3 units. So, the coordinates of D are (2, −2).

31. $x° + 0.25x° + x + 0.25x° = 360°$

$$2.5x = 360$$
$$x = \frac{360}{2.5}$$
$$x = 144$$

$0.25x° = 0.25(144°) = 36°$

The angles are 36° and 144°.

32. $x° + (4x + 50)° + x° + (4x + 50)° = 360°$

$$10x + 100 = 360$$
$$10x = 260$$
$$x = 26$$

$(4x + 50)° = (4 \cdot 26 + 50)° = 154°$

The angles are 154° and 26°.

33. If the points are in the order of $ABCD$, the quadrilateral could not be a parallelogram, because $\angle A$ and $\angle C$ are opposite angles, but $m\angle A \neq m\angle C$.

34. $m\angle J + m\angle K = 180°$

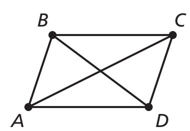
$(3x + 7)° + (5x - 11)° = 180°$

$$8x - 4 = 180$$
$$8x = 184$$
$$x = 23$$

$m\angle J = (3 \cdot 23 + 7)° = 76°$

$m\angle K = (5 \cdot 23 - 11)° = 104°$

35. *Sample answer:*



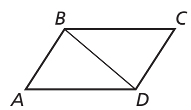
When you fold the parallelogram so that vertex A is on vertex C, the fold will pass through the point where the diagonals intersect, which demonstrates that this point of intersection is also the midpoint of $\overline{AC}$. Similarly, when you fold the parallelogram so that vertex B is on vertex D, the fold will pass through the point where the diagonals intersect, which demonstrates that this point of intersection is also the midpoint of $\overline{BD}$.

36. *Sample answer:*

$m\angle 1 = m\angle G$ because corresponding pairs of congruent figures are congruent. Note that $\overline{AB} \parallel \overleftrightarrow{CD} \parallel \overline{FG}$. So, $m\angle 1 = m\angle BDE$ and $m\angle GDE = m\angle G$ because they are pairs of alternate interior angles, and $m\angle 1 = m\angle GDE$ by the Transitive Property of Equality. Also, by the Angle Addition Postulate (Post. 1.4), $m\angle 2 = m\angle BDE + m\angle GDE$. By substituting, you get $m\angle 2 = m\angle 1 + m\angle 1 = 2m\angle 1$.

37. Given *ABCD* is a parallelogram.

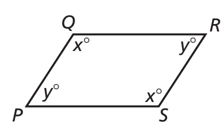
Prove $\angle A \cong \angle C$, $\angle B \cong \angle D$



STATEMENTS	REASONS
1. *ABCD* is a parallelogram.	**1.** Given
2. $\overline{AB} \parallel \overline{DC}, \overline{BC} \parallel \overline{AD}$	**2.** Definition of parallelogram
3. $\angle BDA \cong \angle DBC$, $\angle DBA \cong \angle BDC$	**3.** Alternate Interior Angles Theorem (Thm. 3.2)
4. $\overline{BD} \cong \overline{BD}$	**4.** Reflexive Property of Congruence (Thm. 2.1)
5. $\triangle ABD \cong \triangle CDB$	**5.** ASA Congruence Theorem (Thm. 5.10)
6. $\angle A \cong \angle C$, $\angle B \cong \angle D$	**6.** Corresponding parts of congruent triangles are congruent.

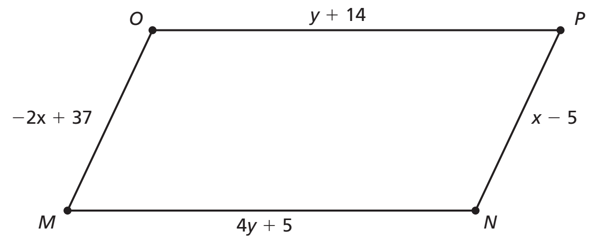
38. Given *PQRS* is a parallelogram.

Prove $x° + y° = 180°$



STATEMENTS	REASONS
1. *PQRS* is a parallelogram.	**1.** Given
2. $\overline{QR} \parallel \overline{PS}$	**2.** Definition of parallelogram
3. $\angle Q$ and $\angle P$ are supplementary.	**3.** Consecutive Interior Angles Theorem (Thm. 3.4)
4. $x° + y° = 180°$	**4.** Definition of supplementary angles

39.



$MQ = NP$

$-2x + 37 = x - 5$

$-3x = -42$

$x = 14$

$MQ = -2 \cdot 14 + 37 = -28 + 37 = 9$

$NP = 14 - 5 = 9$

$QP = MN$

$y + 14 = 4y + 5$

$-3y = -9$

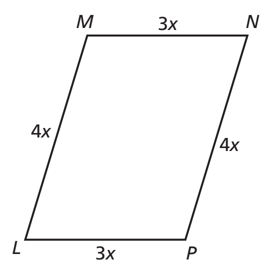
$y = 3$

$QP = 3 + 14 = 17$

$MN = 4 \cdot 3 + 5 = 12 + 5 = 17$

The perimeter of *MNPQ* is $17 + 9 + 17 + 9 = 52$ units.

40. $\dfrac{LM}{MN} = \dfrac{4x}{3x}$



$4x + 3x + 4x + 3x = 28$

$14x = 28$

$x = 2$

$LM = 4 \cdot 2 = 8$ units

41. no; Two parallelograms with congruent corresponding sides may or may not have congruent corresponding angles.

42. a. decreases; Because $\angle P$ and $\angle Q$ are supplementary, as one increases, the other must decrease so that their total is still 180°.

b. increases; As $m\angle Q$ decreases, the parallelogram gets skinnier, which means that *Q* and *S* get farther apart.

c. The mirror gets closer to the wall; As $m\angle Q$ decreases, the parallelograms get skinnier, which means that *P*, *R*, and the other corresponding vertices all get closer together. So, the distance between the mirror and the wall gets smaller.

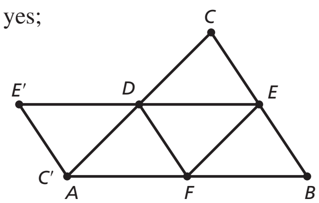
Chapter 7

43. $m\angle USV + m\angle TSU = m\angle TUV$

$$(x^2)^\circ + 32^\circ = 12x^\circ$$
$$x^2 - 12x + 32 = 0$$
$$(x - 8)(x - 4) = 0$$
$$(x - 8) = 0$$
$$x = 8$$
$$m\angle USV = (x^2)^\circ = (8^2)^\circ = 64^\circ$$

$$(x - 4) = 0$$
$$x = 4$$
$$m\angle USV = (x^2)^\circ = (4^2)^\circ = 16^\circ$$

44. yes;



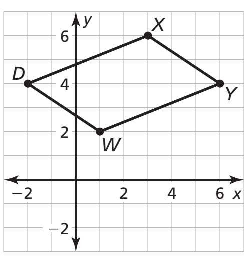
Any triangle, such as $\triangle ABC$, can be partitioned into four congruent triangles by drawing the midsegment triangle, such as $\triangle DEF$. Then, one triangle, such as $\triangle CDE$, can be rotated 180° about a vertex, such as D, to create a parallelogram as shown.

45. Three parallelograms can be created.

Let D be the fourth vertex.

Slope of $\overline{XY} = \dfrac{6 - 4}{3 - 6} = \dfrac{2}{-3} = -\dfrac{2}{3}$

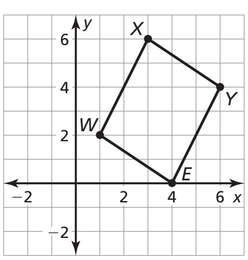
Starting at W, go up 2 units and left 3 units. So, the coordinates of D are $(-2, 4)$.



Let E be the fourth vertex.

Slope of $\overline{YX} = \dfrac{4 - 6}{6 - 3} = \dfrac{-2}{3} = -\dfrac{2}{3}$

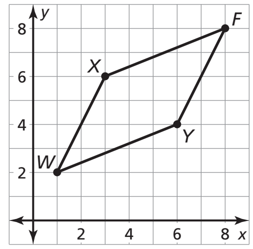
Starting at W, go down 2 units and right 3 units. So, the coordinates of E are $(4, 0)$.



Let F be the fourth vertex.

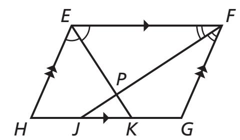
Slope of $\overline{XW} = \dfrac{6 - 2}{3 - 1} = \dfrac{4}{2} = 2$

Starting at Y, go up 4 units and right 2 units. So, the coordinates of F are $(8, 8)$.



46. Given $\overline{EK}$ bisects $\angle FEH$, $\overline{FJ}$ bisects $\angle EFG$, and $EFGH$ is a parallelogram.

Prove $\overline{EK} \perp \overline{FJ}$



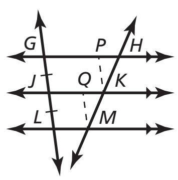
STATEMENTS	REASONS
1. $\overline{EK}$ bisects $\angle FEH$ and $\overline{FJ}$ bisects $\angle EFG$. $EFGH$ is a parallelogram.	**1.** Given
2. $m\angle PEH = m\angle PEF$, $m\angle PFE = m\angle PFG$	**2.** Definition of angle bisector
3. $m\angle HEF = m\angle PEH + m\angle PEF$, $m\angle EFG = m\angle PFE + m\angle PFG$	**3.** Angle Addition Postulate (Post. 1.4)
4. $m\angle HEF = m\angle PEF + m\angle PEF$, $m\angle EFG = m\angle PFE + m\angle PFE$	**4.** Substitution Property of Equality
5. $m\angle HEF = 2(m\angle PEF)$, $m\angle EFG = 2(m\angle PFE)$	**5.** Distributive Property
6. $m\angle HEF + m\angle EFG = 180^\circ$	**6.** Parallelogram Consecutive Angles Theorem (Thm. 7.5)
7. $2(m\angle PEF) + 2(m\angle PFE) = 180^\circ$	**7.** Substitution Property of Equality
8. $2(m\angle PEF + m\angle PFE) = 180^\circ$	**8.** Distributive Property
9. $m\angle PEF + m\angle PFE = 90^\circ$	**9.** Division Property of Equality
10. $m\angle PEF + m\angle PFE + m\angle EPF = 180^\circ$	**10.** Triangle Sum Theorem (Thm. 5.1)
11. $90^\circ + m\angle EPF = 180^\circ$	**11.** Substitution Property of Equality

12. $m\angle EPF = 90°$	12. Subtraction Property of Equality
13. $\angle EPF$ is a right angle.	13. Definition of right angle
14. $\overline{EK} \perp \overline{FJ}$	14. Definition of perpendicular lines

47. Given $\overleftrightarrow{GH} \parallel \overleftrightarrow{JK} \parallel \overleftrightarrow{LM}, \overline{GJ} \cong \overline{JL}$

Prove $\overline{HK} \cong \overline{KM}$

Construct $\overline{KP}$ and $\overline{MQ}$, such that $\overline{KP} \parallel \overline{GJ}$ and $\overline{MQ} \parallel \overline{JL}$, thus *KPGJ* is a parallelogram and *MQJL* is a parallelogram.



STATEMENTS	REASONS
1. $\overleftrightarrow{GH} \parallel \overleftrightarrow{JK} \parallel \overleftrightarrow{LM}, \overline{GJ} \cong \overline{JL}$	1. Given
2. Construct $\overline{PK}$ and $\overline{QM}$ such that $\overline{PK} \parallel \overleftrightarrow{GL} \parallel \overline{QM}$	2. Construction
3. *GPKJ* and *JQML* are parallelograms.	3. Definition of parallelogram
4. $\angle GHK \cong \angle JKM$, $\angle PKQ \cong \angle QML$	4. Corresponding Angles Theorem (Thm. 3.1)
5. $\overline{GJ} \cong \overline{PK}, \overline{JL} \cong \overline{QM}$	5. Parallelogram Opposite Sides Theorem (Thm. 7.3)
6. $\overline{PK} \cong \overline{QM}$	6. Transitive Property of Congruence (Thm. 2.1)
7. $\angle HPK \cong \angle PKQ$, $\angle KQM \cong \angle QML$	7. Alternate Interior Angles Theorem (Thm. 3.2)
8. $\angle HPK \cong \angle QML$	8. Transitive Property of Congruence (Thm. 2.2)
9. $\angle HPK \cong \angle KQM$	9. Transitive Property of Congruence (Thm. 2.2)
10. $\triangle PHK \cong \triangle QKM$	10. AAS Congruence Theorem (Thm. 5.11)
11. $\overline{HK} \cong \overline{KM}$	11. Corresponding sides of congruent triangles are conguent.

Maintaining Mathematical Proficiency

48. yes; $\ell \parallel m$ by the Alternate Interior Angles Converse Theorem (Thm. 3.6).

49. yes; $\ell \parallel m$ by the Alternate Exterior Angles Converse Theorem (Thm. 3.7).

50. no; By the Consecutive Interior Angles Theorem (Thm. 3.4), consecutive interior angles of parallel line are supplementary.

7.3 Explorations (p. 375)

1. a. Check students' work.

b. yes; Slope of $\overline{BC} = \dfrac{7-3}{-8-(-12)} = \dfrac{4}{-8+12} = \dfrac{4}{4} = 1$

Slope of $\overline{AD} = \dfrac{3-(-1)}{-5-(-9)} = \dfrac{4}{-5+9} = \dfrac{4}{4} = 1$

Slope of $\overline{AB} = \dfrac{3-(-1)}{-12-(-9)} = \dfrac{4}{-12+9} = \dfrac{4}{-3} = -\dfrac{4}{3}$

Slope of $\overline{CD} = \dfrac{7-3}{-8-(-5)} = \dfrac{4}{-8+5} = \dfrac{4}{-3} = -\dfrac{4}{3}$

Because the slope of $\overline{BC}$ equals the slope of $\overline{AD}$, $\overline{BC} \parallel \overline{AD}$. Because the slope of $\overline{AB}$ equals the slope of $\overline{CD}$, $\overline{AB} \parallel \overline{CD}$. So, the quadrilateral *ABCD* is a parallelogram.

c. Check students' work. If the opposite sides of a quadrilateral are congruent, then the quadrilateral is a parallelogram.

d. If a quadrilateral is a parallelogram, then its opposite sides are congruent. The converse is true. This is the Parallelogram Opposite Sides Theorem (Thm. 7.3).

2. a. Check students' work.

b. Yes the quadrilateral is a parallelogram. The opposite angles are congruent and the opposite sides have the same slope.

c. Check students' work. If the opposite angles of a quadrilateral are congruent, then the quadrilateral is a parallelogram.

d. If a quadrilateral is a parallelogram, then its opposite angles are congruent. The converse is true. This is the Parallelogram Opposite Angles Theorem (Thm. 7.4).

3. To prove a quadrilateral is a parallelogram, show that the opposite sides are congruent or that the opposite angles are congruent.

4. In the figure $m\angle A = m\angle C$ and $\angle B \cong \angle D$. Because the opposite angles are congruent, you can conclude that *ABCD* is a parallelogram.

7.3 Monitoring Progress (pp. 377–380)

1. *WXYZ* is a parallelogram because opposite angles are congruent; $\angle W \cong \angle Y$ and $\angle X \cong \angle Z$. So, $m\angle Z = 138$.

Chapter 7

2. By the Parallelogram Opposite Angles Converse (Thm. 7.8), if both pairs of opposite angles of a quadrilateral are congruent, then the quadrilateral is a parallelogram. So, solve $3x - 32 = 2x$ for x and $4y = y + 87$ for y.

$3x - 32 = 2x$

$-32 = -x$

$x = 32$

$y + 87 = 4y$

$87 = 3y$

$29 = y$

So, $x = 32$ and $y = 29$.

3. Opposite Sides Parallel and Congruent Theorem (Thm. 7.9).

4. Parallelogram Opposite Sides Converse (Thm. 7.7).

5. Parallelogram Opposite Angles Converse (Thm. 7.8).

6. By the Parallelogram Diagonals Converse (Thm. 7.10), if the diagonals of a quadrilateral bisect each other, then the quadrilateral is a parallelogram. So, solve $2x = 10 - 3x$ for x.

$2x = 10 - 3x$

$5x = 10$

$x = 2$

So, $x = 2$.

7. Slope of $\overline{LM} = \frac{-3 - 3}{3 - 2} = \frac{-6}{1} = -6$

Slope of $\overline{JK} = \frac{-5 - 1}{-3 - (-4)} = \frac{-6}{-3 + 4} = \frac{-6}{1} = -6$

Because the slope of $\overline{LM}$ equals the slope of $\overline{JK}$, $\overline{LM} \parallel \overline{JK}$.

$LM = \sqrt{(3 - 2)^2 + (-3 - 3))^2} = \sqrt{(1)^2 + (-6)^2} = \sqrt{37}$

$JK = \sqrt{(-3-(-4))^2 + (-5 - 1)^2} = \sqrt{(1)^2 + (-6)^2} = \sqrt{37}$

Because $LM = JK = \sqrt{37}$, $\overline{LM} \cong \overline{JK}$.

So, $\overline{JK}$ and $\overline{LM}$ are congruent and parallel, which means that $JKLM$ is a parallelogram by the Opposite Sides Parallel and Congruent Theorem (Thm. 7.9).

