Definition, total pressure, centre of pressure, total pressure on horizontal, vertical and inclined plane surface, total pressure on curved surfaces, water pressure on gravity dams, lock gates, Numerical problems. 03 Hrs.

TOTAL PRESSURE AND CENTRE OF PRESSURE

The normal force exerted by stationary fluid is known as ***hydrostatic force***.

Total Pressure (P):

Total pressure may be defined as pressure exerted by a static fluid on a surface either plane or curved when the fluid comes in contact with the surface. Total pressure always acts normal to the surface.

Whenever we dive in a swimming pool, we feel some uneasiness. As we dive deeper and deeper, we feel more and more uneasiness. This uneasiness is, in fact, due to the total weight (Total pressure) of water above us.

Total Pressure = Pressure Intensity x Area

$= p \times a = kg/cm^2 \times cm^2 = kg$

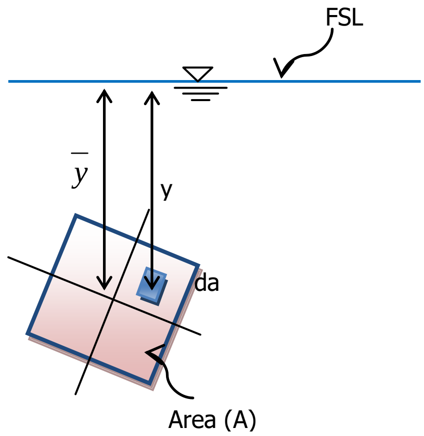
Note: The unit of total pressure is always Kg or KN or N.

June/July – 2013 / 10CV35 – 10 marks:

Prove that for a plate submerged in horizontal position in water the centre of pressure is same as centriod of the plate.

Total pressure on horizontally immersed plane surface:

Consider a plane vertical surface (Lamina) of area 'A' immersed in a liquid of specific weight 'w '. Consider an elemental area of the lamina at a depth 'y' below the free liquid surface.



Intensity of pressure on the elemental area = p = wy

Pressure force on the elemental area = p da = wy da

This pressure intensity is constant over the plane surface since every depth 'y' below the liquid surface.

Total Pressure force on the whole lamina $P = \Sigma wday = w\Sigma day = wA\bar{y}$

Where, $\bar{y}$ = Depth of centroid of the lamina below the free surface

w = specific weight of liquid = 9.81 kN/m^3 for water

A = Area of the surface.

Total pressure on a vertically immersed plane surface:

Problem:®

A rectangular tank 5 m long 2 m wide contains water up to a depth of 2.5 m. calculate the pressure on the base of the tank.

Solution:

Given, base length L = 5 m, breadth = 2, and depth of base from FLS $(\bar{y})$ = 2.5m, Area of base = length x breadth = 5 x 2 = 10 m^2.

Total pressure on the base $(P) = wA\bar{y} = 9.81 \times 10 \times 2.5 =$ 245.25 kN

Centre of Pressure (C_p):

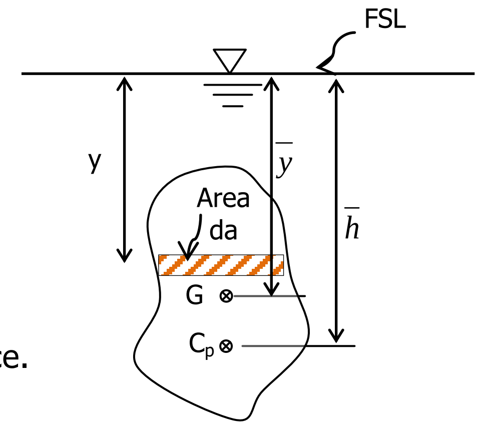
The centre of pressure is defined as the point of application of the total pressure on the surface. It is always expressed in terms of depth from the liquid surface.

The intensity of pressure, on immersed surface is not uniform, but increases with depth, therefore the resultant pressure, on the immersed surface will act at some point, below the centre of gravity of the immersed surface and towards the lower edge of the figure.

If the pressure intensity on the surface is uniform, the resultant pressure (Total pressure) will act at c.g of the surface.

If the pressure intensity on the surface is not uniform, the resultant pressure (Total pressure) will act at the point C_p below c.g of the surface.

Centre of pressure for a vertically immersed surface:



Let, $\bar{h}$ = Depth of centre of pressure of the lamina below the free surface

$\bar{y}$ = Depth of centroid of the lamina below the free surface

w = specific weight of liquid

A = Area of the arbitary shape of plane surface.

Consider an elemental area '*da*' of the lamina at a depth 'y' below the free liquid surface.

Intensity of pressure on the elemental area = p = wy

Pressure force on the elemental area = p x da = wy da

$\therefore$ Moment of this force about the water surface = wy^2da

$\therefore$ Moment of the Total Pressure force on the whole lamina about the water surface

$$= \Sigma w y^2 da = w\Sigma da y^2 = wI_o$$

where, I_o = Moment of inertia of the lamina about the line of intersection of the water surface and the plane of the lamina.

Let, P = Total pressure on the lamina

$$P = wA\bar{y}$$

P acts at the centre of pressure C_p.

$\therefore$ Moment of the total pressure about the free surface $= P \times \bar{h} = (wA\bar{y}) \times \bar{h}$

$$(wA\bar{y}) \times \bar{h} = wI_o$$

$$\bar{h} = \frac{I_o}{A\bar{y}} = \frac{\text{Second moment of the area about the water surface}}{\text{First moment of the area about the water surface}}$$

Let, I_g = Moment of inertia of the lamina about the centroidal horizontal axis in the plane of the lamina.

we know by parallel axes theorem

$$I_o = I_g + A\bar{y}^2$$

$$\bar{h} = \frac{I_o}{A\bar{y}} = \frac{I_g + A\bar{y}^2}{A\bar{y}}$$

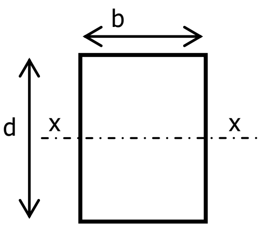
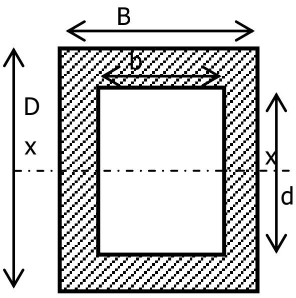
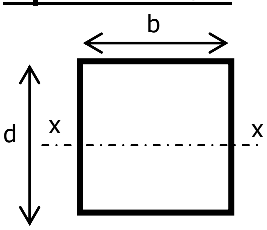
$$\bar{h} = \bar{y} + \frac{I_g}{A\bar{y}}$$

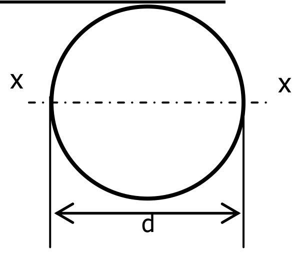
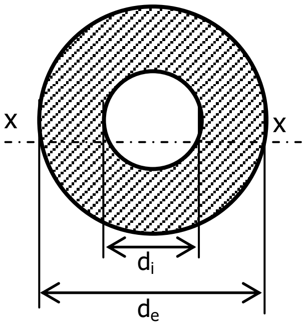
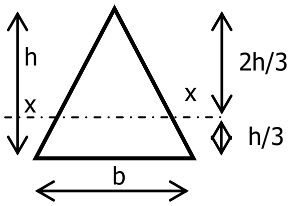
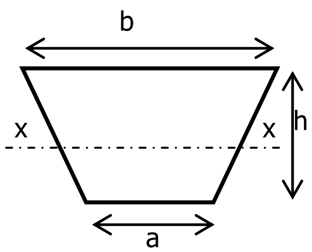
Note: The depth of C_p will always be greater than depth of c.g from the free liquid surface by a distance equal to $\frac{I_g}{A\bar{y}}$.

Dec 2013/jan 2014 – 10CV35 – 06 marks

Derive the expression for total pressure and Centre of pressure for a vertical plate submerged in a static liquid.

<u>Geometrical properties of some laminas:</u>

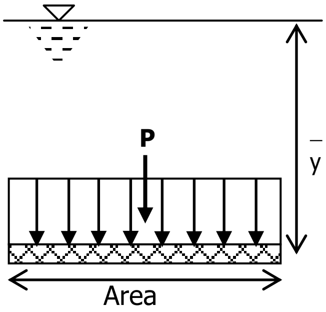
Sl No	Section	Y	M.I about centroidal Axis (Ig)
1	**Rectangular section** 	$\frac{d}{2}$	$I_{xx} = \frac{bd^3}{12}$
2	**Hollow Rectangular section** 	$\frac{D}{2}$	$I_{xx} = \frac{BD^3}{12}\ \frac{bd^3}{12}$ $I_{xx} = \frac{1}{12}\left[BD^3 - bd^3\right]$
3	**Square section:** 	$\frac{b}{2}$	$I_{xx} = \frac{b^4}{12}$

4	**Circular section** 	$\frac{d}{2}$	$I_{xx} = \frac{\pi d^4}{64}$
5	**Hollow circular section** 	$\frac{d_e}{2}$	$I_{xx} = \frac{\pi\left(d_e^4 - d_i^4\right)}{64}$
6	**Triangular section** 	$\frac{h}{3}$ from base $\frac{2h}{3}$ from apex	$I_{XX} = \frac{bh^3}{36}$
7	**Trapezium section** 	$\bar{y}_{bottom} = \frac{(a+2b)}{(a+b)} \times \frac{h}{3}$ $\bar{y}_{Top} = \frac{(2a+b)}{(a+b)} \times \frac{h}{3}$	$I_g = \frac{h^3}{36}(3a+b)$

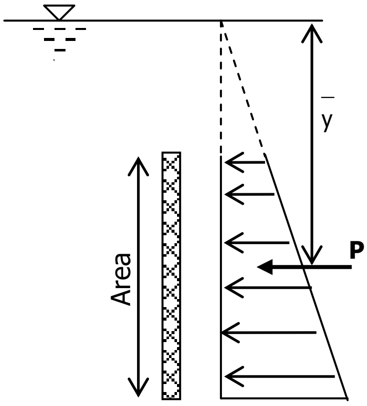
June / July 2011/ 06 CV 35/ 06 marks

Draw the pressure diagram for horizontal, inclined and vertical plane surface and explain briefly.

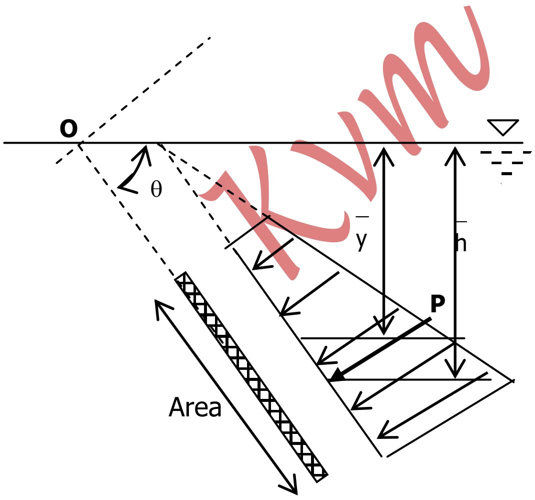
Pressure diagram: A pressure diagram may be defined as a graphical representation of the variation in the intensity of pressure over a surface. Such diagrams are very useful for finding out the total pressure and centre of pressure of a liquid on a vertical surface.



