2. Futures and Forward Markets
2.1. Institutions

1. (Hull 2.3) Suppose that you enter into a short futures contract to sell July silver for \$5.20 per ounce on the New York Commodity Exchange. The size of the contract is 5,000 ounces. The initial margin is \$4,000 and the maintenance margin is \$3,000. What change in the futures price will lead to a margin call?

$$\begin{aligned} 5,000(\$5.20 - F) &= -\$1,000 \\ 26,000 - 5,000F &= -1,000 \\ F &= \tfrac{27,000}{5,000} \\ &= 5.40 \end{aligned}$$

i.e., a price change of $+\$0.20$ *per ounce.*

What happens if you do not meet the margin call?

If you don't meet the margin call, your position gets liquidated.

2. Futures and Forward Markets
2.1. Institutions

2. (Hull 2.11) An investor enters into two long futures contracts on frozen orange juice. Each contract is for the delivery of 15,000 pounds. The current futures price is 160 cents per pound, the initial margin is $6,000 per contract, and the maintenance margin is $4,500 per contract. What price change would lead to a margin call?

There is a margin call if $1,500 is lost on one contract.

$$\begin{aligned} 15,000(F - \$1.60) &= -\$1,500 \\ 15,000F - 24,000 &= -1,500 \\ F &= \tfrac{22,500}{15,000} \\ &= 1.50 \end{aligned}$$

This happens if the futures price falls to $1.50 per pound.

Under what circumstances could $2,000 be withdrawn from the margin account?

$2,000 can be withdrawn from the margin account if the value of one contract rises by $1,000.

$$\begin{aligned} 15,000(F - \$1.60) &= \$1,000 \\ 15,000F - 24,000 &= 1,000 \\ F &= \tfrac{25,000}{15,000} \\ &= 1.6667 \end{aligned}$$

This happens if the futures price rises to $1.6667 per pound.

2. Futures and Forward Markets

2.1. Institutions

3. (Hull 2.15) At the end of one day a clearinghouse member is long 100 contracts, and the settlement price is $50,000 per contract. The original margin is $2,000 per contract. On the following day the member becomes responsible for clearing an additional 20 long contracts, entered into at a price of $51,000 per contract. The settlement price at the end of this day is $50,200. How much does the member have to add to its margin account with the exchange clearinghouse?

$36,000.

From the clearinghouse to the member:
100 old contracts with ($50,200-$50,000) settlement each.

From the member to the clearinghouse:
20 new contracts with $2,000 original margin each.

From the member to the clearinghouse:
20 new contracts with ($50,200-$51,000) settlement each.

2. Futures and Forward Markets

2.1. Institutions

4. (Baby Hull 2.24, Papa Hull 2.26) A company enters into a short futures contract to sell 5,000 bushels of wheat for 250 cents per bushel. The initial margin is \$3,000 and the maintenance margin is \$2,000. What price change would lead to a margin call?

There is a margin call if \$1,000 is lost on the contract.

$$
\begin{aligned}
5,000(-F + \$2.50) &= -\$1,000 \\
5,000F - 12,500 &= 1,000 \\
F &= \tfrac{13,500}{5,000} \\
&= 2.70
\end{aligned}
$$

This will happen if the futures price rises to \$2.70 per bushel.

Under what circumstances could \$1,500 be withdrawn from the margin account?

$$
\begin{aligned}
5,000(-F + \$2.50) &= \$1,500 \\
5,000F - 12,500 &= -1,500 \\
F &= \tfrac{11,000}{5,000} \\
&= 2.20
\end{aligned}
$$

\$1,500 can be withdrawn if the futures price falls to \$2.20 per bushel.

2. Futures and Forward Markets
2.2. Pricing

1. (Baby Hull 5.9, Papa Hull 3.11) A one-year long forward contract on a nondividend-paying stock is entered into when the stock price is \$40 and the risk-free rate of interest is 10 percent per annum with continuous compounding.

(a) What are the forward price and the initial value of the contract?

$$\begin{aligned} F &= Se^{rT} \\ &= 40e^{0.1\frac{12}{12}} \\ &= 44.21 \end{aligned}$$

$$\begin{aligned} f &= (F-K)e^{-rT} \\ &= 0 \end{aligned}$$

(b) Six months later, the price of the stock is \$45 and the risk-free interest rate is still 10 percent. What are the forward price and the value of the forward contract?

$$\begin{aligned} F &= Se^{rT} \\ &= 45e^{0.1\frac{6}{12}} \\ &= 47.31 \end{aligned}$$

$$\begin{aligned} f &= (F-K)e^{-rT} \\ &= (47.31-44.21)e^{-0.1\frac{6}{12}} \\ &= 2.95 \end{aligned}$$

2. Futures and Forward Markets
2.2. Pricing

2. (Baby Hull 5.11, Papa Hull 3.13) Assume that the risk-free interest rate is 9 percent per annum with continuous compounding and that the dividend yield on a stock index varies throughout the year. In February, May, August, and November, it is 5 percent per annum. In other months, it is 2 percent per annum. Suppose that the value of the index on July 31 is 300. What is the futures price for a contract deliverable on December 31?

average dividend yield $\frac{(5+2+2+5+2)}{5} = 3.2\%$

$$\begin{aligned} F = & \quad Se^{(r-q)T} \\ = & \quad 300e^{(0.09-0.032)\frac{5}{12}} \\ = & \quad 307.34 \end{aligned}$$

2. Futures and Forward Markets
2.2. Pricing

3. (Baby Hull 5.13, Papa Hull 3.15) Estimate the difference between short-term interest rates in Mexico and the United States on February 4, 2004 from the following information:

delivery	settle (\$/Peso)
Mar	.08920
June	.08812

$$\begin{aligned}
F &= Se^{(r-r_f)T} \\
\frac{\partial F}{\partial T} &= Se^{(r-r_f)T}(r-r_f) \\
&= F(r-r_f) \\
\frac{0.08812-0.08920}{0.25} &= 0.0.08920(r-r_f) \\
(r-r_f) &= -0.0484
\end{aligned}$$

2. Futures and Forward Markets
2.2. Pricing

4. (Baby Hull 5.23, Papa Hull 3.24) A stock is expected to pay a dividend of \$1 per share in two months and in five months. The stock price is \$50 and the risk-free rate of interest is 8 percent per annum with continuous compounding for all maturities. An investor has just taken a short position in a six-month forward contract on the stock.
(a) What are the forward price and the initial value of the forward contract?

$$\begin{aligned} I = &\ 1e^{-0.08\frac{2}{12}} + 1e^{-0.08\frac{5}{12}} \\ = &\ 0.9868 + 0.9672 \\ = &\ 1.9540 \end{aligned}$$

$$\begin{aligned} F = &\ (S-I)e^{rT} \\ = &\ (50 - 1.9540)e^{0.08\frac{6}{12}} \\ = &\ 50.01 \end{aligned}$$

$$\begin{aligned} f = &\ (K-F)e^{-rT} \\ = &\ 0 \end{aligned}$$

(b) Three months later, the price of the stock is \$48 and the risk-free rate of interest is still 8 percent per annum. What are the forward price and the value of the short position in the forward contract?

$$\begin{aligned} I = &\ 1e^{-0.08\frac{2}{12}} \\ = &\ 0.9868 \end{aligned}$$

$$\begin{aligned} F = &\ (S-I)e^{rT} \\ = &\ (48 - 0.9868)e^{0.08\frac{3}{12}} \\ = &\ 47.96 \end{aligned}$$

2. Futures and Forward Markets
2.2. Pricing

4. (b) (Continued)

$$\begin{aligned} f = & (K - F)e^{-rT} \\ = & (50.01 - 47.96)e^{-0.08\frac{3}{12}} \\ = & 2.01 \end{aligned}$$

2. Futures and Forward Markets
2.2. Pricing

5. (Baby Hull 5.25, Papa Hull 3.26) A company that is uncertain about the exact date when it will pay or receive a foreign currency may try to negotiate with its bank a forward contract that specifies a period during which delivery can be made. The company wants to reserve the right to choose the exact delivery date to fit in with its own cash flows. Put yourself in the position of the bank. How would you price the product that the company wants?