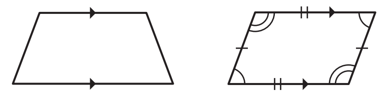
8. *Sample answer:* Find the slopes of all four sides and show that opposite sides are parallel. Another way is to find the point of intersection of the diagonals and show that the diagonals bisect each other.

7.3 Exercises (pp. 381–384)

Vocabulary and Core Concept Check

1. yes; If all four sides are congruent, then both pairs of opposite sides are congruent. So, the quadrilateral is a parallelogram by the Parallelogram Opposite Sides Converse (Thm. 7.7).

2. The statement that is different is "Construct a quadrilateral with one pair of parallel sides".



Monitoring Progress and Modeling with Mathematics

3. Parallelogram Opposite Angles Converse (Thm. 7.8)

4. Parallelogram Opposite Sides Converse (Thm. 7.7)

5. Parallelogram Diagonals Converse (Thm. 7.10)

6. Parallelogram Opposite Angles Converse (Thm. 7.8)

7. Opposite Sides Parallel and Congruent Theorem (Thm. 7.9)

8. Parallelogram Diagonals Converse (Thm. 7.10)

9. $x = 114$ and $y = 66$ by the Parallelogram Opposite Angles Converse (Thm. 7.8).

10. $x = 16$ and $y = 9$, by the Parallelogram Opposite Sides Converse (Thm. 7.7).

11. By the Parallelogram Opposite Sides Converse (Thm. 7.7):

$4x + 6 = 7x - 3$

$6 = 3x - 3$

$9 = 3x$

$3 = x$

$4y - 3 = 3y + 1$

$y - 3 = 1$

$y = 4$

So, $x = 3$ and $y = 4$.

12. By the Parallelogram Opposite Angles Converse (Thm. 7.8):

$(4x + 13)^\circ = (5x - 12)^\circ$

$13 = x - 12$

$25 = x$

$(4y + 7)^\circ = (3x - 8)^\circ$

$4y + 7 = 3 \cdot 25 - 8$

$4y + 7 = 75 - 8$

$4y = 60$

$y = 15$

So, $x = 25$ and $y = 15$.

13. By the Parallelogram Diagonals Converse (Thm. 7.10):

$4x + 2 = 5x - 6$

$2 = x - 6$

$8 = x$

So, $x = 8$.

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14. By the Opposite Sides Parallel and Congruent Theorem (Thm. 7.9):

$2x + 3 = x + 7$

$x + 3 = 7$

$x = 4$

So, $x = 4$.

15. By the Opposite Sides Parallel and Congruent Theorem (Thm. 7.9):

$3x + 5 = 5x - 9$

$-2x + 5 = -9$

$-2x = -14$

$x = 7$

So, $x = 7$.

16. By the Parallelogram Diagonals Converse (Thm. 7.10):

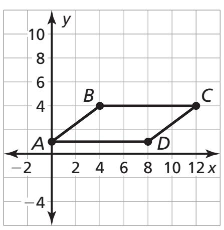
$6x = 3x + 2$

$3x = 2$

$x = \frac{2}{3}$

So, $x = \frac{2}{3}$.

17.



Slope of $\overline{BC} = \frac{4 - 4}{12 - 4} = \frac{0}{8} = 0$

Slope of $\overline{AD} = \frac{1 - 1}{8 - 0} = \frac{0}{8} = 0$

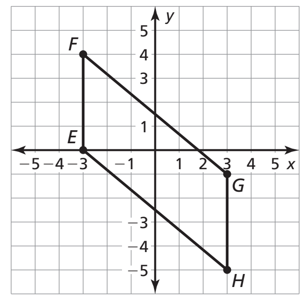
The slope of $\overline{BC}$ equals the slope of $\overline{AD}$, therefore $\overline{BC} \parallel \overline{AD}$.

$BC = \sqrt{(12 - 4)^2 + (4 - 4)^2} = \sqrt{(8)^2 + (0)^2} = \sqrt{64} = 8$

$AD = \sqrt{(8 - 0)^2 + (1 - 1)^2} = \sqrt{(8)^2 + (0)^2} = \sqrt{64} = 8$

Because $BC =$ AD $= 8$, $\overline{BC} \cong \overline{AD}$. $\overline{BC}$ and $\overline{AD}$ are opposite sides that are both congruent and parallel. So, *ABCD* is a parallelogram by the Parallelogram Opposite Sides Parallel and Congruent Theorem (Thm 7.9).

18.



Slope of $\overline{EF} = \frac{4 - 0}{-3 - (-3)} = \frac{4}{0} =$ undefined

Slope of $\overline{GH} = \frac{-5 - (-1)}{3 - 3} = \frac{-4}{0} =$ undefined

The slope of $\overline{EF}$ equals the slope of $\overline{GH}$, therefore $\overline{EF} \parallel \overline{GH}$.

$EF = \sqrt{(-3 - (-3)^2 + (4 - 0)^2} = \sqrt{0^2 + 4^2} = \sqrt{16} = 4$

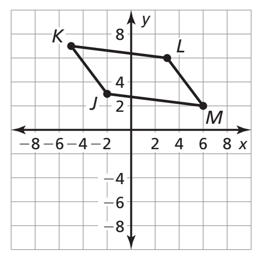
$GH = \sqrt{(3 - 3)^2 + (-5 - (-1))^2} = \sqrt{(0)^2 + (-4)^2}$

$= \sqrt{16} = 4$

Because $EF = GH = 4$, $\overline{EF} \cong \overline{GH}$. $\overline{EF}$ and $\overline{GH}$ are opposite sides that are both congruent and parallel.

So, *EFGH* is a parallelogram by the Opposite Sides Parallel and Congruent Theorem (Thm. 7.9).

19.



Slope of $\overline{KL} = \frac{6 - 7}{3 - (-5)} = \frac{-1}{3 + 5} = -\frac{1}{8}$

Slope of $\overline{JM} = \frac{2 - 3}{6 - (-2)} = \frac{-1}{6 + 2} = -\frac{1}{8}$

The slope of $\overline{JK}$ equals the slope of $\overline{LM}$, therefore $\overline{JK} \parallel \overline{LM}$.

$JK = \sqrt{(-5 - (-2)^2 + (7 - 3)^2} = \sqrt{(-5 + 2)^2 + 4^2}$

$= \sqrt{9 + 16} = \sqrt{25} = 5$

$LM = \sqrt{(6 - 3)^2 + (2 - 6)^2} = \sqrt{(3)^2 + (-4)^2} = \sqrt{9 + 16}$

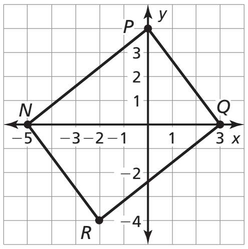
$= \sqrt{25} = 5$

Because $JK = LM = 5$, $\overline{JK} \cong \overline{LM}$. $\overline{JK}$ and $\overline{LM}$ are opposite sides that are both congruent and parallel.

So, *JKLM* is a Parallelogram by the Opposite Sides Parallel and Congruent Theorem (Thm. 7.9).

Chapter 7

20.



Slope of $\overline{NP} = \frac{4 - 0}{0 - (-5)} = \frac{4}{5}$

Slope of $\overline{PQ} = \frac{0 - 4}{3 - 0} = -\frac{4}{3}$

Slope of $\overline{QR} = \frac{-4 - 0}{-2 - 3} = \frac{-4}{-5} = \frac{4}{5}$

Slope of $\overline{NR} = \frac{-4 - 0}{-2 - (-5)} = \frac{-4}{-2 + 5} = -\frac{4}{3}$

The slope of $\overline{NP}$ equals the slope of $\overline{QR}$, therefore $\overline{NP} \parallel \overline{QR}$. The slope of $\overline{PQ}$ equals the slope of $\overline{NR}$, therefore $\overline{PQ} \parallel \overline{NR}$. Because both pairs of opposite sides are parallel, *NPQR* is a parallelogram by definition.

21. In order to be a parallelogram, the quadrilateral must have two pairs of opposite sides that are congruent, not consecutive sides. *DEFG* is not a parallelogram.

22. In order to determine that *JKLM* is a parallelogram by the Opposite Sides Parallel and Congruent Theorem (Thm. 7.9), you would need to know that $\overline{JM} \parallel \overline{KL}$. There is not enough information provided to determine whether *JKLM* is a parallelogram.

23. The diagonals must bisect each other, so solve for x using either $2x + 1 = x + 6$ or $4x - 2 = 3x + 3$. Also, the opposite sides must be congruent, so solve for x using either $3x + 1 = 4x - 4$ or $3x + 10 = 5x$.

$2x + 1 = x + 6$
$x + 1 = 6$
$x = 5$

$4x - 2 = 3x + 3$
$x - 2 = 3$
$x = 5$

$3x + 10 = 5x$
$10 = 2x$
$x = 5$

$3x + 1 = 4x - 4$
$1 = x - 4$
$5 = x$

So, $x = 5$.

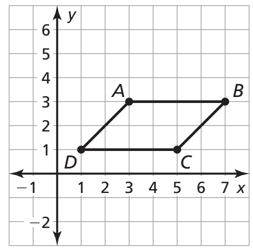
24. yes; By the Consecutive Interior Angles Converse (Thm. 3.8), $\overline{WX} \parallel \overline{ZY}$. Because $\overline{WX}$ and $\overline{ZY}$ are also congruent, *WXYZ* is a parallelogram by the Opposite Sides Parallel and Congruent Theorem (Thm. 7.9).

25. A quadrilateral is a parallelogram if and only if both pairs of opposite sides are congruent.

26. A quadrilateral is a parallelogram if and only if both pairs of opposite angles are congruent.

27. A quadrilateral is a parallelogram if and only if the diagonals bisect each other.

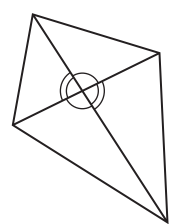
28. *Sample answer:* Draw two horizontal segments that are the same length and connect the endpoints.



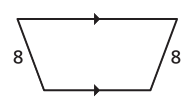
29. Check students' work. Because the diagonals bisect each other, this quadrilateral is a parallelogram by the Parallelogram Diagonals Converse (Thm. 7.10).

30. both; If you show that $\overline{QR} \parallel \overline{TS}$ and $\overline{QT} \parallel \overline{RS}$, then *QRST* is a parallelogram by definition. If you show that $\overline{QR} \cong \overline{TS}$ and $\overline{QT} \cong \overline{RS}$, then *QRST* is a parallelogram by the Parallelogram Opposite Sides Converse (Thm. 7.7).

31. *Sample answer:*



32. *Sample answer:*



33. **a.** Because $m\angle AEF = 63°$ and $\angle EAF$ is a right angle ($m\angle EAF = 90°$), $m\angle AFE = 90° - 63° = 27°$.

b. Because the angle of incident equals the angle of reflection $m\angle AFE = m\angle DFG = 27°$. Because $m\angle FDG = 90°$, $m\angle FGD = 90° - 27° = 63°$.

c. $m\angle GHC = m\angle EHB = 27°$

d. yes; $\angle HEF \cong \angle HGF$ because they both are adjacent to two congruent angles that together add up to 180°, and $\angle EHG \cong \angle GFE$ for the same reason. So, *EFGH* is a parallelogram by the Parallelogram Opposite Angles Converse (Thm. 7.8).

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34. **a.** Because $\overline{JK} \cong \overline{LM}$ and $\overline{KL} \cong \overline{JM}$, $JKLM$ is a parallelogram by the Parallelogram Opposite Sides Converse (Thm. 7.7).

b. Because $m\angle JKL = 60°$, $m\angle JML = 60°$ by the Parallelogram Opposite Angles Converse (Thm. 7.8). $m\angle KJM = 180° - 60° = 120°$ by the Parallelogram Consecutive Angles Theorem (Thm. 7.5). $m\angle KLM = 120°$ by the Parallelogram Opposite Angles Converse (Thm. 7.8).

c. Transitive Property of Parallel Lines (Thm. 3.9)

35. You can use the Alternate Interior Angles Converse (Thm. 3.6) to show that $\overline{AD} \parallel \overline{BC}$. Then, $\overline{AD}$ and $\overline{BC}$ are both congruent and parallel. So, $ABCD$ is a parallelogram by the Opposite Sides Parallel and Congruent Theorem (Thm. 7.9).

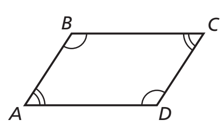
36. You can use the Alternate Interior Angles Converse (Thm. 3.6) to show that $\overline{AB} \parallel \overline{DC}$ and $\overline{AD} \parallel \overline{BC}$. Because both pairs of opposite sides are parallel, $ABCD$ is a parallelogram by definition.

37. First, you can use the Linear Pair Postulate (Post. 2.8) and the Congruent Supplements Theorem (Thm. 2.4) to show that $\angle ABC$ and $\angle DCB$ are supplementary. Then, you can use the Consecutive Interior Angles Converse (Thm. 3.8) to show that $\overline{AB} \parallel \overline{DC}$ and $\overline{AD} \parallel \overline{BC}$. So, $ABCD$ is a parallelogram by definition.

38. By the Parallelogram Opposite Sides Theorem (Thm. 7.3), $\overline{JM} \cong \overline{LK}$. Also, you can use the Linear Pair Postulate (Thm. 2.8) and the Congruent Supplements Theorem (Thm. 2.4) to show that $\angle GJM \cong \angle HLK$. Because $\angle JGM$ and $\angle LHK$ are congruent right angles, you can now state that $\triangle MGJ \cong \triangle KHL$ by the AAS Congruence Theorem (Thm. 5.11).

39. **Given** $\angle A \cong \angle C$, $\angle B \cong \angle D$

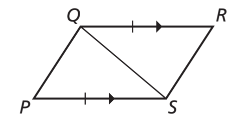
Prove $ABCD$ is a parallelogram.



STATEMENTS	REASONS
1. $\angle A \cong \angle C$, $\angle B \cong \angle D$	**1.** Given
2. Let $m\angle A = m\angle C = x°$ and $m\angle B = m\angle D = y°$	**2.** Definition of congruent angles
3. $m\angle A + m\angle B + m\angle C + m\angle D = x° + y° + x° + y° = 360°$	**3.** Corollary to the Polygon Interior Angles Theorem (Cor. 7.1)
4. $2(x°) + 2(y°) = 360°$	**4.** Simplify
5. $2(x° + y°) = 360°$	**5.** Distributive Property
6. $x° + y° = 180°$	**6.** Division Property of Equality
7. $m\angle A + m\angle B = 180°$, $m\angle A + m\angle D = 180°$	**7.** Substitution Property of Equality
8. $\angle A$ and $\angle B$ are supplementary. $\angle A$ and $\angle D$ are supplementary.	**8.** Definition of supplementary angles
9. $\overline{BC} \parallel \overline{AD}$, $\overline{AB} \parallel \overline{DC}$	**9.** Consecutive Interior Angles Converse (Thm. 3.8)
10. $ABCD$ is a parallelogram.	**10.** Definition of parallelogram

40. **Given** $\overline{QR} \parallel \overline{PS}$, $\overline{QR} \cong \overline{PS}$

Prove $PQRS$ is a parallelogram.

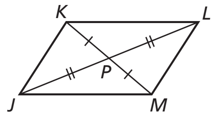


STATEMENTS	REASONS
1. $\overline{QR} \parallel \overline{PS}$, $\overline{QR} \cong \overline{PS}$	**1.** Given
2. $\angle RQS \cong \angle PSQ$	**2.** Alternate Interior Angles Theorem (Thm. 3.2)
3. $\overline{QS} \cong \overline{QS}$	**3.** Reflexive Property of Conguence (Thm. 2.1)
4. $\triangle PQS \cong \triangle RSQ$	**4.** SAS Congruence Theorem (Thm. 5.5)
5. $\angle QSR \cong \angle SQP$	**5.** Corresponding parts of congruent triangles are congruent.
6. $\overline{QP} \parallel \overline{RS}$	**6.** Alternate Interior Angles Converse (Thm. 3.6)
7. $PQRS$ is a parallelogram.	**7.** Definition of parallelogram

Chapter 7

41. Given Diagonals $\overline{JL}$ and $\overline{KM}$ bisect each other.

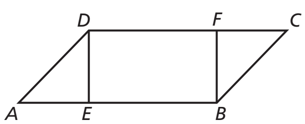
Prove $JKLM$ is a parallelogram.



STATEMENTS	REASONS
1. Diagonals $\overline{JL}$ and $\overline{KM}$ bisect each other.	**1.** Given
2. $\overline{JP} \cong \overline{LP}, \overline{KP} \cong \overline{MP}$	**2.** Definition of segment bisector
3. $\angle KPL \cong \angle MPJ$	**3.** Reflexive Property of Congruence (Thm. 2.1)
4. $\triangle KPL \cong \triangle MPJ$	**4.** SAS Congruence Theorem (Thm. 5.5)
5. $\angle MKL \cong \angle KMJ$, $\overline{KL} \cong \overline{MJ}$	**5.** Corresponding parts of congruent triangles are congruent.
6. $\overline{KL} \parallel \overline{MJ}$	**6.** Alternate Interior Angles Converse (Thm. 3.6)
7. $JKLM$ is a parallelogram.	**7.** Opposite Sides Parallel and Congruent Theorem (Thm. 7.9)

42. Given $DEBF$ is a parallelogram. $AE = CF$

Prove $ABCD$ is a parallelogram.



