Ideological bias and the production of macroeconomic theories

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[PRELIMINARY AND INCOMPLETE]

1 Introduction

The formation of expectations plays a key role in our understanding of the macroeconomy. Historically, economists have moved from a naive, mechanical representation of expectations to a more sphisticated one, where rational agents optimally use their information to forecast the future.

To be able to do so, agents need to use a model, which allows them to compute the expectations of the relevant variables that they need in order to make their decisions. Typically, in the rational expectations literature, it is assumed that one uses the correct model.

In practice, though, the "correct model" is unknown, and, to the extent that it is inevitably an abstraction, the concept of "correct model" is probably meaningless. Instead, we observed different models produced by different economists. Depending on the model one is using, one will act differently. Hence, in such a world, economists have substantial influence over macroeconomic outcomes: they can manipulate them by designing their theory appropriately.

Of course, no economist will ever concede that he or she is motivated by a political or personal agenda. Instead, they would argue that they are pursuing truth in a disinterested fashion. Yet it is not difficult to find a correlation between an economist's personal and political background and the nature of his vision. [Keynes, Hayek]

This does not mean that the expert can say anything he wants. The models been produced must be "credible", in that their predictions fit the data. But, if the expert is unflential, the data will themselves reflect the fact that people use his model to make their decisions. I define a model as "autocoherent" if, conditional on people using it to form expectations, it replicates the joint distribution of the observables. A natural restriction to impose on a model is to be autocoherence. Otherwise, people will eventually abandon it.

When will the expert be able to affect outcomes despite the requirement that his model is autocoherent? Here we have to distinguish between three cases.

First, it may be that all the variables whose expectations matter for private decisions are observable. This means that people do not really need a structural model. All they need to know is the joint distribution between the forecasted variables and the variables in their information set when they form their expectations. One can then solve for a rational expectations equilibrium in a standard way, replacing forecasts by expectations using the actual equilibrium distribution of the variable. If this procedure yields a unique equilibrium, then the economy must be at this equilibrium. This does not mean that one could use several alternative models. But all those models must be autocoherent, and therefore replicate the equilibrium distribution of the observables, implying that one must be at an REE. Since that REE is unique, all autocoherent models are equivalent in that they deliver the same REE.

Second, it may be that the variables that need to be forecast are observable, but that the REE is not unique. One can show (see Saint-Paul (2010) for a simple example) that the expert can then devise a model with a unique equilibrium, such that if everybody uses that model, then (i) the economy settles at the REE preferred by the expert, and (ii) the model is autocoherent. Economists then craft their theories so as to induce the economy to pick the REE they want.

Finally, it may be that the variables of interest are not observed. Different models will relate them differently to the observables, and thus lead to different inferences about those observables. But the models have the consistency requirements that they explain the observables. In such a case, the 'true' structural model is underidentified but which structural model is used affects expectations and thus the behaviour of the economy. Economists can influence those outcomes by proposing alternative structural models; these alternative models are equally good in that they are all autocoherent, but, contrary to the first case, which model is used matters because it will change the expectations of the relevant variables.

This paper investigates, in a simplified macro context, the joint determination of the prevailing model and the equilibrium. I assume that the model is designed by an economist who has his own preferences and knwos the true structural model. This model influences both the people and the government; while the people need to know future prices and can just use the distribution of these prices to form expectations, the government tries to stabilize an unobserved demand shock and will make different inferences about that shock depending on the model it uses. People care about output stability but also the stability of government spending. The greater the loss from government spending volatility, the more "conservative" the individual. I then study how the models devised by the economists varies depending on whether they are "progressive" vs. "conservative".

In the present paper, there is a single expert who sets the theory (intellectual monopoly). In related work (2011b), I also study the case of intellectual competition, when several schools design different models, and each of them influences only a fraction of the population.

The predictions depend greatly on the specifics of the economy being considered. But in many cases, they are plausible. For example, conservative economists will tend to report a lower keynesian multiplier, and a greater long-term inflationary impact of output expansions. On then other hand, the economists' margin of manoeuver is constrained by the autocoherence conditions. In one example, the price to be paid for reporting a too high inflationary cost of output is that one should report a too low relative variance of aggregate supply shock. Hence because of the autocoherence conditions, one cannot always be uniformly conservative or liberal, relative to the correct model, when picking parameters.

The paper is related to several strands of literature. But unlike most of this literature, it focuses on how a given equilibrium is supported by a *theory* $--$ i.e. a formal representation of the world – instead of just *beliefs* – i.e. assumptions about what happens as a consequence of my decisions – and studies which theories arise in equilibrium when the theorists are self-interested.

In an important paper by Piketty (1995), a redistributive problem is studied where people may form different beliefs about the effort elasticity of income. Because of the feedback effects of these beliefs on taxation, they are self reinforcing and multiple equilibria may arise. The concept of autocoherence is related to that of self-confirming equilibria by Fudenberg and Levine (1993, 2007). Such equilibria are supported by beliefs that are true along the equilibrium path but not off the equilibrium path. Here, similarly, if people can adopt random policies (or random expectations), they could observe more of the structural parameters. But the fact that policies are systematically correlated with observed signals reduces the dimension of the set of observed outcomes, increases underidentification and thus makes manipulation by experts more feasible.

Recently, there has been renewed interest on the fact that the true model may be unknown and how this affects the optimal policy (see Hansen et al (2006)). Sargent () evaluates how the model used by the central bank has evolved over time. How beliefs affect policies and how they evolve is also discussed by Buera et al () and Saint-Paul (2010).

Finally the paper is also related to the literature on cheap talk (Crawford and Sobel, 1982). Here, however, a totally different route is taken. In the cheap talk literature, the preferences of the expert are known and any signal can be reverse engineered into the true value of the parameter. However for such reverse engineering to take place, one needs to know the relevant probability distributions in addition to the expert's preferences, that is, one needs a *model.* Since this model can only be obtained by an expert, some expert must be trusted. Here, the expert is trusted, and his preferences are not known. While in the cheap talk literature the expert can only send unbiased signals, here what is constraining him instead is the set of autocoherence conditions: while the signals (i.e. the models' parameters reported by the expert) can be biased, the model's predictions are not falsified in equilibrium.

2 A simple example

I start by considering a simple example of stabilization policy. The economy is driven by the following process:

$$y = ag + u + v; \tag{1}$$

$$z = \omega u + \varepsilon. \tag{2}$$

Here, y is output, g is government spending, and u and v are shocks effecting output. For example, we can think of u as an aggregate demand shock. The variable z is a signal about the state of aggregate demand, which is observed prior to the government deciding on the expenditure level g. It could be some leading indicator such as a business or consumer confidence survey, order or vacancies data, and so forth. By contrast, the shock v cannot be stabilized because no signal of v is drawn by the government prior to setting policy., We will label it a 'supply' shock to distinguish it from u.

The most relevant parameter is a, which can be labelled "the Keynesian Multiplier". As will be clear below, most ideological conflict revolves around its actual value.

The shocks u , v, and ε are uncorrelated and have zero mean and variances σ_u^2, σ_v^2, and σ_ε^2, respectively. To economize on notation, I will impose the following normalization

$$\omega^2 \sigma_u^2 + \sigma_\varepsilon^2 = 1.$$

The government wants to stabilize output but suffers a cost for fiscal activism. Its preferences are

$$\min Ey^2 + \varphi E g^2.$$

The greater φ, the more the government is "right-wing" and averse to fiscal interventions.

In order to figure out how to set g, given the value of z it observes, the government must have *model* which predicts, in particular, how g affects y. In most of the literature, all agents use the right model. Here I am assuming that the model used by the government may be wrong. Thus, while the true model is summarized by $(a, \omega, \sigma_u^2, \sigma_v^2, \sigma_\varepsilon^2)$, the government believes that these parameters are in fact given by $(\hat{a}, \hat{\omega}, \hat{\sigma}_u^2, \hat{\sigma}_v^2, \hat{\sigma}_\varepsilon^2)$. I will refer to this model as the *perceived* model, as opposed to the correct one. I will describe below how the perceived model is determined.

In general one may want to impose plausibility limits on the perceived model parameters instead of allowing any possible value. In this model and the richer model of the next section I will impose that each coefficient has the same sign as its counterpart in the actual model. This means that all these parameters must be positive (for the variances this is actually a feasibility constraint rather than a plausibility one). More generally there is a set of admissible values for the perceived model's parameters. I will refer to the inequalities that define this set as the "plausibility conditions". The correct model's parameters always match those condiitons.

Under the correct model, the government sets a stabilization rule $g(z)$ which is the solution of the first order condition

$$aE(y \mid z) + \varphi g(z) = 0.$$

Furthermore

$$E(y \mid z) = ag(z) + E(u \mid z),$$

and by Bayes' law

$$E(u \mid z) = \frac{1}{\omega} \frac{\omega^2 \sigma_u^2 z}{\omega^2 \sigma_u^2 + \sigma_\varepsilon^2}.$$

Under the perceived model, the stabilization rule satisfies

$$\hat{a}\hat{E}(y \mid z) + \varphi g(z) = 0,$$

where $\hat{E}$ denotes mathematical expectation computed using the perceived model. Here we have $\hat{E}(y \mid z) = \hat{a}g(z) + \hat{E}(u \mid z)$ and therefore the optimal stabilization policy satisfies

$$g(z) = -\frac{\hat{a}\hat{E}(u \mid z)}{\hat{a}^2 + \varphi}. \tag{3}$$

To compute $\hat{E}(u \mid z)$, the government applies Bayes' law using the perceived model. Therefore,

$$\hat{E}(u \mid z) = \frac{\hat{\omega}\hat{\sigma}_u^2}{\hat{\omega}\hat{\sigma}_u^2 + \hat{\sigma}_\varepsilon^2} z = \hat{\mu} z. \tag{4}$$

2.1 Equilibrium

Given the perceived model, it is straightforward to compute the equilibrium by substituting (3) into (1)and using (2):

$$y = u\left(1 - \frac{a\hat{a}\hat{\mu}\omega}{\hat{a}^2 + \varphi}\right) - \frac{a\hat{a}\hat{\mu}\varepsilon}{\hat{a}^2 + \varphi} + \hat{v}.$$

2.2 How is the perceived model determined?

I assume that the perceived model is produced by a school of professional economists. These economists are not disinterested but pursue their own agenda. That is, they want to design their model in such a way that the outcomes maximizes their utility function, which may be different from that of the government. Furthermore, I assume that they know the true model.