It is likely that the bank will price the product on assumption that the company chooses the delivery date least favorable to the bank. If the foreign interest rate is higher than the domestic interest rate then,

1. *The earliest delivery date will be assumed when the company has a long position.*
2. *The latest delivery date will be assumed when the company has a short position.*

If the foreign interest rate is lower than the domestic interest rate then,

1. *The latest delivery date will be assumed when the company has a long position.*
2. *The earliest delivery date will be assumed when the company has a short position.*

2. Futures and Forward Markets
2.3. Hedging Strategies

1. (Baby Hull 3.16, Papa Hull 4.16) The standard deviation of monthly changes in the spot price of live cattle is 1.2 (in cents per pound). The standard deviation of monthly changes in the futures price of live cattle for the closest contract is 1.4. The correlation between the futures price changes and the spot price changes is 0.7. It is now October 15. A beef producer is committed to purchasing 200,000 pounds of live cattle on November 15. The producer wants to use the December live-cattle futures contracts to hedge its risk. Each contract is for the delivery of 40,000 pounds of cattle. What strategy should the beef producer follow?

$$\begin{aligned} h = & \quad \rho\frac{\sigma_S}{\sigma_F} \\ = & \quad 0.7\frac{1.2}{1.4} \\ = & \quad 0.6 \end{aligned}$$

$$\begin{aligned} N = & \quad \frac{hN_S}{Q_F} \\ = & \quad \frac{0.6200{,}000}{40{,}000} \\ = & \quad 3 \end{aligned}$$

The beef producer should long 3 contracts.

2. Futures and Forward Markets
2.3. Hedging Strategies

2. (Baby Hull 3.21, Papa Hull 4.23) It is July 16. A company has a portfolio of stocks worth \$100 million. The beta of the portfolio is 1.2. The company would like to use the CME December futures contract on the S&P 500 to change the beta of the portfolio to 0.5 during the period July 16 to November 16. The index is currently 1000, and each contract is on \$250 times the index.
(a) What position should the company take?

$$\begin{aligned} N = & \quad (\beta_F - \beta)\tfrac{P}{A} \\ = & \quad (0.5 - 1.2)\tfrac{100{,}000{,}000}{250\times 1000} \\ = & \quad -280 \end{aligned}$$

The company should short 280 contracts.

(b) Suppose that the company changes its mind and decides to increase the beta of the portfolio from 1.2 to 1.5. What position in futures contracts should it take?

$$\begin{aligned} N = & \quad (\beta_F - \beta)\tfrac{P}{A} \\ = & \quad (1.5 - 1.2)\tfrac{100{,}000{,}000}{250\times 1000} \\ = & \quad 120 \end{aligned}$$

The company should long 120 contracts.

2. Futures and Forward Markets
2.3. Hedging Strategies

3. (Baby Hull 3.22, Papa Hull 4.22) The following table gives data on monthly changes in the spot price and the futures price for a certain commodity. Use the data to calculate the minimum variance hedge ratio.

Spot Price Change	Futures Price Change
+0.50	+0.56
+0.61	+0.63
−0.22	−0.12
−0.35	−0.44
+0.79	+0.60
+0.04	−0.06
+0.15	+0.01
+0.70	+0.80
−0.51	−0.56
−0.41	−0.46

$$\begin{aligned}\sum x &= 0.96\\ \sum y &= 1.30\\ \sum x^2 &= 2.4474\\ \sum y^2 &= 2.3594\\ \sum xy &= 2.352\end{aligned}$$

$$\begin{aligned}\sigma_x &= \sqrt{\tfrac{2.4474}{9} - \tfrac{0.96^2}{10\times 9}} = 0.5116\\ \sigma_y &= \sqrt{\tfrac{2.3594}{9} - \tfrac{1.30^2}{10\times 9}} = 0.4933\\ \rho &= \frac{10(2.352)-(0.96)(1.30)}{\sqrt{\{10(2.4474)-0.96^2\}\{10(2.3594)-1.30^2\}}} = 0.981\end{aligned}$$

$h = \rho\frac{\sigma_S}{\sigma_F} = 0.946$

3. Swaps

1. (Baby Hull 7.10, Papa Hull 6.8) Companies X and Y have been offered the following rates per annum on a \$5 million 10-year investment:

Company	Fixed Rate	Floating Rate
X	8%	LIBOR
Y	8.8%	LIBOR

Company X requires a fixed-rate investment; company Y requires a floating-rate investment. Design a swap that will net a bank, acting as intermediary, 0.2% per annum and that will appear equally attrative to X and Y.

gains from swap: $|0.8\% - 0| = 0.8\%$

net gains $\frac{0.8\% - 0.2\%}{2} = 0.3\%$

$$\begin{tikzcd} \arrow[r, "\text{LIBOR}"] & X \arrow[r, shift left, "\text{LIBOR}"] & \text{F.I.} \arrow[l, shift left, "8.3\%"] \arrow[r, shift left, "\text{LIBOR}"] & Y \arrow[l, shift left, "8.5\%"] & \arrow[l, "8.8\%"'] \end{tikzcd}$$

X receives:	*LIBOR*	*Y receives:*	*8.8%*
	8.3%		*LIBOR*
	-LIBOR		*-8.5%*
	8.3% > 8% w/o swap		*LIBOR +0.3% > LIBOR w/o swap*

3. Swaps

2. (Baby Hull 7.20, Papa Hull 6.22) Company A, a British manufacturer, wishes to borrow U.S. dollars at a fixed rate of interest. Company B, a U.S. multinational, wishes to borrow sterling at a fixed rate of interest. They have been quoted the following rates per annum (adjusted for tax effects):

Company	Sterling	U.S. Dollars
A	11%	7%
B	10.6%	6.2%

Design a swap that will net a bank, acting as intermediary, 10 basis points per annum and that will produce a gain of 15 basis points per annum for each of the two companies.

gains from swap: $|0.4\% - 0.8\%| = 0.4\%$

net gains $\frac{0.4\% - 0.1\%}{2} = 0.15\%$

$$\begin{array}{ccccccc} & & \textit{6.85\% on \$} & & \textit{6.2\% on \$} & & \\ \textit{11\% on Ł} & & \longrightarrow & & \longrightarrow & & \textit{6.2\% on \$} \\ \longleftarrow & A & & \textit{F.I.} & & B & \longrightarrow \\ & & \longleftarrow & & \longleftarrow & & \\ & & \textit{11\% on Ł} & & \textit{10.45\% on Ł} & & \end{array}$$

A pays: *11% on Ł*
6.85% on $
-11% on Ł
———
6.85% on $ < 7% w/o swap

B pays: *6.2% on $*
10.45% on Ł
-6.2% on $
———
10.45% on Ł< 10.6% w/o swap

3. Swaps

3. (Baby Hull 7.21, Papa Hull 6.20) Under the terms of an interest-rate swap, a financial institution has agreed to pay 10 percent per annum and to receive three-month LIBOR in return on a notional principal of \$100 million with payments being exchanged every three months. The swap has a remaining life of 14 months. The average bid-ask fixed rate currently being swapped for three-month LIBOR is 12 percent per annum for all maturities. The three-month LIBOR one month ago was 11.8 percent per annum. All rates are compounded quarterly. What is the value of the swap?

Value with bonds:

$$\begin{aligned}
e^{r_c T} &= \left(1+\tfrac{r_m}{m}\right)^{mT} \\
e^{r_c} &= \left(1+\tfrac{0.12}{4}\right)^{4} \\
r_c &= 4\ln 1.03 \\
&= 0.1182
\end{aligned}$$

$Q = \$100M$

$k = 0.1 \times \frac{3}{12} \times \$100M = \$2.5M$

$k^* = 0.118 \times \frac{3}{12} \times \$100M = \$2.95M$

$$\begin{aligned}
B_{fx} &= \sum_{i=1}^{n} k e^{-r_i t_i} + Q e^{-r_n t_n} \\
&= 2.5e^{-0.1182\frac{2}{12}} + 2.5e^{-0.1182\frac{5}{12}} + 2.5e^{-0.1182\frac{8}{12}} + 2.5e^{-0.1182\frac{11}{12}} + 102.5e^{-0.1182\frac{14}{12}} \\
&= \$98.68M \\
B_{fl} &= (k^* + Q)e^{-r_1 t_1} \\
&= 102.95e^{-0.1182\frac{2}{12}} \\
&= \$100.94M \\
V^{swap} &= B_{fl} - B_{fx} \\
&= \$2.26M
\end{aligned}$$

3. (Continued)

Value with FRAs:

$$\begin{aligned}
V_1 = &\ \tfrac{3}{12} \times \$100M(0.118 - 0.1)e^{-0.1182\frac{2}{12}} = \$441,200 \\
V_2 = &\ \tfrac{3}{12} \times \$100M(0.12 - 0.1)e^{-0.1182\frac{5}{12}} = \$476,000 \\
V_3 = &\ \tfrac{3}{12} \times \$100M(0.12 - 0.1)e^{-0.1182\frac{8}{12}} = \$462,100 \\
V_4 = &\ \tfrac{3}{12} \times \$100M(0.12 - 0.1)e^{-0.1182\frac{11}{12}} = \$448,700 \\
V_5 = &\ \tfrac{3}{12} \times \$100M(0.12 - 0.1)e^{-0.1182\frac{14}{12}} = \$435,600 \\
V^{swap} = &\ V_1 + V_2 + V_3 + V_4 + V_5 = \$2.26M
\end{aligned}$$

4. Options Markets
4.1. Institutions

1. (Baby Hull 8.9) Suppose that a European call option to buy a share for \$100 costs \$5 and is held until maturity.

