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# 1 Introduction

The widespread application of adhesively bonded joints has necessitated the development of methodology to predict ultimate static joint strength and service life under cyclic loading. Due to the complexity of mathematically modelling adhesive joint response, analytical treatments are limited to highly idealized joint configurations and simplified assumed stress states, applied loading and material behavior. To overcome these limitations, a specialized finite element-based numerical approach is advocated to provide a versatile approach to analyze actual bonded joint concepts with complex geometries, load paths and support conditions. To enhance a finite-element based methodology, special 2-D and 3-D layered elements have been developed in Reference [1] to improve the computational efficiency and accuracy of determining stresses in adhesive joints. The special 2-D and 3-D layered continuum elements are formulated using the hybrid stress technique to explicitly enforce stress equilibrium throughout the element domain and stress continuity conditions at layer interfaces. In an extensive investigation presented in Reference [1], optimum element configurations have been determined and demonstrate improved performance compared to standard displacement-based finite elements in predicting joint stresses.

This report details the use of several special 2-D and 3-D continuum elements in the commercial finite element code ABAQUS through a developed user-defined subroutine. The elements are specialized for the analysis of adhesive joints by incorporating a layered hybrid formulation to accurately model the adhesive/adherend interface and are, thus, referred to as 'adhesive elements'. The adhesive elements are currently restricted to linear elastic behavior and a geometric constraint is imposed which requires that all element layers are rectangular. To permit the representation of composite laminate adherends and property variation through the adhesive layer, material properties are input as orthotropic laminae within each element layer. In addition, 2-D adhesive elements are supported for arbitrary orientation in the global X-Y plane and 3-D elements may be arbitrarily oriented in space. Element stress and strain output may be selected in either global or local coordinate systems.

A brief description of the support of user-defined elements in ABAQUS is presented in the next section followed by a description of the input format established for the adhesive elements. The basic element library is discussed in subsequent sections detailing element configuration, coordinate system convention and comments on their use. Two illustrative numerical examples are presented demonstrating the use of selected 2-D and 3-D adhesive elements. Sample input and output datasets together with the complete FORTRAN source code performing all element computations are presented in separate appendices.

## 2 User-Defined Elements in ABAQUS

New finite elements may be used with ABAQUS via a subroutine denoted *UEL* (for *U*ser *E*lement) which performs the necessary element computations and interfaces with the main ABAQUS program through a standardized parameter list in the subroutine call statement.

The *\*USER SUBROUTINE* statement in the input deck alerts ABAQUS to the presence of user-defined subroutines which either immediately follow this data entry or are contained in a separate file. These subroutines are then compiled and linked to the main ABAQUS executable prior to job execution. A complete description of this and other user-defined capabilities in ABAQUS may be found in [2]. Shown in Figure 1 is the basic format of the *UEL* subroutine with the argument list used by ABAQUS to pass into the user-defined subroutine all necessary information needed to compute element stiffness matrices. Once computed, these matrices are then passed back to ABAQUS for global assembly and problem solution. In static analysis, data recovery is performed during a second pass through the user-defined subroutine after the solution for global displacements has been obtained. During this phase, ABAQUS passes in the nodal displacements for the current element from which all element stresses and strains can be computed. In addition to linear static analysis, the information passed into the *UEL* subroutine is sufficient to support material and geometric nonlinear analysis.

The complete source code supporting linear static analysis for the special adhesive elements in ABAQUS is listed in Appendix A.

```

SUBROUTINE UEL(RHS,AMATRX,SVARS,ENERGY,NDOFEL,NRHS,NSVARS,
1          PROPS,NPROPS,COORDS,MCRD,NNODE,U,DU,V,A,
2          JTYPE,TIME,DTIME,KSTEP,KINC,JELEM,PARAMS,
3          NDLOAD,JDLTYP,ADLMAG,PREDEF,NPREDF,LFLAGS,
4          MLVARX,DDL MAG,MDLOAD,PNEWDT )

C
C      IMPLICIT DOUBLE PRECISION (A-H,O-Z)
C
C      VARIABLE AND ARRAY DECLARATIONS FOR UEL/ABAQUS INTERFACE
C
      DIMENSION RHS(MLVARX,*),AMATRX(NDOFEL,NDOFEL),PROPS(*),
1          SVARS(1),ENERGY(8),COORDS(MCRD,NNODE),U(NDOFEL),
2          DU(MLVARX),V(NDOFEL),A(NDOFEL),TIME(2),
3          PARAMS(*),JDLTYP(MDLOAD,*),ADLMAG(MDLOAD,*),
4          DDL MAG(MDLOAD,*),PREDEF(2,NPREDF,NNODE),LFLAGS(5)

      *****
      *** Source code for user-defined elements ***
      *****

      RETURN
      END

```

Figure 1. Format of the user-defined subroutine *UEL* supporting the special adhesive elements in ABAQUS.

### 3 Use of Special Adhesive Elements in ABAQUS

Three sets of statements are used to describe the adhesive elements in the ABAQUS input deck. Each of these statements may be identified in the sample input files presented in Appendices B and C. The first set, *\*USER ELEMENT*, defines the basic parameters of the user elements. All parameters are mandatory and the user must set the *N*, *n* and *M* values as described below.

**Statement set I:**

- (i) *\*USER ELEMENT, NODES = N, TYPE = Un, PROPERTIES = M*
- (ii) *n<sub>1</sub>, n<sub>2</sub>, ...*

where the various input parameters are:

**Card (i):** *NODES = N* specifies the total number of nodes present in the adhesive element selected. *TYPE = Un* specifies the internal designation of the element through setting the value of *n*. The element type designation, *n*, will be given in the discussions of the various elements in following sections.

*PROPERTIES = M* specifies that a user-defined property list of length *M* is to established for each element as explained below.

**Card(ii):** This entry indicates the active degrees of freedom at each node in the element. For 2-D elements, this list is: 1,2; for 3-D elements, this list is: 1,2,3.



The second entry, *\*UEL PROPERTY*, is the primary list of input data used to compute element quantities for each adhesive element. The material input has been made general to allow the input of composite laminate material for the adherends, or to specify property variations in the adhesive layer. The size of this list is determined by the user as function of the total number of plies in each of the element layers. Only a single ply would be specified for a homogeneous material whereas any number may be specified to define laminate properties. ABAQUS requires for each line in the property list that all quantities be expressed as real numbers in free format with up to eight entries per line - missing entries are simply treated as zeros. In the basic format of the property list shown below, the total length of the property list is calculated as

$$M = 8(1 + \sum_i^k [2NLAY_i + 1])$$

where  $k$  is equal to the number of layers in the element and  $NLAY_i$  is the number of plies used within the  $i^{th}$  layer. This length is then entered as a parameter on the *\*USER ELEMENT* entry.

### Statement set II

- (i) *\*UEL PROPERTY, ELSET = NM*
- (ii) *NVER, IPLANE, OUTPUT, NSIDE*
- (iii) *NPLY<sub>1</sub>, WIDTH<sub>1</sub>*
- (iv) *PTHK<sub>1</sub>,  $\Theta_1$ ,  $E_1$ ,  $E_2$ ,  $E_3$ ,  $\mu_{12}$ ,  $\mu_{23}$ ,  $\mu_{13}$*
- (v)  *$G_{12}$ ,  $G_{23}$ ,  $G_{31}$*
- (vi) *NPLY<sub>2</sub>, WIDTH<sub>2</sub>*
- (vii) *PTHK<sub>2</sub>,  $\Theta_2$ ,  $E_1$ ,  $E_2$ ,  $E_3$ ,  $\mu_{12}$ ,  $\mu_{23}$ ,  $\mu_{13}$*
- (viii)  *$G_{12}$ ,  $G_{23}$ ,  $G_{31}$*
- (ix) *NPLY<sub>3</sub>, WIDTH<sub>3</sub>*
- (x) *PTHK<sub>3</sub>,  $\Theta_3$ ,  $E_1$ ,  $E_2$ ,  $E_3$ ,  $\mu_{12}$ ,  $\mu_{23}$ ,  $\mu_{13}$*
- (xi)  *$G_{12}$ ,  $G_{23}$ ,  $G_{31}$*

where the input parameters are defined by:

- Card(i): *NM* is the set Id of the adhesive element for which the following properties are to be used.
- Card(ii): *NVER* designates a particular version of an element type.  
*IPLANE* is used to select plane stress/plane strain assumptions in the use of 2-D elements.  
 For 3-D elements, this field is ignored.  
*IPLANE* = 1 for plane stress.  
*IPLANE* = 2 for plane strain.  
*OUTPUT* is the element output control flag.  
*OUTPUT* = 0 for suppression of element data output.  
*OUTPUT* = 1 for output of stresses and strains in local element coordinates.  
*OUTPUT* = 2 for output of stresses and strains in global coordinates.  
*NSIDE* indicates the face on which zero tractions are explicitly enforced. This property is only recognized by the element types which support this option.
- Card(iii): *NPLY<sub>1</sub>* is the number of plies in layer 1.  
*WIDTH<sub>1</sub>* is the width of layer number 1 in 2-D elements. The width dimension is defined as normal to the element plane. This entry is left blank in 3-D elements.
- Card(iv): *PTHK<sub>1</sub>* is the ply thickness for the first ply in layer 1.  
 $\Theta_1$  is the orientation of the first ply in layer 1.  
 $E_1 - \mu_{13}$  are layer Youngs moduli and Poisson ratios.
- Card(v):  $G_{12} - G_{31}$  are layer shear moduli.

Cards (iv) and (v) are repeated for each ply specified. The data block represented by cards (vi) through (viii) follow the same format. The data block represented by cards (ix) through (xi) are used only if a third element layer is present in the element.

The last entry is the *\*USER SUBROUTINE* statement. As stated above, this alerts ABAQUS to the presence of source code which is to be included together with the main executable code prior to running the requested job. This data statement is given by

Statement set III:

(i) *\*USER SUBROUTINE, INPUT = uel\_hybrid.f*

where the optional parameter, *INPUT*, specifies the name of an external file containing the source code for the user-defined adhesive elements. If this parameter is omitted, ABAQUS assumes that the source code immediately follows this statement.

The library of adhesive elements is discussed below.

## 4 Special Adhesive Elements

The element library presented herein contains several 2-D and 3-D special adhesive elements for general use in the analysis of bonded joint stresses. These elements are assumed to be used specifically for the numerical representation of the local region encompassing the adhesive bond with standard elements representing all other regions of the joint adherends. The use of 2-layer and 3-layer elements in modelling the bond layer is depicted in figures 2 and 3. Specific details of the specialized adhesive elements are described below.

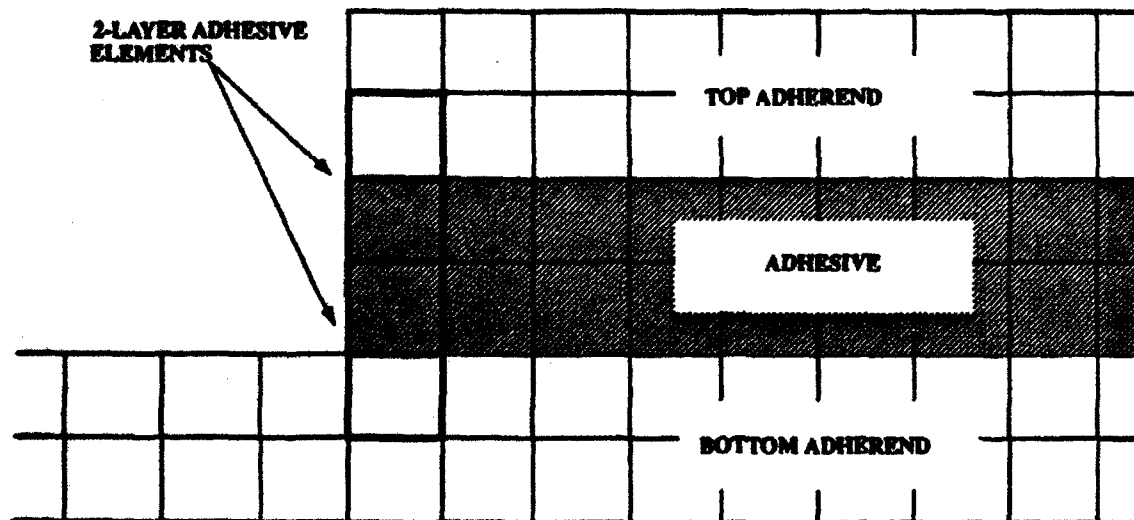


Figure 2. Use of the 2-layered elements in modelling an adhesive layer.

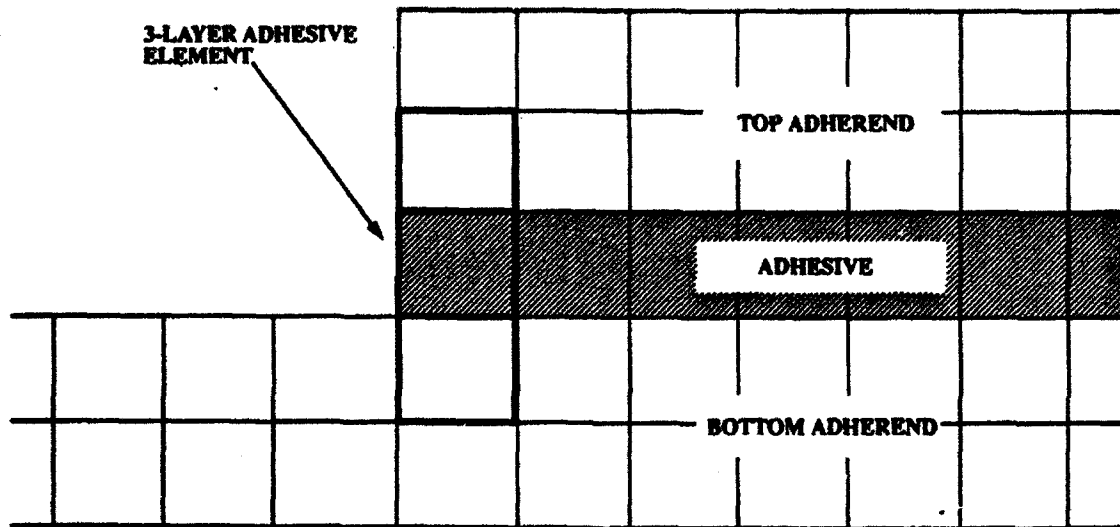


Figure 3. Use of the 3-layered elements in modelling an adhesive layer.

#### 4.1 2-D Adhesive Elements

Several 2-D special adhesive elements have been incorporated in the user-defined subroutine. The elements differ in number of layers, element order, assumed order of stress expansions and applied stress field constraints. An account of their performance in predicting bondline stresses is extensively examined in Reference [1]. Details of the 2-D elements, designated H2L6N, H2L10N, H3L8N and H2L13N, are discussed in the following subsections.

##### 4.1.1 The H2L6N Element

The configuration of the H2L6N element is depicted in Figure 4. As shown, a local element coordinate system is defined at each layer centroid with the local  $\xi$  and  $\eta$  axes parallel to adjacent sides of the layer.

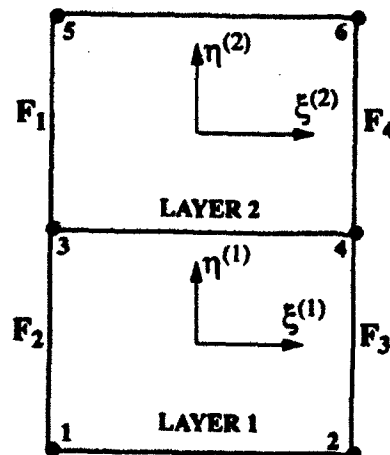


Figure 4. H2L6N element configuration and local layer coordinate system.

The H2L6N element is designated as  $TYPE = U1$  and two versions are available incorporating complete linear and quadratic stress expansions. These versions are selected by setting the element version parameter as  $NVER = 11$  and  $NVER = 12$ , respectively. The H2L6N element is also supported for use as an end-element in which zero traction conditions are enforced in the  $\tau_{xy}$  stress component. This version is selected as  $NVER = 13$  and the input parameter  $NSIDE$  is used to select the traction-free element side by setting the property parameter  $NSIDE = i$  where  $i$  is indicated by the  $F_i$  designation in the above figure.

This element has demonstrated excellent convergence properties in a study of bondline stress prediction in single-lap joints. The linear field used in version 11 yields good convergence behavior but the quadratic field used in version 12 should be selected if a coarse mesh is used along the bond axis. The increase in computational cost is minimal.

#### 4.1.2 The H2L10N Element

The configuration of the H2L10N element is depicted in Figure 5. As shown, a local element coordinate system is defined at each layer centroid with the local  $\xi$  and  $\eta$  axes parallel to adjacent sides of the layer.

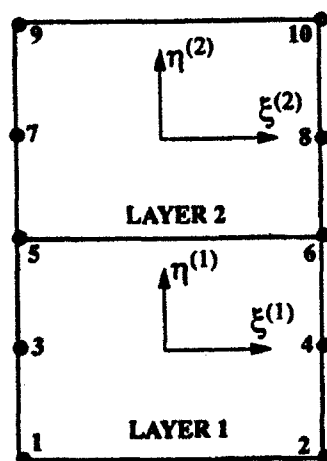


Figure 5. H2L10N element node configuration and local layer coordinate system.

The H2L10N element is designated as  $TYPE = U2$  and two versions are available incorporating complete quadratic and cubic stress expansions. These versions are selected by setting the parameter  $NVER = 11$  and  $NVER = 12$ , respectively.

The H2L10N element formulation has a higher-order strain field representation in the normal bondline direction. However, this selective increase in the degree of freedom representation in the bond thickness direction has not demonstrated an overall improvement in bond stress prediction in the single-lap joint case over the H2L6N element. It is maintained in the element library for a further assessment in analyzing other joint configurations.

### 4.1.3 The H2L13N Element

The configuration of the H2L13N element together with local layer coordinate system convention is depicted in Figure 6.

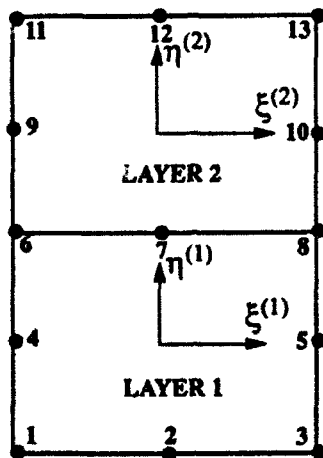


Figure 6. H2L13N element node configuration and local layer coordinate system.

The H2L13N element is designated as  $TYPE = U3$  and two versions are contained in the element library. A difficulty was encountered in Reference [1] in the formulation of this element. It was found that adopting a higher-order displacement field and strictly enforcing all stress field constraints inevitably leads to spurious kinematic deformation modes in the resulting element stiffness matrix. Therefore, selective relaxation of some constraints were made in the two versions of this element. One version, selected using  $NVER = 11$ , incorporates a complete cubic stress field with the addition of two quartic terms in the shear stress expansion which are not constrained to enforce continuity at the element layer interface. These two terms are added to suppress zero energy modes which result from using complete expansions satisfying all equilibrium and continuity constraints. A second version, designated  $NVER = 12$ , is formulated using a complete quadratic field with only stress continuity conditions applied at the layer interface.

The performance of these versions in the analysis of a single-lap joint configuration has shown that both demonstrate accurate stress predictions with element version 12 showing a faster rate of convergence and a highly accurate recovery of bondline stresses. The violation of strict continuity enforcement in element version 11 has been shown to be of minimal consequence due to the high-order of the unconstrained stress expansion terms.

#### 4.1.4 The H3L8N Element

The configuration of the H2L8N element and local coordinate system is depicted in Figure 7.

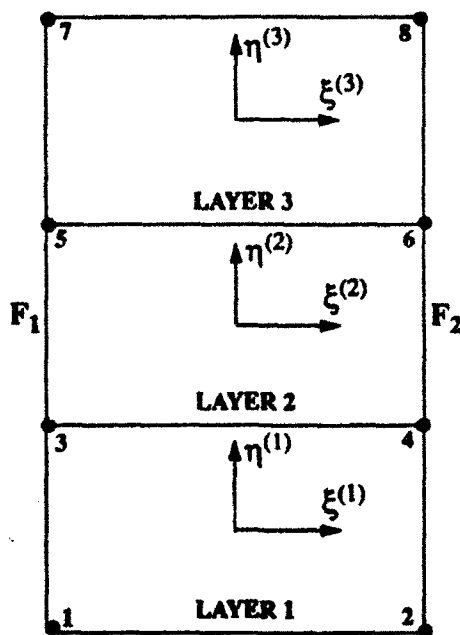


Figure 7. H3L8N element node configuration and local layer coordinate system.

The H3L8N element is designated as  $TYPE = U4$  and two versions are available incorporating complete quadratic and cubic stress expansions. These versions are denoted as  $NVER = 11$  and  $NVER = 12$ , respectively. The H3L8N element is also supported for use as an *end-element* in which zero traction conditions are enforced in the  $\tau_{xy}$  stress component. This version is selected by setting  $NVER = 13$  and the  $NSIDE$  input parameter is set to  $i$  from the  $F_i$  designations shown above to select the traction-free element face.

The performance of H3L8N has been shown to be accurate in the prediction of joint stresses in single-lap configurations with faster convergence rates obtained by using the higher-order cubic stress field in coarse mesh models.

## 4.2 3-D Adhesive Elements

All 2-D elements developed have a theoretical counterpart in a 3-D formulation, however, from the study of 2-D element behavior, a single 3-D solid element has been developed and incorporated into the user-element library.

### 4.3 The H2L12N Element

The H2L12N element configuration and local coordinate system are depicted in Figure 8.

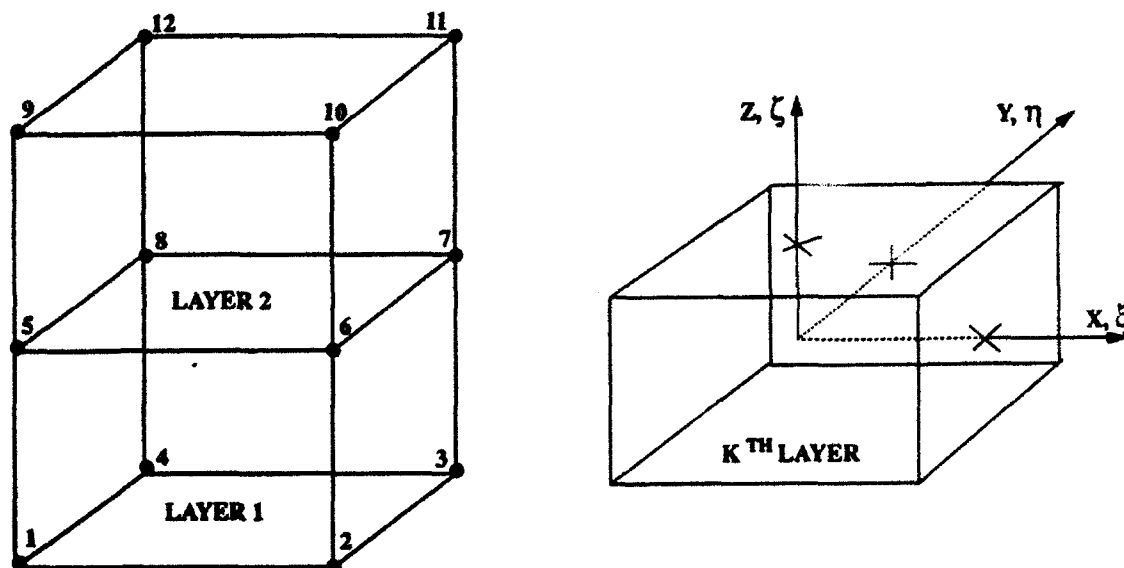


Figure 8. H2L12N element and local layer coordinate system.

The H2L12N element, designated as  $TYPE = U5$ , permits a general adhesive joint planform to be modelled and is available in two versions incorporating complete linear and quadratic stress fields. These versions are designated as  $NVER = 11$  and  $NVER = 12$ , respectively.

Studies have shown that, as in the case of the 2-D H2L6N element, the higher-order quadratic expansion yields improved coarse mesh performance - for finer levels of discretization along the bondline the distinction between the performance of the two versions vanish.

## 5 Demonstration Problems

The analysis of two single-lap joints are presented in this section. Results are taken from Reference [1] and used to demonstrate the use of two representative adhesive elements, namely, the 2-D H2L6N and 3-D H2L12N elements. The material properties selected are given by:

$$\text{Adherend: } E = 69000.0 \quad \mu = 0.32$$

$$\text{Adhesive: } E = 3000.0 \quad \mu = 0.36$$

All stresses are normalized as  $\sigma_{ij}^* = \sigma_{ij} / \sigma_{ref}$  where  $\sigma_{ref} = P/A$  in which  $P$  is a uniformly applied tensile load and  $A$  is the cross-sectional area of the adherend end.

### 5.1 Problem I: 2-D Single-Lap Joint

Figures 9 and 10 show the geometry and boundary conditions, respectively, of a 2-D single-lap joint. A state of plane strain is assumed to exist in the joint and H2L6N elements are used to model the adhesive and locally adjacent regions of the adhesive.

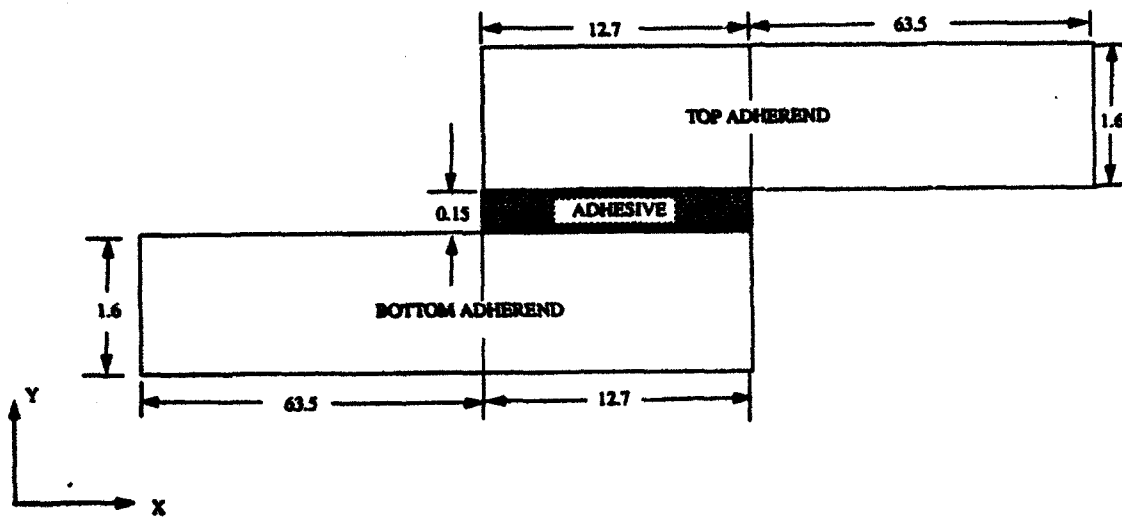


Figure 9. 2-D Single-lap joint geometry.

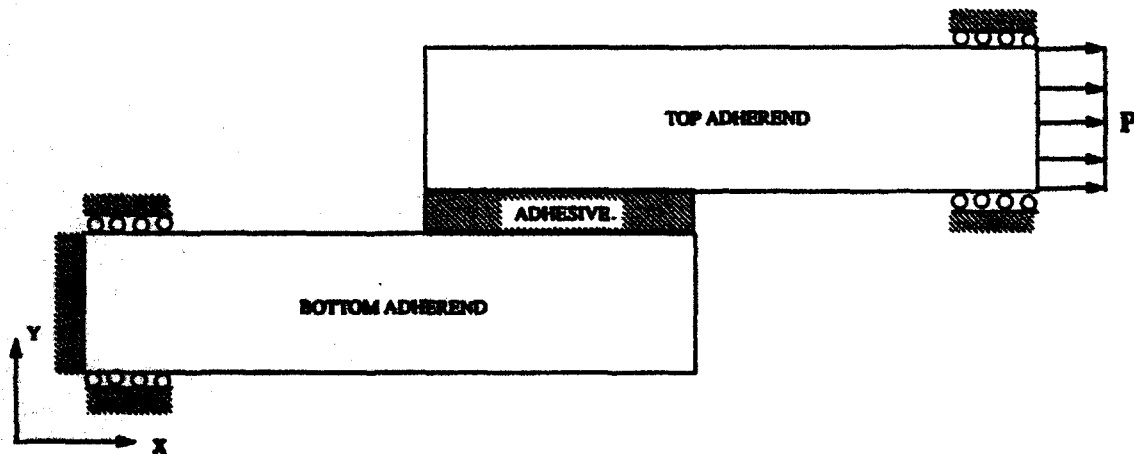


Figure 10. Applied boundary conditions.

Figures 11 and 12 show the convergence of models incorporating 10, 25, 50 and 100 H2L6N elements along the bondline in comparison to a reference solution. Details of the model discretization is presented in Reference [1]. Stress predictions were made for the  $\sigma_{yy}$  and  $\tau_{xy}$  stress components along the adhesive/adherend interface. Element version 12, incorporating complete quadratic stress fields, was selected in generating these results. To show the improvement in element performance over standard displacement-based elements, the same models were used in which the layered adhesive elements were each replaced by two 4-node plane-strain elements (CPE4) from the ABAQUS library. As can be seen in Figures 13 and 14, the purely displacement-based solutions actually converge away from the reference solution which validates the improvement in element efficiency afforded by the layered hybrid formulation. The ABAQUS input deck and selected output associated with the refined model using the H2L6N element is presented in Appendix B.



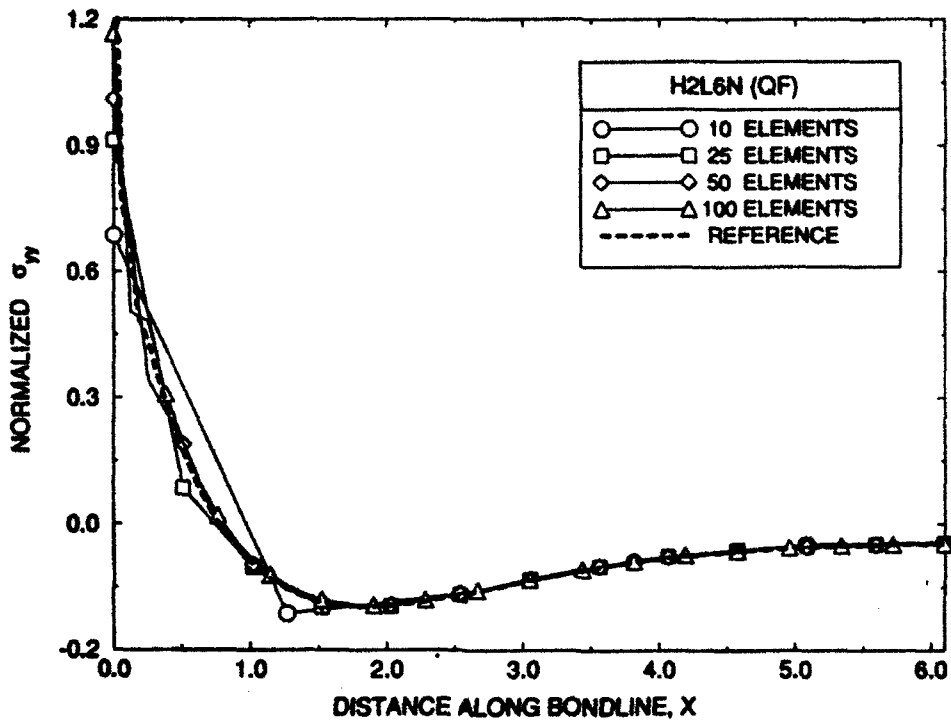


Figure 11. H2L6N prediction of  $\sigma_{yy}$  distribution along the bondline.

