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IDENTIFICATION OF NONLINEAR MULTIVARIABLE SYSTEMS BY ADAPTIVE FUZZY TAKAGI-SUGENO MODEL

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Abstract. This paper investigates the use of a fuzzy method as a tool for model identification of a non linear and multivariable system when the measurement data is available. In fact, the use of fuzzy clustering facilitates automatic generation of Takagi-Sugeno rules and its antecedent parameters. After the determination of the consequent parameters, these are adapted by a recursive least squares algorithm with a forgetting factor in order to use the established model in an adaptive control scheme.

1. INTRODUCTION

For nonlinear dynamic systems, the conventional techniques of modeling and identifications are difficult to implement and sometimes impracticable. However, others techniques based on fuzzy logic are more and more used for modeling this kind of process [4]. Among the different fuzzy methods, the Takagi-Sugeno model (TS) has attracted most attention [18]. In fact, this model consists of if-then rules with fuzzy antecedents and mathematical functions in the consequent part. The task of system identification is to determine both the non linear parameters of the antecedents and the linear parameters of the rules consequent.

In general, there are two ways to obtain this information. Human experts may be able to formulate their process knowledge in fuzzy rules. However, this method is often inefficient because human cannot sense all the details. Therefore, numerous approaches have been proposed [16] which compute non-linear dynamic fuzzy models from input/output measurement

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data, e.g., local linear model tree method (LOLIMOT), tree construction algorithms [17], or neuro-fuzzy approaches [10].

This paper is organized as follows: Section 2 formulates the problem of MIMO systems identification. In Section 3, we present a fuzzy identification method based on fuzzy clustering who allows an automatic generation of (TS) models with constant consequent parameters. In our paper, we use the same fuzzy clustering technique but we obtain an adaptive fuzzy (TS) model. Finally, Section 4 discusses the experimental results obtained with a MIMO non linear system. Section 5 concludes the paper.

2. MODELING

Modeling and identification are important steps in the design of control system [9]. Typical applications of these models are the simulation, the prediction or the control system design.

Generally, modeling process consists to obtain a parametric model with the same dynamic behavior of the real process. However, when the process is complex, it is very difficult to define the different mathematical or physical laws which describe its behavior [11].

In this section, we are interested to the problem of the MIMO process identification [13].

We consider a MIMO system with$n_i$inputs and n_o outputs. This system can be approximated by a set of discrete time fuzzy MISO models.

We consider also:

- two polynomials A and B defined by:

$$\begin{aligned} A &= a_0 + a_1 q + a_2 q^2 + \cdots a_{n_A} q^{n_A} \\ B &= b_0 + b_1 q + b_2 q^2 + \cdots b_{n_B} q^{n_B} \end{aligned} \tag{1}$$

  q is a backward shift operator ($q^n y(k) = y(k-n)$).
- two integers m and n, $m \leq n$ which define a delayed sample of a discrete time signal as:

$$\{y(k)\}_m^n = \left[y(k-m), y(k-m-1), \cdots, y(k-n)\right]. \tag{2}$$

The MISO models are a input-output NARX (**N**on linear **A**uto **R**egressive with e**X**ogenous input) defined by:

$$y_l(k+1) = f_l(x_l(k)), \quad l = 1, 2, \ldots, n_o. \tag{3}$$

where the regression vector is given by:

$$\begin{aligned} x_l(k) = &\Big[\{y_1(k)\}_0^{n_{yl1}}, \{y_2(k)\}_0^{n_{yl2}}, \ldots, \{y_{n_0}(k)\}_0^{n_{yln_0}}, \\ &\{u_1(k)\}_{n_{dl1}}^{n_{ul1}}, \{u_2(k)\}_{n_{dl2}}^{n_{ul2}}, \ldots, \{u_{n_i}(k)\}_{n_{dln_i}}^{n_{uln_i}}\Big]. \end{aligned} \tag{4}$$

n_y and n_u define the number of delayed outputs and inputs respectively. n_d is the number of pure delays. n_y is a $n_o \times n_o$ matrix and n_u , n_d are $n_o \times n_i$ matrices. f_l are unknown non linear functions.

MISO models are estimated independly, so, to simplify the notation, the output index l is omitted and we will be interested only in the multi-input, mono-output case.

In this approach, the process's output can be written as:

$$y_(k+1) = Ay(k) + Bu(k) + \alpha \tag{5}$$

α is an offset coefficient. For a non linear MIMO system, fuzzy Takagi-Sugeno (TS) models represent an efficient tool to model this kind of system [12].

3. FUZZY IDENTIFICATION

The TS model has attracted the attention of many searchers. In fact, this model consists of if-then rules with fuzzy antecedents and mathematical functions in the consequent part [18]. The antecedents fuzzy sets divide the input space into a number of fuzzy regions, while the consequent functions describe the system's behavior in these regions [7].