$X = \$100$
$c = \$5$

Under what circumstances will the holder of the option make a profit?

The holder of the option makes a profit if the share price S_T is above \$105 at maturity.

$$\begin{aligned} \max\{S_T - X, 0\} - c &> 0 \\ S_T - 100 - 5 &> 0 \\ S_T > 105 \end{aligned}$$

Under what circumstances will the option be exercised?

The option is exercised if the stock price at maturity is above \$100.

$$\begin{aligned} \max\{S_T - X, 0\} > 0 \\ S_T - 100 &> 0 \\ S_T > 100 \end{aligned}$$

Draw a diagram illustrating how the profit from a long position in the option depends on the stock price at maturity of the option.

(You get the picture, don't you?)

4. Options Markets
4.1. Institutions

2. (Baby Hull 8.10) Suppose that a European put option to sell a share for \$60 costs \$8 and is held until maturity.

$X = \$60$
$p = \$8$

Under what circumstances will the seller of the option make a profit?

The seller of the option makes a profit if the share price S_T is above \$52 at maturity.

$$\begin{aligned} -\max\{X - S_T, 0\} + p &> 0 \\ -60 + S_T + 8 &> 0 \\ S_T > 52 \end{aligned}$$

Under what circumstances will the option be exercised?

The option is exercised if the stock price at maturity is below \$60.

$$\begin{aligned} \max\{X - S_T, 0\} > 0 \\ 60 - S_T &> 0 \\ S_T < 60 \end{aligned}$$

Draw a diagram illustrating how the profit from a short position in the option depends on the stock price at maturity of the option.

(Ask me if you don't get it.)

4. Options Markets
4.1. Institutions

3. (Baby Hull 8.18, Papa Hull 7.9) Consider an exhange-traded call option contract to buy 500 shares with a strike price of \$40 and maturity in four months.

$N = 500$
$X = 40$

Explain how the terms of the option contract change when there is:
(a) a ten percent stock dividend

A 10% stock dividend is equivalent as a $n = 110$ *for* $m = 100$ *stock split.*
The strike price changes to $\frac{mX}{n} = \frac{100\times40}{110} = 36.36$.
The number of options changes to $\frac{nN}{m} = \frac{110\times500}{100} = 550$.

(b) a ten percent cash dividend

The terms of an option contract are not adjusted for cash dividends.

(c) a four-for-one stock split

$n = 4$ *for* $m = 1$ *split*
The strike price changes to $\frac{mX}{n} = \frac{1\times40}{4} = 10$.
The number of options changes to $\frac{nN}{m} = \frac{4\times500}{1} = 2000$.

4. Options Markets
4.2. Basic Properties

1. (Baby Hull 9.9, Papa Hull 8.8) What is a lower bound for the price of a six-month call option on a non-dividend-paying stock when the stock price is \$80, the strike price is \$75, and the risk-free interest rate is 10 percent per annum?

$T = \frac{6}{12}$
$D = 0$
$S = 80$
$X = 75$
$r = 0.1$

$$\begin{aligned} c &\geq S - Xe^{-rT} \\ &\geq 80 - 75e^{-0.1\frac{6}{12}} \\ &\geq 8.66 \end{aligned}$$

4. Options Markets

4.2. Basic Properties

2. (Baby Hull 9.10, Papa Hull 8.9) What is a lower bound for the price of a two-month European put option on a non-dividend-paying stock when the stock price is \$58, the strike price is \$65, and the risk-free interest rate is 5 percent per annum?

$T = \frac{2}{12}$
$D = 0$
$S = 58$
$X = 65$
$r = 0.05$

$$\begin{aligned} p \geq & \quad Xe^{-rT} - S \\ \geq & \quad 65e^{-0.05\frac{2}{12}} - 58 \\ \geq & \quad 6.46 \end{aligned}$$

4. Options Markets
4.2. Basic Properties

3. (Baby Hull 9.11, Baby Hull 8.10) A four-month European call option on a dividend-paying stock is currently selling for \$5. The stock price is \$64, the strike price is \$60, and a dividend of \$0.80 is expected in one month. The risk-free interest rate is 12 percent per annum for all maturities. What opportunities are there for an arbitrageur?

$T = \frac{4}{12}$
$c = 5$
$S = 64$
$X = 60$
$r = 0.12$
$D = 0.8e^{-0.12\frac{1}{12}} = 0.79$

$$
\begin{aligned}
c &\geq S - D - Xe^{-rT} \\
5 &\geq 64 - 0.79 - 60e^{-0.12\frac{4}{12}} \\
5 &\geq 5.56
\end{aligned}
$$

	0	$\frac{1}{12}$	$\frac{4}{12}$
buy call	-5		$+\max\{S_T - 60, 0\}$
sell stock	$+64$		$-S_T$
	-0.79	$+0.8$	
invest	$-60e^{-0.12\frac{4}{12}}$		$+60$
	$+0.56$		*if* $S_T > 60$, *profit*$_T = S_T - 60 - S_T + 60 = 0$
	upfront profit!		*if* $S_T < 60$, *profit*$_T = 0 - S_T + 60 > 0$

4. Options Markets
4.2. Basic Properties

4. (Baby Hull 9.12, Papa Hull 8.11) A one-month European put option on a non-dividend-paying stock is currently selling for \$2.50. The stock price is \$47, the strike price is \$50, and the risk-free interest rate is 6 percent per annum. What opportunities are there for an arbitrageur?

$T = \frac{1}{12}$
$D = 0$
$p = 2.50$
$S = 47$
$X = 50$
$r = 0.06$

$$\begin{aligned} p &\geq \quad Xe^{-rT} - S \\ 2.50 &\geq \quad 50e^{-0.06\frac{1}{12}} - 47 \\ 2.50 &\geq \quad 2.75 \end{aligned}$$

	0	$\frac{1}{12}$
buy put	-2.50	$+\max\{50 - S_T, 0\}$
buy stock	-47	$+S_T$
borrow	$+49.50$	$-49.50e^{0.06\frac{1}{12}} = -49.75$
	0	

if $S_T < 50$, *profit*$_T = 50 - S_T + S_T - 49.75 = 0.25$
if $S_T > 50$, *profit*$_T = -49.75 + 0 + S_T > 0.25$

4. Options Markets
4.2. Basic Properties

5. (Baby Hull 9.14, Papa Hull 8.13) The price of a European call which expires in six months and has a strike price of \$30 is \$2. The underlying stock price is \$29, and a dividend of \$0.50 is expected in two months and again in five months. The term structure is flat, with all risk-free interest rates being 10 percent. What is the price of a European put option that expires in six months and has a strike price of \$30?

$c = 2$
$T = \frac{6}{12}$
$X = 30$
$S = 29$
$r = 0.1$
$D = 0.5e^{-0.1\frac{2}{12}} + 0.5e^{-0.1\frac{5}{12}} = 0.97$

$$\begin{aligned} c + D + Xe^{-rT} = & \quad p + S \\ 2 + 0.97 + 30e^{-0.1\frac{6}{12}} = & \quad p + 29 \\ p = & \; 2.51 \end{aligned}$$

4. Options Markets
4.2. Basic Properties

6. (Baby Hull 9.15, Papa Hull 8.14) Explain carefully the arbitrage opportunities in the previous problem if the European put price is $3.

if $p = 3 > 2.51$, then there exists an arbitrage opportunity.

	0	$\frac{2}{12}$	$\frac{5}{12}$	$\frac{6}{12}$
buy call	-2			$+\max\{S_T - 30, 0\}$
invest	$-30e{-}0.1\frac{6}{12}$			$+30$
write put	$+3$			$-\max\{30 - S_T, 0\}$
short share	$+29$			$-S_T$
	-0.97	*+0.5*	*+0.5*	
	$+0.49$ *upfront profit!*			*if $S_T > 30$, profit$_T = 0$* *if $S_T < 30$, profit$_T = 0$*

4. Options Markets
4.2. Basic Properties

7. (Baby Hull 9.16, Papa Hull 8.15) The price of an American call on a non-dividend-paying stock is \$4. The stock price is \$31, the strike price is \$30, and the expiration date is in three months. The risk-free interest rate is 8 percent. Derive upper and lower bounds for the price of an American put on the same stock with the same strike price and expiration date.