Pressure diagram on Horizontal plate

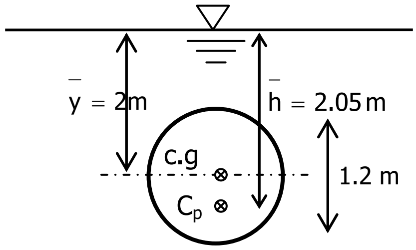


Pressure diagram on Vertical plate



Pressure diagram on Inclined plate

Problem: ®
A circular plate of 1.2 m diameter is placed vertically in water so that the centre of the plate is 2 m below the free surface. Determine the total pressure on the plate and the depth of centre of pressure.
Solution:



$$A = \frac{\pi \times 1.2^2}{4} = 1.13m^2, \qquad \bar{y} = 2m.$$

Total pressure on the plate $P = wA\bar{y}$

$$P = 9.81 \times 1.13 \times 2 = 22.17\ KN$$

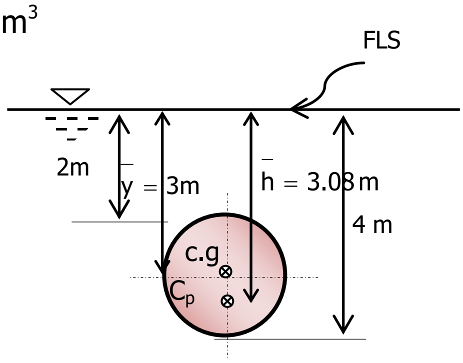
Depth of centre of pressure $\bar{h} = \bar{y} + \frac{I_g}{A\bar{y}}$

$$I_g = \frac{\pi \times 1.2^4}{64} = 0.102\ m^4$$

$$\bar{h} = 2 + \frac{0.102}{1.13 \times 2} = 2.05\ m$$

Problem: ®
A circular plate is immersed vertically in oil of specific gravity 0.9 with its least and greatest depth of immersion being 2m and 4m respectively. Find the total pressure and depth of centre of pressure.
Solution:

Specific gravity $= 0.9$, Sp. wt $= 0.9 \times 9.81 = 8.83\ KN/m^3$



$$\text{dia}(d) = 4 - 2 = 2m, \qquad A = \frac{\pi \times 2^2}{4} = 3.14m^2,$$

$$\bar{y} = 2 + \frac{2}{2} = 3m,$$

Total pressure on the plate $P = wA\bar{y}$

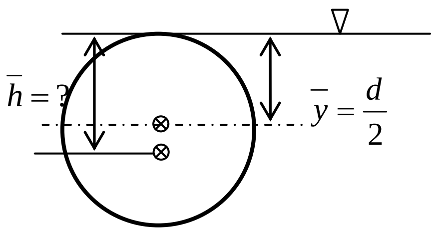
$$P = 8.83 \times 3.14 \times 3 = 83.18\ KN$$

Depth of centre of pressure $\bar{h} = \bar{y} + \frac{I_g}{A\bar{y}}$

$$I_g = \frac{\pi \times 2^4}{64} = 0.785\ m^4$$

$$\bar{h} = 3 + \frac{0.785}{3.14 \times 3} = 3.083\ m$$

Problem: ®
A circular area of diameter 'd' is vertical and submerged in a liquid as shown in fig. find the depth of the centre of pressure.
Solution:



$$A = \frac{\pi \times d^2}{4}, \qquad \bar{y} = \frac{d}{2}, \qquad I_g = \frac{\pi \times d^4}{64}$$

Depth of centre of pressure $\bar{h} = \bar{y} + \frac{I_g}{A\bar{y}}$

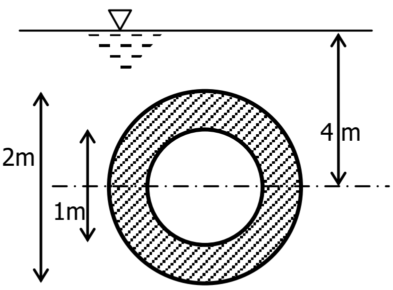
$$\bar{h} = \frac{d}{2} + \frac{\frac{\pi \times d^4}{64}}{\frac{\pi \times d^2}{4} \times \frac{d}{2}} = \frac{d}{2} + \frac{d}{8} = \frac{4d + d}{8} = \frac{5d}{8}$$

Problem: ®

A hollow circular plate of 2m external and 1m internal diameter is immersed vertically in water, such that the centre of the plate is 4m deep from the water surface.

Find the total pressure and the depth of centre of pressure.

Solution: Outer dia d_e = 2 m, Inner dia d_i = 1 m,



$$\therefore \text{Area of the plate } A = \frac{\pi \times (d_e^2 - d_i^2)}{4}$$

$$A = \frac{\pi \times (2^2 - 1^2)}{4} = 2.36 \text{ m}^2$$

$$\bar{y} = 4 \text{ m},$$

$$\text{Total pressure } P = wA\bar{y} = 9.81 \times 2.36 \times 4 = 92.61 \text{KN}.$$

$$\text{Depth of centre of pressure } \quad \bar{h} = \bar{y} + \frac{I_g}{A\bar{y}}$$

$$I_g = \frac{\pi \times (d_e^4 - d_i^4)}{64} = \frac{\pi \times (2^4 - 1^4)}{64} = 0.74 \text{m}^4$$

$$\bar{h} = 4 + \frac{0.74}{2.36 \times 4} = 4.078 \text{ m}.$$

Problem: ®

A hollow circular plate of 70cm external and 30cm internal diameter is immersed vertically in water at a depth 1 m from its upper edge.

Find the total pressure and the depth of centre of pressure.

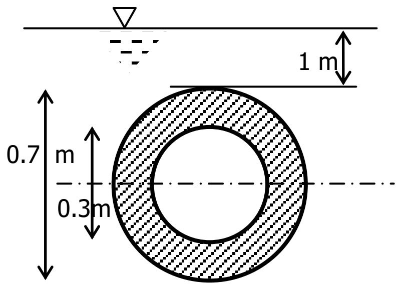
Solution:

Outer dia d_e = 70 cm = 0.7m, Inner dia d_i = 30 cm =0.3 m,

$$\bar{y} = 1 + \frac{0.7}{2} = 1.35 \text{ m},$$

$$\text{Total pressure } P = wA\bar{y} = 9.81 \times 0.314 \times 1.35 = 4.16 \text{KN}.$$

$$\therefore \text{Area of the plate } A = \frac{\pi \times (d_e^2 - d_i^2)}{4}$$



$$A = \frac{\pi \times (0.7^2 - 0.3^2)}{4} = 0.314 \text{ m}^2$$

$$\text{Depth of centre of pressure } \quad \bar{h} = \bar{y} + \frac{I_g}{A\bar{y}}$$

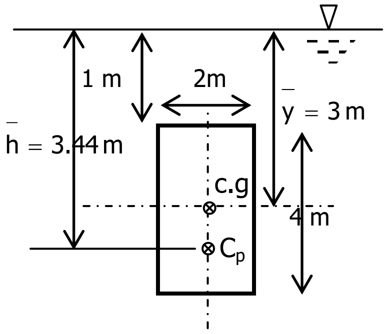
$$I_g = \frac{\pi \times (d_e^4 - d_i^4)}{64} = \frac{\pi \times (0.7^4 - 0.3^4)}{64} = 0.0114 \text{m}^4$$

$$\bar{h} = 1.35 + \frac{0.0114}{0.314 \times 1.35} = 1.38 \text{ m}.$$

1999 supplementary, 2b): ®

A rectangular plate 2 m wide and 4 m deep is immersed vertically in water in such a way that its 2 m side is parallel to the water surface and at 1 metre below it. Find the total pressure on the plate and the position of the centre pressure.

Solution:



$A = 2 \times 4 = 8 m^2$, $\quad \bar{y} = 1 + \frac{4}{2} = 3m$

Total pressure on the plate $P = wA\bar{y}$

$P = 9.81 \times 8 \times 3 = 235.44$ KN

Depth of centre of pressure $\bar{h} = \bar{y} + \frac{I_g}{A\bar{y}}$

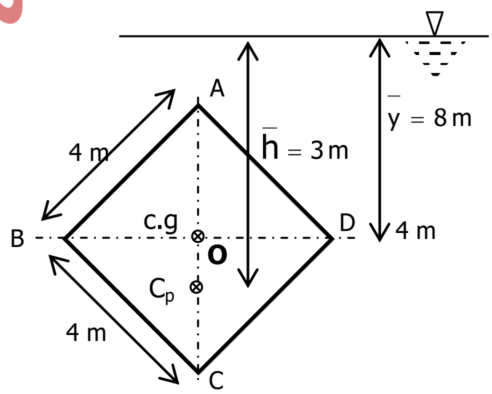
$$I_g = \frac{bd^3}{12} = \frac{2 \times 4^3}{12} = 10.67 \text{ m}^4$$

$$\bar{h} = 3 + \frac{10.67}{8 \times 3} = 3.44 \text{ m}$$

Problem: ®

A square plate 4 m x 4 m in hangs in water from one of its corners and its centroid lies at a depth of 8m from water surfaces. Workout the total pressure on the plate and locate the position of centre of pressure with respect to the plate centroid.

Solution:



Area of plate ABCD $= AB \times BC = 4 \times 4 = 16 \text{ m}^2$, ,

$w = 9.81 \text{ kN/m}^3$

$\bar{y} = 8 \text{ m}$,

Depth of centre of pressure $\bar{h} = \bar{y} + \frac{I_g}{A\bar{y}}$

Diagonal $AC = BD = \sqrt{4^2 + 4^2} = 5.66$ m

$OA = OC = \frac{5.66}{2} = 2.83$ m

I_g = M.I of triangle ABD about BD + M.I of triangle BCD about BD

$$I_g = \frac{BD \times OA^3}{12} + \frac{BD \times OC^3}{12} = \frac{5.66 \times 2.83^3}{12} + \frac{5.66 \times 2.83^3}{12} = 21.38 \text{ m}^4$$

$$\bar{h} = 8 + \frac{21.38}{16 \times 8} = 8.167 \text{ m}$$

Problem: ®

Find the total pressure and the depth of centre of pressure for the triangular lamina as shown in fig

Solution:

$$A = \frac{1}{2} \times 1.5 \times 1.25 = 0.9375 \text{ m}^2, \quad \bar{y} = 1 + \frac{2}{3}(1.25) = 1.83 \text{ m},$$

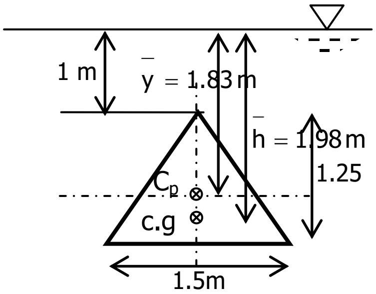
$$I_g = \frac{bh^3}{36} = \frac{1.5 \times 1.25^3}{36} = 0.2441 \text{ m}^4$$

Total pressure on the plate $P = wA\bar{y}$

$P = 9.81 \times 0.9375 \times 1.833 = 16.86$ KN

Depth of centre of pressure $\bar{h} = \bar{y} + \frac{I_g}{A\bar{y}}$

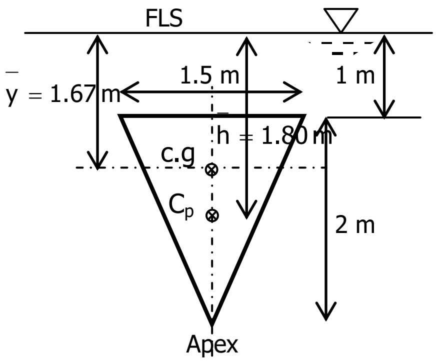
$$\bar{h} = 1.83 + \frac{0.2441}{0.9375 \times 1.83} = 1.98 \text{ m}$$



Problem: ®

A triangular plate of base width 1.5 m and height 2 m lies immersed in water with the apex downwards. The base of the plate is 1 m below and parallel to the free water surface. Calculate the total pressure on the plate and the depth of the centre of pressure.

Solution:



$A = \frac{1}{2} \times 1.5 \times 2 = 1.5 \text{ m}^2$, $\bar{y} = 1 + \frac{1}{3}(2) = 1.67$ m,

Total pressure on the plate $P = wA\bar{y}$

$P = 9.81 \times 1.5 \times 1.67 = 24.53$ KN

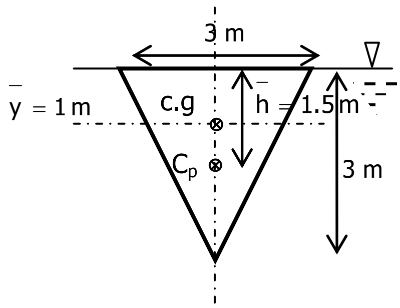
Depth of centre of pressure $\bar{h} = \bar{y} + \frac{I_g}{A\bar{y}}$

$$I_g = \frac{bh^3}{36} = \frac{1.5 \times 2^3}{36} = 0.333 \text{ m}^4$$

$$\bar{h} = 1.67 + \frac{0.333}{1.5 \times 1.67} = 1.80 \text{ m}$$

Problem: ®

An isosceles triangular plate of base 3 m and altitude 3 m is immersed vertically in an oil of specific gravity 0.8 as shown in fig.