The fuzzy rules are defined as:

R_i : **If** $x_(k)$ **is** Ω_i , **then**

$$y^i(k+1) = A_i y(k) + B_i u(k) + \alpha_i, i = 1, 2, \ldots, K. \tag{6}$$

Here Ω_i is the antecedent fuzzy set of the ith rule. $A_i = [A_{i1}, \ldots, A_{in_o}]$, $B_i = [B_{i1}, \ldots, B_{in_i}]$ are vectors of polynomials and K is the rule's number.

The antecedent of (6) can be written:

R_i :**if** $x_1(k)$**is** Ω_{i1} **and . . . and** $x_p(k)$ **is** Ω_{ip} , **then**

$$y^i(k+1) = A_i y(k) + B_i u(k) + \alpha_i \quad i = 1, 2, \ldots, K, \tag{7}$$

where

$$p_= \sum_{j=1}^{n_o} n_{y_j} + \sum_{j=1}^{n_i} n_{u_j} + 1, \tag{8}$$

the output of TS model is computed:

$$y(k+1) = \frac{\sum\limits_{i=1}^{K} \mu_i(x(k)) y^i(k+1)}{\sum\limits_{i=1}^{K} \mu_i(x(k))} \tag{9}$$

or:

(10)
$$y_(k+1) = \sum_{i=1}^{K} y^i(k+1)\Phi_i(x, c_i, \sigma_i)$$

where $\Phi_i(x, c_i, \sigma_i)$ is the validity function for the gaussian membership functions with centers c_i and standard deviations σ_i defined as:

(11)
$$\Phi_j(x_, c_i, \sigma_i) = \frac{\mu_j(x(k))}{\sum\limits_{i=1}^{K} \mu_i(x(k))}$$

(12)
$$\mu_j(x_(k)) = \exp\left(-\frac{1}{2}\frac{(x_1 - c_{j1})^2}{\sigma_{j1}^2}\right) \ldots \exp\left(-\frac{1}{2}\frac{(x_p - c_{jp})^2}{\sigma_{jp}^2}\right)$$

$\mu_j(x_(k))$ is the degree of fulfillment of the rule j.

The structure of the model, i.e., the matrices n_y , n_u and n_d are determined by the user on the basis of system's prior knowledge and/or by comparison of different structures based on an error criteria [17], [18]. Once the structure is fixed, the n_o MISO parameters are estimated independently by fuzzy clustering [2].

The model identification procedure based on the proposed method consists of two distinct steps. In the first step, called off-line identification of the fuzzy model, both non linear parameters of the gaussian membership functions, namely the centers c_i and standard deviationsσ_i , and the linear parameters of the local models are determined by fuzzy clustering method. In the second step, called on-line adaptation of the fuzzy model, the consequence's parameters of fuzzy rules are adapted by a recursive least squares method [15].

3.1. **Off-Line Identification of the Fuzzy Model.** This procedure is carried into four steps:

- construction of the regression data,
- determination of the clusters corresponding to a set of local linear submodels,
- determination of the antecedent membership function from the cluster parameters,
- estimation of rule's consequent parameters.

3.1.1. *Regression Data.* The available data samples are collected in matrix Z formed by concatenating the regression matrix X and the output vector Y:

$$X = \begin{bmatrix} \dots \\ x(k) \\ \dots \\ x(N-1) \end{bmatrix}, Y = \begin{bmatrix} \dots \\ y(k+1) \\ \dots \\ y(N) \end{bmatrix}, \quad Z^T = [X\ Y]. \tag{13}$$

N is the number of data samples.

3.1.2. *Construction of the Fuzzy Clusters.* There is different algorithms to construct the fuzzy clusters such as: the C-means algorithm [3], the Gath-Geva algorithm [6] and the Gustafson-Kessel algorithm [8]which will be used in our contribution.

Through clustering, the data set Z is partitioned into Nc clusters. In this paper, Nc is determined by testing many values according to an error criterion. The result is a fuzzy partition matrix $U = [\mu_{ik}]_{Nc\times N}$, whose element $\mu_{ik} \in [0,1]$ represents the degree of membership of the observation in cluster i, a prototype matrix $V = [v_1, \dots, v_{Nc}]$ and a set of cluster covariance matrices $F = [F_1, \dots, F_{Nc}](\{F_i\}$ are defined positive matrices).

Once the triplet (U, V, F) is determined, the parameters of the rule's premises (c_i and σ_i) and the consequent parameters (A_i ,B_i and α_i) are computed. For more details, see [1].