$C = 4$
$D = 0$
$S = 31$
$X = 30$
$T = \frac{3}{12}$
$r = 0.08$

$$\begin{aligned} S - X < C - P &< S - Xe^{-rT} \\ 31 - 30 < 4 - P &< 31 - 30e^{-0.08\frac{3}{12}} \\ 1 < 4 - P &< 1.59 \\ -3 < -P &< -2.41 \\ 3 > P &> 2.41 \end{aligned}$$

4. Options Markets
4.3. Trading Strategies

1. (Baby Hull 10.10, Papa Hull 9.10) Suppose that put options on a stock with strike prices \$30 and \$35 cost \$4 and \$7 respectively. Construct a table that shows the profit and payoff. How can the options be used to create
(a) a bull spread?

bull spread: buy a \$30 put and write a \$35 put
upfront flow: $-4+7=+3$

stock price	*payoff* $= +\max\{30-S_T,0\}-\max\{35-S_T,0\}$	*profit* =*payoff*+3
$S_T \geq 35$	$+0-0=0$	$0+3=3$
$30 \leq S_T < 35$	$+0-(35-S_T)=S_T-35$	$(S_T-35)+3=S_T-32$
$S_T < 30$	$+(30-S_T)-(35-S_T)=-5$	$-5+3=-2$

(b) a bear spread?

bear spread: buy a \$35 put and write a \$30 put
upfront flow: $-7+4=-3$

stock price	*payoff* $= +\max\{35-S_T,0\}-\max\{30-S_T,0\}$	*profit* =*payoff*−3
$S_T \geq 35$	$+0-0=0$	$0-3=-3$
$30 \leq S_T < 35$	$+(35-S_T)-0=35-S_T$	$(35-S_T)-3=32-S_T$
$S_T < 30$	$+(35-S_T)-(30-S_T)=5$	$5-3=2$

4. Options Markets
4.3. Trading Strategies

2. (Baby Hull 10.12, Papa Hull 9.12) A call with a strike price of \$60 costs \$6. A put with the same strike price and expiration date costs \$4. Construct a table that shows the profit from a straddle.

straddle: buy a call and a put
upfront flow: $-6-4=-10$

stock price	*payoff* $= +\max\{S_T-60,0\}+\max\{60-S_T,0\}$	*profit* $=$*payoff*-10
$S_T \geq 60$	$(S_T-60)+0=S_T-60$	$(S_T-60)-10=S_T-70$
$S_T < 60$	$0+(60-S_T)=60-S_T$	$(60-S_T)-10=50-S_T$

For what range of stock prices would the straddle lead to a loss?

The straddle leads to a loss when the final stock price is between \$50 and \$70.

4. Options Markets
4.3. Trading Strategies

3. (Baby Hull 10.19, Papa Hull 9.19) Three put options on a stock have the same expiration date and strike prices of \$55, \$60, and \$65. The market prices are \$3, \$5, and \$8 respectively. Explain how a butterfly spread can be created. Construct a table showing the profit from the strategy.

butterfly spread: buy a \$55 put, write two \$60 put, and buy a \$65 put
upfront flow: $-3 + 2(5) - 8 = -1$

stock price	*payoff* $= +\max\{55 - S_T, 0\} - 2\max\{60 - S_T, 0\} + \max\{65 - S_T, 0\}$	*profit* =*payoff*−1
$S_T \geq 65$	$+0 - 2(0) + 0 = 0$	$0 - 1 = -1$
$60 \leq S_T < 65$	$+0 - 2(0) + (65 - S_T) = 65 - S_T$	$(65 - S_T) - 1 = 64 - S_T$
$55 \leq S_T < 60$	$+0 - 2(60 - S_T) + (65 - S_T) = S_T - 55$	$(S_T - 55) - 1 = S_T - 56$
$S_T < 55$	$+(55 - S_T) - 2(60 - S_T) + (65 - S_T) = 0$	$0 - 1 = -1$

For what range of stock prices would the butterfly spread lead to a loss?

The butterfly spread leads to a loss when the final stock price is greater than \$64 or less than \$56.

4. Options Markets
4.4. Binomial Trees

1. (Baby Hull 11.9, Papa Hull 10.8) A stock price is currently \$50. It is known that at the end of two months it will be either \$53 or \$48. The risk-free interest rate is 10 percent per annum with continuous compounding. What is the value of a two-month European call option with a strike price of \$49?

$r = 0.1$
step$= \frac{2}{12}$
$X = 49$

$$\begin{array}{lll} & & \mathit{53} \\ & & \max\{53-49,0\}=4 \\ & \nearrow & \\ \mathit{50} & & \\ f? & & \\ & \searrow & \\ & & \mathit{48} \\ & & \max\{48-49,0\}=0 \end{array}$$

$$\begin{aligned} u = & \quad \frac{53}{50} = 1.06 \\ d = & \quad \frac{48}{50} = 0.96 \\ p = & \quad \frac{e^{rT}-d}{u-d} = \frac{e^{0.1\frac{2}{12}}-0.96}{1.06-0.96} = 0.5681 \\ 1-p = & \quad 0.4319 \\ f = & \quad (0.5681 \times 4 + 0.4319 \times 0)e^{-0.1\frac{2}{12}} = 2.23 \end{aligned}$$

4. Options Markets
4.4. Binomial Trees

2. (Baby Hull 11.10, Papa Hull 10.9) A stock price is currently \$80. It is known that at the end of four months it will be either \$75 or \$85. The risk-free interest rate is 5 percent per annum with continuous compounding. What is the value of a four-month European put option with a strike price of \$80?

$r = 0.05$
step$= \frac{4}{12}$
$X = 80$

85
$\max\{80 - 85, 0\} = 0$

$\nearrow$

80
f?

$\searrow$

75
$\max\{80 - 75, 0\} = 5$

$$
\begin{aligned}
u &= \frac{85}{80} = 1.0625 \\
d &= \frac{75}{80} = 0.9375 \\
p &= \frac{e^{rT} - d}{u - d} = \frac{e^{0.05\frac{4}{12}} - 0.9375}{1.0625 - 0.9375} = 0.6345 \\
1 - p &= 0.3655 \\
f &= (0.6345 \times 0 + 0.3655 \times 5)e^{-0.05\frac{4}{12}} = 1.80
\end{aligned}
$$

4. Options Markets
4.4. Binomial Trees

3. (Baby Hull 11.17, Papa Hull 10.15) A stock price is currently \$40. Over each of the next two three-month periods it is expected to go up by 10 percent or down by 10 percent. The risk-free interest rate is 12 percent per annum with continuous compounding.
(a) What is the value of a six-month European put option with a strike price of \$42?

$r = 0.12$
$step = \frac{3}{12}$
$u = 1.1$
$d = 0.9$
$S = 40$
$X = 42$

$$
\begin{tikzcd}[row sep=small]
 & & \begin{array}{l}\mathit{48.4}\\ \max\{42-48.4,0\}=0\end{array} \\
 & \begin{array}{l}\mathit{44}\\ \mathit{(B)}\end{array} \arrow[ur] \arrow[dr] & \\
\begin{array}{l}\mathit{40}\\ \mathit{(A)}\end{array} \arrow[ur] \arrow[dr] & & \begin{array}{l}\mathit{39.6}\\ \max\{42-39.6,0\}=2.4\end{array} \\
 & \begin{array}{l}\mathit{36}\\ \mathit{(C)}\end{array} \arrow[ur] \arrow[dr] & \\
 & & \begin{array}{l}\mathit{32.4}\\ \max\{42-32.4,0\}=9.6\end{array}
\end{tikzcd}
$$

$$
\begin{aligned}
p &= \frac{e^{rT}-d}{u-d} = \frac{e^{0.12\frac{3}{12}}-0.9}{1.1-0.9} = 0.6523 \\
1-p &= 0.3477 \\
f_{(B)} &= (0.6523 \times 0 + 0.3477 \times 2.4)e^{-0.12\frac{3}{12}} = 0.81 \\
f_{(C)} &= (0.6523 \times 2.4 + 0.3477 \times 9.6)e^{-0.12\frac{3}{12}} = 4.76 \\
f_{(A)} &= (0.6523 \times 0.81 + 0.3477 \times 4.76)e^{-0.12\frac{3}{12}} = 2.12
\end{aligned}
$$

3. (Continued) (b) What is the value of a six-month American put option with a strike price of \$42?

- *Exercise at (B)? No*

  $\max\{42-44, 0\} = 0 < 0.81$

- *Exercise at (C)? Yes*

  $\max\{42-36, 0\} = 6 > 4.76$

  $f_{(A)} = (0.6523 \times 0.81 + 0.3477 \times 6)e^{-0.12\frac{3}{12}} = 2.54$

- *Exercise at (A)? NO*

  $\max\{42-40, 0\} = 2 < 2.54$

4. Options Markets
4.5. Black-Scholes

1. (Baby Hull 12.13, Papa Hull 12.13) What is the price of a European call option on a non-dividend-paying stock when the stock price is \$52, the strike price is \$50, the risk-free interest rate is 12 percent per annum, the volatility is 30 percent per annum, and the time to maturity is three months?