Why would the government believe the economists and not treat their predictions as cheap talk? The answer is that the government *has* to do something and has to use *some* model in order to design its policy. It cannot escape the necessity of trusting an expert and using his model. It is the production of such models by a trusted expert that the present paper analyses[1].

This does not mean that the economist can produce an arbitrary model. Any model must be consistent with the *data.* In some cases, the data will be rich enough (and the set of admissible models small enough) that the data will force identification of the correct model. Here, however, the model is underidentified, and which model is selected matters because different models lead to different policies.

Consistency with the data imposes restrictions on the models that the economists can produce. I will say that a model is *autocoherent* if, conditional on all agents using it to make their decisions, it correctly predicts the moments of the observables. This concept is akin to that of a self-confirming equilibrium in game theory (See Fudenberg and Levine (xxxx)), but autocoherence is a property of a model in addition to that of an equilibrium. In Saint-Paul (2011a), I provide some formal definitions and some results.

2.3 The autocoherence conditions

In the present case, people observe output y and the signal z. This means that in equilibrium, the variance-covariance matrix of (y, z) as predicted by the perceived model must be the one observed in the data.

The actual elements of that matrix are:

[1] In Crawford and Sobel (1982), an informed party observes a signal and can send a message to an uninformed one. The uninformed one knows the true distribution of the signal as well as the preferences of the informed party. In such a setting, any attempt to manipulate the recipient can be reversed engineered and equilibria are either fully revealing or partially revealing in an unbiased way (that is, the same message is being sent for a cluster of signals, and the recipient makes an unbiased inference conditional on the signal being in that cluster). Here the government does now know the right model nor does it know the experts' preferences. It needs to rely on some expert to be able to use a model and make a decision.

$$
\begin{aligned}
Ey^2 &= \left(1-\frac{a\hat{a}\hat{\mu}\omega}{\hat{a}^2+\varphi}\right)^2\sigma_u^2+\left(\frac{a\hat{a}\hat{\mu}}{\hat{a}^2+\varphi}\right)^2\sigma_\varepsilon^2+\sigma_v^2;\\
Ez^2 &= \omega^2\sigma_u^2+\sigma_\varepsilon^2=1;\\
Eyz &= \left(1-\frac{a\hat{a}\hat{\mu}\omega}{\hat{a}^2+\varphi}\right)\omega\sigma_u^2-\frac{a\hat{a}\hat{\mu}}{\hat{a}^2+\varphi}\sigma_\varepsilon^2.
\end{aligned}
$$

But people believe that the data are generated by the perceived model, in which case these moments would be equal to

$$
\begin{aligned}
\hat{E}y^2 &= \left(1-\frac{\hat{a}^2\hat{\mu}\hat{\omega}}{\hat{a}^2+\varphi}\right)^2\hat{\sigma}_u^2+\left(\frac{\hat{a}^2\hat{\mu}}{\hat{a}^2+\varphi}\right)^2\hat{\sigma}_\varepsilon^2+\hat{\sigma}_v^2;\\
\hat{E}z^2 &= \hat{\omega}^2\hat{\sigma}_u^2+\hat{\sigma}_\varepsilon^2;\\
\hat{E}yz &= \left(1-\frac{\hat{a}^2\hat{\mu}\hat{\omega}}{\hat{a}^2+\varphi}\right)\hat{\omega}\hat{\sigma}_u^2-\frac{\hat{a}^2\hat{\mu}}{\hat{a}^2+\varphi}\hat{\sigma}_\varepsilon^2\\
&= \frac{\varphi\hat{\omega}\hat{\sigma}_u^2}{\hat{a}^2+\varphi}.
\end{aligned}
$$

The autocoherence conditions are

$$
\begin{aligned}
Ey^2 &= \hat{E}y^2;\\
Ez^2 &= \hat{E}z^2;\\
Eyz &= \hat{E}yz.
\end{aligned}
$$

Computing, it can be seen that they are equivalent to

$$
\varphi(\hat{\sigma}_u^2\hat{\omega}-\sigma_u^2\omega) = \hat{a}(\hat{a}\omega\sigma_u^2-a\hat{\omega}\hat{\sigma}_u^2); \tag{5}
$$

$$
\hat{\sigma}_\varepsilon^2 = 1-\hat{\omega}^2\hat{\sigma}_u^2; \tag{6}
$$

$$
\hat{\sigma}_v^2 = \sigma_v^2+\frac{\hat{a}^2\hat{\omega}^2\hat{\sigma}_u^4}{(\hat{a}^2+\varphi)}(a^2-\hat{a}^2)+\sigma_u^2-\hat{\sigma}_u^2-\frac{2\hat{a}\hat{\omega}\sigma_u^2}{\hat{a}^2+\varphi}\left(a\omega\sigma_u^2-\hat{a}\hat{\omega}\hat{\sigma}_u^2\right) \tag{7}
$$

*

Hence, the autocoherence conditions leave the expert with two degrees of freedom. He can pick a triplet $(\hat{a},\hat{\omega},\hat{\sigma}_u)$ which satisfies (5) and then $\hat{\sigma}_\varepsilon$ and $\hat{\sigma}_v$ are determined residually by (6) and (7). More generally, in this class of linear models where all shocks and endogenous variables have a zero mean, and these means are common knowledge, if the dimension of the vector space spanned by the observables is n and there are p parameters, then there are $p - n(n+1)/2$ degrees of freedom in choosing the model. Here $n = 2$ and $p = 5$.

I assume that the economist's objective is similar to the policymaker's, but the weight on the stabilization of public expenditure is different. Thus the economist's objective is

$$\min Ey^2 + \bar{\varphi} Eg^2.$$

If $\bar{\varphi} > \varphi$, the economist is more "right-wing" than the government.

Given the linear quadratic structure of the problem, the optimal policy is of the form $g = -\gamma z$, and the policy problem amounts to picking γ. Given his two degrees of freedom, the economist is a quasi-dictator. That is, he can design his model so as to induce the government to select the value of γ that he would pick if he were setting γ directly. This value is clearly equal to

$$\gamma = \frac{-a\omega\sigma_u^2}{a^2 + \bar{\varphi}}.$$

Comparing with (3)-(4), we see that to induce this desired policy the economist must select a model which satisfies

$$\frac{-a\omega\sigma_u^2}{a^2 + \bar{\varphi}} = \frac{-\hat{a}\hat{\omega}\hat{\sigma}_u^2}{\hat{a}^2 + \varphi},$$

or equivalently

$$a\hat{a}(\hat{a}\omega\sigma_u^2 - a\hat{\omega}\hat{\sigma}_u^2) = \bar{\varphi}\hat{a}\hat{\omega}\hat{\sigma}_u^2 - \varphi a\omega\sigma_u^2. \tag{8}$$

This is an optimality condition for the model's parameters. Thus, we have a theory which predicts which models will prevail. There are the models that satisfy the autocoherence conditions (5)-(7) along with the optimality condition (8).

2.4 Properties of the equilibrium

Since we have 4 equations with 5 unknowns, there is still one degree of freedom. But ω and σ_u^2 only appear through their product. Thus this degree of freedom is irrelevant and I will now assume that $\omega = \hat{\omega} = 1$, in effect getting rid of parameter ω. The equilibrium value of $\hat{a}$ can then be obtained by substituting (5) into (8). This delivers the following cubic equation:

$$\bar{\varphi}\hat{a}^3 - (a\varphi)\hat{a}^2 + \varphi\bar{\varphi}\hat{a} - \varphi^2 a = 0. \tag{9}$$

Proposition 1 characterizes this equilibrium:

Proposition 1 – Assume

$$\bar{\varphi} \geq \frac{a\sqrt{\varphi}}{\sqrt{3}}.$$

Then there exists a unique non-negative solution $\hat{a}$ to (9). Furthermore, $\hat{a} = a$ if $\bar{\varphi} = \varphi$. Finally, $\frac{\partial \hat{a}}{\partial \bar{\varphi}} < 0$.

Proof– See Appendix.

Proposition 1 implies that if the economist has the same preferences as the government, then he will reveal the true model. A similar result obtains in communication games, but we will see below that this result breaks down in richer model where the public's expectations enter the model; as in the credibility literature, one may then want to mainpulate people even if everybody agrees on a common social welfare function.

The property that $\frac{\partial \hat{a}}{\partial \bar{\varphi}} < 0$ implies that the more right-wing (resp. left-wing) the economist relative to the government, the more he will understate (resp. overstate) the value of a. That is, conservative economists will produce theories where the Keynesian multiplier is low in order to deter activist policies, while left-wing ones will prefer to get a large Keynesian multiplier. The smaller the Keynesian multiplier, the more costly its is in terms of welfare to implement an activist policy (because of the aversion to public expenditure volatility in the government's preferences), and the less activist the policy. This is the reason why conservative economists have an interest in under-reporting the Keynesian multiplier, while left-wing ones want to over-report it.

