PHYSICS 1401 (1) homework solutions

14-30 Calculate the amount of energy required to escape from

(a) Earth's moon and

(b) Jupiter

relative to that required to escape from Earth.

14-30 Kinetic energy required to escape (from surface) must equal the potential energy at surface. Ratio, relative to Earth

(a) Moon:

$$\frac{K_M}{K_E} = \left|\frac{U_M}{U_E}\right| = \frac{\cancel{G}\frac{M_M \cancel{m}}{R_M}}{\cancel{G}\frac{M_E \cancel{m}}{R_E}} = \frac{M_M}{M_E}\frac{R_E}{R_M}$$

(Appendix C)

$$\frac{K_M}{K_E} = \frac{7.36\times10^{22}\text{ kg}}{5.98\times10^{24}\text{ kg}} \cdot \frac{6.37\times10^{6}\text{ m}}{1.74\times10^{6}\text{ m}} = 0.045$$

(b) Jupiter

Appendix C

gives mass and escape velocity

$$\frac{K_J}{K_E} = \frac{\cancel{\frac{1}{2}}\cancel{M} v_J^2}{\cancel{\frac{1}{2}}\cancel{M} v_E^2}$$

$$K_J = \left(\frac{v_J}{v_E}\right)^2 = \left(\frac{59.5\text{ km/s}}{11.2\text{ km/s}}\right)^2 = 28.2$$

Check $\frac{K_J}{K_E} = \frac{M_J}{M_E}\frac{R_E}{R_M} = 318\,\frac{12{,}800}{143{,}000} = 28.5$) ok!

14-34 Planet Roton, with a mass of 7.0×10^{24} kg and a radius of 1600 km, gravitationally attracts a meteorite that is initially at rest relative to the planet, at a great enough distance to take as infinite. The meteorite falls toward the planet. Assuming the planet is airless, find the speed of the meteorite when it reaches the planet's surface.

14-34

Mechanical energy $(K+U)$ is conserved, where

$$K = \frac{1}{2}mv^2 \qquad U = -G\frac{Mm}{r}$$

Initially $r_0 = \infty$ $\quad v_0 = 0$ $\quad$ so $\quad K + U = 0$

At surface, $r_2 = R$

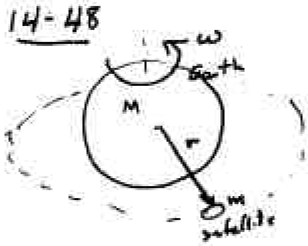
$$\frac{1}{2}\cancel{m}v_s^2 - G\frac{M\cancel{m}}{R} = 0$$

$$v_s = \sqrt{2\frac{GM}{R}} = \sqrt{2\,\frac{(6.67\times10^{-11})(7.0\times10^{24})}{1600\times10^3}}$$

$$v_s = 2.42\times10^4\ m/s$$

14-48 A satellite hovers over a certain spot on the equator of (rotating) Earth. What is the altitude of its orbit (called a geosynchronous orbit)?

14-48



The plane of satellite orbit, in order to be geosynchronous, must lay along the equator, and have the same angular velocity as the earth.

$$\omega = \frac{2\pi}{T} \qquad \text{where } T = 1 \text{ day}$$

$$\cancel{m}r\omega^2 = G\frac{M\cancel{m}}{r^2}$$

$$r^3 = \frac{GM}{\omega^2} = \frac{GM}{(4\pi^2)}T^2 = \frac{6.67\times10^{-11}\times6.0\times10^{24}}{(4\pi^2)}\left(1\,\text{day}\times24\frac{\text{hr}}{\text{day}}\times3600\frac{\text{s}}{\text{hr}}\right)^2$$

$r^3 =$ $\qquad r = 41.6\times10^6$ m

height (altitude) $= r - R_e = (41.6 - 6.4)\times10^6 = 35.2\times10^6$ m

14-51 In 1610, Galileo used his telescope to discover four prominent moons around Jupiter. Their mean orbital radii a and periods T are as follows:

Name	*a (10^8 m)*	*T (days)*
Io	4.22	1.77
Europa	6.71	3.55
Ganymede	10.7	7.16
Callisto	18.8	16.7

(a) Plot log a (y axis) against log T (x axis) and show that you get a straight line.
(b) Measure the slope of the line and compare it with the value that you expect from Kepler's third law.
(c) Find the mass of Jupiter from the intercept of this line with the y axis.