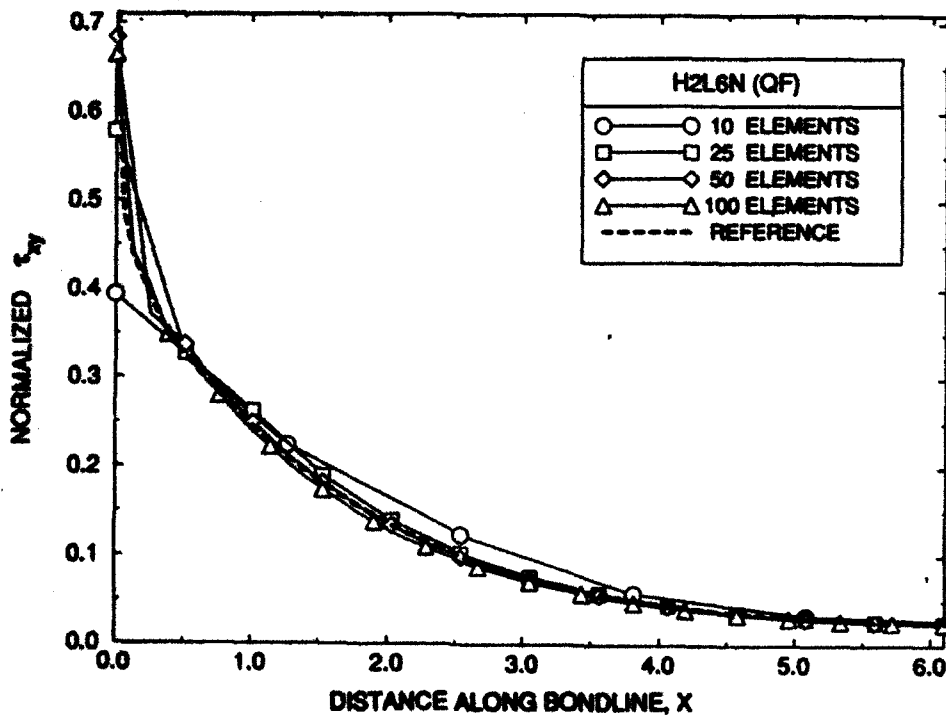


Figure 12. H2L6N prediction of  $\tau_{xy}$  distribution along the bondline.

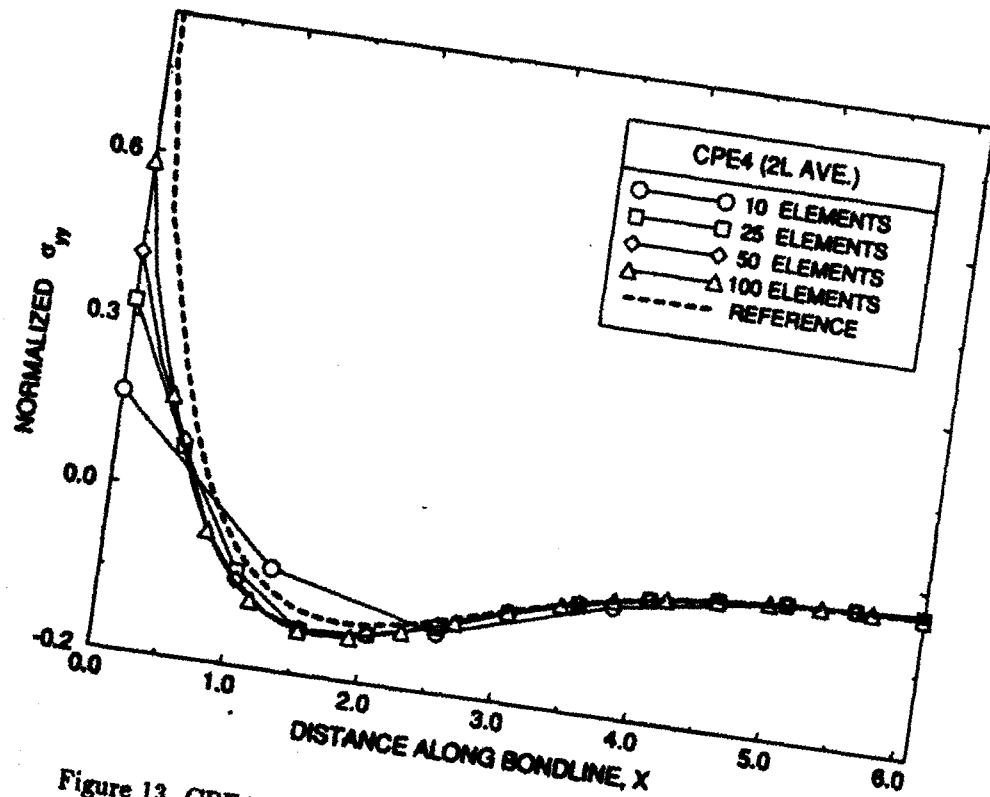


Figure 13. CPE4 Prediction of  $\sigma_{yy}$  distribution along the bondline.

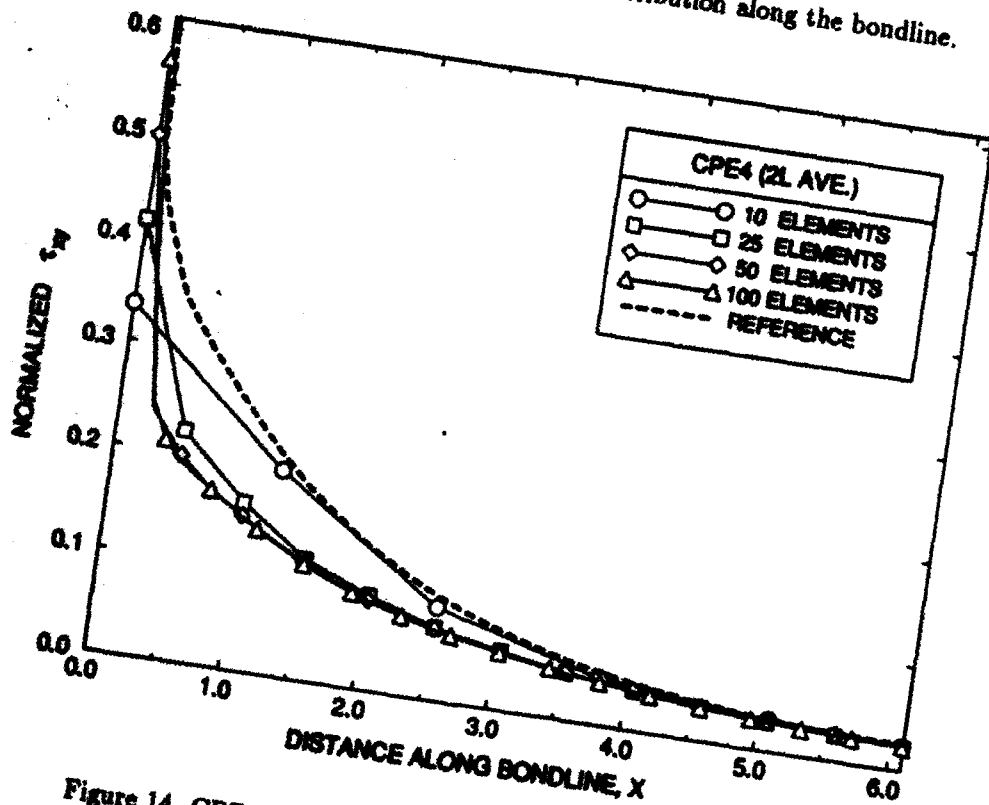


Figure 14. CPE4 prediction of  $\tau_{xy}$  distribution along the bondline.

## 5.2 Problem II: 3-D Single-Lap Joint

A rectangular 3-D single lap joint is analyzed using H2L12N elements to model the bond region along the adhesive/adherent interface. The geometry is depicted in Figure 15 and the applied boundary conditions are identical to those presented above in figure 10.

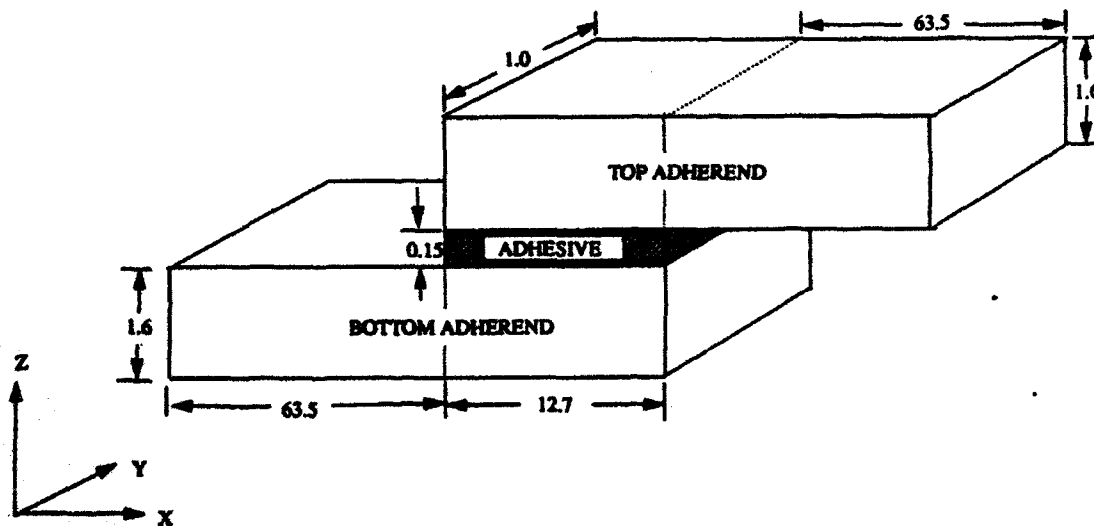


Figure 15. 3-D Single-lap joint geometry.

The convergence behavior is identical to that shown in the 2-D lap joint example presented above. For clarity, 3-D solutions are depicted for a model incorporating 100 elements along the bondline showing comparisons between the special layered hybrid adhesive element and standard displacement-based elements with a reference solution. In generating the displacement-based solution, the same model was used in which the layered H2L12N elements were each replaced by two 8-node brick elements (C3D8) from the ABAQUS library. Figures 16 and 17 show predictions for  $\sigma_{zz}$  and  $\tau_{xz}$  over the bond interface using the H2L12N element. The  $\tau_{yz}$  shear stress component is essentially zero for this particular joint problem and is, therefore, not shown. The purely displacement-based element solution is shown in figures 18 and 19 and demonstrates a convergence away from the reference solution. The ABAQUS input deck and selected output is presented in Appendix C.

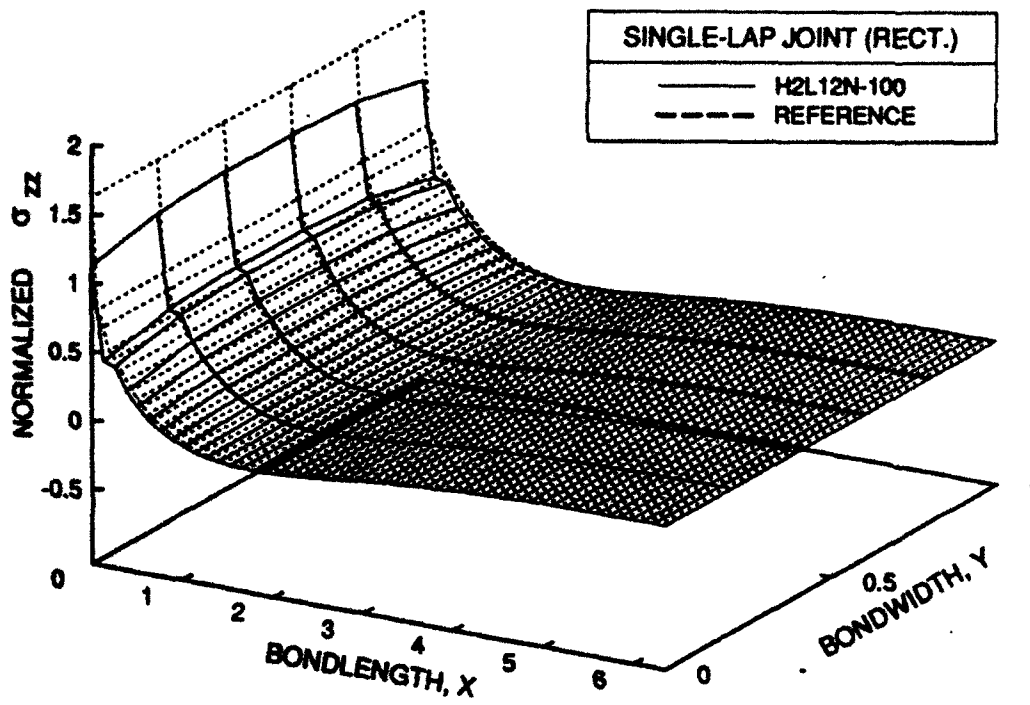


Figure 16. H2L12N prediction of  $\sigma_{zz}$  distribution along the bondline.

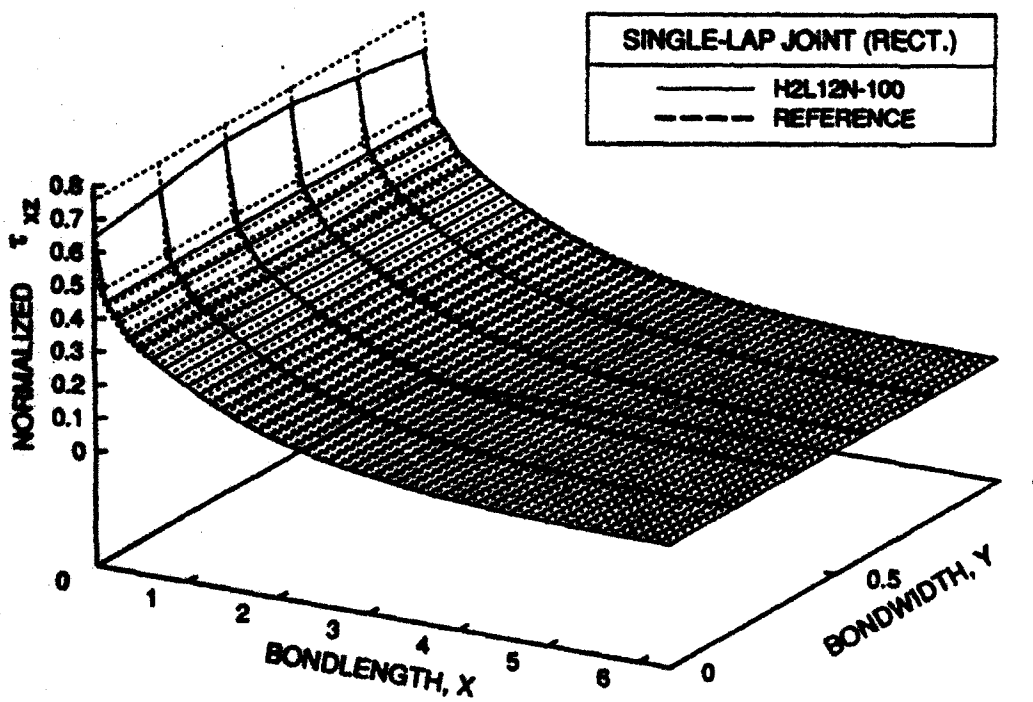


Figure 17. H2L12N prediction of  $\tau_{xz}$  distribution along the bondline.

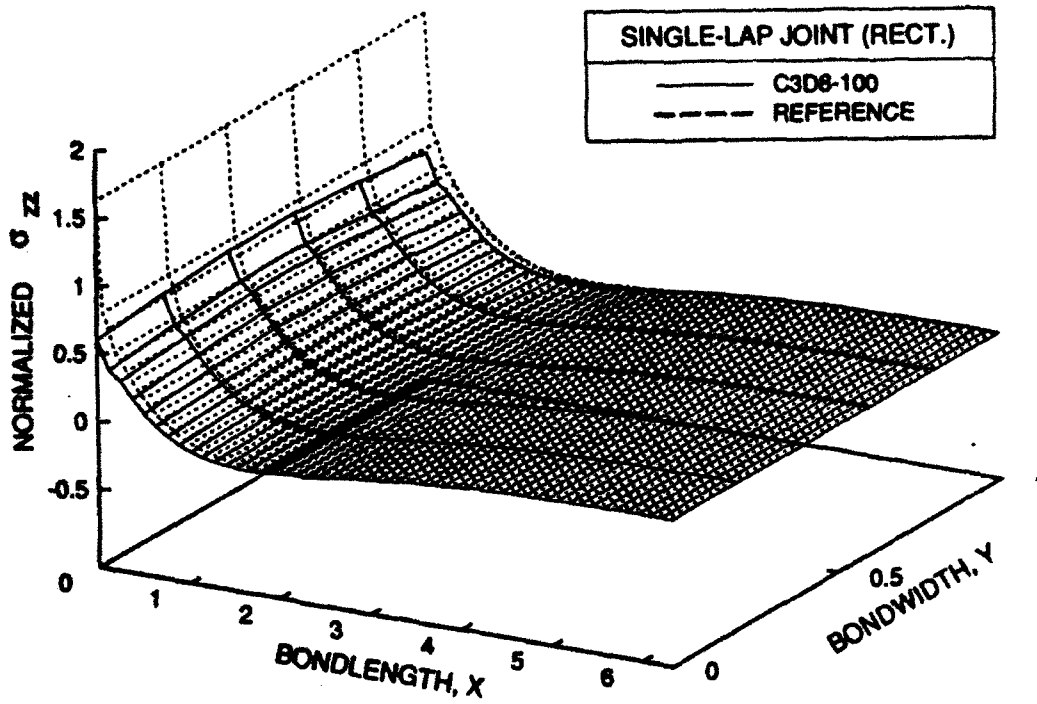


Figure 18. C3D8 prediction of  $\sigma_{zz}$  distribution along the bondline.

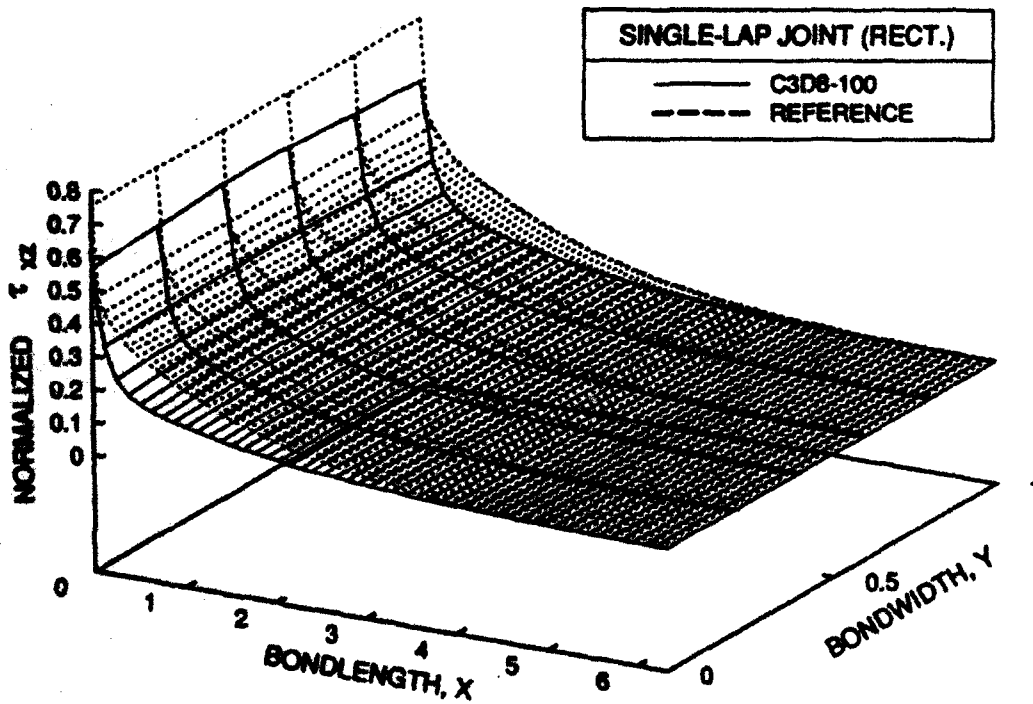


Figure 19. C3D8 prediction of  $\tau_{xz}$  distribution along the bondline.

## 6 Conclusion

A variety of 2-D and 3-D special layered hybrid element formulations have been developed for the analysis of bondline stresses in adhesive joints. The hybrid stress method was selected to allow the explicit enforcement of layer domain equilibrium and interface continuity constraints. In addition, for the H2L6N and H3L8N elements, stress fields have been derived to enforce zero traction conditions along element sides. The elements demonstrate improved efficiency over similar displacement-based elements and are fully supported for use in the commercial finite element code ABAQUS through the development of a user-defined subroutine. The required input format has been detailed and element performance demonstrated in two example problems. Sample input and output datasets together with the complete source code performing all element computations have been included in separate appendices. The developed special adhesive elements provide an ideal basis for further enhancements such as the incorporation of nonlinear material and geometric capabilities to accurately model bondline stresses in complex adhesive joint designs.

## References

- [1] E. Saether and K. Weight, 'Special hybrid stress finite elements for the analysis of interface stress distribution in adhesive joints,' ARL-TR-449, U.S. Army Research Laboratory, June, (1994).
- [2] Hibbit, Karlsson and Sorensen, Inc., ABAQUS USER'S MANUAL, Version 5.3, 1994.

## APPENDIX A

Source code listing of subroutine UEL  
supporting special adhesive elements  
in ABAQUS.





```

C      **
C      ** ADHESIVE ELEMENT STIFFNESS GENERATION **
C      **
C      *****
C
C      IMPLICIT DOUBLE PRECISION (A-H,O-Z)
C
C      DIMENSION AMATRX(NDOFEL,NDOFEL), PROPS(*), COORDS(MCRD,NNODE)
C
C      DIMENSION HMAT(100,100), HINV(100,100), AJINV(3,3),
1      XH(20), YH(20), ZH(20), TMP1(100,100), TMP2(100,100),
2      DXSI(20), DELTA(20), DCEE(20), BMATS(6,60)
C      DIMENSION SMAT(6,6), PMAT(6,100), THETA(100),
1      GMAT(100,36), INDX1(36), GSMBL(100,36), HSMBL(100,100),
2      PTHK(100), E1(100), E2(100), E3(100), V12(100),
3      V23(100), V13(100), G12(100), G23(100), G31(100)
C      DIMENSION WDT(3), NLAY(3), THK(3), ETRN(6,6), STRN(6,6), TRI(3,3),
1      CTRN(36,36)
C
C      DATA EPS / 1.0D-8 /
C
C      READ IN ELEMENT DATA FROM PROPS ARRAY AND
C      SET ELEMENT PARAMETERS
C
C      CALL ELDATA( PROPS, PTHK, THETA, E1, E2, E3, V12, V23, V13, G12, G23, G31,
1      WDT, THK, NORD, IPLANE, IOTYPE, NLAYR, NELDIM, NDOFN, INTNOD,
2      NODNUM, JTYPE, NSIDE, NDV, INTDOF, NDOFT, NDOFL, MORD, NLAY,
3      NVER )
C
C      CALL MXINT( GSMBL, 100, 36, 0.0D0 )
C      CALL MXINT( HSMBL, 100, 100, 0.0D0 )
C
C      CHECK GEOMETRY OF ELEMENTS ON INITIAL PAST
C
C      IF ( TEST .LT. EPS ) CALL VCHECK ( COORDS, MCRD, NNODE, JTYPE, JELEM )
C
C      OBTAIN TRANSFORMATION MATRIX BETWEEN GLOBAL AND
C      LOCAL ELEMENT COORDINATES
C
C      CALL TRANS ( COORDS, CTRN, STRN, ETRN, TRI, JTYPE, MCRD, NNODE )
C
C      LOOP OVER AND ASSEMBLE ALL ELEMENT LAYERS
C
C      DO K = 1, NLAYR
C
C          LAYER = K
C          TFAC = 1.0
C          IF ( NELDIM .EQ. 2 ) TFAC = WDT(K)
C
C          N = (NODNUM-INTNOD) * (K-1)
C          DO I = 1, NODNUM
C              N = (NODNUM-INTNOD) * (K-1)
C              AX = COORDS(1, I+N)
C              BY = COORDS(2, I+N)
C              CZ = COORDS(3, I+N)
C
C              TRANSFORM TO LOCAL COORDINATES
C
C              XH(I) = TRI(1,1)*AX+TRI(1,2)*BY+TRI(1,3)*CZ
C              YH(I) = TRI(2,1)*AX+TRI(2,2)*BY+TRI(2,3)*CZ
C              ZH(I) = TRI(3,1)*AX+TRI(3,2)*BY+TRI(3,3)*CZ
C          END DO
C
C      COMPUTE ELEMENT MATERIAL PROPERTY MATRICES
C
C      CALL MATPROP(PTHK, THETA, E1, E2, E3, G12, G23, G31, V12, V23, V13,
1      SMAT, NELDIM, IPLANE, MORD, LAYER, NLAY)

```

```
CALL MXINT( HMAT, 100, 100, 0.0D0 )
CALL MXINT( GMAT, 100, 36, 0.0D0 )
```

```
STRAIN VECTOR CONVENTION: (EX,EY,EZ,TYZ,TXZ,TTY)
```

```
NRDZ = NORD
IF ( NELDIM .EQ. 2 ) NRDZ = 1
```

```
DO IXSI = 1, NORD
  DO JETA = 1, NORD
    DO KCEE = 1, NRDZ
```

```
      OBTAIN GAUSS POINTS AND WEIGHTS
```

```
      CALL GAUSS(NORD,NELDIM,IXSI,JETA,KCEE,XSI,ETA,CEE,WEIGHT)
```

```
      COMPUTE SHAPE FUNCTIONS AND THEIR DERIVATIVES AT
      THE CURRENT GAUSS POINT
```

```
      CALL SHAPE(NODNUM,NELDIM,XSI,ETA,CEE,DXSI,DETA,DCEE)
```

```
      COMPUTE JACOBIAN MATRIX, ITS DETERMINANT AND
      INVERSE
```

```
      CALL JACOB(NODNUM,NELDIM,XH,YH,ZH,DXSI,DETA,DCEE,
1          AJINV,DETJ )
```

```
      COMPUTE MATRIX OF ASSUMED STRESS FUNCTIONS AT
      CURRENT GAUSS POINT
```

```
      CALL ASTRSS(XSI,ETA,CEE,JTYPE,LAYER,NELDIM,NODNUM,
1          PMAT,THK,NBVAL,XH,YH,ZH,NSIDE,NVER)
```

```
      COMPUTE THE STRAIN-DISPLACEMENT MATRIX BMATS
```

```
      CALL BMAT(NELDIM,MORD,NODNUM,NDOFL,AJINV,DXSI,DETA,
1          DCEE,BMATS)
```

```
      FORM G AND H MATRICES
```

```
      CALL MXATB(PMAT,BMATS,TMP2,6,6,100,NBVAL,MORD,NDOFL)
```

```
      INTEGRATE GMAT COEFFICIENTS
```

```
      DO II = 1, NBVAL
      DO JJ = 1, NDOFL
        GMAT(II,JJ) = GMAT(II,JJ)+DETJ*WEIGHT*TFAC*TMP2(II,JJ)
      END DO
      END DO
```

```
      CALL MXMUL(SMAT,PMAT,TMP1,6,6,100,MORD,MORD,NBVAL)
      CALL MXATB(PMAT,TMP1,TMP2,6,100,100,NBVAL,MORD,NBVAL)
```

```
      INTEGRATE HMAT COEFFICIENTS
```

```
      DO II = 1, NBVAL
      DO JJ = 1, NBVAL
        HMAT(II,JJ) = HMAT(II,JJ)+DETJ*WEIGHT*TFAC*TMP2(II,JJ)
      END DO
      END DO
```

```
      END DO
      END DO
      END DO
```

```
ASSEMBLE GMAT AND HMAT
```

```
N = (NODNUM-INTNOD)*(K-1)*NDOFN
```

```

DO I = 1, NBVAL
  DO J = 1, NDOFL
    GSMBL(I,J+N) = GSMBL(I,J+N) + GMAT(I,J)
  END DO
  DO J = 1, NBVAL
    HSMBL(I,J) = HSMBL(I,J) + HMAT(I,J)
  END DO
END DO

C
END DO

C
C
C
COMPUTE THE INVERSE OF HSMBL

NDIM = 100
CALL INVERS(HSMBL,HINV,INDX1,NDIM,NBVAL)

C
IF ( TEST .GT. EPS ) RETURN

C
C
C
COMPUTE STIFFNESS COEFFICIENTS

CALL MXMUL(HINV,GSMBL,TMP1,100,100,100,NBVAL,NBVAL,NDOFT)
CALL MXATB(GSMBL,TMP1,AMATRX,100,100,NDOFT,NDOFT,NBVAL,NDOFT)

C
C
C
TRANSFORM ELEMENT STIFFNESSES TO GLOBAL COORDINATES

CALL MXMUL(AMATRX,CTRN,TMP1,NDOFEL,36,100,NDOFEL,NDOFEL,NDOFEL)
CALL MXATB(CTRN,TMP1,AMATRX,36,100,NDOFEL,NDOFEL,NDOFEL,NDOFEL)

C
RETURN
END

C
C
C
SUBROUTINE RECOV( COORDS,PROPS,DU,HINV,GMAT,MLVARX,NDOFEL,
1 NPROPS,MCRD,NNODE,JTYPE,JELEM,NBVAL )

C
C
C
*****
**                                     **
**   PERFORM REQUESTED ELEMENT DATA RECOVERY   **
**                                     **
*****

C
IMPLICIT DOUBLE PRECISION (A-H,O-Z)

C
DIMENSION PROPS(*),COORDS(MCRD,NNODE),DU(MLVARX)

C
DIMENSION HINV(100,100),TMP2(100,100),TMP1(100,100),AJINV(3,3),
1 STRN(6,6),ETRN(6,6),XH(20),YH(20),ZH(20),
2 ULI(36),UL(36),DXSI(20),DETA(20),DCEE(20)
DIMENSION PMAT(6,100),BMATS(6,60),GMAT(100,36),BETA(100),
1 THETA(100),NDT(3),THK(3),PTHK(100),E1(100),
2 E2(100),E3(100),V12(100),V23(100),V13(100),
3 G12(100),G23(100),G31(100),TRI(3,3)
DIMENSION CTRN(36,36),SCOMP(27,6),ECOMP(27,6),NLAY(3)

C
CALL ELDATA( PROPS,PTHK,THETA,E1,E2,E3,V12,V23,V13,G12,G23,G31,
1 WDT,THK,NORD,IPLANE,IOTYPE,NLAYR,NELDIM,NDOFN,INTNOD,
2 NODNUM,JTYPE,NSIDE,NDV,INTDOF,NDOFT,NDOFL,MORD,NLAY,
3 NVER )

C
IF ( IOTYPE .EQ. 0 ) RETURN

C
WRITE(6,6794) JELEM

C
C
C
OBTAIN ORTHOGONAL AND TENSORIAL TRANSFORMATION MATRICES FOR
DISPLACEMENTS, STRESSES AND STRAINS

CALL TRANS( COORDS,CTRN,STRN,ETRN,TRI,JTYPE,MCRD,NNODE )

```





```

C          DO IET = 1, NTPS
C
C          CALL IOPNTS(IET,NTPS,JTYPE,NELDIM,NORD,LAYER,
1             XSI,ETA,CEE)
C
C          WRITE(6,995) XSI,ETA,CEE,(SCOMP(IET,I),I=1,6)
C
C          END DO
C
C          IF ( IOTYPE .EQ. 1 ) WRITE(6,896) LAYER
C          IF ( IOTYPE .EQ. 2 ) WRITE(6,897) LAYER
C
C          OUTPUT LAYER STRAINS
C
C          DO IET = 1, NTPS
C
C          CALL IOPNTS(IET,NTPS,JTYPE,NELDIM,NORD,LAYER,
1             XSI,ETA,CEE)
C
C          WRITE(6,995) XSI,ETA,CEE,(ECOMP(IET,I),I=1,6)
C
C          END DO
C
C          END IF
C
C          END DO
C
C          FORMAT STATEMENTS FOR HYBRID ELEMENT OUTPUT
C
855  FORMAT(///,45X,'H Y B R I D   E L E M E N T   D A T A   ',//)
848  FORMAT(//,' ELEMENT ID: ',I5,//)
800  FORMAT(/,20X,'HYBRID STIFFNESS MATRIX:',/)
801  FORMAT(/,10I11)
815  FORMAT(1X,I3,2X,10(E9.3,2X))
892  FORMAT(20X,/, 'STRESS/STRAIN OUTPUT IN LOCAL COORDINATES FOR LAYER'
1, I5, //, 2X, 'RECOVERY POINTS', 24X, 'STRESS/STRAIN
2COMPONENTS', /, 3X, ' CI      CJ', 9X, 'SXX', 8X, 'EXX', 8X, 'SYY', 8X,
3' EYY', 8X, 'SKY', 8X, 'EXY')
893  FORMAT(20X,/, 'STRESS/STRAIN OUTPUT IN GLOBAL COORDINATES FOR LAYER
1', I5, //, 2X, 'RECOVERY POINTS', 24X, 'STRESS/STRAIN
2COMPONENTS', /, 3X, ' CI      CJ', 9X, 'SXX', 8X, 'EXX', 8X, 'SYY', 8X,
3' EYY', 8X, 'SKY', 8X, 'EXY')
894  FORMAT(20X,/, 'STRESS OUTPUT IN LOCAL COORDINATES FOR LAYER', I5, //,
19X, 'RECOVERY POINTS', 29X, 'STRESS COMPONENTS', /,
23X, ' CI      CJ      CK', 9X, 'SXX', 8X, 'SYY', 8X, 'SZZ', 8X,
3' SYZ', 8X, 'SZX', 8X, 'SKY')
895  FORMAT(20X,/, 'STRESS OUTPUT IN GLOBAL COORDINATES FOR LAYER', I5, //,
1, 9X, 'RECOVERY POINTS', 29X, 'STRESS COMPONENTS', /,
23X, ' CI      CJ      CK', 9X, 'SXX', 8X, 'SYY', 8X, 'SZZ', 8X,
3' SYZ', 8X, 'SZX', 8X, 'SKY')
896  FORMAT(20X,/, 'STRAIN OUTPUT IN LOCAL COORDINATES FOR LAYER', I5, //,
19X, 'RECOVERY POINTS', 29X, 'STRAIN COMPONENTS', /,
23X, ' CI      CJ      CK', 9X, 'EXX', 8X, 'EYY', 8X,
3' EZZ', 8X, 'EYZ', 8X, 'EZX', 8X, 'EXY')
897  FORMAT(20X,/, 'STRAIN OUTPUT IN GLOBAL COORDINATES FOR LAYER', I5, //,
1, 9X, 'RECOVERY POINTS', 29X, 'STRAIN COMPONENTS', /,
23X, ' CI      CJ      CK', 9X, 'EXX', 8X, 'EYY', 8X,
3' EZZ', 8X, 'EYZ', 8X, 'EZX', 8X, 'EXY')
994  FORMAT(2(F7.4,2X),2X,6(E9.3,2X))
995  FORMAT(3(F7.4,2X),2X,12(E9.3,2X))
6794 FORMAT(/,' ELEMENT ID ', I5, /)
C
C          RETURN
C          END
C
C
C