Solution:

$A = \frac{1}{2} \times 3 \times 3 = 4.5 \text{m}^2$, $w = 0.8 \times 9.81 = 7.848 \text{ kN/m}^3$,

$\bar{y} = \frac{1}{3}(3) = 1 \text{m}$,

Total pressure on the plate $P = wA\bar{y}$

$P = 7.848 \times 4.5 \times 1 = 35.316$ KN

Depth of centre of pressure $\bar{h} = \bar{y} + \frac{I_g}{A\bar{y}}$

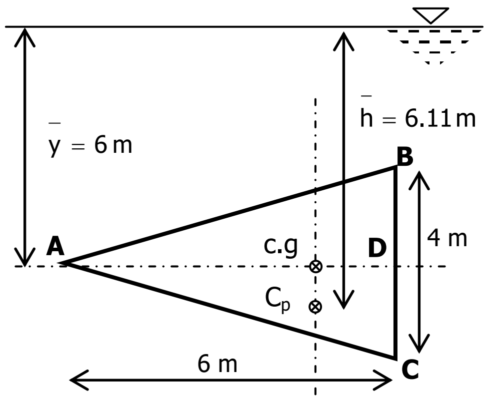
$$I_g = \frac{bh^3}{36} = \frac{3 \times 3^3}{36} = 2.25 \text{ m}^4$$

$$\bar{h} = 1 + \frac{2.25}{4.5 \times 1} = 1.5 \text{ m}$$

Problem: ®

An isosceles triangular plate of base 4 m and altitude 6 m is immersed vertically in water. Its axis of symmetry is parallel to and at a depth of 6 m from the free water surface. Calculate the magnitude and location of total pressure force.

Solution:



$$\text{Area of ABC} = \frac{1}{2}\times BC\times AD = \frac{1}{2}\times 4\times 6 = 12\,m^2, ,$$

$$w = 9.81\ kN/m^3$$

$\bar{y} = 6\,m,$

$$\text{Total pressure on the plate } P = wA\bar{y}$$

$$P = 9.81\times 12\times 6 = 706.32\ KN$$

$$\text{Depth of centre of pressure}\quad \bar{h} = \bar{y} + \frac{I_g}{A\bar{y}}$$

I_g = M.I of triangle ABD about AD + M.I of triangle ACD about AD

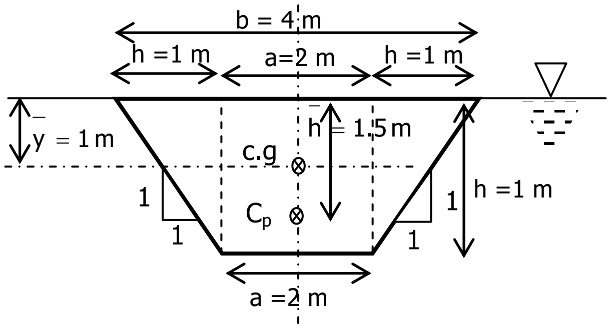
$$I_g = \frac{AD\times BD^3}{12} + \frac{AD\times CD^3}{12} = \frac{6\times 2^3}{12} + \frac{6\times 2^3}{12} = 8\,m^4$$

$$\bar{h} = 6 + \frac{8}{12\times 6} = 6.11\,m$$

June/July -2014- 10CV35 – 08 marks: ®

A trapezoidal channel 2 m wide at the bottom and 1 m deep has side 1:1. Determine

i. Total pressure
ii. Centre of pressure, when it is full of water



Solution: a = 2m, b = a + 2h = 2 + 2x1 = 4m

$$A = \frac{1}{2}(a+b)\times h = \frac{1}{2}(2+4)\times 1 = 3m^2,$$

$$\bar{y}_{from\,top} = \left(\frac{b+2a}{a+b}\right)\times\frac{h}{3} = \left(\frac{4+2\times 2}{2+4}\right)\times\frac{1}{3} = 0.444\ m,$$

$$\text{Total pressure on the plate } P = wA\bar{y}$$

$$P = 9.81\times 3\times 0.444 = 13.07\ KN$$

$$\text{Depth of centre of pressure}\quad \bar{h} = \bar{y} + \frac{I_g}{A\bar{y}}$$

$$I_g = \frac{h^3}{36}(3a+b) = \frac{1^3}{36}\times(3\times 2+4) = 0.278\,m^4$$

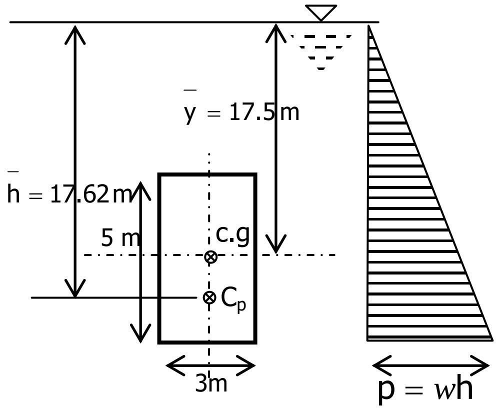
$$\bar{h} = 0.444 + \frac{0.278}{3\times 0.444} = 0.653\,m \quad \text{or}$$

$$\bar{h} = \left(\frac{4a+b}{6a+2b}\right)\times h = \left(\frac{4\times 2+4}{6\times 2+2\times 4}\right)\times 1 = 0.60\ m,$$

June/July-2011 06CV35 – 06 marks/1999 (Annual), 3a BU ®

A vertical gate closes a horizontal tunnel 5m high and 3m wide running full with water. The pressure at the bottom of the gate is 196.20 kN/m^2. Determine the total pressure on the gate and position of the center of the pressure.

Solution:



Pressure at bottom (p) = 196.20 kN/m^2

P = wh

$$\text{Pressure head (h)} = \frac{p}{w} = \frac{196.20}{9.81} = 20 \text{ m of water}$$

$$\text{Total pressure on the gate } P = wA\bar{y}$$

$$A = 3 \times 5 = 15 \text{ m}^2,$$

$$\bar{y} = h - \frac{5}{2} = 17.5 \text{ m}$$

$$P = 9.81 \times 15 \times 17.5 = 2575.125 \text{ KN}$$

$$\text{Depth of centre of pressure} \quad \bar{h} = \bar{y} + \frac{I_g}{A\bar{y}}$$

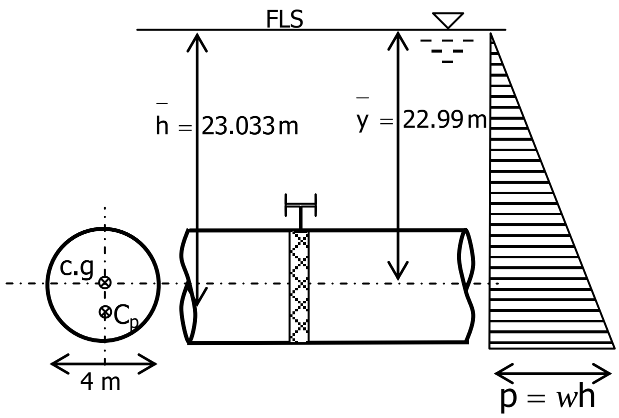
$$I_g = \frac{bd^3}{12} = \frac{3 \times 5^3}{12} = 31.25 \text{ m}^4$$

$$\bar{h} = 17.5 + \frac{31.25}{15 \times 17.5} = 17.62 \text{ m}$$

Problem: ®

A pipe which is 4 m in diameter contains a gate value. The pressure at the centre of pipe is 196.2 kPa. if the pipe is filled with oil of specific gravity 0.87, find the force exerted by the oil upon the gate and the position of the centre of pressure.

Solution:



Pressure at bottom (p) = 196.2 kPa

=196.2 kN/m^2

P = wh

$$\text{Pressure head (h)} = \frac{p}{w} = \frac{196.2}{0.87 \times 9.81}$$

$$= 22.99 \text{ m of water}$$

$$\text{Total pressure on the gate } P = wA\bar{y}$$

$$A = \frac{\pi \times d^2}{4} = \frac{\pi \times 4^2}{4} = 12.57 \text{ m}^2$$

$$\text{Height of oil surface above centre of pipe } (\bar{y}) = h = 22.99 \text{ m}$$

$$P = 9.81 \times 12.57 \times 22.99 = 2834.94 \text{ KN}$$

$$\text{Depth of centre of pressure} \quad \bar{h} = \bar{y} + \frac{I_g}{A\bar{y}}$$

$$I_g = \frac{\pi \times d^4}{64} = \frac{\pi \times 4^4}{64} = 12.57 \text{ m}^4$$

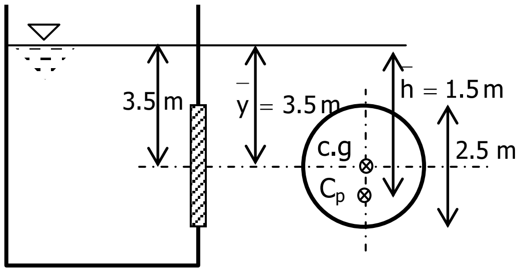
$$\bar{h} = 22.99 + \frac{12.57}{12.57 \times 22.99} = 23.033 \text{ m}$$

December 2011, 10CV35 - ®

A Circular opening 2.5m diameter, in a vertical side of tank is closed by a disc of 2.5m diameter which can rotate about a horizontal diameter. Determine

a) The force on the disk.
b) The torque required to maintain the disc in equilibrium in vertical position when the head of water above the horizontal diameter is 3.5.

Solution: Given



Diameter of opening = 2.5m,

$$\text{Area of opening }(A) = \frac{\pi \times d^2}{4} = \frac{\pi \times 2.5^2}{4} = 4.91\ m^2$$

w = 9.81 kN/m^3

Depth of centre of aperture from the surface.

$$\bar{y} = 3.5\text{m}$$

(i) The force on the disc is given by

$$P = wA\bar{h} = 9.81 \times 4.91 \times 3.5 = 168.60\ \text{kN}$$

(ii) Centre of pressure ($\bar{h}$) is given by

$$\bar{h} = \bar{y} + \frac{I_G}{A\bar{y}}$$

$$I_g = \frac{\pi \times d^4}{64} = \frac{\pi \times 2.5^4}{64} = 1.917\ m^4$$

$$\bar{h} = 3.5 + \frac{1.917}{4.91 \times 3.5} = 3.61\ \text{m}$$

Distance between the centre of pressure and centroid = Lever arm

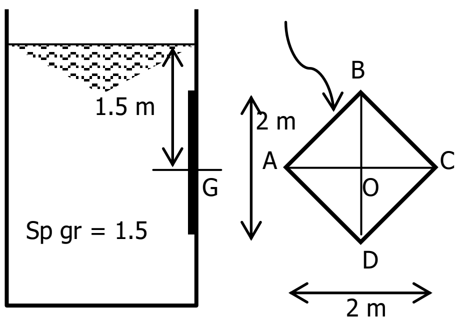
$$= \quad \bar{h} - \bar{y} = 3.61 - 3.5 = 0.11\text{m}$$

$\therefore$ Torque on the disc $= (T) =$ Force $\times$ Lever arm $= 168.60 \times 0.11 = 18.55$ kN - m

December 2010 - 06CV35 – 10 marks: ®

A square aperture in a vertical side of a tank has one diagonal vertical and is completely covered by a plane plate hinged along the upper side of the aperture. The diagonals of aperture are 2 m long and the tank contains a liquid of relative density 1.5. The centre of aperture is 1.5 m below the free surface. Calculate the thrust exerted on plate by the liquid and position of it.

Square Aperture



Solution: Given

Diagonals of aperture AC=BD=2 m

$\therefore$Area of square aperture A= Area of angle ABCD

$$= \frac{AC \times BO}{2} + \frac{AC \times OD}{2} = \frac{2 \times 1}{2} + \frac{2 \times 1}{2} = 1 + 1 = 2.0\text{m}^2$$

Sp.gr. of liquid = 1.5

$\therefore$ w = 1.5 x 9.81 = 14.715 kN/m^3

Depth of centre of aperture from the surface.

$$\bar{y} = 1.5\text{m}$$

i) The thrust on the plate is given by

$$P = wA\bar{h} = 14.715 \times 2 \times 1.5 = 44.145 \text{ kN}$$

ii) Centre of pressure ($\bar{h}$) is given by

$$\bar{h} = \bar{y} + \frac{I_G}{A\bar{y}}$$

Where I_G = M.O.I. of ABCD about diagonal AC.