14-51

Kepler's 3rd Law says

$$T^2 = \left(\frac{4\pi^2}{GM_J}\right) a^3$$

Here T and a are the periods and orbit radii of Jupiters moons and M_J = mass of Jupiter

Applied to Earth's motion around the Sun (mass $M_\odot$)

$$T_E^{\,2} = \left(\frac{4\pi^2}{GM_\odot}\right) r_E^3 \qquad \left[\text{Here } T_E = 365.25\text{ day},\ r_E = 1.50\times10^{11}\text{ m}\right]$$

Ratio $$\left(\frac{T}{T_E}\right)^2 = \left(\frac{M_\odot}{M_J}\right)\left(\frac{a}{r_E}\right)^3$$

or $$\left(\frac{r_E}{a}\right) = \left(\frac{M_\odot}{M_J}\right)^{1/3}\left(\frac{T_E}{T}\right)^{2/3}$$

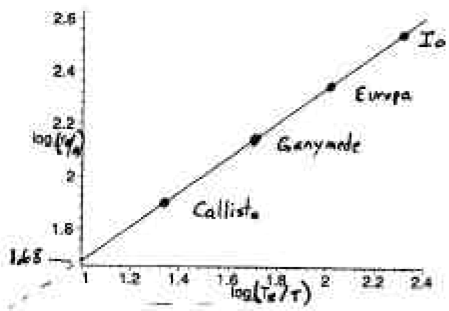
$$\boxed{\log\left(\frac{r_E}{a}\right) = \frac{1}{3}\log\left(\frac{M_\odot}{M_J}\right) + \frac{2}{3}\log\left(\frac{T_E}{T}\right)}$$

Prediction

Name	a (10^8 m)	T (days)	$\log(r_E/a)$	$\log(T_E/T)$
Io	4.22	1.77	2.55	2.31
Europa	6.71	3.55	2.35	2.01
Ganymede	10.7	7.16	2.14	1.708
Callisto	18.8	16.7	1.90	1.340

logs are to base 10! Graph is linear!

(a) graph



(b) Slope (between Io + Callisto)

$$\text{Slope} = \frac{2.55 - 1.90}{2.31 - 1.34} = 0.67$$

very close to the predicted 2/3

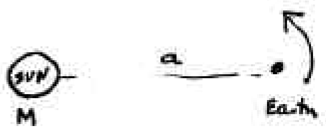
(c) Extrapolate straight line to $\log(T_E/T) = 0$ (intercept) of $\log\left(\frac{r_E}{a}\right)$ is $1.68 - 0.67 = 1.01$

So, $$\frac{1}{3}\log\left(\frac{M_\odot}{M_J}\right) = 1.01 \Rightarrow \frac{M_\odot}{M_J} = 10^{3.03} = 1072$$

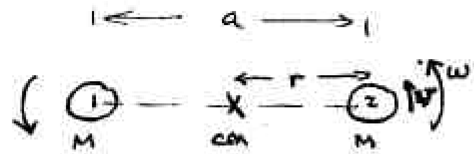
$$M_J = \frac{M_\odot}{1072} = \frac{2.0\times10^{27}\text{ kg}}{1072} = 1.87\times10^{27}\text{ kg}$$

14-53 In a certain binary-star system, each star has the same mass as our Sun, and they revolve about their center of mass. The distance between them is the same as the distance between Earth and the Sun. What is their period of revolution in years?