```

```

C
SUBROUTINE VCHECK( COORDS,MCRD,NNODE,JTYPE,JELEM )
C
C   IMPLICIT DOUBLE PRECISION (A-H,O-Z)
C
C   DIMENSION COORDS(MCRD,NNODE),X(20),Y(20),Z(20)
C
C   DATA NONE / 0 /
C
C   TEST FOR IRREGULAR ELEMENT GEOMETRY BY CHECKING
C   INTERNAL ANGLES IN ELEMENT LAYERS
C
DO I = 1, NNODE
  X(I) = COORDS(1,I)
  Y(I) = COORDS(2,I)
  Z(I) = COORDS(3,I)
END DO
C
C   INITIALIZE LAYER ERROR FLAGS
C
NERRL1 = 0
NERRL2 = 0
NERRL3 = 0
C
IF ( JTYPE .EQ. 1 ) THEN
C
  CALL ANGLE ( X,Y,Z,1,2,3,JELEM,NERRL1 )
  CALL ANGLE ( X,Y,Z,4,3,2,JELEM,NERRL1 )
  CALL ANGLE ( X,Y,Z,3,4,5,JELEM,NERRL2 )
  CALL ANGLE ( X,Y,Z,6,5,4,JELEM,NERRL1 )
C
ELSE IF ( JTYPE .EQ. 2 ) THEN
C
  CALL ANGLE ( X,Y,Z,1, 2,5,JELEM,NERRL1 )
  CALL ANGLE ( X,Y,Z,6, 5,2,JELEM,NERRL1 )
  CALL ANGLE ( X,Y,Z,5, 6,9,JELEM,NERRL2 )
  CALL ANGLE ( X,Y,Z,10,9,6,JELEM,NERRL2 )
C
ELSE IF ( JTYPE .EQ. 3 ) THEN
C
  CALL ANGLE ( X,Y,Z,1, 3, 6,JELEM,NERRL1 )
  CALL ANGLE ( X,Y,Z,8, 6, 3,JELEM,NERRL1 )
  CALL ANGLE ( X,Y,Z,6, 8,11,JELEM,NERRL2 )
  CALL ANGLE ( X,Y,Z,13,11,8,JELEM,NERRL2 )
C
ELSE IF ( JTYPE .EQ. 4 ) THEN
C
  CALL ANGLE ( X,Y,Z,1,2,3,JELEM,NERRL1 )
  CALL ANGLE ( X,Y,Z,3,4,5,JELEM,NERRL2 )
  CALL ANGLE ( X,Y,Z,4,3,2,JELEM,NERRL1 )
  CALL ANGLE ( X,Y,Z,6,5,4,JELEM,NERRL2 )
  CALL ANGLE ( X,Y,Z,5,6,7,JELEM,NERRL3 )
  CALL ANGLE ( X,Y,Z,8,7,6,JELEM,NERRL3 )
C
ELSE IF ( JTYPE .EQ. 5 ) THEN
C
  CALL ANGLE ( X,Y,Z,1, 2, 5,JELEM,NERRL1 )
  CALL ANGLE ( X,Y,Z,6, 5, 2,JELEM,NERRL1 )
  CALL ANGLE ( X,Y,Z,2, 3, 6,JELEM,NERRL1 )
  CALL ANGLE ( X,Y,Z,7, 6, 3,JELEM,NERRL1 )
  CALL ANGLE ( X,Y,Z,3, 4, 7,JELEM,NERRL1 )
  CALL ANGLE ( X,Y,Z,8, 7, 4,JELEM,NERRL1 )
  CALL ANGLE ( X,Y,Z,4, 1, 8,JELEM,NERRL1 )
  CALL ANGLE ( X,Y,Z,5, 8, 1,JELEM,NERRL1 )
  CALL ANGLE ( X,Y,Z,5, 6, 9,JELEM,NERRL2 )
  CALL ANGLE ( X,Y,Z,10,9, 6,JELEM,NERRL2 )
  CALL ANGLE ( X,Y,Z,6, 7,10,JELEM,NERRL2 )
  CALL ANGLE ( X,Y,Z,11,10,7,JELEM,NERRL2 )
  CALL ANGLE ( X,Y,Z,7, 8,11,JELEM,NERRL2 )

```



```

CALL ANGLE ( X,Y,Z,12,11,8,JELEM,NERRL2 )
CALL ANGLE ( X,Y,Z,8, 5,12,JELEM,NERRL2 )
CALL ANGLE ( X,Y,Z,9,12, 5,JELEM,NERRL2 )
C
END IF
C
IF ( NERRL1 .EQ. 1 .OR. NERRL2 .EQ. 1 .OR. NERRL3 .EQ. 1 ) THEN
  IF ( NONE .EQ. 0 ) THEN
    WRITE(6,100)
    NONE = 1
  END IF
END IF
C
IF ( NERRL1 .NE. 0 ) WRITE(6,10) JELEM
IF ( NERRL2 .NE. 0 ) WRITE(6,20) JELEM
IF ( NERRL3 .NE. 0 ) WRITE(6,30) JELEM
C
10 FORMAT(' ERROR - ELEMENT #',I10,' IS DEFORMED IN LAYER 1')
20 FORMAT(' ERROR - ELEMENT #',I10,' IS DEFORMED IN LAYER 2')
30 FORMAT(' ERROR - ELEMENT #',I10,' IS DEFORMED IN LAYER 3')
100 FORMAT('/', ' ELEMENT LAYERS MUST BE OF RECTANGULAR GEOMETRY',/)
C
RETURN
END
C
C
SUBROUTINE ANGLE ( X,Y,Z,N1,N2,N3,JELEM,NERR )
C
IMPLICIT DOUBLE PRECISION (A-H,O-Z)
C
DIMENSION X(20),Y(20),Z(20)
C
PI = ACOS(-1.0D0)
C
V1 = X(N2) - X(N1)
V2 = Y(N2) - Y(N1)
V3 = Z(N2) - Z(N1)
V4 = X(N3) - X(N1)
V5 = Y(N3) - Y(N1)
V6 = Z(N3) - Z(N1)
C
DOT = V1*V4 + V2*V5 + V3*V6
ADA = (V1*V1 + V2*V2 + V3*V3)**0.5
BDB = (V4*V4 + V5*V5 + V6*V6)**0.5
THETA = ABS(180*ACOS(DOT/(ADA*BDB))/PI)
C
IF ( THETA .GE. 95.0 .OR. THETA .LE. 85.0 ) NERR = 1
C
RETURN
END
C
C
SUBROUTINE ELDATA( PROPS,PTHK,THETA,E1,E2,E3,V12,V23,V13,G12,
1 G23,G31,WDT,THK,NORD,IPLANE,IOTYPE,NLAYR,
2 NELDIM,NDOFN,INTNOD,NODNUM,JTYPE,NSIDE,NDV,
3 INTDOF,NDOFT,NDOFL,MORD,NLAY,NVER )
C
IMPLICIT DOUBLE PRECISION (A-H,O-Z)
C
DIMENSION E1(100),E2(100),E3(100),G12(100),G23(100),G31(100),
1 V12(100),V23(100),V13(100),PTHK(100),THETA(100),
2 PROPS(1),WDT(3),NLAY(3),THK(3)
C
NOTE: ALL PROPERTY VALUES MUST BE INPUTTED AS REAL
NUMBERS ON THE UEL PROPERTY INPUT BLOCK
C

```

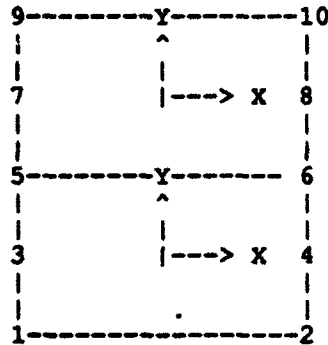




ELSE IF ( JTYPE .EQ. 2 ) THEN

\*\*\*\*\*  
\* H2L10N ELEMENT \*  
\*\*\*\*\*

LOCAL LAYER COORDINATE SYSTEM CONVENTION:



NLAYR = 2  
NELDIM = 2  
NDOFN = 2  
INTNOD = 2  
NODNUM = 6  
NDV = NDOFN\*NELDIM  
INTDOF = INTNOD\*NDOFN  
NDOFT = 20  
NDOFL = NODNUM\*NDOFN  
MORD = 3

ELEMENT INPUT PROPERTIES:

NVER - ELEMENT VERSION DESIGNATION  
IPLANE - 0 FOR PLANE STRESS  
          1 FOR PLANE STRAIN  
IOTYPE - 0 TO SUPPRESS OUTPUT OF ELEMENT DATA  
          1 FOR ELEMENT OUTPUT IN LOCAL COORDINATES  
          2 FOR ELEMENT OUTPUT IN GLOBAL COORDINATES  
NLAY1 - NUMBER OF PLYS IN ELEMENT LAYER 1  
NLAY2 - NUMBER OF PLYS IN ELEMENT LAYER 2  
WDT(1) - DEPTH DIMENSION (WIDTH) OF LAYER 1  
WDT(2) - DEPTH DIMENSION (WIDTH) OF LAYER 2  
THK(1) - THICKNESS OF LAYER 1 (CALCULATED FROM PLY THICKNESS)  
THK(2) - THICKNESS OF LAYER 2 (CALCULATED FROM PLY THICKNESS)

PTHK - PLY THICKNESS  
THETA - PLY ORIENTATION  
E1, E2, E3 - NORMAL MATERIAL MODULII  
G12, G23, G31 - SHEAR MODULII  
V12, V23, V13 - POISSON RATIOS

PROPERTY LIST FORMAT:

- 1) NVER, IPLANE, IOTYPE
  - 2) NLAY1, WDT1,
  - 3) PHTK, THETA, E1, E2, E3, V12, V23, V13
  - 4) G12, G23, G31
- . REPEAT FOR EACH PLY IN LAYER 1
- I) NLAY2, WDT2
  - J) PHTK, THETA, E1, E2, E3, V12, V23, V13







```

NLAYR = 3
NELDIM = 2
NDOFN = 2
INTNOD = 2
NODNUM = 4
NDV = NDOFN*NELDIM
INTDOF = INTNOD*NDOFN
NDOFT = 16
NDOFL = NODNUM*NDOFN
MORD = 3

```

ELEMENT INPUT PROPERTIES:

```

NVER - ELEMENT VERSION DESIGNATION
IPLANE - 0 FOR PLANE STRESS
        1 FOR PLANE STRAIN
IOTYPE - 0 TO SUPPRESS OUTPUT OF ELEMENT DATA
        1 FOR ELEMENT OUTPUT IN LOCAL COORDINATES
        2 FOR ELEMENT OUTPUT IN GLOBAL COORDINATES
NSIDE - ELEMENT SIDE DESIGNATION FOR ZERO TRACTIONS
NLAY1 - NUMBER OF PLYS IN ELEMENT LAYER 1
NLAY2 - NUMBER OF PLYS IN ELEMENT LAYER 2
NLAY3 - NUMBER OF PLYS IN ELEMENT LAYER 3
WDT(1) - DEPTH DIMENSION (WIDTH) OF LAYER 1
WDT(2) - DEPTH DIMENSION (WIDTH) OF LAYER 2
WDT(3) - DEPTH DIMENSION (WIDTH) OF LAYER 3
THK(1) - THICKNESS OF LAYER 1 (CALCULATED FROM PLY THICKNESS)
THK(2) - THICKNESS OF LAYER 2 (CALCULATED FROM PLY THICKNESS)
THK(3) - THICKNESS OF LAYER 3 (CALCULATED FROM PLY THICKNESS)

PTHK - PLY THICKNESS
THETA - PLY ORIENTATION
E1, E2, E3 - NORMAL MATERIAL MODULI
G12, G23, G31 - SHEAR MODULI
V12, V23, V13 - POISSON RATIOS

```

PROPERTY LIST FORMAT:

```

1) NTYPE, IPLANE, IOTYPE
2) NLAY1, WDT1,
3) PHTK, THETA, E1, E2, E3, V12, V23, V13
4) G12, G23, G31
   . REPEAT FOR EACH PLY IN LAYER 1
I) NLAY2, WDT2
J) PHTK, THETA, E1, E2, E3, V12, V23, V13
K) G12, G23, G31
   . REPEAT FOR EACH PLY IN LAYER 2
L) NLAY3, WDT3
M) PHTK, THETA, E1, E2, E3, V12, V23, V13
N) G12, G23, G31
   . REPEAT FOR EACH PLY IN LAYER 3

```

EXTRACT ELEMENT DATA OFF PROPS ARRAY

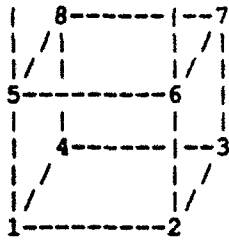
```

NVER = INT (PROPS (1))
IPLANE = INT (PROPS (2))
IOTYPE = INT (PROPS (3))
NSIDE = INT (PROPS (4))
NLAY (1) = INT (PROPS (9))
WDT (1) = PROPS (10)
DO I = 1, NLAY (1)
  N = I*16

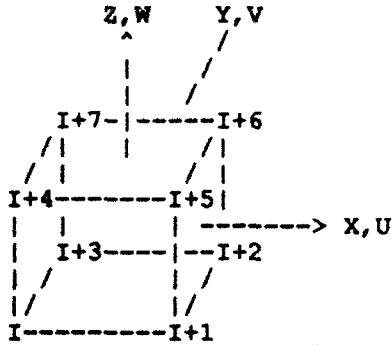
```







LOCAL LAYER COORDINATE SYSTEM CONVENTION:



NLAYR = 2  
 NELDIM = 3  
 NDOFN = 3  
 INTNOD = 4  
 NODNUM = 8  
 NDV = NDOFN\*NELDIM  
 INTDOF = INTNOD\*NDOFN  
 NDOFL = NODNUM\*NDOFN  
 NDOFT = 36  
 MORD = 6

ELEMENT INPUT PROPERTIES:

NVER - ELEMENT VERSION DESIGNATION  
 IOTYPE - 0 TO SUPPRESS OUTPUT OF ELEMENT DATA  
           1 FOR ELEMENT OUTPUT IN LOCAL COORDINATES  
           2 FOR ELEMENT OUTPUT IN GLOBAL COORDINATES  
 NLAY1 - NUMBER OF PLIES IN ELEMENT LAYER 1  
 NLAY2 - NUMBER OF PLIES IN ELEMENT LAYER 2  
 THK(1) - THICKNESS OF LAYER 1 (CALCULATED FROM PLY THICKNESS)  
 THK(2) - THICKNESS OF LAYER 2 (CALCULATED FROM PLY THICKNESS)  
 PTHK - PLY THICKNESS  
 THETA - PLY ORIENTATION  
 E1, E2, E3 - NORMAL MATERIAL MODULII  
 G12, G23, G31 - SHEAR MODULII  
 V12, V23, V13 - POISSON RATIOS

PROPERTY LIST FORMAT:

- 1) NVER, IOTYPE
  - 2) NLAY1
  - 3) PTHK, THETA, E1, E2, E3, V12, V23, V13
  - 4) G12, G23, G31
- . REPEAT FOR EACH PLY IN LAYER 1
- I) NLAY2
  - J) PTHK, THETA, E1, E2, E3, V12, V23, V13
  - K) G12, G23, G31

C  
C  
C  
C  
C

REPEAT FOR EACH PLY IN LAYER 2

EXTRACT ELEMENT DATA OFF PROPS ARRAY

```
NVER = INT(PROPS(1))
IOTYPE = INT(PROPS(2))
NLAY(1) = INT(PROPS(9))
DO I = 1, NLAY(1)
  N = I*16
  PTHK(I) = PROPS(N+1)
  THK(1) = THK(1) + PTHK(I)
  THETA(I) = PROPS(N+2)
  E1(I) = PROPS(N+3)
  E2(I) = PROPS(N+4)
  E3(I) = PROPS(N+5)
  V12(I) = PROPS(N+6)
  V23(I) = PROPS(N+7)
  V13(I) = PROPS(N+8)
  G12(I) = PROPS(N+9)
  G23(I) = PROPS(N+10)
  G31(I) = PROPS(N+11)
END DO
```

```
NLAY(2) = INT(PROPS(16*NLAY(1)+17))
M = NLAY(1)
DO I = 1, NLAY(2)
  N = 24+16*(M+I-1)
  PTHK(I+M) = PROPS(N+1)
  THK(2) = THK(2) + PTHK(I+M)
  THETA(I+M) = PROPS(N+2)
  E1(I+M) = PROPS(N+3)
  E2(I+M) = PROPS(N+4)
  E3(I+M) = PROPS(N+5)
  V12(I+M) = PROPS(N+6)
  V23(I+M) = PROPS(N+7)
  V13(I+M) = PROPS(N+8)
  G12(I+M) = PROPS(N+9)
  G23(I+M) = PROPS(N+10)
  G31(I+M) = PROPS(N+11)
END DO
```

C  
C  
C

SET GAUSSIAN INTEGRATION ORDER

```
IF ( NVER .EQ. 11 ) NORD = 2
IF ( NVER .EQ. 12 ) NORD = 3
```

C  
C

END IF

RETURN  
END

C  
C  
C

SUBROUTINE BMAT (ND, MORD, NODNUM, NDOFL, AJINV, DXSI, DETA,  
1 DCEE, BMATS)

C  
C

IMPLICIT DOUBLE PRECISION (A-H, O-Z)

DIMENSION AJINV(3,3), DXSI(20), DETA(20), DCEE(20), BMATS(6,60)  
DIMENSION TMP1(36,36), HMATS(6,9), TMAT(9,9), UMAT(9,60)

C  
C

```
CALL MXINT ( HMATS, 6, 9, 0.0D0 )
CALL MXINT ( TMAT, 9, 9, 0.0D0 )
CALL MXINT ( UMAT, 6, 60, 0.0D0 )
```

C  
C

IF ( ND .EQ. 2 ) THEN

HMATS(1,1) = 1.0

```

      HMATS(2,4) = 1.0
      HMATS(3,2) = 1.0
      HMATS(3,3) = 1.0
C
C
      ELSE IF ( ND .EQ. 3 ) THEN
C
      HMATS(1,1) = 1.0
      HMATS(2,5) = 1.0
      HMATS(3,9) = 1.0
      HMATS(4,6) = 1.0
      HMATS(4,8) = 1.0
      HMATS(5,3) = 1.0
      HMATS(5,7) = 1.0
      HMATS(6,2) = 1.0
      HMATS(6,4) = 1.0
C
      END IF
C
      DO I = 1, ND
      DO J = 1, ND
      IF ( ND .EQ. 2 ) THEN
      TMAT(I,J) = AJINV(I,J)
      TMAT(I+ND,J+ND) = AJINV(I,J)
      ELSE IF ( ND .EQ. 3 ) THEN
      TMAT(I,J) = AJINV(I,J)
      TMAT(I+ND,J+ND) = AJINV(I,J)
      TMAT(I+2*ND,J+2*ND) = AJINV(I,J)
      END IF
      END DO
      END DO
C
C
      COMPUTE THE TRANSFORMATION MATRIX UMAT
C
      IF ( ND .EQ. 2 ) THEN
      DO J = 1, NODNUM
      UMAT(1,2*(J-1)+1) = DXSI(J)
      UMAT(2,2*(J-1)+1) = DETA(J)
      UMAT(3,2*(J-1)+2) = DXSI(J)
      UMAT(4,2*(J-1)+2) = DETA(J)
      END DO
      ELSE IF ( ND .EQ. 3 ) THEN
      DO J = 1, NODNUM
      UMAT(1,3*(J-1)+1) = DXSI(J)
      UMAT(2,3*(J-1)+1) = DETA(J)
      UMAT(3,3*(J-1)+1) = DCEE(J)
      UMAT(4,3*(J-1)+2) = DXSI(J)
      UMAT(5,3*(J-1)+2) = DETA(J)
      UMAT(6,3*(J-1)+2) = DCEE(J)
      UMAT(7,3*(J-1)+3) = DXSI(J)
      UMAT(8,3*(J-1)+3) = DETA(J)
      UMAT(9,3*(J-1)+3) = DCEE(J)
      END DO
      END IF
C
      NDV = ND**2
      CALL MXMUL(TMAT,UMAT,TMP1,9,9,36,NDV,NDV,NDOFL)
      CALL MXMUL(HMATS,TMP1,BMATS,6,36,6,MORD,NDV,NDOFL)
C
      RETURN
      END
C
C
      SUBROUTINE TRANS( COORDS,CTRN,STRN,ETRN,TRI,JTYPE,MCRD,NNODE )
      IMPLICIT REAL*8 (A-H,O-Z)
C
      CALCULATE TRANSFORMATION MATRICES FOR CONVERTING QUANTITIES

```

```

C      BETWEEN ELEMENT AND GLOBAL COORDINATE SYSTEMS
C
1      DIMENSION X(20),      Y(20),      Z(20),      EO(3,3),
2      EP(3,3),      TRI(3,3), STRN(6,6),
      ETRN(6,6), CTRN(36,36), COORDS(MCRD,NNODE)
C
      CALL MXINT ( CTRN, 36, 36, 0.0D0 )
      CALL MXINT ( STRN, 6, 6, 0.0D0 )
      CALL MXINT ( ETRN, 6, 6, 0.0D0 )
C
      DO I = 1, NNODE
        X(I) = COORDS(1,I)
        Y(I) = COORDS(2,I)
        Z(I) = COORDS(3,I)
      END DO
C
C      UNIT VECTORS IN GLOBAL SYSTEM
C
      EO(1,1) = 1.0
      EO(1,2) = 0.0
      EO(1,3) = 0.0
      EO(2,1) = 0.0
      EO(2,2) = 1.0
      EO(2,3) = 0.0
      EO(3,1) = 0.0
      EO(3,2) = 0.0
      EO(3,3) = 1.0
C
C      DETERMINE ELEMENT COORDINATE VECTORS
C
      IF ( JTYPE .EQ. 1 .OR. JTYPE .EQ. 2 .OR.
1      JTYPE .EQ. 4 ) THEN
C
      AL = SQRT( (X(2)-X(1))**2+(Y(2)-Y(1))**2 )
      EP(1,1) = (X(2)-X(1))/AL
      EP(1,2) = (Y(2)-Y(1))/AL
      AL = SQRT( (X(3)-X(1))**2+(Y(3)-Y(1))**2 )
      EP(2,1) = (X(3)-X(1))/AL
      EP(2,2) = (Y(3)-Y(1))/AL
C
      NDIM = 2
C
      ELSE IF ( JTYPE .EQ. 3 ) THEN
C
      AL = SQRT( (X(3)-X(1))**2+(Y(3)-Y(1))**2 )
      EP(1,1) = (X(3)-X(1))/AL
      EP(1,2) = (Y(3)-Y(1))/AL
      AL = SQRT( (X(6)-X(1))**2+(Y(6)-Y(1))**2 )
      EP(2,1) = (X(6)-X(1))/AL
      EP(2,2) = (Y(6)-Y(1))/AL
C
      NDIM = 2
C
      ELSE IF ( JTYPE .EQ. 5 ) THEN
C
      AL = SQRT( (X(2)-X(1))**2+(Y(2)-Y(1))**2+(Z(2)-Z(1))**2 )
      EP(1,1) = (X(2)-X(1))/AL
      EP(1,2) = (Y(2)-Y(1))/AL
      EP(1,3) = (Z(2)-Z(1))/AL
      AL = SQRT( (X(4)-X(1))**2+(Y(4)-Y(1))**2+(Z(4)-Z(1))**2 )
      EP(2,1) = (X(4)-X(1))/AL
      EP(2,2) = (Y(4)-Y(1))/AL
      EP(2,3) = (Z(4)-Z(1))/AL
      AL = SQRT( (X(5)-X(1))**2+(Y(5)-Y(1))**2+(Z(5)-Z(1))**2 )
      EP(3,1) = (X(5)-X(1))/AL
      EP(3,2) = (Y(5)-Y(1))/AL
      EP(3,3) = (Z(5)-Z(1))/AL
C

```



```

STRN(1,6) = 2*TRI(1,1)*TRI(1,2)
STRN(2,1) = TRI(2,1)**2
STRN(2,2) = TRI(2,2)**2
STRN(2,3) = TRI(2,3)**2
STRN(2,4) = 2*TRI(2,2)*TRI(2,3)
STRN(2,5) = 2*TRI(2,3)*TRI(2,1)
STRN(2,6) = 2*TRI(2,1)*TRI(2,2)
STRN(3,1) = TRI(3,1)**2
STRN(3,2) = TRI(3,2)**2
STRN(3,3) = TRI(3,3)**2
STRN(3,4) = 2*TRI(3,2)*TRI(3,3)
STRN(3,5) = 2*TRI(3,3)*TRI(3,1)
STRN(3,6) = 2*TRI(3,1)*TRI(3,2)
STRN(4,1) = TRI(2,1)*TRI(3,1)
STRN(4,2) = TRI(2,2)*TRI(3,2)
STRN(4,3) = TRI(2,3)*TRI(3,3)
STRN(4,4) = TRI(2,2)*TRI(3,3) + TRI(3,2)*TRI(2,3)
STRN(4,5) = TRI(2,3)*TRI(3,1) + TRI(3,3)*TRI(2,1)
STRN(4,6) = TRI(2,1)*TRI(3,2) + TRI(3,1)*TRI(2,2)
STRN(5,1) = TRI(3,1)*TRI(1,1)
STRN(5,2) = TRI(3,2)*TRI(1,2)
STRN(5,3) = TRI(3,3)*TRI(1,3)
STRN(5,4) = TRI(3,2)*TRI(1,3) + TRI(1,2)*TRI(3,3)
STRN(5,5) = TRI(3,3)*TRI(1,1) + TRI(1,3)*TRI(3,1)
STRN(5,6) = TRI(3,1)*TRI(1,2) + TRI(1,1)*TRI(3,2)
STRN(6,1) = TRI(1,1)*TRI(2,1)
STRN(6,2) = TRI(1,2)*TRI(2,2)
STRN(6,3) = TRI(1,3)*TRI(2,3)
STRN(6,4) = TRI(1,2)*TRI(2,3) + TRI(2,2)*TRI(1,3)
STRN(6,5) = TRI(1,3)*TRI(2,1) + TRI(2,3)*TRI(1,1)
STRN(6,6) = TRI(1,1)*TRI(2,2) + TRI(2,1)*TRI(1,2)

```

```

C
END IF

```

```

C
C
C
C
STRAIN TRANSFORMATION MATRIX, [ETRN]:

```

```

DO I = 1, NDIM
  DO J = 1, NDIM
    ETRN(I,J) = STRN(I,J)
    ETRN(I,J+NDIM) = STRN(I,J+NDIM)/2.0
    ETRN(I+NDIM,J) = STRN(I+NDIM,J)/0.5
    ETRN(I+NDIM,J+NDIM) = STRN(I+NDIM,J+NDIM)
  END DO
END DO

```

```

C
RETURN
END

```

```

C
C
C
C
SUBROUTINE MATPROP(PTHK, THETA, E1, E2, E3, G12, G23, G31, V12, V23, V13,
1 SMAT, NELDIM, IPLANE, MORD, K, NLAY )

```

```

C
C
C
C
COMPUTATION OF ELEMENT MATERIAL PROPERTY MATRICES

```

```

C
IMPLICIT DOUBLE PRECISION (A-H,O-Z)

```

```

DIMENSION E1(100), E2(100), E3(100), V12(100), V23(100), V13(100),
1 G12(100), G23(100), G31(100), V21(100), V32(100), V31(100),
2 THETA(100), PTHK(100), DMAT(6,6), SMAT(6,6), QBAR(6,6),
3 CL(6,6), INDEX(6), NLAY(3)

```

```

C
PI = ACOS(-1.0D0)
TLM = 0.0
CALL MXINT( DMAT, 6, 6, 0.0D0 )
CALL MXINT( QBAR, 6, 6, 0.0D0 )

```

```

C
NOFF = 1

```

```

DO I = 1, K-1
  NOFF = NOFF + NLAY(I)
END DO
NL = NLAY(K) + NOFF - 1

```

```

IF ( NELDIM .EQ. 2 ) THEN

```

```

  [EX,EY,GXY]

```

```

  IF ( IPLANE .EQ. 0 ) THEN

```

```

    PLANE STRESS

```

```

    DO I = NOFF, NL

```

```

      PHI = THETA(I) * PI / 180.0
      C   = COS(PHI)
      S   = SIN(PHI)
      C2  = C*C
      S2  = S*S
      C3  = C2*C
      S3  = S2*S
      C4  = C2*C2
      S4  = S2*S2

```

```

      TLM = TLM + PTHK(I)

```

```

      V21(I) = V12(I)*E2(I)/E1(I)

```

```

      Q11 = E1(I)/(1.0-V12(I)*V21(I))

```

```

      Q12 = V21(I)*Q11

```

```

      Q22 = E2(I)/(1.0-V12(I)*V21(I))

```

```

      Q66 = G12(I)

```

```

      QBAR(1,1) = Q11*C4+2.*(Q12+2.*Q66)*C2*S2+Q22*S4

```

```

      QBAR(1,2) = (Q11+Q22-4.*Q66)*S2*C2+Q12*(C4+S4)

```

```

      QBAR(1,3) = (Q11-Q12-2.*Q66)*S*C3+(Q12-Q22+2.*Q66)*S3*C

```

```

      QBAR(2,1) = QBAR(1,2)

```

```

      QBAR(2,2) = Q11*S4+2.*(Q12+2.*Q66)*S2*C2+Q22*C4

```

```

      QBAR(2,3) = (Q11-Q12-2.*Q66)*S3*C+(Q12-Q22+2.*Q66)*S*C3

```

```

      QBAR(3,1) = QBAR(1,3)

```

```

      QBAR(3,2) = QBAR(2,3)

```

```

      QBAR(3,3) = (Q11+Q22-2.*Q12-2.*Q66)*S2*C2+Q66*(S4+C4)

```

```

      DO L = 1, 3

```

```

        DO J = 1, 3

```

```

          DMAT(L,J) = DMAT(L,J) + QBAR(L,J) * PTHK(I)

```

```

        END DO

```

```

      END DO

```

```

    END DO

```

```

  DO L = 1, 3

```

```

    DO J = 1, 3

```

```

      DMAT(L,J) = DMAT(L,J)/TLM

```

```

    END DO

```

```

  END DO

```

```

ELSE

```

```

  PLANE STRAIN

```

```

  DO I = NOFF, NL

```

```

    PHI = THETA(I) * PI / 180.0

```

```

    C   = COS(PHI)

```

```

    S   = SIN(PHI)