STATEMENTS	REASONS
1. $DEBF$ is a parallelogram, $AE = CF$	**1.** Given
2. $\overline{DE} \cong \overline{BF}, \overline{FD} \cong \overline{EB}$	**2.** Parallelogram Opposite Sides Theorem (Thm. 7.3)
3. $\angle DFB \cong \angle DEB$	**3.** Parallelogram Opposite Angles Theorem (Thm. 7.4)
4. $\angle AED$ and $\angle DEB$ form a linear pair. $\angle CFB$ and $\angle DFB$ form a linear pair.	**4.** Definition of linear pair
5. $\angle AED$ and $\angle DEB$ are supplementary. $\angle CFB$ and $\angle DFB$ are supplementary.	**5.** Linear Pair Postulate (Post. 2.8)
6. $\angle AED \cong \angle CFB$	**6.** Congruent Supplements Theorem (Thm. 2.4)
7. $\overline{AE} \cong \overline{CF}$	**7.** Definition of congruent segments
8. $\triangle KPL \cong \triangle MPJ$	**8.** SAS Congruence Theorem (Thm. 5.5)
9. $\overline{AD} \cong \overline{CB}$	**9.** Corresponding parts of congruent triangles are congruent.
10. $AB = AE + EB$, $DC = CF + FD$	**10.** Segment Addition Postulate (Post. 1.2)
11. $FD = EB$	**11.** Definition of congruent segments
12. $AB = CF + FD$	**12.** Substitution Property of Equality
13. $AB = DC$	**13.** Transitive Property of Equality
14. $\overline{AB} \cong \overline{DC}$	**14.** Definition of congruent segments
15. $ABCD$ is a parallelogram.	**15.** Parallelogram Opposite Sides Converse (Thm. 7.7)

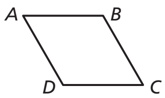
43. no; The fourth angle will be 113° because of the Corollary to the Polygon Interior Angles Theorem (Cor. 7.1), but these could also be the angle measures of an isosceles trapezoid with base angles that are each 67°.

44. The segments that remain parallel as the stand is folded are $\overline{AD} \parallel \overline{EF} \parallel \overline{BC}, \overline{AE} \parallel \overline{DF}$, and $\overline{BE} \parallel \overline{CF}$.

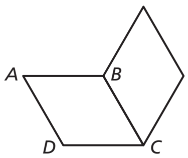
45. By the Parallelogram Opposite Sides Theorem (Thm. 7.3), $\overline{AB} \cong \overline{CD}$. Also, $\angle ABE$ and $\angle CDF$ are congruent alternate interior angles of parallel segments $\overline{AB}$ and $\overline{CD}$. Then, you can use the Segment Addition Postulate (Post. 1.2), the Substitution Property of Equality, and the Reflexive Property of Congruence (Thm. 2.1) to show that $\overline{DF} \cong \overline{BE}$. So, $\triangle ABE \cong \triangle CDF$ by the SAS Congruence Theorem (Thm. 5.5), which means that $AE = CF = 8$ because corresponding parts of congruent triangles are congruent.

Chapter 7

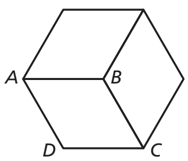
46. Draw the first parallelogram:



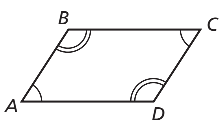
Rotate *ABCD* 120° about *B*:



Then rotate *ABCD* −120° about *B*:



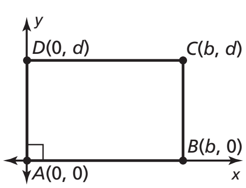
47. Converse to the Parallelogram Consecutive Angles Theorem: If every pair of consecutive angles of a quadrilateral are supplementary, then the quadrilateral is a parallelogram.



In *ABCD*, you are given that $\angle A$ and $\angle B$ are supplementary, and $\angle B$ and $\angle C$ are supplementary. So, $m\angle A = m\angle C$. Also, $\angle B$ and $\angle C$ are supplementary, and $\angle C$ and $\angle D$ are supplementary. So, $m\angle B = m\angle D$. So, *ABCD* is a parallelogram by the Parallelogram Opposite Angles Converse (Thm. 7.8).

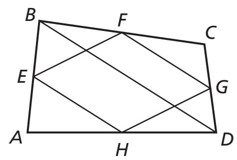
48. **Given** *ABCD* is a parallelogram and $\angle A$ is a right angle.

Prove $\angle B$, $\angle C$, and $\angle D$ are right angles.



By the definition of a right angle, $m\angle A = 90°$. Because *ABCD* is a parallelogram, and opposite angles of a parallelogram are congruent, $m\angle A = m\angle C = 90°$. Because consecutive angles of a parallelogram are supplementary, $\angle C$ and $\angle B$ are supplementary, and $\angle C$ and $\angle D$ are supplementary. So, $90° + m\angle B = 180°$ and $90° + m\angle D = 180°$. This gives you $m\angle B = m\angle D = 90°$. So, $\angle B$, $\angle C$, and $\angle D$ are right angles.

49. Given quadrilateral *ABCD* with midpoints *E*, *F*, *G*, and *H* that are joined to form a quadrilateral, you can construct diagonal $\overline{BD}$. Then $\overline{FG}$ is a midsegment of $\triangle BCD$, and $\overline{EH}$ is a midsegment of $\triangle DAB$. So, by the Triangle Midsegment Theorem (Thm. 6.8), $\overline{FG} \parallel \overline{BD}$, $FG = \frac{1}{2}BD$, $\overline{EH} \parallel \overline{BD}$, and $EH = \frac{1}{2}BD$. So, by the Transitive Property of Parallel Lines (Thm. 3.9), $\overline{EH} \parallel \overline{FG}$ and by the Transitive Property of Equality, $EH = FG$. Because one pair of opposite sides is both congruent and parallel, *EFGH* is a parallelogram by the Opposite Sides Parallel and Congruent Theorem (Thm. 7.9).



50. Based on the given information, $\overline{GH}$ is a midsegment of $\triangle EBC$, and $\overline{FJ}$ is a midsegment of $\triangle EAD$. So, by the Triangle Midsegment Theorem (Thm. 6.8), $\overline{GH} \parallel \overline{BC}$, $GH = \frac{1}{2}BC$, $\overline{FJ} \parallel \overline{AD}$, and $FJ = \frac{1}{2}AD$. Also, by the Parallelogram Opposite Sides Theorem (Thm. 7.3) and the definition of a parallelogram, $\overline{BC}$ and $\overline{AD}$ are congruent and parallel. So, by the Transitive Property of Parallel Lines (Thm. 3.9), $\overline{AD} \parallel \overline{FJ} \parallel \overline{GH} \parallel \overline{BC}$ and by the Transitive Property of Equality, $\frac{1}{2}BC = GH = FJ = \frac{1}{2}AD$. Because one pair of opposite sides is both congruent and parallel, *FGHJ* is a parallelogram by the Opposite Sides Parallel and Congruent Theorem (Thm. 7.9).

Maintaining Mathematical Proficiency

51. The quadrilateral is a parallelogram by the definition of a parallelogram (a quadrilateral with both pairs of opposite sides parallel).

52. The quadrilateral is a rectangle by the definition of a rectangle (a quadrilateral with four right angles).

53. The quadrilateral is a square by the definition of a square (a quadrilateral with four right angles and four congruent sides).

54. The quadrilateral is a rhombus by the definition of a rhombus (a quadrilateral with four congruent sides).

7.1–7.3 What Did You Learn? (p. 385)

1. The relationship between the 540° increase and the answer is that the interior angle value added is 540° or $3 \cdot 180°$.

2. By the Parallelogram Diagonals Theorem (Thm. 7.6), the diagonals of a parallelogram bisect each other. So, the diagonals will have the same midpoint, and it will also be the point where the diagonals intersect. Therefore, with any parallelogram, you can find the midpoint of either diagonal, and it will be the coordinates of the intersection of the diagonals; Instead of this method, you could also find the equations of the lines that define each diagonal, set them equal to each other and solve for the values of the coordinates where the lines intersect.

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3. You could start by setting the two parts of either diagonal equal to each other by the Parallelogram Diagonals Theorem (Thm. 7.6) or you could start by setting either pair of opposite sides equal to each other by the Parallelogram Opposite Sides Theorem (Thm. 7.3).

7.1–7.3 Quiz (p. 386)

1. $115° + 95° + 70° + x° = 360°$
$$280 + x = 360$$
$$x = 80$$

2. $(5 - 2) \cdot 180° = 3 \cdot 180° = 540°$
$$60° + 120° + 150° + 75° + x° = 540°$$
$$405 + x = 540$$
$$x = 135$$

3. $x° + 60° + 30° + 72° + 46° + 55° = 360°$
$$x + 263 = 360$$
$$x = 97$$

4. Interior angle $= \dfrac{(10 - 2) \cdot 180°}{10} = \dfrac{8 \cdot 180°}{10} = \dfrac{1440°}{10} = 144°$

Exterior angle $= \dfrac{360°}{10} = 36°$

In a regular decagon, the measure of each interior angle is 144° and the measure of each exterior angle is 36°.

5. Interior angle $= \dfrac{(15 - 2) \cdot 180°}{15} = \dfrac{13 \cdot 180°}{15} = \dfrac{2340°}{15} = 156°$

Exterior angle $= \dfrac{360°}{15} = 24°$

In a regular 15-gon, the measure of each interior angle is 156° and the measure of each exterior angle is 24°.

6. Interior angle $= \dfrac{(24 - 2) \cdot 180°}{24} = \dfrac{22 \cdot 180°}{24} = \dfrac{3960°}{24} = 165°$

Exterior angle $= \dfrac{360°}{24} = 15°$

In a regular 24-gon, the measure of each interior angle is 165° and the measure of each exterior angle is 15°.

7. Interior angle $= \dfrac{(60 - 2) \cdot 180°}{60} = \dfrac{58 \cdot 180°}{60} = \dfrac{10{,}440°}{60} = 174°$

Exterior angle $= \dfrac{360°}{60} = 6°$

In a regular 60-gon, the measure of each interior angle is 174° and the measure of each exterior angle is 6°.

8. $CD = 16$; By the Parallelogram Opposite Sides Theorem (Thm. 7.3), $AB = CD$.

9. $AD = 7$; By the Parallelogram Opposite Sides Theorem (Thm. 7.3), $AD = BC$.

10. $AE = 7$; By the Parallelogram Diagonals Theorem (Thm. 7.6), $AE = EC$.

11. $BD = 2 \cdot 10.2 = 20.4$; By the Parallelogram Diagonals Theorem (Thm. 7.6), $BE = ED$.

12. $m\angle BCD = 120$; By the Parallelogram Opposite Angles Theorem (Thm. 7.4), $m\angle DAB = m\angle BCD$.

13. By the Parallelogram Consecutive Angles Theorem (Thm. 7.5), $\angle DAB$ and $\angle ABC$ are supplementary. So, $m\angle ABC = 180° - 120° = 60°$.

14. By the Parallelogram Consecutive Angles Theorem (Thm. 7.5), $\angle DAB$ and $\angle ADC$ are supplementary. So, $m\angle ADC = 180° - 120° = 60°$.

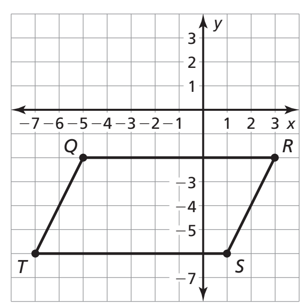
15. $m\angle ADB = 39°$; By the Alternate Interior Angles Theorem (Thm. 3.2), $m\angle DBC = m\angle ADB$.

16. The quadrilateral is a parallelogram by the Opposite Sides Parallel and Congruent Theorem (Thm. 7.9).

17. The quadrilateral is a parallelogram by the Parallelogram Diagonals Converse (Thm. 7.10).

18. The quadrilateral is a parallelogram by the Parallelogram Opposite Angles Converse (Thm. 7.8).

19.



Slope of $\overline{QR} = \dfrac{-2 - (-2)}{3 - (-5)} = \dfrac{-2 + 2}{3 + 5} = \dfrac{0}{8} = 0$

Slope of $\overline{ST} = \dfrac{-6 - (-6)}{-7 - 1} = \dfrac{-6 + 6}{-8} = \dfrac{0}{-8} = 0$

The slope of $\overline{QR}$ equals slope of $\overline{ST}$, therefore $\overline{QR} \parallel \overline{ST}$.

$QR = \sqrt{(3 - (-5))^2 + (-2 - (-2))^2}$
$= \sqrt{(3 + 5)^2 + (-2 + 2)^2} = \sqrt{8^2} = \sqrt{64} = 8$

Because $QR = ST = 8$, $\overline{QR} \cong \overline{ST}$. $\overline{QR}$ and $\overline{ST}$ are opposite sides that are both congruent and parallel.

So, $QRST$ is a parallelogram by the Opposite Sides Parallel and Congruent Theorem (Thm. 7.9).

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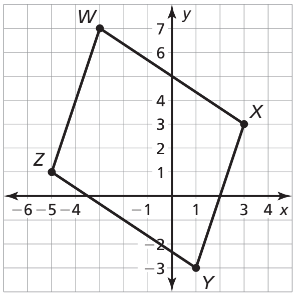
20. 

$$\text{Slope of } \overline{WX} = \frac{3-7}{3-(-3)} = \frac{-4}{3+3} = \frac{-4}{6} = -\frac{2}{3}$$

$$\text{Slope of } \overline{XY} = \frac{-3-3}{1-3} = \frac{-6}{-2} = \frac{-3}{-1} = 3$$

$$\text{Slope of } \overline{YZ} = \frac{1-(-3)}{-5-1} = \frac{1+3}{-6} = \frac{4}{-6} = -\frac{2}{3}$$

$$\text{Slope of } \overline{WZ} = \frac{1-7}{-5-(-3)} = \frac{-6}{-5+3} = \frac{-6}{-2} = 3$$

Because the slope of $\overline{WX}$ equals slope of $\overline{YZ}$, $\overline{WX} \parallel \overline{YZ}$ and because the slope of $\overline{XY}$ equals the slope of $\overline{WZ}$, $\overline{XY} \parallel \overline{WZ}$. Because both pairs of opposite sides are parallel, *WXYZ* is a parallelogram by definition.

21. a. The stop sign is a regular octagon.

b. $\frac{(8-2) \cdot 180^\circ}{8} = \frac{6 \cdot 180^\circ}{8} = \frac{1080^\circ}{8} = 135^\circ$

The measure of each interior angle is 135°.

$\frac{360^\circ}{8} = 45^\circ$

The measure of each exterior angle is 45°.

22. a. $\overline{JK} \cong \overline{ML}$ by the Parallelogram Opposite Sides Theorem (Thm. 7.3). $\overline{JM} \cong \overline{KL}$ by the Parallelogram Opposite Sides Theorem (Thm. 7.3), $\angle J \cong \angle KLM$ by the Parallelogram Opposite Angles Theorem (Thm. 7.4), $\angle M \cong \angle JKL$ by the Parallelogram Opposite Angles Theorem (Thm. 7.4)

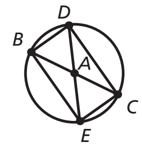
b. Because $\overline{QT} \parallel \overline{RS}$ and $QT = RS$, *QRST* is a parallelogram by the Opposite Sides Parallel and Congruent Theorem (Thm. 7.9).

c. $ST = 3$ feet, because $ST = QR$ by the Parallelogram Opposite Sides Theorem (Thm. 7.3). $m\angle QTS = 123^\circ$, because $m\angle QTS = m\angle QRS$ by the Parallelogram Opposite Angles Theorem (Thm. 7.4). $m\angle TQR = 57^\circ$, because $\angle TQR$ and $\angle QTS$ are consecutive interior angles and they are supplementary. So, $m\angle TQR = 180^\circ - 123^\circ = 57^\circ$. Because $\angle TSR$ and $\angle TQR$ are opposite angles by the Parallelogram Opposite Angles Theorem (Thm. 7.4), $m\angle TSR = 57^\circ$.

7.4 Explorations (p. 387)

1. a. Check students' work.

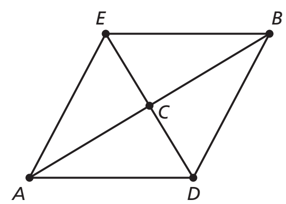
b. Check students' work.

c. 

d. yes; yes; no; no; Because all points on a circle are the same distance from the center, $\overline{AB} \cong \overline{AE} \cong \overline{AC} \cong \overline{AD}$. So, the diagonals of quadrilateral *BDCE* bisect each other, which means it is a parallelogram by the Parallelogram Diagonals Converse (Thm. 7.10). Because all 4 angles of *BDCE* are right angles, it is a rectangle *BDCE* is neither a rhombus nor a square because $\overline{BD}$ and $\overline{EC}$ are not necessarily the same length as $\overline{BE}$ and $\overline{DC}$.

e. Check students' work. The quadrilateral formed by the endpoints of two diameters is a rectangle (and a parallelogram). In other words, a quadrilateral is a rectangle if and only if its diagonals are congruent and bisect each other.

2. a. Check students' work.

b. 

c. yes; no; yes; no; Because the diagonals bisect each other, *AEBD* is a parallelogram by the Parallelogram Diagonals Converse (Thm. 7.10). Because $EB = BD = AD = AE$, *AEBD* is a rhombus. *AEBD* is neither a rectangle nor a square because its angles are not necessarily right angles.

d. Check students' work. A quadrilateral is a rhombus if and only if the diagonals are perpendicular bisectors of each other.

