Algebra I Unit 2 Linear Relationships

Chapter 6 Systems of Linear Equations and Inequalities

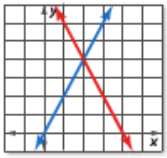
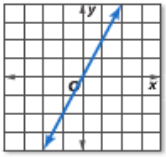
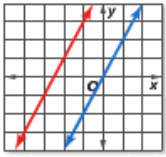
Lesson 6-1 Graphing Systems of Equations

Objectives: I can determine the number of solutions a system of linear equations has, if any.

I can solve systems of linear equations by graphing.

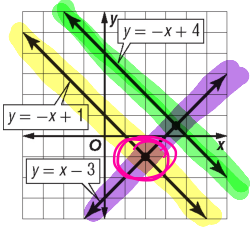
CCSS: A.CED.3, A.REI.6, MP.3, MP.8

ConceptSummary Possible Solutions

Number of Solutions	exactly one	infinite	no solution
Terminology	consistent and independent	consistent and dependent	inconsistent
Graph			

Example 1: Number of Solutions

Use the graph to determine whether each system is *consistent* or *inconsistent* and if it is *independent* or *dependent*.



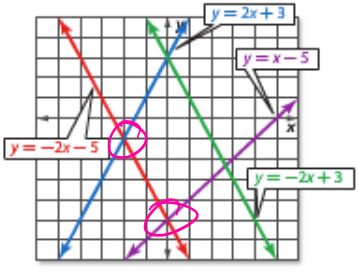
a. $y = -x + 1$, $y = -x + 4$ → inconsistent

b. $y = x - 3$, $y = -x + 1$

Consistent independent

Guided Practice 1: Number of Solutions

Use the graph to determine whether each system is *consistent* or *inconsistent* and if it is *independent* or *dependent*.



a. $\begin{cases} y = 2x + 3 \\ y = -2x - 5 \end{cases}$ Consistent Indepedent

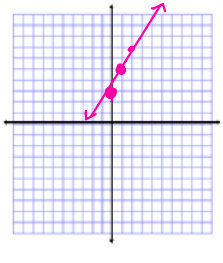
b. $\begin{cases} y = x - 5 \\ y = -2x - 5 \end{cases}$ Consistent independent

Example 2: Solve by Graphing

Graph each system and determine the number of solutions that it has. If it has one solution, name it.

a. $\begin{cases} y = -2x + 3 \\ 8x - 4y = -12 \end{cases}$ (0,3)

infinitely ∞ly many



x	y
0	$8(0) - 4y = -12$ $\frac{-4y}{-4} = \frac{-12}{-4}$ $y = 3$ (0,3)
1	$8(1) - 4y = -12$ $\frac{-4y}{-4} = \frac{-20}{-4}$ $y = 5$ (1,5)

NO solution

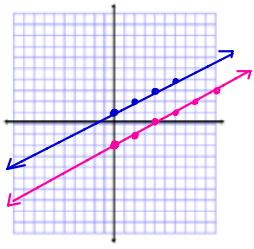
b. $\begin{cases} x - 2y = 4 \\ x - 2y = -2 \end{cases}$

$-x \quad -x$

$\frac{-2y}{-2} = \frac{-x - 2}{-2} \quad \frac{-2y}{-2} = \frac{-x + 4}{-2}$

$y = \frac{1}{2}x - 2 \quad y = \frac{1}{2}x + 1$

rise 1, run 2

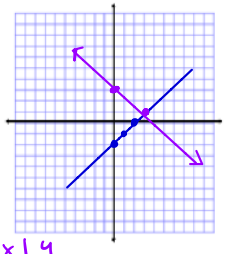


Guided Practice 2: Solve by Graphing

Graph each system and determine the number of solutions that it has. If it has one solution, name it.