```

```

    C2  = C*C

```

```

    S2  = S*S

```

```

    C3  = C2*C

```

```

    S3  = S2*S

```





```

C
C
C
CL(6,6) = G12(I)
COMPUTE THE KTH REDUCED C MATRIX (PLATE COORDINATES)

C0 = COS(PHI)
C2 = C0*C0
C3 = C0*C0*C0
C4 = C2*C2
S0 = SIN(PHI)
S2 = S0*S0
S3 = S0*S0*S0
S4 = S2*S2
C2T = COS(2.*PHI)

C
1 QBAR(1,1) = CL(1,1)*C4 + CL(2,2)*S4 + 2*CL(1,2)*S2*C2 +
1 4*CL(6,6)*C2*S2
1 QBAR(2,2) = CL(1,1)*S4 + CL(2,2)*C4 + 2*(CL(1,2) +
1 2*CL(6,6))*S2*C2
1 QBAR(3,3) = CL(3,3)
1 QBAR(4,4) = CL(4,4)*C2 + CL(5,5)*S2
1 QBAR(5,5) = CL(5,5)*C2 + CL(4,4)*S2
1 QBAR(6,6) = CL(6,6) + (CL(1,1) + CL(2,2) - 2*CL(1,2) -
1 4*CL(6,6))*S2*C2
1 QBAR(1,2) = CL(1,2) + (CL(1,1) + CL(2,2) - 2*CL(1,2) -
1 4*CL(6,6))*S2*C2
1 QBAR(2,1) = QBAR(1,2)
1 QBAR(1,3) = CL(1,3)*C2 + CL(2,3)*S2
1 QBAR(3,1) = QBAR(1,3)
1 QBAR(2,3) = CL(1,3)*S2 + CL(2,3)*C2
1 QBAR(3,2) = QBAR(2,3)
1 QBAR(1,6) = C0*S0*(CL(1,1)*C2 - CL(2,2)*S2 - C2T*(CL(1,2) +
1 2*CL(6,6)))
1 QBAR(6,1) = QBAR(1,6)
1 QBAR(2,6) = S0*C0*(CL(1,1)*S2 - CL(2,2)*C2 + C2T*(CL(1,2) +
1 2*CL(6,6)))
1 QBAR(6,2) = QBAR(2,6)
1 QBAR(3,6) = S0*C0*(CL(1,3) - CL(2,3))
1 QBAR(6,3) = QBAR(3,6)
1 QBAR(4,5) = S0*C0*(CL(5,5) - CL(4,4))
1 QBAR(5,4) = QBAR(4,5)

C
C
TLM = TLM + PTHK(I)

DO L = 1, 6
DO J = 1, 6
DMAT(L,J) = DMAT(L,J) + QBAR(L,J) * PTHK(I)
END DO
END DO

C
C
DO L = 1, 6
DO J = 1, 6
DMAT(L,J) = DMAT(L,J) / TLM
END DO
END DO

C
C
END IF

C
C
C
C
COMPUTE SMAT = DMAT-1

NDIM = 6
CALL INVERS (DMAT, SMAT, INDEX, NDIM, MORD)

C
RETURN
END
C

```





```

DCEE(5) = 0.125*(1.-XSI)*(1.-ETA)
DCEE(6) = 0.125*(1.+XSI)*(1.-ETA)
DCEE(7) = 0.125*(1.+XSI)*(1.+ETA)
DCEE(8) = 0.125*(1.-XSI)*(1.+ETA)

```

```

C
C
END IF

```

```

C
C
RETURN
END

```

```

C
C
SUBROUTINE JACOB( NODNUM, NELDIM, X, Y, Z, DXSI, DETA, DCEE,
1 AJINV, DETJ )

```

```

C
C
COMPUTE JACOBIAN MATRIX, ITS DETERMINANT AND INVERSE

```

```

C
C
IMPLICIT DOUBLE PRECISION (A-H,O-Z)

```

```

C
C
DIMENSION X(20), Y(20), Z(20), DXSI(20), DETA(20), DCEE(20),
1 AJMT(3,3), AJINV(3,3)

```

```

C
C
CALL MXINT( AJMT, 3, 3, 0.0D0 )

```

```

C
C
IF ( NELDIM .EQ. 2 ) THEN

```

```

C
C
DO I = 1, NODNUM

```

```

AJMT(1,1) = AJMT(1,1) + DXSI(I)*X(I)
AJMT(1,2) = AJMT(1,2) + DXSI(I)*Y(I)
AJMT(2,1) = AJMT(2,1) + DETA(I)*X(I)
AJMT(2,2) = AJMT(2,2) + DETA(I)*Y(I)

```

```

C
C
END DO

```

```

C
C
DETJ = AJMT(1,1)*AJMT(2,2) - AJMT(2,1)*AJMT(1,2)

```

```

C
C
COMPUTE INVERSE OF JACOBIAN

```

```

AJINV(1,1) = AJMT(2,2)/DETJ
AJINV(2,1) = -AJMT(2,1)/DETJ
AJINV(1,2) = -AJMT(1,2)/DETJ
AJINV(2,2) = AJMT(1,1)/DETJ

```

```

C
C
ELSE IF ( NELDIM .EQ. 3 ) THEN

```

```

C
C
DO I = 1, NODNUM

```

```

AJMT(1,1) = AJMT(1,1) + DXSI(I)*X(I)
AJMT(1,2) = AJMT(1,2) + DXSI(I)*Y(I)
AJMT(1,3) = AJMT(1,3) + DXSI(I)*Z(I)
AJMT(2,1) = AJMT(2,1) + DETA(I)*X(I)
AJMT(2,2) = AJMT(2,2) + DETA(I)*Y(I)
AJMT(2,3) = AJMT(2,3) + DETA(I)*Z(I)
AJMT(3,1) = AJMT(3,1) + DCEE(I)*X(I)
AJMT(3,2) = AJMT(3,2) + DCEE(I)*Y(I)
AJMT(3,3) = AJMT(3,3) + DCEE(I)*Z(I)

```

```

C
C
END DO

```

```

C
C
DETJ = AJMT(1,1)*(AJMT(2,2)*AJMT(3,3) - AJMT(2,3)*AJMT(3,2)) -
1 AJMT(1,2)*(AJMT(2,1)*AJMT(3,3) - AJMT(2,3)*AJMT(3,1)) +
2 AJMT(1,3)*(AJMT(2,1)*AJMT(3,2) - AJMT(2,2)*AJMT(3,1))

```

```

C
C
COMPUTE INVERSE OF JACOBIAN

```

```

AJINV(1,1) = (AJMT(2,2)*AJMT(3,3) - AJMT(2,3)*AJMT(3,2))/DETJ
AJINV(2,1) = -(AJMT(2,1)*AJMT(3,3) - AJMT(2,3)*AJMT(3,1))/DETJ
AJINV(3,1) = (AJMT(2,1)*AJMT(3,2) - AJMT(2,2)*AJMT(3,1))/DETJ
AJINV(1,2) = -(AJMT(1,2)*AJMT(3,3) - AJMT(1,3)*AJMT(3,2))/DETJ
AJINV(2,2) = (AJMT(1,1)*AJMT(3,3) - AJMT(1,3)*AJMT(3,1))/DETJ

```

```

AJINV(3,2) = -(AJMT(1,1)*AJMT(3,2)-AJMT(1,2)*AJMT(3,1))/DETJ
AJINV(1,3) = (AJMT(1,2)*AJMT(2,3)-AJMT(1,3)*AJMT(2,2))/DETJ
AJINV(2,3) = -(AJMT(1,1)*AJMT(2,3)-AJMT(1,3)*AJMT(2,1))/DETJ
AJINV(3,3) = (AJMT(1,1)*AJMT(2,2)-AJMT(1,2)*AJMT(2,1))/DETJ

```

```

C
C
END IF

```

```

C
C
RETURN
END

```

```

C
C
SUBROUTINE ASTRSS(XSI,ETA,CEE,JTYPE,LAYER,NELDIM,NODNUM,
1 PMAT,THK,NBVAL,XE,YE,ZE,NSIDE,NVER)

```

```

C
C
IMPLICIT DOUBLE PRECISION (A-H,O-Z)

```

```

C
C
DIMENSION PMAT(6,100),XE(20),YE(20),ZE(20),THK(3)

```

```

C
C
IF ( NELDIM .EQ. 2 ) THEN

```

```

C
C
IF ( NODNUM .EQ. 4 ) THEN

```

```

C
C
AC = (-XE(1)+XE(2)-XE(3)+XE(4))/4.0
BC = (-YE(1)-YE(2)+YE(3)+YE(4))/4.0

```

```

C
C
X = AC*XSI
Y = BC*ETA
A = (XE(2)-XE(1))/2.0

```

```

C
C
ELSE IF ( NODNUM .EQ. 6 ) THEN

```

```

C
C
AC = (-XE(3)+XE(4))/2.0
BC = (-YE(1)-YE(2)+YE(5)+YE(6))/4.0

```

```

C
C
X = AC*XSI
Y = BC*ETA

```

```

C
C
ELSE IF ( NODNUM .EQ. 8 ) THEN

```

```

C
C
AC = (-XE(4)+XE(5))/2.0
BC = (-YE(2)+YE(7))/2.0

```

```

C
C
X = AC*XSI
Y = BC*ETA
A = (XE(2)-XE(1))/2.0

```

```

C
C
END IF

```

```

C
C
ELSE IF ( NELDIM .EQ. 3 ) THEN

```

```

C
C
IF ( NODNUM .EQ. 8 ) THEN

```

```

C
C
AC = (-XE(1)+XE(2)+XE(3)-XE(4)-XE(5)+XE(6)+XE(7)-XE(8))/8.0
BC = (-YE(1)-YE(2)+YE(3)+YE(4)-YE(5)-YE(6)+YE(7)+YE(8))/8.0
CC = (-ZE(1)-ZE(2)-ZE(3)-ZE(4)+ZE(5)+ZE(6)+ZE(7)+ZE(8))/8.0

```

```

C
C
X = AC*XSI
Y = BC*ETA
Z = CC*CEE

```

```

C
C
END IF

```

```

C
C
END IF

```

```

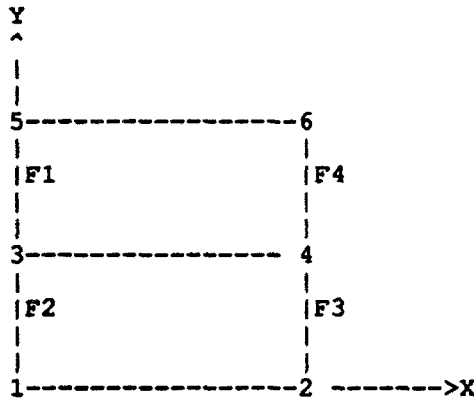
C
C
CALL MXINT(PMAT, 6, 100, 0.0D0)

```

IF ( JTYPE .EQ. 1 ) THEN

2-D 2-LAYERED 6-NODE HYBRID ELEMENT.

OPTIONAL ZERO TRACTION CONDITIONS MAY BE  
SPECIFIED ON DESIGNATED ELEMENT SIDES.



T1 = THK(1)/2.

T2 = THK(2)/2.

IF ( NVER .EQ. 11 ) THEN

LINEAR STRESS FIELD

NBVAL = 10

IF ( LAYER .EQ. 1 ) THEN

PMAT(1,1) = 1.0  
PMAT(1,2) = X  
PMAT(1,3) = Y  
PMAT(2,4) = 1.0  
PMAT(2,5) = X  
PMAT(2,7) = -Y  
PMAT(3,6) = 1.0  
PMAT(3,7) = X  
PMAT(3,2) = -Y

ELSE IF ( LAYER .EQ. 2 ) THEN

PMAT(1,8) = 1.0  
PMAT(1,9) = X  
PMAT(1,10) = Y  
PMAT(2,4) = 1.0  
PMAT(2,5) = X  
PMAT(2,7) = -(T1+T2+Y)  
PMAT(3,6) = 1.0  
PMAT(3,2) = -T1  
PMAT(3,7) = X  
PMAT(3,9) = -(T2+Y)

END IF

ELSE IF ( NVER .EQ. 12 ) THEN

QUADRATIC STRESS FIELD

NBVAL = 18

IF ( LAYER .EQ. 1 ) THEN

PMAT(1,1) = 1.0  
PMAT(1,2) = -X  
PMAT(1,3) = Y  
PMAT(1,4) = -2\*X\*Y  
PMAT(1,5) = -0.5\*X\*\*2  
PMAT(1,6) = Y\*\*2  
PMAT(2,7) = 1.0  
PMAT(2,8) = 2\*(T1+T2)\*X - 2\*X\*Y  
PMAT(2,9) = X  
PMAT(2,5) = T1\*Y - 0.5\*Y\*\*2  
PMAT(2,10) = -Y  
PMAT(2,11) = T2\*Y  
PMAT(2,12) = X\*\*2  
PMAT(3,13) = 1.0  
PMAT(3,5) = -T1\*X + X\*Y  
PMAT(3,10) = X  
PMAT(3,11) = -T2\*X  
PMAT(3,2) = Y  
PMAT(3,8) = X\*\*2  
PMAT(3,4) = Y\*\*2

ELSE IF ( LAYER .EQ. 2 ) THEN

PMAT(1,14) = 1.0  
PMAT(1,15) = -X  
PMAT(1,16) = Y  
PMAT(1,17) = -2\*X\*Y  
PMAT(1,11) = -0.5\*X\*\*2  
PMAT(1,18) = Y\*\*2  
PMAT(2,7) = 1.0  
PMAT(2,5) = 0.5\*T1\*\*2  
PMAT(2,10) = -(T1+T2) - Y  
PMAT(2,11) = (T1\*T2+0.5\*T2\*\*2) - 0.5\*Y\*\*2  
PMAT(2,9) = X  
PMAT(2,8) = -2\*X\*Y  
PMAT(2,12) = X\*\*2  
PMAT(3,13) = 1.0  
PMAT(3,2) = T1  
PMAT(3,4) = T1\*\*2  
PMAT(3,15) = T2 + Y  
PMAT(3,17) = -T2\*\*2 + Y\*\*2  
PMAT(3,10) = X  
PMAT(3,11) = X\*Y  
PMAT(3,8) = X\*\*2

END IF

ELSE IF ( NVER .EQ. 13 .AND. NSIDE .EQ. 1 ) THEN

TRACTIONS SX & TXY SET TO ZERO ON FACE F1

NBVAL = 15

IF ( LAYER .EQ. 1 ) THEN

PMAT(1,1) = 1.0  
PMAT(1,2) = -X  
PMAT(1,3) = Y  
PMAT(1,4) = -2\*X\*Y  
PMAT(1,5) = -0.5\*X\*\*2  
PMAT(1,6) = Y\*\*2  
PMAT(2,5) = -0.5\*T1\*\*2 + T1\*Y - 0.5\*Y\*\*2  
PMAT(2,7) = -(T1+T2) + Y  
PMAT(2,8) = 1.0  
PMAT(2,9) = -T1\*T2 + T2\*Y  
PMAT(2,10) = 2\*T1\*X - 2\*X\*Y



```

PMAT (2,11) = X
PMAT (2,12) = -T2*X
PMAT (2,13) = X**2
PMAT (3,2) = -T1 + Y
PMAT (3,4) = -T1**2 + Y**2
PMAT (3,7) = -A - X
PMAT (3,10) = -A**2 + X**2
PMAT (3,9) = -A*T2 - T2*X
PMAT (3,5) = -T1*X + X*Y

```

```

ELSE IF ( LAYER .EQ. 2 ) THEN

```

```

PMAT (1,9) = -A*(X+A)
PMAT (1,14) = (X**2-A**2)
PMAT (1,15) = Y*(X+A)
PMAT (2,8) = 1.0
PMAT (2,11) = X
PMAT (2,7) = Y
PMAT (2,12) = X*Y
PMAT (2,13) = X**2
PMAT (3,7) = -(X+A)
PMAT (3,10) = (X**2-A**2)
PMAT (3,9) = Y*(X+A)

```

```

END IF

```

```

ELSE IF ( NVER .EQ. 13 .AND. NSIDE .EQ. 2 ) THEN

```

```

TRACTIONS SX & TXY SET TO ZERO ON FACE F2

```

```

NBVAL = 15

```

```

IF ( LAYER .EQ. 1 ) THEN

```

```

PMAT (1,9) = -A*(X+A)
PMAT (1,14) = (X**2-A**2)
PMAT (1,15) = Y*(X+A)
PMAT (2,8) = 1.0
PMAT (2,11) = X
PMAT (2,7) = Y
PMAT (2,12) = X*Y
PMAT (2,13) = X**2
PMAT (3,7) = -(X+A)
PMAT (3,10) = (X**2-A**2)
PMAT (3,9) = Y*(X+A)

```

```

ELSE IF ( LAYER .EQ. 2 ) THEN

```

```

PMAT (1,1) = 1.0
PMAT (1,2) = -X
PMAT (1,3) = Y
PMAT (1,4) = -2*X*Y
PMAT (1,5) = -0.5*X**2
PMAT (1,6) = Y**2
PMAT (2,5) = -0.5*T2**2 - T2*Y - 0.5*Y**2
PMAT (2,7) = (T1+T2) + Y
PMAT (2,8) = 1.0
PMAT (2,9) = -T1*T2 - T1*Y
PMAT (2,10) = -2*T2*X - 2*X*Y
PMAT (2,11) = X
PMAT (2,12) = T1*X
PMAT (2,13) = X**2
PMAT (3,2) = T2 + Y
PMAT (3,4) = -T2**2 + Y**2
PMAT (3,7) = -A - X
PMAT (3,10) = -A**2 + X**2
PMAT (3,9) = A*T1 + T1*X
PMAT (3,5) = T2*X + X*Y

```



ELSE IF ( LAYER .EQ. 2 ) THEN

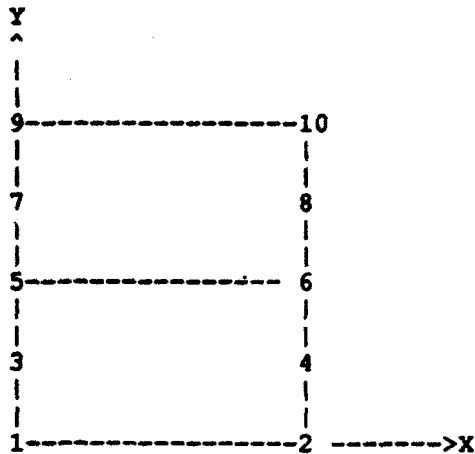
```
PMAT(1,1) = 1.0
PMAT(1,2) = -X
PMAT(1,3) = Y
PMAT(1,4) = -2*X*Y
PMAT(1,5) = -0.5*X**2
PMAT(1,6) = Y**2
PMAT(2,5) = -0.5*T2**2 - T2*Y - 0.5*Y**2
PMAT(2,7) = (T1+T2) + Y
PMAT(2,8) = 1.0
PMAT(2,9) = -T1*T2 - T1*Y
PMAT(2,10) = -2*T2*X - 2*X*Y
PMAT(2,11) = X
PMAT(2,12) = T1*X
PMAT(2,13) = X**2
PMAT(3,2) = T2 + Y
PMAT(3,4) = -T2**2 + Y**2
PMAT(3,7) = A - X
PMAT(3,10) = -A**2 + X**2
PMAT(3,9) = -A*T1 + T1*X
PMAT(3,5) = T2*X + X*Y
```

END IF

END IF

ELSE IF ( JTYPE .EQ. 2 ) THEN

2-D 2-LAYERED 10-NODE HYBRID ELEMENT.



```
T1 = THK(1)/2.
T2 = THK(2)/2.
```

IF ( NVER .EQ. 11 ) THEN

QUADRATIC STRESS FIELD

NBVAL = 18

IF ( LAYER .EQ. 1 ) THEN

```
PMAT(1,1) = 1.0
PMAT(1,2) = -X
PMAT(1,3) = Y
PMAT(1,4) = -2*X*Y
PMAT(1,5) = -0.5*X**2
PMAT(1,6) = Y**2
```

```

PMAT (2,7) = 1.0
PMAT (2,8) = 2*(T1+T2)*X - 2*X*Y
PMAT (2,9) = X
PMAT (2,5) = T1*Y - 0.5*Y**2
PMAT (2,10) = -Y
PMAT (2,11) = T2*Y
PMAT (2,12) = X**2
PMAT (3,13) = 1.0
PMAT (3,5) = -T1*X + X*Y
PMAT (3,10) = X
PMAT (3,11) = -T2*X
PMAT (3,2) = Y
PMAT (3,8) = X**2
PMAT (3,4) = Y**2

```

```

ELSE IF ( LAYER .EQ. 2 ) THEN

```

```

PMAT (1,14) = 1.0
PMAT (1,15) = -X
PMAT (1,16) = Y
PMAT (1,17) = -2*X*Y
PMAT (1,11) = -0.5*X**2
PMAT (1,18) = Y**2
PMAT (2,7) = 1.0
PMAT (2,5) = 0.5*T1**2
PMAT (2,10) = -(T1+T2) - Y
PMAT (2,11) = (T1*T2+0.5*T2**2) - 0.5*Y**2
PMAT (2,9) = X
PMAT (2,8) = -2*X*Y
PMAT (2,12) = X**2
PMAT (3,13) = 1.0
PMAT (3,2) = T1
PMAT (3,4) = T1**2
PMAT (3,15) = T2 + Y
PMAT (3,17) = -T2**2 + Y**2
PMAT (3,10) = X
PMAT (3,11) = X*Y
PMAT (3,8) = X**2

```

```

END IF

```

```

ELSE IF ( NVER .EQ. 12 ) THEN

```

```

CUBIC ORDER EXPANSION

```

```

NVAL = 28

```

```

IF ( LAYER .EQ. 1 ) THEN

```

```

PMAT (1,1) = 1.0
PMAT (1,2) = -X
PMAT (1,3) = Y
PMAT (1,4) = -2*X*Y
PMAT (1,5) = -0.5*X**2
PMAT (1,6) = Y**2
PMAT (1,7) = -3*X*Y**2
PMAT (1,8) = -Y*X**2
PMAT (1,9) = -X**3/3.
PMAT (1,10) = Y**3
PMAT (2,5) = -T1**2/2 + T1*Y - 0.5*Y**2
PMAT (2,8) = -2*T1**3/3 + T1**2*Y - Y**3/3.
PMAT (2,13) = T1+T2 - Y
PMAT (2,12) = 1.0
PMAT (2,14) = -(T1*T2+0.5*T2**2) + T2*Y
PMAT (2,15) = (T1*T2**2+T2**3/3.) - T2**2*Y
PMAT (2,9) = -T1**2*X + 2*T1*X*Y - X*Y**2
PMAT (2,16) = X

```

```

PMAT (2,17) = 2*(T1+T2)*X - 2*X*Y
PMAT (2,18) = -(2*T1*T2+T2**2)*X + 2*T2*X*Y
PMAT (2,19) = 3*(T1+T2)*X**2 - 3*Y*X**2
PMAT (2,11) = X**2
PMAT (2,20) = X**3
PMAT (3,2) = -T1 + Y
PMAT (3,4) = -T1**2 + Y**2
PMAT (3,7) = -T1**3 + Y**3
PMAT (3,21) = 1.0
PMAT (3,22) = -T2
PMAT (3,23) = T2**2
PMAT (3,24) = -T2**3
PMAT (3,5) = -T1*X + X*Y
PMAT (3,8) = -T1**2*X + X*Y**2
PMAT (3,13) = X
PMAT (3,14) = -T2*X
PMAT (3,15) = T2**2*X
PMAT (3,9) = -T1*X**2 + Y*X**2
PMAT (3,17) = X**2
PMAT (3,18) = -T2*X**2
PMAT (3,19) = X**3

```

C  
C

```
ELSE IF ( LAYER .EQ. 2 ) THEN
```

```

PMAT (1,25) = 1.0
PMAT (1,22) = -X
PMAT (1,26) = Y
PMAT (1,23) = -2*X*Y
PMAT (1,14) = -0.5*X**2
PMAT (1,27) = Y**2
PMAT (1,24) = -3*X*Y**2
PMAT (1,15) = -Y*X**2
PMAT (1,18) = -X**3/3.
PMAT (1,28) = Y**3
PMAT (2,12) = 1.0
PMAT (2,16) = X
PMAT (2,13) = -Y
PMAT (2,17) = -2*X*Y
PMAT (2,11) = X**2
PMAT (2,14) = -0.5*Y**2
PMAT (2,18) = -X*Y**2
PMAT (2,19) = -3*Y*X**2
PMAT (2,20) = X**3
PMAT (2,15) = -Y**3/3.
PMAT (3,21) = 1.0
PMAT (3,13) = X
PMAT (3,22) = Y
PMAT (3,14) = X*Y
PMAT (3,17) = X**2
PMAT (3,23) = Y**2
PMAT (3,15) = X*Y**2
PMAT (3,18) = Y*X**2
PMAT (3,19) = X**3
PMAT (3,24) = Y**3

```

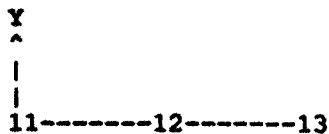
C  
C  
C  
C  
C  
C  
C  
C  
C  
C  
C

```
END IF
```

```
END IF
```

```
ELSE IF ( JTYPE .EQ. 3 ) THEN
```

```
2-D 2-LAYERED 13-NODE HYBRID ELEMENT.
```





```

PMAT (1,10) = 1.0
PMAT (1,13) = X
PMAT (1,15) = Y
PMAT (1,17) = X**2
PMAT (1,19) = X*Y
PMAT (1,21) = Y**2
PMAT (1,22) = X**3
PMAT (1,23) = 3*X**2*Y
PMAT (1,24) = 3*X*Y**2
PMAT (2,11) = 1.0
PMAT (2,14) = X
PMAT (2,16) = Y
PMAT (2,17) = Y**2
PMAT (2,18) = X**2
PMAT (2,20) = X*Y
PMAT (2,22) = 3*X*Y**2
PMAT (2,23) = Y**3
PMAT (2,25) = 3*X**2*Y
PMAT (3,12) = 1.0
PMAT (3,13) = -Y
PMAT (3,16) = -X
PMAT (3,17) = -2*X*Y
PMAT (3,19) = -0.5*Y**2
PMAT (3,20) = -0.5*X**2
PMAT (3,22) = -3*X**2*Y
PMAT (3,23) = -3*X*Y**2
PMAT (3,24) = -Y**3
PMAT (3,25) = -X**3
PMAT (3,27) = X**4

```

```

END IF

```

```

ELSE IF ( NVER .EQ. 12 ) THEN

```

```

    QUADRATIC STRESS FIELD

```

```

    (ONLY STRESS CONTINUITY CONDITIONS AT INTERFACE ENFORCED)

```

```

    NBVAL = 30

```

```

    IF ( LAYER .EQ. 1 ) THEN

```

```

        PMAT (1,1) = Y**2
        PMAT (1,2) = X**2
        PMAT (1,3) = X*Y
        PMAT (1,4) = Y
        PMAT (1,5) = X
        PMAT (1,6) = 1.0
        PMAT (2,7) = X*Y-X*T1
        PMAT (2,8) = Y
        PMAT (2,9) = 1.0
        PMAT (2,10) = X**2
        PMAT (2,11) = -X*T2
        PMAT (2,12) = X
        PMAT (2,13) = Y**2
        PMAT (3,14) = Y**2
        PMAT (3,15) = X*Y
        PMAT (3,16) = X**2
        PMAT (3,17) = Y
        PMAT (3,18) = X
        PMAT (3,19) = 1.0

```

```

    ELSE IF ( LAYER .EQ. 2 ) THEN

```

```

        PMAT (1,20) = 1.0
        PMAT (1,21) = Y**2
        PMAT (1,22) = X**2
        PMAT (1,23) = X*Y

```

```

PMAT (1,24) = Y
PMAT (1,25) = X
PMAT (2,26) = Y**2-T2**2
PMAT (2,27) = T2+Y
PMAT (2,8) = T1
PMAT (2,9) = 1.0
PMAT (2,10) = X**2
PMAT (2,11) = X*Y
PMAT (2,12) = X
PMAT (2,13) = T1**2
PMAT (3,28) = Y**2-T2**2
PMAT (3,29) = X*Y+T2*X
PMAT (3,30) = T2+Y
PMAT (3,14) = T1**2
PMAT (3,15) = T1*X
PMAT (3,16) = X**2
PMAT (3,17) = T1
PMAT (3,18) = X
PMAT (3,19) = 1.0

```

```

END IF

```

```

END IF

```

```

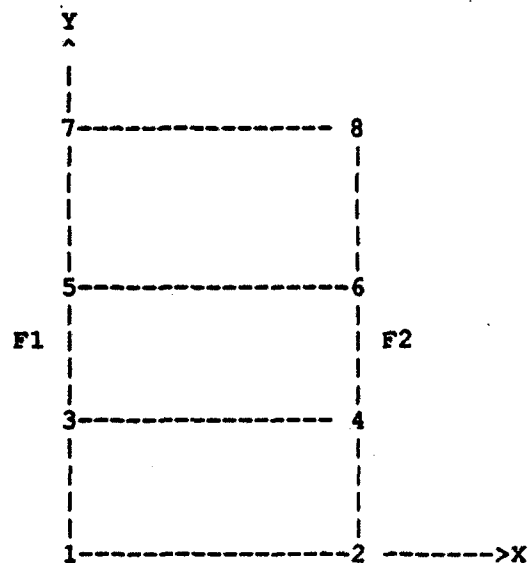
ELSE IF ( JTYPE .EQ. 4 ) THEN

```

```

2-D 3-LAYERED 8-NODE HYBRID ELEMENT.

```



```

T1 = THK(1)/2.
T2 = THK(2)/2.
T3 = THK(3)/2.