3. Because rectangles, rhombuses, and squares are all parallelograms, their diagonals bisect each other by the Parallelogram Diagonals Theorem (Thm. 7.6). The diagonals of a rectangle are congruent. The diagonals of a rhombus are perpendicular. The diagonals of a square are congruent and perpendicular.

4. yes; no; yes; no; *RSTU* is a parallelogram because the diagonals bisect each other. *RSTU* is not a rectangle because the diagonals are not congruent. *RSTU* is a rhombus because the diagonals are perpendicular. *RSTU* is not a square because the diagonals are not congruent.

5. A rectangle has congruent diagonals that bisect each other.

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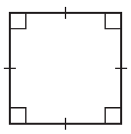
7.4 Monitoring Progress (pp. 389–392)

1. For any square $JKLM$, it is always true that $\overline{JK} \perp \overline{KL}$, because by definition, a square has four right angles.

2. For any rectangle $EFGH$, it is sometimes true that $\overline{FG} \cong \overline{GH}$, because some rectangles are squares.

3. The quadrilateral is a square.



4. $m\angle ADC = 2 \cdot 29° = 58°$ because each diagonal of a rhombus bisects a pair of opposite angles.

$m\angle BCD = 2 \cdot 61° = 122°$ because each diagonal of a rhombus bisects a pair of opposite angles.

5. $m\angle EDG = 180° - 118° = 62°$ by the Parallelogram Consecutive Angles Theorem (Thm. 7.5).

$m\angle 1 = \frac{62°}{2} = 31°$ because each diagonal of a rhombus bisects a pair of opposite angles.

$m\angle 2 = m\angle 1 = 31°$ because each diagonal of a rhombus bisects a pair of opposite angles.

$m\angle EFG = m\angle EDG = 62°$ because opposite angles of a parallelogram are congruent, and a rhombus is a parallelogram.

$m\angle 3 = 31°$ because each diagonal of a rhombus bisects a pair of opposite angles.

$m\angle 4 = m\angle 3 = 31°$ because each diagonal of a rhombus bisects a pair of opposite angles.

6. no; The quadrilateral might not be a parallelogram.

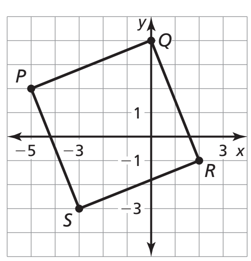
7.
$$QS = RT$$
$$4x - 15 = 3x + 8$$
$$x - 15 = 8$$
$$x = 23$$

Lengths of the diagonals:

$QS = 4 \cdot 23 - 15 = 92 - 15 = 77$

$RT = 3 \cdot 23 + 8 = 69 + 8 = 77$

8. $P(-5, 2)$, $Q(0, 4)$, $R(2, -1)$, $S(-3, -3)$



$PR = \sqrt{(-5 - 2)^2 + (2 - (-1))^2} = \sqrt{49 + 9} = \sqrt{58}$

$QS = \sqrt{(0 - (-3))^2 + (4 - (-3))^2} = \sqrt{9 + 49} = \sqrt{58}$

Because $PR = QS$, the diagonals are congruent, so the quadrilateral is either a square or rectangle.

$PQ = \sqrt{(-5 - 0)^2 + (2 - 4)^2} = \sqrt{25 + 4} = \sqrt{29}$

$QR = \sqrt{(0 - 2)^2 + (4 - (-1))^2} = \sqrt{4 + 25} = \sqrt{29}$

Because $PQ = QR$, the adjacent sides $\overline{PQ}$ and $\overline{QR}$ are congruent. So, $PQRS$ is a square, a rectangle, and a rhombus.

7.4 Exercises (pp. 393–396)

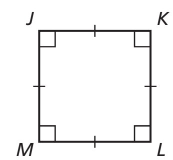
Vocabulary and Core Concept Check

1. Another name for an equilateral rectangle is a square.

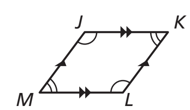
2. If two consecutive sides of a parallelogram are congruent, then the parallelogram is also a rhombus.

Monitoring Progress and Modeling with Mathematics

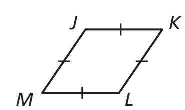
3. $\angle L$ is sometimes congruent to $\angle M$. Some rhombuses are squares.



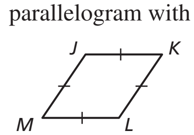
4. $\angle K$ is always congruent to $\angle M$. A rhombus is a Parallelogram and the opposite angles of a parallelogram are congruent.



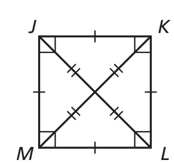
5. $\overline{JM}$ is always congruent to $\overline{KL}$. By definition, a rhombus is a parallelogram, and opposite sides a parallelogram are congruent.



6. $\overline{JK}$ is always congruent to $\overline{KL}$. By definition, a rhombus is a parallelogram with four congruent sides.

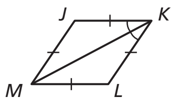


7. $\overline{JL}$ is sometimes congruent to $\overline{KM}$. Some rhombuses are squares.



Chapter 7

8. $\angle JKM$ is always congruent to $\angle LKM$. Each diagonal of a rhombus bisects a pair of opposite angles.



9. square; All of the sides are congruent, and all of the angles are congruent.

10. rectangle; Opposite sides are congruent and the angles are 90°.

11. rectangle; Opposite sides are parallel and the angles are 90°.

12. rhombus; Opposite angles are congruent and adjacent sides are congruent.

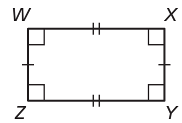
13. A rhombus is a parallelogram with four congruent sides. So, $m\angle 1 = m\angle FEG = 27°$, by the Base Angles Theorem (Thm. 5.6). By the Rhombus Opposite Angles Theorem (Thm. 7.12), $m\angle 2 = m\angle 1 = 27°$. By the Rhombus Diagonals Theorem (Thm. 7.11), $m\angle 3 = 90°$. By the Rhombus Opposite Angles Theorem (Thm. 7.12), $m\angle 4 = m\angle FEG = 27°$. By the Rhombus Opposite Angles Theorem (Thm. 7.12), $m\angle FED = 2 \cdot 27° = 54°$. By the Parallelogram Consecutive Angles Theorem (Thm. 7.5), $m\angle GFE = 180° - m\angle FED = 180° - 54° = 126°$. So, $m\angle 5 = m\angle 6 = 63°$, by the Rhombus Opposite Angles Theorem (Thm. 7.12).

14. By the Rhombus Diagonals Theorem (Thm. 7.11), $m\angle 1 = 90°$. By the Rhombus Opposite Angles Theorem (Thm. 7.12), $m\angle EDG = 2 \cdot 48° = 96°$. By the Parallelogram Consecutive Angles Theorem (Thm. 7.5), $m\angle DGF = 180° - m\angle EDG = 180° - 96° = 84°$. So, by the Rhombus Opposite Angles Theorem (Thm. 7.12), $m\angle 2 = m\angle 3 = 42°$. By the Rhombus Opposite Angles Theorem (Thm. 7.12), $m\angle 4 = 48°$. By the definition of a parallelogram $\overline{DE} \parallel \overline{GF}$. So, $m\angle 5 = 48°$, by the Alternate Interior Angle Theorem (Thm. 3.2).

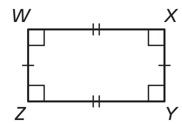
15. By the Parallelogram Consecutive Angles Theorem (Thm. 7.5), $m\angle EDG = 180° - 106° = 74°$, So, by the Rhombus Opposite Angles Theorem (Thm. 7.12), $m\angle 1 = m\angle 2 = 37°$. By the definition of a parallelogram $\overline{DE} \parallel \overline{GF}$. So, $m\angle 3 = 37°$, by the Alternate Interior Angles Theorem (Thm. 3.2). By the Rhombus Opposite Angles Theorem (Thm. 7.12), $m\angle 4 = 37°$. By the Parallelogram Opposite Angles Theorem (Thm. 7.3), $m\angle 5 = 106°$.

16. By the Rhombus Opposite Angles Theorem (Thm. 7.12), $m\angle EDG = 2 \cdot 72° = 144°$. So, $m\angle 1 = 180° - 144° = 36°$, by the Parallelogram Consecutive Angles Theorem (Thm. 7.5). By the Triangle Sum Theorem (Thm. 5.1), $m\angle 1 + m\angle 2 + 72° = 180°$. So, $m\angle 2 + 108° = 180°$ and $m\angle 2 = 72°$. By the Rhombus Opposite Angles Theorem (Thm. 7.12), $m\angle 3 = 72°$. By the Rhombus Opposite Angles Theorem (Thm. 7.12), $m\angle 4 = 72°$. By the Parallelogram Opposite Angles Theorem (Thm. 7.3), $m\angle 5 = 36°$.

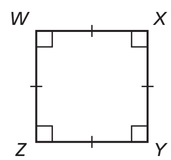
17. $\angle W$ is always congruent to $\angle X$. All angles of a rectangle are congruent.



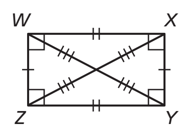
18. $\overline{WX}$ is always congruent to $\overline{YZ}$. Opposite sides of a rectangle are congruent.



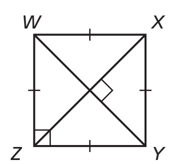
19. $\overline{WX}$ is sometimes congruent to $\overline{XY}$. Some rectangles are squares.



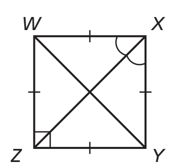
20. $\overline{WY}$ is always congruent to $\overline{XZ}$. The diagonals of a rectangle are congruent.



21. $\overline{WY}$ is sometimes perpendicular to $\overline{XZ}$. Some rectangles are squares.



22. $\angle WXZ$ is sometimes congruent to $\angle YXZ$. Some rectangles are squares.



23. The quadrilateral is not a rectangle. All four angles are not congruent.

24. The quadrilateral is a rectangle. Opposite sides are congruent and the angles are 90°.

Chapter 7

25. $WY = XZ$

$6x - 7 = 3x + 2$

$3x = 9$

$x = 3$

$WY = 6 \cdot 3 - 7 = 18 - 7 = 11$

$XZ = 3 \cdot 3 + 2 = 9 + 2 = 11$

26. $WY = XZ$

$14x + 10 = 11x + 22$

$3x = 12$

$x = 4$

$WY = 14 \cdot 4 + 10 = 56 + 10 = 66$

$XZ = 11 \cdot 4 + 22 = 44 + 22 = 66$

27. $WY = XZ$

$24x - 8 = -18x + 13$

$42x = 21$

$x = \frac{21}{42} = \frac{1}{2}$

$WY = 24 \cdot \frac{1}{2} - 8 = 12 - 8 = 4$

$XZ = -18 \cdot \frac{1}{2} + 13 = -9 + 13 = 4$

28. $WY = XZ$

$16x + 2 = 36x - 6$

$-20x = -8$

$x = \frac{-8}{-20} = \frac{2}{5}$

$WY = 16 \cdot \frac{2}{5} + 2 = \frac{32}{5} + \frac{10}{5} = \frac{42}{5} = 8.4$

$XZ = 36 \cdot \frac{2}{5} - 6 = \frac{72}{5} - \frac{30}{5} = \frac{42}{5} = 8.4$

29. Quadrilaterals that are equiangular are squares and rectangles.

30. Quadrilaterals that are equiangular and equilateral are squares.

31. Quadrilaterals where the diagonals are perpendicular are squares and rhombuses.

32. Opposite sides are congruent are true for all parallelograms, rectangles, rhombuses, and squares.

33. Diagonals bisect each other are true for all parallelograms, rectangles, rhombuses, and squares.

34. Diagonals bisect opposite angles are true for rhombuses and squares.

35. Diagonals do not necessarily bisect opposite angles of a rectangle. The sum of the two angles equals 90°.

$m\angle QSP = 90° - m\angle RQS$

$x° = 90° - 58°$

$x = 32$

36. $\angle QSP$ and $\angle RQS$ should be complementary because they are the two acute angles of a right triangle.

$m\angle QSP = 90° - m\angle RQS$

$x° = 90° - 37°$

$x = 53$

37. $m\angle DAC = 53°$ by the Rhombus Opposite Angles Theorem (Thm. 7.12).

38. $m\angle AED$ 90° by the Rhombus Diagonals Theorem (Thm. 7.11).

39. $m\angle ADC = 180° - 2 \cdot 53°$

$= 180° - 106° = 74°$

40. $DB = 16$ by the Parallelogram Diagonals Theorem (Thm. 7.6).

41. $AE = 6$ by the Parallelogram Diagonals Theorem (Thm. 7.6).

42. $AC = 12$ by the Parallelogram Diagonals Theorem (Thm. 7.6).

43. $m\angle QTR = 90° - 34° = 56°$

44. $m\angle QRT = 34°$ by the Alternate Interior Angles Theorem (Thm. 3.2).

45. $m\angle SRT = 180° - 34° - 90° = 56°$

46. $QP = 5$ by the Parallelogram Diagonals Theorem (Thm. 7.6).

47. $RT = 10$ by the Parallelogram Diagonals Theorem (Thm. 7.6).

48. $RP = 5$ by the Parallelogram Diagonals Theorem (Thm. 7.6).

49. $m\angle MKN = 90°$ because the diagonals of a square are perpendicular.

50. $m\angle LMK = 45°$ because the diagonals of a square bisect opposite angles.

51. $m\angle LPK = 45°$ because the diagonals of a square bisect opposite angles.

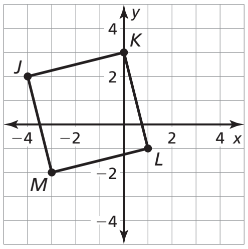
52. $KN = 1$ because the diagonals of a square bisect each other.

53. $LN = 2$ because the diagonals of a square bisect each other.

54. $MP = 2$ because the diagonals of a square are congruent.

Chapter 7

55.



Diagonals:

$JL = \sqrt{(1-(-4))^2 + (-1-2)^2} = \sqrt{(1+4)^2 + (-3)^2}$

$= \sqrt{25+9} = \sqrt{34}$

$KM = \sqrt{(0-(-3))^2 + (3-(-2))^2} = \sqrt{(3)^2 + (5)^2}$

$= \sqrt{9+25} = \sqrt{34}$

Sides:

$JK = \sqrt{(0-(-4))^2 + (3-2)^2} = \sqrt{(4)^2 + (1)^2} = \sqrt{16+1}$

$= \sqrt{17}$

$JM = \sqrt{(-3-(-4))^2 + (-2-2)^2} = \sqrt{(1)^2 + (-4)^2}$

$= \sqrt{1+16} = \sqrt{17}$

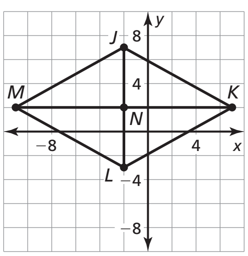
Slope of $\overline{JM} = \dfrac{-2-2}{-3-(-4)} = \dfrac{-4}{1} = -4$

Slope of $\overline{JK} = \dfrac{3-2}{0-(-4)} = \dfrac{1}{4}$

Because $-4 \bullet \dfrac{1}{4} = -1$, $\overline{JM} \perp \overline{JK}$ at $\angle J$.

So, quadrilateral *JKLM* is a square, a rhombus, and a rectangle.

56.



Diagonals:

$JL = \sqrt{(-2-(-2))^2 + (7-(-3))^2} = \sqrt{(0)^2 + (10)^2}$

$= \sqrt{100} = 10$

$KM = \sqrt{(-11-7)^2 + (2-2)^2}$

$= \sqrt{(-18)^2 + 0^2}$

$= \sqrt{324}$

$= 18$

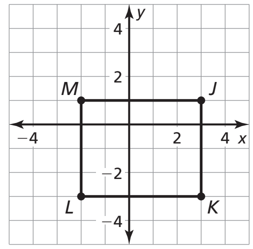
Slope of $\overline{JL} = \dfrac{7-(-3)}{-2-(-2)} = \dfrac{10}{0} =$ undefined

Slope of $\overline{MK} = \dfrac{2-2}{-11-7} = \dfrac{0}{-18} = 0$

$\overline{JL} \perp \overline{MK}$

The diagonals are perpendicular and not congruent, so *JKLM* is a rhombus.

57.



Sides:

$ML = \sqrt{(-2-(-2))^2 + (1-(-3))^2} = \sqrt{(0)^2 + (4)^2}$

$= \sqrt{16} = 4$

$LK = \sqrt{(3-(-2))^2 + (-3-(-3))^2} = \sqrt{(5)^2 + (0)^2}$

$= \sqrt{25} = 5$

$KJ = \sqrt{(3-3)^2 + (1-(-3))^2} = \sqrt{(0)^2 + (4)^2} = \sqrt{16} = 4$

$JM = \sqrt{(3-(-2))^2 + (1-1)^2} = \sqrt{(5)^2 + (0)^2} = \sqrt{25} = 5$

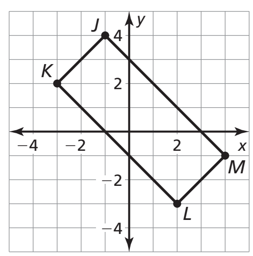
Slope of $\overline{ML} = \dfrac{1-(-3)}{-2-(-2)} = \dfrac{4}{0} =$ undefined

Slope of $\overline{JM} = \dfrac{1-1}{3-(-2)} = \dfrac{0}{5} = 0$

$\overline{ML} \perp \overline{JM}$

The sides are perpendicular and not congruent. So, *JKLM* is a rectangle.