a. $\begin{cases} x - y = 2 \\ 3y + 2x = 9 \end{cases}$ $y = x - 2$ $y = \frac{-2}{3}x + 3$ 1 solution (3, 1)

x	y
0	-2
1	-1
2	0
3	1



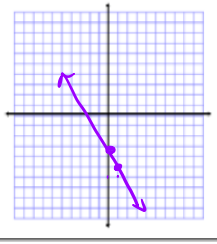
x	y
0	3
3	1
6	-1

b. $\begin{cases} y = -2x - 3 \\ 6x + 3y = -9 \end{cases}$ $y = -2x - 3$

Pg 338 14, 15, 19-31 odd, 42

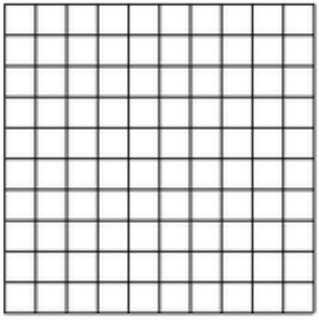
infinitely many solution

x	y
0	-3
1	-5
2	-7



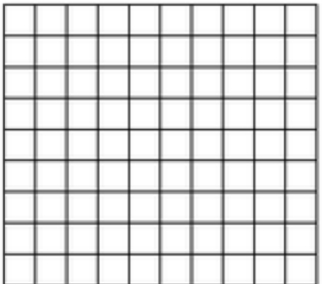
Real-World Example 3: Write and Solve a System of Equations

Alex rode 20 miles last week and plans to ride 35 miles per week. David rode 50 miles last week and plans to ride 25 miles per week. Predict the week in which Alex and David will have ridden the same number of miles.



Guided Practice 3: Write and Solve a System of Equations

Joe and Josh want to buy a video game. Joe has $14 and saves $10 a week. Josh has $26 and saves $7 a week. In how many weeks will they have the same amount?



Lesson 6-2 Substitution

Objectives: I can solve systems of equations by using substitution.
I can solve real-world problems involving systems of equations by using substitution.

CCSS: A.CED.3, A.REI.6, MP.2

KeyConcept Solving by Substitution

Step 1 When necessary, solve at least one equation for one variable.

Step 2 Substitute the resulting expression from Step 1 into the other equation to replace the variable. Then solve the equation.

Step 3 Substitute the value from Step 2 into either equation, and solve for the other variable. Write the solution as an ordered pair.

Example 1: Solve a system by Substitution

Use substitution to solve the system of equations.

$\begin{cases} y = -4x + 12 \\ 2x + y = 2 \end{cases}$

$2x + (-4x + 12) = 2$

$-2x + 12 = 2$

$-12 \quad -12$

$\frac{-2x}{-2} = \frac{-10}{-2}$

$x = 5$

$y = -4(5) + 12$

$y = -20 + 12$

$y = -8$

Guided Practice 1: Solve a system by Substitution

Use substitution to solve the system of equations.

$\begin{cases} 2x + 5y = -1 \\ y = 3x + 10 \end{cases}$

$2x + 5(3x + 10) = -1$

$2x + 15x + 50 = -1$

$17x + 50 = -1$

$-50 \quad -50$

$\frac{17x}{17} = \frac{-51}{17}$

$x = -3$

$y = 3(-3) + 10$

$y = -9 + 10$

$y = 1$

Example 2: Solve and Then Substitute

Use substitution to solve the system of equations.

$$\begin{cases} x - 2y = -3 \\ 3x + 5y = 24 \end{cases}$$

$x = 2y - 3$

$3(2y - 3) + 5y = 24$

$6y - 9 + 5y = 24$

$11y - 9 = 24$

$+9 \quad +9$

$11y = 33$

$y = 3$

$x = 2(3) - 3$

$x = 6 - 3$

$x = 3$

Guided Practice 2: Solve and Then Substitute

Use substitution to solve the system of equations.

$$\begin{cases} 4x + 5y = 11 \\ y - 3x = -13 \end{cases}$$

$+3x \quad +3x$

$y = 3x - 13$

$4x + 5(3x - 13) = 11$

$4x + 15x - 65 = 11$

$y = -1$

$19x - 65 = 11$

$+65 \quad +65$

$\frac{19x}{19} = \frac{76}{19}$

$x = 4$

$y = 3(4) - 13$

$12 - 13$

Example 3: No Solution or Infinitely Many Solutions

Use substitution to solve the system of equations.