$S = 52$
$X = 50$
$r = 0.12$
$\sigma = 0.3$
$T = \frac{3}{12}$

$$\begin{aligned} d_1 = & \frac{\ln\left(\frac{S}{X}\right)+\left(r+\frac{\sigma^2}{2}\right)T}{\sigma\sqrt{T}} \\ = & \frac{\ln\left(\frac{52}{50}\right)+\left(0.12+\frac{0.3^2}{2}\right)\frac{3}{12}}{0.3\sqrt{\frac{3}{12}}} = 0.5365 \approx 0.54 \end{aligned}$$

$$\begin{aligned} d_2 = & \quad d_1 - \sigma\sqrt{T} \\ = & \quad 0.5365 - 0.3\sqrt{\frac{3}{12}} = 0.3865 \approx 0.39 \end{aligned}$$

$$\begin{aligned} N(d_1) = & \quad N(0.54) \\ = & \quad 0.7054 \end{aligned}$$

$$\begin{aligned} N(d_2) = & \quad N(0.39) \\ = & \quad 0.6517 \end{aligned}$$

$$\begin{aligned} c = & \quad SN(d_1) - Xe^{-rT}N(d_2) \\ = & \quad 52 \times 0.7054 - 50e^{-0.12\frac{3}{12}} \times 0.6517 \\ = & \quad 5.06 \end{aligned}$$

4. Options Markets
4.5. Black-Scholes

2. (Baby Hull 12.14, Papa Hull 12.14) What is the price of a European put option on a non-dividend-paying stock when the stock price is \$69, the strike price is \$70, the risk-free interest rate is 5 percent per annum, the volatility is 35 percent per annum, and the time to maturity is six months?

$S = 69$
$X = 70$
$r = 0.05$
$\sigma = 0.35$
$T = \frac{6}{12}$

$$\begin{aligned} d_1 &= \frac{\ln\left(\frac{S}{X}\right)+\left(r+\frac{\sigma^2}{2}\right)T}{\sigma\sqrt{T}} \\ &= \frac{\ln\left(\frac{69}{70}\right)+\left(0.05+\frac{0.35^2}{2}\right)\frac{6}{12}}{0.35\sqrt{\frac{6}{12}}} = 0.1666 \approx 0.17 \end{aligned}$$

$$\begin{aligned} d_2 &= d_1 - \sigma\sqrt{T} \\ &= 0.1666 - 0.35\sqrt{\frac{6}{12}} = -0.0809 \approx -0.08 \end{aligned}$$

$$\begin{aligned} N(-d_1) &= N(-0.17) \\ &= 0.4325 \end{aligned}$$

$$\begin{aligned} N(-d_2) &= N(0.08) \\ &= 0.5319 \end{aligned}$$

$$\begin{aligned} p &= Xe^{-rT}N(-d_2) - SN(-d_1) \\ &= 70e^{-0.05\frac{6}{12}} \times 0.5319 - 69 \times 0.4325 \\ &= 6.47 \end{aligned}$$

4. Options Markets
4.5. Black-Scholes

3. (Baby Hull 12.25, Papa Hull 12.27) Consider an option on a non-dividend-paying stock when the stock price is \$30, the exercise price is \$29, the risk-free interest rate is 5 percent per annum, the volatility is 25 percent per annum, and the time to maturity is four months.

$S = 30$
$X = 29$
$r = 0.05$
$\sigma = 0.25$
$T = \frac{4}{12}$

$$\begin{aligned} d_1 &= \frac{\ln\left(\frac{S}{X}\right)+\left(r+\frac{\sigma^2}{2}\right)T}{\sigma\sqrt{T}} \\ &= \frac{\ln\left(\frac{30}{29}\right)+\left(0.05+\frac{0.25^2}{2}\right)\frac{4}{12}}{0.25\sqrt{\frac{4}{12}}} = 0.4225 \approx 0.42 \end{aligned}$$

$$\begin{aligned} d_2 &= d_1 - \sigma\sqrt{T} \\ &= 0.4225 - 0.25\sqrt{\frac{4}{12}} = 0.2782 \approx 0.28 \end{aligned}$$

$$\begin{aligned} N(d_1) &= N(0.42) \\ &= 0.6628 \end{aligned}$$

$$\begin{aligned} N(d_2) &= N(0.28) \\ &= 0.6103 \end{aligned}$$

$$\begin{aligned} N(-d_1) &= 1 - N(d_1) \\ &= 1 - 0.6628 \\ &= 0.3372 \end{aligned}$$

$$\begin{aligned} N(-d_2) &= 1 - N(d_2) \\ &= 1 - 0.6103 \\ &= 0.3897 \end{aligned}$$

3. (Continued)
(a) What is the price of the option if it is a European call?

$$\begin{aligned} c = & \quad SN(d_1) - Xe^{-rT}N(d_2) \\ = & \quad 30 \times 0.6628 - 29e^{-0.05\frac{4}{12}} \times 0.6103 \\ = & \quad 2.48 \end{aligned}$$

(b) What is the price of the option if it is an American call?

$C = c = 2.48$

(c) What is the price of the option if it is a European put?

$$\begin{aligned} p = & \quad Xe^{-rT}N(-d_2) - SN(-d_1) \\ = & \quad 29e^{-0.05\frac{4}{12}} \times 0.3897 - 30 \times 0.3372 \\ = & \quad 1.00 \end{aligned}$$

(d) Verify that put-call parity holds.

$$\begin{aligned} p + S = & \quad c + Xe^{-rT} \\ 1 + 30 = & \quad 2.48 + 29e^{-0.05\frac{4}{12}} \\ 31 = & \quad 31 \end{aligned}$$

4. Options Markets
4.5. Black-Scholes

4. (Baby Hull 12.26, Papa Hull 12.28) Assume that the stock in the previous problem is due to go ex-dividend in 1.5 months. The expected dividend is 50 cents.

$$D = 0.5e^{-0.05\frac{1.5}{12}} = 0.4969$$

$$\begin{aligned} d_1 &= \frac{\ln\left(\frac{S-D}{X}\right)+\left(r+\frac{\sigma^2}{2}\right)T}{\sigma\sqrt{T}} \\ &= \frac{\ln\left(\frac{30-0.4969}{29}\right)+\left(0.05+\frac{0.25^2}{2}\right)\frac{4}{12}}{0.25\sqrt{\frac{4}{12}}} = 0.3068 \approx 0.31 \end{aligned}$$

$$\begin{aligned} d_2 &= d_1 - \sigma\sqrt{T} \\ &= 0.3068 - 0.25\sqrt{\frac{4}{12}} = 0.1625 \approx 0.16 \end{aligned}$$

$$\begin{aligned} N(d_1) &= N(0.31) \\ &= 0.6217 \end{aligned}$$

$$\begin{aligned} N(d_2) &= N(0.16) \\ &= 0.5636 \end{aligned}$$

$$\begin{aligned} N(-d_1) &= 1 - N(d_1) \\ &= 1 - 0.6271 \\ &= 0.3783 \end{aligned}$$

$$\begin{aligned} N(-d_2) &= 1 - N(d_2) \\ &= 1 - 0.5636 \\ &= 0.4364 \end{aligned}$$

4. (Continued)
(a) What is the price of the option if it is a European call?

$$\begin{aligned} c = & \quad (S-D)N(d_1) - Xe^{-rT}N(d_2) \\ = & \quad (30 - 0.4969) \times 0.6217 - 29e^{-0.05\frac{4}{12}} \times 0.5636 \\ = & \quad 2.27 \end{aligned}$$

(b) What is the price of the option if it is a European put?

$$\begin{aligned} p = & \quad Xe^{-rT}N(-d_2) - (S-D)N(-d_1) \\ = & \quad 29e^{-0.05\frac{4}{12}} \times 0.4364 - (30 - 0.4969) \times 0.3783 \\ = & \quad 1.29 \end{aligned}$$

4. Options Markets
4.6. Applications

1. (Papa Hull 13.15) The S&P 100 index currently stands at 696 and has a volatility of 30 percent per annum. The risk-free rate of interest is 7 percent per annum and the index provides a dividend yield of 4 percent per annum. Calculate the value of a three-month European put with an exercise price of 700.

$S = 696$
$\sigma = 0.3$
$r = 0.07$
$q = 0.04$
$T = \frac{3}{12}$
$X = 700$

$$\begin{aligned} d_1 = & \frac{\ln\left(\frac{S}{X}\right)+\left(r-q+\frac{\sigma^2}{2}\right)T}{\sigma\sqrt{T}} \\ = & \frac{\ln\left(\frac{696}{700}\right)+\left(0.07-0.04+\frac{0.3^2}{2}\right)\frac{3}{12}}{0.3\sqrt{\frac{3}{12}}} = 0.0868 \approx 0.09 \end{aligned}$$

$$\begin{aligned} d_2 = & d_1 - \sigma\sqrt{T} \\ = & 0.0868 - 0.3\sqrt{\frac{3}{12}} = -0.0632 \approx -0.06 \end{aligned}$$

$$\begin{aligned} N(-d_1) = & N(-0.09) \\ = & 0.4641 \end{aligned}$$

$$\begin{aligned} N(-d_2) = & N(0.06) \\ = & 0.5239 \end{aligned}$$

$$\begin{aligned} p = & Xe^{-rT}N(-d_2) - Se^{-qT}N(-d_1) \\ = & 700e^{-0.07\frac{3}{12}} \times 0.5239 - 696e^{-0.04\frac{3}{12}} \times 0.4641 = 40.6 \end{aligned}$$

4. Options Markets
4.6. Applications

2. (Baby Hull 13.10, Papa Hull 13.18) Consider a stock index currently standing at 250. The dividend yield on the index is 4 percent per annum and the risk-free rate is 6 percent per annum. A three-month European call option on the index with a strike price of 245 is currently worth \$10. What is the value of a three-month put option on the index with a strike price of 245?