58.



Diagonals:

$MK = \sqrt{(4-(-3))^2 + (-1-2)^2} = \sqrt{(7)^2 + (-3)^2}$

$= \sqrt{49+9} = \sqrt{58}$

$JL = \sqrt{(-1-2)^2 + (4-(-3))^2} = \sqrt{(-3)^2 + (7)^2}$

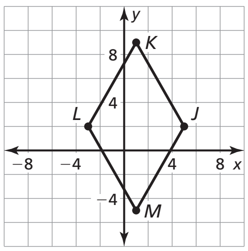
$= \sqrt{9+49} = \sqrt{58}$

Slope of $\overline{MK} = \dfrac{-1-2}{4-(-3)} = -\dfrac{3}{7}$

Slope of $\overline{JL} = \dfrac{4-(-3)}{-1-2} = -\dfrac{7}{3}$

The diagonals are congruent and not perpendicular, so *JKLM* is a rectangle.

Chapter 7

59. 

Diagonals:

$KM = \sqrt{(1 - 1)^2 + (9 - (-5))^2} = \sqrt{(0)^2 + (14)^2}$

$= \sqrt{196} = 14$

$LJ = \sqrt{(-3 - 5)^2 + (2 - 2)^2} = \sqrt{(-8)^2 + (0)^2}$

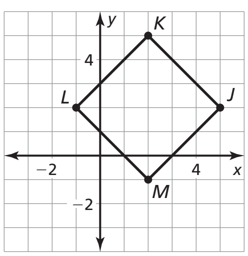
$= \sqrt{64} = 8$

Slope of $\overline{KM} = \dfrac{9 - (-5)}{1 - 1} = \dfrac{14}{0} =$ undefined

Slope of $\overline{LJ} = \dfrac{2 - 2}{-3 - 5} = \dfrac{0}{-8} = 0$

$\overline{KM} \perp \overline{LJ}$

The diagonals are perpendicular and not congruent, so *JKLM* is a rhombus.

60. 

Diagonals:

$LJ = \sqrt{(-1 - 5)^2 + (2 - 2)^2} = \sqrt{(-6)^2 + (0)^2}$

$= \sqrt{36} = 6$

$KM = \sqrt{(2 - 2)^2 + (5 - (-1))^2} = \sqrt{(0)^2 + (6)^2}$

$= \sqrt{36} = 6$

Slope of $\overline{LJ} = \dfrac{2 - 2}{-1 - 5} = \dfrac{0}{-6} = 0$

Slope of $\overline{KM} = \dfrac{5 - (-1)}{2 - 2} = \dfrac{6}{0} =$ undefined

$\overline{LJ} \perp \overline{KM}$

The diagonals are perpendicular and congruent, so *JKLM* is a rectangle, rhombus, and square.

61. *ABCD* is a rhombus, because the sides are congruent.

$104° + x° = 180°$

$x = 76$

$y + 8 = 3y$

$8 = 2y$

$4 = y$

62. *PQRS* is a square because all four angles are 90° and the diagonals are perpendicular.

$5x° = (3x + 18)°$

$2x = 18$

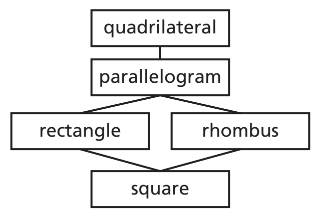
$x = 9$

$2y = 10$

$y = 5$

63. **a.** *HBDF* is a rhombus, because $\overline{BD} \cong \overline{DF} \cong \overline{BH} \cong \overline{HF}$. *ACEG* is a rectangle because $\angle HAB$, $\angle BCD$, $\angle DEF$, and $\angle FGH$ are right angles.

b. $AE = GC$ and $AJ = JE = CJ = JG$, because the diagonals of a rectangle are congruent and bisect each other.

64. 

All of the shapes have 4 sides. So, quadrilateral is at the top of the diagram. Because the rest all have two pairs of parallel sides, they are all parallelograms. Then, parallelograms with four right angles make one category, while those with four congruent sides make another, and if a parallelograms is both a rhombus and a rectangle, then it is a square.

65. A square is always a rhombus. By the Square Corollary (Cor. 7.4), a square is a rhombus.

66. A rectangle is sometimes a square. A rectangle with four congruent sides is a square.

67. A rectangle always has congruent diagonals. The diagonals of a rectangle are congruent by the Rectangle Diagonals Theorem (Thm. 7.13).

68. The diagonals of a square always bisect its angles. A square is a rhombus.

69. A rhombus sometimes has four congruent angles. Some rhombuses are squares.

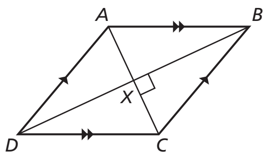
70. A rectangle sometimes has perpendicular diagonals. Some rectangles are rhombuses.

71. Measure the diagonals to see if they are congruent. They should be $\sqrt{12.5} \approx 3.54$ meters in length.

Chapter 7

72. Given $ABCD$ is a parallelogram, $\overline{AC} \perp \overline{BD}$.

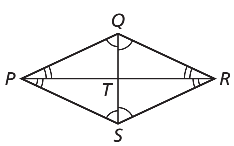
Prove $ABCD$ is a rhombus.



Because $ABCD$ is a parallelogram, its diagonals bisect each other by the Parallelogram Diagonals Theorem (Thm. 7.6). So, $\overline{BX} \cong \overline{DX}$ by definition of segment bisector. Because $\overline{AC} \perp \overline{BD}$, $\angle DXC \cong \angle BXC$. By the Reflexive Property of Congruence (Thm. 2.1), $\overline{XC} \cong \overline{XC}$. So $\triangle BXC \cong \triangle DXC$ by the SAS Congruence Theorem (Thm. 5.5). So, $\overline{BC} \cong \overline{DC}$ because corresponding parts of congruent triangles are congruent. Also, $\overline{AD} \cong \overline{BC}$ and $\overline{DC} \cong \overline{AB}$ because opposite sides of a parallelogram are congruent. So, by the Transitive Property of Congruence (Thm. 2.1), $\overline{AB} \cong \overline{BC} \cong \overline{DC} \cong \overline{AD}$, which means that by the Rhombus Corollary (Cor. 7.2), $ABCD$ is a rhombus.

73. Given $PQRS$ is a parallelogram, $\overline{PR}$ bisects $\angle SPQ$ and $\angle QRS$, and $\overline{SQ}$ bisects $\angle PSR$ and $\angle RQP$.

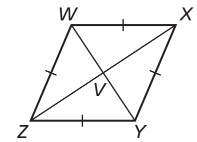
Prove $PQRS$ is a rhombus.



STATEMENTS	REASONS
1. $PQRS$ is a parallelogram. $\overline{PR}$ bisects $\angle SPQ$ and $\angle QRS$. $\overline{SQ}$ bisects $\angle PSR$ and $\angle RQP$.	**1.** Given
2. $\angle SRT \cong \angle QRT$, $\angle RQT \cong \angle RST$	**2.** Definition of angle bisector
3. $\overline{TR} \cong \overline{TR}$	**3.** Reflexive Property of Congruence (Thm. 2.1)
4. $\triangle QRT \cong \triangle SRT$	**4.** AAS Congruence Theorem (Thm. 5.11)
5. $\overline{QR} \cong \overline{SR}$	**5.** Corresponding parts of congruent triangles are congruent.
6. $\overline{QR} \cong \overline{PS}$, $\overline{PQ} \cong \overline{SR}$	**6.** Parallelogram Opposite Sides Theorem (Thm. 7.3)
7. $\overline{PS} \cong \overline{QR} \cong \overline{SR} \cong \overline{PQ}$	**7.** Transitive Property of Congruence (Thm. 2.1)
8. $PQRS$ is a rhombus.	**8.** Definition of rhombus

74. Given $WXYZ$ is a rhombus.

Prove $\overline{WY}$ bisects $\angle ZWX$ and $\angle XYZ$. $\overline{ZX}$ bisects $\angle WZY$ and $\angle YXW$.



STATEMENTS	REASONS
1. $WXYZ$ is a rhombus.	**1.** Given
2. $\overline{WX} \cong \overline{XY} \cong \overline{YZ} \cong \overline{WZ}$	**2.** Definition of a rhombus
3. $\overline{XV} \cong \overline{XV}$, $\overline{YV} \cong \overline{YV}$, $\overline{ZV} \cong \overline{ZV}$, and $\overline{WV} \cong \overline{WV}$	**3.** Reflexive Property of Congruence (Thm. 2.1)
4. $WXYZ$ is a parallelogram.	**4.** Definition of a rhombus
5. $\overline{XZ}$ bisects $\overline{WY}$, $\overline{WY}$ bisects $\overline{XZ}$.	**5.** Parallelogram Diagonals Theorem (Thm. 7.6)
6. $\overline{WV} \cong \overline{YV}$, $\overline{XV} \cong \overline{ZV}$	**6.** Definition of segment bisector
7. $\triangle WXV \cong \triangle YXV \cong \triangle YZV \cong \triangle WZV$	**7.** SSS Congruence Theorem (Thm. 5.8)
8. $\angle WXV \cong \angle YXV$, $\angle XYV \cong \angle ZYV$, $\angle YZV \cong \angle WZV$, $\angle ZWV \cong \angle XWV$	**8.** Corresponding parts of congruent triangles are congruent.
9. $\overline{WY}$ bisects $\angle ZWX$ and $\angle XYZ$, $\overline{ZX}$ bisects $\angle WZY$ and $\angle YXW$.	**9.** Definition of angle bisector

75. A diagonal will never divide a square into an equilateral triangle because then the diagonals of a square always create two right triangles.

76. The diagonal of a rhombus can divide the rhombus into two equilateral triangles. If the angles of a rhombus are 60°, 120°, 60°, and 120°, then the diagonal that bisects the opposite 120° angles will divide the rhombus into two equilateral triangles.

77. A square can be called a regular quadrilateral because it has four congruent sides and four congruent angles.

78. *Sample answer:* You need to know whether the figure is a parallelogram.

79. All rhombuses are not similar because corresponding angles from two rhombuses might not be congruent.

All squares are similar because corresponding angles of two squares are congruent.

Chapter 7

80. Because the line connecting a point with its preimage in a reflection is always perpendicular to the line of reflection, when a diagonal connecting two vertices is perpendicular to the other diagonal, both can be a line of symmetry.

81. Conditional statement: If a quadrilateral is a rhombus, then it has four congruent sides.

Converse: If a quadrilateral has four congruent sides, then it is a rhombus.

The conditional statement is true by the definition of rhombus.

The converse is true because if a quadrilateral has four congruent sides, then both pairs of opposite sides are congruent. So, by the Parallelogram Opposite Sides Converse (Thm. 7.7), it is a parallelogram with four congruent sides, which is the definition of a rhombus.

82. Conditional statement: If a quadrilateral is a rectangle, then it has four right angles.

Converse: If a quadrilateral has four right angles, then it is a rectangle.

The conditional statement is true by definition of rectangle. The converse is true because if a quadrilateral has four right angles, then both pairs of opposite angles are congruent. So, by the Parallelogram Opposite Angles Converse (Thm. 7.8), it is a parallelogram with four right angles, which is the definition of a rectangle.

83. Condition statement: If a quadrilateral is a square, then it is a rhombus and a rectangle.

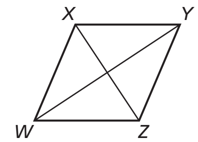
Converse: If a quadrilateral is a rhombus and a rectangle, then it is a square.

If a quadrilateral is a square, then by definition of a square, it has four congruent sides, which makes it a rhombus by the Rhombus Corollary (Cor. 7.2), and it has four right angles, which makes it a rectangle by the Rectangle Corollary (Cor. 7.3). If a quadrilateral is a rhombus and a rectangle, then by the Rhombus Corollary (Cor. 7.2), it has four congruent sides, and by the Rectangle Corollary (Cor. 7.3), it has four right angles. So, by the definition, it is a square.

84. no; If a rhombus is a square, then it is also a rectangle.

85. Given $\triangle XYZ \cong \triangle XWZ$, $\angle XYW \cong \angle ZWY$

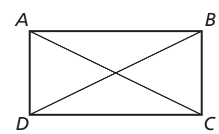
Prove $XYZW$ is a rhombus.



STATEMENTS	REASONS
1. $\triangle XYZ \cong \triangle XWZ$, $\angle XYW \cong \angle ZWY$	**1.** Given
2. $\angle YXZ \cong \angle WXZ$, $\angle YZX \cong \angle WZX$, $\overline{XY} \cong \overline{XW}$, $\overline{YZ} \cong \overline{WZ}$	**2.** Corresponding parts of congruent triangles are congruent.
3. $\overline{XZ}$ bisects $\angle WXY$ and $\angle WZY$.	**3.** Definition of angle bisector
4. $\angle XWY \cong \angle XYW$, $\angle WYZ \cong \angle ZWY$	**4.** Base Angles Theorem (Thm. 5.6)
5. $\angle XYW \cong \angle WYZ$, $\angle XWY \cong \angle ZWY$	**5.** Transitive Property of Congruence (Thm. 2.2)
6. $\overline{WY}$ bisects $\angle XWZ$ and $\angle XYZ$.	**6.** Definition of angle bisector
7. $XYZW$ is a rhombus.	**7.** Rhombus Opposite Angles Theorem (Thm. 7.12)

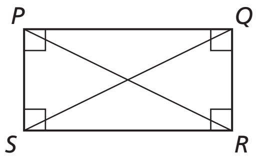
86. Given $\overline{BC} \cong \overline{AD}$, $\overline{BC} \perp \overline{DC}$, $\overline{AD} \perp \overline{DC}$

Prove $ABCD$ is a rectangle.



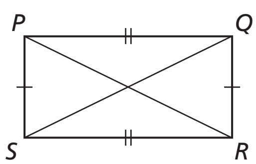
STATEMENTS	REASONS
1. $\overline{BC} \cong \overline{AD}$, $\overline{BC} \perp \overline{DC}$, $\overline{AD} \perp \overline{DC}$	**1.** Given
2. $\overline{BC} \parallel \overline{AD}$	**2.** Lines Perpendicular to a Transversal Theorem (Thm. 3.12)
3. $ABCD$ is a parallelogram.	**3.** Opposite Sides Parallel and Congruent Theorem (Thm. 7.9)
4. $m\angle DAB = m\angle BCD$, $m\angle ABC = m\angle ADC$	**4.** Parallelogram Opposite Angles Theorem (Thm. 7.4)
5. $m\angle BCD = m\angle ADC = 90°$	**5.** Definition of perpendicular lines
6. $m\angle DAB = m\angle BCD = m\angle ABC = m\angle ADC = 90°$	**6.** Transitive Property of Equality
7. $ABCD$ has four right angles.	**7.** Definition of a right angle
8. $ABCD$ is a rectangle.	**8.** Definition of a rectangle

87. Given *PQRS* is a rectangle.
Prove $\overline{PR} \cong \overline{SQ}$



STATEMENTS	REASONS
1. *PQRS* is a rectangle.	1. Given
2. *PQRS* is a parallelogram.	2. Definition of a rectangle
3. $\overline{PS} \cong \overline{QR}$	3. Parallelogram Opposite Sides Theorem (Thm. 7.3)
4. $\angle PQR$ and $\angle QPS$ are right angles.	4. Definition of a rectangle
5. $\angle PQR \cong \angle QPS$	5. Right Angle Congruence Theorem (Thm. 2.3)
6. $\overline{PQ} \cong \overline{PQ}$	6. Reflexive Property of Congruence (Thm. 2.1)
7. $\triangle PQR \cong \triangle QPS$	7. SAS Congruence Theorem (Thm. 5.5)
8. $\overline{PR} \cong \overline{SQ}$	8. Corresponding parts of congruent triangles are congruent.

88. Given *PQRS* is a parallelogram.
$\overline{PR} \cong \overline{SQ}$
Prove *PQRS* is a rectangle.



STATEMENTS	REASONS
1. *PQRS* is a parallelogram, $\overline{PR} \cong \overline{SQ}$	1. Given
2. $\overline{PS} \cong \overline{QR}$	2. Parallelogram Opposite Sides Theorem (Thm. 7.3)
3. $\overline{PQ} \cong \overline{PQ}$	3. Reflexive Property of Congruence (Thm. 2.1)
4. $\triangle PQR \cong \triangle QPS$	4. SSS Congruence Theorem (Thm. 5.8)
5. $\angle SPQ \cong \angle RQP$	5. Corresponding parts of congruent triangles are congruent.
6. $m\angle SPQ = m\angle RQP$	6. Definition of congruent angles
7. $m\angle SPQ + m\angle RQP = 180°$	7. Parallelogram Consecutive Angles Theorem (Thm. 7.5)
8. $2m\angle SPQ = 180°$ and $2m\angle RQP = 180°$	8. Substitution Property of Equality
9. $m\angle SPQ = 90°$ and $m\angle RQP = 90°$	9. Division Property of Equality
10. $m\angle RSP = 90°$ and $m\angle QRS = 90°$	10. Parallelogram Opposite Angles Theorem (Thm. 7.4)
11. $\angle SPQ$, $\angle RQP$, $\angle RSP$, and $\angle QRS$ and right angles.	11. Definition of a right angle
12. *PQRS* is a rectangle.	12. Definition of a rectangle

Maintaining Mathematical Proficiency

89. $AE = EC$

$x = 10$

$ED = \frac{1}{2}CB$

$y = \frac{1}{2} \cdot 16 = 8$

90. $DE = \frac{1}{2}BC$

$7 = \frac{1}{2}x$

$14 = x$

$BD = DA$

$y = 6$

91. $AD = DB$

$x = 9$

$DE = \frac{1}{2}AC$

$13 = \frac{1}{2} \cdot y$

$26 = y$

7.5 Explorations (p. 397)

1. a. Check students' work.
b. yes; $AD = BC$
c. If the base angles of a trapezoid are congruent, the trapezoid is isosceles.