= M.O.I. of triangle ABC about AC + M.O.I. of triangle ACD about AC.

$$= \frac{AC \times OB^3}{12} + \frac{AC \times OD^3}{12}$$

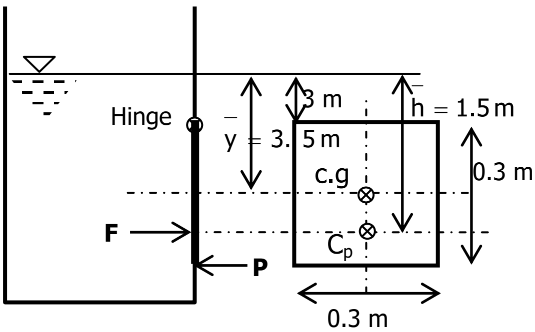
$$= \frac{2 \times 1^3}{12} + \frac{2 \times 1^3}{12} = \frac{1}{6} + \frac{1}{6} = \frac{1}{3}\text{m}^4$$

$$\bar{h} = 1.5 + \frac{1/3}{2 \times 1.5} = 1.5 + \frac{1}{3 \times 2 \times 1.5} = 1.61 \text{ m}$$

Problem: ®

A square door with side dimensions 30 cm is provided in the side wall of a tank which is filled with water of specific weight 9790 N/m^3. What force must be applied at the lower depth of 3 m from the free water surface?

Solution: Given



Size of door = 0.3 m x 0.3 m

Area of door $(A) = 0.3 \times 0.3 = 0.09 \text{ m}^2$

Specific weight = w = 9790 N/m^3 = 9.79 kN/m^3

Depth of centre of aperture from the surface.

$$\bar{y} = 3 + \frac{0.3}{2} = 3.15 \text{ m}$$

(i) The force on the disc is given by

$$P = wA\bar{h} = 9.79 \times 0.09 \times 3.15 = 2.78 \text{ kN}$$

(ii) Centre of pressure ($\bar{h}$) is given by

$$\bar{h} = \bar{y} + \frac{I_G}{A\bar{y}}$$

$$I_g = \frac{b \times d^3}{12} = \frac{0.3 \times 0.3^3}{12} = 6.75 \times 10^{-4} \text{ m}^4$$

$$\bar{h} = 3.15 + \frac{6.75 \times 10^{-4}}{0.09 \times 3.15} = 3.152 \text{ m}$$

Distance between the centre of pressure and hinge $= 3.152 - 3.0 = 0.152$ m

Taking moments about the hinged end,

$$P \times 0.3 = F \times 0.152$$

$$P = \frac{2.78 \times 0.152}{0.3} = 1.41 \text{ kN}$$

Thus a horizontal force equivalent to 1.41 kN should be applied at the lower end to keep the door closed.

Problem:

A tank with vertical sides is square in plan with sides 2.0 m x 2.0 m and depth 1.5 m. The tank contains water upto a height of 0.6 m and an immiscible oil of specific gravity 0.8 floats on the water top for the remaining 0.9 m height. Calculate the hydrostatic pressure force on one vertical side of the tank and height of its centre of pressure above the base.

Solution:

Pressure intensity at top = 0

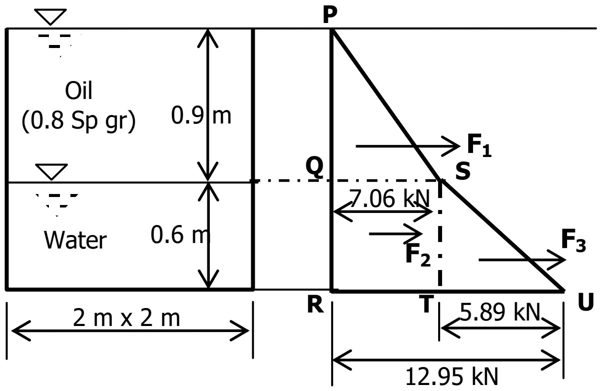
Pressure intensity at the oil – water interface = wh = 0.8 x 9.81 x 0.9

=7.06 kN/m^2

Pressure intensity at the base

= 7.06 + 9.81 x 0.6

= 12.95 kN/m^2



From the pressure diagram,

RT = QS = 7.06 kN/m^2

TU = 12.95 - 7.06 = 5.89 kN/m^2

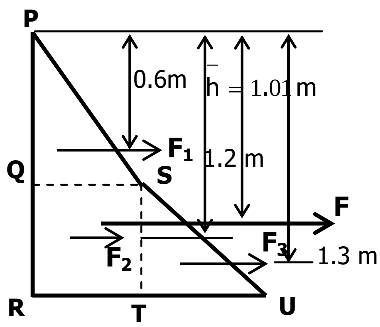
Total force F = F_1 + F_2 + F_3

Pressure force F_1

F_1 = (Area of triangle PQS x Tank width) acting at 2/3rd of 0.9 from free liquid surface

$$F_1 = \left(\frac{1}{2}\times 7.06\times 0.9\right)\times 2 = 6.35 \text{ kN}$$

$$\text{acting at } \frac{2}{3}\times 0.9 = 0.6 \text{ m from FLS}$$



Pressure force F_2

F_2 = (Area of rectangle QSTR x Tank width) acting at 0.9 + 0.6/2 from free liquid surface.

$$F_2 = (7.06\times 0.6)\times 2 = 8.48 \text{ kN} \quad \text{acting at } 0.9+\left(\frac{1}{2}\times 0.6\right) = 1.2 \text{ m from FLS}$$

Pressure force F_3

F_3 = (Area of triangle STU x Tank width) acting at 0.9 + 2/3rd of 0.6 from free liquid surface.

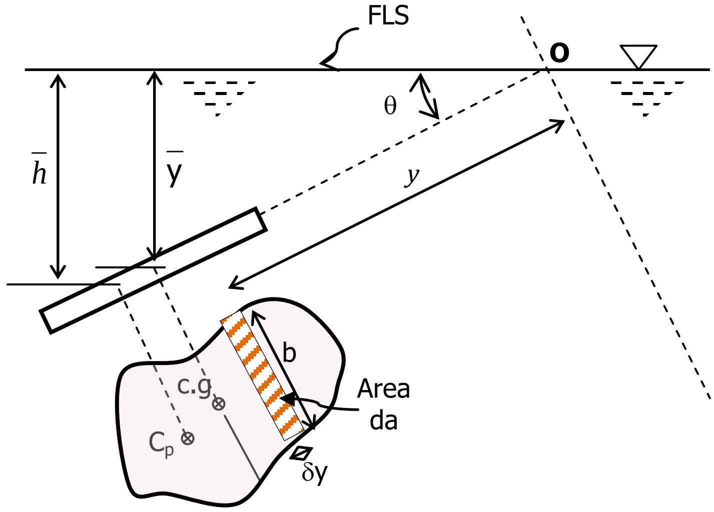
$$F_3 = \left(\frac{1}{2}\times 5.89\times 0.6\right)\times 2 = 3.534 \text{ kN} \quad \text{acting at } 0.9+\frac{2}{3}\times 0.6 = 1.3 \text{ m from FLS}$$

Total force F = F_1 + F_2 + F_3 = 6.35 + 8.48 + 3.534 = 18.724 kN

Position of resultant pressure force F can be obtained by taking moments of relevant forces about the free oil surface.

$$\bar{h} = \frac{6.35\times 0.6 + 8.48\times 1.2 + 3.534\times 1.3}{18.36} = 1.01 \text{ m from FLS}$$

Total Pressure and Centre of pressure for Inclined immersed plane surface:



Let us consider a strip of thickness of δy, width 'b' and at a distance 'y' from 'O' (the point on the liquid surface where the immersed surface will meet, if produced).

The intensity of pressure on the strip = $wy\sin\theta$

Area of the strip (da) = b x δy

∴ Pressure on the strip = Intensity of pressure x Area

$= wy\sin\theta \times b\delta y$

Moment of this pressure about O

$= (wy\sin\theta \times b\delta y)y$

$= wy^2\sin\theta \times b\delta y$

Now sum of moment of all such pressures about O,

$$M = \int wy^2 \sin\theta\, b\delta y$$

$$= w\sin\theta \int y^2 \times b\delta y$$

But $\int y^2 \times b\delta y = I_o =$ Moment of inertia of the surface about the point O, or second moment of area.

∴ $M = w\sin\theta \times I_o$ ……. (i)

We know that the sum of the moments of all such pressures about O is also equal to $\dfrac{P \times \bar{h}}{\sin\theta}$ ………(ii)

Where P = Total pressure on the surface, and

H= Depth of centre of pressure from the liquid surface.

Equating (i) and (ii),

$$\frac{P \times \bar{h}}{\sin\theta} = w\sin\theta \times I_o$$

$$\frac{wA\bar{y} \times \bar{h}}{\sin\theta} = w\sin\theta \times I_o$$

$$\bar{h} = \frac{I_o \times \sin^2\theta}{A\bar{y}}$$

We know from the theorem of parallel axis that

$$I_o = I_G + Ah^2 \quad \text{-----(iii)}$$

Where, I_G = Moment of inertia of the figure about horizontal axis through its centre of gravity, and

h= Distance between 'O' and the centre of gravity of the figure ('y' in the case).

Rearranging the equation (iii),

$$\bar{h} = \frac{\sin^2\theta}{A\bar{y}}(I_G + Ay^2)$$

$$\bar{y} = \frac{\sin^2\theta}{A\bar{y}}\left[I_G + A\left(\frac{\bar{y}}{\sin\theta}\right)^2\right] \qquad \because \sin\theta = \frac{\bar{y}}{y}$$

$$= \frac{I_G \sin^2\theta}{A\bar{y}} + \bar{y}$$

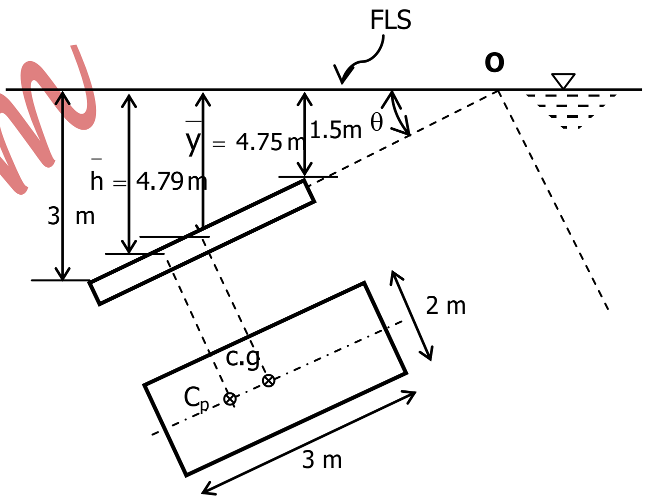
$$\bar{h} = \bar{y} + \frac{I_G \sin^2\theta}{A\bar{y}}$$

Thus the centre of pressure is always below the centre of gravity of the area by a distance equal to $\frac{I_G \sin^2\theta}{A\bar{y}}$.

June 2013 – 10CV35 – 08 marks: ®

A rectangle plate 2m wide and 3m depth is immersed in water such that its ends are at depth of 1.5 m and 3m respectively. Determine the total pressure acting on the plate and locate centre of pressure.