3.1.3. *Determination of the Antecedent Membership Function from the Cluster Parameters.* In this paper, Gaussian membership functions are used to represent the fuzzy sets Ω_{ij}:

$$\Omega_{ij}(x_j(k)) = \exp\left(-\frac{1}{2}\frac{(x_j - c_{ij})^2}{\sigma_{ij}^2}\right) \tag{14}$$

This choice leads to the following compact formula for (12):

$$\mu_j(x(k)) = \Omega_j(x(k)) = \exp\left(-\frac{1}{2}(x(k) - c_i^x)^T (F_j^{xx})^{-1}(x(k) - c_i^x)\right) \tag{15}$$

with $c_i^x = [c_{1i}, \dots, c_{pi}]$ is the center vector and $(F_j^{xx})^{-1}$ is the inverse of the matrix containing the variances on its diagonal:

$$F_j^{xx} = \begin{bmatrix} \sigma_{1j}^2 & 0 & \cdots & 0 \\ 0 & \sigma_{2j}^2 & \cdots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & \cdots & \sigma_{pj}^2 \end{bmatrix} \tag{16}$$

3.1.4. *Estimation of Consequent Parameters.* The consequent parameters in each rule are estimated separately by the weighted least squares method by minimizing the following criterion [1]:

$$\min_{\theta_i} \frac{1}{N}(Y - X_e\theta_i)^T Q_i (Y - X_e\theta_i) \tag{17}$$

where$X_e = [X \quad 1]$ is the regression matrix extended by a unitary column and Q_i is a matrix containing the values of the validity functions Φ_i of the ith local model for each data sample:

$$Q_i = \begin{bmatrix} \Phi_i(x(1), c_i, \sigma_i) & 0 & \cdots & \\ 0 & \Phi_i(x(2), c_i, \sigma_i) & \cdots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & \cdots & \Phi_i(x(N), c_i, \sigma_i) \end{bmatrix} \tag{18}$$

The weighted least-squares estimate of the consequent parameters ($\theta_i = [A_i, B_i, \alpha_i]$) is given by:

$$\theta_i = [X_e^T Q_i X_e]^{-1} X_e^T Q_i Y \tag{19}$$

3.2. **On-Line Adaptation of the Fuzzy Model.** We are interesting in this paper on a non-linear system with time-variant behavior. Therefore, an on-line adaptation is necessary to obtain a "good" model able to describe the process in a large operating points to be used in a schema of adaptive control [5].

Generally, the fuzzy TS models obtained by clustering are constant consequent parameters, i.e., a rule's consequent is written as:

$$y^i(k+1) = A_i y(k) + B_i u(k) + \alpha_i \tag{20}$$

but in our case, these parameters are updated. It means that at every moment k, one obtains a TS model:

$$y^i(k+1) = A_i(k)y(k) + B_i(k)u(k) + \alpha_i(k) \tag{21}$$

In this phase, the rule premises are kept fixed and only the rule consequence are adapted for each local model by a recursive version of the weighted least-squares algorithm with forgetting factor λ :

$$\theta_j(k) = \theta_j(k-1) + \delta_j(k)(y(k) - x^\top(k)(\theta_j(k-1)) \tag{22}$$

$$\delta_j(k) = \frac{P_j(k-1)x(k)}{x^\top(k)P_j(k-1)x(k) + \lambda/\Phi_j(x(k), c_j, \sigma_j)} \tag{23}$$

$$P_j(k) = \frac{1}{\lambda}[I - \delta_j(k)x^\top(k)]P_j(k-1). \tag{24}$$

In (22), the parameter vector θ_j is the same as for off-line identification in (19). It is updated by adding a correction vector to the old estimate $\theta_j(k-1)$. In (23) and (24), λ is a forgetting factor that implements forgetting of the old measurements, Φ_j is the weighting of the actual data with the rule activation and P_j is a matrix of the adaptation gain.

4. SIMULATION EXAMPLE

Consider a MIMO process [14] described by the equations:

$$y_{p1}(k+1) = \frac{y_{p1}(k)}{1 + y_{p2}^2(k)} + u_1(k) \tag{25}$$

$$y_{p2}(k+1) = \frac{y_{p1}(k)y_{p2}(k)}{1 + y_{p2}^2(k)} + u_2(k)$$

The inputs are u_1 and u_2 , and the outputs are y_{p1} and y_{p2} . The identification procedure is carried out with random inputs $u_1(k)$ and $u_2(k)$ uniformly distributed in the interval $[-1 \quad 1]$ and three clusters for each output.

The input signals are shown in Figure 1. From inputs data, we obtain the two outputs y_{p1} and y_{p2} according to (25). Once the input-output data is available, we compute the matrix U, V and F according to (15) and (16) by Gustafson-Kessel algorithm, then we determine the consequent parameters of each rule generated by fuzzy clustering according to (19).

The responses of the plant and the identification model are shown in Figure 2.