2. a. Check students' work.
b. $\angle B \cong \angle C$
c. If a quadrilateral is a kite, it has exactly one pair of congruent opposite angles.

3. A trapezoid is a quadrilateral with only one pair of parallel sides. A trapezoid that has congruent base angles is isosceles. A kite is a quadrilateral with exactly one pair of congruent opposite angles.

4. yes; When the base angles are congruent, the opposite sites are also congruent.

5. no; In a kite, only one pair of opposite angles is congruent.

Chapter 7

7.5 Monitoring Progress (pp. 398–402)

1. Slope of $\overline{AB} = \dfrac{9-6}{4-(-5)} = \dfrac{3}{9} = \dfrac{1}{3}$

Slope of $\overline{CD} = \dfrac{4-2}{4-(-2)} = \dfrac{2}{6} = \dfrac{1}{3}$

Slope of $\overline{AD} = \dfrac{2-6}{-2-(-5)} = \dfrac{-4}{3} = -\dfrac{4}{3}$

Slope of $\overline{CB} = \dfrac{4-9}{4-4} = \dfrac{5}{0} =$ undefined

$AD = \sqrt{(-2-(-5))^2 + (2-6)^2} = \sqrt{(-2+5)^2 + (-4)^2}$
$= \sqrt{3^2 + 16} = \sqrt{9+16} = \sqrt{25} = 5$

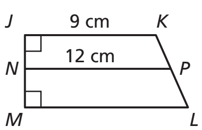
$CB = \sqrt{(4-4)^2 + (4-9)^2} = \sqrt{(0)^2 + (-5)^2} = \sqrt{25} = 5$

The slope of $\overline{AB}$ equals the slope of $\overline{DC}$, and the slope of $\overline{AD}$ is not equal to the slope of $\overline{BC}$. Because *ABCD* has exactly one pair of parallel sides, it is a trapezoid. Also, $AD = BC$. So, *ABCD* is isosceles.

2. If $EG = FH$, then by the Isosceles Trapezoid Diagonals Theorem (Thm. 7.16) the trapezoid is an isosceles trapezoid.

3. Because $m\angle FGH = 110°$, $m\angle GFE = 70°$ by the Consecutive Interior Angles Theorem (Thm. 3.4). So, $\angle HEF \cong \angle GFE$ and the trapezoid is isosceles.

4.



$NP = \frac{1}{2}(JK + ML)$

$12 = \frac{1}{2}(9 + ML)$

$24 = 9 + ML$

$15 = ML$

So, $ML = 15$ centimeters.

5. *Sample answer:* Find the coordinates of *Y* and *Z* and calculate the distance between the points.

Midpoint $Y = \left(\dfrac{0+8}{2}, \dfrac{6+10}{2}\right) = \left(\dfrac{8}{2}, \dfrac{16}{2}\right) = (4, 8)$

Midpoint $Z = \left(\dfrac{2+12}{2}, \dfrac{2+2}{2}\right) = \left(\dfrac{14}{2}, \dfrac{4}{2}\right) = (7, 2)$

$YZ = \sqrt{(4-7)^2 + (8-2)^2} = \sqrt{(-3)^2 + 6^2} = \sqrt{9+36}$
$= \sqrt{9 \cdot 5} = 3\sqrt{5}$ units

6. $3x° + 75° + 90° + 120° = 360°$

$3x + 285 = 360$

$3x = 75$

$x = 25$

The angles are $3 \cdot 25 = 75°$, $75°$, $90°$, and $120°$.

The value of *x* is 25. The measure of the congruent angles is 75°.

7. Quadrilateral *DEFG* could be an isosceles trapezoid, parallelogram, rectangle, square, or rhombus.

8. Quadrilateral *RSTU* is a kite, because it has two pairs of consecutive congruent sides and the opposite sides are not congruent.

9. Quadrilateral *YVWX* is a trapezoid because two sides are parallel and the diagonals do not bisect each other.

10. Quadrilateral *CDEF* is a quadrilateral because the markings are not sufficient to give it a more specific name.

7.5 Exercises (pp. 403–406)

Vocabulary and Core Concept Check

1. A trapezoid has exactly one pair of parallel sides and a kite has two pairs of consecutive congruent sides.

2. The question that is different is "Is there enough information to prove that $\overline{AB} \cong \overline{DC}$?" There is not enough information to prove $AB \cong CD$, but there is enough information to prove the other three.

Monitoring Progress and Modeling with Mathematics

3. Slope of $\overline{YZ} = \dfrac{9-3}{-3-(-3)} = \dfrac{6}{-3+3} = \dfrac{6}{0} =$ undefined

Slope of $\overline{XW} = \dfrac{8-4}{1-1} = \dfrac{4}{0} =$ undefined

Slope of $\overline{YX} = \dfrac{9-8}{-3-1} = -\dfrac{1}{4}$

Slope of $\overline{ZW} = \dfrac{3-4}{-3-1} = \dfrac{-1}{-4} = \dfrac{1}{4}$

$YX = \sqrt{(-3-1)^2 + (9-8)^2} = \sqrt{(-4)^2 + (1)^2}$
$= \sqrt{16+1} = \sqrt{17}$

$ZW = \sqrt{(-3-1)^2 + (3-4)^2} = \sqrt{(-4)^2 + (-1)^2}$
$= \sqrt{16+1} = \sqrt{17}$

The slope of $\overline{YZ}$ equals the slope of $\overline{XW}$, and the slope of $\overline{YX}$ is not equal to the slope of $\overline{ZW}$. Because *WXYZ* has exactly one pair of parallel sides, it is a trapezoid. Also, $YX = ZW$. So, *WXYZ* is isosceles.

Chapter 7

4. Slope of $\overline{DG} = \frac{3-0}{-3-(-3)} = \frac{3}{-3+3} = \frac{3}{0} =$ undefined

Slope of $\overline{EF} = \frac{1-(-4)}{-1-1} = \frac{1+4}{-2} = -\frac{5}{2}$

Slope of $\overline{DE} = \frac{3-1}{-3-(-1)} = \frac{2}{-3+1} = \frac{2}{-2} = -1$

Slope of $\overline{GF} = \frac{0-(-4)}{-3-1} = \frac{4}{-4} = -1$

$DG = \sqrt{(-3-(-3))^2 + (3-0)^2} = \sqrt{(-3+3)^2 + (3)^2}$
$= \sqrt{9} = 3$

$EF = \sqrt{(-1-1)^2 + (1-(-4))^2} = \sqrt{(-2)^2 + (1+4)^2}$
$= \sqrt{4+25} = \sqrt{29}$

The slope of $\overline{DE}$ equals the slope of $\overline{GF}$, and the slope of $\overline{DG}$ is not equal to the slope of $\overline{EF}$. Because *DEFG* has exactly one pair of parallel sides, it is a trapezoid. Also, $DG \neq EF$. So, *DEFG* is not isosceles.

5. Slope of $\overline{NP} = \frac{4-4}{5-0} = \frac{0}{5} = 0$

Slope of $\overline{MQ} = \frac{0-0}{8-(-2)} = \frac{0}{10} = 0$

Slope of $\overline{NM} = \frac{4-0}{0-(-2)} = \frac{4}{2} = 2$

Slope of $\overline{PQ} = \frac{4-0}{5-8} = -\frac{4}{3}$

$NM = \sqrt{(0-(-2))^2 + (4-0)^2} = \sqrt{(2)^2 + (4)^2} = \sqrt{4+16}$
$= \sqrt{20} = 2\sqrt{5}$

$PQ = \sqrt{(5-8)^2 + (4-0)^2} = \sqrt{(-3)^2 + (4)^2} = \sqrt{9+16}$
$= \sqrt{25} = 5$

The slope of $\overline{NP}$ is equal to the slope of $\overline{MQ}$, and the slope of $\overline{NM}$ is not equal to the slope of $\overline{PQ}$. Because *NMPQ* has exactly one pair of parallel sides it is a trapezoid. Also, $NM \neq PQ$. So, *NMPQ* is not isosceles.

6. Slope of $\overline{HL} = \frac{9-9}{8-1} = \frac{0}{7} = 0$

Slope of $\overline{JK} = \frac{2-2}{5-4} = \frac{0}{1} = 0$

Slope of $\overline{HJ} = \frac{9-2}{1-4} = -\frac{7}{3}$

Slope of $\overline{LK} = \frac{9-2}{8-5} = \frac{7}{3}$

$HJ = \sqrt{(1-4)^2 + (9-2)^2} = \sqrt{(-3)^2 + (7)^2}$
$= \sqrt{9+49} = \sqrt{58}$

$LK = \sqrt{(8-5)^2 + (9-2)^2} = \sqrt{(3)^2 + (7)^2}$
$= \sqrt{9+49} = \sqrt{58}$

The slope of $\overline{HL}$ equals the slope of $\overline{JK}$, and the slope of $\overline{HJ}$ is not equal to the slope of $\overline{LK}$. Because *HJKL* has exactly one pair of parallel sides, it is a trapezoid. Also, $HJ = LK$. So, *HJKL* is isosceles.

7. $m\angle K + m\angle L = 180°$
$118° + m\angle L = 180°$
$m\angle L = 180° - 118° = 62°$

Quadrilateral *JKLM* is isosceles, so $m\angle J = m\angle K = 118°$ and $m\angle M = m\angle K = 62°$.

8. $m\angle R + m\angle S = 180°$
$m\angle R + 82° = 180°$
$m\angle R = 180° - 82° = 78°$

Quadrilateral *JKLM* is isosceles, so $m\angle S = m\angle R = 82°$ and $m\angle Q = m\angle T = 98°$.

9. $MN = \frac{1}{2}(10 + 18)$

$MN = \frac{1}{2} \cdot 28$

$MN = 14$

The length of midsegment $\overline{MN}$ is 14.

10. $MN = \frac{1}{2}(76 + 57)$

$MN = \frac{1}{2} \cdot 133$

$MN = 66.5$

The length of midsegment $\overline{MN}$ is 66.5.

11. $MN = \frac{1}{2}(AB + DC)$

$7 = \frac{1}{2}(AB + 10)$

$14 = AB + 10$

$4 = AB$

The length of midsegment $\overline{AB}$ is 4.

12. $MN = \frac{1}{2}(AB + DC)$

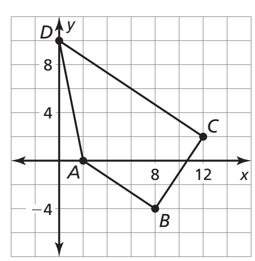
$18.7 = \frac{1}{2}(AB + 11.5)$

$37.4 = AB + 11.5$

$25.9 = AB$

The length of midsegment $\overline{AB}$ is 25.9.

13.



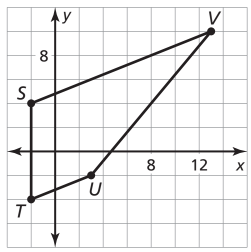
$AB = \sqrt{(2-8)^2 + (0-(-4))^2} = \sqrt{(-6)^2 + (4)^2}$
$= \sqrt{36+16} = \sqrt{52} = 2\sqrt{13}$

$CD = \sqrt{(12-0)^2 + (2-10)^2} = \sqrt{(12)^2 + (-8)^2}$
$= \sqrt{144+64} = \sqrt{208} = \sqrt{16 \cdot 13} = 4\sqrt{13}$

Midsegment $= \frac{1}{2}\left(4\sqrt{13} + 2\sqrt{13}\right) = \frac{1}{2}\left(6\sqrt{13}\right) = 3\sqrt{13}$

Chapter 7

14.



$TU = \sqrt{(3 - (-2))^2 + (-2 - (-4))^2}$

$= \sqrt{(3 + 2)^2 + (-2 + 4)^2} = \sqrt{25 + 4} = \sqrt{29}$

$SV = \sqrt{(13 - (-2))^2 + (10 - 4)^2} = \sqrt{(15)^2 + (6)^2}$

$= \sqrt{225 + 36} = \sqrt{261} = \sqrt{9 \bullet 29} = 3\sqrt{29}$

Midsegment $= \frac{1}{2}\left(\sqrt{29} + 3\sqrt{29}\right) = \frac{1}{2}\left(4\sqrt{29}\right) = 2\sqrt{29}$

15. $m\angle G + m\angle H + m\angle E + m\angle F^\circ = 360^\circ$

$x^\circ + 100^\circ + x^\circ + 40^\circ = 360^\circ$

$2x + 140 = 360$

$2x = 220$

$x = 110$

So, $m\angle G = 110^\circ$.

16. $m\angle G + m\angle H + m\angle E + m\angle F = 360^\circ$

$x^\circ + 90^\circ + x^\circ + 150^\circ = 360^\circ$

$2x + 240 = 360$

$2x = 120$

$x = 60$

So, $m\angle G = 60^\circ$.

17. $m\angle G + m\angle H + m\angle E + m\angle F = 360^\circ$

$x^\circ + 110^\circ + 60^\circ + 110^\circ = 360^\circ$

$x + 280 = 360$

$x = 80$

So, $m\angle G = 80^\circ$.

18. $m\angle G + m\angle H + m\angle E + m\angle F = 360^\circ$

$x^\circ + 90^\circ + 110^\circ + 90^\circ = 360^\circ$

$x + 290 = 360$

$x = 70$

So, $m\angle G = 70^\circ$.

19. Because $MN = \frac{1}{2}(AB + DC)$, when you solve for DC, you should get

$DC = 2(MN) - AB$.

$DC = 2(8) - 14$

$DC = 16 - 14$

$DC = 2$

20. In the kite shown, $\angle B \cong \angle D$. Find $m\angle A$ by subtracting the measures of the other three angles from 360°.

$m\angle A + m\angle B + m\angle C + m\angle D = 360^\circ$

$m\angle A + 120^\circ + 50^\circ + 120^\circ = 360^\circ$

$m\angle A = 360^\circ - 50^\circ - 2(120^\circ)$

$m\angle A = 70^\circ$

21. Quadrilateral $JKLM$ is a rectangle because it is a quadrilateral with four right angles.

22. Quadrilateral $RSPQ$ is a trapezoid, because $\overline{PS} \parallel \overline{QR}$, and $\angle QPS$ and $\angle PSR$ are not supplementary.

23. Quadrilateral $ABCD$ is a square because it has four congruent sides and four right angles.

24. Quadrilateral $XYZW$ is a kite because it has two pairs of consecutive congruent sides and opposite sides are not congruent.

25. no; Even though the diagonals are perpendicular, it does not indicate that the quadrilateral is a rhombus. It could be a kite.

26. no; A square has four right angles and the diagonals bisect each other, but this could also describe a rectangle.

27. $12.5 = \frac{1}{2}(3x + 1 + 15)$

$25 = 3x + 16$

$9 = 3x$

$3 = x$

28. $15 = \frac{1}{2}(3x + 2 + 2x - 2)$

$15 = \frac{1}{2}(5x)$

$30 = 5x$

$6 = x$

29. $MQ = \frac{1}{2}(NP + LR)$

$MQ = \frac{1}{2}(8 + 20)$

$MQ = \frac{1}{2}(28) = 14$

$LR = \frac{1}{2}(MQ + KS)$

$20 = \frac{1}{2}(14 + KS)$

$40 = 14 + KS$

$26 = KS$

The diameter of the bottom layer of the cake is 26 inches.

30. The length of the stick from X to W is 18 inches, and the length of the stick from W to Z is 29 inches. A kite is a quadrilateral that has two pairs of consecutive congruent sides, but opposite sides are not congruent.

Chapter 7

31. For $ABCD$ to be an isosceles trapezoid, $m\angle D = 70°$ and $m\angle C = 110°$. If a trapezoid has a pair of congruent base angles, then it is an isosceles trapezoid.

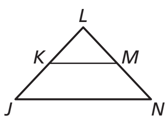
32. *Sample answer:* For $ABCD$ to be an kite, $\overline{BC} \cong \overline{DC}$. Then $\triangle ABC \cong \triangle ADC$ and $ABCD$ has two pairs of consecutive congruent sides.

33. *Sample answer:* For $ABCD$ to be a parallelogram, $\overline{BE} \cong \overline{DE}$. Then the diagonals bisect each other.

34. *Sample answer:* For $ABCD$ to be a square, $\overline{AB} \cong \overline{BC}$. A rectangle with a pair of congruent adjacent sides is a square.