However, this cannot be done independently of the rest of the theory, because the theory as a whole must match that data. The autocoherence condition can be rewritten

$$\hat{\sigma}_u^2 = \frac{\hat{a}^2 + \varphi}{a\hat{a} + \varphi}\sigma_u^2.$$

Clearly, we have $\hat{\sigma}_u^2 > \sigma_u^2$ for $\hat{a} > a$, and conversely for $\hat{a} < a$. Conservative economists downplay the contribution of demand shocks to GDP, while progressive ones overstate it. Why is that so?

Assume $\hat{a} < a$. Then the response of government spending to the demand shock u will have a stronger effect on output than what people believe. This means that government spending stabilizes output more than what people think, implying that the overall response of y to the demand shock u is weaker in reality than in the model used by the people. As such, this effect leads people to overestimate the covariance between y and z relative to the data. Similarly, output reacts more to the measurement error ε than what people believe. Since output reacts negatively to ε, this effect also induces people to overestimate the covariance between y and z. In order to compensate for those biases, the economist's model must underestimate σ_u^2 and accordingly overestimate σ_ε^2. This way, the positive contribution of the demand shock to Exy is being deflated, while the negative contribution of the measurement error is inflated. Consequently, these additional biases tend to offset the biases induced by the low value of $\hat{a}$ and restore the consistency between the predicted and actual values of Exy.

As for matching the variance of output, it can always be done by picking the appropriate variance of the 'supply' shocks σ_v^2.[2]

[2] As long as a is not too remote from $\hat{a}$, the model variance $\hat{\sigma}_v^2$ will remain positive.

We thus see how because of under-identification, the same evidence can be interpreted differently depending on the theorist's political preferences.

3 A richer model

In the preceding model, there is only one relevant economic actor: the government. The expert wants to induce a given government policy by promoting a certain model of the economy. In reality, other economic agents also make decisions. The models they use will also influence their behavior and this offers another way for the expert to influence the outcome.

To understand those issues, I extend the above model into a richer description of the economy. The model consists of three equations:

$$\begin{aligned} y &= -\beta i + \alpha g + u_0 + \theta v \\ i &= p + y \\ y &= \delta p - \mu p^e + v \end{aligned}$$

The additional endogenous variables are i, the interest rate, p, the price level, and p^e, the expected price level (to make the discussion more realistic I will interchangeably refer to p as the inflation rate). The economy is subjected to an aggregate demand shock u_0 and an aggregate supply shock v. The first equation is an "IS" curve, the second one can be interpreted as either an LM curve or a Taylor rule, and the third equation is an aggregate supply (or Phillips) curve. Note that the supply shock also affects aggregate demand. This makes it harder to identify the true model's parameters and raises the expert's degrees of freedom in designing his model. There is no dearth of theoretical mechanisms for supply shocks to affect aggregate demand as well; in most models greater productivity will change investment and consumption plans through its realtive price and wealth effects. The coefficients of the interest rate equation are assumed to be common knowledge and normalized to one for simplicity.

I assume $0 \leq \mu \leq \delta$. Roughly, δ is the slope of the short-run Phillips curve and $\delta - \mu$ is the slope of the long-run Phillips curve. If $\delta = \mu$, we have a Lucas supply curve, and there is no long-run trade-off between output and inflation. If $\mu = 0$, we have an old fashioned Phillips curve which ignores expectations. The output-inflation trade-off is more "favorable", the greater δ and the smaller μ. Thus we expect more "progressive" experts to favor models with large values of δ and small values of μ.

Eliminating interest rates, the model can be re-expressed as the following recursive form:

$$\begin{aligned} y &= -bp^e + ag + u + \rho v; & (10) \\ p &= \frac{\mu}{\delta} p^e - \frac{v}{\delta} + \frac{y}{\delta}. & (11) \end{aligned}$$

Here, a, b, and ρ are composite parameters given by

$$\begin{aligned} a &= \frac{\alpha\delta}{\delta + \beta(1+\delta)}; \\ b &= \frac{\beta\mu}{\delta + \beta(1+\delta)} \leq \mu; \\ \rho &= \frac{\beta + \theta\delta}{\delta + \beta(1+\delta)} \geq \frac{b}{\mu}. \end{aligned}$$

To save on notation, the aggregate demand shock is redefined as $u = \frac{\delta}{\delta + \beta(1+\delta)} u_0$.

Both expectations and government policy are formed upon observing a signal of the demand shock,

$$z = \omega u + \varepsilon.$$

I will again impose the normalization $\omega^2 \sigma_u^2 + \sigma_\varepsilon^2 = 1$.

After the equilibrium is realized, people observe the output level y and the price level p. Given that the monetary policy rule is known and the interest rate only depends on p and y, there is no additional information in observing the interest rate.

Thus, we distinguish between two information sets: The information set prevailing when expectations and government policy are formed, which is given by $\{z\}$, and the information set which determines the data against which any credible model must be validated. That information set is given by $\{y, p, z\}$.[3] Government spending is also observed but since it will be proportional to z, that knowledge is redundant.

There is a unique economist who produces a model which will be used by both the people and the government. As in the preceding section, this model's parameters will be denoted by a hat, and a hat over the expectations operator will mean that the expectation is computed using that model. The perceived model must satisfy the plausibility conditions that all its parameters are non-negative and that $0 \leq \hat{\mu} \leq \hat{\delta}$. Since, given the other parameters, any plausible target value for $(\hat{a}, \hat{b}, \hat{\rho})$ that satisfies $\hat{\rho} \geq \frac{\hat{b}}{\hat{\mu}}$ and $\hat{b} \leq \hat{\mu}$ can be matched by an appropriate choice of $(\hat{\alpha}, \hat{\beta}, \hat{\theta})$, I will consider that the theorist can directly set the three composite parameters $(\hat{a}, \hat{b}, \hat{\rho})$, and accordingly add the inequalities $\hat{\rho} \geq \frac{\hat{b}}{\hat{\mu}}$ and $\hat{b} \leq \hat{\mu}$ to the set of plausibility conditions.

I will proceed as follows. First, I solve for the equilibrium, given the model used by the people and the level of government spending. Second, I derive the optimal government policy. Third, I spell out the autocoherence conditions that the model must satisfy.

3.1 Solving for p and y.

The first step in solving for the equilibrium consists in computing p^e. Substituting (10) into (11) we get that

$$p = \frac{\mu}{\delta} p^e - \frac{v}{\delta} - \frac{b}{\delta} p^e + \frac{a}{\delta} g + \frac{u}{\delta} + \frac{\rho v}{\delta}.$$

People believe that the following relationship holds instead:

$$p = \frac{\hat{\mu}}{\hat{\delta}} p^e - \frac{\hat{v}}{\hat{\delta}} - \frac{\hat{b}}{\hat{\delta}} p^e + \frac{\hat{a}}{\hat{\delta}} g + \frac{\hat{u}}{\hat{\delta}} + \frac{\hat{\rho} \hat{v}}{\hat{\delta}}.$$

[3] Note that I require the model to match those data despite that it will be used prior to their realization. While the model is one-shot, I want it to take into account the fact that the people's forecasting model will be used repeatedly and therefore must match the data.

Note the hats on u and v : the realization of the shocks that would be inferred from the people's model differ from the actual ones, unless the model is correct.

Taking hatted expectations on both sides, we get that

$$p^e = \frac{1}{\hat{\delta} + \hat{b} - \hat{\mu}} \hat{E}(u \mid z) + \frac{\hat{a}}{\hat{\delta} + \hat{b} - \hat{\mu}} g. \tag{12}$$

Substituting into (10), we get a reduced form equation for output

$$y = -\frac{b}{\hat{\delta} + \hat{b} - \hat{\mu}} \hat{E}(u \mid z) + \left(a - \frac{b\hat{a}}{\hat{\delta} + \hat{b} - \hat{\mu}} \right) g + u + \rho v. \tag{13}$$

Plugging (12) and (13) into (11), we then get

$$p = \frac{\mu - b}{\delta(\hat{\delta} + \hat{b} - \hat{\mu})} \hat{E}(u \mid z) + \left(\frac{a}{\delta} + \frac{\hat{a}(\mu - b)}{\delta(\hat{\delta} + \hat{b} - \hat{\mu})} \right) g + \frac{u}{\delta} + \frac{\rho - 1}{\delta} v. \tag{14}$$

3.2 Optimal government policy

As above, the government wants to stabilize output and government spending. Again, its objective function is $\min \hat{E}(y^2 + \varphi g^2)$. I could also allow for the government to stabilize prices, but since the government can only react to demand shocks – there is no supply signal at the time of setting policy – that additional objective is similar to stabilizing output, and I ignore it for simplicity.[4]

Upon realization of the signal z, the government sets g so as to minimize

$$\hat{E}(y^2 + \varphi g^2 \mid z) = \hat{E}(y^2 \mid z) + \varphi g^2.$$

I assume g is observed at the time of setting inflationary expectations. Therefore, there is no credibility problem and the government will internalize the entire feedback effect of fiscal stimulus on output through inflation and its monetary policy response when setting its policy. Therefore, the first-order condition is

$$\frac{\hat{d}y}{\hat{d}g} \hat{E}(y \mid z) + \varphi g = 0. \tag{15}$$

[4] One could extend the model by assuming that a signal of the supply shock is also observed. Responding to that signal would involve a trade-off between price stability and output stability. In this paper focus is instead over price/output stability vs. government expenditure stability.

