Identifying Proportional and Non-Proportional Relationships in Graphs Lesson

Lesson by Common Core, Inc., on behalf of New York State Education Department, annotation by Student Achievement Partners

GRADE LEVEL Seventh

IN THE STANDARDS 7.RP.A.2A

WHAT WE LIKE ABOUT THIS LESSON

Mathematically:

- Requires students to make connections between the different representations of a situation, and requires students to create tables from ratios in various formats
- Requires students to use multiple methods and reasoning to determine whether a relationship is proportional; provides examples and non-examples
- Allows students to draw conclusions based on mathematical evidence

In the classroom:

- Offers an engaging exploration that connects students' procedural skill and conceptual understanding to real-life situations
- Gives students the opportunity to work collaboratively in groups
- Provides students with an opportunity to critique each other's work
- Gives formal and informal opportunities for teachers to check for understanding
- Includes a problem set that can be used for homework or for additional practice, as well as an exit ticket that summarizes the mathematics of the lesson

MAKING THE SHIFTS[1]

Focus	Belongs to the major work[2] of seventh grade
Coherence	Builds on key understandings of ratios, rates, and unit rates (6.RP.A), and prior understanding of proportional relationships in grade 7
Rigor[3]	This lesson touches on all three aspects of rigor: conceptual understanding, procedural skill and fluency, and application.

[1]For more information read Shifts for Mathematics.
[2]For more information, see Focus in Grade Seven in the Supplemental Resources below.
[3]Lessons may target one or more aspect(s) of rigor.

ADDITIONAL THOUGHTS

It's important to note that this sample lesson is the last of a 6-lesson series on "Proportional Relationships", which is part of a 22-lesson unit on Ratios and Proportional Relationships. This sample lesson lays a strong foundation for the work that is to come in the unit, but it is not intended for students to meet the full expectations of the standards through only this lesson. In subsequent lessons, students explore ratios and rates involving fractions, as well as ratios of scale drawings.

In this particular lesson, students work in groups to demonstrate their understanding of proportional relationships. The "art gallery" provides an opportunity for students to showcase their work and thinking, as well as to reflect on each other's representations and reasoning. This lesson could be strengthened by concluding the lesson with questions that ask students to discuss and compare various strategies for determining the proportionality of relationships and having them make connections between the different representations of the situations.

This activity has students think about the structure of the situations to determine whether a proportional relationship exists, as opposed to using the traditional method of "cross-multiplying" for solving proportions ($a/b = c/d$). For more insight on the grade-level concepts addressed in this lesson, read page 8 of the progression document, Grade 6–7, Ratios and Proportional Relationships.

The structure of these lessons and the unit overall have some interesting aspects to highlight. Each unit is divided into topics (a set of lessons) that are connected to prior learning and also point to the lesson that follows in the learning progression. Within individual lessons, there are a number of components that add to their strength including variety in questioning techniques and frequent opportunities for students to debrief about their learning. Through the series of lessons, students have the opportunity to engage in all three aspects of rigor.

Topic A:

Proportional Relationships

7.RP.2a

Focus Standard:	7.RP.2a	Recognize and represent proportional relationships between quantities. a. Decide whether two quantities are in a proportional relationship, e.g., by testing for equivalent ratios in a table or graphing on a coordinate plane and observing whether the graph is a straight line through the origin.
Instructional Days:	6	
Lesson 1:	An Experience in Relationships as Measuring Rate (P)[1]	
Lesson 2:	Proportional Relationships (P)	
Lessons 3–4:	Identifying Proportional and Non-Proportional Relationships in Tables (P)	
Lessons 5–6:	Identifying Proportional and Non-Proportional Relationships in Graphs (E)	

In Lesson 1 of Topic A, students are reintroduced to the meanings of value of a ratio, equivalent ratios, rate, and unit rate through a collaborative work task where they record their rates choosing an appropriate unit of rate measurement. In Lesson 2, students conceptualize that two quantities are proportional to each other when there exists a constant such that each measure in the first quantity multiplied by this constant gives the corresponding measure in the second quantity (**7.RP.2**). They then apply this basic understanding in Lessons 3–6 by examining situations to decide whether two quantities are in a proportional or non-proportional relationship by first checking for a constant multiple between measures of the two quantities, when given a table, and then by graphing on a coordinate plane. Students recognize that the graph of a proportional relationship must be a straight line through the origin (**7.RP.2a**).