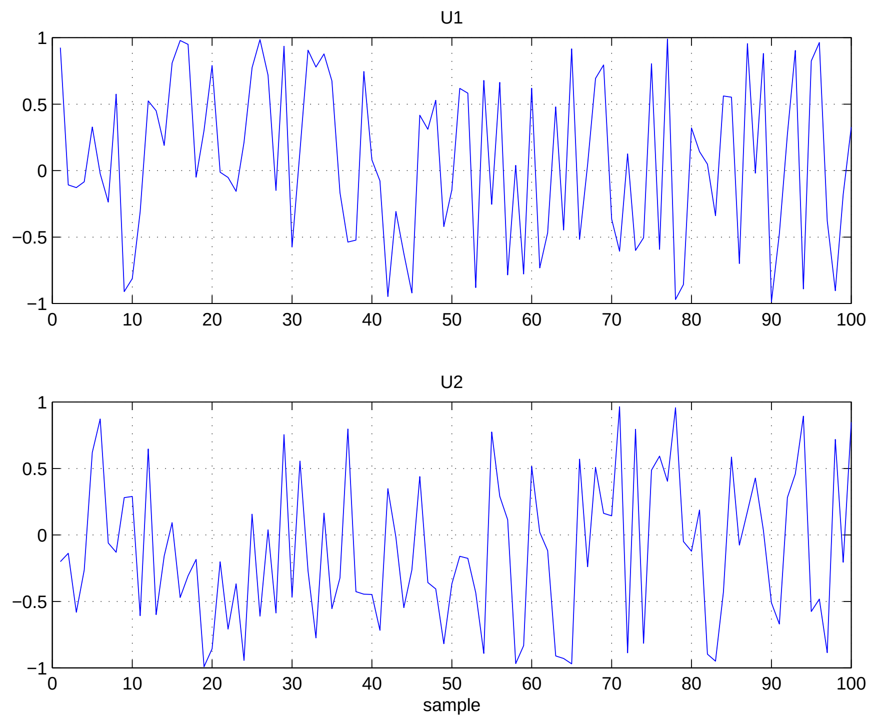


FIGURE 1. Input data for identification.

To valid the established model, we apply this input vector

$$[\sin(2\pi k/25),\ \cos(2\pi k/25)]^\top.$$

The responses of the plant and the identification model to these inputs are shown in Figure 3.

These responses are obtained from local models computed by fuzzy clustering without adaptation of the consequent parameters. It is noticed that the estimated outputs can't follow the process's outputs and the error resultant is rather big. To improve quality of the established fuzzy model, the parameters of the consequences of the rules are adapted by a recursive least squares algorithm with forgetting factor ($\lambda = 0.99$) according to (22). Figure 4 shows the responses of the plant and the identification model for a vector input $[\sin(2\pi k/25),\ \cos(2\pi k/25)]^\top$ but with adaptation:

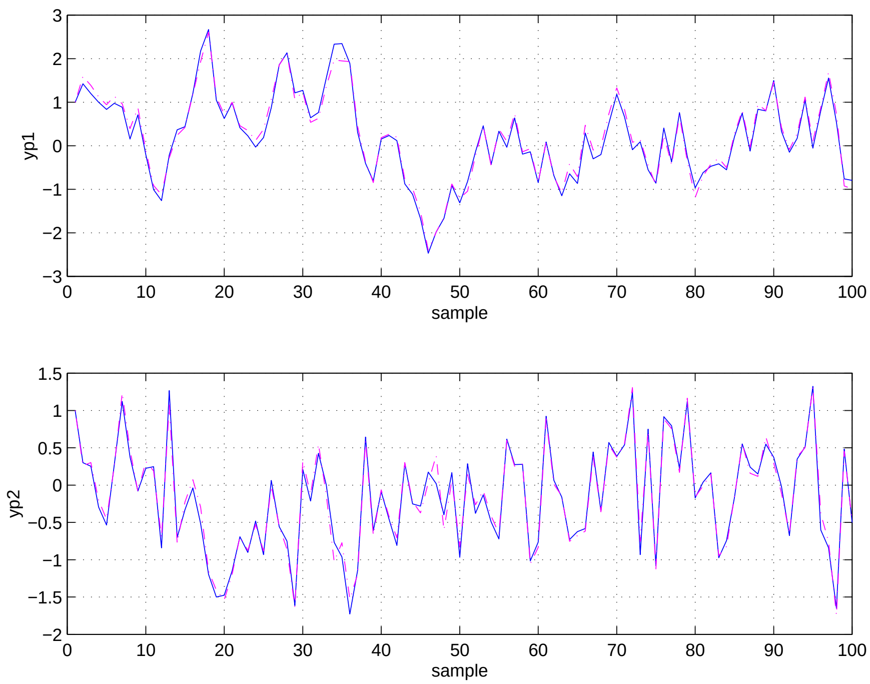


FIGURE 2. Comparison of the process output (solid line) with the fuzzy model output (dashed-dotted line) for identification.