Solution:



$$\sin\theta = \frac{\text{Opposite side}}{\text{Hypotunse}} = \frac{3-1.5}{3} = 0.5$$

$$\theta = \sin^{-1} 0.5 = 30^0$$

Depth of centroid of plate

$$\bar{y} = 4 + \frac{3}{2} \times \sin 30^0 = 4.75 \text{ m}$$

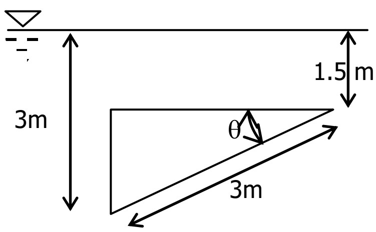
Area of plate A = 2 x 3 = 6 m^2

$$\text{Total pressure on the plate } P = wA\bar{y}$$

$$P = 9.81 \times 6 \times 4.75 = 279.60 \text{ KN}$$

This force acts at centre of pressure whose depth from the free water surface is

$$\text{Depth of centre of pressure} \quad \bar{h} = \bar{y} + \frac{I_g \times \sin^2\theta}{A\bar{y}}$$



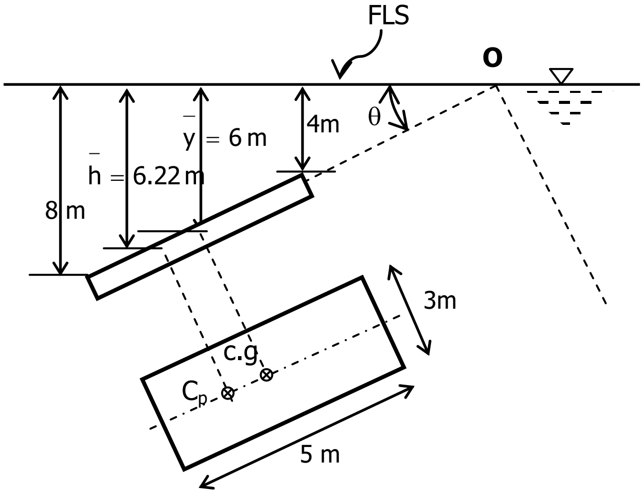
$$I_g = \frac{bd^3}{12} = \frac{2 \times 3^3}{12} = 4.5 \text{ m}^4$$

$$\bar{h} = 4.75 + \frac{4.5 \times (0.5)^2}{6 \times 4.75} = 4.79 \text{ m}$$

June 2012 – 10CV35 – 10 marks: ®

A rectangle gate 5m x 3m is placed under water such that the 3m edges are parallel to the free surface. The top and bottom edges are 4.0m and 8.0m below the water surface respectively. Determine the total pressure and the position of the centre of pressure on the gate.

Solution:



$$\sin\theta = \frac{\text{Opposite side}}{\text{Hypotunse}} = \frac{8-4}{5} = 0.8$$

$$\theta = \sin^{-1} 0.8 = 53.13^0$$

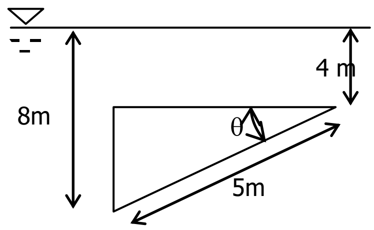
Depth of centroid of plate

$$\bar{y} = 4 + \frac{5}{2} \times \sin 53.13^0 = 6.0 \text{ m}$$

Area of plate A = 3 x 5 = 15 m^2

$$\text{Total pressure on the plate } P = wA\bar{y}$$

$$P = 9.81 \times 15 \times 6 = 882.90 \text{ KN}$$



This force acts at centre of pressure whose depth from the free water surface is

$$\text{Depth of centre of pressure} \quad \bar{h} = \bar{y} + \frac{I_g \times \sin^2\theta}{A\bar{y}}$$

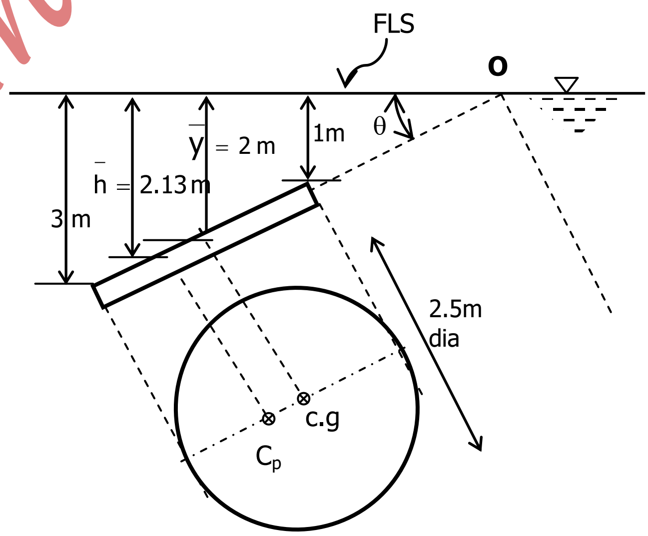
$$I_g = \frac{bd^3}{12} = \frac{3 \times 5^3}{12} = 31.25 \text{ m}^4$$

$$\bar{h} = 6 + \frac{31.25 \times (0.8)^2}{15 \times 6} = 6.22 \text{ m}$$

Dec 2013 / Jan 2014 – 08 marks ®

A circular plate 2.5 m diameter is immersed in water, its greatest and least depth below the free surface being 3 m and 1 m respectively. Find

i) The total pressure on one face of the plate and
ii) The position of centre of pressure.



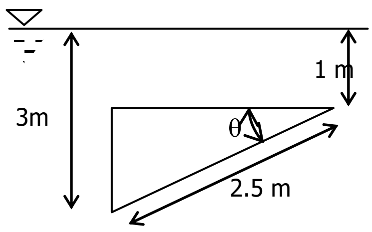
Solution:

$$\sin\theta = \frac{\text{Opposite side}}{\text{Hypotunse}} = \frac{3-1}{2.5} = 0.8$$

$$\theta = \sin^{-1} 0.8 = 53.13^0$$

Depth of centroid of plate

$$\bar{y} = 1 + \frac{2.5}{2} \times \sin 53.13^0 = 2.0 \text{ m}$$



$$\text{Area of plate (A)} = \frac{\pi \times 2.5^2}{4} = 4.91 \text{ m}^2$$

$$\text{Total pressure on the plate } P = wA\bar{y}$$

$$P = 9.81 \times 4.91 \times 2 = 96.33 \text{ KN}$$

This force acts at centre of pressure whose depth from the free water surface is

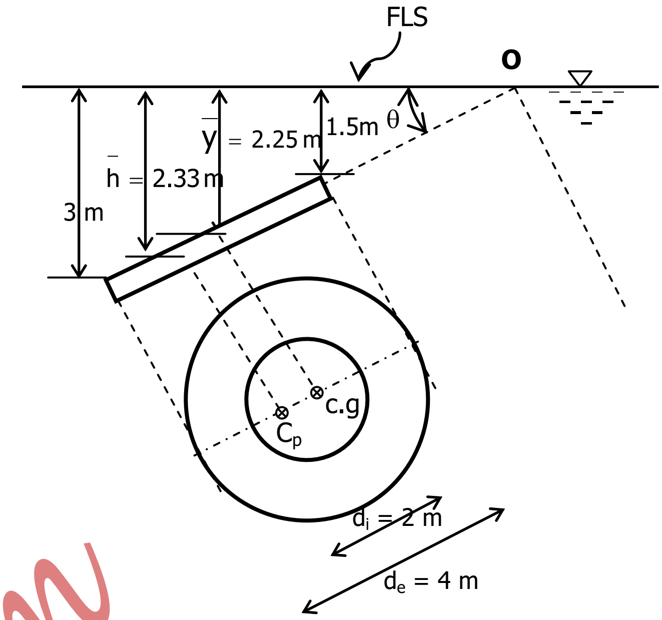
$$\text{Depth of centre of pressure} \quad \bar{h} = \bar{y} + \frac{I_g \times \sin^2\theta}{A\bar{y}}$$

$$I_g = \frac{\pi d^4}{64} = \frac{\pi \times 2.5^4}{64} = 1.92\ m^4$$

$$\bar{h} = 2 + \frac{1.92 \times (0.8)^2}{4.91 \times 2} = 2.13\ m$$

Problem: ®

An annular plate 4m external diameter and 2 m internal diameter with its greatest and least depths below the surface being 3m and 1.5m respectively. Calculate the magnitude, direction and location of the force acting upon one side of the plate due to water pressure.



Solution:

$$\sin\theta = \frac{3-1.5}{4} = 0.375$$

$$\theta = \sin^{-1} 0.375 = 22.02^0$$

Area of the plate

$$A = \frac{\pi}{4}\left(4^2 - 2^2\right) = 9.42\ m^2$$

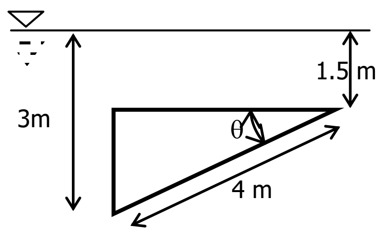
$$\text{Depth of centriod } \bar{y} = \frac{3+1.5}{2} = 2.25\ m$$

$$\text{Total pressure on the plate } P = wA\bar{y}$$

$$P = 9.81 \times 9.42 \times 2.25 = 207.92\ KN$$

This force acts at centre of pressure whose depth from the free water surface is

$$\text{Depth of centre of pressure } \bar{h} = \bar{y} + \frac{I_g \times \sin^2\theta}{A\bar{y}}$$

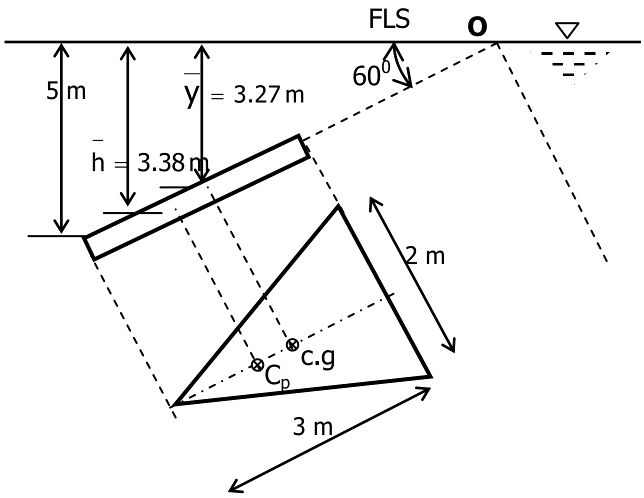


$$I_g = \frac{\pi\left(d_e^2 - d_i^2\right)}{64} = \frac{\pi\left(4^2 - 2^2\right)}{64} = 11.78\ m^4$$

$$\bar{h} = 2.25 + \frac{11.78 \times (0.375)^2}{9.42 \times 2.25} = 2.328\ m$$

Problem: ®

A triangular plate of base width 2m and height 3m is immersed in water with its plan making an angle of 60^0 with the free surface of water. Determine the hydrostatic pressure force and the position of centre of pressure when the apex of the triangle lies 5m below the free water surface.



Solution:

$$A = \frac{1}{2} \times \text{base} \times \text{height} = \frac{1}{2} \times 2 \times 3 = 3\ m^2,$$

Centre of gravity of a triangular plate from apex

$$c\,g = \frac{2}{3} \times \text{height} = \frac{2}{3} \times 3 = 2\text{ m}$$

Depth of centroid of plate

$$\bar{y} = 5 - 2 \times \sin 60^0 = 3.27\text{ m}$$

Total pressure on the plate $P = wA\bar{y}$

$$P = 9.81 \times 3 \times 3.27 = 96.24\text{ kN}$$

This force acts at centre of pressure whose depth from the free water surface is

Depth of centre of pressure $\bar{h} = \bar{y} + \dfrac{I_g \times \sin^2\theta}{A\bar{y}}$

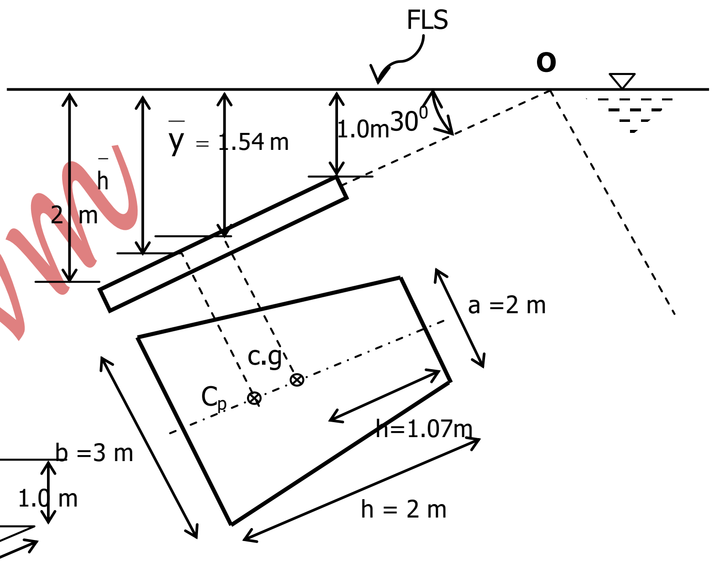
$$I_g = \frac{bh^3}{36} = \frac{2 \times 3^3}{36} = 1.5\text{ m}^4$$

$$\sin 60^0 = 0.866$$

$$\bar{h} = 3.27 + \frac{1.5 \times (0.866)^2}{3 \times 3.27} = 3.38\text{ m}$$

Problem: ®

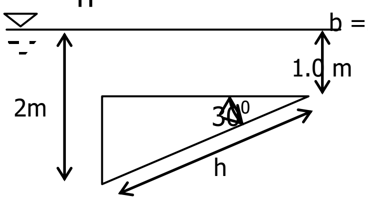
A trapezoidal plate measuring 2m at top edge and 3m at the bottom edge is immersed in water with the plan making an angle of 30^0 to the free surface of water. The top and the bottom edges lie at 1m and 2m respectively from the surface. Determine the hydrostatic force on the plate.