35. Given $\overline{JL} \cong \overline{LN}$, $\overline{KM}$ is a midsegment of $\triangle JLN$.

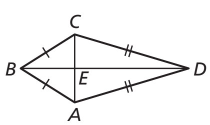
Prove Quadrilateral $JKMN$ is an isosceles trapezoid.



STATEMENTS	REASONS
1. $\overline{JL} \cong \overline{LN}$, $\overline{KM}$ is a midsegment of $\triangle JLN$.	**1.** Given
2. $\overline{KM} \parallel \overline{JN}$	**2.** Triangle Midsegment Theorem (Thm. 6.8)
3. $KMNJ$ is a trapezoid.	**3.** Definition of trapezoid
4. $\angle LJN \cong \angle LNJ$	**4.** Base Angles theorem (Thm. 5.6)
5. $KMNJ$ is an isosceles triangle.	**5.** Isosceles Trapezoid Base Angles Converse (Thm. 7.15)

36. Given $ABCD$ is a kite. $\overline{AB} \cong \overline{CB}$, $\overline{AD} \cong \overline{CD}$

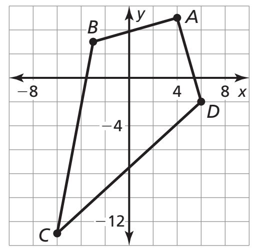
Prove $\overline{CE} \cong \overline{AE}$



STATEMENTS	REASONS
1. $ABCD$ is a kite, $\overline{AD} \cong \overline{CD}$, $\overline{AB} \cong \overline{CB}$	**1.** Given
2. $\overline{BD} \cong \overline{BD}$, $\overline{ED} \cong \overline{ED}$	**2.** Reflexive Property of Congruence (Thm. 2.1)
3. $\triangle ABD \cong \triangle CBD$	**3.** SSS Congruence Theorem (Thm. 5.8)
4. $\angle CDE \cong \angle ADE$	**4.** Corresponding parts of congruent triangles are congruent.
5. $\triangle CED \cong \triangle AED$	**5.** SAS Congruence Theorem (Thm. 5.5)
6. $\overline{CE} \cong \overline{AE}$	**6.** Corresponding parts of congruent triangles are congruent.

37. $RSTU$ is a kite for which S is any point on $\overleftrightarrow{UV}$ such that $UV \neq SV$.

38.



Slope of $\overline{AD} = \frac{-2-5}{6-4} = -\frac{7}{2}$

Slope of $\overline{AB} = \frac{5-3}{4-(-3)} = \frac{2}{7}$

Slope of $\overline{BC} = \frac{3-(-13)}{-3-(-6)} = \frac{16}{-3+6} = \frac{16}{3}$

Slope of $\overline{CD} = \frac{-2-(-13)}{6-(-6)} = \frac{-2+13}{6+6} = \frac{11}{12}$

There are no parallel segments.

$AD = \sqrt{(6-4)^2 + (-2-5)^2} = \sqrt{(2)^2 + (-7)^2}$
$= \sqrt{4+49} = \sqrt{53}$

$AB = \sqrt{(4-(-3))^2 + (5-3)^2} = \sqrt{(4+3)^2 + (2)^2}$
$= \sqrt{7^2 + 4} = \sqrt{49+4} = \sqrt{53}$

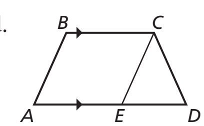
$BC = \sqrt{(-3-(-6))^2 + (3-(-13))^2}$
$= \sqrt{(-3+6)^2 + (16)^2} = \sqrt{9+256} = \sqrt{265}$

$CD = \sqrt{(6-(-6))^2 + (-2-(-13))^2}$
$= \sqrt{(6+6)^2 + (-2+13)^2} = \sqrt{(12)^2 + (11)^2}$
$= \sqrt{144+121} = \sqrt{265}$

Consecutive sides are equal. So, by the definition of a kite, quadrilateral $ABCD$ is a kite.

39. Given $ABCD$ is an isosceles trapezoid. $\overline{BC} \parallel \overline{AD}$

Prove $\angle A \cong \angle D$, $\angle B \cong \angle BCD$

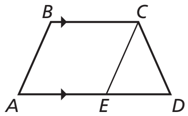


Given isosceles trapezoid $ABCD$ with $\overline{BC} \parallel \overline{AD}$, construct $\overline{CE}$ parallel to $\overline{BA}$. Then, $ABCE$ is a parallelogram by definition, so $\overline{AB} \cong \overline{EC}$. Because $\overline{AB} \cong \overline{CD}$ by the definition of an isosceles trapezoid, $\overline{CE} \cong \overline{CD}$ by the Transitive Property of Congruence (Thm. 2.1). So, $\angle CED \cong \angle D$ by the Base Angles Theorem (Thm. 5.6) and $\angle A \cong \angle CED$ by the Corresponding Angles Theorem (Thm. 3.1). So, $\angle A \cong \angle D$ by the Transitive Property of Congruence (Thm. 2.2). Next, by the Consecutive Interior Angles Theorem (Thm. 3.4), $\angle B$ and $\angle A$ are supplementary and so are $\angle BCD$ and $\angle D$. So, $\angle B \cong \angle BCD$ by the Congruent Supplements Theorem (Thm. 2.4).

Chapter 7

40. Given $ABCD$ is a trapezoid.
$\angle A \cong \angle D$, $\overline{BC} \parallel \overline{AD}$

Prove $ABCD$ is an isosceles trapezoid.



Given trapezoid $ABCD$ with $\angle A \cong \angle D$ and $\overline{BC} \parallel \overline{AD}$, construct $\overline{CE}$ parallel to $\overline{BA}$. Then, $ABCE$ is a parallelogram by definition, so $\overline{AB} \cong \overline{EC}$. $\angle A \cong \angle CED$ by the Corresponding Angles Theorem (Thm. 3.1), so $\angle CED \cong \angle D$ by the Transitive Property of Congruence (Thm. 2.2). Then by the Converse of the Base Angles Theorem (Thm. 5.7), $\overline{EC} \cong \overline{DC}$. So, $\overline{AB} \cong \overline{DC}$ by the Transitive Property of Congruence (Thm. 2.1), and trapezoid $ABCD$ is isosceles.

41. no; It could be a square.

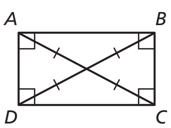
42. $y = \frac{1}{2}(b_1 + b_2)$

$y = \frac{1}{2}(2x + 7 + 2x - 5)$

$y = \frac{1}{2}(4x + 2)$

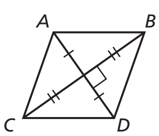
$y = 2x + 1$

43. a.



rectangle; The diagonals are congruent, but not perpendicular.

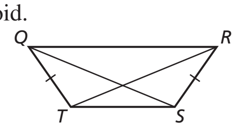
b.



rhombus; The diagonals are perpendicular, but not congruent.

44. Given $QRST$ is an isosceles trapezoid.

Prove $\angle TQS \cong \angle SRT$



STATEMENTS	REASONS
1. $QRST$ is an isosceles trapezoid.	1. Given
2. $\angle QTS \cong \angle RST$	2. Isosceles Trapezoid Base Angles Theorem (Thm. 7.14)
3. $\overline{QT} \cong \overline{RS}$	3. Definition of an isosceles trapezoid
4. $\overline{TS} \cong \overline{TS}$	4. Reflexive Property of Congruence (Thm. 2.1)
5. $\triangle QST \cong \triangle RTS$	5. SAS Congruence Theorem (Thm. 5.5)
6. $\angle TQS \cong \angle SRT$	6. Corresponding parts of congruent triangles are congruent.

45. a. yes; Because $\overline{AQ}$ is not parallel to $\overline{BP}$, $\angle ABX \cong \angle BAX$ and $\overline{AB} \parallel \overline{PQ}$.

b. Because $360° \div 12 = 30°$, $m\angle AXB = 30°$. Because $\angle ABX \cong \angle BAX$, $m\angle ABX = m\angle BAX = \frac{180° - 30°}{2} = 75°$. So, $m\angle AQP = m\angle BPQ = 180° - 75° = 105°$.

46. A; Midsegment: $MN = \frac{1}{2}(b_1 + b_2)$

$MN = \frac{1}{2}(PQ + RS)$

$MN = \frac{1}{2}(PQ + 5PQ)$

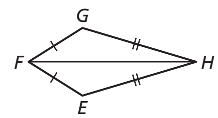
$MN = \frac{1}{2}(6PQ)$

$MN = 3PQ$

Ratio: $\frac{MN}{RS} = \frac{3PQ}{5PQ} = \frac{3}{5}$

47. Given $EFGH$ is a kite.
$\overline{EF} \cong \overline{FG}$, $\overline{EH} \cong \overline{GH}$

Prove $\angle E \cong \angle G$, $\angle F \neq \angle H$



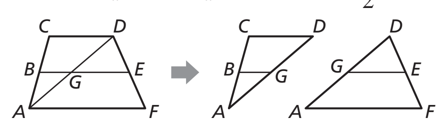
Given like $EFGH$ with $\overline{EF} \cong \overline{FG}$ and $\overline{EH} \cong \overline{GH}$, construct diagonal $\overline{FH}$, which is congruent to itself by the Reflexive Property of Congruence (Thm. 2.1). So, $\triangle FGH \cong \triangle FEH$ by the SSS Congruence Theorem (Thm. 5.8), and $\angle E \cong \angle G$ because corresponding parts of congruent triangles are congruent. Next, assume temporarily that $\angle F \cong \angle H$. Then $EFGH$ is a parallelogram by the Parallelogram Opposite Angles Converse (Thm. 7.8), and opposite sides are congruent. However, this contradicts the definition of a kite, which says that opposite sides cannnot be congruent. So, the assumption cannot be true and $\angle F$ is not congruent to $\angle H$.

48. a. $ABCD$ is a trapezoid.

b. $DEFG$ is an isosceles trapezoid.

49. a. Given $\overline{BG}$ is the midsegment of $\triangle ACD$.
$\overline{GE}$ is the midsegment of $\triangle ADF$.

Prove $\overline{BE} \parallel \overline{CD}$, $\overline{BE} \parallel \overline{AF}$, and $BE = \frac{1}{2}(CD + AF)$



By the Triangle Midsegment Theorem (Thm. 6.8), $\overline{BG} \parallel \overline{CD}$, $BG = \frac{1}{2}CD$, $\overline{GE} \parallel \overline{AF}$, and $GE = \frac{1}{2}AF$. By the Transitive Property of Parallel Lines (Thm. 3.9), $\overline{CD} \parallel \overline{BE} \parallel \overline{AF}$. Also, by the Segment Addition Postulate (Post. 1.2), $BE = BG + GE$. So, by the Substitution Property of Equality, $BE = \frac{1}{2}CD + \frac{1}{2}AF = \frac{1}{2}(CD + AF)$.

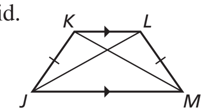
50. no; A concave kite and a convex kite can have congruent corresponding sides and a pair of congruent corresponding angles, but the kites are not congruent.

Chapter 7

51. a. Given $JKLM$ is an isosceles trapezoid.

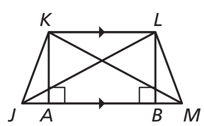
$\overline{KL} \parallel \overline{JM}, \overline{JK} \cong \overline{LM}$

Prove $\overline{JL} \cong \overline{KM}$



STATEMENTS	REASONS
1. $JKLM$ is an isosceles trapezoid, $\overline{KL} \parallel \overline{JM}$, $\overline{JK} \cong \overline{LM}$	**1.** Given
2. $\angle JKL \cong \angle MLK$	**2.** Isosceles Trapezoid Base Angles Theorem (Thm. 7.14)
3. $\overline{KL} \cong \overline{KL}$	**3.** Reflexive Property of Congruence (Thm. 2.1)
4. $\triangle JKL \cong \angle MLK$	**4.** SAS Congruence Theorem (Thm. 5.5)
5. $\overline{JL} \cong \overline{KM}$	**5.** Corresponding parts of congruent triangles are congruent.

b. If the diagonals of a trapezoid are congruent, then the trapezoid is isosceles. Let $JKLM$ be a trapezoid, $\overline{KL} \parallel \overline{JM}$ and $\overline{JL} \cong \overline{KM}$. Construct line segments through K and L perpendicular to $\overline{JM}$ as shown below.



Because $\overline{KL} \parallel \overline{JM}$, $\angle AKL$ and $\angle KLB$ are right angles, so $KLBA$ is a rectangle and $\overline{AK} \cong \overline{BL}$. Then $\triangle JLB \cong \triangle MKA$ by the HL Congruence Theorem (Thm. 5.9). So, $\angle LJB \cong \angle KMA$. $\overline{JM} \cong \overline{JM}$, by the Reflexive Property of Congruence (Thm. 2.1). So, $\triangle KJM \cong \triangle LMJ$ by the SAS Congruence Theorem (Thm. 5.5). Then $\angle KJM \cong LMJ$, and the trapezoid is isosceles by the Isosceles Trapezoid Base Angles Converse (Thm. 7.15).

52. You are given $\overline{JK} \cong \overline{LM}$, E is the midpoint of $\overline{JL}$, F is the midpoint of $\overline{KL}$, G is the midpoint of $\overline{KM}$, and H is the midpoint of $\overline{JM}$. By the definition of midsegment, $\overline{EK} \parallel \overline{JK}$ and $\overline{EF}$ is the midsegment of $\triangle JKL$, $\overline{FG} \parallel \overline{LM}$ and $\overline{FG}$ is the midsegment of $\triangle KML$, $\overline{GH} \parallel \overline{JK}$ and $\overline{GH}$ is the midsegment of $\triangle KML$, and $\overline{EH} \parallel \overline{LM}$ and $\overline{EH}$ is the midsegment of $\triangle JML$. You know that $\overline{EF} \parallel \overline{GH}$ and $\overline{FG} \parallel \overline{EH}$ by the Transitive Property of Parallel Lines (Thm. 3.9). $EFGH$ is a parallelogram by the definition of a parallelogram. By the Trapezoid Midsegment Theorem (Thm. 7.17), $EF = \frac{1}{2}JK$, $FG = \frac{1}{2}LM$, $GH = \frac{1}{2}JK$, and $EH = \frac{1}{2}LM$. You can conclude $JK = LM$ by the definition of congruent segments. Then by the Substitution Property of Equality, $FG = \frac{1}{2}JK$ and $EH = \frac{1}{2}JK$. It follows from the Transitive Property of Equality that $EF = FG = GH = EH$. Then by the definition of congruent segments, $\overline{EF} \cong \overline{FG} \cong \overline{GH} \cong \overline{EH}$. Therefore, by the definition of a rhombus, $EFGH$ is a rhombus.

Maintaining Mathematical Proficiency

53. *Sample answer:* A similarity transformation that maps the blue preimage to the green image is a translation 1 unit right followed by a dilation with a scale factor of 2.

54. *Sample answer:* A similarity transformation that maps the blue preimage to the green image is a reflection in the y-axis followed by a dilation with a scale factor of $\frac{1}{3}$.

7.4–7.5 What Did You Learn? (p. 407)

1. They are congruent base angles of congruent isosceles triangles, $\triangle DEF$ and $\triangle DGF$.

2. If one type of quadrilateral is under another in the diagram, then a quadrilateral from the lower category will always fit into the category above it.

3. Find the difference between the length of the midsegment and the length of the given base, and then either add or subtract that amount to the midsegment to find the other base.

Chapter 7 Review (pp. 408–410)

1. The sum of the measures of the interior angles of a regular 30-gon is $(30 - 2) \cdot 180° = 28 \cdot 180° = 5040°$.

The measure of each interior angle is

$$\frac{(30 - 2) \cdot 180°}{30} = \frac{28 \cdot 180°}{30} = \frac{5040°}{30} = 168°.$$

The measure of each exterior angle is $\frac{360°}{30} = 12°$.

2. The sum of the measures of the interior angles is $(6 - 2) \cdot 180° = 4 \cdot 180° = 720°$.

$$120° + 97° + 130° + 150° + 90° + x° = 720°$$
$$587 + x = 720$$
$$x = 133$$

3. The sum of the measures of the interior angles is $(7 - 2) \cdot 180° = 5 \cdot 180° = 900°$.

$$x° + 160° + 2x° + 125° + 110° + 112° + 147° = 900°$$
$$3x + 654 = 900$$
$$3x = 246$$
$$x = 82$$

4. $49° + 7x° + 83° + 33° + 6x° = 360°$

$$13x + 165 = 360$$
$$13x = 195$$
$$x = 15$$

5. $a° + 101° = 180°$

$$a = 79$$

$b° = 101°$

6. $a - 10 = 18$

$$a = 28$$

$(b + 16)° = 103°$

$$b = 87$$

Chapter 7

7. $c + 5 = 11$

$c = 6$

$d + 4 = 14$

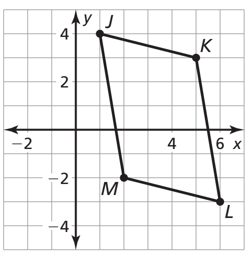
$d = 10$

8. Midpoint of $\overline{TR}$: $\left(\frac{-6+2}{2}, \frac{-3+1}{2}\right) = \left(\frac{-4}{2}, \frac{-2}{2}\right) = (-2, -1)$

Midpoint of $\overline{QS}$: $\left(\frac{-8+4}{2}, \frac{1+(-3)}{2}\right) = \left(\frac{-4}{2}, \frac{-2}{2}\right) = (-2, -1)$

The coordinates of the intersection of the diagonals are $(-2, -1)$.

