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Symbols in the list are sometimes also used temporarily for other purposes. . .

G. H. Hardy and E. M. Wright, *An Introduction to the Theory of Numbers*, 5th ed.

Summary of Notation

Because the material in the book draws on a variety of fields, there are some resulting conflicts or ambiguities in the notation. In general, these ambiguities can be cleared up by context, and the authors have attempted to avoid situations where like notation overlaps in the same context. Some examples are:

(1) The symbol $[\cdot,\cdot]$ can have three meanings: bilinear form, commutator, and homotopy classes of maps.
(2) χ can have three meanings: Euler characteristic, stream function for a surface current, or a gauge function.
(3) π can be a permutation map or the ratio of circumference to diameter of a circle. In addition, π_k signifies the kth homotopy group, while π_1^k signifies the kth term in the lower central series of the fundamental group.
(4) R can be a resistance matrix, the de Rham map, or a region in $\mathbb{R}^3$.
(5) Pullbacks and pushforwards of many varieties can be induced from a single map. For example, an inclusion map i can induce maps denoted by i^*, i_*, $i^\#$, $i_\flat$, $\tilde{\imath}$, etc.
(6) The symbols ξ, λ, α, β, η, ρ, ν, and θ have meanings particular to Chapter 7 (see Figure 7.4, page 211).

Other multiple uses of notation are noted below.

$\beta_p(R)$	pth Betti number = Rank $H_p(R)$
δ^{ij}	Kronecker delta; 1 if $i = j$, 0 otherwise
δ	Inner product space adjoint to the exterior derivative
δ	Connecting homomorphism in a long exact sequence
∂	Boundary operator
∂^T	Coboundary operator.
$\breve{\partial}$	Boundary operator on dual mesh (related to ∂^T)
ε	Dielectric permittivity
ζ_j^i	jth 1-cocycle on dual mesh, indexed on 1-cells of DK: $1 \leq i \leq \breve{m}_1$

η	Wave impedance
θ	Normalized angle of $f : R \longrightarrow S^1$
θ^i_j	θ discretized on nodes of unassembled mesh
λ	Wavelength
λ_i	Barycentric coordinates, $1 \leq i \leq 4$
μ	Magnetic permeability
π	Ratio of circumference to diameter of a circle
π	Permutation map
π_i	ith homotopy group (but π_0 distinguishes path components and is not a group)
ρ	Volume electrical charge density
σ	Electrical conductivity
σ_s	Surface electrical charge density
$\sigma_{p,i}$	ith p-simplex in a triangulation of R
τ_e	Dielectric relaxation time, $\tau_e = \varepsilon/\sigma$
Φ_i	ith magnetic flux
ϕ	Electric scalar potential
χ	Euler characteristic
χ	Stream function for surface current distribution
χ	Gauge function
χ_e, χ_m	Electric and magnetic susceptibilities
ψ	Magnetic scalar potential
ψ^+ (ψ^-)	Value of ψ on plus (minus) side of a cut
ω	Radian frequency
Ω	Subset of $\mathbb{R}^n$
$\boldsymbol{A}$	Magnetic vector potential
$\boldsymbol{B}$	Magnetic flux density vector
$B^p(K;R)$	p-coboundary group of K with coefficients in module R
$B_p(K;R)$	p-boundary group or K with coefficients in module R
$B^p(K,S;R)$	Relative p-coboundary group of K (relative to S)
$B_p(K,S;R)$	Relative p-boundary group of K (relative to S)
$B^p_c(M-S)$	Relative exact form defined via compact supports; $S \subset \partial M$
$\tilde{B}_p(M,S_1)$	Coexact p-forms in $\tilde{C}_p(M,S_1)$
c	Speed of light in a vacuum, $(\varepsilon_0\mu_0)^{-1/2}$
c	Curve (or contour of integration)
c_p	p-chain
c^p	p-cochain
curl	Curl operator
$\overline{\mathrm{curl}}$	Adjoint to the curl operator in two dimensions
C	Capacitance matrix
C	Constitutive law (see Figure 7.4)