$$\begin{cases} 2x + 2y = 8 \\ x + y = -2 \end{cases} \cdot 2$$

$2x + 2y = 8$

$2x + 2y = -4$

$8 \neq -4$

No solution

Guided Practice 3: No Solution or Infinitely Many Solutions

Use substitution to solve the system of equations.

a. $\begin{cases} 2x - y = 8 \\ y = 2x - 3 \end{cases}$

$2x - (2x - 3) = 8$

$2x - 2x + 3 \neq 8$

No solution

b. $\begin{cases} 4x - 3y = 1 \\ 6y - 8x = -2 \end{cases} \cdot -2$

$6y - 8x = -2$

#

∞ly many

Real-World Example 4: Write and Solve a System of Equations

A nature center charges $35.25 for a yearly membership and $6.25 for a single admission. Last week it sold a combined total of 50 yearly memberships and single admissions for $660.50. How many memberships and how many single admissions were sold?

x = single admission

y = yearly membership

$x + y = 50$

$35.25y + 6.25x = 660.50$

$x = 50 - y$

$35.25y + 6.25(50 - y) = 660.50$

$x + y = 50$

$12 + x = 50$

$-12 \quad -12$

$x = 38$

Guided Practice 4: Write and Solve a System of Equations

As of 2009, the New York Yankees and the Cincinnati Reds together had won a total of 32 World Series. The Yankees had won 5.4 times as many as the Reds. How many World Series had each team won?

x = Red's world series

y = Yankee's world series

$x + y = 32$

$y = 5.4x$

$x + 5.4x = 32$

$6.4x = 32$

$x = 5$

$y = 27$

$35.25y + 312.50 - 6.25y = 660.5$

$29y + 312.5 = 660.5$

$\frac{29y}{29} = \frac{348}{29}$

$y = 12$

Lesson 6-3 Elimination Using Addition and Subtraction

Objectives: I can solve systems of equations by using elimination with addition.

I can solve systems of equations by using elimination with subtraction

CCSS: A.CED.2, A.REI.6, MP.7

KeyConcept Solving by Elimination

Step 1 Write the system so like terms with the same or opposite coefficients are aligned.

Step 2 Add or subtract the equations, eliminating one variable. Then solve the equation.

Step 3 Substitute the value from Step 2 into one of the equations and solve for the other variable. Write the solution as an ordered pair.

Example 1: Elimination Using Addition
Use elimination to solve the system of equations.

$$\begin{cases} 3x+4y=12 \\ 3x-6y=18 \end{cases}$$

$-2y = 30$, $\frac{-2y}{-2} = \frac{30}{-2}$, $y = -15$

$3x - 6(-15) = 18$

$3x + 90 = 18$

$-90 \quad -90$

$\frac{3x}{3} = \frac{-72}{3}$

$x = -24$

Guided Practice 1: Elimination Using Addition
Use elimination to solve the system of equations.

$$\begin{cases} 4y+3x=22 \\ 3x-4y=14 \end{cases}$$

$\frac{6x}{6} = \frac{36}{6}$, $x = 6$

$4y + 3(6) = 22$

$4y + 18 = 22$

$-18 \quad -18$

$\frac{4y}{4} = \frac{4}{4}$, $y = 1$

Example 2: Write and Solve a System of Equations
Four times one number minus three times another number is 12. Two times the first number added to three times the second number is 6. Find the numbers.

$4x - 3y = 12$

$2x + 3y = 6$

$\frac{6x}{6} = \frac{18}{6}$, $x = 3$, $y = 0$

$2(3) + 3y = 6$

$6 + 3y = 6$

$-6 \quad -6$

$3y = 0$

Guided Practice 2: Write and Solve a System of Equations
The sum of two numbers is –10. Negative three times the first number minus the second number equals 2. Find the numbers.

$x + y = -10$

$-3x - y = 2$

$\frac{-2x}{-2} = \frac{-8}{-2}$, $x = 4$

$4 + y = -10$

$-4 \quad -4$

$y = -14$

Standardized Test Example 3:
Use elimination to solve the system of equations.