14-53 Solve from basic principles and Kepler's Law



Our solar system



Accelerations are

$$a = \frac{v^2}{r}$$

So $$M\frac{v^2}{r} = G\frac{M^2}{(2r)^2}$$

gives $$\boxed{v^2 = G\frac{M}{4r}}$$

Since the orbit is circular, the period of the binary about the c. of M. is T_B, where $\boxed{v = \frac{2\pi r}{T_B}}$

Combining $$\frac{4\pi^2 r^2}{T_B^2} = G\frac{M}{4r}$$

or $$T_B^2 = \frac{16\pi^2 r^3}{GM}$$, and putting in $r = a/2$

$$T_B^2 = \frac{2\pi^2 a^3}{GM}$$ to compare with Earth period $$T_E^2 = \frac{4\pi^2 a^2}{GM}$$

$$\left(\frac{T_B}{T_E}\right)^2 = \frac{1}{2} \qquad T_B = \frac{1}{\sqrt{2}} T_E$$

$$T_B = (.707)(1\ \text{year})$$

$$T_B = 0.71\ \text{year}$$

PHYSICS 1401 (1) homework solutions

15-12

(a) Assuming the density of seawater is 1.03 g/cm3, find the total weight of water on top of a nuclear submarine at a depth of 200 m if its (horizontal cross-sectional) hull area is 3000 m^2.

(b) In atmospheres, what water pressure would a diver experience at this depth? Do you think that occupants of a damaged submarine at this depth could escape without special equipment?

15-12

Weight of water on $3000 m^2$ at a depth of $h = 200 m$ is

$$W = \rho g h A = (1.03 \times 10^3 \tfrac{kg}{m^3})(9.8 \tfrac{m}{s^2})(200 m)(3000 m^2)$$

$$W = 6.06 \times 10^9 \text{ Newtons} \qquad (\text{or about } 1.4 \times 10^9 \text{ lbs})$$

The pressure is

$$p = \rho g h = \frac{W}{A} = \frac{6.06 \times 10^9 N}{3000 m^2} = 2.02 \times 10^6 \text{ Paschal} \times \frac{1 \text{ atm}}{1.01 \times 10^5 Pa}$$

$$p = 20 \text{ atmospheres}$$

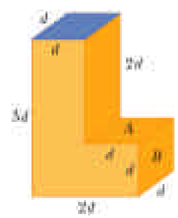
(or about 300 lbs on every square inch. Human body would implode!)

15-18

The L-shaped tank shown in Fig. 15-32 is filled with water and is open at the top. If d=5.0 m, what are

(a) the force on face A and

(b) the force on face B due to the water?



15-18

(a) Surface A is $h_A = 2d$ below the top. Hence, the pressure at this depth is

$$P_A = \rho g h_A \qquad \text{so} \qquad P_A = 2\rho g d$$

and the net force is $F_A = P_A (d)^2 = 2\rho g d^3 = 2(1.0 \times 10^3)(9.8)(5.0)^3$

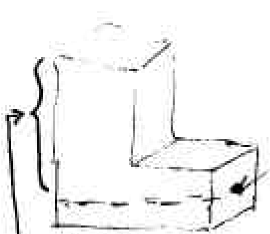
$$F_A = 2.45 \times 10^6 \text{ Newtons}$$

(b) The pressure on surface B varies with distance (vertical). The variation, however, is linear, so we can use the average position ½ way up.... This position is $\frac{5}{2}d$ from the top. Hence,

$$F_B = P_{av} A$$

$$F_B = \rho g \left(\tfrac{5}{2} d\right) d^2 = \tfrac{5}{2} \rho g d^3$$

$$\text{So} \quad F_B = \frac{5}{4} F_A = 3.06 \times 10^6 \text{ Newtons}$$



$3d - \frac{1}{2}d = \frac{5}{2}d$

15-28 A blimp is cruising slowly at low altitude, filled as usual with helium gas. Its maximum useful payload, including crew and cargo, is 1280 kg. The volume of the helium-filled interior space is 5000 m^3. The density of helium gas is 0.16 kg/m^3, and the density of hydrogen is 0.081 kg/m^3. How much more payload could the blimp carry if you replaced the helium with hydrogen? (Why not do it?)