<u>Solution:</u>

$$\sin 30^0 = \frac{\text{Opposite side}}{\text{Hypotunse}} = \frac{2-1}{h}$$

$$h = \frac{2-1}{\sin 30^0} = 2\text{ m}$$



$$\bar{y}_{\text{from top}} = \left(\frac{a+2b}{a+b}\right) \times \frac{h}{3} = \left(\frac{2+2\times 3}{2+3}\right) \times \frac{2}{3} = 1.07\text{ m}$$

Depth of centroid of plate

$$\bar{y} = 1 + 1.07 \times \sin 30^0 = 1.54\text{ m}$$

$$A = \frac{1}{2}(a+b) \times h = \frac{1}{2}(2+3) \times 2 = 5\text{ m}^2,$$

Total pressure on the plate $P = wA\bar{y}$

$$P = 9.81 \times 5 \times 1.54 = 75.54\text{ kN}$$

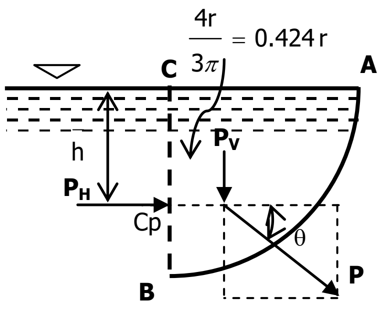
Total pressure on curved surface:

Consider a curved surface AB immersed in a liquid. Let BC be the vertical projection, and AC the horizontal projection of the curved surface.

The horizontal pressure, P_H will be the total horizontal pressure on the projection BC of the curved surface, and will act through the centre of pressure of the surface.

The vertical pressure, P_V will be the total weight of the liquid in the portion ABC, and will act through the centre of gravity of the volume ABC.

Now the total pressure or resultant pressure may be found out by the relation,



Curved surface

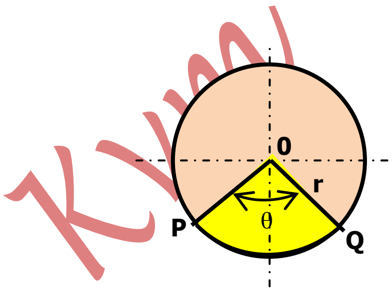
$$P = \sqrt{P_H^{\ 2} + p_V^{\ 2}}$$

The inclination of the resultant pressure with the horizontal will be given by the equation,

$$\tan\theta = \frac{P_V}{P_H}$$

Where 'θ' is the angle, which the resultant pressure makes with the horizontal.

Note: Area of sector POQ is determined as shown in fig.



$$\frac{\text{Area of the sector POQ}}{\text{Area of the circle}} = \frac{\theta}{360^0}$$

$$\text{Area of the sector POQ} = \frac{\theta}{360^0} \times \pi \times r^2$$

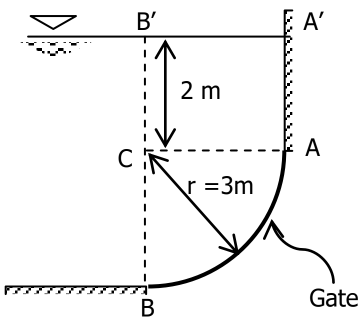
Problem: ®

Figure shows a gate having a quadrant shape of 6m diameter, supporting water. If the gate is 2m long, find the horizontal and vertical components of the resultant pressure on the gate. Find also the resultant force on the gate.

Solution:

Diameter = 6 m, Radius (r) = 3 m, length = 2 m,

Horizontal force on curve surface ACB = Horizontal force acting on area projected on to the vertical plane BC.



$$\text{Total pressure on the Gate (BC)} = wA\bar{y}$$

Area (A) = BC x length = 3 x 2 = 6 m^2

$$w = 9.81\ \text{KN/m}^3,\ \bar{y} = 2 + \frac{3}{2} = 3.5\ \text{m}$$

$$P_H = 9.81 \times 6 \times 3.5 = 206.01\ \text{KN}$$

Vertical force exerted in water (P_V) = Weight of block A′ABCB′ of water

$$\left.\begin{array}{l}\text{Vertical force}\\ \text{exerted in water}\end{array}\right\} P_V = w \times \left\{\begin{array}{c}\text{volume of rectangular portion B'A'AC}\\ + \\ \text{volume of portion CAB}\end{array}\right\}$$

$$\text{Volume of portion CAB} = \text{Area of the sector CAB} \times \text{Length} = \frac{\theta}{360^0} \times \pi \times r^2 \times \text{Length}$$

$$P_V = 9.81 \times \left\{(3 \times 2)\times 2 + \left(\frac{90^0 \times \pi \times 3^2}{360^0}\right) \times 2\right\} = 256.41\ \text{KN}$$

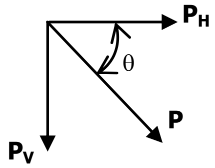
Total pressure or resultant pressure

$$P = \sqrt{P_H^{\,2} + p_V^{\,2}} = \sqrt{206.01^2 + 256.41^2} = 328.92\ \text{kN}$$

The inclination of the resultant pressure with the horizontal will be given by the equation,

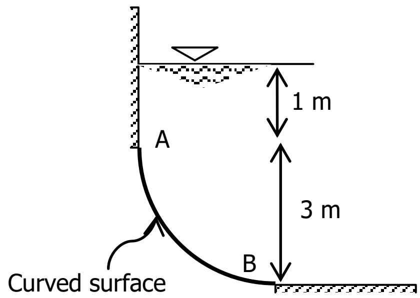
$$\tan\theta = \frac{P_V}{P_H} = \frac{256.41}{206.01} = 1.245$$

$$\theta = \tan^{-1}(1.245) = 51.23^0$$



June -2012- 06CV35 – 10 marks:®

Fig shows a curved surface AB, which is in the form of a quadrant of a circle of radius 3 m, immersed in the water. If the width of the gate is unity, calculate the horizontal component of the total force acting on the curved surface.



Solution:

Radius (r) = 3 m, Length = 1 m

Horizontal force on curve surface ACB = Horizontal force acting on area projected on to the vertical plane BC.

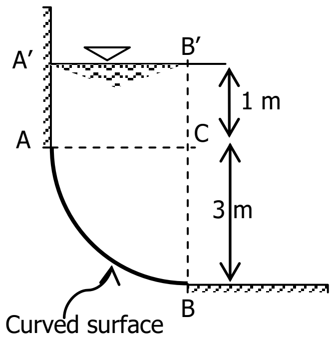
$$\text{Total pressure on the Gate (BC)} = wA\bar{y}$$

Area (A) = BC x length = 3 x 1 = 3 m^2

$$w = 9.81\ \text{KN/m}^3,\ \bar{y} = 1 + \frac{3}{2} = 2.5\ \text{m}$$

$$P_H = 9.81 \times 3 \times 2.5 = 73.58\ \text{KN}$$

Vertical force exerted in water (P_V) = Weight of block A'ABCB' of water



$$\left.\begin{array}{l}\text{Vertical force}\\ \text{exerted in water}\end{array}\right\} P_V = w \times \left\{\begin{array}{c}\text{volume of rectangular portion B'A'AC}\\ + \\ \text{volume of portion CAB}\end{array}\right\}$$

$$\text{Volume of portion CAB} = \text{Area of the sector CAB} \times \text{Length} = \frac{\theta}{360^0} \times \pi \times r^2 \times \text{Length}$$

$$P_V = 9.81 \times \left\{ (3 \times 1) \times 1 + \left(\frac{90^0 \times \pi \times 3^2}{360^0} \right) \times 1 \right\} = 98.77 \text{ KN}$$

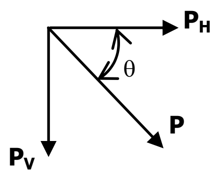
Total pressure or resultant pressure

$$P = \sqrt{{P_H}^2 + {p_V}^2} = \sqrt{73.58^2 + 98.77^2} = 123.16 \text{ kN}$$

The inclination of the resultant pressure with the horizontal will be given by the equation,

$$\tan\theta = \frac{P_V}{P_H} = \frac{98.77}{73.58} = 1.34$$

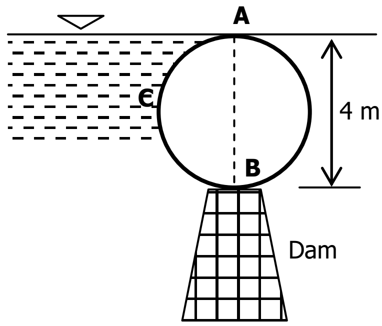
$$\theta = \tan^{-1}(1.34) = 53.27^0$$



June/July 2016/10CV35: ®

Find the magnitude and direction of the resultant force due to water acting on a roller gate of cylindrical form of 4m diameter, when the gate is placed on the dam in such a way that water is just going to spill. Take the length of the gate as 8 m.

Solution:



Diameter = 4 m,

Horizontal force on curve surface ACB = Horizontal force acting on area projected on to the vertical plane AB.

$$\text{Total pressure on the Gate (AB)} = wA\bar{y}$$

Area (A) = AB x length = 4 x 8 = 32 m^2

$$w = 9.81 \text{ KN/m}^3, \quad \bar{y} = \frac{AB}{2} = \frac{4}{2} = 2 \text{ m}$$

$$P_H = 9.81 \times 32 \times 2 = 627.84 \text{ KN}$$

Vertical force due to water (P_V) = Weight of imaginary water lying in the curved portion ACB.

$$\left.\begin{array}{l}\text{Vertical force}\\ \text{exerted in water}\end{array}\right\} P_V = w \times \text{volume} = w \times \text{area of ACB portion} \times \text{Length}$$

$$P_V = 9.81 \times \left\{ \frac{1}{2} \times \left(\frac{\pi \times 4^2}{4} \right) \right\} \times 8 = 493.10 \text{ KN}$$

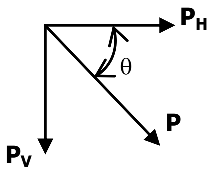
Total pressure or resultant pressure

$$P = \sqrt{{P_H}^2 + {p_V}^2} = \sqrt{627.84^2 + 493.10^2} = 798.33 \text{ kN}$$

The inclination of the resultant pressure with the horizontal will be given by the equation,

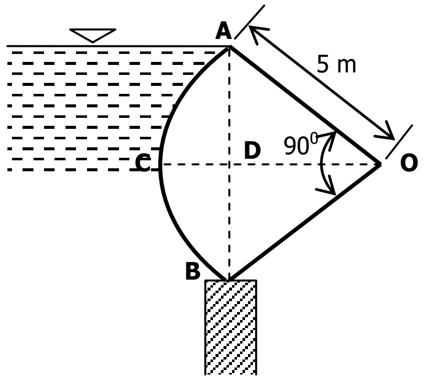
$$\tan\theta = \frac{P_V}{P_H} = \frac{493.10}{627.84} = 0.785$$

$$\theta = \tan^{-1}(0.785) = 38.15^0$$



Problem: ®

A tainter gate of 90^0 sectors is subjected to water pressure.
Determine horizontal and vertical pressure acting on the force of the gate.