9.



Slope of $\overline{KL} = \frac{3-(-3)}{5-6} = \frac{3+3}{-1} = \frac{-6}{1} = -6$

Starting at J, go down 6 units and right 1 unit. So, the coordinates of M are $(2, -2)$.

10. Parallelogram Opposite Sides Converse (Thm. 7.7)

11. Parallelogram Diagonals Converse (Thm. 7.10)

12. Parallelogram Opposite Angles Converse (Thm. 7.8)

13. By the Parallelogram Opposite Sides Converse (Thm. 7.7):

$4x + 7 = 12x - 1$

$-8x + 7 = -1$

$-8x = -8$

$x = 1$

$y + 1 = 3y - 11$

$-2y + 1 = -11$

$-2y = -12$

$y = 6$

The quadrilateral is a parallelogram when $x = 1$ and $y = 6$.

14. By the Parallelogram Diagonals Converse (Thm. 7.10):

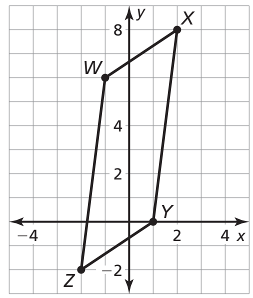
$6x - 8 = 4x$

$-8 = -2x$

$4 = x$

The quadrilateral is a parallelogram when $x = 4$.

15.



Slope of $\overline{WX} = \frac{8-6}{2-(-1)} = \frac{2}{3}$

Slope of $\overline{XY} = \frac{0-8}{1-2} = \frac{-8}{-1} = 8$

Slope of $\overline{YZ} = \frac{-2-0}{-2-1} = \frac{-2}{-3} = \frac{2}{3}$

Slope of $\overline{WZ} = \frac{-2-6}{-2-(-1)} = \frac{-8}{-2+1} = \frac{-8}{-1} = 8$

The slope of $\overline{WX}$ equals the slope of $\overline{YZ}$, therefore $\overline{WX} \parallel \overline{YZ}$. The slope of $\overline{XY}$ equals the slope of $\overline{WZ}$, therefore $\overline{XY} \parallel \overline{WZ}$. Because both pairs of opposite sides are parallel, $WXYZ$ is a parallelogram by definition.

16. The special quadrilateral is a rhombus because it has four congruent sides.

17. The special quadrilateral is a parallelogram because it has two pairs of parallel sides.

18. The special quadrilateral is a square because it has four congruent sides and four right angles.

19. $WY = XZ$

$-2x + 34 = 3x - 26$

$-5x + 34 = -26$

$-5x = -60$

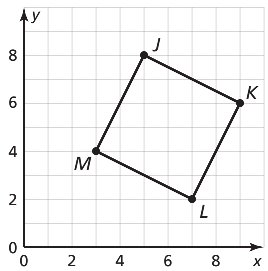
$x = 12$

$WY = -2 \bullet 12 + 34 = -24 + 34 = 10$

$XZ = 3 \bullet 12 - 26 = 36 - 26 = 10$

Chapter 7

20.



Slope of $\overline{JK} = \dfrac{8-6}{5-9} = \dfrac{2}{-4} = -\dfrac{1}{2}$

Slope of $\overline{KL} = \dfrac{6-2}{9-7} = \dfrac{4}{2} = 2$

Slope of $\overline{LM} = \dfrac{2-4}{7-3} = \dfrac{-2}{4} = -\dfrac{1}{2}$

Slope of $\overline{MJ} = \dfrac{4-8}{3-5} = \dfrac{-4}{-2} = 2$

Because the product of the two slopes is $\left(-\dfrac{1}{2}\right)(2) = -1$, there are four right angles ($\overline{JK} \perp \overline{KL}$, $\overline{KL} \perp \overline{LM}$, $\overline{LM} \perp \overline{MJ}$, and $\overline{MJ} \perp \overline{JK}$). So, quadrilateral $JKLM$ is a rectangle.

$JK = \sqrt{(5-9)^2 + (8-6)^2} = \sqrt{(-4)^2 + 2^2} = \sqrt{16+4}$
$= \sqrt{20} = \sqrt{4 \cdot 5} = 2\sqrt{5}$

$KL = \sqrt{(9-7)^2 + (6-2)^2} = \sqrt{(2)^2 + 4^2} = \sqrt{4+16}$
$= \sqrt{20} = \sqrt{4 \cdot 5} = 2\sqrt{5}$

Because $JK = KL$, and $JKLM$ is a rectangle, which is also a parallelogram, opposite sides are equal. So, $JK = KL = LM = MJ$. By definition of a rhombus and square, quadrilateral $JKLM$ is also a rhombus and a square.

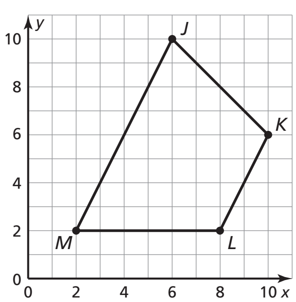
21. $m\angle Z = m\angle Y = 58^\circ$

$m\angle X = 180^\circ - 58^\circ = 122^\circ$

$m\angle W = m\angle X^\circ = 122^\circ$

22. The length of the midsegment is $\dfrac{1}{2}(39 + 13) = \dfrac{1}{2} \cdot 52 = 26$.

23.



$KL = \sqrt{(10-8)^2 + (6-2)^2} = \sqrt{(2)^2 + 4^2} = \sqrt{4+16}$
$= \sqrt{20} = \sqrt{4 \cdot 5} = 2\sqrt{5}$

$JM = \sqrt{(6-2)^2 + (10-2)^2} = \sqrt{(4)^2 + 8^2} = \sqrt{16+64}$
$= \sqrt{80} = \sqrt{16 \cdot 5} = 4\sqrt{5}$

Midsegment $= \dfrac{1}{2}(KL + JM) = \dfrac{1}{2}(2\sqrt{5} + 4\sqrt{5}) = \dfrac{1}{2}(6\sqrt{5})$
$= 3\sqrt{5}$

24. $7x^\circ + 65^\circ + 85^\circ + 105^\circ = 360^\circ$

$7x + 255 = 360$

$7x = 105$

$x = 15$

The two congruent angles are 105°.

28. yes; By the Isosceles Trapezoid Base Angles Converse (Thm. 7.15), if a trapezoid has a pair of congruent base angles, then the trapezoid is isosceles.

26. The quadrilateral is a trapezoid because it has exactly one pair of parallel sides.

27. The quadrilateral is a rhombus because it has four congruent sides.

28. The quadrilateral is a rectangle because it has four right angles.

Chapter 7 Test (p. 411)

1. The diagonals bisect each other, so $r = 6$ and $s = 3.5$.

2. In a parallelogram opposite angles are equal, therefore $b = 101$. Consecutive interior angles are supplementary, therefore $a = 180 - 101 = 79$.

3. In a parallelogram opposite sides are congruent. So, $p = 5$ and $q = 6 + 3 = 9$.

4. If consecutive interior angles are supplementary, then the lines that form those angles are parallel. So, the quadrilateral is a trapezoid.

5. The quadrilateral is a kite because it has two pairs of consecutive congruent sides, but opposite sides are not congruent.

6. The quadrilateral is an isosceles trapezoid because it has congruent base angles.

7. $3x^\circ + 5(2x + 7)^\circ = 360^\circ$

$3x + 10x + 35 = 360$

$13x + 35 = 360$

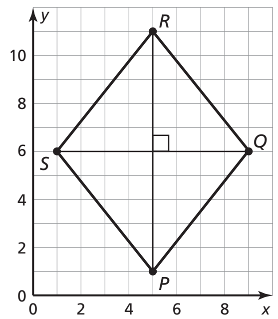
$13x = 325$

$x = 25$

$2x + 7 = 2 \cdot 25 + 7 = 50 + 7 = 57$

The measurement of each exterior angle of the octagon is 25°, 25°, 25°, 57°, 57°, 57°, 57°, and 57°.

Chapter 7

8. 

Slope of $\overline{RP} = \frac{11 - 1}{5 - 5} = \frac{10}{0}$ = undefined

Slope of $\overline{SQ} = \frac{6 - 6}{9 - 1} = \frac{0}{8} = 0$

So, $\overline{RP} \perp \overline{SQ}$, and quadrilateral $PQRS$ is a rhombus.

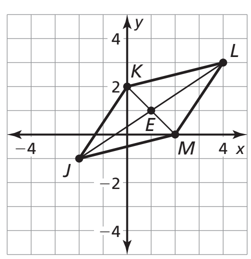
9. yes; the diagonals bisect each other.

10. no; $\overline{JK}$ and $\overline{ML}$ might not be parallel.

11. yes; $m\angle Y = 360° - (120° + 120° + 60°) = 360° - 300° = 60°$. Because opposite angles are congruent, the quadrilateral is a parallelogram.

12. If one angle in a parallelogram is a right angle, then consecutive angles are supplementary. So, the parallelogram is a rectangle.

13. Show that a quadrilateral is a parallelogram with four congruent sides and four right angles, or show that a quadrilateral is both a rectangle and a rhombus.

14. 

a. Slope of $\overline{JK} = \frac{3 - 0}{4 - 2} = \frac{3}{2}$

Starting at L, go down 3 units and left 2 units. So, the coordinates of M are (2, 0).

b. Midpoint of $\overline{JL} = \left(\frac{4 + (-2)}{2}, \frac{3 + (-1)}{2}\right) = \left(\frac{2}{2}, \frac{2}{2}\right) = (1, 1)$

Midpoint of $\overline{KM} = \left(\frac{0 + 2}{2}, \frac{2 + 0}{2}\right) = \left(\frac{2}{2}, \frac{2}{2}\right) = (1, 1)$

The coordinates of the intersection of the diagonals are (1, 1).

15. Midsegment $= \frac{1}{2}(6 + 15) = \frac{1}{2}(21) = 10.5$

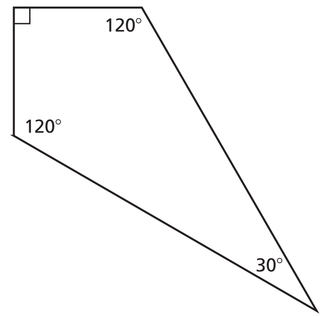
The middle shelf will have a diameter of 10.5 inches.

16. $\frac{(n - 2) \cdot 180°}{n} = \frac{(5 - 2) \cdot 180°}{5} = \frac{3 \cdot 180°}{5} = \frac{540°}{5} = 108°$

The measure of each interior angle of a regular pentagon is 108°.

17. Design $AB = DC$ and $AD = BC$, then $ABCD$ is a parallelogram.

18. *Sample answer:*



Chapter 7 Standards Assessment (p. 412)

1. Definition of parallelogram; Alternate Interior Angles Theorem (Thm. 3.2); Reflexive Property of Congruence (Thm. 2.1); Definition of congruent angles; Angle Addition Postulate (Post. 1.4); Transitive Property of Equality; Definition of congruent angles; ASA Congruence Theorem (Thm. 5.10); Corresponding parts of congruent triangles are congruent.

2. In steps 1 and 2, an angle bisector is drawn for $\angle A$ and $\angle C$. The point of intersection D is the incenter of $\triangle ABC$ and the center of the inscribed circle. By constructing a perpendicular segment to $\overline{AB}$ from D, the radius of the circle is $\overline{ED}$. An inscribed circle touches each side of the triangle.

3. no; No theorem can be used to prove itself.

4. $UV = \sqrt{(-2 - 1)^2 + (2 - 1)^2} = \sqrt{(-3)^2 + 1}$
$= \sqrt{9 + 1} = \sqrt{10}$

$VQ = \sqrt{(-2 - (-5))^2 + (1 - 2)^2} = \sqrt{(-2 + 5)^2 + (-1)^2}$
$= \sqrt{3^2 + 1} = \sqrt{10}$

$QR = \sqrt{(-4 - (-5))^2 + (5 - 2)^2} = \sqrt{(-4 + 5)^2 + 3^2}$
$= \sqrt{1^2 + 9} = \sqrt{10}$

$RS = \sqrt{(-1 - (-4))^2 + (6 - 5)^2} = \sqrt{(-1 + 4)^2 + 1^2}$
$= \sqrt{9 + 1} = \sqrt{10}$

$ST = \sqrt{(2 - (-1))^2 + (5 - 6)^2} = \sqrt{(3)^2 + (-1)^2}$
$= \sqrt{9 + 1} = \sqrt{10}$

$TU = \sqrt{(2 - 1)^2 + (5 - 2)^2} = \sqrt{(1)^2 + (3)^2} = \sqrt{1 + 9}$
$= \sqrt{10}$

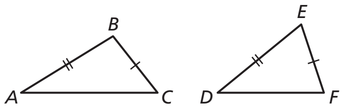
Perimeter: $UV + VQ + QR + RS + ST + TU = \sqrt{10} + \sqrt{10} + \sqrt{10} + \sqrt{10} + \sqrt{10} + \sqrt{10} = 6\sqrt{10}$

The perimeter is $6\sqrt{10}$. The polygon $QRSTUV$ is equilateral. For the hexagon to be equiangular each angle must be $\frac{(6 - 2) \cdot 180°}{6} = \frac{4 \cdot 180°}{6} = \frac{720°}{6} = 120°$. Because $m\angle Q = 90°$, the hexagon is not equiangular. So, it is not a regular polygon.

Chapter 7

5. Given $\overline{AB} \cong \overline{DE}, \overline{BC} \cong \overline{EF}, AC > DF$

Prove $m\angle B > m\angle E$



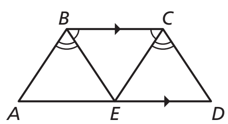
Indirect Proof

Step 1: Assume temporarily that $m\angle B \not> m\angle E$. Then it follows that either $m\angle B = m\angle E$ or $m\angle B < m\angle E$.

Step 2: If $m\angle B < m\angle E$, then $AC < DF$ by the Hinge Theorem (Theorem 6.12). If $m\angle B = m\angle E$, then $\angle B \cong \angle E$. So, $\triangle ABC \cong \triangle DEF$ by the SAS Congruence Theorem (Theorem 5.5) and $AC = DF$.

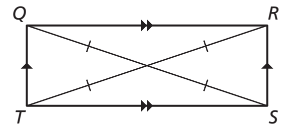
Step 3: Both conclusions contradict the given statement that $AC > DF$. So, the temporary assumption that $m\angle B \not> m\angle E$ cannot be true. This proves that $m\angle B > m\angle E$.

6. Given $\overline{BC} \parallel \overline{AD}$, $\angle EBC \cong \angle ECB$, $\angle ABE \cong \angle DCE$
Prove *ABCD* is an isosceles trapezoid.



STATEMENTS	REASONS
1. $\overline{BC} \parallel \overline{AD}$, $\angle EBC \cong \angle EBC$, $\angle ABE \cong \angle DCE$	**1.** Given
2. *ABCD* is a trapezoid.	**2.** Definition of trapezoid
3. $m\angle EBC = m\angle ECB$, $m\angle ABE = m\angle DCE$	**3.** Definition of congruent angles
4. $m\angle ABE + m\angle EBC = m\angle ABC$, $m\angle DCE + m\angle ECB = m\angle DCB$	**4.** Angle Addition Postulate (Post. 1.4)
5. $m\angle ABE + m\angle EBC = m\angle ABE + m\angle EBC$	**5.** Reflexive Property of Equality
6. $m\angle ABE + m\angle EBC = m\angle DCE + m\angle ECB$	**6.** Substitution Property of Equality
7. $m\angle ABC = m\angle DCB$	**7.** Transititve Property of Equality
8. $\angle ABC \cong \angle DCB$	**8.** Definition of congruent angles
9. *ABCD* is an isosceles trapezoid.	**9.** Isosceles Trapezoid Base Angles Converse (Thm. 7.15)

7. Given *QRST* is a parallelogram, $\overline{QS} \cong \overline{RT}$
Prove *QRST* is a rectangle.



Sample answer:

STATEMENTS	REASONS
1. $\overline{QS} \cong \overline{RT}$	**1.** Given
2. $\overline{QT} \cong \overline{RS}, \overline{QR} \cong \overline{TS}$	**2.** Parallelogram Opposite Sides Theorem (Thm. 7.3)
3. $\triangle TQR \cong \Delta SRQ$	**3.** SSS Congruence Theorem (Thm. 5.8)
4. $\angle TQR \cong \angle SRQ$	**4.** Corresponding parts of congruent triangles are congruent.
5. $m\angle TQR + m\angle SRQ = 180°$	**5.** Parallelogram Consecutive Angles Theorem (Thm. 7.5)
6. $m\angle TQR = m\angle SRQ = 90°$	**6.** Congruent supplementary angles have the same measure.
7. $90° + m\angle QTS = 180°$ $90° + m\angle RST = 180°$	**7.** Parallelogram Consecutive Angles Theorem (Thm. 7.5)
8. $m\angle QTS = 90°$; $m\angle RST = 90°$	**8.** Subtraction Property of Equality
9. $\angle TQR$, $\angle SRQ$, $\angle QTS$, and $\angle RST$ are right angles.	**9.** Definition of a right angle
10. *QRST* is a rectangle.	**10.** Definition of a rectangle