$S = 250$
$q = 0.04$
$r = 0.06$
$T = \frac{3}{12}$
$X = 245$
$c = 10$

$$\begin{aligned} p + Se^{-qT} &= c + Xe^{-rT} \\ p + 250e^{-0.04\frac{3}{12}} &= 10 + 245e^{-0.06\frac{3}{12}} \\ p &= 3.84 \end{aligned}$$

3. (Baby Hull 13.16, Papa Hull 13.24) Suppose that a portfolio is worth \$60 million and the S&P 500 is at 1200. If the value of the portfolio mirrors the value of the index, what options should be purchased to provide protection against the value of the portfolio falling below \$54 million in one year's time?

500 put contracts should be bought with a strike of 1080.

$$\begin{aligned} N &= \beta \tfrac{V_p}{100 V_m} \\ &= 1 \tfrac{60{,}000{,}000}{100 \times 1200} \\ &= 500 \end{aligned}$$

$$\begin{aligned} N &= \beta \tfrac{V_p}{100 X} \\ 500 &= 1 \tfrac{54{,}000{,}000}{100 \times X} \\ X &= 1080 \end{aligned}$$

4. (Baby Hull 13.17, Papa Hull 13.25) Consider again the situation in the previous problem. Suppose that the portfolio has a beta of 2.0, the risk-free interest rate is 5 percent per annum, and the dividend yield on both the portfolio and the index is 3 percent per annum. What options should be purchased to provide protection against the value of the portfolio falling below $54 million?

portfolio value: 60,000,000
S&P500: 1200
$\beta = 2$
$r = 0.05$
$q = 0.03$
portfolio value floor: 54,000,000
$T = 1$

- *From the Portfolio...*

  capital gains: $\frac{54M-60M}{60} = -0.1$

  dividends: 0.03

  rate of return: $E[r_p] = -0.07$

- *To the Index...*

  rate of return: $E[r_m]$*?*

$$
\begin{aligned}
E[r_p] - r = &\quad \beta(E[r_m] - r) \\
-0.07 - 0.05 = &\quad 2(E[r_m] - 0.05) \\
E[r_m] = &\quad -0.01
\end{aligned}
$$

dividends: 0.03

capital gains: -0.04

- *You should purchase 1000 put contracts with a 1152 strike price.*

  $X = 1200(1 + (-0.04)) = 1152$

  $N = 2\frac{60{,}000{,}000}{100 \times 1200} = 1000$

4. (Continued)

To check that the answer is correct, consider what happens when the value of the portfolio declines to \$48M.

- *From the Portfolio...*

  capital gains: $\frac{48M-60M}{60} = -0.2$

  dividends: 0.03

  rate of return: $E[r_p] = -0.17$

- *To the Index...*

  rate of return: $E[r_m]$?

$$\begin{aligned} E[r_p] - r &= \beta(E[r_m] - r) \\ -0.17 - 0.05 &= 2(E[r_m] - 0.05) \\ E[r_m] &= -0.06 \end{aligned}$$

  dividends: 0.03

  capital gains: -0.09

- *The total value is insured at \$54,000,000.*

  value of the S&500: $S = 1200(1 + (-0.09)) = 1092$

  value of the portfolio: 48,000,000

  value of put options: $1000 \times 100 \times \max\{1152 - 1092, 0\} = 6,000,000$

  total value: 48,000,000+6,000,000=54,000,000

4. Options Markets
4.6. Applications

5. (Baby Hull 13.19, Papa Hull 13.42) A stock index currently stands at 300. It is expected to increase or decrease by 10 percent over each of the next two time periods of three months. The risk-free interest rate is 8 percent and the dividend yield on the index is 3 percent. What is the value of a six-month put option on the index with a strike price of 300 if it is
(a) European?

$S = 300$
$u = 1.1$
$d = 0.9$
step$= \frac{3}{12}$
$r = 0.08$
$q = 0.03$
$X = 300$

$$
\begin{array}{ccccc}
 & & & & \begin{array}{l} 363 \\ \max\{300-363,0\}=0 \end{array} \\
 & & & \nearrow & \\
 & & \begin{array}{c} 330 \\ (B) \end{array} & & \\
 & \nearrow & & \searrow & \\
\begin{array}{c} 300 \\ (A) \end{array} & & & & \begin{array}{l} 297 \\ \max\{300-297,0\}=3 \end{array} \\
 & \searrow & & \nearrow & \\
 & & \begin{array}{c} 270 \\ (C) \end{array} & & \\
 & & & \searrow & \\
 & & & & \begin{array}{l} 243 \\ \max\{300-243,0\}=57 \end{array}
\end{array}
$$

$$
\begin{array}{rcl}
p = & \frac{e^{(r-q)T}-d}{u-d} = \frac{e^{(0.08-0.03)\frac{3}{12}}-0.9}{1.1-0.9} & = 0.5629 \\
1-p = & 0.4371 &
\end{array}
$$

5. (a) (Continued)

$$
\begin{aligned}
f_{(B)} &= (0.5629 \times 0 + 0.4371 \times 3)e^{-0.08\frac{3}{12}} = 1.29 \\
f_{(C)} &= (0.5629 \times 3 + 0.4371 \times 57)e^{-0.08\frac{3}{12}} = 26.08 \\
f_{(A)} &= (0.5629 \times 1.29 + 0.4371 \times 26.08)e^{-0.08\frac{3}{12}} = 11.89
\end{aligned}
$$

(b) American?

- *Exercise at (B)? No*

  $\max\{300 - 330, 0\} = 0 < 1.29$

- *Exercise at (C)? Yes*

  $\max\{300 - 270, 0\} = 30 > 26.08$

  $f_{(A)} = (0.5629 \times 1.29 + 0.4371 \times 30)e^{-0.08\frac{3}{12}} = 13.57$

- *Exercise at (A)? NO*

  $\max\{300 - 300, 0\} = 0 < 13.57$

4. Options Markets
4.6. Applications

6. (Baby Hull 13.20, Papa Hull 13.43) Suppose that the spot price of the Canadian dollar is U.S.$0.75 and that the Canadian dollar/U.S. dollar exchange rate has a volatility of 4 percent per annum. The risk-free rates of interest in Canada and the United States are 9 percent and 7 percent per annum, respectively. Calculate the value of a European call option with an exercise price of $0.75 and an exercise date in 9 months.

$S = 0.75$
$\sigma = 0.04$
$r_f = 0.09$
$r = 0.07$
$X = 0.75$
$T = \frac{9}{12}$

$$\begin{aligned} d_1 &= \frac{\ln\left(\frac{S}{X}\right)+\left(r-r_f+\frac{\sigma^2}{2}\right)T}{\sigma\sqrt{T}} \\ &= \frac{\ln\left(\frac{0.75}{0.75}\right)+\left(0.07-0.09+\frac{0.04^2}{2}\right)\frac{9}{12}}{0.04\sqrt{\frac{9}{12}}} = -0.4157 \approx -0.42 \end{aligned}$$

$$\begin{aligned} d_2 &= d_1 - \sigma\sqrt{T} \\ &= -0.4157 - 0.04\sqrt{\frac{9}{12}} = -0.4503 \approx -0.45 \end{aligned}$$

$$\begin{aligned} N(d_1) &= N(-0.42) \\ &= 0.3372 \end{aligned}$$

$$\begin{aligned} N(d_2) &= N(-0.45) \\ &= 0.3264 \end{aligned}$$

$$\begin{aligned} c &= Se{-}r_fTN(d_1) - Xe^{-rT}N(d_2) \\ &= 0.75e^{-0.09\frac{9}{12}} \times 0.3372 - 0.75e^{-0.07\frac{9}{12}} \times 0.3264 = 0.0041 \end{aligned}$$

7. (Baby Hull 14.10, Papa Hull 13.29) Consider a two-month call futures option with a strike price of 40 when the risk-free interest rate is 10 percent per annum. The current futures price is 47.