Solution:

Radius (r) = 5 m, $\theta = 90^0$, Length = 1m. (Assumed per metre length)

Horizontal force on curve surface ACB = Horizontal force acting on area projected on to the vertical plane AB.

$$AB = AD + DB$$

$$\text{Sin } 45^0 = \frac{AD}{AO}$$

$$AD = AO \times \sin 45^0 = 5 \times \sin 45^0 = 3.54 \text{ m} = DB$$

$$AB = 3.54 + 3.54 = 7.08 \text{ m}$$

$$\text{Total pressure on the Gate } (AB) = wA\bar{y}$$

Area (A) = AB x length = 7.08 x 1 = 7.08 m^2

$$\bar{y} = \frac{7.08}{2} = 3.54 \text{ m}$$

$$P_H = 9.81 \times 7.08 \times 3.54 = 245.87 \text{ KN}$$

Vertical pressure due to water = Weight of imaginary water lying in the curved portion ACB.

$$\left.\begin{array}{l}\text{Vertical force}\\ \text{exerted in water}\end{array}\right\} P_V = w \times \text{volume} = w \times \text{area of ACB portion} \times \text{Length}$$

$$P_V = w \times \{\text{Area of sector ACBO - Area of Triangle AOB}\} \times \text{Length}$$

$$P_V = 9.81 \times \left\{ \frac{\theta \times \pi \times r^2}{360} - \frac{1}{2} \times AB \times DO \right\} \times 1$$

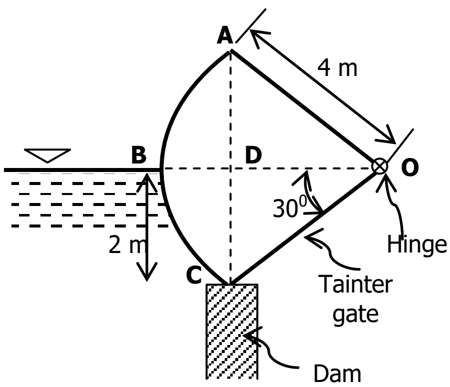
$$AB = 7.08 \text{ m}$$

$$DO = AO \times \cos 45^0 = 5 \times \cos 45^0 = 3.54 \text{ m}$$

$$P_V = 9.81 \times \left\{ \frac{90^0 \times \pi \times 5^2}{360^0} - \frac{1}{2} \times 7.08 \times 3.54 \right\} \times 1 = 7.10 \text{ KN}$$

Problem: ®

Figure shows a tainter gate retaining water, Calculate for 1 meter run of the gate the horizontal and vertical components of the force exerted on the gate.



Solution:

Radius of tainter (r) = 4 m, $\theta = 30^0$

Horizontal force on curve surface BC = Horizontal force acting on area projected on to the vertical plane DC

$$\text{Horizontal pressure force } (P_H) = wA\bar{y}$$

Area of the gate per metre length

A = DC x length = 2 x 1 = 2 m^2

$$\bar{y} = \frac{2}{2} = 1\,\text{m}$$

$$P_H = 9.81 \times 2 \times 1 = 19.62\ \text{KN}$$

Vertical force (upward force) = Weight of block BCD of water.

$$\left.\begin{array}{l}\text{Vertical force}\\ \text{exerted in water}\end{array}\right\} P_V = w \times \text{volume} = w \times \text{area of BCD portion} \times \text{Length}$$

$$P_V = w \times \{\text{Area of sector OBC - Area of Triangle DOC}\} \times \text{Length}$$

$$P_V = 9.81 \times \left\{ \frac{\theta \times \pi \times r^2}{360} - \frac{1}{2} \times OD \times DC \right\} \times 1$$

$$P_V = 9.81 \times \left\{ \frac{30^0 \times \pi \times 4^2}{360^0} - \frac{1}{2} \times 4 \times \cos 30^0 \times 2 \right\} \times 1 = 7.11\ \text{KN}$$

Problem: ®

For the tainter gate shown in the figure compute

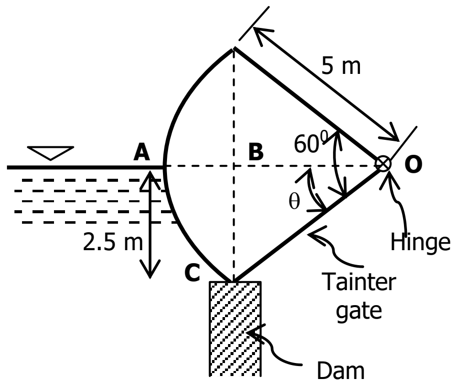
(i) The total horizontal push of Water on the gate.
(ii) The total vertical component of water pressure on the gate.
(iii) The resultant water pressure on the gate.

It may be assumed that the gate has a length of 0.8m perpendicular to the plane of the paper.

Solution:

Radius of tainter (r) = 5 m, θ = 30^0

Horizontal force on curve surface AC = Horizontal force acting on area projected on to the vertical plane BC



Horizontal pressure force $(P_H) = wA\bar{y}$

Area of the gate for 0.8 m length

A = BC x length = 2.5 x 0.8 = 2 m^2

$$\bar{y} = \frac{2.5}{2} = 1.25\,\text{m}$$

$$P_H = 9.81 \times 2 \times 1.25 = 24.53\ \text{KN}$$

Vertical force (upward force) = Weight of block ACB of water.

$$\left.\begin{array}{l}\text{Vertical force}\\ \text{exerted in water}\end{array}\right\} P_V = w \times \text{volume} = w \times \text{area of ACB portion} \times \text{Length}$$

$$P_V = w \times \{\text{Area of sector OAC - Area of Triangle OBC}\} \times \text{Length}$$

$$P_V = 9.81 \times \left\{ \frac{\theta \times \pi \times r^2}{360} - \frac{1}{2} \times OB \times BC \right\} \times 0.8$$

$$P_V = 9.81 \times \left\{ \frac{30^0 \times \pi \times 5^2}{360^0} - \frac{1}{2} \times 5 \times \cos 30^0 \times 2.5 \right\} \times 0.8 = 8.89\ \text{KN}$$

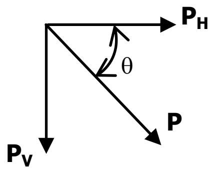
Total pressure or resultant pressure

$$P = \sqrt{P_H^{\ 2} + p_V^{\ 2}} = \sqrt{24.53^2 + 8.89^2} = 26.10\ \text{kN}$$

The inclination of the resultant pressure with the horizontal will be given by the equation,

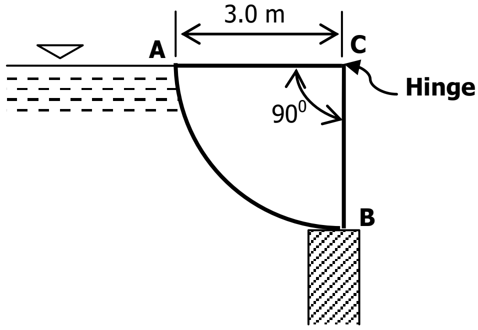
$$\tan\theta = \frac{P_V}{P_H} = \frac{8.89}{24.53} = 0.362$$

$$\theta = \tan^{-1}(0.362) = 19.90^0$$



Problem: ®

Find the resultant pressure due to water per metre length acting on the gate of radius 3 m as shown in fig. Also find out the angle at which the total pressure will act.



Solution:

Radius of gate (r) = 3.0 m, θ = 90^0, length = 1m.

Horizontal force on curve surface AB = Horizontal force acting on area projected on to the vertical plane BC = AC = 3.0m.

Horizontal pressure force $(P_H) = wA\bar{y}$

Area of the gate per m length

A = BC x length = 3.0 x 1 = 3.0 m^2

$$\bar{y} = \frac{3.0}{2} = 1.5\text{ m}$$

$$P_H = 9.81 \times 3 \times 1.5 = 44.15\text{ KN}$$

Vertical force (upward force) = Weight of block ABC of water.

$$\left.\begin{matrix}\text{Vertical force}\\ \text{exerted in water}\end{matrix}\right\} P_V = w \times \text{volume} = w \times \text{area of ABC portion} \times \text{Length}$$

$$P_V = 9.81 \times \left\{\frac{\theta \times \pi \times r^2}{360}\right\} \times 1$$

$$P_V = 9.81 \times \left\{\frac{90^0 \times \pi \times 3.0^2}{360^0}\right\} \times 1 = 69.34\text{ KN}$$

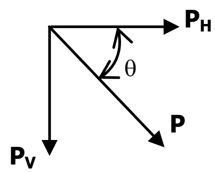
Total pressure or resultant pressure

$$P = \sqrt{P_H{}^2 + p_V{}^2} = \sqrt{44.15^2 + 69.34^2} = 82.20\text{ kN}$$

The inclination of the resultant pressure with the horizontal will be given by the equation,

$$\tan\theta = \frac{P_V}{P_H} = \frac{69.34}{44.15} = 1.57$$

$$\theta = \tan^{-1}(1.57) = 57.51^0$$



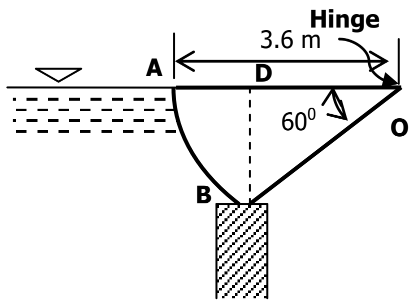
Problem: ®

A 60^0 sector gate of 3.6m radius is mounted on the spillway of a dam. Its hinge and one of its end radius arms are at the same horizontal level as the water surface. What is the magnitude and direction of the resultant pressure on the gate, if the length of the gate is 3m.

Solution:

Radius of gate (r) = 3.6 m, θ = 60^0, length = 3m.

Horizontal force on curve surface AB = Horizontal force acting on area projected on to the vertical plane BD = BO x cos 60^0 = 3.6 x cos 60^0 = 3.12 m.



$$\text{Horizontal pressure force } (P_H) = w A \bar{y}$$

Area of the gate for 3 m length

A = BD x length = 3.12 x 3= 9.36m^2

$$\bar{y} = \frac{3.12}{2} = 1.56 \text{ m}$$

$$P_H = 9.81 \times 9.36 \times 1.56 = 143.24 \text{ KN}$$

Vertical force (upward force) = Weight of block AB of water.

$$\left.\begin{matrix}\text{Vertical force}\\ \text{exerted in water}\end{matrix}\right\} P_V = w \times \text{volume} = w \times \text{area of ABD portion} \times \text{Length}$$

$$P_V = w \times \{\text{Area of sector OAB - Area of Triangle ODB}\} \times \text{Length}$$

$$P_V = 9.81 \times \left\{ \frac{\theta \times \pi \times r^2}{360} - \frac{1}{2} \times OD \times BD \right\} \times 3$$

$$\cos 60^0 = \frac{OD}{BO} \quad \Rightarrow \quad OD = BO \times \cos 60^0 = 3.6 \times \cos 60^0 = 1.8 \text{ m}$$

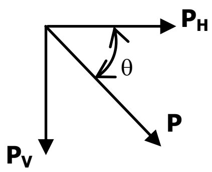
$$\sin 60^0 = \frac{BD}{BO} \quad \Rightarrow \quad BD = BO \times \sin 60^0 = 3.6 \times \sin 60^0 = 3.12 \text{ m}$$

$$P_V = 9.81 \times \left\{ \frac{60^0 \times \pi \times 3.6^2}{360^0} - \frac{1}{2} \times 1.8 \times 3.12 \right\} \times 3 = 117.07 \text{ KN}$$

Total pressure or resultant pressure

$$P = \sqrt{P_H{}^2 + p_V{}^2} = \sqrt{143.24^2 + 117.07^2} = 185 \text{ kN}$$

The inclination of the resultant pressure with the horizontal will be given by the equation,



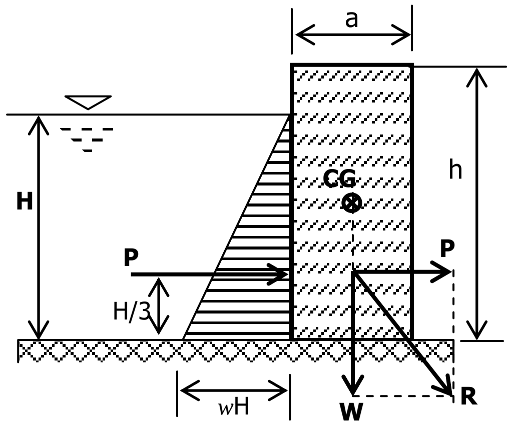
$$\tan\theta = \frac{P_V}{P_H} = \frac{117.07}{143.24} = 0.817$$

$$\theta = \tan^{-1}(0.817) = 39.26^0$$

Water pressure on gravity dams:
The dams are constructed in order to store large quantities of water, for the purpose of irrigation and power generation. A dam may be of any cross-section, but the following are important from the subjected point of view.