The derivative $\frac{\hat{dy}}{\hat{dg}}$ – the reduced form Keynesian multiplier – is computed by the government using the perceived model. Its true value can be obtained from (13):

$$\frac{dy}{dg} = a - \frac{b\hat{a}}{\hat{\delta} + \hat{b} - \hat{\mu}}. \tag{16}$$

Two aspects are noteworthy. First, the true Keynesian multiplier not only depends on the true model but also on the perceived one. This is because part of the expansionary effect of government spending is dissipated by greater inflationary expectations, which in turn generate greater inflation and a contractionary response of the interest rate. For example, the more people believe that government policy is effective (the greater $\hat{a}$), the more they think it will be inflationary, and the smaller the Keynesian multiplier given a. For the same reason, the more people believe the output/inflation trade-off is unfavorable (the smaller $\hat{\delta}$), the smaller $\frac{dy}{dg}$. Second, the Keynesian multiplier is not identified, because g is endogenous and always proportional to z. If there was a random, exogenous component to g, and if that component were observable, it would make it possible to identify the Keynesian multiplier. That is, the vector space spanned by g and z would be of dimension 2 instead of 1. Here, though, people cannot disentangle the sensitivity of output to government spending from the direct effect of demand shocks. This would remain true in richer models provided that the number of parameters is large enough relative to the dimension of the observables space.[5]|

[5] Let us go back to the model of the preceding section. Assume now that $g = \gamma z + \upsilon$, where υ is noise. If it is common knowledge that the government uses the same model as the people, and observes the same signal, then γ will be common knowledge. Thus, as long as υ is independent of the other shocks, the only additional parameter that can differ between the perceived and the correct model is the variance of υ. We now have $p = 6$, but, since g is no longer colinear with z, the dimension of the observables space is $n = 3$ instead of $n = 2$. the set of autocoherent models is of dimension $6 - 3 \times 4/2 = 0$. This suggests that the correct model is identified, although since the autocoherence conditions are non linear, they may have several solutions: some isolated incorrect autocoherent models may exist.

Now assume that the level of expenditure is itself measured with noise – that is, we observe $g + \psi$ instead of g – this adds an additional parameter which restores one degree of freedom in the set of autocoherent models. In other words, my claim that the Keynesian multiplier is underidentified does not rest on the specific assumption that g is colinear with z. It will typically hold in all models such that $p > n(n+1)/2$.

The government uses the perceived model to compute the Keynesian multiplier. To get the perceived multiplier, one just has to replace a and b with $\hat{a}$ and $\hat{b}$, respectively, in (16), getting

$$\frac{\hat{d}y}{\hat{d}g} = \frac{\hat{a}(\hat{\delta} - \hat{\mu})}{\hat{\delta} + \hat{b} - \hat{\mu}}. \tag{17}$$

To compute g, we can compute $\hat{E}(y \mid z)$ by applying hatted expectations to (13), yielding

$$\hat{E}(y \mid z) = \frac{\hat{a}(\hat{\delta} - \hat{\mu})}{\hat{\delta} + \hat{b} - \hat{\mu}} g + \frac{\hat{\delta} - \hat{\mu}}{\hat{\delta} + \hat{b} - \hat{\mu}} \hat{E}(u \mid z). \tag{18}$$

Note that (4) still holds. Furthermore, I will anticipate and already make use of the autocoherence condition

$$1 = \hat{\omega}^2 \hat{\sigma}_u^2 + \hat{\sigma}_\varepsilon^2.$$

Then

$$\hat{E}(u \mid z) = \hat{\omega} \hat{\sigma}_u^2 z. \tag{19}$$

Substituting (19),(18), and (17) into (15), we eventually get

$$g = \gamma z,$$

where

$$\gamma = -\hat{a} \frac{(\hat{\delta} - \hat{\mu})^2}{\varphi \left(\hat{\delta} + \hat{b} - \hat{\mu}\right)^2 + \hat{a}^2 \left(\hat{\delta} - \hat{\mu}\right)^2} \hat{\omega} \hat{\sigma}_u^2 < 0. \tag{20}$$

Inspection of this formula reveals that government activism is larger, i.e. $|\gamma|$ is larger,

- The more people believe in a favorable "long-term" phillips curve, i.e. the greater $\hat{\delta} - \hat{\mu}$
- The more they believe the interest response of aggregate demand is low, i.e. the smaller $\hat{b}$

As for the effect of $\hat{a}$, there is an "income effect" and a"substitution" effect, implying that γ is not monotonic in $\hat{a}$. For small value of $\hat{a}$, the substitution effect dominates; a more efficient fiscal policy generates greater activism. For large values of $\hat{a}$, though, the income effect dominates: the government takes advantage of an increase in $\hat{a}$ to reduce its activism, since that increase has a direct favorable impact of the degree of stabilization which is being achieved.

3.3 The reduced form model

The preceding subsection allows to compute the variables of interest p and y as a function of the realization of the shocks u, v and ε. This solution determines the reduced form model, which is summarized in Table 1. Then, by replacing non hatted parameters (other than γ) by their hatted counterparts, one can compute the reduced form perceived model, which is reported in Table 2. These expressions introduce composite coefficients that capture the response of output and prices to the demand shock u and the error ε.

For example, the coefficient of u on y,

$$a_{yu} = -\frac{b}{\hat{\delta} + \hat{b} - \hat{\mu}} \omega \hat{\omega} \hat{\sigma}_u^2 + \omega \gamma (a - \frac{\hat{a} b}{\hat{\delta} + \hat{b} - \hat{\mu}}) + 1, \tag{21}$$

has three components. The constant 1 captures the direct effect of the aggregate demand shock on output. The term $\omega\gamma(a - \frac{\hat{a}b}{\hat{\delta}+\hat{b}-\hat{\mu}})$ is typically negative and captures the stabilizing effect of fiscal policy. In it, a captures the direct effect of fiscal policy, and $-\frac{\hat{a}b}{\hat{\delta}+\hat{b}-\hat{\mu}}$ captures the dissipation due to the effect of fiscal policy on price expectations: A fiscal expansion boosts expectations of inflation, which in turn increases actual inflation and interest rates, which in turn reduces the efficiency of the expansion[6]. This reaction is stronger, the greater the perceived effect of fiscal policy on output ($\hat{a}$), the greater the actual effect of interest rates on output (b), and the more "unfavorable" the perceived Phillips curve (the

[6] If people think that fiscal policy is much more efficient than it actually is, i.e. if $\hat{a}$ is large, then the fiscal term becomes positive, implying that fiscal expansions are contractionary because of their (excessive) adverse effect on inflation expectations.

greater $\hat{\mu}$ and the smaller $\hat{\delta}$). Finally, the term $-\frac{b}{\hat{\delta}+\hat{b}-\hat{\mu}}\omega\hat{\omega}\hat{\sigma}_u^2$ is the effect of the direct reaction of price expectations to the signal about the demand shock. Since this effect would not exist absent a reaction of interest rates to inflation, we will label it the "monetary component" of the reaction of output to demand shocks. It is also negative, since that effect tends to dampen demand shocks.

Observable	Expression
Output	$y = a_{yu}u + a_{y\varepsilon}\varepsilon + \rho v$
Price	$p = a_{pu}u + a_{p\varepsilon}\varepsilon + \frac{\rho-1}{\delta}v$
Coefficients	**Expression**
a_{yu}	$-\frac{b}{\hat{\delta}+\hat{b}-\hat{\mu}}\omega\hat{\omega}\hat{\sigma}_u^2 + \omega\gamma(a - \frac{\hat{a}b}{\hat{\delta}+\hat{b}-\hat{\mu}}) + 1$
$a_{y\varepsilon}$	$-\frac{b}{\hat{\delta}+\hat{b}-\hat{\mu}}\hat{\omega}\hat{\sigma}_u^2 + \gamma(a - \frac{\hat{a}b}{\hat{\delta}+\hat{b}-\hat{\mu}})$
a_{pu}	$\frac{\mu-b}{\delta(\hat{\delta}+\hat{b}-\hat{\mu})}\omega\hat{\omega}\hat{\sigma}_u^2 + \omega\gamma\left(\frac{a}{\delta} + \frac{\hat{a}(\mu-b)}{\delta(\hat{\delta}+\hat{b}-\hat{\mu})}\right) + \frac{1}{\delta}$
$a_{p\varepsilon}$	$\frac{\mu-b}{\delta(\hat{\delta}+\hat{b}-\hat{\mu})}\hat{\omega}\hat{\sigma}_u^2 + \left(\frac{a}{\delta} + \frac{\hat{a}(\mu-b)}{\delta(\hat{\delta}+\hat{b}-\hat{\mu})}\right)\gamma$

Table 1 – The correct reduced form model

Observable	Expression
Output	$y = \hat{a}_{yu}\hat{u} + \hat{a}_{y\varepsilon}\hat{\varepsilon} + \hat{\rho}\hat{v}$
Price	$p = \hat{a}_{pu}\hat{u} + \hat{a}_{p\varepsilon}\hat{\varepsilon} + \frac{\hat{\rho}-1}{\hat{\delta}}v$
Coefficients	**Expression**
$\hat{a}_{yu}$	$-\frac{\hat{b}}{\hat{\delta}+\hat{b}-\hat{\mu}}\hat{\omega}^2\hat{\sigma}_u^2 + \gamma\hat{\omega}\frac{\hat{a}(\hat{\delta}-\hat{\mu})}{\hat{\delta}+\hat{b}-\hat{\mu}} + 1$
$\hat{a}_{y\varepsilon}$	$-\frac{\hat{b}}{\hat{\delta}+\hat{b}-\hat{\mu}}\hat{\omega}\hat{\sigma}_u^2 + \gamma\frac{\hat{a}(\hat{\delta}-\hat{\mu})}{\hat{\delta}+\hat{b}-\hat{\mu}}$
$\hat{a}_{pu}$	$\frac{\hat{\mu}-\hat{b}}{\hat{\delta}(\hat{\delta}+\hat{b}-\hat{\mu})}\hat{\omega}^2\hat{\sigma}_u^2 + \frac{\hat{a}}{\hat{\delta}+\hat{b}-\hat{\mu}}\hat{\omega}\gamma + \frac{1}{\hat{\delta}}$
$\hat{a}_{p\varepsilon}$	$\frac{\hat{\mu}-\hat{b}}{\hat{\delta}(\hat{\delta}+\hat{b}-\hat{\mu})}\hat{\omega}\hat{\sigma}_u^2 + \frac{\hat{a}}{\hat{\delta}+\hat{b}-\hat{\mu}}\gamma$