$T = \frac{2}{12}$
$X = 40$
$r = 0.1$
$F = 47$

What is a lower bound for the value of the futures option if it is
(a) European?

$$\begin{aligned} c \geq & \quad (F - X)e^{-rT} \\ \geq & \quad (47 - 40)e^{-0.1\frac{2}{12}} \\ \geq & \quad 6.88 \end{aligned}$$

(b) American?

$$\begin{aligned} C \geq & \quad (F - X) \\ \geq & \quad (47 - 40) \\ \geq & \quad 7 \end{aligned}$$

4. Options Markets
4.6. Applications

8. (Baby Hull 14.11, Papa Hull 13.30) Consider a four-month put futures option with a strike price of 50 when the risk-free interest rate is 10 percent per annum. The current futures price is 47.

$T = \frac{4}{12}$
$X = 50$
$r = 0.1$
$F = 47$

What is a lower bound for the value of the futures option if it is
(a) European?

$$\begin{aligned} p \geq & \quad (X - F)e^{-rT} \\ \geq & \quad (50 - 47)e^{-0.1\frac{4}{12}} \\ \geq & \quad 2.90 \end{aligned}$$

(b) American?

$$\begin{aligned} P \geq & \quad (X - F) \\ \geq & \quad (50 - 47) \\ \geq & \quad 3 \end{aligned}$$

4. Options Markets
4.6. Applications

9. (Baby Hull 14.12, Papa Hull 13.31) A futures price is currently 60. It is known that over each of the next two three-month periods it will either rise by 10 percent or fall by 10 percent. The risk-free interest rate is 8 percent per annum. What is the value of a six-month European call option on the futures with a strike price of 60? If the call were American, would it ever be worth exercising early?

$step = \frac{3}{12}$
$r = 0.08$
$X = 60$

$$
\begin{array}{ccccc}
 & & & & \begin{array}{l} 72.6 \\ \max\{72.6-60,0\}=12.6 \end{array} \\
 & & & \nearrow & \\
 & & \begin{array}{l} 66 \\ (B) \end{array} & & \\
 & \nearrow & & \searrow & \\
\begin{array}{l} 60 \\ (A) \end{array} & & & & \begin{array}{l} 59.4 \\ \max\{59.4-60,0\}=0 \end{array} \\
 & \searrow & & \nearrow & \\
 & & \begin{array}{l} 54 \\ (C) \end{array} & & \\
 & & & \searrow & \\
 & & & & \begin{array}{l} 48.6 \\ \max\{48.6-60,0\}=0 \end{array}
\end{array}
$$

$$
\begin{aligned}
p &= \frac{1-d}{u-d} = \frac{1-0.9}{1.1-0.9} = 0.5 \\
1-p &= 0.5 \\
f_{(B)} &= (0.5 \times 12.6 + 0.5 \times 0)e^{-0.08\frac{3}{12}} = 6.18 \\
f_{(C)} &= (0.5 \times 0 + 0.5 \times 0)e^{-0.08\frac{3}{12}} = 0 \\
f_{(A)} &= (0.5 \times 6.18 + 0.5 \times 0)e^{-0.08\frac{3}{12}} = 3.03
\end{aligned}
$$

9. (Continued)

- *Exercise at (B)? No*

  $\max\{66 - 60, 0\} = 6 < 6.18$

- *Exercise at (C)? No*

  $\max\{54 - 60, 0\} = 0 = 0$

- *Exercise at (A)? NO*

  $\max\{60 - 60, 0\} = 0 < 3.03$

10. (Baby Hull 14.13, Papa Hull 13.32) In the previous problem what is the value of a six-month European put option on futures with a strike price of 60? If the put were American, would it ever be worth exercising it early? Verify that the call prices calculated in the previous problem and the put prices calculated here satisfy put-call parity relationships.

step= $\frac{3}{12}$
$r = 0.08$
$X = 60$

$$
\begin{array}{ccccc}
 & & & & \begin{array}{l}72.6\\ \max\{60-72.6,0\}=0\end{array}\\
 & & & \nearrow & \\
 & & \begin{array}{l}66\\ \textit{(B)}\end{array} & & \\
 & \nearrow & & \searrow & \\
\begin{array}{l}60\\ \textit{(A)}\end{array} & & & & \begin{array}{l}59.4\\ \max\{60-59.4,0\}=0.6\end{array}\\
 & \searrow & & \nearrow & \\
 & & \begin{array}{l}54\\ \textit{(C)}\end{array} & & \\
 & & & \searrow & \\
 & & & & \begin{array}{l}48.6\\ \max\{60-48.6,0\}=11.4\end{array}
\end{array}
$$

$$
\begin{aligned}
p &= \frac{1-d}{u-d} = \frac{1-0.9}{1.1-0.9} = 0.5\\
1-p &= 0.5\\
f_{(B)} &= (0.5\times 0 + 0.5\times 0.6)e^{-0.08\frac{3}{12}} = 0.29\\
f_{(C)} &= (0.5\times 0.6 + 0.5\times 11.4)e^{-0.08\frac{3}{12}} = 5.88\\
f_{(A)} &= (0.5\times 0.29 + 0.5\times 5.88)e^{-0.08\frac{3}{12}} = 3.02
\end{aligned}
$$

10. (Continued)

- *Exercise at (B)? No*

  $\max\{60 - 66, 0\} = 0 < 0.29$

- *Exercise at (C)? Yes*

  $\max\{60 - 54, 0\} = 6 > 5.88$

  $f_{(A)} = (0.5 \times 0.29 + 0.5 \times 6)e^{-0.08\frac{3}{12}} = 3.08$

- *Exercise at (A)? NO*

  $\max\{60 - 60, 0\} = 0 < 3.08$

European put-call parity:

$$\begin{aligned} p + Fe^{-rT} &= \quad c + Xe^{-rT} \\ 3.02 + 60e^{-0.08\frac{6}{12}} &= \quad 3.03 + 60e^{-0.08\frac{6}{12}} \\ 60.67 &\approx \quad 60.68 \end{aligned}$$

American put-call relationship:

$$\begin{aligned} Fe^{-rT} - X &< \quad C - P < F - Xe^{-rT} \\ 60e^{-0.08\frac{6}{12}} - 60 &< \quad 3.03 - 3.08 < 60 - 60e^{-0.08\frac{6}{12}} \\ -2.35 &< \quad -0.05 < 2.35 \end{aligned}$$

11. (Baby Hull 14.14, Papa Hull 13.33) A futures price is currently 25, its volatility is 30 percent per annum, and the risk-free interest rate is 10 percent per annum. What is the value of a nine-month European call on the futures with a strike price of 26?

$F = 25$
$\sigma = 0.3$
$r = 0.1$
$T = \frac{9}{12}$
$X = 26$

$$\begin{aligned} d_1 = & \frac{\ln\left(\frac{F}{X}\right)+\frac{\sigma^2}{2}T}{\sigma\sqrt{T}} \\ = & \frac{\ln\left(\frac{25}{26}\right)+\frac{0.3^2}{2}\frac{9}{12}}{0.3\sqrt{\frac{9}{12}}} = -0.0211 \approx -0.02 \end{aligned}$$

$$\begin{aligned} d_2 = & \quad d_1 - \sigma\sqrt{T} \\ = & -0.0211 - 0.3\sqrt{\frac{9}{12}} = -0.2808 \approx -0.28 \end{aligned}$$

$$\begin{aligned} N(d_1) = & \ N(-0.02) \\ = & \ 0.4920 \end{aligned}$$

$$\begin{aligned} N(d_2) = & \ N(-0.28) \\ = & \ 0.3897 \end{aligned}$$

$$\begin{aligned} c = & \ e^{-rT}[FN(d_1) - XN(d_2)] \\ = & \ e^{-0.1\frac{9}{12}}[25 \times 0.4920 - 26 \times 0.3897] \\ = & \ 2.01 \end{aligned}$$

4. Options Markets
4.6. Applications

12. (Baby Hull 14.15, Papa Hull 13.34) A futures price is currently 70, its volatility is 20 percent per annum, and the risk-free interest rate is 6 percent per annum. What is the value of a five-month European put on the futures with a strike price of 65?