We define a function VAF which computes the percentile *Variance Accounted For* between two signals as follows:

$$VAF = 100\% \left[1 - \frac{var(y_1 - y_2)}{var(y_1)}\right].$$

y_1 is the output of the process and y_2 is the output of the model. The VAF of two equal signals is 100%. If the signals differ, VAF is lower.

Table 1 gives the VAF performance indices for the responses of the plant and the identification model in the identification phase, the validation phase without adaptation and in the validation phase with adaptation.

From Table 1, we can see that the adapted fuzzy TS model is more accurate than a non-adapted fuzzy TS model.

To verify the applicability of our method to real process, we must show the evolution of parameters of polynomial A_i ,B_i and α_i during the adaptation. In fact, the parameters of polynomial B_i correspond to the gain of

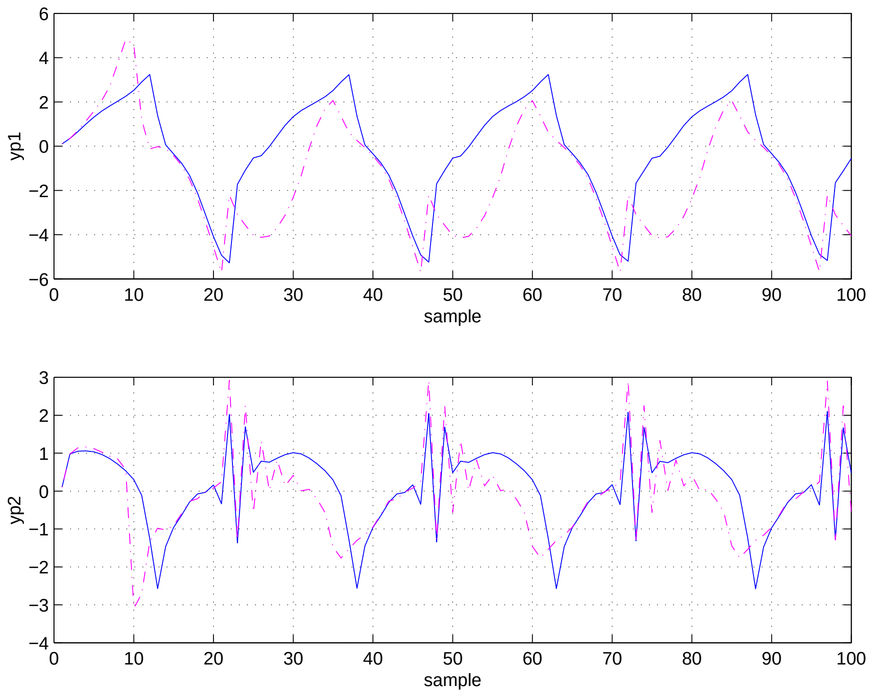


FIGURE 3. Comparison of the process output (solid line) with the fuzzy model output (dashed-dotted line) for validation.

TABLE 1. Comparison of the prediction accuracy of the TS Fuzzy model in three phases.

	Identification	Validation without adaptation	Validation with adaptation
VAF _yp1 (%)	98.43	49.79	88.81
VAF _yp2 (%)	98.38	38.61	69.62

command u_1 and u_2 which must not change their values in great proportions.

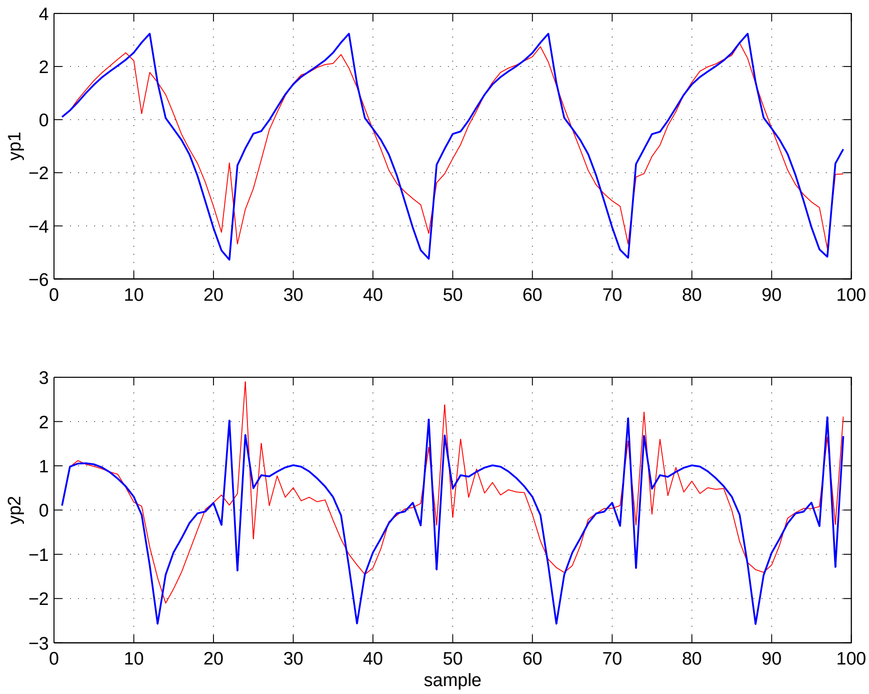


FIGURE 4. Comparison of the process output and the fuzzy model output with parameters's adaptation.