$\begin{cases} 4x+2y=28 \\ 4x-3y=18 \end{cases}$ $\cdot(-1)$

$4x+2y=28$
$-4x+3y=-18$
$5y=10$
$y=2$
$x=6$

$4x+2(2)=28$
$4x+4=28$
$4x=24$
$\frac{4x}{4}=\frac{24}{4}$

A. (6, −2) **B.** (6, 2) **C.** (−6, −2) **D.** (−6, 2)

Guided Practice 3: Use elimination to solve the system of equations.

$\begin{cases} 8b+3c=11 \\ 8b+7c=7 \end{cases}$ $\cdot(-1)$

$8b+3c=11$
$-8b-7c=-7$
$-4c=4$
$\frac{-4c}{-4}=\frac{4}{-4}$
$c=-1$

$8b+3(-1)=11$
$8b-3=11$
$+3 \quad +3$
$\frac{8b}{8}=\frac{14}{8}$
$b=1\frac{3}{4}$

F. (1.5, −1) **G.** (1.75, −1) **H.** (1.75, 1) **J.** (1.5, 1)

Real-World Example 4: Write and Solve a System of Equations

A hardware store earned $956.50 from renting ladders and power tools last week. The store charged customers for a total of 36 days for ladders and 85 days for power tools. This week the store charged 36 days for ladders, 70 days for power tools, and earned $829. How much does the store charge per day for ladders and for power tools?

L = ladder price
P = power tool price

$956.50=36L+85P$
$-1(829=36L+70P)$

$956.5=36L+85P$
$-829=-36L-70P$
$127.5=15P$
$\frac{127.5}{15}=\frac{15P}{15}$
$P=\$8.50$

$829=36L+70(8.5)$
$829=36L+595$
$36L=234$
$L=\$6.50$

Guided Practice 4: Write and Solve a System of Equations

Tamera and Addie are throwing a birthday party for their friend. Tamera invited 5 fewer friends than Addie. Together they invited 47 guests. How many guests did each girl invite?

A = Addie's friends
T = Tamera's friends

$A+T=47$
$A-T=5$
$2A=52$
$\frac{2A}{2}=\frac{52}{2}$
$A=26$

$26-T=5$
$-26 \quad -26$
$-T=-21$
$T=21$

Lesson 6-4 Elimination Using Multiplication

Objectives: I can solve systems of equations by using elimination with multiplication.

I can solve real-world problems involving systems of equations.

CCSS: A.REI.5, A.REI.6, MP.1

KeyConcept Solving by Elimination Using Multiplication

Step 1 Multiply at least one equation by a constant to get two equations that contain opposite terms.

Step 2 Add the equations, eliminating one variable. Then solve the equation.

Step 3 Substitute the value from Step 2 into one of the equations and solve for the other variable. Write the solution as an ordered pair.

Example 1: Multiply One Equation to Eliminate a Variable

Use elimination to solve the system of equations.

$2x + y = 23$ $\quad -2$

$3x + 2y = 37$

$-4x - 2y = -46$

$3x + 2y = 37$

$-x = -9$

$x = 9$

$2(9) + y = 23$

$18 + y = 23$

$-18 \quad -18$

$y = 5$

Guided Practice 1: Multiply One Equation to Eliminate a Variable

Use elimination to solve the system of equations.

$6r + q = 13$ $\quad -2$

$3r + 2q = -4$

$-12r - 2q = -26$

$3r + 2q = -4$

$-15r = -30$

$-15 \quad -15$

$r = 2$

$3(2) + 2q = -4$

$6 + 2q = -4$

$-6 \quad -6$

$2q = -10$

$q = -5$

Example 2: Multiply Both Equations to Eliminate a Variable

Use elimination to solve the system of equations.

$4x + 3y = 8$ $\quad 5$

$3x - 5y = -23$ $\quad 3$

$20x + 15y = 40$

$9x - 15y = -69$

$29x = -29$

$29 \quad 29$

$x = -1$

$4(-1) + 3y = 8$

$-4 + 3y = 8$

$+4 \quad +4$

$3y = 12$

$3 \quad 3$

$y = 4$

Guided Practice 2: Multiply Both Equations to Eliminate a Variable

Use elimination to solve the system of equations.