1. Rectangular dams.
2. Trapezoidal dams.

<u>Water pressure on rectangular dams.</u>



$$\text{Total pressure of water } (P) = \frac{1}{2} \times wH \times H$$

$$P = \frac{wH^2}{2}$$

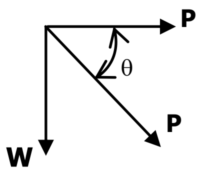
H = Height of water stored by the dam.
Pressure P acts at a distance H/3 from base.

W = weight of the masonry (concrete) dam per metre length.
W = volume x density = breadth(a) x depth(h) x 1 x Density of dam

Resultant pressure of the force (P) and weight (W) is given by the relation.

$$R = \sqrt{P^2 + W^2}$$

The inclination of the resultant pressure with the horizontal will be given by the equation,

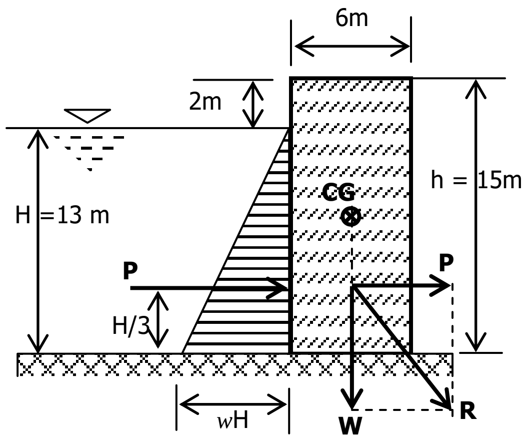


$$\tan\theta = \frac{W}{P} \qquad \Rightarrow \theta = \tan^{-1}\left(\frac{W}{P}\right)$$

Problem:
A concrete dam of rectangular section, 15 metres high and 6 metres wide has water standing 2 metres below its top. Find:

a) Total pressure of water on one metre length of the dam.
b) Height of centre of pressure above base.
c) Resultant force.

Assume weight of concrete = 25.30 KN/m^3



Solution:
a = 6 m h = 15 m, H = 15 - 2 = 13 m,
w_m = 25.30 kN/m^3, w_w = 9.81 kN/m^3

$$\text{Total pressure of water } (P) = \frac{w_w H^2}{2}$$

$$P = \frac{9.81 \times 13^2}{2} = 828.95 \text{ kN}$$

Total pressure acts at H/3 = 13/3 = 4.33 m from base.

$$\text{Weight of masonry } (W) = a \times h \times 1 \times \text{Density}$$

$$W = 6 \times 15 \times \times 1 \times 25.30 = 2277 \text{ kN}$$

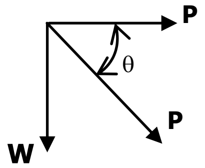
Resultant pressure of the force (P) and weight (W) is given by the relation.

$$R = \sqrt{P^2 + W^2} = \sqrt{828.95^2 + 2277^2} = 2423.20 \text{ kN}$$

The inclination of the resultant pressure with the horizontal will be given by the equation,

$$\tan\theta = \frac{W}{P} = \frac{2277}{828.95} = 2.75$$

$$\theta = \tan^{-1}(2.75) = 70^0$$

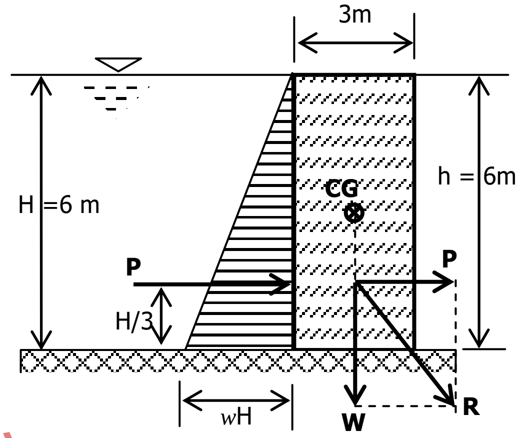


Problem:

A masonry dam 6 m high and 3 m wide has water level with its top. Find:

a) Total pressure of water on one metre length of the dam.
b) Depth of centre of pressure.
c) Resultant force.

Assume weight of masonry = 20 KN/m^3



Solution:

a = 3 m, H = h = 6 m,

w_m = 20 kN/m^3, w_w = 9.81 kN/m^3

$$\text{Total pressure of water }(P) = \frac{w_w H^2}{2}$$

$$P = \frac{9.81 \times 6^2}{2} = 176.58 \text{ kN}$$

Total pressure acts at H/3 = 6/3 = 2 m from base.

$$\text{Weight of masonry }(W) = a \times h \times 1 \times \text{Density}$$

$$W = 3 \times 6 \times 1 \times 20 = 360 \text{ kN}$$

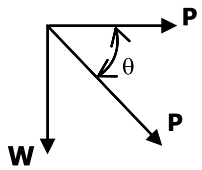
Resultant pressure of the force (P) and weight (W) is given by the relation.

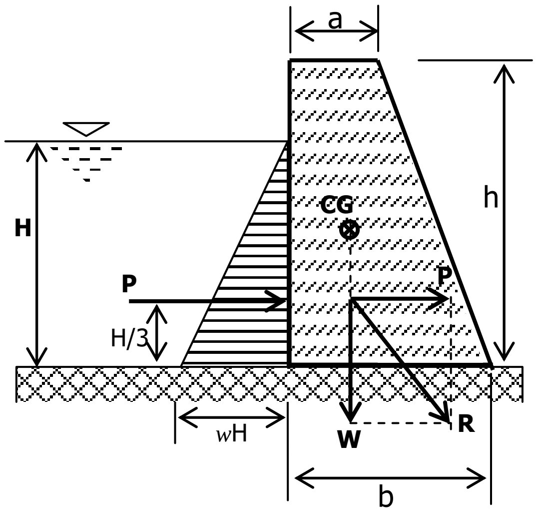
$$R = \sqrt{P^2 + W^2} = \sqrt{176.58^2 + 360^2} = 400.97 \text{ kN}$$

The inclination of the resultant pressure with the horizontal will be given by the equation,

$$\tan\theta = \frac{W}{P} = \frac{360}{176.58} = 2.04$$

$$\theta = \tan^{-1}(2.04) = 63.87^0$$





Water pressure on Trapezoidal dams:

A trapezoidal dam is more economical and also easier to construct than rectangular dam.

$$\text{Total pressure of water } (P) = \frac{1}{2} \times wH \times H$$

$$P = \frac{wH^2}{2}$$

H = Height of water stored by the dam.
Pressure P acts at a distance H/3 from base.

W = weight of the masonry (concrete) dam per metre length.
W = volume x density

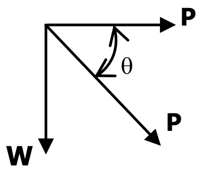
$$W = \left\{\frac{1}{2} \times (\text{Sum of the parallel sides}) \times \text{perpendicular distance between them}\right\} \times 1 \times \text{Density}$$

$$W = \left\{\frac{1}{2} \times (a + b) \times h\right\} \times 1 \times \text{Density}$$

Resultant pressure of the force (P) and weight (W) is given by the relation.

$$R = \sqrt{P^2 + W^2}$$

The inclination of the resultant pressure with the horizontal will be given by the equation,

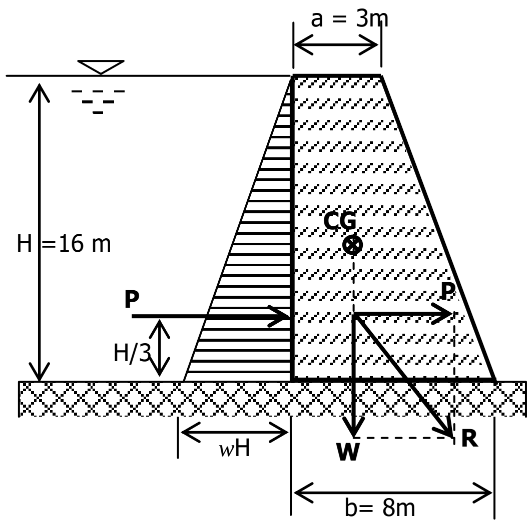


$$\tan\theta = \frac{W}{P} \qquad \Rightarrow \theta = \tan^{-1}\left(\frac{W}{P}\right)$$

Problem:

A concrete dam of horizontal section having water on vertical face is 16 m high. The base of the dam is 8 m wide and top 3 m wide. Find the resultant thrust on the base per metre length of the dam.

Take weight of masonry as 24 kN/m^3 and the water level coincides with the top of the dam.



Solution:

Given: a = 3 m, b = 8 m, H = h = 16 m,
w_m = 24 kN/m^3, w_w = 9.81 kN/m^3

$$\text{Total pressure of water } (P) = \frac{w_w H^2}{2}$$

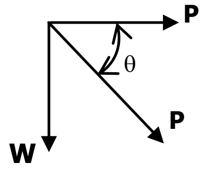
$$P = \frac{9.81 \times 16^2}{2} = 1255.68 \text{ kN}$$

$$\text{Weight of masonry } (W) = \left\{\frac{1}{2} \times (a + b) \times h\right\} \times 1 \times \text{Density}$$

$$W = \left\{\frac{1}{2} \times (3 + 8) \times 16\right\} \times 1 \times 24 = 2112 \text{ kN}$$

Resultant pressure of the force (P) and weight (W) is given by the relation.

$$R = \sqrt{P^2 + W^2} = \sqrt{1255.68^2 + 2112^2} = 2457.10 \text{ kN}$$



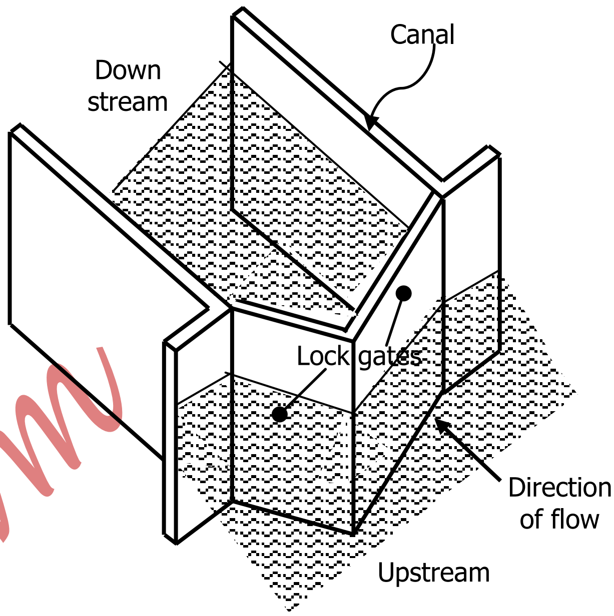
The inclination of the resultant pressure with the horizontal will be given by the equation,

$$\tan\theta = \frac{W}{P} = \frac{2112}{1255.68} = 1.68$$

$$\theta = \tan^{-1}(1.68) = 59.27^0$$

Water pressure on lock gates:

Whenever a dam or a weir is constructed across a river or a canal, the water levels on both the sides of the dam will be different. If it is desired to have navigation or boating in such a river or a canal, then a chamber, known as lock, is constructed between these two different water levels. Two sets of lock gates (one on the upstream side and the other on downstream side of the dam) are provided.



In order to transfer a boat from the upstream (i.e., from a higher water level to the downstream) (i.e., to a lower water level) the upstream gates are opened (while the downstream gates are closed) and water level in the chamber rises up to the upstream water level. The boat is then admitted in the chamber. Then upstream gates are closed and downstream gates are opened and the water level in the chamber is lowered to the downstream water level. Now the boat can proceed further downwards. If the boat is to be transferred from downstream to upstream side, the above procedure is reversed.