Table 2 – The perceived reduced form model

3.4 Autocoherence conditions

The reduced form models can then be used to derive the autocoherence conditions. The autocoherence property requires that the variance-covariance matrix of $(y, p, z)'$ computed using that perceived model matches the actual one. This determines six independent autocoherence conditions that are derived in the Appendix (equations (31)-(36)). There are nine parameters: $(\hat{a}, \hat{b}, \hat{\rho}, \hat{\sigma}_u^2, \hat{\delta}, \hat{\mu}, \hat{\sigma}_v^2, \hat{\omega}, \hat{\sigma}_\varepsilon^2)$ and therefore three degrees of freedom.

Since the joint distribution of p and z is observed, the autocoherence conditions always imply that $\hat{E}(p \mid z) = E(p \mid z)$. In other words, in equilibrium expectations are rational in the usual sense[7]. If government policy were fixed, we could then solve for a unique rational expectations equilibrium (REE) for model (10)-(11) in the usual way. All autocoherent models would then be equivalent in that they deliver the same REE equilibrium[8], leaving no room for the economists to manipulate outcomes. However, government policy does depend on the perceived model, because to set its optimal policy the government must know structural parameters (in particular the multiplier a) that are not identified from the joint distribution of (p, y, z). This opens the possibility for the expert to manipulate government policy.

However, not all parameters can be used to manipulate policy. The autocoherence conditions imply that the parameters of the Phillips curve are useless for pursuing an agenda.

Proposition 2 — The autocoherence conditions imply

$$\hat{\delta} - \hat{\mu} = \delta - \mu.$$

Proof – See Appendix.

Corollary – Given $\hat{a}$, and $\hat{b}$, γ is independent of the choice of $\hat{\delta}$ and $\hat{\mu}$, and so is the equilibrium.

Proof – Immediate from (20).

[7] Algebraically, we have that $E(p \mid z) = a_{pu}E(u \mid z) + a_{p\varepsilon}E(u \mid z)$ $= (a_{pu}\omega\sigma_u^2 + a_{p\varepsilon}\sigma_\varepsilon^2)z$. Similarly, $\hat{E}(p \mid z) = (\hat{a}_{pu}\hat{\omega}\hat{\sigma}_u^2 + \hat{a}_{p\varepsilon}\hat{\sigma}_\varepsilon^2)z$. Therefore, the condition $E(u \mid z) = \hat{E}(u \mid z)$ is equivalent to $a_{pu}\omega\sigma_u^2 + a_{p\varepsilon}\sigma_\varepsilon^2 = \hat{a}_{pu}\hat{\omega}\hat{\sigma}_u^2 + \hat{a}_{p\varepsilon}\hat{\sigma}_\varepsilon^2$, i.e. autocoherence condition (AC3) in the appendix.

[8] Again, this can be checked algebraically. Note that $\frac{\omega\sigma_u^2}{\delta} + a_{p\varepsilon} = \frac{\omega\sigma_u^2}{\delta} + \frac{\gamma a}{\delta} + \frac{\mu - b}{\delta}\left[\frac{\hat{\omega}\hat{\sigma}_u^2 + \gamma\hat{a}}{\hat{\delta} + \hat{b} - \hat{\mu}}\right]$ and that $\frac{\hat{\omega}\hat{\sigma}_u^2}{\hat{\delta}} + \hat{a}_{p\varepsilon} = \frac{\hat{\omega}\hat{\sigma}_u^2 + \gamma\hat{a}}{\hat{\delta} + \hat{b} - \hat{\mu}}$. Therefore, condition (33) is equivalent to $\frac{\hat{\omega}\hat{\sigma}_u^2 + \gamma\hat{a}}{\hat{\delta} + \hat{b} - \hat{\mu}} = \frac{\omega\sigma_u^2 + \gamma a}{\delta + b - \mu}$. Next, note that all the hatted terms in $a_{p\varepsilon}, a_{pu}$, $a_{y\varepsilon}$ and a_{yu} can be grouped in the ratio $\frac{\hat{\omega}\hat{\sigma}_u^2 + \gamma\hat{a}}{\hat{\delta} + \hat{b} - \hat{\mu}}$. Since that ratio must be equal to $\frac{\omega\sigma_u^2 + \gamma a}{\delta + b - \mu}$, if γ is exogenous, none of those coefficients depend on the perceived model. Consequently, the equilibrium is unique and must be identical to the REE equilibrium.

The policymaker cares about the ultimate effect of output of government spending, which only depends on price formation through the difference between the output response to prices δ and its (adverse) response to price expectations μ. But to match the covariances between output and the demand signal and prices and the demand signal, the economist is forced to reveal this difference. Thus given $\hat{a}$ and $\hat{b}$, he cannot influence policy through the design of the price block of his model.[9] Intutitively, this is because the demand signal z, which is not polluted by the supply shock, acts as an instrumental variable allowing agents to infer $\delta - \mu$ from $cov(y, z)$ and $cov(p, z)$, while they cannot get it from $cov(y, p)$ which is affected by both demand and supply shocks.

Since there is little room for the perceived Phillips curve to be used by expert to influence outcomes, in what follows I will assume that δ and μ are known. Furthermore, to simplify the analysis, I will also assume that ω and σ_u are known.

3.5 The price block is revealed

3.5.1 Simplifying the autocoherence conditions

The first case I focus on is when ω, σ_u and δ are known. Then it must be that $\hat{\omega} = \omega, \hat{\sigma}_u = \sigma_u$, and $\hat{\delta} = \delta$, implying also $\mu = \hat{\mu}$. It is then shown in the Appendix that in such a case, autocoherence implies that the perceived reduced form model must match the correct reduced form model, that is:

[9] Remember, though, that $\hat{a}$ and $\hat{b}$ are themselves composite parameters and their expression depends on $\hat{\delta}$ and $\hat{\mu}$. While given $\hat{\delta}$ and $\hat{\mu}$, any target for those parameters can be reached by picking the appropriate $\hat{\alpha}$ and $\hat{\beta}$, if for example α is known it may be necessary to choose a particular value of $\hat{\delta}$ to get the desired value of $\hat{a}$.

$$
\begin{aligned}
a_{y\varepsilon} &= \hat{a}_{y\varepsilon}, \\
a_{yu} &= \hat{a}_{yu}, \\
a_{p\varepsilon} &= \hat{a}_{p\varepsilon}, \\
a_{pu} &= \hat{a}_{pu}, \\
\rho &= \hat{\rho}.
\end{aligned}
$$

Nevertheless, because the correct reduced form coefficients themselves depend on beliefs, through the government policy parameter γ, it does not follow that the perceived structural model should be the same as the correct one. And which perceived model is picked matters, because different perceived models will lead to different stabilization policies and thus different outcomes.

3.5.2 The trade-off between the fiscal and monetary output responses

Experts are left with only one degree of freedom in designing their model, which is captured by a trade-off between $\hat{a}$ and $\hat{b}$, the perceived effects on output of government spending and price expectations[10]. This trade-off is defined by the following formulae:

$$
\begin{aligned}
(\hat{b}-b)\omega\sigma_u^2 &= \gamma\left[(\hat{a}-a)(\delta-\mu)+\hat{a}b-a\hat{b}\right]; && (22) \\
\gamma &= -\hat{a}\frac{(\delta-\mu)^2}{\varphi\left(\delta+\hat{b}-\mu\right)^2+\hat{a}^2\left(\delta-\mu\right)^2}\omega\sigma_u^2. && (23)
\end{aligned}
$$

Eliminating γ between these two yields a cubic equation for $\hat{b}$, as a function of $\hat{a}$, which can be solved analytically, although numerical analysis is necessary

[10] This degree of freedom comes from the fact that in this special case, one autocoherence condition becomes redundant. Thus one degree of freedom is left despite that the number of free parameters has been reduced to the number of autocoherence conditions.

Why is one autocoherence condition redundant here? Basically, if one only imposes that $\hat{\omega}=\omega, \hat{\sigma}_u=\sigma_u$, one can derive a condition involving $\hat{\delta}$ of which δ is a solution, although other values may also be solution in principle. Thus the condition $\hat{\delta}=\delta$ is almost endogenously derived from $\hat{\omega}=\omega, \hat{\sigma}_u=\sigma_u$. Imposing it rules out some other values of $\hat{\delta}$ but is redundant as long as δ is selected as the solution to the nonlinear equation which determines $\hat{\delta}$.

to find out how $\hat{b}$ varies with $\hat{a}$ and the other parameters. Whenever there are three values of $\hat{b}$ that solve this equation, the largest root was selected. Given the requirement that $\hat{b} > 0$, if that largest root is negative, then there is no plausible autocoherent model for this value of $\hat{a}$.