```

```

IF ( NVER .EQ. 11 ) THEN

```

```

QUADRATIC FIELD EXPANSION

```

```

NVAL = 24

```

```

IF ( LAYER .EQ. 1 ) THEN

```

```

PMAT (1,1) = 1.0
PMAT (1,2) = X
PMAT (1,3) = Y
PMAT (1,4) = X*Y
PMAT (1,5) = -0.5*X**2

```



```

PMAT(1,6) = Y**2
PMAT(2,5) = -0.5*T1**2 + T1*Y - 0.5*Y**2
PMAT(2,7) = -(T1+T2) + Y
PMAT(2,8) = -(T1*T2+0.5*T2**2) + T2*Y
PMAT(2,9) = 1.0
PMAT(2,10) = X
PMAT(2,11) = -(T1+T2)*X + X*Y
PMAT(2,12) = X**2
PMAT(3,2) = T1 - Y
PMAT(3,4) = 0.5*T1**2 - 0.5*Y**2
PMAT(3,13) = 1.0
PMAT(3,14) = T2
PMAT(3,15) = -0.5*T2**2
PMAT(3,5) = -T1*X + X*Y
PMAT(3,7) = -X
PMAT(3,8) = -T2*X
PMAT(3,11) = -0.5*X**2

```

```

C
C
ELSE IF ( LAYER .EQ. 2 ) THEN

```

```

PMAT(1,16) = 1.0
PMAT(1,14) = X
PMAT(1,17) = Y
PMAT(1,15) = X*Y
PMAT(1,8) = -0.5*X**2
PMAT(1,18) = Y**2
PMAT(2,9) = 1.0
PMAT(2,10) = X
PMAT(2,7) = Y
PMAT(2,11) = X*Y
PMAT(2,12) = X**2
PMAT(2,8) = -0.5*Y**2
PMAT(3,13) = 1.0
PMAT(3,7) = -X
PMAT(3,14) = -Y
PMAT(3,8) = X*Y
PMAT(3,11) = -0.5*X**2
PMAT(3,15) = -0.5*Y**2

```

```

C
C
ELSE IF ( LAYER .EQ. 3 ) THEN

```

```

PMAT(1,19) = 1.0
PMAT(1,20) = X
PMAT(1,21) = Y
PMAT(1,22) = X*Y
PMAT(1,23) = -0.5*X**2
PMAT(1,24) = Y**2
PMAT(2,9) = 1.0
PMAT(2,7) = (T2+T3) + Y
PMAT(2,8) = -(T2*T3+0.5*T2**2) - T2*Y
PMAT(2,23) = -0.5*T3**2 - T3*Y - 0.5*Y**2
PMAT(2,10) = X
PMAT(2,11) = (T2+T3)*X + X*Y
PMAT(2,12) = X**2
PMAT(3,13) = 1.0
PMAT(3,14) = -T2
PMAT(3,15) = -0.5*T2**2
PMAT(3,20) = -T3 - Y
PMAT(3,22) = 0.5*T3**2 - 0.5*Y**2
PMAT(3,7) = -X
PMAT(3,8) = T2*X
PMAT(3,23) = T3*X + X*Y
PMAT(3,11) = -0.5*X**2

```

```

C
C
END IF

```

```

C
ELSE IF ( NVER .EQ. 12 ) THEN

```

C  
C  
C  
C

CUBIC FIELD EXPANSION

NBVAL = 38

IF ( LAYER .EQ. 1 ) THEN

PMAT(1,1) = 1.0  
PMAT(1,2) = X  
PMAT(1,3) = Y  
PMAT(1,4) = X\*Y  
PMAT(1,5) = -X\*\*2/2.  
PMAT(1,6) = Y\*\*2  
PMAT(1,7) = -3.\*X\*Y\*\*2  
PMAT(1,8) = -Y\*X\*\*2  
PMAT(1,9) = -X\*\*3/3.  
PMAT(1,10) = Y\*\*3  
PMAT(2,5) = -0.5\*T1\*\*2 + T1\*Y - 0.5\*Y\*\*2  
PMAT(2,8) = -2\*T1\*\*3/3 + T1\*\*2\*Y - Y\*\*3/3.  
PMAT(2,21) = 1.0  
PMAT(2,23) = T1+T2 - Y  
PMAT(2,15) = -T1\*T2-0.5\*T2\*\*2 + T2\*Y  
PMAT(2,18) = T1\*T2\*\*2+T2\*\*3/3. - T2\*\*2\*Y  
PMAT(2,9) = -T1\*\*2\*X + 2\*T1\*X\*Y - X\*Y\*\*2  
PMAT(2,22) = X  
PMAT(2,24) = 2\*(T1+T2)\*X - 2\*X\*Y  
PMAT(2,19) = -(2\*T1\*T2+T2\*\*2)\*X + 2\*T2\*X\*Y  
PMAT(2,26) = 3\*(T1+T2)\*X\*\*2 - 3\*Y\*X\*\*2  
PMAT(2,25) = X\*\*2  
PMAT(2,27) = X\*\*3  
PMAT(3,2) = T1 - Y  
PMAT(3,4) = 0.5\*T1\*\*2 - 0.5\*Y\*\*2  
PMAT(3,7) = -T1\*\*3 + Y\*\*3  
PMAT(3,28) = 1.0  
PMAT(3,12) = T2  
PMAT(3,14) = -0.5\*T2\*\*2  
PMAT(3,17) = -T2\*\*3  
PMAT(3,5) = -T1\*X + X\*Y  
PMAT(3,8) = -T1\*\*2\*X + X\*Y\*\*2  
PMAT(3,23) = X  
PMAT(3,15) = -T2\*X  
PMAT(3,18) = T2\*\*2\*X  
PMAT(3,9) = -T1\*X\*\*2 + Y\*X\*\*2  
PMAT(3,24) = X\*\*2  
PMAT(3,19) = -T2\*X\*\*2  
PMAT(3,26) = X\*\*3

C  
C

ELSE IF ( LAYER .EQ. 2 ) THEN

PMAT(1,11) = 1.0  
PMAT(1,12) = X  
PMAT(1,13) = Y  
PMAT(1,14) = X\*Y  
PMAT(1,15) = -0.5\*X\*\*2  
PMAT(1,16) = Y\*\*2  
PMAT(1,17) = -3\*X\*Y\*\*2  
PMAT(1,18) = -Y\*X\*\*2  
PMAT(1,19) = -X\*\*3/3.  
PMAT(1,20) = Y\*\*3  
PMAT(2,21) = 1.0  
PMAT(2,22) = X  
PMAT(2,23) = -Y  
PMAT(2,24) = -2\*X\*Y  
PMAT(2,25) = X\*\*2  
PMAT(2,15) = -0.5\*Y\*\*2  
PMAT(2,19) = -X\*Y\*\*2  
PMAT(2,26) = -3\*Y\*X\*\*2  
PMAT(2,27) = X\*\*3  
PMAT(2,18) = -Y\*\*3/3.

```

PMAT (3,28) = 1.0
PMAT (3,23) = X
PMAT (3,12) = -Y
PMAT (3,15) = X*Y
PMAT (3,24) = X**2
PMAT (3,14) = -0.5*Y**2
PMAT (3,18) = X*Y**2
PMAT (3,19) = Y*X**2
PMAT (3,26) = X**3
PMAT (3,17) = Y**3

```

```

ELSE IF ( LAYER .EQ. 3 ) THEN

```

```

PMAT (1,29) = 1.0
PMAT (1,30) = X
PMAT (1,31) = Y
PMAT (1,32) = X*Y
PMAT (1,33) = -0.5*X**2
PMAT (1,34) = Y**2
PMAT (1,35) = -3*X*Y**2
PMAT (1,36) = -Y*X**2
PMAT (1,37) = -X**3/3.
PMAT (1,38) = Y**3
PMAT (2,23) = -Y - (T2+T3)
PMAT (2,15) = -(T2*T3+0.5*T2**2) - T2*Y
PMAT (2,18) = -(T3*T2**2+T2**3/3.) - T2**2*Y
PMAT (2,33) = -0.5*T3**2 - T3*Y - 0.5*Y**2
PMAT (2,36) = 2*T3**3/3. + T3**2*Y - Y**3/3.
PMAT (2,21) = 1.0
PMAT (2,24) = -2*(T2+T3)*X - 2*X*Y
PMAT (2,19) = -2*(T2*T3+0.5*T2**2)*X - 2*T2*X*Y
PMAT (2,37) = (T3**2-2*T2*T3)*X - 2*T2*X*Y - X*Y**2
PMAT (2,22) = X
PMAT (2,25) = X**2
PMAT (2,26) = -3*(T2+T3)*X**2 - 3*Y*X**2
PMAT (2,27) = X**3
PMAT (3,12) = -T2
PMAT (3,14) = -0.5*T2**2
PMAT (3,17) = T2**3
PMAT (3,28) = 1.0
PMAT (3,30) = -T3 - Y
PMAT (3,32) = 0.5*T3**2 - 0.5*Y**2
PMAT (3,35) = T3**3 + Y**3
PMAT (3,23) = X
PMAT (3,15) = T2*X
PMAT (3,18) = T2**2*X
PMAT (3,33) = T3*X + X*Y
PMAT (3,36) = -T3**2*X + X*Y**2
PMAT (3,24) = X**2
PMAT (3,19) = T2*X**2
PMAT (3,37) = T3*X**2 + Y*X**2
PMAT (3,26) = X**3

```

```

END IF

```

```

ELSE IF ( NVER .EQ. 13 .AND. NSIDE .EQ. 1 ) THEN

```

```

ZERO TRACTION CONDITION OF SXX AND TXY
IMPOSED ON ELEMENT SIDE F1

```

```

NVAL = 21

```

```

IF ( LAYER .EQ. 1 ) THEN

```

```

PMAT (1,1) = 1.0
PMAT (1,2) = -X
PMAT (1,3) = Y
PMAT (1,4) = -2*X*Y

```

```

PMAT(1,5) = -X**2
PMAT(1,6) = Y**2
PMAT(2,5) = -0.5*T1**2 + Y*T1 - 0.5*Y**2
PMAT(2,7) = -(T1+T2) + Y
PMAT(2,8) = 1.0
PMAT(2,9) = -T1*T2 + Y*T2
PMAT(2,10) = 2*X*T1 - 2*X*Y
PMAT(2,11) = X
PMAT(2,12) = -X*T2
PMAT(2,13) = X**2
PMAT(3,2) = -T1 + Y
PMAT(3,4) = -T1**2 + Y**2
PMAT(3,7) = -A - X
PMAT(3,10) = -A**2 + X**2
PMAT(3,9) = -T2*A - X*T2
PMAT(3,5) = -X*T1 + X*Y

```

```

ELSE IF ( LAYER .EQ. 2 ) THEN

```

```

PMAT(1,9) = -A*(X+A)
PMAT(1,14) = (X**2-A**2)
PMAT(1,15) = Y*(X+A)
PMAT(2,8) = 1.0
PMAT(2,11) = X
PMAT(2,7) = Y
PMAT(2,12) = X*Y
PMAT(2,13) = X**2
PMAT(3,7) = -(X+A)
PMAT(3,10) = (X**2-A**2)
PMAT(3,9) = Y*(X+A)

```

```

ELSE IF ( LAYER .EQ. 3 ) THEN

```

```

PMAT(1,16) = 1.0
PMAT(1,17) = -X
PMAT(1,18) = Y
PMAT(1,19) = -X*Y
PMAT(1,20) = -0.5*X**2
PMAT(1,21) = Y**2
PMAT(2,20) = -0.5*T3**2 - T3*Y - 0.5*Y**2
PMAT(2,7) = (T2+T3) + Y
PMAT(2,8) = 1.0
PMAT(2,9) = -T2*T3 - T2*Y
PMAT(2,10) = -2*T3*X - 2*X*Y
PMAT(2,11) = X
PMAT(2,12) = T2*X
PMAT(2,13) = X**2
PMAT(3,17) = T3 + Y
PMAT(3,19) = -T3**2 + Y**2
PMAT(3,7) = -A - X
PMAT(3,10) = -A**2 + X**2
PMAT(3,9) = T2*A + T2*X
PMAT(3,20) = T3*X + X*Y

```

```

END IF

```

```

ELSE IF ( NVER .EQ. 13 .AND. NSIDE .EQ. 2 ) THEN

```

```

ZERO TRACTION CONDITION OF SXX AND TXY
IMPOSED ON FACE F2

```

```

NBVAL = 21

```

```

IF ( LAYER .EQ. 1 ) THEN

```

```

PMAT(1,1) = 1.0
PMAT(1,2) = -X
PMAT(1,3) = Y

```

```

PMAT(1,4) = -2*X*Y
PMAT(1,5) = -X**2
PMAT(1,6) = Y**2
PMAT(2,5) = -0.5*T1**2 + Y*T1 - 0.5*Y**2
PMAT(2,7) = -(T1+T2) + Y
PMAT(2,8) = 1.0
PMAT(2,9) = -T1*T2 + Y*T2
PMAT(2,10) = 2*X*T1 - 2*X*Y
PMAT(2,11) = X
PMAT(2,12) = -X*T2
PMAT(2,13) = X**2
PMAT(3,2) = -T1 + Y
PMAT(3,4) = -T1**2 + Y**2
PMAT(3,7) = A - X
PMAT(3,10) = -A**2 + X**2
PMAT(3,9) = T2*A - X*T2
PMAT(3,5) = -X*T1 + X*Y

```

```

C
C
ELSE IF ( LAYER .EQ. 2 ) THEN

```

```

PMAT(1,9) = A*(X-A)
PMAT(1,14) = (X**2-A**2)
PMAT(1,15) = Y*(X-A)
PMAT(2,8) = 1.0
PMAT(2,11) = X
PMAT(2,7) = Y
PMAT(2,12) = X*Y
PMAT(2,13) = X**2
PMAT(3,7) = -(X-A)
PMAT(3,10) = (X**2-A**2)
PMAT(3,9) = Y*(X-A)

```

```

C
C
ELSE IF ( LAYER .EQ. 3 ) THEN

```

```

PMAT(1,16) = 1.0
PMAT(1,17) = -X
PMAT(1,18) = Y
PMAT(1,19) = -X*Y
PMAT(1,20) = -0.5*X**2
PMAT(1,21) = Y**2
PMAT(2,20) = -0.5*T3**2 - T3*Y - 0.5*Y**2
PMAT(2,7) = (T2+T3) + Y
PMAT(2,8) = 1.0
PMAT(2,9) = -T2*T3 - T2*Y
PMAT(2,10) = -2*T3*X - 2*X*Y
PMAT(2,11) = X
PMAT(2,12) = T2*X
PMAT(2,13) = X**2
PMAT(3,17) = T3 + Y
PMAT(3,19) = -T3**2 + Y**2
PMAT(3,7) = A - X
PMAT(3,10) = -A**2 + X**2
PMAT(3,9) = -T2*A + T2*X
PMAT(3,20) = T3*X + X*Y

```

```

C
END IF

```

```

C
END IF

```

```

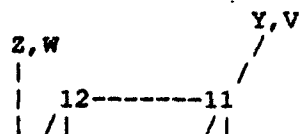
C
ELSE IF ( JTYPE .EQ. 5 ) THEN

```

```

C
C
C
C
C
C
C
C
C
C
C
2-LAYER 3-D HYBRID ELEMENT.

```





```
PMAT(6,26) = Z
PMAT(6,15) = X*Z
PMAT(6,7) = Z*Y
```

C  
C

```
ELSE IF ( LAYER .EQ. 2 ) THEN
```

```
PMAT(1,35) = 1.0
PMAT(1,36) = -X
PMAT(1,32) = -X
PMAT(1,37) = Y
PMAT(1,38) = Z
PMAT(1,34) = -X*Y
PMAT(1,39) = -X*Z
PMAT(1,40) = Y*Z
PMAT(2,41) = 1.0
PMAT(2,42) = X
PMAT(2,47) = Y
PMAT(2,43) = Z
PMAT(2,30) = X*Y
PMAT(2,44) = X*Z
PMAT(2,45) = -Z*Y
PMAT(3,18) = 1.0
PMAT(3,20) = X
PMAT(3,22) = Y
PMAT(3,16) = -Z
PMAT(3,17) = -Z
PMAT(3,23) = X*Y
PMAT(3,19) = -X*Z
PMAT(3,21) = -Z*Y
PMAT(4,27) = 1.0
PMAT(4,29) = X
PMAT(4,17) = Y
PMAT(4,28) = Z
PMAT(4,19) = X*Y
PMAT(4,30) = -X*Z
PMAT(5,31) = 1.0
PMAT(5,16) = X
PMAT(5,33) = Y
PMAT(5,32) = Z
PMAT(5,21) = X*Y
PMAT(5,34) = Y*Z
PMAT(6,46) = 1.0
PMAT(6,47) = -X
PMAT(6,28) = -X
PMAT(6,36) = Y
PMAT(6,48) = Z
PMAT(6,45) = X*Z
PMAT(6,39) = Z*Y
```

C  
C  
C  
C  
C  
C  
C

```
END IF
```

```
ELSE IF ( NVER .EQ. 12 ) THEN
```

```
QUADRATIC STRESS FIELD
```

```
NVAL = 78
```

```
IF ( LAYER .EQ. 1 ) THEN
```

```
PMAT(1,1) = Y*Z
PMAT(1,2) = X*Z
PMAT(1,3) = Z
PMAT(1,4) = Y**2
PMAT(1,5) = X**2
PMAT(1,6) = X*Y
PMAT(1,7) = Y
PMAT(1,8) = X
PMAT(1,9) = Z**2
```

PMAT (1, 10) = 1.0  
 PMAT (2, 11) = Y\*Z  
 PMAT (2, 12) = X\*Z  
 PMAT (2, 13) = Z  
 PMAT (2, 14) = Y\*\*2  
 PMAT (2, 15) = X\*\*2  
 PMAT (2, 16) = X\*Y  
 PMAT (2, 17) = Y  
 PMAT (2, 18) = X  
 PMAT (2, 19) = 1.0  
 PMAT (2, 20) = Z\*\*2  
 PMAT (3, 21) = Z\*\*2-2\*Z\*T1  
 PMAT (3, 22) = Y\*Z  
 PMAT (3, 23) = X\*Z  
 PMAT (3, 24) = Z  
 PMAT (3, 25) = Y\*\*2  
 PMAT (3, 26) = X\*\*2  
 PMAT (3, 27) = X\*Y  
 PMAT (3, 28) = Y  
 PMAT (3, 29) = X  
 PMAT (3, 30) = 1.0  
 PMAT (3, 31) = -2\*Z\*T2  
 PMAT (4, 21) = 2\*Y\*T1-Y\*Z  
 PMAT (4, 23) = -X\*Y  
 PMAT (4, 24) = -Y  
 PMAT (4, 14) = -Y\*Z  
 PMAT (4, 34) = -2\*X\*Z  
 PMAT (4, 16) = -X\*Z  
 PMAT (4, 17) = -Z  
 PMAT (4, 41) = -Y  
 PMAT (4, 42) = X  
 PMAT (4, 36) = -Z  
 PMAT (4, 43) = 1.0  
 PMAT (4, 31) = 2\*Y\*T2  
 PMAT (4, 39) = Z\*\*2  
 PMAT (4, 5) = Y\*Z  
 PMAT (4, 44) = Y\*\*2  
 PMAT (4, 45) = -2\*X\*Y  
 PMAT (4, 46) = X\*\*2  
 PMAT (5, 22) = -X\*Y  
 PMAT (5, 33) = -2\*Y\*Z  
 PMAT (5, 14) = X\*Z  
 PMAT (5, 47) = Y  
 PMAT (5, 35) = -Z  
 PMAT (5, 41) = X  
 PMAT (5, 48) = 1.0  
 PMAT (5, 38) = Z\*\*2  
 PMAT (5, 5) = -X\*Z  
 PMAT (5, 6) = -Y\*Z  
 PMAT (5, 21) = -X\*Z  
 PMAT (5, 8) = -Z  
 PMAT (5, 49) = Y\*\*2  
 PMAT (5, 44) = -2\*X\*Y  
 PMAT (5, 45) = X\*\*2  
 PMAT (6, 2) = -Y\*Z  
 PMAT (6, 11) = -X\*Z  
 PMAT (6, 32) = Z  
 PMAT (6, 33) = Y\*\*2  
 PMAT (6, 14) = -X\*Y  
 PMAT (6, 34) = X\*\*2  
 PMAT (6, 35) = Y  
 PMAT (6, 36) = X  
 PMAT (6, 37) = 1.0  
 PMAT (6, 38) = -2\*Y\*Z  
 PMAT (6, 39) = -2\*X\*Z  
 PMAT (6, 5) = -X\*Y  
 PMAT (6, 40) = Z\*\*2  
 PMAT (6, 21) = X\*Y



C  
C

ELSE IF ( LAYER .EQ. 2 ) THEN

```
PMAT (1,50) = Y*Z
PMAT (1,51) = X*Z
PMAT (1,52) = Z
PMAT (1,53) = -2*X*Y
PMAT (1,54) = Y**2
PMAT (1,55) = X**2
PMAT (1,56) = Y
PMAT (1,57) = X
PMAT (1,58) = 1.0
PMAT (1,59) = Z**2
PMAT (1,60) = -X*Y
PMAT (2,61) = Y*Z
PMAT (2,62) = X*Z
PMAT (2,63) = Z
PMAT (2,64) = Y**2
PMAT (2,65) = X**2
PMAT (2,66) = X*Y
PMAT (2,67) = Y
PMAT (2,68) = X
PMAT (2,69) = 1.0
PMAT (2,70) = Z**2
PMAT (3,31) = (-T2**2+Z**2-2*T1*T2)
PMAT (3,22) = (Y*(T2+T1)+Y*Z)
PMAT (3,23) = (X*(T2+T1)+X*Z)
PMAT (3,24) = (T2+T1+Z)
PMAT (3,25) = Y**2
PMAT (3,26) = X**2
PMAT (3,27) = X*Y
PMAT (3,28) = Y
PMAT (3,29) = X
PMAT (3,30) = 1.0
PMAT (3,21) = -T1**2
PMAT (4,77) = (Z**2-T2**2)
PMAT (4,55) = (Y*Z+T2*Y)
PMAT (4,67) = (-T2-Z)
PMAT (4,31) = (T2*Y-Y*Z)
PMAT (4,64) = (-Y*Z-T2*Y)
PMAT (4,66) = (-X*Z-T2*X)
PMAT (4,73) = (-T2-Z)
PMAT (4,72) = (-2*X*Z-2*T2*X)
PMAT (4,23) = -X*Y
PMAT (4,24) = -Y
PMAT (4,14) = -T1*Y
PMAT (4,34) = -2*T1*X
PMAT (4,16) = -T1*X
PMAT (4,17) = -T1
PMAT (4,41) = -Y
PMAT (4,42) = X
PMAT (4,36) = -T1
PMAT (4,43) = 1.0
PMAT (4,39) = T1**2
PMAT (4,5) = T1*Y
PMAT (4,21) = T1*Y
PMAT (4,44) = Y**2
PMAT (4,45) = -2*X*Y
PMAT (4,46) = X**2
PMAT (5,76) = (Z**2-T2**2)
PMAT (5,60) = (Y*Z+T2*Y)
PMAT (5,64) = (X*Z+T2*X)
PMAT (5,55) = (-X*Z-T2*X)
PMAT (5,31) = (-X*Z-T2*X)
PMAT (5,75) = (T2+Z)
PMAT (5,22) = -X*Y
PMAT (5,33) = -2*T1*Y
PMAT (5,14) = T1*X
```



```

C      XSI = -1.0
      ETA = 1.0
C
C      ELSE IF ( IET .EQ. 3 ) THEN
C
C      XSI = 1.0
      ETA = -1.0
C
C      ELSE IF ( IET .EQ. 4 ) THEN
C
C      XSI = 1.0
      ETA = 1.0
C
C      ELSE IF ( IET .EQ. 5 ) THEN
C
C      XSI = 0.0
      ETA = 0.0
C
C      ELSE IF ( IET .EQ. 6 ) THEN
C
C      XSI = -0.577350269189626
      ETA = -0.577350269189626
C
C      ELSE IF ( IET .EQ. 7 ) THEN
C
C      XSI = 0.577350269189626
      ETA = -0.577350269189626
C
C      ELSE IF ( IET .EQ. 8 ) THEN
C
C      XSI = -0.577350269189626
      ETA = 0.577350269189626
C
C      ELSE IF ( IET .EQ. 9 ) THEN
C
C      XSI = 0.577350269189626
      ETA = 0.577350269189626
C
C      END IF
C
C      ELSE IF ( JTYPE .EQ. 5 ) THEN
C
C      3-D ADHESIVE ELEMENT
C
C      IF ( IET .EQ. 1 ) THEN
C
C      XSI = -1.0
      ETA = -1.0
      CEE = -1.0
C
C      ELSE IF ( IET .EQ. 2 ) THEN
C
C      XSI = -1.0
      ETA = -1.0
      CEE = 1.0
C
C      ELSE IF ( IET .EQ. 3 ) THEN
C
C      XSI = -1.0
      ETA = 1.0
      CEE = -1.0
C
C      ELSE IF ( IET .EQ. 4 ) THEN
C
C      XSI = -1.0
      ETA = 1.0
      CEE = 1.0

```

```
C
C      ELSE IF ( IET .EQ. 5 ) THEN
C
C          XSI = 1.0
C          ETA = -1.0
C          CEE = -1.0
C
C      ELSE IF ( IET .EQ. 6 ) THEN
C
C          XSI = 1.0
C          ETA = -1.0
C          CEE = 1.0
C
C      ELSE IF ( IET .EQ. 7 ) THEN
C
C          XSI = 1.0
C          ETA = 1.0
C          CEE = -1.0
C
C      ELSE IF ( IET .EQ. 8 ) THEN
C
C          XSI = 1.0
C          ETA = 1.0
C          CEE = 1.0
C
C      ELSE IF ( IET .EQ. 9 ) THEN
C
C          XSI = 0.0
C          ETA = 0.0
C          CEE = 0.0
C
C      ELSE IF ( IET .EQ. 10 ) THEN
C
C          XSI = -0.577350269189626
C          ETA = -0.577350269189626
C          CEE = -0.577350269189626
C
C      ELSE IF ( IET .EQ. 11 ) THEN
C
C          XSI = 0.577350269189626
C          ETA = -0.577350269189626
C          CEE = -0.577350269189626
C
C      ELSE IF ( IET .EQ. 12 ) THEN
C
C          XSI = -0.577350269189626
C          ETA = 0.577350269189626
C          CEE = -0.577350269189626
C
C      ELSE IF ( IET .EQ. 13 ) THEN
C
C          XSI = 0.577350269189626
C          ETA = 0.577350269189626
C          CEE = -0.577350269189626
C
C      ELSE IF ( IET .EQ. 14 ) THEN
C
C          XSI = -0.577350269189626
C          ETA = -0.577350269189626
C          CEE = 0.577350269189626
C
C      ELSE IF ( IET .EQ. 15 ) THEN
C
C          XSI = 0.577350269189626
C          ETA = -0.577350269189626
C          CEE = 0.577350269189626
C
C      ELSE IF ( IET .EQ. 16 ) THEN
```

```

C
      XSI = -0.577350269189626
      ETA = 0.577350269189626
      CEE = 0.577350269189626
C
      ELSE IF ( IET .EQ. 17 ) THEN
C
      XSI = 0.577350269189626
      ETA = 0.577350269189626
      CEE = 0.577350269189626
C
      END IF
C
      END IF
C
      RETURN
      END
C
C
C
      SUBROUTINE INVERS (A, B, INDEX, NDIM, NFT)
C
      IMPLICIT DOUBLE PRECISION (A-H, O-Z)
C
      DIMENSION A (NDIM, NDIM) , B (NDIM, NDIM) , INDEX (NDIM)
C
      DO 12 I = 1, NFT
        DO 11 J = 1, NFT
          B(I, J) = 0.0
11        CONTINUE
          B(I, I) = 1.0
12        CONTINUE
      CALL LUDCMP (A, NFT, NDIM, INDEX, D)
      DO 13 J = 1, NFT
        CALL LUBKSB (A, NFT, NDIM, INDEX, B(1, J))
13        CONTINUE
C
      RETURN
      END
C
C
C
      SUBROUTINE LUDCMP (A, N, NP, INDX, D)
C
      IMPLICIT DOUBLE PRECISION (A-H, O-Z)
C
      PARAMETER (NMAX=100, TINY=1.0E-20)
      DIMENSION A (NP, NP) , INDX (NP) , VV (NMAX)
      D=1.
      DO 12 I=1, N
        AAMAX=0.
        DO 11 J=1, N
          IF (ABS(A(I, J)) .GT. AAMAX) AAMAX=ABS(A(I, J))
11        CONTINUE
          IF (AAMAX.EQ.0.) PAUSE 'Singular matrix.'
          VV(I)=1./AAMAX
12        CONTINUE
      DO 19 J=1, N
        IF (J.GT.1) THEN
          DO 14 I=1, J-1
            SUM=A(I, J)
            IF (I.GT.1) THEN
              DO 13 K=1, I-1
                SUM=SUM-A(I, K) *A(K, J)
13              CONTINUE
            A(I, J)=SUM
          ENDOF
14        CONTINUE

```

```

ENDIF
AAMAX=0.
DO 16 I=J,N
  SUM=A(I,J)
  IF (J.GT.1) THEN
    DO 15 K=1,J-1
      SUM=SUM-A(I,K)*A(K,J)
15    CONTINUE
      A(I,J)=SUM
    ENDIF
    DUM=VV(I)*ABS(SUM)
    IF (DUM.GE.AAMAX) THEN
      IMAX=I
      AAMAX=DUM
    ENDIF
16  CONTINUE
    IF (J.NE.IMAX) THEN
      DO 17 K=1,N
        DUM=A(IMAX,K)
        A(IMAX,K)=A(J,K)
        A(J,K)=DUM
17    CONTINUE
      D=-D
      VV(IMAX)=VV(J)
    ENDIF
    INDX(J)=IMAX
    IF (J.NE.N) THEN
      IF (A(J,J).EQ.0.) A(J,J)=TINY
      DUM=1./A(J,J)
      DO 18 I=J+1,N
        A(I,J)=A(I,J)*DUM
18    CONTINUE
    ENDIF
19  CONTINUE
    IF (A(N,N).EQ.0.) A(N,N)=TINY
    RETURN
  END

```

C  
C  
C  
C  
C

```

SUBROUTINE LUBKSB(A,N,NP,INDX,B)
IMPLICIT DOUBLE PRECISION(A-H,O-Z)
DIMENSION A(NP,NP),INDX(NP),B(N)
II=0
DO 12 I=1,N
  LL=INDX(I)
  SUM=B(LL)
  B(LL)=B(I)
  IF (II.NE.0) THEN
    DO 11 J=II,I-1
      SUM=SUM-A(I,J)*B(J)
11    CONTINUE
    ELSE IF (SUM.NE.0.) THEN
      II=I
    ENDIF
  B(I)=SUM
12  CONTINUE
  DO 14 I=N,1,-1
    SUM=B(I)
    IF (I.LT.N) THEN
      DO 13 J=I+1,N
        SUM=SUM-A(I,J)*B(J)
13    CONTINUE
    ENDIF
    B(I)=SUM/A(I,I)
14  CONTINUE

```



```
ETA = 0.861136311594053
WE = 0.347854845137454
ELSE IF ( JETA .EQ. 2 ) THEN
ETA = 0.339981043584856
WE = 0.652145154862546
ELSE IF ( JETA .EQ. 3 ) THEN
ETA = -.339981043584856
WE = 0.652145154862546
ELSE IF ( JETA .EQ. 4 ) THEN
ETA = -.861136311594053
WE = 0.347854845137454
END IF
WEIGHT = WX*WE
```

C

```
ELSE IF ( NORD .EQ. 5 ) THEN
```

C

```
IF ( IXSI .EQ. 1 ) THEN
XSI = 0.906179845938664
WX = 0.236926885056189
ELSE IF ( IXSI .EQ. 2 ) THEN
XSI = 0.538469310105683
WX = 0.478628670499366
ELSE IF ( IXSI .EQ. 3 ) THEN
XSI = 0.000000000000000
WX = 0.568888888888889
ELSE IF ( IXSI .EQ. 4 ) THEN
XSI = -.538469310105683
WX = 0.478628670499366
ELSE IF ( IXSI .EQ. 5 ) THEN
XSI = -.906179845938664
WX = 0.236926885056189
END IF
```

```
IF ( JETA .EQ. 1 ) THEN
ETA = 0.906179845938664
WE = 0.236926885056189
ELSE IF ( JETA .EQ. 2 ) THEN
ETA = 0.538469310105683
WE = 0.478628670499366
ELSE IF ( JETA .EQ. 3 ) THEN
ETA = 0.000000000000000
WE = 0.568888888888889
ELSE IF ( JETA .EQ. 4 ) THEN
ETA = -.538469310105683
WE = 0.478628670499366
ELSE IF ( JETA .EQ. 5 ) THEN
ETA = -.906179845938664
WE = 0.236926885056189
END IF
```

```
WEIGHT = WX*WE
```

C

```
END IF
```

C

```
ELSE IF ( NELDIM .EQ. 3 ) THEN
```

C

```
IF ( NORD .EQ. 1 ) THEN
```

C

```
XSI = 0.0
ETA = 0.0
CEE = 0.0
WEIGHT = 8.0
```

C

```
ELSE IF ( NORD .EQ. 2 ) THEN
```

C

```
WEIGHT = 1.000
XSI = 0.577350269189626
ETA = 0.577350269189626
CEE = 0.577350269189626
IF ( IXSI .EQ. 2 ) XSI = -.577350269189626
```



IF ( JETA .EQ. 2 ) ETA = -.577350269189626  
IF ( KCEE .EQ. 2 ) CEE = -.577350269189626

C  
C

ELSE IF ( NORD .EQ. 3 ) THEN

IF ( IXSI .EQ. 1 ) THEN  
XSI = 0.774596669241483  
WX = 0.555555555555556  
ELSE IF ( IXSI .EQ. 2 ) THEN  
XSI = 0.000000000000000  
WX = 0.888888888888889  
ELSE IF ( IXSI .EQ. 3 ) THEN  
XSI = -.774596669241483  
WX = 0.555555555555556  
END IF  
IF ( JETA .EQ. 1 ) THEN  
ETA = 0.774596669241483  
WE = 0.555555555555556  
ELSE IF ( JETA .EQ. 2 ) THEN  
ETA = 0.000000000000000  
WE = 0.888888888888889  
ELSE IF ( JETA .EQ. 3 ) THEN  
ETA = -.774596669241483  
WE = 0.555555555555556  
END IF  
IF ( KCEE .EQ. 1 ) THEN  
CEE = 0.774596669241483  
WC = 0.555555555555556  
ELSE IF ( KCEE .EQ. 2 ) THEN  
CEE = 0.000000000000000  
WC = 0.888888888888889  
ELSE IF ( KCEE .EQ. 3 ) THEN  
CEE = -.774596669241483  
WC = 0.555555555555556  
END IF  
WEIGHT = WX\*WE\*WC

C  
C

ELSE IF ( NORD .EQ. 4 ) THEN

IF ( IXSI .EQ. 1 ) THEN  
XSI = 0.861136311594053  
WX = 0.347854845137454  
ELSE IF ( IXSI .EQ. 2 ) THEN  
XSI = 0.339981043584856  
WX = 0.652145154862546  
ELSE IF ( IXSI .EQ. 3 ) THEN  
XSI = -.339981043584856  
WX = 0.652145154862546  
ELSE IF ( IXSI .EQ. 4 ) THEN  
XSI = -.861136311594053  
WX = 0.347854845137454  
END IF  
IF ( JETA .EQ. 1 ) THEN  
ETA = 0.861136311594053  
WE = 0.347854845137454  
ELSE IF ( JETA .EQ. 2 ) THEN  
ETA = 0.339981043584856  
WE = 0.652145154862546  
ELSE IF ( JETA .EQ. 3 ) THEN  
ETA = -.339981043584856  
WE = 0.652145154862546  
ELSE IF ( JETA .EQ. 4 ) THEN  
ETA = -.861136311594053  
WE = 0.347854845137454  
END IF  
IF ( KCEE .EQ. 1 ) THEN  
CEE = 0.861136311594053  
WC = 0.347854845137454

```

ELSE IF ( KCEE .EQ. 2 ) THEN
  CEE = 0.339981043584856
  WC = 0.652145154862546
ELSE IF ( KCEE .EQ. 3 ) THEN
  CEE = -.339981043584856
  WC = 0.652145154862546
ELSE IF ( KCEE .EQ. 4 ) THEN
  CEE = -.861136311594053
  WC = 0.347854845137454
END IF
WEIGHT = WX*WE*WC