[1] Lesson Structure Key: **P**-Problem Set Lesson, **M**-Modeling Cycle Lesson, **E**-Exploration Lesson, **S**-Socratic Lesson

Lesson 6: Identifying Proportional and Non-Proportional Relationships in Graphs

Student Outcomes

- Students examine situations carefully to decide whether two quantities are proportional to each other by graphing on a coordinate plane and observing whether all the points would fall on a line that passes through the origin.
- Students study examples of relationships that are not proportional as well as those that are.

Today's activity is an extension of Lesson 5. You will be working in groups to table, graph and identify whether or not the two quantities are proportional to each other.

Classwork

Preparation (7 minutes)

Place students in groups of four. Hand out markers, poster paper, graph paper, and envelopes containing 5 ratios each. (Each group will have identical contents.)

- Have recorder fold the poster paper in quarters and label as follows: Quad 1- Table, Quad 2- Problem, Quad 3- Graph, Quad 4- Proportional or Not?, Explain.

MP.1 & MP.2

- Instruct the reader to take out the contents of the envelopes and the group to arrange them on a table and graph.
- Instruct the reader to state the problem. Students use multiple methods to show whether the quantities represented in the envelope are proportional to each other or not.

Collaborative Work (20 minutes)

Within the groups, give students 15 minutes to discuss the problem and record their responses onto the poster paper. For the last 5 minutes, have groups adhere their posters on the wall and circulate around the room looking for the group that has the same ratios. Have groups with the same ratios identify and discuss the differences of their posters.

Art Gallery (8 minutes)

In groups, have students observe each poster, write any thoughts on sticky notes and adhere them to the posters. Also, have students answer the following questions on their worksheets:

- Were there any differences found in groups that had the same ratios?
- Did you notice any common mistakes? How might they be fixed?
- Was there a group that stood out by representing their problems and findings exceptionally clearly?

Closing (10 minutes)

- Why make posters with others? Why not do this activity in your student books?
  - *We can dialogue with others and learn from their thought processes. When we share information with others, our knowledge is tested and questioned.*
- What does it mean for a display to be both visually appealing and informative?
  - *For a display to be both visually appealing and informative, the reader should be able to find data and results fairly quickly and somewhat enjoyably.*
- How much time did your group spend on the content of your poster, and how much time was spent making it visually appealing? What factors determined these time lengths?
  - *The discussion and dialogue take the most time and then the outline of the poster the next.*
- Suppose we invited people from another school, state or country to walk through our gallery. Would they be able to learn about ratio and proportion from our posters?
  - *Hopefully, after looking through the series of posters, people can learn and easily determine for themselves if graphs represent proportional and non-proportional relationships.*

Lesson Summary

Graphs of Proportional Relationships: The graph of two quantities that are proportional fall on a straight line that passes through the origin.

Exit Ticket (5 minutes)

Lesson 6: Identifying Proportional and Non-Proportional Relationships in Graphs

Ratios for Groups

Cut and place in labeled envelopes prior to instructional time.

Group 1 A local frozen yogurt shop is known for their monster sundaes to be shared by a group. The ratios represent the number of toppings to total cost. Create a table then graph and explain if the quantities are proportional to each other or not.	Group 2 The school library receives money for every book sold at the school's book fair. The ratios represent the number of books sold to the amount of money the library receives. Create a table then graph and explain if the quantities are proportional to each other or not.	Group3 Your uncle just bought a hybrid car and wants to take you and your siblings camping. The ratios represent the number of gallons remaining to hours of driving. Create a table then graph and explain if the quantities are proportional to each other or not.	Group 4 For a Science project Eli decided to study colonies of mold. He observed a piece of bread that was molding. The ratios represent the number of days passed to colonies of mold on the bread. Create a table then graph and explain if the quantities are proportional to each other or not.
4 to 0	1 to 5	8 to 0	1 to 1
6:3	2 to 10	After 1 hour of driving, there are 6 gallons of gas left in the tank.	2 to 4
8/6	The library received $15 for selling 3 books.	4 : 4	3:9
The total cost of a 10-topping sundae is $9.	4:20	2 to 7	4/16
12 to 12	5: 25	0/8	Twenty five colonies were found on the fifth day.