But much can be learned by considering the following approximation. Assume this is a "quasi-Lucas" economy, that is, $\delta - \mu << 1$. Then (23) is equivalent to

$$\gamma \approx -\hat{a}\frac{(\delta-\mu)^2}{\varphi\hat{b}^2}\omega\sigma_u^2 \tag{24}$$

and substituting it into (22) we get

$$\hat{b} \approx b - \frac{\hat{a}(\hat{a}-a)}{\varphi b}(\delta-\mu)^2. \tag{25}$$

This trade-off has the following properties

- $(\hat{b}-b)(\hat{a}-a) < 0$ and for $\hat{a} > a/2$, $d\hat{b}/d\hat{a} < 0$. Thus, the more the economist claims that government spending has a large impact on output, the lower the theoretical impact of interest rates. The only exception is if $\hat{a}$ is very low compared to a.

- The trade-off is flatter, the smaller $\delta - \mu$, the greater φ and the greater b. That is, the more the government is averse to stabilization, the less favorable the phillips curve, and the greater the true impact of interest rates, the more the theoretical effect of interest rates must be close to the actual one, and the more arbitrary the theoretical impact of government spending.

How can we make sense of these effects? In order to understand them we can focus on how $\hat{a}$ and $\hat{b}$ affect output's reaction to demand shocks, as captured by the value of a_{yu} and its perceived counterpart

$$\hat{a}_{yu} = -\frac{\hat{b}}{\hat{\delta}+\hat{b}-\hat{\mu}}\hat{\omega}^2\hat{\sigma}_u^2 + \gamma\hat{\omega}\frac{\hat{a}(\hat{\delta}-\hat{\mu})}{\hat{\delta}+\hat{b}-\hat{\mu}} + 1.$$

As stated above, autocoherence implies that the perceived model must correctly predict this elasticity. Furthermore, we also know that because of rational expectations this correct value only depends on the perceived model through the policy parameter γ. Consider an increase in $\hat{a}$ and hold γ constant (the effect of the change in γ is more complex and discussed in Remark 1 below). Then the output response a_{yu} is unchanged. On the other hand, people will believe that it has fallen, since they think that the direct expansionary effect of fiscal policy (which outweighs its indirect contractionary effect through inflation expectations) is now stronger. This is captured by the fiscal component in $\hat{a}_{yu}$, $\gamma\hat{\omega}\frac{\hat{a}(\hat{\delta}-\hat{\mu})}{\hat{\delta}+\hat{b}-\hat{\mu}}$, which, since $\hat{\mu} < \hat{\delta}$, clearly falls in algebraic value as $\hat{a}$ goes up. This discrepancy would invalidate the model empirically unless $\hat{b}$ is changed so as to restore the equality between the actual and perceived elasticity of output to demand shocks. The dominant effect of a reduction in $\hat{b}$ (in a quasi-Lucas economy) is to increase the algebraic value of the perceived monetary component of $\hat{a}_{yu}$, given by $-\frac{\hat{b}}{\hat{\delta}+\hat{b}-\hat{\mu}}\hat{\omega}^2\hat{\sigma}_u^2$;[11] the lower $\hat{b}$, the lower the perceived output response to interest rates, and the lower the perceived stabilizing effect of monetary reactions to demand shocks. This effect raises the perceived response of output to demand shocks, thus restoring the model's autocoherence. This explains why there is a negative trade-off between $\hat{a}$ and $\hat{b}$. Since $\hat{b}$ is the interest elasticity of output, this means that experts face a trade-off between believing in fiscal policy effectiveness versus believing in monetary policy effectiveness. An economist who would underpredict both elasticities would also underpredict output volatility and could not empirically validate his model.

Remark 1: An increase in $\hat{a}$ also increases γ, the degree of fiscal activism. This magnifies the discrepancy between the perceived and actual fiscal components of a_{yu}–because government expenditures are more reactive to the demand shock signal. This discrepancy is negative if $\hat{a} > a$, i.e. people expect more

[11] $\hat{b}$, also appears in the fiscal component but in a quasi-lucas economy this contribution is very small since that component is proportional to $(\hat{\delta} - \hat{\mu})^3$.

fiscal stabilization than actually happens. In this case, the increase in γ further widens the gap between actual and perceived fiscal components, thus reinforcing the negative required response of $\hat{b}$ to the increase in $\hat{a}$. On the other hand, if $\hat{a} < a$, the discrepancy is positive: people expect greater volatility of output coming from the fiscal component than in reality. While the direct effect of a greater $\hat{a}$ tends to make this discrepancy less positive, the indirect effect on γ which magnifies the difference tends to make it larger. For $\hat{a} < a/2$ this effect dominates, which explains why $d\hat{b}/d\hat{a} > 0$ in this zone.

The size of the effects I just discussed is proportional to $|\gamma|$, the degree of fiscal activism. The lower $|\gamma|$, the lower the discrepancy between the actual and perceived fiscal components and less reactive it will be to an increase in $\hat{a}$. Thus, the less fiscal policy is active, the lower the deviation between b and $\hat{b}$ that must be implemented to compensate a given deviation between a and $\hat{a}$. In the limit case where $\gamma = 0$, there is no variation is fiscal policy that would allow to identify a, and the only unidentified parameter that affects the output elasticity to demand shock is b, through the monetary component. Thus, in that limit case, $\hat{b} = b$ and $\hat{a}$ is arbitrary. In turn, fiscal activism is greater, the more favorable the output-inflation trade-off – the larger $\delta - \mu$ – and the smaller the welfare cost φ of fiscal volatility. This explains why the trade-off is flatter, the smaller $\delta - \mu$ and the greater φ.

Remark 2: Effect of b. The equilibrium output response a_{yu} falls more with $\hat{a}$, the greater b. This is because the greater b, the greater the stabilizing effects of the monetary response to inflation. This reduces the reduction in $\hat{b}$ that is needed to offset an increase in $\hat{a}$, since the correct output response to demand shock that one has to match is now lower. Consequently, a greater value of b makes tre trade-off between $\hat{a}$ and $\hat{b}$ flatter.

Figure 1 depicts numerical simulations of the actual trade-off for four different sets of the parameters a and $\delta - \mu$ (Note that the trade-off only depends on

δ and μ through the diffence $\delta - \mu$).[12] The results are very similar to what the above discussion based on the quasi-Lucas economy suggests. For $\hat{a} > a/2$ the trade-off is decreasing and concave. It stops at a maximum value of $\hat{a}$ beyond which the plausibility condition $\hat{b} > 0$ is violated. In most cases this corresponds to a catastrophy, mathematically speaking, in that the number of roots of the cubic equation defining $\hat{b}$ falls from 1 to 2 in such a way that the two largest roots disappear. Because of this discontinuity, the curves on Figure 1 stop before hitting the horizontal axis. As in the quasi-Lucas case, it is flatter, the less favorable the Phillips curve, i.e. the smaller $\delta - \mu$. Furthermore, it shifts up and its slope becomes larger algebraically when a, the actual Keynesian multiplier, goes up, which is also implied by (25).

3.5.3 The optimal model

Which model will the expert select? As in the preceding section, I assume his objective is $\bar{W} = \min \hat{E}(y^2 + \bar{\varphi} g^2)$. In equilibrium, this is equal to (ignoring constants that are independent of the perceived model)

$$\bar{W} = a_{yu}^2 \sigma_u^2 + a_{y\varepsilon}^2 \sigma_\varepsilon^2 + \bar{\varphi}\gamma^2. \tag{26}$$

Since the reduced form elasticities a_{yu} and $a_{y\varepsilon}$ only depend on the perceived model through γ, as long as the point chosen on the $(\hat{a}, \hat{b})$ trade-off is interior, the corresponding value of γ is the one that would be obtained by directly maximizing $\bar{W}$ with respect to γ. In other words, the intellectual is again a quasi-dictator as in the preceding example, unless plausibility constraints force him into an corner solution. This preferred value of γ is the one that would prevail if the intellectual were setting policy using the right model:[13]

[12] The other parameters in Figures 1 and 2 are $b = 0.5$, $\varphi = 0.8$, $\omega = 1, \sigma_u^2 = 0.1, \sigma_v^2 = 0.5$.

[13] This can again be checked algebraically. The crucial autocoherence condition $\frac{\hat{\omega}\hat{\sigma}_u^2 + \gamma\hat{a}}{\hat{\delta} + \hat{b} - \hat{\mu}} = \frac{\omega\sigma_u^2 + \gamma a}{\delta + b - \mu}$ implies that $a_{yu} = \frac{\omega\gamma a(\delta - \mu)}{\delta + b - \mu} + 1 - \frac{b\omega^2\sigma_u^2}{\delta + b - \mu}$ and that $a_{y\varepsilon} = \frac{\gamma a(\delta - \mu)}{\delta + b - \mu} - \frac{b\omega\sigma_u^2}{\delta + b - \mu}$. Substituting these expressions into (26) and deriving the first-order conditions with respect ton γ delivers (27).

$$\gamma = \bar{\gamma} = -a \frac{(\delta - \mu)^2}{\bar{\varphi}\,(\delta + b - \mu)^2 + a^2\,(\delta - \mu)^2} \omega \sigma_u^2. \tag{27}$$

This equality allows us to find out how the perceived model depends on the economist's preferences. From this equality we have

$$\frac{d\hat{a}}{d\bar{\varphi}} = \frac{\partial \bar{\gamma}/\partial \bar{\varphi}}{\partial \gamma/\partial \hat{a} + \partial \gamma/\partial \hat{b}.d\hat{b}/d\hat{a}},$$

where the derivative $d\hat{b}/d\hat{a}$ is taken along the autocoherence trade-off between $\hat{b}$ and $\hat{a}$. We know that $\partial \gamma/\partial \hat{b} > 0, \partial \bar{\gamma}/\partial \bar{\varphi} > 0$, and $\partial \gamma/\partial \hat{a} < 0$ if the substitution effect dominates. Then, in the 'normal' part of the trade-off where $d\hat{b}/d\hat{a} < 0$, we have that $\frac{d\hat{a}}{d\bar{\varphi}} < 0$. More conservative economists will understate the impact of public interest rates and accordingly, to remain autocoherent, overstate that of interest rates. Furthermore, it is again the case that if the economist's preferences are aligned with that of the government, then the correct model is revealed, since by using it the government will then select $\gamma = \bar{\gamma}$. Since autocoherence imposes rational inflation expectations, there is no scope for manipulating the public and an economist aligned with the government cannot do better than reveal the truth.