C^i_{jk}	Connection matrix, $1 \leq i \leq m_3$, $1 \leq j \leq 4$, $1 \leq k \leq m_0$
$C^i_{p,jk}$	Connection matrix of p-dimensional mesh
$C^p(K;R)$	p-cochain group of K with coefficients in module R
$C_p(K;R)$	p-chain group or K with coefficients in module R
$C^p(K,S;R)$	Relative p-cochain group of K (relative to S)
$C_p(K,S;R)$	Relative p-chain group or K (relative to S)
$C^p_c(M-S)$	Differential forms with compact support on $M-S$; $S \subset \partial M$
$\tilde{C}_p(M,S_1)$	p-forms in the complex defined by δ, the formal adjoint of d in $C^*_c(M-S_2)$
d	Coboundary operator; exterior derivative
d	Thickness of current-carrying sheet
div	Divergence operator
div_S	Divergence operator on a surface
D	Differential operator
D, δ	Skin depth
$\boldsymbol{D}$	Electric displacement field
DK	Dual cell complex of simplicial complex K
$\boldsymbol{E}$	Electric field intensity
E_M	Magnetic energy
f_p	"Forcing function" associated with the pth cut (a vector with entries f_{pi})
f	Frequency
f	Generic function
$f^*(\mu)$	Pullback of μ by f
F	Rayleigh dissipation function
F_p	Free subgroup of pth homology group
F^p	Free subgroup of pth cohomology group
F	Primary functional
$F^\perp$	Secondary functional needed for convexity
F^s_0	Number of FLOPs per CG iteration for node-based interpolation of scalar Laplace equation
F_0	Number of FLOPs per CG iteration for node-based vector interpolation
F_1	Number of FLOPs per CG iteration for edge-based vector interpolation
$\mathcal{F}$, $\mathcal{G}$	Spaces of vector fields with elements F and G, respectively
grad	Gradient operator
G	Convex functional
$\boldsymbol{H}$	Magnetic field intensity
$H^p(R;\mathbb{Z})$	pth cohomology group of R with coefficients in $\mathbb{Z}$
$H_p(R;\mathbb{Z})$	pth homology group of R, coefficients in $\mathbb{Z}$

$H^p(R, \partial R; \mathbb{Z})$	pth cohomology group of R relative to ∂R, coefficients in $\mathbb{Z}$
$H_p(R, \partial R; \mathbb{Z})$	pth homology group of R relative to ∂R, coefficients in $\mathbb{Z}$
$H_c^p(M - S)$	$Z_c^p(M - S)/B_c^p(M - S)$; harmonic forms
$\mathcal{H}^p(M, S_1)$	$\tilde{Z}_c^p(M, S_2) \cap Z^p(M - S_1)$; harmonic fields
i	inclusion map
im	Image of map
I	Electrical current
I_i	ith current
I_f,I_p	Free and prescribed lumped-parameter currents
Int $(\cdot, \cdot)$	Oriented intersection number
$\mathcal{I}$	Intersection number matrix
$\mathcal{I}_p(m, l)$	Indicator function, $1 \leq p \leq \beta_1(R)$, $1 \leq m \leq 4$, $1 \leq l \leq m_3$
j	Map inducing a third map in a long exact sequence
$\boldsymbol{J}$	(Volumetric) current density vector
$\boldsymbol{J}_{\mathrm{av}}$	Average current density in effective depth of current sheet
$\mathcal{J}_j^i \in \mathbb{Z}$	Nodal jumps on each element, $1 \leq i \leq m_3$, $1 \leq j \leq 4$
ker	Kernel of map
$\boldsymbol{K}$	Surface current density vector
K	Simplicial complex
$\mathcal{K}_{mn}^k$	Stiffness matrix for kth element in mesh
$\mathcal{K}$	Global finite element stiffness matrix
l_{max}	"Characteristic length" of electromagnetic system
L	Inductance matrix
L	Lagrangian
$L^2\Lambda^q(X)$	Space of square-integrable differential q-forms on manifold X
Link$(\cdot, \cdot)$	Linking number of two curves
m_p	Number of p-simplexes in a triangulation of R
$\check{m}_p$	Number of p-cells in dual complex
$\boldsymbol{M}$	Magnetization
M	Manifold
n_c, n_v	Number of prescribed currents and number of prescribed voltages
n_p	Number of p-simplexes in a triangulation of ∂R
$\boldsymbol{n}$	Normal vector to a codimension 1 surface
$\boldsymbol{n}'$	Normal to a two-dimensional manifold with boundary embedded in $\mathbb{R}^3$
nz(A)	Number of nonzero entries of a matrix A
$\mathcal{O}(n^\alpha)$	Order n^α
$\boldsymbol{P}$	Polarization density
$\boldsymbol{P}$	Poynting vector
P	Period matrix