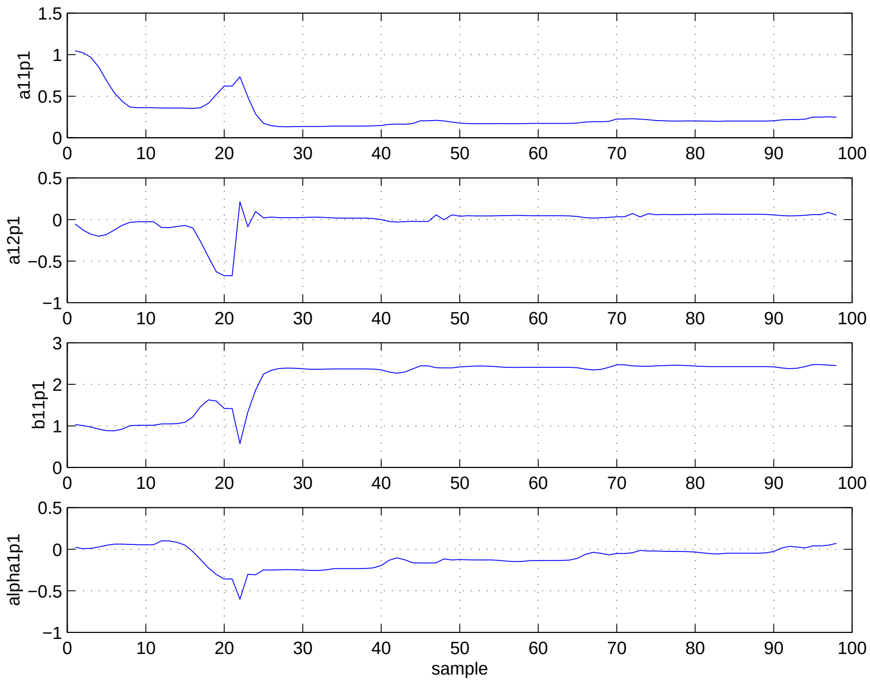
The rules are linear conclusions of system's inputs, for example for the rule i:

$$\begin{aligned} y_{p1i}(k+1) &= a_{i1p1}(k)y_{p1i}(k) + a_{i2p1}(k)y_{p2i}(k) + b_{i1p1}(k)u_1(k) + \alpha_{ip1} \\ y_{p2i}(k+1) &= a_{i1p2}(k)y_{p1i}(k) + a_{i2p2}(k)y_{p2i}(k) + b_{i2p2}(k)u_{2p2}(k) + \alpha_{ip2} \end{aligned}$$

Figure 5 shows the evolution of parameters a_{i1p1}, b_{i1p1} and α_{ip1} of the first output y_{p1} for the three clusters (rules):

Figure 6 shows the evolution of parameters a_{i1p2} ,b_{i1p2} and α_{ip2} of the second output y_{p2} for the three rules:

From Figure 5 and Figure 6, we can notice that the linear parameters vary until the sample 30. After that, their variation is weak. They are practically constant.



Rule 1

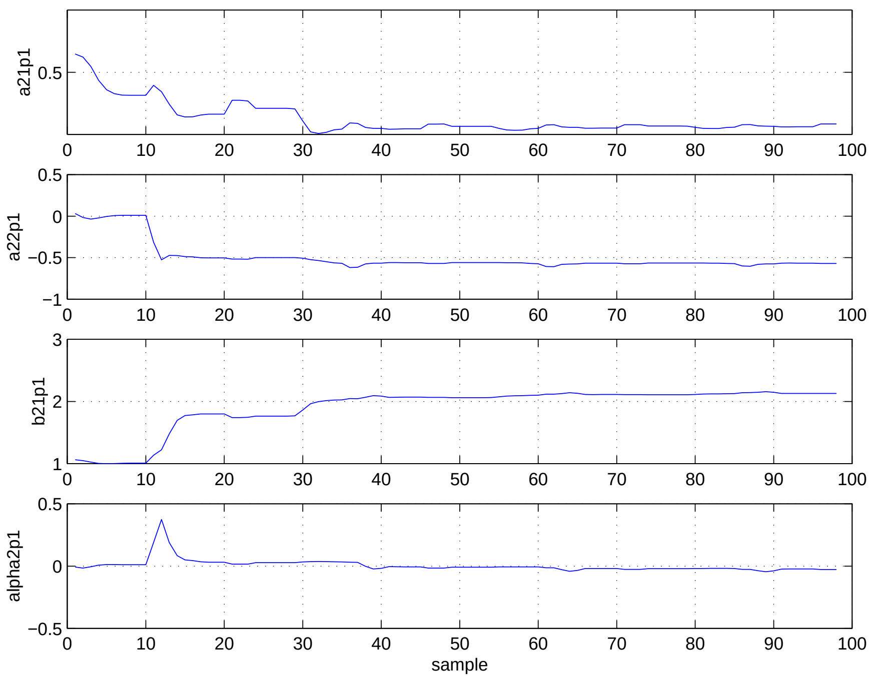
FIGURE 5. Evolution of parameters a_{i1p1}, b_{i1p1}, and α_{ip1}.