Table 3 presents numerical simulations for various values of $\bar{\varphi}$, the degree of conservatism of the economist (the parameter values are the same as in Figure 1 and in particular $b = 0.5$). It confirms that the more conservative the economist, the lower his theoretical Keynesian multiplier $\hat{a}$, and the larger the interest elasticity of output $\hat{b}$. Note also that a corner solution prevails for very progressive economists: the largest plausible value of a is selected.

$\bar{\varphi}$	$a = 0.2, \delta - \mu = 0.4$		$a = 0.2, \delta - \mu = 0.1$		$a = 0.8, \delta - \mu = 0.4$		$a = 0.8, \delta - \mu = 0.1$	
	$\hat{a}$	$\hat{b}$	$\hat{a}$	$\hat{b}$	$\hat{a}$	$\hat{b}$	$\hat{a}$	$\hat{b}$
0.08	1.1*	0.117	1.78	0.43	1.48*	0.08	3.11*	0.21
0.4	0.39	0.48	0.34	0.498	1.29	0.33	1.53	0.48
$0.8 = \varphi$	0.2	0.5	0.2	0.5	0.8	0.5	0.8	0.5
1.2	0.13	0.502	0.13	0.5	0.55	0.53	0.54	0.502
1.6	0.1	0.502	0.1	0.5	0.42	0.534	0.4	0.503

Table 3 –Ideological preferences and the expert's preferred perceived model.* = maximum possible value.

4 Two variants

4.1 Credibility

If the government sets policy once price expectations are formed, and if it cannot commit to a contigent rule, policy will be too activist relative to the optimum (that is, γ is larger absent commitment than under commitment). This is because the government now considers that the effect of government spending on output is now given by $\hat{a}$, the perceived impact multiplier, rather than the lower $\frac{\hat{a}(\hat{\delta}-\hat{\mu})}{\hat{\delta}+\hat{b}-\hat{\mu}}$ which reflects the output losses due to the rise in inflationary expectations. As a result, economists will tend to design more conservative models than if the government could commit. By manipulating the beliefs of the government, they indirectly tie its hands as though policy had been delegated to a conservative policymaker. Even an economist who has the same preferences as the government will act as a benevolent paternalist and promote conservative views so as to du reduce government activism.

The value of γ that prevails under no commitment, denoted by γ_{NC}, is readily obtained by replacing the first-order condition (15) with

$$\hat{a}\hat{E}(y \mid z) + \varphi g = 0,$$

yielding

$$\gamma_{NC} = -\hat{a}\frac{\hat{\delta}-\hat{\mu}}{\varphi\left(\hat{\delta}+\hat{b}-\hat{\mu}\right)+\hat{a}^2\left(\hat{\delta}-\hat{\mu}\right)}\hat{\omega}\hat{\sigma}_u^2.$$

Clearly, $|\gamma_{NC}| > |\gamma|$. In a quasi-Lucas economy, γ_{NC} is one order of magnitude larger than γ. Instead of (24) we have

$$\gamma_{NC} \approx -\frac{\hat{a}(\hat{\delta}-\hat{\mu})}{\varphi b}\hat{\omega}\hat{\sigma}_u^2.$$

Assume again that ω and σ_u^2 are known. Under commitment, a quasi-dictator economist with preference parameter $\bar{\varphi}$ will equate $\gamma \approx -\hat{a}\frac{(\delta-\mu)^2}{\varphi b^2}\omega\sigma_u^2$ with $\bar{\gamma} \approx -a\frac{(\delta-\mu)^2}{\varphi b^2}\omega\sigma_u^2$ and select

$$\hat{a} \approx a\frac{\varphi}{\bar{\varphi}}.$$

But absent commitment, he will equate $\bar{\gamma}$ with γ_{NC} and select instead

$$\hat{a} \approx a\frac{\varphi}{\bar{\varphi}}\frac{\delta-\mu}{b} << 1.$$

Thus all economists, even very left-wing ones, will be "very conservative" in such a situation. In a quasi-Lucas economy, the scope for activism is very reduced because most of the benefits of a fiscal expansion are dissipated by their induced hike in inflationary expectations. But because such expansion takes place after expectations have been set, as in Barro and Gordon (1982) the government is tempted to take advantage of the output-inflation trade-off even though that delivers very little in equilibrium. This is the reason why activism is much larger than under commitment, and why to restore commitments the economists want to promote a model with an impact Keynesian multiplier one order of magnitude smaller than in reality.

4.2 An example where the price block can be successfully manipulated

I now study an example where the signal z upon which forecasts are based does not allow to identify the slope of the Phillips curve $\delta - \mu$. That is, I assume that z is now an aggregate of the demand and supply shock:

$$z = \omega u - \lambda v.$$

Again I assume $\lambda, \omega > 0$. The signal z is interpreted as a signal about the aggregate price level. Thus this signal goes up with demand shocks but down with supply shocks.

I impose again the following normalization:

$$E(z^2) = \omega^2\sigma_u^2 + \lambda^2\sigma_v^2 = 1.$$

To solve the model we now note that[14] $\hat{E}(u \mid z) = \hat{\omega}\hat{\sigma}_u^2 z$ and $\hat{E}(v \mid z) = -\hat{\lambda}\hat{\sigma}_v^2 z$. Performing the same steps as in section 3.1 and using those expressions, we get that

$$p^e = \frac{\hat{a}}{\hat{\delta} + \hat{b} - \hat{\mu}} g + \hat{c} z,$$

with

$$\hat{c} = \frac{\hat{\omega}\hat{\sigma}_u^2 - \hat{\lambda}(\hat{\rho} - 1)\hat{\sigma}_v^2}{\hat{\delta} + \hat{b} - \hat{\mu}}.$$

Therefore the solution is

$$y = u + \rho v - b\hat{c}z + \left(a - \frac{b\hat{a}}{\hat{\delta} + \hat{b} - \hat{\mu}} \right) g \tag{28}$$

$$p = \frac{\mu - b}{\delta}\hat{c}z + \left(\frac{a}{\delta} + \frac{\hat{a}(\mu - b)}{\delta(\hat{\delta} + \hat{b} - \hat{\mu})} \right) g + \frac{u}{\delta} + \frac{\rho - 1}{\delta} v.$$

How is government policy determined in this variant? Conditions (16) and (17) as well as the FOC (15) still hold. But applying hatted expectations to both sides of (28) we now get

$$\hat{E}(y \mid z) = \frac{\hat{a}(\hat{\delta} - \hat{\mu})}{\hat{\delta} + \hat{b} - \hat{\mu}} g + \left[\hat{\omega}\hat{\sigma}_u^2 \frac{\hat{\delta} - \hat{\mu}}{\hat{\delta} + \hat{b} - \hat{\mu}} - \hat{\lambda}\hat{\sigma}_v^2 \left(\hat{\rho} - \frac{(\hat{\rho} - 1)\hat{b}}{\hat{\delta} + \hat{b} - \hat{\mu}} \right) \right] z.$$

Consequently optimal fiscal policy is now given by $g = \gamma z$, with

$$\gamma = \hat{a} \frac{\hat{\lambda}\hat{\sigma}_v^2 \left[\hat{\rho}\left(\hat{\delta} - \hat{\mu}\right)^2 + \hat{b}\left(\hat{\delta} - \hat{\mu}\right) \right] - (\hat{\delta} - \hat{\mu})^2 \hat{\omega}\hat{\sigma}_u^2}{\varphi\left(\hat{\delta} + \hat{b} - \hat{\mu}\right)^2 + \hat{a}^2\left(\hat{\delta} - \hat{\mu}\right)^2} \lesseqgtr 0. \tag{29}$$

Note that the sign of γ depends on the relative importance of supply and demand shocks. If supply shocks are perceived to be more important ($\hat{\sigma}_v^2$ large enough relative to $\hat{\sigma}_u^2$), an indication of price pressure ($z > 0$) signals a contraction and will be met with expansionary policies ($\gamma > 0$).

[14] This again anticipates on the autocoherence condition $E(z^2) = \hat{E}(z^2) = 1$.

The model's new solution is now given in Tables 4 and 5.