Group 5	Group 6	Group 7	Group 8
For a Science project Eli decided to study colonies of mold. He observed a piece of bread that was molding. The ratios represent the number of days passed to colonies of mold on the bread. Create a table then graph and explain if the quantities are proportional to each other or not.	Your uncle just bought a hybrid car and wants to take you and your siblings camping. The ratios represent the number of gallons remaining to hours of driving. Create a table then graph and explain if the quantities are proportional to each other or not.	The school library receives money for every book sold at the school's book fair. The ratios represent the number of books sold to the amount of money the library receives. Create a table then graph and explain if the quantities are proportional to each other or not.	A local frozen yogurt shop is known for their monster sundaes to be shared by a group. The ratios represent the number of toppings to total cost. Create a table then graph and explain if the quantities are proportional to each other or not.
1 to 1	8 to 0	1 to 5	4 to 0
2 to 4	After 1 hour of driving, there are 6 gallons of gas left in the tank.	2 to 10	6:3
3:9	4 : 4	The library received $15 for selling 3 books.	8/6
4/16	2 to 7	4:20	The total cost of a 10-topping sundae is $9.
Twenty five colonies were found on the fifth day.	0/8	5: 25	12 to 12

Collaborative Work Sample Solutions

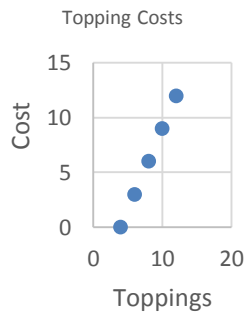
Group 1 and 8

Problem: *A local frozen yogurt shop is known for their monster sundaes. Create a table then graph and explain if the quantities are proportional to each other or not.*

Table:

Number of Toppings	*Cost of Toppings ($)*
4	*0*
6	*3*
8	*6*
10	*9*
12	*12*

Graph:



Explanation: *Although the graph lies in a straight line, the quantities are not proportional to each other because the line does not go through the origin. Each topping does not have the same unit cost.*

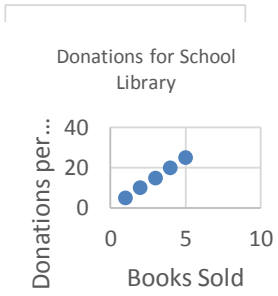
Group 2 and 7

Problem: *The school library receives money for every book sold at the school's book fair. Create a table then graph and explain if the quantities are proportional to each other or not.*

Table:

Number of Books Sold	*Donations per Sponsor ($)*
1	*5*
2	*10*
3	*15*
4	*20*
5	*25*

Graph:



Explanation: *The quantities are proportional to each other because the points lie in a straight line and go through the origin. Each book sold brings in $5.00 no matter how many books are sold.*

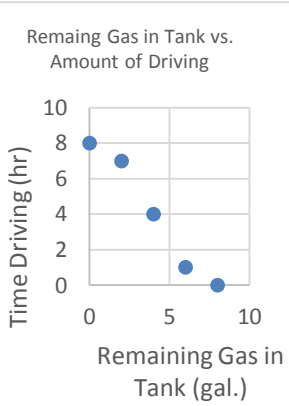
Group 3 and 6

Problem: *Your uncle just bought a hybrid car and wants to take you and your siblings camping. Create a table then graph and explain if the quantities are proportional to each other or not.*

Table:

Gallons of Gas left in tank	*Hours of Driving*
8	*0*
6	*1*
4	*4*
2	*7*
0	*8*

Graph:



Explanation: *The graph is not a straight line passing through the origin so the quantities are not proportional to each other. The number of gallons of gas vary depending on how fast or slow the car is driven.*

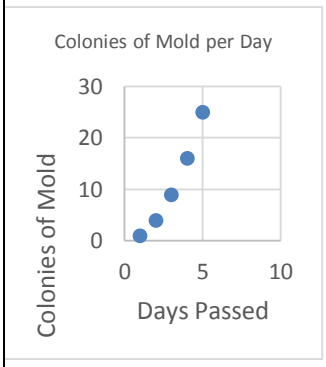
Group 4 and 5

Problem: *For a Science project Eli decided study colonies of mold. He observed a piece of bread that was molding. Create a table then graph and explain if the quantities are proportional to each other or not.*

Table:

Number of Days	*Colonies of Mold*
1	*1*
2	*4*
3	*9*
4	*16*
5	*25*

Graph:



Explanation: *Although the graph looks as though it goes through the origin, the quantities are not proportional to each other because the points do not lie on a straight line. Each day does not produce the same amount of colonies as the other days.*

Name _____ Date_____

Lesson 6: Identifying Proportional and Non-Proportional Relationships in Graphs

Exit Ticket

1. Which graphs in the art gallery walk represented proportional relationships and which did not? List the group number.

Proportional Relationship	Non-proportional Relationship

2. What are the characteristics of the graphs that represent proportional relationships?

3. For the graphs representing proportional relationships, what does (0,0) mean in the context of the given situation?

Exit Ticket Sample Solutions

The following solutions indicate an understanding of the objectives of this lesson:

1. **Which graphs in the art gallery walk represented proportional relationships and which did not? List the group number.**

Proportional Relationship	Non-proportional Relationship
Group 2	*Group 1*
Group 7	*Group 3*
	Group 4
	Group 5
	Group 6
	Group 8

2. **What are the characteristics of the graphs that represent proportional relationships?**

   Graphs of groups 2 and 7 fall in a straight line and go through the origin.

3. **For the graphs representing proportional relationships, what does (0,0) mean in the context of the situation?**

   For zero books sold, the library received zero dollars of donations.

Problem Set Sample Solutions

1. **Sally's aunt put money in a savings account for her on the day Sally was born. The savings account pays interest for keeping her money in the bank. The ratios below represent years to amount of money in her savings account.**

   - **After one year, the interest had accumulated, and the total was $312 in Sally's account.**
   - **After three years, the total was $340. After six years, the total was $380.**
   - **After nine years, the total was $430. After 12 years, the total amount in Sally's savings account was $480.**

   Using the same four-fold method from class, create a table then graph, and determine whether the amount of money accumulated and time elapsed are proportional to each other or not. Use your table and graph to support your reasoning.

Problem: ***Sally's aunt put money in a savings account for her on the day Sally was born. The savings account pays interest for keeping her money in the bank. The ratios represent years to amount of money in her savings account. Create a table then graph and explain if the quantities are proportional to each other or not.***

Table:

Years	*Savings ($)*
1	*312*
3	*340*
6	*380*
9	*430*
12	*480*

Graph:

Explanation: ***The graph is not proportional because although it seems to pass through a line, it is not a line that goes through the origin. The amount of interest collected is not the same every year.***

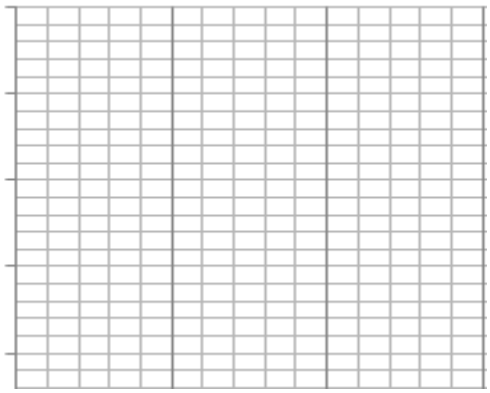
Lesson 6: Identifying Proportional and Non-Proportional Relationships in Graphs

Today's activity is an extension of Lesson 5. You will be working in groups to table, graph and identify whether or not the two quantities are proportional to each other.

Classwork

Poster Layout

Use for notes

Problem	Table
Graph 	Proportional or not? Explain.

Gallery Walk: Take notes and answer the following questions:

-
-
-

Poster 1:

Poster 2:

Poster 3:

Poster 4:

Lesson Summary:

Graphs of Proportional Relationships: The graph of two quantities that are proportional fall on a straight line that passes through the origin.

Note about Lesson Summary

Problem Set

1. Sally's aunt put money in a savings account for her on the day Sally was born. The savings account pays interest for keeping her money in the bank. The ratios below represent years to amount of money in her savings account.
   - After one year, the interest had accumulated and the total was $312 in Sally's account.
   - After three years, the total was $340. After six years, the total was $380.
   - After nine years, the total was $430. After 12 years, the total amount in Sally's savings account was $480.

   Using the same four-fold method from class, create a table then graph and determine whether the amount of money accumulated and time elapsed are proportional to each other or not. Use your table and graph to support your reasoning.