```

C

```
END IF
```

C

```
END IF
```

C

```
RETURN
```

```
END
```

C

C

C

```
SUBROUTINE MXMUL (A,B,C, IDIM, JDIM, KDIM, IROW, JCOL, KCOL)
```

C

```
MATRIX (A) TIMES (B)
```

C

```
IMPLICIT DOUBLE PRECISION (A-H,O-Z)
```

C

```
DIMENSION A (IDIM, 1), B (JDIM, 1), C (KDIM, 1)
```

C

```
DO I = 1, IROW
```

```
  DO K = 1, KCOL
```

```
    SUM = 0.0
```

```
      DO J = 1, JCOL
```

```
        SUM = SUM + A(I, J) * B(J, K)
```

```
      END DO
```

```
      C(I, K) = SUM
```

```
    END DO
```

```
  END DO
```

C

```
RETURN
```

```
END
```

C

C

C

```
SUBROUTINE MXATB (A,B,C, IDIM, JDIM, KDIM, IROW, JCOL, KCOL)
```

C

```
MATRIX (A) TRANSPOSE TIMES (B)
```

C

```
IMPLICIT DOUBLE PRECISION (A-H,O-Z)
```

C

```
DIMENSION A (IDIM, 1), B (JDIM, 1), C (KDIM, 1)
```

C

```
DO I = 1, IROW
```

```
  DO K = 1, KCOL
```

```
    SUM = 0.0
```

```
      DO J = 1, JCOL
```

```
        SUM = SUM + A(J, I) * B(J, K)
```

```
      END DO
```

```
      C(I, K) = SUM
```

```
    END DO
```

```
  END DO
```

C

```
RETURN
```

```
END
```

C

C

C

```
SUBROUTINE MXINT (A, IDIM, JDIM, VAL)
```



**APPENDIX B**

**Demonstration problem I: 2-D analysis of a single-lap joint.**

# ABAQUS INPUT FILE

```
*HEADING
2-D SINGLE-LAP JOINT. 100 H2L6N ELEMENTS ALONG BONDLINE.
**
*NODE
1, 0.0, 0.0
51, 63.5, 0.0
151, 76.2, 0.0
605, 0.0, 1.6
655, 63.5, 1.6
755, 76.2, 1.6
1001, 63.5, 1.75
1101, 76.2, 1.75
1151, 139.7, 1.75
1605, 63.5, 3.35
1705, 76.2, 3.35
1755, 139.7, 3.35
2001, 63.5, 1.675
2101, 76.2, 1.675
**
*NGEN, NSET=BL
1,605,151
*NGEN, NSET=BM
51,655,151
*NGEN, NSET=BR
151,755,151
*NGEN, NSET=TL
1001,1605,151
*NGEN, NSET=TM
1101,1705,151
*NGEN, NSET=TR
1151,1755,151
*NGEN, NSET=MIDDLE
2001,2101,1
**
*NFILL
BL, BM, 50, 1
BM, BR, 100, 1
TL, TM, 100, 1
TM, TR, 50, 1
*ELEMENT, TYPE=CPE4
1, 1, 2, 153, 152
451, 454, 455, 606, 605
1101, 1101, 1102, 1253, 1252
1151, 1152, 1153, 1304, 1303
**
** DEFINE ADHESIVE ELEMENT H2L6N
**
*USER ELEMENT, NODES=6, TYPE=U1, PROPERTIES=56
1,2
*ELEMENT, TYPE=U1
2001, 504, 505, 655, 656, 2001, 2002
2101, 2001, 2002, 1001, 1002, 1152, 1153
*ELGEN, ELSET=TOP
1101, 50, 1, 1, 1
1151,150, 1, 1, 3, 151, 150
*ELGEN, ELSET=BOT
1, 150, 1,1,3,151,150
451,50, 1,1,1
*ELGEN, ELSET=MID4
2001, 100, 1, 1, 1
```

```

*ELGEN, ELSET=MID5
2101, 100, 1, 1, 1
*ELSET, ELSET=ONE
1
**
**      USER DEFINED SUBROUTINE:
**
**      USER SUBROUTINE, INPUT=uel_hybrid.f
**
**      ELEMENT PROPERTIES
**
**      *SOLID SECTION, ELSET=TOP, MATERIAL=MID1
**      *SOLID SECTION, ELSET=BOT, MATERIAL=MID3
**      *MATERIAL, NAME=MID1
**      *ELASTIC, TYPE=ISO
0.69000E+05, 0.32E+00, 0.000000000E+00
**
**
**      *MATERIAL, NAME=MID3
**      *ELASTIC, TYPE=ISO
0.69000E+05, 0.32E+00, 0.000000000E+00
**
**      USER DEFINED ELEMENT PROPERTY LIST:
**
**      *BOTTOM ROW
**
**      *UEL PROPERTY, ELSET=MID4
11.0, 1.0, 1.0
1.0, 1.0
0.4, 0.0, 69000.0, 69000.0, 69000.0, 0.32, 0.32, 0.32
26136.3636, 26136.3636, 26136.3636
1.0, 1.0
0.15, 0.0, 3000.0, 3000.0, 3000.0, 0.36, 0.36, 0.36
1102.9412, 1102.9412, 1102.9412,
**
**      *TOP ROW
**
**      *UEL PROPERTY, ELSET=MID5
11.0, 1.0, 1.0
1.0, 1.0
0.15, 0.0, 3000.0, 3000.0, 3000.0, 0.36, 0.36, 0.36
1102.9412, 1102.9412, 1102.9412,
1.0, 1.0
0.4, 0.0, 69000.0, 69000.0, 69000.0, 0.32, 0.32, 0.32
26136.3636, 26136.3636, 26136.3636
**
**
**      *NSET, NSET=HOLD
1, 152, 303, 454, 605
**      *ELSET, ELSET=PULL
1150, 1300, 1450, 1600
**      *ELSET, ELSET=NAVE
500
**      *NGEN, NSET=ROLLER
2, 5, 1
606, 609, 1
1147, 1151, 1
1751, 1755, 1
**
**
**      *BOUNDARY
HOLD, 1, 2
ROLLER, 2
**
**
**      *STEP, PERTURBATION
*STATIC
**

```

```
**          LOAD CASE SPECIFICATION:
**
*DLOAD, OP=NEW
PULL, P2, -93.75
**
**
*NODE PRINT
U
RF
*EL PRINT, ELSET=NAVE, POSITION=AVERAGED AT NODES
S
*END STEP
```

# ABAQUS OUTPUT FILE

```

AAAAAA      BBBB BBBB      AAAAA      QQQQQQQQ      U      U      SSSSSSSS
A      A      B      B      A      A      Q      Q      U      U      S
A      A      B      B      A      A      Q      Q      U      U      S
A      A      B      B      A      A      Q      Q      U      U      S
AAAAAAAAA    BBBB BBBB    AAAAAAAAAA  Q      Q      U      U      SSSSSSSS
A      A      B      B      A      A      Q      Q      U      U      S
A      A      B      B      A      A      Q      Q      U      U      S
A      A      B      B      A      A      Q      Q      U      U      S
A      A      BBBB BBBB    A      A      QQQQQQQQ    UUUUUUUU    SSSSSSSS

```

```

<|> <|> <|> <|> <|> <|> <|> <|> <|> <|>
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-----
| <|> | <|> | | <|> <|> <|> <|>
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<|> <|> <|> <|> <|> <|> <|> | <|> |
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```

THIS PROGRAM HAS BEEN DEVELOPED BY  
HIBBITT, KARLSSON AND SORENSEN, INC.  
1080 MAIN STREET  
PAWTUCKET, R.I. 02860

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```

*****
*
* *****
*   N O T I C E   *
* *****
*
*
* THIS IS ABAQUS VERSION 5.3.
*
* PLEASE MAKE SURE YOU ARE USING VERSION 5.3 MANUALS
* PLUS THE NOTES ACCOMPANYING THIS RELEASE. THESE NOTES
* CAN BE OBTAINED BY USING THE INFORMATION OPTION ON THE
* ABAQUS COMMAND LINE.
*
*****

```

A B A Q U S   I N P U T   E C H O

```

      5    10    15    20    25    30    35    40    45    50    55    60    65    70    75
-----
*HEADING
*NODE
1, 0.0, 0.0
51, 63.5, 0.0
151, 76.2, 0.0

```

CARD      5



```

605, 0.0, 1.6
655, 63.5, 1.6
755, 76.2, 1.6
CARD 10 1001, 63.5, 1.75
1101, 76.2, 1.75
1151, 139.7, 1.75
1605, 63.5, 3.35
1705, 76.2, 3.35
CARD 15 1755, 139.7, 3.35
2001, 63.5, 1.675
2101, 76.2, 1.675
*NGEN, NSET=BL
1, 605, 151
CARD 20 *NGEN, NSET=BM
51, 655, 151
*NGEN, NSET=BR
151, 755, 151
*NGEN, NSET=TL
CARD 25 1001, 1605, 151
*NGEN, NSET=TM
1101, 1705, 151
*NGEN, NSET=TR
1151, 1755, 151
*NGEN, NSET=MIDDLE
CARD 30 2001, 2101, 1
*NFILL
BL, BM, 50, 1
BM, BR, 100, 1
CARD 35 TL, TM, 100, 1
TM, TR, 50, 1
*ELEMENT, TYPE=CPE4
1, 1, 2, 153, 152
451, 454, 455, 606, 605
CARD 40 1101, 1101, 1102, 1253, 1252
1151, 1152, 1153, 1304, 1303
*USER ELEMENT, NODES=6, TYPE=U1, PROPERTIES=24, COORDINATES=3, VARIABLES=1
1, 2
*ELEMENT, TYPE=U1
CARD 45 2001, 504, 505, 655, 656, 2001, 2002
2101, 2001, 2002, 1001, 1002, 1152, 1153
*ELGEN, ELSET=TOP
1101, 50, 1, 1, 1
1151, 150, 1, 1, 3, 151, 150
*ELGEN, ELSET=BOTTOM
CARD 50 1, 150, 1, 1, 3, 151, 150
451, 50, 1, 1, 1
*ELGEN, ELSET=MID4
2001, 100, 1, 1, 1
*ELGEN, ELSET=MID5
CARD 55 2101, 100, 1, 1, 1
*ELSET, ELSET=ONE
1
**
**
CARD 60 ** USER DEFINED SUBROUTINE:
**
** *USER SUBROUTINE, INPUT=uel_hybrid.f
**
**
**
CARD 65 ** ELEMENT PROPERTIES
**
** *SOLID SECTION, ELSET=TOP, MATERIAL=MID1
** *SOLID SECTION, ELSET=BOTTOM, MATERIAL=MID3
** *MATERIAL, NAME=MID1
** *ELASTIC, TYPE=ISO
CARD 70 0.69000E+05, 0.32E+00, 0.000000000E+00
**
**
** *MATERIAL, NAME=MID3
** *ELASTIC, TYPE=ISO
CARD 75 0.69000E+05, 0.32E+00, 0.000000000E+00
**
**
** USER DEFINED ELEMENT PROPERTY LIST:
**
**
CARD 80 ** BOTTOM ROW
** *UEL PROPERTY, ELSET=MID4
0.69E5, 0.32, 1.0
0.3E4, 0.36, 1.0
6.0, 1.0, 3.0, 0.0
** TOP ROW
CARD 85 *UEL PROPERTY, ELSET=MID5
0.3E4, 0.36, 1.0
0.69E5, 0.32, 1.0
6.0, 1.0, 3.0, 0.0
** *NSET, NSET=HOLD
CARD 90 1, 152, 303, 454, 605

```

```

*ELSET, ELSET=PULL
1150, 1300, 1450, 1600
*ELSET, ELSET=NAVE
500
CARD 95 *NGEN, NSET=ROLLER
2, 5, 1
606, 609, 1
1147, 1151, 1
1751, 1755, 1
CARD 100 **
**
*BOUNDARY
HOLD, 1, 2
ROLLER, 2
CARD 105 *STEP, PERTURBATION
*STATIC
**
** LOAD CASE SPECIFICATION:
**
CARD 110 *DLOAD, OP=NEW
PULL, P2, -93.75
*NODE PRINT
U
RF
CARD 115 *EL PRINT, ELSET=NAVE, POSITION=AVERAGED AT NODES
S
*END STEP
-----
5 10 15 20 25 30 35 40 45 50 55 60 65 70 75
-----

```

OPTIONS BEING PROCESSED  
\*\*\*\*\*

```

*HEADING
*NODE
*NGEN, NSET=BL
*NGEN, NSET=BM
*NGEN, NSET=BR
*NGEN, NSET=TL
*NGEN, NSET=TM
*NGEN, NSET=TR
*NGEN, NSET=MIDDLE
*NFILL

```

THE FOLLOWING NODES WILL BE USED IN THE NFILL GENERATION

```

BOUND 1      1      152      303      454      605
BOUND 2      51      202      353      504      655

```

THE FOLLOWING NODES WILL BE USED IN THE NFILL GENERATION

```

BOUND 1      51      202      353      504      655
BOUND 2     151      302      453      604      755

```

THE FOLLOWING NODES WILL BE USED IN THE NFILL GENERATION

```

BOUND 1     1001     1152     1303     1454     1605
BOUND 2     1101     1252     1403     1554     1705

```

THE FOLLOWING NODES WILL BE USED IN THE NFILL GENERATION

```

BOUND 1     1101     1252     1403     1554     1705
BOUND 2     1151     1302     1453     1604     1755

```

```

*ELEMENT, TYPE=CPE4
*USER ELEMENT, NODES=6, TYPE=U1, PROPERTIES=24, COORDINATES=3, VARIABLES=1
*ELEMENT, TYPE=U1
*ELGEN, ELSET=TOP
*ELGEN, ELSET=BOTTOM
*ELGEN, ELSET=MID4
*ELGEN, ELSET=MID5
*ELSET, ELSET=ONE

```

```

* NSET, NSET=HOLD
* ELSET, ELSET=PULL
* ELSET, ELSET=NAVE
* NGEN, NSET=ROLLER
* MATERIAL, NAME=MID1
* ELASTIC, TYPE=ISO
* MATERIAL, NAME=MID3
* ELASTIC, TYPE=ISO
* USER ELEMENT, NODES=6, TYPE=U1, PROPERTIES=24, COORDINATES=3, VARIABLES=1
* SOLID SECTION, ELSET=TOP, MATERIAL=MID1
* SOLID SECTION, ELSET=BOTTOM, MATERIAL=MID3
* UEL PROPERTY, ELSET=MID4
* UEL PROPERTY, ELSET=MID5
* STEP, PERTURBATION
* STATIC
* DLOAD, OP=NEW
* EL PRINT, ELSET=NAVE, POSITION=AVERAGED AT NODES
* END STEP
* BOUNDARY
* STEP, PERTURBATION
* STATIC
* NODE PRINT
* END STEP

```

E L E M E N T D E F I N I T I O N S

NUMBER	TYPE	PROPERTY REFERENCE	NODES FORMING ELEMENT					
1	CPE4	2	1	2	153	152		
2	CPE4	2	2	3	154	153		
3	CPE4	2	3	4	155	154		
4	CPE4	2	4	5	156	155		
5	CPE4	2	5	6	157	156		
6	CPE4	2	6	7	158	157		
7	CPE4	2	7	8	159	158		
8	CPE4	2	8	9	160	159		
9	CPE4	2	9	10	161	160		
10	CPE4	2	10	11	162	161		
.	.	.	.	.	.	.	.	.
.	.	.	.	.	.	.	.	.
1590	CPE4	1	1593	1594	1745	1744		
1591	CPE4	1	1594	1595	1746	1745		
1592	CPE4	1	1595	1596	1747	1746		
1593	CPE4	1	1596	1597	1748	1747		
1594	CPE4	1	1597	1598	1749	1748		
1595	CPE4	1	1598	1599	1750	1749		
1596	CPE4	1	1599	1600	1751	1750		
1597	CPE4	1	1600	1601	1752	1751		
1598	CPE4	1	1601	1602	1753	1752		
1599	CPE4	1	1602	1603	1754	1753		
1600	CPE4	1	1603	1604	1755	1754		
2001	U1	3	504	505	655	656	2001	2002
2002	U1	3	505	506	656	657	2002	2003
2003	U1	3	506	507	657	658	2003	2004
2004	U1	3	507	508	658	659	2004	2005
2005	U1	3	508	509	659	660	2005	2006
2006	U1	3	509	510	660	661	2006	2007
2007	U1	3	510	511	661	662	2007	2008
2008	U1	3	511	512	662	663	2008	2009
2009	U1	3	512	513	663	664	2009	2010
2010	U1	3	513	514	664	665	2010	2011
.	.	.	.	.	.	.	.	.
.	.	.	.	.	.	.	.	.
2190	U1	4	2090	2091	1090	1091	1241	1242
2191	U1	4	2091	2092	1091	1092	1242	1243
2192	U1	4	2092	2093	1092	1093	1243	1244
2193	U1	4	2093	2094	1093	1094	1244	1245
2194	U1	4	2094	2095	1094	1095	1245	1246
2195	U1	4	2095	2096	1095	1096	1246	1247
2196	U1	4	2096	2097	1096	1097	1247	1248
2197	U1	4	2097	2098	1097	1098	1248	1249
2198	U1	4	2098	2099	1098	1099	1249	1250
2199	U1	4	2099	2100	1099	1100	1250	1251
2200	U1	4	2100	2101	1100	1101	1251	1252

U S E R E L E M E N T S

ELEMENT TYPE  
NUMBER OF NODES

U1  
6

NUMBER OF COORDINATES 3  
 NUMBER OF PROPERTIES 24  
 NUMBER OF VARIABLES 1

DEGREES OF FREEDOM  
 NODE D.O.F.  
 1 1 2  
 2 1 2  
 3 1 2  
 4 1 2  
 5 1 2  
 6 1 2

SOLID SECTION

PROPERTY NUMBER 1  
 MATERIAL NAME MID1  
 ATTRIBUTES 1.0000 .00000E+00 .00000E+00

HOURLASS CONTROL STIFFNESS PARAMETER 130.68

(USED WITH LOWER ORDER REDUCED INTEGRATED SOLID ELEMENTS LIKE CPS4R,CPE4RH,C3D8R)

PROPERTY NUMBER 2  
 MATERIAL NAME MID3  
 ATTRIBUTES 1.0000 .00000E+00 .00000E+00

HOURLASS CONTROL STIFFNESS PARAMETER 130.68

(USED WITH LOWER ORDER REDUCED INTEGRATED SOLID ELEMENTS LIKE CPS4R,CPE4RH,C3D8R)

USER ELEMENT PROPERTY

PROPERTY NUMBER 3  
 PROPERTIES  
 11.00 1.000 1.000 .0000E+00 .0000E+00 .0000E+00 .0000E+00 .0000E+00  
 1.000 1.000 .0000E+00 .0000E+00 .0000E+00 .0000E+00 .0000E+00 .0000E+00  
 .4000 .0000E+00 6.9000E+04 6.9000E+04 6.9000E+04 .3200 .3200 .3200  
 2.6136E+04 2.6136E+04 2.6136E+04 .0000E+00 .0000E+00 .0000E+00 .0000E+00 .0000E+00  
 1.000 1.000 .0000E+00 .0000E+00 .0000E+00 .0000E+00 .0000E+00 .0000E+00  
 .1500 .0000E+00 3000. 3000. 3000. .3600 .3600 .3600  
 1103. 1103. 1103. .0000E+00 .0000E+00 .0000E+00 .0000E+00 .0000E+00

PROPERTY NUMBER 4  
 PROPERTIES  
 11.00 1.000 1.000 .0000E+00 .0000E+00 .0000E+00 .0000E+00 .0000E+00  
 1.000 1.000 .0000E+00 .0000E+00 .0000E+00 .0000E+00 .0000E+00 .0000E+00  
 .1500 .0000E+00 3000. 3000. 3000. .3600 .3600 .3600  
 1103. 1103. 1103. .0000E+00 .0000E+00 .0000E+00 .0000E+00 .0000E+00  
 1.000 1.000 .0000E+00 .0000E+00 .0000E+00 .0000E+00 .0000E+00 .0000E+00  
 .4000 .0000E+00 6.9000E+04 6.9000E+04 6.9000E+04 .3200 .3200 .3200  
 2.6136E+04 2.6136E+04 2.6136E+04 .0000E+00 .0000E+00 .0000E+00 .0000E+00 .0000E+00

MATERIAL DESCRIPTION

MATERIAL NAME: MID1

ELASTIC YOUNG'S POISSON'S  
 MODULUS RATIO  
 69000. .32000

MATERIAL NAME: MID3

ELASTIC YOUNG'S POISSON'S  
 MODULUS RATIO  
 69000. .32000

ELEMENT SETS

SET TOP MEMBERS 1101 1102 1103 1104 1105 1106 1107 1108 11  
 1113 1114 1115 1116 1117 1118 1119 1120 11  
 1125 1126 1127 1128 1129 1130 1131 1132 11  
 1581 1582 1583 1584 1585 1586 1587 1588 15  
 1593 1594 1595 1596 1597 1598 1599 1600

SET	BOTTOM	MEMBERS	1 13	2 14	3 15	4 16	5 17	6 18	7 19	8 20	
			481 493	482 494	483 495	484 496	485 497	486 498	487 499	488 500	4
SET	MID4	MEMBERS	2001 2013	2002 2014	2003 2015	2004 2016	2005 2017	2006 2018	2007 2019	2008 2020	20 20
			2097	2098	2099	2100					
SET	MID5	MEMBERS	2101 2113	2102 2114	2103 2115	2104 2116	2105 2117	2106 2118	2107 2119	2108 2120	21 21
			2197	2198	2199	2200					
SET	ONE	MEMBERS	1								
SET	PULL	MEMBERS	1150	1300	1450	1600					
SET	NAVE	MEMBERS	500								

N O D E S E T S

SET	BL	MEMBERS	1	152	303	454	605				
SET	BM	MEMBERS	51	202	353	504	655				
SET	BR	MEMBERS	151	302	453	604	755				
SET	TL	MEMBERS	1001	1152	1303	1454	1605				
SET	TM	MEMBERS	1101	1252	1403	1554	1705				
SET	TR	MEMBERS	1151	1302	1453	1604	1755				
SET	MIDDLE	MEMBERS	2001 2013 2025 2037 2049 2061 2073 2085 2097	2002 2014 2026 2038 2050 2062 2074 2086 2098	2003 2015 2027 2039 2051 2063 2075 2087 2099	2004 2016 2028 2040 2052 2064 2076 2088 2100	2005 2017 2029 2041 2053 2065 2077 2089 2101	2006 2018 2030 2042 2054 2066 2078 2090	2007 2019 2031 2043 2055 2067 2079 2091	2008 2020 2032 2044 2056 2068 2080 2092	20 20 20 20 20 20 20 20 20
SET	HOLD	MEMBERS	1	152	303	454	605				
SET	ROLLER	MEMBERS	2 1151	3 1751	4 1752	5 1753	606 1754	607 1755	608	609	11

N O D E D E F I N I T I O N S

NODE NUMBER	COORDINATES			SINGLE POINT CONSTRAINTS		
				TYPE	PLUS	DOF
1	.00000E+00	.00000E+00	.00000E+00			1 2
2	1.2700	.00000E+00	.00000E+00			2
3	2.5400	.00000E+00	.00000E+00			2
4	3.8100	.00000E+00	.00000E+00			2
5	5.0800	.00000E+00	.00000E+00			2
6	6.3500	.00000E+00	.00000E+00			
7	7.6200	.00000E+00	.00000E+00			
8	8.8900	.00000E+00	.00000E+00			
9	10.160	.00000E+00	.00000E+00			
10	11.430	.00000E+00	.00000E+00			
.	.	.	.			
2091	74.930	1.6750	.00000E+00			
2092	75.057	1.6750	.00000E+00			
2093	75.184	1.6750	.00000E+00			
2094	75.311	1.6750	.00000E+00			
2095	75.438	1.6750	.00000E+00			
2096	75.565	1.6750	.00000E+00			
2097	75.692	1.6750	.00000E+00			
2098	75.819	1.6750	.00000E+00			
2099	75.946	1.6750	.00000E+00			
2100	76.073	1.6750	.00000E+00			
2101	76.200	1.6750	.00000E+00			

STEP 1 STATIC ANALYSIS

FIXED TIME INCREMENTS  
 TIME INCREMENT IS 2.220E-16  
 TIME PERIOD IS 2.220E-16

THIS IS A LINEAR PERTURBATION STEP.  
 ALL LOADS ARE DEFINED AS CHANGE IN LOAD TO THE REFERENCE STATE

EXTRAPOLATION WILL NOT BE USED

CHARACTERISTIC ELEMENT LENGTH .492

PRINT OF INCREMENT NUMBER, TIME, ETC., EVERY 1 INCREMENTS

ELEMENT PRINT

SUMMARIES WILL BE PRINTED WHERE APPLICABLE

TABLE 1 S11 S22 S33 S12

NODE PRINT

THE FOLLOWING TABLE IS PRINTED FOR ALL NODES AT EVERY 1 INCREMENT

SUMMARIES WILL BE PRINTED

TABLE 1 U1 U2

THE FOLLOWING TABLE IS PRINTED FOR ALL NODES AT EVERY 1 INCREMENT

SUMMARIES WILL BE PRINTED

TABLE 2 RF1 RF2

DISTRIBUTED LOADS

ELEMENT	LOAD TYPE	AMP. REF.	MAGNITUDE	ELEMENT	LOAD TYPE	AMP. REF.	MAGNITUDE
1150	P2		-93.750	1300	P2		-93.750
1600	P2		-93.750	1450	P2		-93.750

BOUNDARY CONDITIONS

NODE	DOF	AMP. REF.	MAGNITUDE	NODE	DOF	AMP. REF.	MAGNITUDE
1	1	(RAMP)	.00000E+00	1	2	(RAMP)	.00000E+00
2	2	(RAMP)	.00000E+00	3	2	(RAMP)	.00000E+00
4	2	(RAMP)	.00000E+00	5	2	(RAMP)	.00000E+00
152	1	(RAMP)	.00000E+00	152	2	(RAMP)	.00000E+00
303	1	(RAMP)	.00000E+00	303	2	(RAMP)	.00000E+00
454	1	(RAMP)	.00000E+00	454	2	(RAMP)	.00000E+00
605	1	(RAMP)	.00000E+00	605	2	(RAMP)	.00000E+00
606	2	(RAMP)	.00000E+00	607	2	(RAMP)	.00000E+00
608	2	(RAMP)	.00000E+00	609	2	(RAMP)	.00000E+00
1147	2	(RAMP)	.00000E+00	1148	2	(RAMP)	.00000E+00
1149	2	(RAMP)	.00000E+00	1150	2	(RAMP)	.00000E+00
1151	2	(RAMP)	.00000E+00	1751	2	(RAMP)	.00000E+00
1752	2	(RAMP)	.00000E+00	1753	2	(RAMP)	.00000E+00
1754	2	(RAMP)	.00000E+00	1755	2	(RAMP)	.00000E+00

- (RAMP) OR (STEP) - INDICATE USE OF DEFAULT AMPLITUDES ASSOCIATED WITH THE STEP

WAVEFRONT MINIMIZATION

WAVEFRONT MINIMIZATION METHOD 1 WILL BE USED.

NUMBER OF NODES 1611  
 NUMBER OF ELEMENTS 1200  
 ORIGINAL MAXIMUM D.O.F WAVEFRONT ESTIMATED AS 714  
 ORIGINAL RMS D.O.F WAVEFRONT ESTIMATED AS 429  
 PERIPHERAL DIAMETER IS DEFINED BY NODES 1 1151

WAVEFRONT OPTIMIZED BY CHOOSING 1151 AS THE STARTING NODE

MINIMUM WAVEFRONT OBTAINED USING METHOD 1. USE

\*WAVEFRONT MINIMIZATION, NODES, METHOD=1

1, 1151  
TO REDUCE THE CPU TIME ON SUBSEQUENT JOBS USING THIS SAME MESH.