5. CONCLUSIONS

This paper proposes a study on the extension of the application of fuzzy clustering to the identification of Takagi-Sugeno (TS) fuzzy models. These local models correspond to the different rules generated automatically which have variable consequent linear parameters in contrast to the common approach using fixed consequent linear parameters.

The performance of the proposed modeling technique was demonstrated on benchmarks from the literature.

The results obtained are satisfactory and we think that we are able to more improve the fuzzy TS model by the automatic determination of the number of clusters and the optimal value of the forgetting factor. This will be the object of our forthcoming work.



Rule 2
Fig. 6 (*Continued*).

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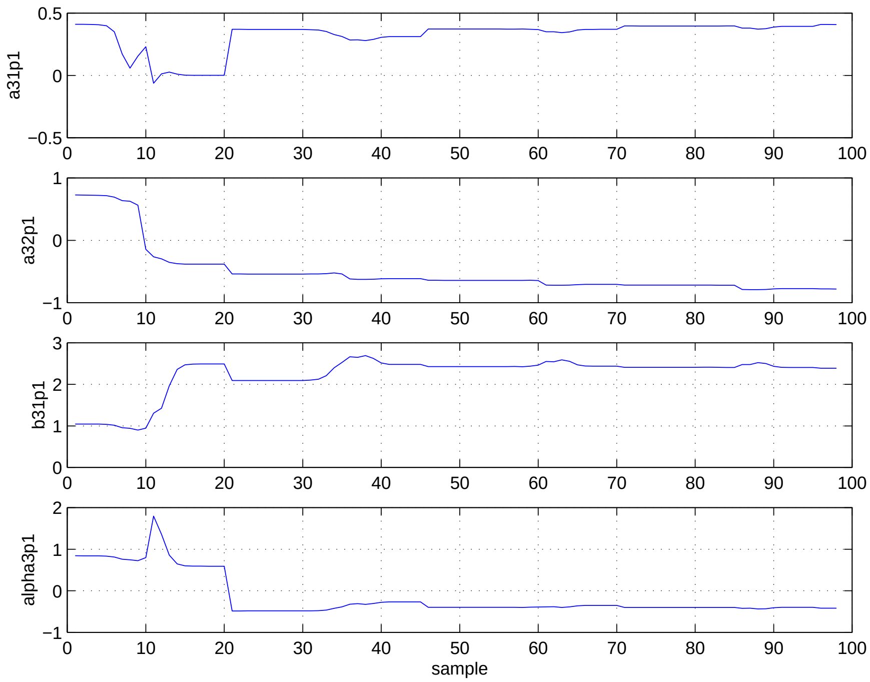

Rule 3

Fig. 6 (*Continued*).

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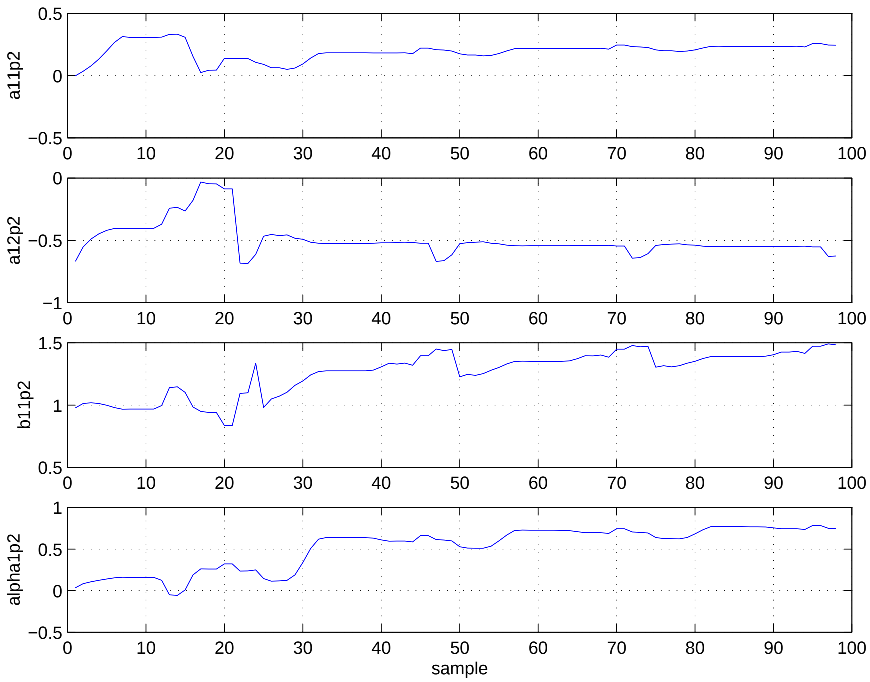

Rule 1

FIGURE 6. Evolution of parameters a_{i1p2}, b_{i1p2}, and α_{ip2}.