$F = 70$
$\sigma = 0.2$
$r = 0.06$
$T = \frac{5}{12}$
$X = 65$

$$\begin{aligned} d_1 &= \frac{\ln\left(\frac{F}{X}\right)+\frac{\sigma^2}{2}T}{\sigma\sqrt{T}} \\ &= \frac{\ln\left(\frac{70}{65}\right)+\frac{0.2^2}{2}\frac{5}{12}}{0.2\sqrt{\frac{5}{12}}} = 0.6386 \approx 0.64 \end{aligned}$$

$$\begin{aligned} d_2 &= d_1 - \sigma\sqrt{T} \\ &= 0.6386 - 0.2\sqrt{\frac{5}{12}} = 0.5095 \approx 0.51 \end{aligned}$$

$$\begin{aligned} N(-d_1) &= N(-0.64) \\ &= 0.2611 \end{aligned}$$

$$\begin{aligned} N(-d_2) &= N(-0.51) \\ &= 0.3050 \end{aligned}$$

$$\begin{aligned} p &= e^{-rT}[XN(-d_2) - FN(-d_1)] \\ &= e^{-0.06\frac{5}{12}}[65 \times 0.3050 - 70 \times 0.2611] \\ &= 1.51 \end{aligned}$$

4. Options Markets
4.7. Hedging Strategies

1. (Baby Hull 15.11, Papa Hull 14.10) What is the delta of a short position in 1000 European call options on silver futures? The options mature in eight months, and the futures contract underlying the option matures in nine months. The current nine-month futures price is \$8 per ounce, the exercise price of the options is \$8, the risk-free interest rate is 12 percent per annum, and the volatility of silver is 18 percent per annum.

$F = 8$
$X = 8$
$r = 0.12$
$\sigma = 0.18$
$T = \frac{8}{12}$
$T^* = \frac{9}{12}$

$$\begin{aligned} d_1 = & \quad \frac{\ln\left(\frac{F}{X}\right)+\frac{\sigma^2}{2}T}{\sigma\sqrt{T}} \\ = & \quad \frac{\ln\left(\frac{8}{8}\right)+\frac{0.18^2}{2}\frac{8}{12}}{0.18\sqrt{\frac{8}{12}}} = 0.0735 \approx 0.07 \end{aligned}$$

$$\begin{aligned} N(d_1) = & \quad N(0.07) \\ = & \quad 0.5279 \end{aligned}$$

$$\begin{aligned} \Delta = & \quad N(d_1)e^{-rT} \\ = & \quad 0.5279 \times e^{-0.12\frac{8}{12}} \\ = & \quad 0.4873 \end{aligned}$$

The Δ of a short position in 1000 futures options is $-1000 \times 0.4873 = -487.3$.

4. Options Markets
4.7. Hedging Strategies

2. (Baby Hull 15.12, Papa Hull 14.11) Assume no storage costs for silver. In the previous problem, what initial position in nine-month silver futures is necessary for delta hedging?

$$\Delta = \frac{\partial f}{\partial F} = 0.4873$$

A long position in $0.4873 \times 1000oz = 487.3oz$ *of nine-month silver futures.*

If silver itself is used, what is the initial position?

$$\Delta = \frac{\partial f}{\partial S} = \frac{\partial f}{\partial F}\frac{\partial F}{\partial S}$$

$$\begin{aligned} F &= Se^{rT^*} \\ \frac{\partial F}{\partial S} &= e^{rT^*} \end{aligned}$$

$$\begin{aligned} \Delta = \frac{\partial f}{\partial S} &= \tfrac{\partial f}{\partial F}\tfrac{\partial F}{\partial S} \\ &= 0.4873e^{0.12\frac{9}{12}} = 0.5332 \end{aligned}$$

A long position in $0.5332 \times 1000oz = 533.2oz$ *of silver.*

If one-year silver futures are used, what is the initial position?

$$\Delta = \frac{\partial f}{\partial F_{1yr}} = \frac{\partial f}{\partial S}\frac{\partial S}{\partial F_{1yr}}$$

$$\frac{\partial S}{\partial F} = e^{-rT_{1yr}}$$

$$\begin{aligned} \Delta = \frac{\partial f}{\partial F_{1yr}} &= \tfrac{\partial f}{\partial S}\tfrac{\partial S}{\partial F_{1yr}} \\ &= 0.5332e^{-0.12\frac{12}{12}} = 0.4729 \end{aligned}$$

A long position in $0.4729 \times 1000oz = 472.9oz$ *of one-year silver futures.*

3. (Baby Hull 15.17, Papa Hull 14.16) A fund manager has a well-diversified portfolio that mirrors the performance of the S&P 500 and is worth \$360 million. The value of the S&P 500 is at 1200, and the portfolio manager would like to buy insurance against a reduction of more than 5 percent in the value of the portfolio over the next six months. The risk-free interest rate is 6 percent per annum. The dividend yield on both the portfolio and the S&P 500 is 3 percent, and the volatility of the index is 30 percent per annum.
(a) If the fund manager buys traded European put options, how much would the insurance cost?

3000 put contracts should be bought with a strike of 1140.

$$\begin{aligned} N &= \beta \tfrac{V_p}{100 V_m} \\ &= 1 \tfrac{360{,}000{,}000}{100 \times 1200} \\ &= 3000 \end{aligned}$$

$$\begin{aligned} N &= \beta \tfrac{V_p}{100 X} \\ 3000 &= 1 \tfrac{342{,}000{,}000}{100 \times X} \\ X &= 1140 \end{aligned}$$

$S = 1200$
$r = 0.06$
$\sigma = 0.3$
$T = \frac{6}{12}$
$q = 0.03$

3. (Continued)

$$
\begin{aligned}
d_1 &= \frac{\ln\left(\frac{S}{X}\right)+\left(r-q+\frac{\sigma^2}{2}\right)T}{\sigma\sqrt{T}} \\
&= \frac{\ln\left(\frac{1200}{1140}\right)+\left(0.06-0.03+\frac{0.3^2}{2}\right)\frac{6}{12}}{0.3\sqrt{\frac{6}{12}}} = 0.4186 \approx 0.42
\end{aligned}
$$

$$
\begin{aligned}
d_2 &= d_1 - \sigma\sqrt{T} \\
&= 0.4186 - 0.3\sqrt{\frac{6}{12}} = 0.2064 \approx 0.21
\end{aligned}
$$

$$
\begin{aligned}
N(d_1) &= N(0.42) \\
&= 0.6628
\end{aligned}
$$

$$
\begin{aligned}
N(d_2) &= N(0.21) \\
&= 0.5832
\end{aligned}
$$

$$
\begin{aligned}
N(-d_1) &= 1 - N(d_1) \\
&= 1 - 0.6628 \\
&= 0.3372
\end{aligned}
$$

$$
\begin{aligned}
N(-d_2) &= 1 - N(d_2) \\
&= 1 - 0.5832 \\
&= 0.4168
\end{aligned}
$$

$$
\begin{aligned}
p &= Xe^{-rT}N(-d_2) - Se^{-qT}N(-d_1) \\
&= 1140e^{-0.06\frac{6}{12}} \times 0.4168 - 1200e^{-0.03\frac{6}{12}} \times 0.3372 \\
&= 62.49
\end{aligned}
$$

The insurance costs $\$62.49 \times 3000 \times 100 = \$18,747,000.$

3. (Continued)
(b) Explain carefully alternative strategies open to the fund manager involving traded European call options, and show that they lead to the same result.

$$\begin{aligned} c = & \quad Se^{-qT}N(d_1) - Xe^{-rT}N(d_2) \\ = & \quad 1200e^{-0.03\frac{6}{12}} \times 0.6628 - 1140e^{-0.06\frac{6}{12}} \times 0.5832 \\ = & \quad 138.32 \end{aligned}$$

From put-call parity $p = c + Xe^{-rT} - Se^{-qT}$.

	0	$\frac{6}{12}$
sell index	$+1200e^{-0.03\frac{6}{12}}$	$-S_T$
buy call	-138.32	$+\max\{S_T - 1140, 0\}$
invest	$-1140e^{-0.06\frac{6}{12}}$	$+1140$
buy put	-62.49	$+\max\{0, 1140 - S_T\}$

(c) If the fund manager decides to provide insurance by keeping part of the portfolio in risk-free securities, what should the initial position be?

$$\begin{aligned} \Delta = & \quad [N(d_1) - 1]e^{-qT} \\ = & \quad [0.6628 - 1]e^{-0.03\frac{6}{12}} \\ = & \quad -0.3322 \end{aligned}$$

33.22% of the portfolio $(360,000,000 \times 0.3322 = 119,592,000)$ *should be sold and invested in risk-free securities.*

4. Options Markets
4.7. Hedging Strategies

3. (Continued)
(d) If the fund manager decides to provide insurance by using nine-month index futures, what should the initial position be?

$$\Delta = \frac{\partial f}{\partial F} = \frac{\partial f}{\partial S}\frac{\partial S}{\partial F}$$

$$\begin{aligned} F &= Se^{(r-q)T^*} \\ S &= Fe^{-(r-q)T^*} \\ \frac{\partial S}{\partial F} &= e^{-(r-q)T^*} \end{aligned}$$

$$\begin{aligned} \Delta = \frac{\partial f}{\partial F} &= \tfrac{\partial f}{\partial S}\tfrac{\partial S}{\partial F} \\ &= -0.3322e^{-(0.06-0.03)\frac{9}{12}} \\ &= -0.3248 \end{aligned}$$

A short position of 32.48% of the portfolio $(360,000,000 \times 0.3248 = 116,928,000)$ *should be taken in index futures.*