### PROBLEM SIZE

NUMBER OF ELEMENTS IS	1200
NUMBER OF NODES IS	1611
NUMBER OF NODES DEFINED BY THE USER	1611
NUMBER OF INTERNAL NODES GENERATED BY THE PROGRAM	0
TOTAL NUMBER OF VARIABLES IN THE MODEL (DEGREES OF FREEDOM PLUS ANY LAGRANGE MULTIPLIER VARIABLES)	3222
MAXIMUM D.O.F. WAVEFRONT ESTIMATED AS	32
RMS WAVEFRONT ESTIMATED AS	24

FILE SIZES - THESE VALUES ARE IN WORDS AND ARE CONSERVATIVE UPPER BOUNDS

UNIT	LENGTH
21	167400
22	167400

IF THE RESTART FILE IS WRITTEN ITS LENGTH WILL BE APPROXIMATELY  
107747 WORDS WRITTEN IN THE PRE PROGRAM  
PLUS 60720 WORDS WRITTEN AT THE BEGINNING OF EACH STEP  
PLUS 223141 WORDS FOR EACH INCREMENT WRITTEN TO THE RESTART FILE

ALLOCATED WORKSPACE 436165  
\*USER SUBROUTINE, INPUT=uel\_hybrid.f

END OF USER INPUT PROCESSING

JOB TIME SUMMARY  
CPU TIME (SEC) = 5.8000

### STEP 1 STATIC ANALYSIS

FIXED TIME INCREMENTS  
TIME INCREMENT IS 2.220E-16  
TIME PERIOD IS 2.220E-16

THIS IS A LINEAR PERTURBATION STEP.  
ALL LOADS ARE DEFINED AS CHANGE IN LOAD TO THE REFERENCE STATE

ELEMENT ID 2101

STRESS OUTPUT IN LOCAL COORDINATES FOR LAYER 1

STRESS POINTS		STRESS COMPONENTS		
CI	CJ	SXX	SYX	SXY
-1.0000	-1.0000	-.797E+00	.426E+02	.247E+02
-1.0000	1.0000	.498E+01	.317E+02	.161E+02
1.0000	-1.0000	.138E+02	.693E+02	.402E+02
1.0000	1.0000	.332E+02	.619E+02	.438E+02
.0000	.0000	.159E+02	.531E+02	.330E+02
-.5774	-.5774	.616E+01	.462E+02	.297E+02
.5774	-.5774	.962E+01	.633E+02	.364E+02
-.5774	.5774	.178E+02	.412E+02	.263E+02
.5774	.5774	.258E+02	.594E+02	.370E+02

STRESS OUTPUT IN LOCAL COORDINATES FOR LAYER 2

STRESS POINTS		STRESS COMPONENTS		
CI	CJ	SXX	SYX	SXY
-1.0000	-1.0000	-.265E+02	.317E+02	.161E+02
-1.0000	1.0000	-.111E+02	-.170E+01	.110E+02
1.0000	-1.0000	.528E+02	.619E+02	.438E+02
1.0000	1.0000	.203E+02	.242E+02	.772E+00
.0000	.0000	.349E+00	.158E+02	-.107E+01
-.5774	-.5774	.166E+00	.234E+02	-.907E+01
.5774	-.5774	.122E+02	.384E+02	.354E+01

- .5774	.5774	.212E+01	.275E+01	.133E+02
.5774	.5774	-.174E+01	.163E+02	.133E+02

ELEMENT STRAIN ENERGY = .801E-02

ELEMENT ID 2001

STRESS OUTPUT IN LOCAL COORDINATES FOR LAYER 1

STRESS POINTS		STRESS COMPONENTS		
CI	CJ	SXX	SYX	SXY
-1.0000	-1.0000	.202E+03	.242E+02	-.459E+01
-1.0000	1.0000	.445E+03	.115E+03	.647E+02
1.0000	-1.0000	.169E+03	.177E+02	-.627E+01
1.0000	1.0000	.368E+03	.509E+02	.512E+02
.0000	.0000	.294E+03	.475E+02	.692E+01
-.5774	-.5774	.229E+03	.239E+02	.147E+02
.5774	-.5774	.227E+03	.320E+02	.563E+01
-.5774	.5774	.369E+03	.756E+02	.231E+02
.5774	.5774	.353E+03	.645E+02	.101E+02

STRESS OUTPUT IN LOCAL COORDINATES FOR LAYER 2

STRESS POINTS		STRESS COMPONENTS		
CI	CJ	SXX	SYX	SXY
-1.0000	-1.0000	.847E+02	.115E+03	.647E+02
-1.0000	1.0000	.408E+02	.127E+03	.623E+02
1.0000	-1.0000	.432E+02	.509E+02	.512E+02
1.0000	1.0000	.735E+01	.592E+02	.417E+02
.0000	.0000	.422E+02	.866E+02	.547E+02
-.5774	-.5774	.607E+02	.106E+03	.584E+02
.5774	-.5774	.479E+02	.623E+02	.498E+02
-.5774	.5774	.364E+02	.113E+03	.610E+02
.5774	.5774	.262E+02	.674E+02	.500E+02

ELEMENT STRAIN ENERGY = .476E-01

ELEMENT ID 2100

STRESS OUTPUT IN LOCAL COORDINATES FOR LAYER 1

STRESS POINTS		STRESS COMPONENTS		
CI	CJ	SXX	SYX	SXY
-1.0000	-1.0000	.203E+02	.242E+02	.772E+00
-1.0000	1.0000	.528E+02	.619E+02	.438E+02
1.0000	-1.0000	-.111E+02	-.170E+01	.110E+02
1.0000	1.0000	-.265E+02	.317E+02	.161E+02
.0000	.0000	.349E+00	.158E+02	-.177E+01
-.5774	-.5774	-.174E+01	.163E+02	.133E+02
.5774	-.5774	.212E+01	.275E+01	.133E+02
-.5774	.5774	.122E+02	.384E+02	.354E+01
.5774	.5774	.166E+00	.234E+02	-.907E+01

STRESS OUTPUT IN LOCAL COORDINATES FOR LAYER 2

STRESS POINTS		STRESS COMPONENTS		
CI	CJ	SXX	SYX	SXY
-1.0000	-1.0000	.332E+02	.619E+02	.438E+02
-1.0000	1.0000	.138E+02	.693E+02	.402E+02
1.0000	-1.0000	.498E+01	.317E+02	.161E+02
1.0000	1.0000	-.797E+00	.426E+02	.247E+02
.0000	.0000	.159E+02	.531E+02	.330E+02
-.5774	-.5774	.258E+02	.594E+02	.370E+02
.5774	-.5774	.178E+02	.412E+02	.263E+02
-.5774	.5774	.962E+01	.633E+02	.364E+02
.5774	.5774	.616E+01	.462E+02	.297E+02

ELEMENT ID 2200

STRESS OUTPUT IN LOCAL COORDINATES FOR LAYER 1

STRESS POINTS		STRESS COMPONENTS		
CI	CJ	SXX	SYX	SXY
-1.0000	-1.0000	.735E+01	.592E+02	.417E+02
-1.0000	1.0000	.432E+02	.509E+02	.512E+02
1.0000	-1.0000	.408E+02	.127E+03	.623E+02



1.0000	1.0000	.847E+02	.115E+03	.647E+02
.0000	.0000	.422E+02	.866E+02	.547E+02
-.5774	-.5774	.262E+02	.674E+02	.500E+02
.5774	-.5774	.364E+02	.113E+03	.610E+02
-.5774	.5774	.479E+02	.623E+02	.498E+02
.5774	.5774	.607E+02	.106E+03	.584E+02

STRESS OUTPUT IN LOCAL COORDINATES FOR LAYER 2

STRESS POINTS		STRESS COMPONENTS		
CI	CJ	SXX	SYX	SXY
-1.0000	-1.0000	.368E+03	.509E+02	.512E+02
-1.0000	1.0000	.169E+03	.177E+02	-.627E+01
1.0000	-1.0000	.445E+03	.115E+03	.647E+02
1.0000	1.0000	.202E+03	.242E+02	-.459E+01
.0000	.0000	.294E+03	.475E+02	.692E+01
-.5774	-.5774	.353E+03	.645E+02	.101E+02
.5774	-.5774	.369E+03	.756E+02	.231E+02
-.5774	.5774	.227E+03	.320E+02	.563E+01
.5774	.5774	.229E+03	.239E+02	.147E+02

ELEMENT STRAIN ENERGY = .476E-01

INCREMENT 1 SUMMARY

TIME INCREMENT COMPLETED	2.220E-16,	FRACTION OF STEP COMPLETED	1.00
STEP TIME COMPLETED	2.220E-16,	TOTAL TIME COMPLETED	.000E+00

ELEMENT OUTPUT

THE FOLLOWING TABLE IS PRINTED FOR ELSET NAME AND ELEMENT TYPE CPE4 AVERAGED AT THE NODE

NODE	FOOT-NOTE	S11	S22	S33	S12
503		248.5	22.90	86.84	-57.95
504		235.6	35.79	86.84	66.43
654		287.7	-16.28	86.84	-53.89
655		274.8	-3.389	86.84	70.49
MAXIMUM		287.7	35.79	86.84	70.49
NODE		654	504	504	655
MINIMUM		235.6	-16.28	86.84	-57.95
NODE		504	654	655	503

NODE OUTPUT

THE FOLLOWING TABLE IS PRINTED FOR ALL NODES

NODE	FOOT-NOTE	U1	U2
2		1.2104E-03	.0000E+00
3		2.3850E-03	.0000E+00
4		3.8386E-03	.0000E+00
5		6.3518E-03	.0000E+00
6		9.7042E-03	4.8198E-03
7		1.3104E-02	1.1474E-02
8		1.6396E-02	2.1403E-02
9		1.9556E-02	3.3845E-02
10		2.2586E-02	4.8795E-02
.		.	.
.		.	.
.		.	.
2091		.1302	-.2619
2092		.1304	-.2684
2093		.1307	-.2748
2094		.1309	-.2812
2095		.1312	-.2875
2096		.1314	-.2939
2097		.1317	-.3003
2098		.1321	-.3066
2099		.1324	-.3129
2100		.1325	-.3189
2101		.1315	-.3263
MAXIMUM		.2527	.6578

AT NODE	1755	189
MINIMUM	.0000E+00	-.6578
AT NODE	1	1567

THE FOLLOWING TABLE IS PRINTED FOR ALL NODES

NODE	FOOT- NOTE	RF1	RF2
1		-18.66	-27.60
2		.0000E+00	-56.85
3		.0000E+00	-49.30
4		.0000E+00	-42.34
5		.0000E+00	-62.37
152		-37.51	-.7832
303		-37.62	.6980
454		-37.46	-.2781
605		-18.74	28.11
606		.0000E+00	55.35
607		.0000E+00	62.71
608		.0000E+00	68.27
609		.0000E+00	21.39
1147		.0000E+00	-21.38
1148		.0000E+00	-68.27
1149		.0000E+00	-62.72
1150		.0000E+00	-55.34
1151		.0000E+00	-28.06
1751		.0000E+00	62.36
1752		.0000E+00	42.35
1753		.0000E+00	49.26
1754		.0000E+00	56.86
1755		.0000E+00	27.92
MAXIMUM		.0000E+00	68.27
AT NODE		2	608
MINIMUM		-37.62	-68.27
AT NODE		303	1148

THE ANALYSIS HAS BEEN COMPLETED

**APPENDIX C**

**Demonstration problem II: 3-D analysis of a single-lap joint.**

# ABAQUS INPUT FILE

## \*HEADING

3-D SINGLE-LAP JOINT. 100 H2L12N ELEMENTS ALONG BONDLINE.

\*PREPRINT, ECHO = NO, HISTORY = NO, MODEL = NO

## \*NODE

1,	0.0,	0.0,	0.0
301,	63.5,	0.0,	0.0
401,	76.2,	0.0,	0.0
4001,	0.0,	0.0,	1.6
4301,	63.5,	0.0,	1.6
4401,	76.2,	0.0,	1.6
5001,	63.5,	0.0,	1.75
5101,	76.2,	0.0,	1.75
5401,	139.7,	0.0,	1.75
9001,	63.5,	0.0,	3.35
9101,	76.2,	0.0,	3.35
9401,	139.7,	0.0,	3.35
4701,	63.5,	0.0,	1.675
4801,	76.2,	0.0,	1.675
50001,	0.0,	1.0,	0.0
50301,	63.5,	1.0,	0.0
50401,	76.2,	1.0,	0.0
54001,	0.0,	1.0,	1.6
54301,	63.5,	1.0,	1.6
54401,	76.2,	1.0,	1.6
55001,	63.5,	1.0,	1.75
55101,	76.2,	1.0,	1.75
55401,	139.7,	1.0,	1.75
59001,	63.5,	1.0,	3.35
59101,	76.2,	1.0,	3.35
59401,	139.7,	1.0,	3.35
54701,	63.5,	1.0,	1.675
54801,	76.2,	1.0,	1.675

\*\*

\*NGEN, NSET=FBL

1, 4001, 1000

\*NGEN, NSET=FBM

301, 4301, 1000

\*NGEN, NSET=FBR

401, 4401, 1000

\*NGEN, NSET=FTL

5001, 9001, 1000

\*NGEN, NSET=FTM

5101, 9101, 1000

\*NGEN, NSET=FTR

5401, 9401, 1000

\*NGEN, NSET=FMIDDLE

4701, 4801, 1

\*NGEN, NSET=BBL

50001, 54001, 1000

\*NGEN, NSET=BBM

50301, 54301, 1000

\*NGEN, NSET=BBR

50401, 54401, 1000

\*NGEN, NSET=BTL

55001, 59001, 1000

\*NGEN, NSET=BTM

55101, 59101, 1000

```

*NGEN, NSET=BTR
55401,59401,1000
*NGEN, NSET=BMIDDLE
54701,54801, 1
**
*NFILL, NSET=FRONT
FBL,FBM,300
FBM,FBR,100
FTL,FTM,100
FTM,FTR,300
*NFILL, NSET=BACK
BBL,BBM,300
BBM,BBR,100
BTL,BTM,100
BTM,BTR,300
*NFILL
FRONT, BACK, 5, 10000
FMIDDLE, BMIDDLE, 5, 10000
**
*ELEMENT, TYPE=C3D8
1, 1, 3,10003,10001,1001,1003,11003,11001
151, 301, 302,10302,10301,1301,1302,11302,11301
4001,3001,3003,13003,13001,4001,4003,14003,14001
5001,5101,5103,15103,15101,6101,6103,16103,16101
6001,6001,6002,16002,16001,7001,7002,17002,17001
6101,6101,6103,16103,16101,7101,7103,17103,17101
**
*ELGEN, ELSET=BOT
1, 150,2,1,3,1000,1000,5,10000,10000
151, 100,1,1,3,1000,1000,5,10000,10000
4001,150,2,1,1,,,5,10000,10000
**
*ELGEN, ELSET=TOP
5001,150,2,1,1,,,5,10000,10000
6001,100,1,1,3,1000,1000,5,10000,10000
6101,150,2,1,3,1000,1000,5,10000,10000
**
** DEFINE ADHESIVE ELEMENT H2L12N
**
*USER ELEMENT, NODES=12, TYPE=U5, PROPERTIES=100, COORDINATES=3, VARIABLES=1
1, 2, 3
*ELEMENT, TYPE=U5
4701,3301,3302,13302,13301,4301,4302,14302,14301,4701,4702,14702,14701
4801,4701,4702,14702,14701,5001,5002,15002,15001,6001,6002,16002,16001
*ELGEN, ELSET=ADHBOT
4701,100,1,1,1,,,5,10000,10000
*ELGEN, ELSET=ADHTOP
4801,100,1,1,1,,,5,10000,10000
**
** USER DEFINED SUBROUTINE:
**
*USER SUBROUTINE, INPUT=uel_report.f
**
** ELEMENT PROPERTIES
**
*SOLID SECTION, ELSET=TOP , MATERIAL=MID1
*MATERIAL, NAME=MID1
*ELASTIC, TYPE=ISO
69000.0, 0.32, 0.0
**
*SOLID SECTION, ELSET=BOT , MATERIAL=MID3
*MATERIAL, NAME=MID3
*ELASTIC, TYPE=ISO
69000.0, 0.32, 0.0
**
** USER DEFINED ELEMENT PROPERTY LIST:
**
** TOP ROW

```

```

**
*UEL PROPERTY,ELSET=ADHTOP
11.0, 1.0, 1.0
1.0
0.15, 0.0, 3000.0, 3000.0, 3000.0, 0.36, 0.36, 0.36
1102.9412, 1102.9412, 1102.9412
1.0
0.4, 0.0, 69000.0, 69000.0, 69000.0, 0.32, 0.32, 0.32
26136.3636, 26136.3636, 26136.3636
**
** BOTTOM ROW
**
*UEL PROPERTY,ELSET=ADHBOT
11.0, 1.0, 1.0
1.0
0.4, 0.0, 69000.0, 69000.0, 69000.0, 0.32, 0.32, 0.32
26136.3636, 26136.3636, 26136.3636
1.0
0.15, 0.0, 3000.0, 3000.0, 3000.0, 0.36, 0.36, 0.36
1102.9412, 1102.9412, 1102.9412
*NSET,NSET=L,GENERATE
10001, 14001, 1000
20001, 24001, 1000
30001, 34001, 1000
40001, 44001, 1000
**
*NSET,NSET=LSIDE
L,FBL,BBL
*NSET,NSET=ROLLB,GENERATE
1, 24, 1
10001, 10024, 1
20001, 20024, 1
30001, 30024, 1
40001, 40024, 1
50001, 50024, 1
4001, 4024, 1
14001, 14024, 1
24001, 24024, 1
34001, 34024, 1
44001, 44024, 1
54001, 54024, 1
*NSET,NSET=ROLLE,GENERATE
5378, 5401, 1
15378, 15401, 1
25378, 25401, 1
35378, 35401, 1
45378, 45401, 1
55378, 55401, 1
9378, 9401, 1
19378, 19401, 1
29378, 29401, 1
39378, 39401, 1
49378, 49401, 1
59378, 59401, 1
*ELSET,ELSET=PULL
5150, 6250, 7250, 8250
15150, 16250, 17250, 18250
25150, 26250, 27250, 28250
35150, 36250, 37250, 38250
45150, 46250, 47250, 48250
*ELSET,ELSET=ONE
1
**
** BOUNDARY CONDITIONS:
**
*BOUNDARY
LSIDE, 1, 3
ROLLE, 3

```

ROLLB, 3  
\*STEP, PERTURBATION  
\*STATIC  
\*\*  
\*\* LOAD CASE SPECIFICATION:  
\*\*  
\*DLOAD, OP=NEW  
PULL, P4, -93.75  
\*NODE PRINT  
U  
\*EL PRINT, ELSET=ONE  
MISES,  
\*END STEP

# ABAQUS OUTPUT FILE

\*PREPRINT, ECHO = NO, HISTORY = NO, MODEL = NO

```

AAAAAA      BBBB BBBB      AAAAA      QQQQQQQQ      U      U      SSSSSSSS
A      A      B      B      A      A      Q      Q      U      U      S
A      A      B      B      A      A      Q      Q      U      U      S
A      A      B      B      A      A      Q      Q      U      U      S
AAAAAAAAAA  BBBB BBBB      AAAAAAAAA  Q      Q      U      U      SSSSSSSS
A      A      B      B      A      A      Q      Q      U      U      S
A      A      B      B      A      A      Q      Q      U      U      S
A      A      B      B      A      A      Q      Q      U      U      S
A      A      BBBB BBBB      A      A      QQQQQQQQ      UUUUUUUU      SSSSSSSS

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THIS PROGRAM HAS BEEN DEVELOPED BY  
HIBBITT, KARLSSON AND SORENSEN, INC.  
1080 MAIN STREET  
PAWTUCKET, R.I. 02860

THIS IS A PROPRIETARY PROGRAM. IT MAY ONLY BE  
USED UNDER THE TERMS OF THE LICENSE AGREEMENT  
BETWEEN HIBBITT, KARLSSON & SORENSEN, INC.  
AND ARMY RESEARCH LABORATORY.

```

*****
*
*
*          *****
*        * NOTICE *
*          *****
*
*
*          THIS IS ABAQUS VERSION 5.3.
*
*
* PLEASE MAKE SURE YOU ARE USING VERSION 5.3 MANUALS
* PLUS THE NOTES ACCOMPANYING THIS RELEASE. THESE NOTES
* CAN BE OBTAINED BY USING THE INFORMATION OPTION ON THE
* ABAQUS COMMAND LINE.
*
*****

```

OPTIONS BEING PROCESSED  
\*\*\*\*\*

```

*HEADING
3-D SINGLE-LAP JOINT. 100 H2L12N ELEMENTS ALONG BONDLINE.