$(5x - 3y = 6)\,5$

$(2x + 5y = -10)\,3$

$25x - 15y = 30$

$6x + 15y = -30$

$31x = 0$

$x = 0$

$5(0) - 3y = 6$

$-3y = 6$

$y = -2$

D=rt

Real-World Example 3: Solve a System of Equations

A fishing boat travels 10 miles down-stream in 30 minutes. The return trip takes the boat 40 minutes. Find the rate in miles per hour of the boat in still water.

x = Boat Speed

y = current Speed

$(20 = x + y)\,2$

$30 = 2x - 2y$

$(10 = \frac{1}{2}x + \frac{1}{2}y)\,2$

$(10 = \frac{2}{3}x - \frac{2}{3}y)\,3$

$40 = 2x + 2y$

$30 = 2x - 2y$

$70 = 4x$

$x = 17.5$ mph

Guided Practice 3: Solve a System of Equations

D=RT

A canoeist travels 4 miles downstream in 1 hour. The return trip takes the canoeist 1.5 hours. Find the rate of the boat in still water.

x = ~~Boat~~ canoe speed

y = current speed

$(4 = 1x + 1y)\,1.5$

$4 = 1.5x - 1.5y$

$6 = 1.5x + 1.5y$

$4 = 1.5x - 1.5y$

$10 = 3x$

$x = 3\frac{1}{3}$ mph

Lesson 6-5 Applying Systems of Linear Equations

Objectives: I can determine the best method for solving systems of equations.

I can apply systems of equations.

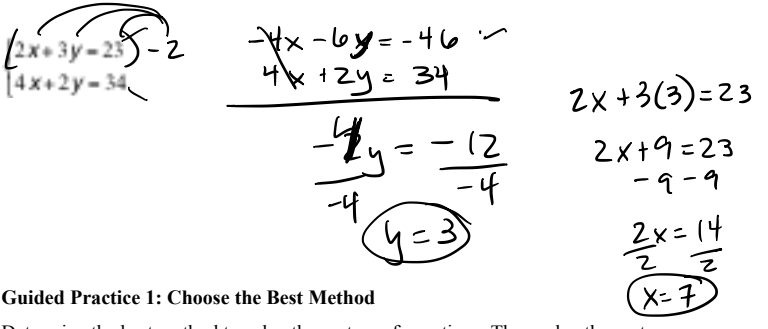
CCSS: A.REI.6, MP.2, MP.4

ConceptSummary Solving Systems of Equations

Method	The Best Time to Use
Graphing	To estimate solutions, since graphing usually does not give an exact solution.
Substitution	If one of the variables in either equation has a coefficient of 1 or −1.
Elimination Using Addition	If one of the variables has opposite coefficients in the two equations.
Elimination Using Subtraction	If one of the variables has the same coefficient in the two equations.
Elimination Using Multiplication	If none of the coefficients are 1 or −1 and neither of the variables can be eliminated by simply adding or subtracting the equations.

Example 1: Choose the Best Method

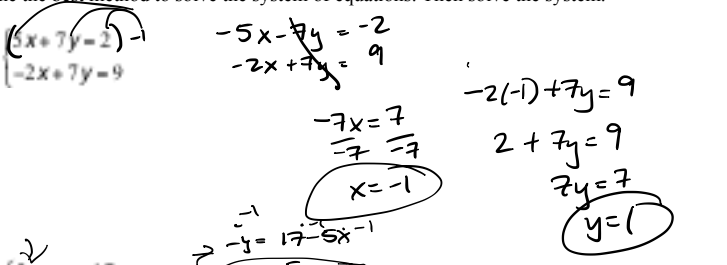
Determine the best method to solve the system of equations. Then solve the system.