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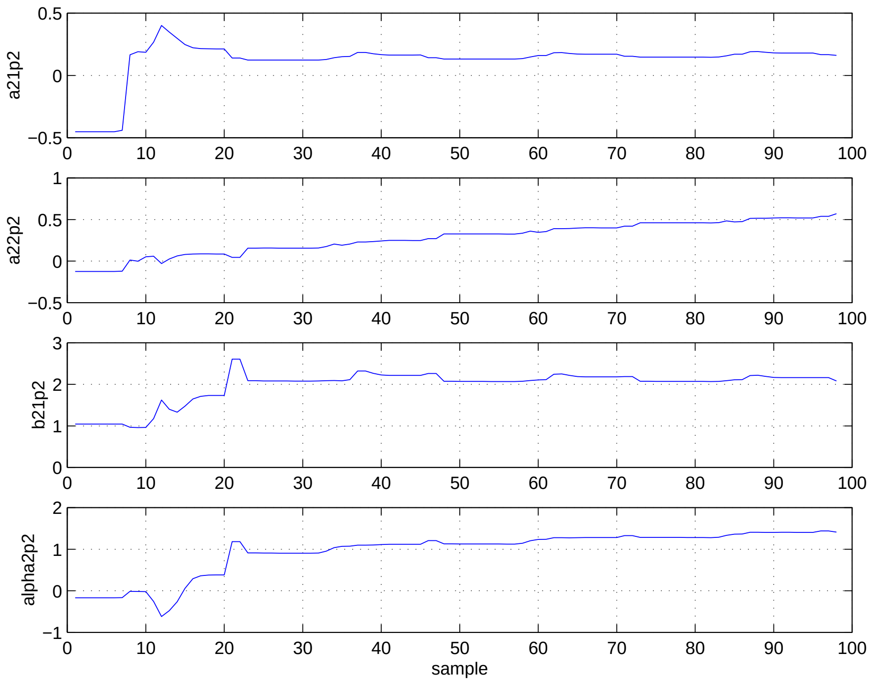

Rule 2
Fig. 6 (*Continued*).

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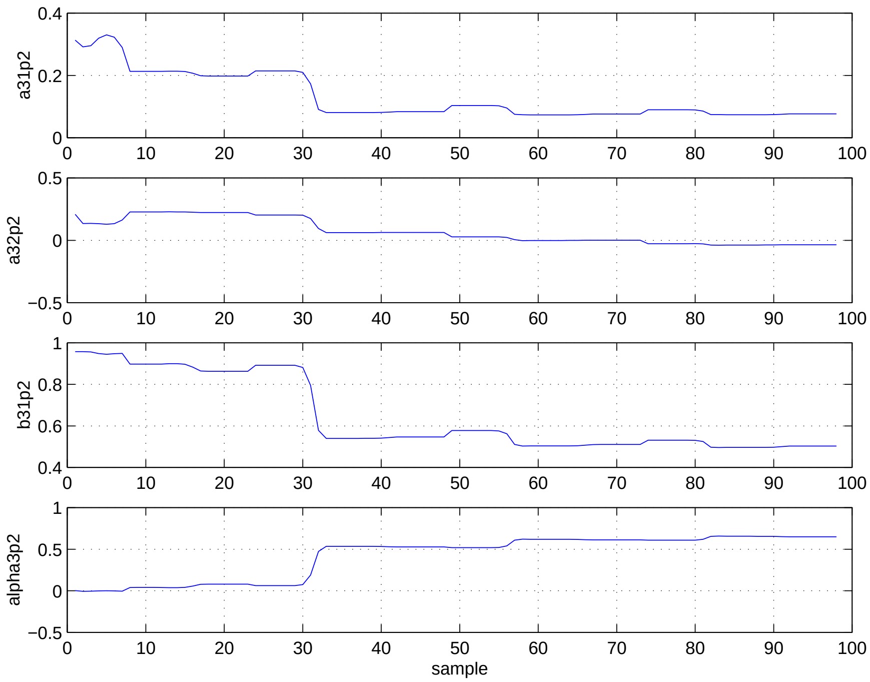

Rule 3
Fig. 6 (*Continued*).