*NODE
*NGEN, NSET-FBL
*NGEN, NSET-FBM
*NGEN, NSET-FBR
*NGEN, NSET-PTL
*NGEN, NSET-PTM
*NGEN, NSET-PTR
*NGEN, NSET-FMIDDLE
*NGEN, NSET-BBL
*NGEN, NSET-BBM
*NGEN, NSET-BBR
*NGEN, NSET-BTL
*NGEN, NSET-BTM
*NGEN, NSET-BTR

```



\*NGEN, NSET=BMIDDLE  
 \*NFILL, NSET=FRONT

THE FOLLOWING NODES WILL BE USED IN THE NFILL GENERATION  
 3-D SINGLE-LAP JOINT. 100 H2L12N ELEMENTS ALONG BONDLINE.

BOUND 1	1	1001	2001	3001	4001
BOUND 2	301	1301	2301	3301	4301

THE FOLLOWING NODES WILL BE USED IN THE NFILL GENERATION

BOUND 1	301	1301	2301	3301	4301
BOUND 2	401	1401	2401	3401	4401

THE FOLLOWING NODES WILL BE USED IN THE NFILL GENERATION

BOUND 1	5001	6001	7001	8001	9001
BOUND 2	5101	6101	7101	8101	9101

THE FOLLOWING NODES WILL BE USED IN THE NFILL GENERATION

BOUND 1	5101	6101	7101	8101	9101
BOUND 2	5401	6401	7401	8401	9401

\*NFILL, NSET=BACK

THE FOLLOWING NODES WILL BE USED IN THE NFILL GENERATION

BOUND 1	50001	51001	52001	53001	54001
BOUND 2	50301	51301	52301	53301	54301

THE FOLLOWING NODES WILL BE USED IN THE NFILL GENERATION

BOUND 1	50301	51301	52301	53301	54301
BOUND 2	50401	51401	52401	53401	54401

THE FOLLOWING NODES WILL BE USED IN THE NFILL GENERATION

BOUND 1	55001	56001	57001	58001	59001
BOUND 2	55101	56101	57101	58101	59101

THE FOLLOWING NODES WILL BE USED IN THE NFILL GENERATION

3-D SINGLE-LAP JOINT. 100 H2L12N ELEMENTS ALONG BONDLINE.

BOUND 1	55101	56101	57101	58101	59101
BOUND 2	55401	56401	57401	58401	59401

\*NFILL

THE FOLLOWING NODES WILL BE USED IN THE NFILL GENERATION

BOUND 1	1	2	3	4	5	6	7	8	9
	11	12	13	14	15	16	17	18	19
	21	22	23	24	25	26	27	28	29
	.	.	.	.	.	.	.	.	.
	.	.	.	.	.	.	.	.	.
	9372	9373	9374	9375	9376	9377	9378	9379	9380
	9382	9383	9384	9385	9386	9387	9388	9389	9390
	9392	9393	9394	9395	9396	9397	9398	9399	9400
BOUND 2	50001	50002	50003	50004	50005	50006	50007	50008	50009
	50011	50012	50013	50014	50015	50016	50017	50018	50019
	50021	50022	50023	50024	50025	50026	50027	50028	50029
	.	.	.	.	.	.	.	.	.
	.	.	.	.	.	.	.	.	.
	59372	59373	59374	59375	59376	59377	59378	59379	59380
	59382	59383	59384	59385	59386	59387	59388	59389	59390

59392 59393 59394 59395 59396 59397 59398 59399 59400

THE FOLLOWING NODES WILL BE USED IN THE NFILL GENERATION

BOUND 1	4701	4702	4703	4704	4705	4706	4707	4708	4709
	4711	4712	4713	4714	4715	4716	4717	4718	4719
	4721	4722	4723	4724	4725	4726	4727	4728	4729
	4731	4732	4733	4734	4735	4736	4737	4738	4739
	4741	4742	4743	4744	4745	4746	4747	4748	4749
	4751	4752	4753	4754	4755	4756	4757	4758	4759
	4761	4762	4763	4764	4765	4766	4767	4768	4769
	4771	4772	4773	4774	4775	4776	4777	4778	4779
	4781	4782	4783	4784	4785	4786	4787	4788	4789
	4791	4792	4793	4794	4795	4796	4797	4798	4799
	4801								

BOUND 2	54701	54702	54703	54704	54705	54706	54707	54708	54709
	54711	54712	54713	54714	54715	54716	54717	54718	54719
	54721	54722	54723	54724	54725	54726	54727	54728	54729
	54731	54732	54733	54734	54735	54736	54737	54738	54739
	54741	54742	54743	54744	54745	54746	54747	54748	54749
	54751	54752	54753	54754	54755	54756	54757	54758	54759
	54761	54762	54763	54764	54765	54766	54767	54768	54769
	54771	54772	54773	54774	54775	54776	54777	54778	54779
	54781	54782	54783	54784	54785	54786	54787	54788	54789
	54791	54792	54793	54794	54795	54796	54797	54798	54799
	54801								

\*ELEMENT, TYPE=C3D8  
\*ELGEN, ELSET=BOT  
\*ELGEN, ELSET=TOP  
\*USER ELEMENT, NODES=12, TYPE=U5, PROPERTIES=100, COORDINATES=3, VARIABLES=1  
\*ELEMENT, TYPE=U5  
\*ELGEN, ELSET=ADHBTOP  
\*ELGEN, ELSET=ADHTOP  
\*NSET, NSET=L, GENERATE  
\*NSET, NSET=LSIDE  
\*NSET, NSET=ROLLB, GENERATE  
\*NSET, NSET=ROLLE, GENERATE  
\*ELSET, ELSET=PULL  
\*ELSET, ELSET=ONE  
\*MATERIAL, NAME=MID1  
\*ELASTIC, TYPE=ISO  
\*MATERIAL, NAME=MID3  
\*ELASTIC, TYPE=ISO  
\*USER ELEMENT, NODES=12, TYPE=U5, PROPERTIES=100, COORDINATES=3, VARIABLES=1  
\*SOLID SECTION, ELSET=TOP, MATERIAL=MID1  
\*SOLID SECTION, ELSET=BOT, MATERIAL=MID3  
\*UEL PROPERTY, ELSET=ADHTOP  
\*UEL PROPERTY, ELSET=ADHBTOP  
\*STEP, PERTURBATION  
\*STATIC  
\*DLOAD, OP=NEW  
\*EL PRINT, ELSET=ONE  
\*END STEP  
\*BOUNDARY  
\*STEP, PERTURBATION  
\*STATIC  
\*NODE PRINT  
\*END STEP

WAVEFRONT MINIMIZATION

WAVEFRONT MINIMIZATION METHOD 1 WILL BE USED.  
NUMBER OF NODES 15666  
NUMBER OF ELEMENTS 10000  
ORIGINAL MAXIMUM D.O.F WAVEFRONT ESTIMATED AS 8592  
ORIGINAL RMS D.O.F WAVEFRONT ESTIMATED AS 6967

PERIPHERAL DIAMETER IS DEFINED BY NODES 1 5401

WAVEFRONT OPTIMIZED BY CHOOSING 1 AS THE STARTING NODE

MINIMUM WAVEFRONT OBTAINED USING METHOD 2. USE  
\*WAVEFRONT MINIMIZATION, NODES, METHOD=2  
1, 5401

TO REDUCE THE CPU TIME ON SUBSEQUENT JOBS USING THIS SAME MESH.

PROBLEM SIZE

NUMBER OF ELEMENTS IS

10000

NUMBER OF NODES IS 15666  
 NUMBER OF NODES DEFINED BY THE USER 15666  
 NUMBER OF INTERNAL NODES GENERATED BY THE PROGRAM 0  
 TOTAL NUMBER OF VARIABLES IN THE MODEL 46998  
 (DEGREES OF FREEDOM PLUS ANY LAGRANGE MULTIPLIER VARIABLES)  
 MAXIMUM D.O.F. WAVEFRONT ESTIMATED AS 273  
 RMS WAVEFRONT ESTIMATED AS 172

FILE SIZES - THESE VALUES ARE IN WORDS AND ARE CONSERVATIVE UPPER BOUNDS

UNIT	LENGTH
2	8815647
10	2282000
19	3328000
21	3320000
22	3320000
25	2220000

IF THE RESTART FILE IS WRITTEN ITS LENGTH WILL BE APPROXIMATELY  
 3323232 WORDS WRITTEN IN THE PRE PROGRAM  
 PLUS 2772000 WORDS WRITTEN AT THE BEGINNING OF EACH STEP  
 PLUS 4134521 WORDS FOR EACH INCREMENT WRITTEN TO THE RESTART FILE

ALLOCATED WORKSPACE 2002539  
 \*USER SUBROUTINE, INPUT=uel\_report.f

END OF USER INPUT PROCESSING

STEP 1 STATIC ANALYSIS

FIXED TIME INCREMENTS  
 TIME INCREMENT IS 2.220E-16  
 TIME PERIOD IS 2.220E-16

THIS IS A LINEAR PERTURBATION STEP.  
 ALL LOADS ARE DEFINED AS CHANGE IN LOAD TO THE REFERENCE STATE

ELEMENT ID 44900

STRESS OUTPUT IN LOCAL COORDINATES FOR

LAYER 1

RECOVERY POINTS			STRESS COMPONENTS					
CI	CJ	CK	SXX	SYX	SZZ	SYZ	SZX	SKY
-1.0000	-1.0000	-1.0000	.546E+01	.245E+01	.506E+02	.652E+01	.552E+02	.165E+01
-1.0000	-1.0000	1.0000	.485E+02	.269E+02	.463E+02	.594E+01	.411E+02	.119E+01
-1.0000	1.0000	-1.0000	.548E+01	.129E+01	.418E+02	-.193E+01	.551E+02	-.235E+01
-1.0000	1.0000	1.0000	.470E+02	.263E+02	.396E+02	-.251E+01	.439E+02	-.197E+01
1.0000	-1.0000	-1.0000	.318E+02	.438E+02	.135E+03	-.936E+01	.678E+02	.338E+01
1.0000	-1.0000	1.0000	.743E+02	.629E+02	.119E+03	-.926E+01	.538E+02	.254E+01
1.0000	1.0000	-1.0000	.269E+02	.408E+02	.125E+03	.139E+02	.641E+02	-.626E+00
1.0000	1.0000	1.0000	.679E+02	.605E+02	.111E+03	.140E+02	.530E+02	-.618E+00
.0000	.0000	.0000	.384E+02	.331E+02	.836E+02	.215E+01	.543E+02	.399E+00
-.5774	-.5774	-.5774	.198E+02	.158E+02	.652E+02	.270E+01	.548E+02	.109E+01
.5774	-.5774	-.5774	.344E+02	.388E+02	.113E+03	-.252E+01	.617E+02	.204E+01
-.5774	.5774	-.5774	.190E+02	.150E+02	.602E+02	.169E+01	.547E+02	-.112E+01
.5774	.5774	-.5774	.320E+02	.374E+02	.107E+03	.702E+01	.604E+02	-.164E+00
-.5774	-.5774	.5774	.444E+02	.294E+02	.616E+02	.245E+01	.471E+02	.884E+00
.5774	-.5774	.5774	.588E+02	.506E+02	.105E+03	-.255E+01	.539E+02	.171E+01
-.5774	.5774	.5774	.432E+02	.287E+02	.573E+02	.143E+01	.479E+02	-.104E+01
.5774	.5774	.5774	.559E+02	.493E+02	.100E+03	.699E+01	.536E+02	-.217E+00

STRAIN OUTPUT IN LOCAL COORDINATES FOR

LAYER 1

RECOVERY POINTS			STRAIN COMPONENTS					
CI	CJ	CK	EXX	EYY	EZZ	EYZ	EZX	EXY
-1.0000	-1.0000	-1.0000	-.773E-02	-.558E-02	.989E-02	-.229E-02	.336E-01	-.126E-03
-1.0000	-1.0000	1.0000	.515E-02	-.202E-02	.989E-02	-.333E-02	.461E-01	.149E-03
-1.0000	1.0000	-1.0000	-.713E-02	-.558E-02	.710E-02	.721E-02	.333E-01	.673E-03
-1.0000	1.0000	1.0000	.491E-02	-.202E-02	.710E-02	-.616E-02	.462E-01	.519E-03
1.0000	-1.0000	-1.0000	-.773E-02	-.507E-02	.310E-01	-.178E-02	.554E-01	.254E-03
1.0000	-1.0000	1.0000	.515E-02	-.178E-02	.310E-01	-.257E-02	.679E-01	-.228E-06
1.0000	1.0000	-1.0000	-.713E-02	-.507E-02	.289E-01	.698E-02	.537E-01	.105E-02
1.0000	1.0000	1.0000	.491E-02	-.178E-02	.289E-01	.620E-02	.666E-01	.370E-03
.0000	.0000	.0000	-.120E-02	-.361E-02	.192E-01	.207E-02	.504E-01	.362E-03
-.5774	-.5774	-.5774	-.492E-02	-.473E-02	.138E-01	-.416E-03	.407E-01	.138E-03
.5774	-.5774	-.5774	-.492E-02	-.447E-02	.261E-01	-.181E-03	.532E-01	.294E-03
-.5774	.5774	-.5774	-.467E-02	-.473E-02	.123E-01	.498E-02	.405E-01	.548E-03
.5774	.5774	-.5774	-.467E-02	-.447E-02	.248E-01	.497E-02	.524E-01	.703E-03

-.5774	-.5774	.5774	.241E-02	-.271E-02	.138E-01	-.988E-03	.480E-01	.180E-03
.5774	-.5774	.5774	.241E-02	-.254E-02	.261E-01	-.667E-03	.604E-01	.159E-03
-.5774	.5774	.5774	.238E-02	-.271E-02	.123E-01	.440E-02	.479E-01	.446E-03
.5774	.5774	.5774	.238E-02	-.254E-02	.248E-01	.448E-02	.598E-01	.425E-03

STRESS OUTPUT IN LOCAL COORDINATES FOR

LAYER 2

RECOVERY POINTS			STRESS COMPONENTS					
CI	CJ	CK	SXX	SYX	SZZ	SYZ	SZX	SXY
-1.0000	-1.0000	-1.0000	.378E+03	-.663E+01	.441E+02	.564E+01	.341E+02	.443E+01
-1.0000	-1.0000	1.0000	.190E+03	.285E+01	.211E+02	.966E+01	-.150E+02	-.158E+01
-1.0000	1.0000	-1.0000	.360E+03	-.905E+01	.385E+02	-.281E+01	.384E+02	.885E+01
-1.0000	1.0000	1.0000	.186E+03	.247E+01	.270E+02	.121E+01	-.997E+01	-.369E+01
1.0000	-1.0000	-1.0000	.391E+03	.334E+02	.111E+03	-.921E+01	.468E+02	.469E+01
1.0000	-1.0000	1.0000	.207E+03	-.271E+01	.249E+02	-.108E+02	-.229E+01	-.261E+01
1.0000	1.0000	-1.0000	.372E+03	.338E+02	.104E+03	.140E+02	.474E+02	.912E+01
1.0000	1.0000	1.0000	.203E+03	-.284E+00	.292E+02	.124E+02	-.946E+00	-.472E+01
.0000	.0000	.0000	.286E+03	.674E+01	.500E+02	.251E+01	.173E+02	.181E+01
-.5774	-.5774	-.5774	.338E+03	.151E+01	.499E+02	.273E+01	.272E+02	.380E+01
.5774	-.5774	-.5774	.346E+03	.194E+02	.807E+02	-.266E+01	.341E+02	.379E+01
-.5774	.5774	-.5774	.329E+03	.701E+00	.479E+02	.172E+01	.293E+02	.556E+01
.5774	.5774	-.5774	.337E+03	.195E+02	.781E+02	.687E+01	.349E+02	.555E+01
-.5774	-.5774	.5774	.232E+03	.166E+01	.303E+02	.437E+01	-.105E+01	-.623E+00
.5774	-.5774	.5774	.241E+03	.436E+01	.400E+02	-.290E+01	.581E+01	-.106E+01
-.5774	.5774	.5774	.228E+03	.154E+01	.321E+02	.335E+01	.129E+01	-.104E+01
.5774	.5774	.5774	.237E+03	.517E+01	.412E+02	.664E+01	.695E+01	-.148E+01

STRAIN OUTPUT IN LOCAL COORDINATES FOR

LAYER 2

RECOVERY POINTS			STRAIN COMPONENTS					
CI	CJ	CK	EXX	EYY	EZZ	EYZ	EZX	EXY
-1.0000	-1.0000	-1.0000	.515E-02	-.202E-02	-.600E-03	-.295E-03	.921E-03	.149E-03
-1.0000	-1.0000	1.0000	.277E-02	-.945E-03	-.600E-03	-.227E-04	.552E-03	.266E-04
-1.0000	1.0000	-1.0000	.491E-02	-.202E-02	-.463E-03	.240E-03	.125E-02	.519E-03
-1.0000	1.0000	1.0000	.268E-02	-.945E-03	-.463E-03	.513E-03	.519E-03	-.212E-03
1.0000	-1.0000	-1.0000	.515E-02	-.178E-02	-.716E-03	-.153E-03	.165E-03	-.228E-06
1.0000	-1.0000	1.0000	.277E-02	-.110E-02	-.716E-03	-.113E-03	-.203E-03	-.287E-04
1.0000	1.0000	-1.0000	.491E-02	-.178E-02	-.696E-03	.188E-03	.544E-03	.370E-03
1.0000	1.0000	1.0000	.268E-02	-.110E-02	-.696E-03	.229E-03	-.189E-03	-.268E-03
.0000	.0000	.0000	.388E-02	-.146E-02	-.619E-03	.732E-04	.445E-03	.693E-04
-.5774	-.5774	-.5774	.460E-02	-.176E-02	-.601E-03	-.114E-03	.739E-03	.146E-03
.5774	-.5774	-.5774	.460E-02	-.167E-02	-.682E-03	-.836E-04	.309E-03	.720E-04
-.5774	.5774	-.5774	.448E-02	-.176E-02	-.536E-03	.172E-03	.892E-03	.286E-03
.5774	.5774	-.5774	.448E-02	-.167E-02	-.656E-03	.137E-03	.477E-03	.211E-03
-.5774	-.5774	.5774	.325E-02	-.119E-02	-.601E-03	.155E-04	.482E-03	.132E-04
.5774	-.5774	.5774	.325E-02	-.123E-02	-.682E-03	-.319E-04	.515E-04	-.301E-04
-.5774	.5774	.5774	.318E-02	-.119E-02	-.536E-03	.301E-03	.513E-03	-.504E-04
.5774	.5774	.5774	.318E-02	-.123E-02	-.656E-03	.189E-03	.983E-04	-.938E-04

ELEMENT ID 34900

STRESS OUTPUT IN LOCAL COORDINATES FOR

LAYER 1

RECOVERY POINTS			STRESS COMPONENTS					
CI	CJ	CK	SXX	SYX	SZZ	SYZ	SZX	SXY
-1.0000	-1.0000	-1.0000	.115E+02	.225E+02	.575E+02	.488E+01	.538E+02	.300E+01
-1.0000	-1.0000	1.0000	.538E+02	.337E+02	.530E+02	.641E+01	.411E+02	.203E+01
-1.0000	1.0000	-1.0000	.132E+02	.195E+02	.562E+02	-.991E+01	.526E+02	-.162E+01
-1.0000	1.0000	1.0000	.517E+02	.306E+02	.513E+02	-.838E+01	.387E+02	-.116E+01
1.0000	-1.0000	-1.0000	.360E+02	.604E+02	.141E+03	-.102E+02	.708E+02	.235E+01
1.0000	-1.0000	1.0000	.773E+02	.675E+02	.125E+03	-.958E+01	.581E+02	.140E+01
1.0000	1.0000	-1.0000	.398E+02	.597E+02	.141E+03	.639E+01	.703E+02	-.227E+01
1.0000	1.0000	1.0000	.774E+02	.668E+02	.124E+03	.703E+01	.563E+02	-.180E+01
.0000	.0000	.0000	.451E+02	.451E+02	.936E+02	-.167E+01	.552E+02	.241E+00
-.5774	-.5774	-.5774	.259E+02	.322E+02	.734E+02	.249E+00	.544E+02	.175E+01
.5774	-.5774	-.5774	.401E+02	.538E+02	.120E+03	-.474E+01	.643E+02	.137E+01
-.5774	.5774	-.5774	.267E+02	.307E+02	.727E+02	-.446E+01	.537E+02	-.745E+00
.5774	.5774	-.5774	.416E+02	.531E+02	.120E+03	.101E+01	.638E+02	-.112E+01
-.5774	-.5774	.5774	.497E+02	.381E+02	.693E+02	.102E+01	.469E+02	.136E+01
.5774	-.5774	.5774	.637E+02	.584E+02	.112E+03	-.427E+01	.568E+02	.994E+00
-.5774	.5774	.5774	.492E+02	.366E+02	.685E+02	-.368E+01	.458E+02	-.654E+00
.5774	.5774	.5774	.639E+02	.577E+02	.112E+03	.149E+01	.559E+02	-.102E+01

STRAIN OUTPUT IN LOCAL COORDINATES FOR

LAYER 1

RECOVERY POINTS			STRAIN COMPONENTS					
CI	CJ	CK	EXX	EYY	EZZ	EYZ	EZX	EXY
-1.0000	-1.0000	-1.0000	-.896E-02	-.131E-02	.979E-02	-.114E-02	.337E-01	.416E-04
-1.0000	-1.0000	1.0000	.513E-02	-.188E-02	.979E-02	-.111E-02	.463E-01	.625E-04
-1.0000	1.0000	-1.0000	-.773E-02	-.131E-02	.989E-02	-.266E-02	.336E-01	-.158E-03
-1.0000	1.0000	1.0000	.515E-02	-.188E-02	.989E-02	-.262E-02	.461E-01	.138E-03
1.0000	-1.0000	-1.0000	-.896E-02	-.144E-02	.312E-01	-.102E-02	.575E-01	.820E-03
1.0000	-1.0000	1.0000	.513E-02	-.183E-02	.312E-01	-.111E-02	.702E-01	.753E-04
1.0000	1.0000	-1.0000	-.773E-02	-.144E-02	.310E-01	-.207E-02	.554E-01	.621E-03

1.0000	1.0000	1.0000	.515E-02	-.183E-02	.310E-01	-.216E-02	.679E-01	.151E-03
.0000	.0000	.0000	-.160E-02	-.161E-02	.205E-01	-.174E-02	.513E-01	.219E-03
-.5774	-.5774	-.5774	-.577E-02	-.145E-02	.143E-01	-.142E-02	.413E-01	.146E-03
.5774	-.5774	-.5774	-.577E-02	-.150E-02	.267E-01	-.130E-02	.548E-01	.503E-03
-.5774	.5774	-.5774	-.521E-02	-.145E-02	.143E-01	-.223E-02	.410E-01	.647E-04
.5774	.5774	-.5774	-.521E-02	-.150E-02	.266E-01	-.197E-02	.538E-01	.421E-03
-.5774	-.5774	.5774	.221E-02	-.175E-02	.143E-01	-.141E-02	.486E-01	.987E-04
.5774	-.5774	.5774	.221E-02	-.175E-02	.267E-01	-.134E-02	.621E-01	.200E-03
-.5774	.5774	.5774	.237E-02	-.175E-02	.143E-01	-.223E-02	.482E-01	.109E-03
.5774	.5774	.5774	.237E-02	-.175E-02	.266E-01	-.200E-02	.610E-01	.209E-03

STRESS OUTPUT IN LOCAL COORDINATES FOR

LAYER 2

RECOVERY POINTS			STRESS COMPONENTS					
CI	CJ	CK	SXX	SYX	SZZ	SYZ	SZX	SXY
-1.0000	-1.0000	-1.0000	.380E+03	.899E+01	.507E+02	.718E+01	.347E+02	.372E+01
-1.0000	-1.0000	1.0000	.194E+03	-.278E+01	.268E+02	.123E+02	-.152E+02	-.130E+01
-1.0000	1.0000	-1.0000	.380E+03	.709E+01	.488E+02	-.761E+01	.317E+02	.228E+01
-1.0000	1.0000	1.0000	.190E+03	-.500E+01	.228E+02	-.247E+01	-.175E+02	-.288E+00
1.0000	-1.0000	-1.0000	.397E+03	.385E+02	.117E+03	-.926E+01	.517E+02	.329E+01
1.0000	-1.0000	1.0000	.209E+03	-.768E+00	.299E+02	-.558E+01	.180E+01	-.153E+01
1.0000	1.0000	-1.0000	.397E+03	.373E+02	.116E+03	.735E+01	.494E+02	.186E+01
1.0000	1.0000	1.0000	.205E+03	-.225E+01	.270E+02	.110E+02	.202E+00	-.516E+00
.0000	.0000	.0000	.294E+03	.101E+02	.548E+02	.162E+01	.171E+02	.939E+00
-.5774	-.5774	-.5774	.344E+03	.111E+02	.563E+02	.300E+01	.272E+02	.238E+01
.5774	-.5774	-.5774	.353E+03	.249E+02	.868E+02	-.283E+01	.371E+02	.216E+01
-.5774	.5774	-.5774	.344E+03	.101E+02	.551E+02	-.171E+01	.256E+02	.185E+01
.5774	.5774	-.5774	.353E+03	.241E+02	.860E+02	.292E+01	.357E+02	.163E+01
-.5774	-.5774	.5774	.236E+03	.937E+00	.346E+02	.579E+01	-.153E+01	-.195E+00
.5774	-.5774	.5774	.245E+03	.555E+01	.442E+02	-.531E+00	.836E+01	-.350E+00
-.5774	.5774	.5774	.234E+03	-.214E+00	.327E+02	.109E+01	-.286E+01	.929E-01
.5774	.5774	.5774	.243E+03	.464E+01	.426E+02	.523E+01	.726E+01	-.630E-01

STRAIN OUTPUT IN LOCAL COORDINATES FOR

LAYER 2

RECOVERY POINTS			STRAIN COMPONENTS					
CI	CJ	CK	EXX	EYX	EZZ	EYZ	EXZ	EXY
-1.0000	-1.0000	-1.0000	.513E-02	-.188E-02	-.534E-03	.727E-05	.111E-02	.625E-04
-1.0000	-1.0000	1.0000	.280E-02	-.106E-02	-.534E-03	-.124E-03	.572E-03	.167E-04
-1.0000	1.0000	-1.0000	.515E-02	-.188E-02	-.600E-03	.414E-03	.921E-03	.138E-03
-1.0000	1.0000	1.0000	.277E-02	-.106E-02	-.600E-03	.283E-03	.552E-03	-.655E-04
1.0000	-1.0000	-1.0000	.513E-02	-.183E-02	-.703E-03	-.103E-03	.367E-03	.753E-04
1.0000	-1.0000	1.0000	.280E-02	-.112E-02	-.703E-03	-.130E-03	-.166E-03	-.418E-05
1.0000	1.0000	-1.0000	.515E-02	-.183E-02	-.716E-03	.254E-03	.165E-03	.151E-03
1.0000	1.0000	1.0000	.277E-02	-.112E-02	-.716E-03	.227E-03	-.203E-03	-.864E-04
.0000	.0000	.0000	.396E-02	-.147E-02	-.638E-03	.104E-03	.414E-03	.359E-04
-.5774	-.5774	-.5774	.464E-02	-.170E-02	-.581E-03	.447E-04	.804E-03	.630E-04
.5774	-.5774	-.5774	.464E-02	-.170E-02	-.673E-03	-.126E-04	.376E-03	.662E-04
-.5774	.5774	-.5774	.464E-02	-.170E-02	-.613E-03	.274E-03	.716E-03	.873E-04
.5774	.5774	-.5774	.464E-02	-.168E-02	-.687E-03	.200E-03	.281E-03	.906E-04
-.5774	-.5774	.5774	.329E-02	-.124E-02	-.581E-03	-.181E-04	.517E-03	.132E-04
.5774	-.5774	.5774	.329E-02	-.126E-02	-.673E-03	-.05E-04	.881E-04	.521E-05
-.5774	.5774	.5774	.328E-02	-.124E-02	-.613E-03	.211E-03	.483E-03	-.151E-04
.5774	.5774	.5774	.328E-02	-.126E-02	-.687E-03	.172E-03	.487E-04	-.230E-04

ELEMENT ID 14701

STRESS OUTPUT IN LOCAL COORDINATES FOR

LAYER 1

RECOVERY POINTS			STRESS COMPONENTS					
CI	CJ	CK	SXX	SYX	SZZ	SYZ	SZX	SXY
-1.0000	-1.0000	-1.0000	.205E+03	-.225E+01	.270E+02	.110E+02	.202E+00	-.516E+00
-1.0000	-1.0000	1.0000	.397E+03	.373E+02	.116E+03	.735E+01	.494E+02	.196E+01
-1.0000	1.0000	-1.0000	.209E+03	-.768E+00	.299E+02	-.558E+01	.180E+01	-.153E+01
-1.0000	1.0000	1.0000	.397E+03	.385E+02	.117E+03	-.926E+01	.517E+02	.329E+01
1.0000	-1.0000	-1.0000	.190E+03	-.500E+01	.228E+02	-.247E+01	-.175E+02	-.288E+00
1.0000	-1.0000	1.0000	.380E+03	.709E+01	.488E+02	-.761E+01	.317E+02	.228E+01
1.0000	1.0000	-1.0000	.194E+03	-.278E+01	.268E+02	.123E+02	-.152E+02	-.130E+01
1.0000	1.0000	1.0000	.380E+03	.899E+01	.507E+02	.718E+01	.347E+02	.372E+01
.0000	.0000	.0000	.294E+03	.101E+02	.548E+02	.162E+01	.171E+02	.939E+00
-.5774	-.5774	-.5774	.243E+03	.464E+01	.426E+02	.523E+01	.726E+01	-.630E-01
.5774	-.5774	-.5774	.234E+03	-.214E+00	.327E+02	.109E+01	-.286E+01	.929E-01
-.5774	.5774	-.5774	.245E+03	.555E+01	.442E+02	-.531E+00	.836E+01	-.350E+00
.5774	.5774	-.5774	.236E+03	.937E+00	.346E+02	.579E+01	-.153E+01	-.195E+00
-.5774	-.5774	.5774	.353E+03	.241E+02	.860E+02	.292E+01	.357E+02	.163E+01
.5774	-.5774	.5774	.344E+03	.101E+02	.551E+02	-.171E+01	.256E+02	.185E+01
-.5774	.5774	.5774	.353E+03	.249E+02	.868E+02	-.283E+01	.371E+02	.216E+01
.5774	.5774	.5774	.344E+03	.111E+02	.563E+02	.300E+01	.272E+02	.238E+01

## STRAIN OUTPUT IN LOCAL COORDINATES FOR

LAYER 1

RECOVERY POINTS			STRAIN COMPONENTS					
CI	CJ	CK	EXX	EYY	EZZ	EYZ	EZX	EXY
-1.0000	-1.0000	-1.0000	.277E-02	-.112E-02	-.716E-03	.227E-03	-.203E-03	-.864E-04
-1.0000	-1.0000	1.0000	.515E-02	-.183E-02	-.716E-03	.254E-03	.165E-03	.151E-03
-1.0000	1.0000	-1.0000	.280E-02	-.112E-02	-.703E-03	-.130E-03	-.166E-03	-.418E-05
-1.0000	1.0000	1.0000	.513E-02	-.183E-02	-.703E-03	-.103E-03	.367E-03	.753E-04
1.0000	-1.0000	-1.0000	.277E-02	-.106E-02	-.600E-03	.283E-03	.552E-03	-.655E-04
1.0000	-1.0000	1.0000	.515E-02	-.188E-02	-.600E-03	.414E-03	.921E-03	.138E-03
1.0000	1.0000	-1.0000	.280E-02	-.106E-02	-.534E-03	-.124E-03	.572E-03	.167E-04
1.0000	1.0000	1.0000	.513E-02	-.188E-02	-.534E-03	.727E-05	.111E-02	.625E-04
.0000	.0000	.0000	.396E-02	-.147E-02	-.638E-03	.104E-03	.414E-03	.359E-04
-.5774	-.5774	-.5774	.328E-02	-.126E-02	-.687E-03	.172E-03	.487E-04	-.230E-04
-.5774	-.5774	-.5774	.328E-02	-.124E-02	-.613E-03	.211E-03	.483E-03	-.151E-04
-.5774	.5774	-.5774	.329E-02	-.126E-02	-.673E-03	-.405E-04	.881E-04	.521E-05
.5774	.5774	-.5774	.329E-02	-.124E-02	-.581E-03	-.181E-04	.517E-03	.132E-04
-.5774	-.5774	.5774	.464E-02	-.168E-02	-.687E-03	.200E-03	.281E-03	.906E-04
.5774	-.5774	.5774	.464E-02	-.170E-02	-.613E-03	.274E-03	.716E-03	.873E-04
-.5774	.5774	.5774	.464E-02	-.168E-02	-.673E-03	-.126E-04	.376E-03	.662E-04
.5774	.5774	.5774	.464E-02	-.170E-02	-.581E-03	.447E-04	.804E-03	.630E-04

## STRESS OUTPUT IN LOCAL COORDINATES FOR

LAYER 2

RECOVERY POINTS			STRESS COMPONENTS					
CI	CJ	CK	SXX	SYX	SZZ	SYZ	SZX	SXY
-1.0000	-1.0000	-1.0000	.774E+02	.668E+02	.124E+03	.703E+01	.563E+02	-.180E+01
-1.0000	-1.0000	1.0000	.398E+02	.597E+02	.141E+03	.639E+01	.703E+02	-.227E+01
-1.0000	1.0000	-1.0000	.773E+02	.675E+02	.125E+03	-.958E+01	.581E+02	.140E+01
-1.0000	1.0000	1.0000	.360E+02	.604E+02	.141E+03	-.102E+02	.708E+02	.235E+01
1.0000	-1.0000	-1.0000	.517E+02	.306E+02	.513E+02	-.838E+01	.387E+02	-.116E+01
1.0000	-1.0000	1.0000	.132E+02	.195E+02	.562E+02	-.991E+01	.526E+02	-.162E+01
1.0000	1.0000	-1.0000	.538E+02	.337E+02	.530E+02	.641E+01	.411E+02	.203E+01
1.0000	1.0000	1.0000	.115E+02	.225E+02	.575E+02	.488E+01	.538E+02	.300E+01
.0000	.0000	.0000	.451E+02	.451E+02	.936E+02	-.167E+01	.552E+02	.241E+00
-.5774	-.5774	-.5774	.639E+02	.577E+02	.112E+03	.149E+01	.559E+02	-.102E+01
.5774	-.5774	-.5774	.492E+02	.366E+02	.685E+02	-.368E+01	.458E+02	-.654E+00
-.5774	.5774	-.5774	.637E+02	.584E+02	.112E+03	-.427E+01	.568E+02	.994E+00
.5774	.5774	-.5774	.497E+02	.381E+02	.693E+02	.102E+01	.469E+02	.136E+01
-.5774	-.5774	.5774	.416E+02	.531E+02	.120E+03	.101E+01	.638E+02	-.112E+01
.5774	-.5774	.5774	.267E+02	.307E+02	.727E+02	-.446E+01	.537E+02	-.745E+00
-.5774	.5774	.5774	.401E+02	.538E+02	.120E+03	-.474E+01	.643E+02	.137E+01
.5774	.5774	.5774	.259E+02	.322E+02	.734E+02	.249E+00	.544E+02	.175E+01

## STRAIN OUTPUT IN LOCAL COORDINATES FOR

LAYER 2

RECOVERY POINTS			STRAIN COMPONENTS					
CI	CJ	CK	EXX	EYY	EZZ	EYZ	EZX	EXY
-1.0000	-1.0000	-1.0000	.515E-02	-.183E-02	.310E-01	-.216E-02	.679E-01	.151E-03
-1.0000	-1.0000	1.0000	-.773E-02	-.144E-02	.310E-01	-.207E-02	.554E-01	.621E-03
-1.0000	1.0000	-1.0000	.513E-02	-.183E-02	.312E-01	-.111E-02	.702E-01	.753E-04
-1.0000	1.0000	1.0000	-.896E-02	-.144E-02	.312E-01	-.102E-02	.575E-01	.820E-03
1.0000	-1.0000	-1.0000	.515E-02	-.188E-02	.989E-02	-.262E-02	.461E-01	.138E-03
1.0000	-1.0000	1.0000	-.773E-02	-.131E-02	.989E-02	-.266E-02	.336E-01	-.158E-03
1.0000	1.0000	-1.0000	.513E-02	-.188E-02	.979E-02	-.111E-02	.463E-01	.625E-04
1.0000	1.0000	1.0000	-.896E-02	-.131E-02	.979E-02	-.114E-02	.337E-01	.416E-04
.0000	.0000	.0000	-.160E-02	-.161E-02	.205E-01	-.174E-02	.513E-01	.219E-03
-.5774	-.5774	-.5774	.237E-02	-.175E-02	.266E-01	-.200E-02	.610E-01	.209E-03
.5774	-.5774	-.5774	.237E-02	-.175E-02	.143E-01	-.223E-02	.482E-01	.109E-03
-.5774	-.5774	-.5774	.221E-02	-.175E-02	.267E-01	-.134E-02	.621E-01	.200E-03
.5774	.5774	-.5774	.221E-02	-.175E-02	.143E-01	-.141E-02	.486E-01	.987E-04
-.5774	-.5774	.5774	-.521E-02	-.150E-02	.266E-01	-.197E-02	.538E-01	.421E-03
.5774	-.5774	.5774	-.521E-02	-.145E-02	.143E-01	-.225E-02	.410E-01	.647E-04
-.5774	.5774	.5774	-.577E-02	-.150E-02	.267E-01	-.130E-02	.548E-01	.503E-03
.5774	.5774	.5774	-.577E-02	-.145E-02	.143E-01	-.142E-02	.413E-01	.146E-03

ELEMENT ID 4701

## STRESS OUTPUT IN LOCAL COORDINATES FOR

LAYER 1

RECOVERY POINTS			STRESS COMPONENTS					
CI	CJ	CK	SXX	SYX	SZZ	SYZ	SZX	SXY
-1.0000	-1.0000	-1.0000	.203E+03	-.284E+00	.292E+02	.124E+02	-.946E+00	-.472E+01
-1.0000	-1.0000	1.0000	.372E+03	.338E+02	.104E+03	.140E+02	.474E+02	.912E+01
-1.0000	1.0000	-1.0000	.207E+03	-.271E+01	.249E+02	-.108E+02	-.229E+01	-.261E+01
-1.0000	1.0000	1.0000	.391E+03	.334E+02	.111E+03	-.921E+01	.468E+02	.469E+01
1.0000	-1.0000	-1.0000	.186E+03	.247E+01	.270E+02	.121E+01	-.997E+01	-.369E+01
1.0000	-1.0000	1.0000	.360E+03	-.905E+01	.385E+02	-.281E+01	.384E+02	.885E+01
1.0000	1.0000	-1.0000	.190E+03	.285E+01	.211E+02	.966E+01	-.150E+02	-.158E+01
1.0000	1.0000	1.0000	.378E+03	-.663E+01	.441E+02	.564E+01	.341E+02	.443E+01
.0000	.0000	.0000	.286E+03	.674E+01	.500E+02	.251E+01	.173E+02	.181E+01
-.5774	-.5774	-.5774	.237E+03	.517E+01	.412E+02	.664E+01	.695E+01	-.148E+01
.5774	-.5774	-.5774	.228E+03	.154E+01	.321E+02	.335E+01	.129E+01	-.104E+01
-.5774	.5774	-.5774	.241E+03	.436E+01	.400E+02	-.290E+01	.581E+01	-.106E+01

.5774	.5774	-.5774	.232E+03	.166E+01	.303E+02	.437E+01	-.105E+01	-.623E+00
-.5774	-.5774	.5774	.337E+03	.195E+02	.781E+02	.687E+01	.349E+02	.555E+01
.5774	-.5774	.5774	.329E+03	.701E+00	.479E+02	.172E+01	.293E+02	.556E+01
-.5774	.5774	.5774	.346E+03	.194E+02	.807E+02	-.266E+01	.341E+02	.379E+01
.5774	.5774	.5774	.338E+03	.151E+01	.499E+02	.273E+01	.272E+02	.380E+01

STRAIN OUTPUT IN LOCAL COORDINATES FOR

LAYER 1

RECOVERY POINTS			STRAIN COMPONENTS					
CI	CJ	CK	EXX	EYY	EZZ	EYZ	EZX	EXY
-1.0000	-1.0000	-1.0000	.268E-02	-.110E-02	-.696E-03	.229E-03	-.189E-03	-.268E-03
-1.0000	-1.0000	1.0000	.491E-02	-.178E-02	-.696E-03	.188E-03	.544E-03	.370E-03
-1.0000	1.0000	-1.0000	.277E-02	-.110E-02	-.716E-03	-.113E-03	-.203E-03	-.287E-04
-1.0000	1.0000	1.0000	.515E-02	-.178E-02	-.716E-03	-.153E-03	.165E-03	-.228E-06
1.0000	-1.0000	-1.0000	.268E-02	-.945E-03	-.463E-03	.513E-03	.519E-03	-.212E-03
1.0000	-1.0000	1.0000	.491E-02	-.202E-02	-.463E-03	.240E-03	.125E-02	.519E-03
1.0000	1.0000	-1.0000	.277E-02	-.945E-03	-.600E-03	-.227E-04	.552E-03	.266E-04
1.0000	1.0000	1.0000	.515E-02	-.202E-02	-.600E-03	-.295E-03	.921E-03	.149E-03
.0000	.0000	.0000	.388E-02	-.146E-02	-.619E-03	.732E-04	.445E-03	.693E-04
-.5774	-.5774	-.5774	.318E-02	-.123E-02	-.656E-03	.189E-03	.983E-04	-.938E-04
.5774	-.5774	-.5774	.318E-02	-.119E-02	-.536E-03	.301E-03	.513E-03	-.504E-04
-.5774	.5774	-.5774	.325E-02	-.123E-02	-.682E-03	-.319E-04	.515E-04	-.301E-04
.5774	.5774	-.5774	.325E-02	-.119E-02	-.601E-03	.155E-04	.482E-03	.132E-04
-.5774	-.5774	.5774	.448E-02	-.167E-02	-.656E-03	.137E-03	.477E-03	.211E-03
.5774	-.5774	.5774	.448E-02	-.176E-02	-.536E-03	.172E-03	.892E-03	.286E-03
-.5774	.5774	.5774	.460E-02	-.167E-02	-.682E-03	-.836E-04	.309E-03	.720E-04
.5774	.5774	.5774	.460E-02	-.176E-02	-.601E-03	-.114E-03	.739E-03	.146E-03

STRESS OUTPUT IN LOCAL COORDINATES FOR

LAYER 2

RECOVERY POINTS			STRESS COMPONENTS					
CI	CJ	CK	SXX	SYX	SZZ	SYZ	SZX	SKY
-1.0000	-1.0000	-1.0000	.679E+02	.605E+02	.111E+03	.140E+02	.530E+02	-.618E+00
-1.0000	-1.0000	1.0000	.269E+02	.408E+02	.125E+03	.139E+02	.641E+02	-.626E+00
-1.0000	1.0000	-1.0000	.743E+02	.629E+02	.119E+03	-.926E+01	.538E+02	.254E+01
-1.0000	1.0000	1.0000	.318E+02	.438E+02	.135E+03	-.936E+01	.678E+02	.338E+01
1.0000	-1.0000	-1.0000	.470E+02	.263E+02	.396E+02	-.251E+01	.439E+02	-.197E+01
1.0000	-1.0000	1.0000	.548E+01	.129E+01	.418E+02	-.193E+01	.551E+02	-.235E+01
1.0000	1.0000	-1.0000	.485E+02	.269E+02	.463E+02	.594E+01	.411E+02	.119E+01
1.0000	1.0000	1.0000	.546E+01	.245E+01	.506E+02	.652E+01	.552E+02	.165E+01
.0000	.0000	.0000	.384E+02	.331E+02	.836E+02	.215E+01	.543E+02	.399E+00
-.5774	-.5774	-.5774	.559E+02	.493E+02	.100E+03	.699E+01	.536E+02	-.217E+00
.5774	-.5774	-.5774	.432E+02	.287E+02	.573E+02	.143E+01	.479E+02	-.104E+01
-.5774	.5774	-.5774	.588E+02	.506E+02	.105E+03	-.255E+01	.539E+02	.171E+01
.5774	.5774	-.5774	.444E+02	.294E+02	.616E+02	.245E+01	.471E+02	.884E+00
-.5774	-.5774	.5774	.320E+02	.374E+02	.107E+03	.702E+01	.604E+02	-.164E+00
.5774	-.5774	.5774	.190E+02	.150E+02	.602E+02	.169E+01	.547E+02	-.112E+01
-.5774	.5774	.5774	.344E+02	.388E+02	.113E+03	-.252E+01	.617E+02	.204E+01
.5774	.5774	.5774	.198E+02	.158E+02	.652E+02	.270E+01	.548E+02	.109E+01

STRAIN OUTPUT IN LOCAL COORDINATES FOR

LAYER 2

RECOVERY POINTS			STRAIN COMPONENTS					
CI	CJ	CK	EXX	EYY	EZZ	EYZ	EZX	EXY
-1.0000	-1.0000	-1.0000	.491E-02	-.178E-02	.289E-01	.620E-02	.666E-01	.370E-03
-1.0000	-1.0000	1.0000	-.713E-02	-.507E-02	.289E-01	.698E-02	.537E-01	.105E-02
-1.0000	1.0000	-1.0000	.515E-02	-.178E-02	.310E-01	-.257E-02	.679E-01	-.228E-06
-1.0000	1.0000	1.0000	-.773E-02	-.507E-02	.310E-01	-.178E-02	.554E-01	.254E-03
1.0000	-1.0000	-1.0000	.491E-02	-.202E-02	.710E-02	.616E-02	.462E-01	.519E-03
1.0000	-1.0000	1.0000	-.713E-02	-.558E-02	.710E-02	.721E-02	.333E-01	.673E-03
1.0000	1.0000	-1.0000	.515E-02	-.202E-02	.989E-02	-.333E-02	.461E-01	.149E-03
1.0000	1.0000	1.0000	-.773E-02	-.558E-02	.989E-02	-.229E-02	.336E-01	-.126E-03
.0000	.0000	.0000	-.120E-02	-.361E-02	.192E-01	.207E-02	.504E-01	.362E-03
-.5774	-.5774	-.5774	.238E-02	-.254E-02	.248E-01	.448E-02	.598E-01	.425E-03
.5774	-.5774	-.5774	.238E-02	-.271E-02	.123E-01	.440E-02	.479E-01	.446E-03
-.5774	.5774	-.5774	.241E-02	-.254E-02	.261E-01	-.667E-03	.604E-01	.159E-03
.5774	.5774	-.5774	.241E-02	-.271E-02	.138E-01	-.988E-03	.480E-01	.180E-03
-.5774	-.5774	.5774	-.467E-02	-.447E-02	.248E-01	.497E-02	.524E-01	.703E-03
.5774	-.5774	.5774	-.467E-02	-.473E-02	.123E-01	.498E-02	.405E-01	.548E-03
-.5774	.5774	.5774	-.492E-02	-.447E-02	.261E-01	-.181E-03	.532E-01	.294E-03
.5774	.5774	.5774	-.492E-02	-.473E-02	.138E-01	-.416E-03	.407E-01	.138E-03

INCREMENT 1 SUMMARY

TIME INCREMENT COMPLETED 2.220E-16, FRACTION OF STEP COMPLETED 1.00  
 STEP TIME COMPLETED 2.220E-16, TOTAL TIME COMPLETED .000E+00

ELEMENT OUTPUT

THE FOLLOWING TABLE IS PRINTED FOR ELSET ONE AND ELEMENT TYPE C3D8 AT THE INTEGRATION

ELEMENT PT FOOT- MISES  
 NOTE

1	1	71.96
	2	83.45
	3	65.04
	4	77.85
1	5	71.94
1	6	83.43
1	7	65.02
1	8	77.83

MAXIMUM ELEMENT 83.45  
1

MINIMUM ELEMENT 65.02  
1

N O D E O U T P U T

THE FOLLOWING TABLE IS PRINTED FOR ALL NODES

NODE FOOT-NOTE	U1	U2	U3
3	5.1742E-04	3.1125E-04	.0000E+00
5	9.9136E-04	2.9044E-04	.0000E+00
7	1.5074E-03	2.9095E-04	.0000E+00
9	2.0354E-03	2.8574E-04	.0000E+00
11	2.5695E-03	2.8841E-04	.0000E+00
13	3.1237E-03	2.9103E-04	.0000E+00
15	3.7166E-03	2.9437E-04	.0000E+00
17	4.3891E-03	3.1594E-04	.0000E+00
19	5.1916E-03	3.4756E-04	.0000E+00
21	6.2068E-03	5.1528E-04	.0000E+00
.	.	.	.
59385	.2942	-3.1596E-04	.0000E+00
59387	.2949	-2.9427E-04	.0000E+00
59389	.2955	-2.9117E-04	.0000E+00
59391	.2960	-2.8800E-04	.0000E+00
59393	.2966	-2.8749E-04	.0000E+00
59395	.2971	-2.8704E-04	.0000E+00
59397	.2976	-2.8693E-04	.0000E+00
59399	.2981	-2.8668E-04	.0000E+00
59401	.2987	-2.8657E-04	.0000E+00
MAXIMUM AT NODE	.2987 5401	1.4847E-03 4800	.8600 1225
MINIMUM AT NODE	.0000E+00 1	-1.4846E-03 54702	-.8600 58177

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