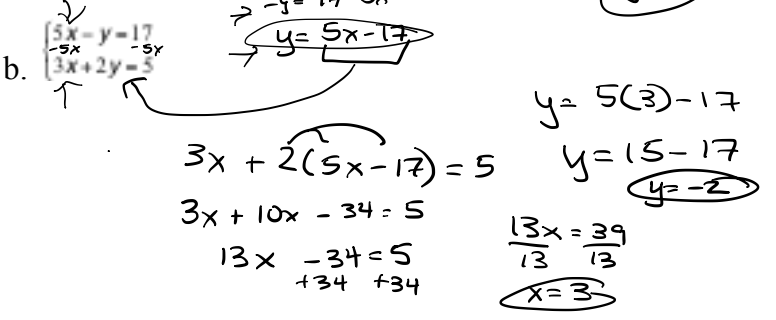
Guided Practice 1: Choose the Best Method

Determine the best method to solve the system of equations. Then solve the system.

a.



b.



Real-World Example 2: Apply Systems of Linear Equations

Ace Car Rental rents a car for $45 and $0.25 per mile. Star Car Rental rents a car for $35 and $0.30 per mile. How many miles would a driver need to drive before the cost of renting a car at Ace Car Rental and renting a car at Star Car Rental were the same?

x = miles
y = Cost

$y = .25x + 45$
$y = .30x + 35$

$.25x + 45 = .30x + 35$
$-.25x \quad -35 \quad -.25x - 35$

$\frac{10}{.05} = \frac{.05x}{.05}$

$x = 200$ miles

Guided Practice 2: Apply Systems of Linear Equations

Jared has volunteered 50 hours and plans to volunteer 3 hours in each coming week. Maddie is a new volunteer who plans to volunteer 5 hours each week. Write and solve a system of equations to find how long it will be before they will have volunteered the same number of hours.

T = total hours
W = weeks

$T = 3W + 50$
$T = 5W$

$2W = 50$
$W = 25$ weeks

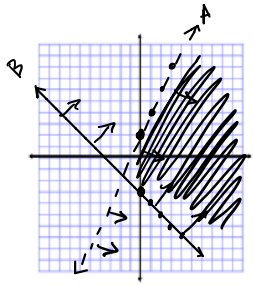
$5W = 3W + 50$
$-3W \quad -3W$

Lesson 6-6 Systems of Inequalities

Objectives: I can solve systems of linear inequalities by graphing.

I can apply systems of linear inequalities.

CCSS: A.REI.12, MP.1, MP.6



① line
② Dashed/solid (< >) (≤ ≥)
③ test pt
④ shading

Example 1: Solve by graphing

Solve the system of inequalities by graphing.

$\begin{cases} y < 2x + 2 & A \\ y \geq -x - 3 & B \end{cases}$

test pt (0,0)

$0 < 2(0) + 2$
$0 < 2$ True!

$0 \geq -0 - 3$
$0 \geq -3$ True!

Guided Practice 1: Solve by graphing

Solve the system of inequalities by graphing.

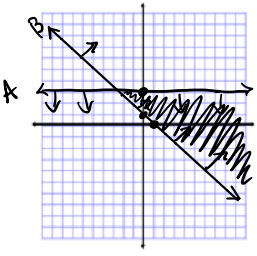
a. $\begin{cases} y \le 3 & A \\ x + y \ge 1 & B \end{cases}$

x	y
0	1
1	0

Test Pt (0,0)

$0 \le 3$ True

$0 + 0 \ge 1$ False



Test pt (0,0)

$2(0) + 0 \ge -2$

$0 \ge -2$ True!

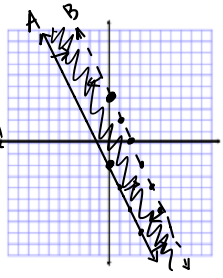
$2(0) + 0 < 4$

$0 < 4$ true!

b. $\begin{cases} 2x + y \ge -2 & A \\ 2x + y < 4 & B \end{cases}$ $\quad -2x \quad -2x$

$y \ge -2x - 2$ A

$y < -2x + 4$ B



Example 2: No Solution

Solve the system of inequalities by graphing.