P_J	Eddy current power dissipation
Q_i	ith charge
R	Resistance matrix
R	de Rham map, $R : L^2\Lambda^q(X) \to C^q(K)$ (K a triangulation of X)
R	Region in $\mathbb{R}^3$, free of conduction currents
$\tilde{R}$	Three-dimensional manifold with boundary, subset of $\mathbb{R}^3$
R_S	Surface resistivity
S	Surface
S', S'_{ck}	Current-carrying surface after cuts for stream function have been removed, and the kth connected component of S'
S_q	qth cut
S^1	Unit circle, $S^1 = \{p \in \mathbb{C} \mid \ \|p\| = 1\}$
T	Kinetic energy
$\boldsymbol{T}$	Vector potential for volumetric current distributions
T_p	Torsion subgroup of pth homology group
T^p	Torsion subgroup of pth cohomology group
T^*	Cotangent space
u_k	Nodal potential, $1 \leq k \leq m_0$
v	vertex
V	Voltage
V_j	Prescribed voltage, $1 \leq i \leq n_v$
V	Potential energy
w_e	Electric field energy density
w_m	Magnetic field energy density
W	Whitney map $W : C^q(K) \to L^2\Lambda^q(X)$
W_e	Electric field energy
W_m	Magnetic field energy
X	Riemannian manifold
X_0^s	# nonzero entries in stiffness matrix for node-based scalar interpolation
X_0	# nonzero entries in stiffness matrix for node-based vector interpolation
X_1	# nonzero entries in stiffness matrix for edge-based vector interpolation
$\bar{z}$	Complex conjugate of z
z^p	p-cocycle
z_p	p-cycle
$Z^p(K;R)$	p-cocycle group of K with coefficients in module R
$Z_p(K;R)$	p-cycle group or K with coefficients in module R
$Z^p(K,S;R)$	Relative p-cocycle group of K (relative to S)

$Z_p(K, S; R)$	Relative p-cycle group or K (relative to S)
$Z_c^p(M - S)$	Relative closed form defined via compact supports; $S \subset \partial M$
$\tilde{Z}_p(M, S_1)$	Coclosed p-forms in $\tilde{C}_p(M, S_1)$
$S^{+(-)}$	Positive (negative) side of an orientable surface with respect to a normal defined on the surface
A^c	Set-theoretic complement of A
$[A, B]$	Homotopy classes of maps $f : A \to B$, i.e. $\pi_0(\mathrm{Map}(A, B))$
$[\cdot, \cdot]$	Homotopy classes of maps
$[\cdot, \cdot]$	Commutator
$[\cdot, \cdot]$	Bilinear pairing
$[\cdot]$	Equivalence class of element $\cdot$
$\wedge$	Exterior multiplication
$*$	Hodge star
$\cap$	Set-theoretic intersection
$\cup$	Set-theoretic union
$\cup$	Cup product

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