15-28

In general, the bouyant force is

$$B = \rho g V$$

So the payload (or weight) difference comes from the difference between hydrogen (ρ_H) and helium (ρ_{He}) densities at STP.

$$\Delta W = B_{He} - B_H = (\rho_{He} - \rho_H) g V$$

$$\Delta W = \left[(0.16 - 0.081)\frac{kg}{m^3}\right](9.8 \tfrac{m}{s^2})(5000\ m^3)$$

$$\Delta W = 3.87 \times 10^3 \text{ Newton}$$

The mass of additional payload would be

$$\Delta M = \frac{\Delta W}{g} = \frac{3.87\times10^3}{9.8} = 395 \text{ kg} \qquad \left(\sim 900 \text{ lbs of weight}\right)$$

This is not done (since catostrophic accidents in 1930's) because of hydrogen's flammability --> though this is really a chemistry issue!

PHYSICS 1401 (1) homework solutions

15-40 Figure 15-38 shows the merging of two streams to form a river. One stream has a width of 8.2 m, depth of 3.4 m, and current speed of 2.3 m/s. The other stream is 6.8 m wide and 3.2 m deep, and flows at 2.6 m/s. The width of the river is 10.5 m, and the current speed is 2.9 m/s. What is its depth?

15-40

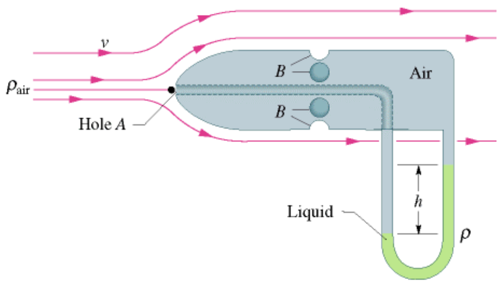
Continuity $\sum \rho v A = \text{constant}$

$$(2.3)(8.2)(3.4) + (2.6)(6.8)(3.2) = (2.9)(10.5)\,h$$

$$h = \frac{(2.3)(8.2)(3.4) + (2.6)(6.8)(3.2)}{(2.9)(10.5)} = \frac{64.1 + 56.6}{30.5}$$

$$h = 3.96 \text{ m}$$

15-57 A pitot tube (Fig. 15-44) is used to determine the airspeed of an airplane. It consists of an outer tube with a number of small holes B (four are shown) that allow air into the tube; that tube is connected to one arm of a U-tube. The other arm of the U-tube is connected to hole A at the front end of the device, which points in the direction the plane is headed. At A the air becomes stagnant so that $v_A = 0$. At B, however, the speed of the air presumably equals the airspeed v of the aircraft.



(a) Use Bernoulli's equation to show that

$$v = \sqrt{\frac{2 r g h}{r_{air}}}$$

where ρ is the density of the liquid in the U-tube and h is the difference in the fluid levels in that tube.

(b) Suppose that the tube contains alcohol and indicates a level difference h of 26.0 cm. What is the plane's speed relative to the air? The density of the air is 1.03 kg/m^3 and that of alcohol is 810 kg/m^3.

15-57

(a) At the entrance (A), the air is deflected to the sides. So, the velocity of the air ~~at~~ in the entry to the tube is

$P_A = 0$ (atmospheric)

On the other hand, at (B) the air is rushing by at speed v. From Bernoulli equation

$P_A = P_B + \frac{1}{2}\rho_a v^2$ where ρ_a = air density

This So the pressure at (B) is $\frac{1}{2}\rho v^2$ less than atmospheric.

This balanced by liquid pressure $\Delta p = \rho g h = \frac{1}{2}\rho v^2$

Hence, $v^2 = \frac{2\rho g h}{\rho_a}$ or $v = \sqrt{\frac{2\rho g h}{\rho_a}}$

(b) For case given

$$v = \sqrt{\frac{2(810)(9.8)(0.26)}{1.03}}$$

$v = 63.3$ m/s