$\begin{cases} y \ge -3x + 1 & A \\ y \le -3x - 2 & B \end{cases}$

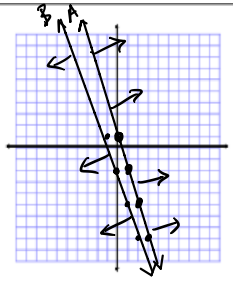
Test Pt (0,0)

$0 \ge -3(0) + 1$

$0 \ge 1$ False!

$0 \le -3(0) - 2$

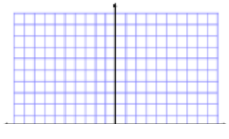
$0 \le -2$ False!

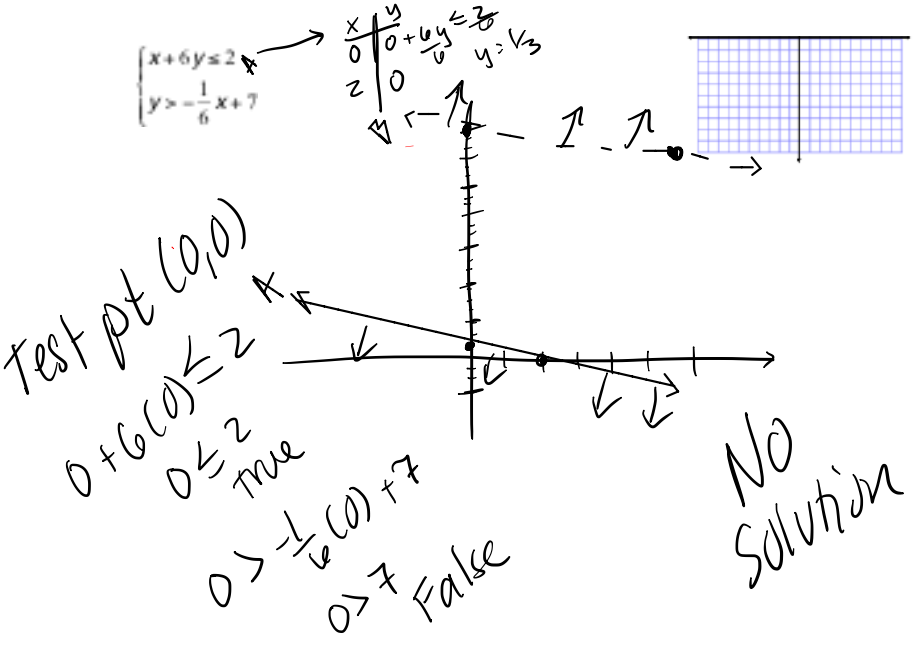


NO Solution

Guided Practice 2: No Solution

Solve the system of inequalities by graphing.

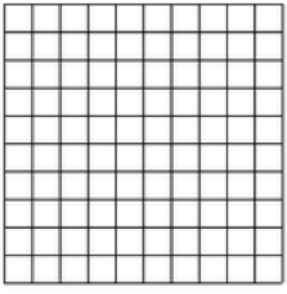




Real-World Example 3: Whole-Number Solutions

A college service organization requires that its members maintain at least a 3.0 grade point average and volunteer at least 10 hours a week.

a. Define the variables and write a system of inequalities to represent this situation. Then graph the system.



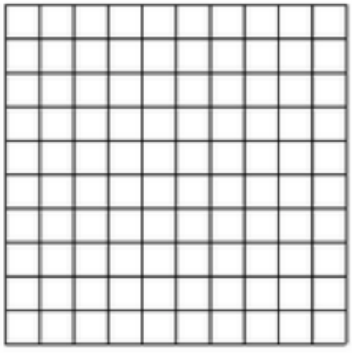
b. Name one possible solution.

(4, 11)

Guided Practice 3: Whole-Number Solutions

The Theater Club is selling shirts. They have only enough supplies to print 120 shirts. They will sell sweatshirts for $22 and T-shirts for $15, with a goal of at least $2000 in sales.

a. Define the variables, and write a system of inequalities to represent this situation. Then graph the system.



b. Name one possible solution.

c. Is (45,30) a solution? Explain.