Observable	Expression
Output	$y = a_{yu}u + a_{yv}v$
Price	$p = a_{pu}u + a_{pv}v$
Coefficients	**Expression**
a_{yu}	$1 - b\omega\hat{c} + \omega\gamma(a - \frac{\hat{a}b}{\hat{\delta}+\hat{b}-\hat{\mu}})$
a_{yv}	$\rho + b\lambda\hat{c} - \gamma\lambda(a - \frac{\hat{a}b}{\hat{\delta}+\hat{b}-\hat{\mu}})$
a_{pu}	$\omega\gamma\left(\frac{a}{\delta} + \frac{\hat{a}(\mu-b)}{\delta(\hat{\delta}+\hat{b}-\hat{\mu})}\right) + (\mu - b)\frac{\omega\hat{c}}{\delta} + \frac{1}{\delta}$
a_{pv}	$\frac{\rho-1}{\delta} - (\mu - b)\frac{\lambda\hat{c}}{\delta} - \lambda\gamma\left(\frac{a}{\delta} + \frac{\hat{a}(\mu-b)}{\delta(\hat{\delta}+\hat{b}-\hat{\mu})}\right).$

Table 4 – The correct reduced form model, Variant B

Observable	Expression
Output	$y = \hat{a}_{yu}\hat{u} + \hat{a}_{yv}\hat{v}$
Price	$p = \hat{a}_{pu}\hat{u} + \hat{a}_{pv}\hat{v}$
Coefficients	**Expression**
$\hat{a}_{yu}$	$1 - \hat{b}\hat{\omega}\hat{c} + \hat{\omega}\gamma\frac{\hat{a}\left(\hat{\delta}-\hat{\mu}\right)}{\hat{\delta}+\hat{b}-\hat{\mu}}$
a_{yv}	$\hat{\rho} + \hat{b}\hat{\lambda}\hat{c} - \gamma\hat{\lambda}\frac{\hat{a}\left(\hat{\delta}-\hat{\mu}\right)}{\hat{\delta}+\hat{b}-\hat{\mu}}$
a_{pu}	$\hat{\omega}\gamma\frac{\hat{a}}{\hat{\delta}+\hat{b}-\hat{\mu}} + (\hat{\mu} - \hat{b})\frac{\hat{\omega}\hat{c}}{\hat{\delta}} + \frac{1}{\hat{\delta}}$
a_{pv}	$\frac{\hat{\rho}-1}{\hat{\delta}} - (\hat{\mu} - \hat{b})\frac{\hat{\lambda}\hat{c}}{\hat{\delta}} - \hat{\lambda}\gamma\frac{\hat{a}}{\hat{\delta}+\hat{b}-\hat{\mu}}.$

Table 5 – The perceived reduced form model, Variant B

There are again six autocoherence solutions and nine parameters. In contrast to the previous section, I will now assume that the key parameters of the output block are common knowledge: $\hat{a} = a$ and $\hat{b} = b$. The autocherence conditions now leave us with one degree of freedom: they define a 1-dimensional manifold in a 7-dimensional space. Rather than solving those highly nonlinear equations, I linearize the system of autocoherence conditions locally around the correct model. For such "quasi-correct" models, the autocoherence conditions are thus a straight line in that space. Define $\Delta\hat{\delta} = \hat{\delta} - \delta << 1$ and similarly for other parameters. Then we can reexpress the AC conditions in the following fashion

$$v = \Delta\hat{\delta}.q,$$

where $v = (\Delta(\hat{\delta} - \hat{\mu}), \Delta\hat{\lambda}, \Delta\hat{\sigma}_v, \Delta\hat{\omega}, \Delta\hat{\sigma}_u, \Delta\hat{\rho})'$ and q is a 6-dimensional vector whose ith coefficient gives us the slope of the trade-off between $\hat{\delta}$ and the ith parameter in v.[15] Of special interest is the first coefficient of q since it defines the set of parameters of the Phillips curve that the economist may promote while remaining autocoherent.

The algebraic steps to derive the q vector are described in the Appendix, and these formulas can be used to numerically compute q in a given economy.

Which point is going to be selected by the economist along this autocoherence locus? Again, he will be a quasi-dictator and it is natural, given our approximation, to assume that his preferences differ only marginally from those of the government: $\bar{\varphi} = \varphi + \Delta\varphi$, $\Delta\varphi << 1$. Let γ_0 be the value of γ prevailing if the perceived model is correct, then the target value of γ for the economist is given by $\tilde{\gamma} \approx \gamma_0 + \frac{\partial\gamma}{\partial\varphi}\Delta\varphi = \gamma_0 + \Delta\tilde{\gamma}$. On the other hand, the value of γ pursued by the government given the perceived model can be expressed as $\gamma \approx \gamma_0 + (\nabla_v\gamma).v = \gamma_0 + \Delta\gamma$. where $\nabla_v\gamma$ is the appropriate vectors of derivatives[16]. They and $\frac{\partial\gamma}{\partial\varphi}$ are computed in the Appendix. The economist will pick the model that satisfies $\Delta\gamma = \Delta\tilde{\gamma}$, implying that the perceived model can be summarized by a relationship between $\Delta\hat{\delta}$ and $\Delta\varphi$:

$$\Delta\hat{\delta} = m\Delta\varphi,$$

where

$$m = \frac{\frac{\partial\gamma}{\partial\varphi}}{(\nabla_v\gamma).q}. \tag{30}$$

I will now use those results to analyze the structure of the perceived model and how it depends on the underlying parameters of the economy. In order to organize the discussion, I will focus on four intuitive characteristics of a theory:

1. The short-term inflationary cost of output (STC). This is equal to $1/\hat{\delta}$.
2. The long-term inflationary cost of output (LTC), equal to $1/(\hat{\delta} - \hat{\mu})$.

[15] Given the particular importance of the parameter $\delta - \mu$, I prefer to use $\hat{\delta} - \hat{\mu}$ rather than $\hat{\mu}$.

[16] There is no contribution of $\Delta\hat{\delta}$ in the differentiation γ with respect to the perceived parameters once one also differentiates wioth respect to the parameters in v, since $\hat{\delta}$ ionly appears through $\hat{\delta} - \hat{\mu}$.

3. The relative importance of supply shocks (RIS), equal to $\frac{\hat{\sigma}_v^2}{\hat{\sigma}_u^2}$.

4. The supply-intensity of the price indicator (SIP), equal to $\hat{\lambda}^2 \hat{\sigma}_v^2$.

5. The share of output fluctuations explained by supply shocks (SSO); given by $\frac{\hat{a}_{yv}^2 \hat{\sigma}_v^2}{\hat{a}_{yv}^2 \hat{\sigma}_v^2 + \hat{a}_{yu}^2 \hat{\sigma}_u^2}$.

For each of these parameters, its *ideological sensitivity* is defined as its derivative with respect to φ. A positive ideological sensitivity means that the parameter goes up, the more conservative the economist. The greater the absolute value of ideological sensitivity, the more the parameter will deviate from its true value as a result of the economist's own agenda (and, intuitively, economists with different ideological positions will disagree more). The expressions for the ideological sensitivities are given by the following table.

Parameter	Ideological sensitivity
STC	$-m/\delta^2$
LTC	$-mq_1/(\delta-\mu)^2$
RIS	$\frac{2\sigma_v}{\sigma_u^2} m(q_3 - \frac{\sigma_v}{\sigma_u} q_5)$
SIP	$2m(\lambda\sigma_v^2 q_2 + \lambda^2 \sigma_v q_3)$

Table 5 – Ideological sensitivities of key perceived parameters

Figures 4 to 9 report ideological sensitivities, as μ varies, for 5 sets of values for the other parameters. We observe the following:

- Typically, the ideological sensitivity of LTC is positive: more conservative economists will report a higher inflationary cost of output in the long run. This makes sense as it will deter activist stabilization policies. However, there are exceptions: on Figure 9 where b is quite low (b=0.1), LTC has a positive sensitivity only if μ is large enough, i.e. on the right of the asymptote.

- However, for other parameters, things are less clear-cut. For example, the STC is always negative except on Figure 9. A conservative economist wants to downplay the efficiency of stabilization through public expenditures, but cannot act on all margins simultaneously because he is bound by the autocoherence conditions. This sometimes forces him to appear progressive on some fronts, as is the case for the short-term inflationary effects of inflation.

- Nevertheless, a pattern emerges: the ideological sensitivity of STC is always small, implying that the truth is almost revealed about δ regardless of the economist's ideological position, while there is much more ideological polarization with respect to the value of μ. A conservative economist will overemphasize the negative impact of inflationary expectations on output, in a way reminiscent of Friedman and Lucas, while the left-wing economist will produce models that understate μ, in a fashion not unlike that of Akerlof and Dickens.

- We also note that in many simulations the share of output fluctuations explained by supply shocks has a positive ideological sensitivity; however this does not happen because of the RIS, which tends to have a negative sensitivity, but through the perceived responses of output to these shocks $\hat{a}_{yu}$ and $\hat{a}_{yv}$. An exception arises when σ_u is very large (Figure 5), or b very low (Figure 9). In Figure 9, the conservative economist believes in a mildly more favorable Phillips curve for μ low, but also promotes the view that supply shocks are relatively important. If μ is high, the pattern is similar to the other figures.

- An economy can be "critical", meaning that the denominator of (30) is close to zero. This happens on Figure 9 around $\mu \approx 0.59$, and on Figure 5 around $\mu = 0.66$. In a critical economy, parameters happen to be such that ideology is uninfluential. To compensate for that and act as quasi-dictators, economists will tend to pick very large deviations between the perceived and actual parameters: ideological sensitivities become very large, as captured by the asymptotes in our figures. This result would be overturned, however, if there was some convex cost to the expert of deviating from the truth; economists would then no longer be quasi-dictators and in a critical economy, the benefits of manipulation would be negligible relative to the costs of deviating from the truth. Instead of becoming infinite, ideological sensitivities would then fall to zero in a critical economy[17].

5 An empirical investigation

In this section, I use the Survey of Professional Forecasters (SPF) to compare the ideas developed above with the data. As the preceding analysis makes clear, the models that will arise depend on the ideological stance of the expert as well as on the autocherence conditions and on the correct model. We have found that the outcome is highly sensitive to the parameters of the correct model and to the set of parameters that are known. This makes it hard to come up with a tight prediction about, say, the value of a parameter.

On the other hand, the analysis tells us that we expect models to be disciplined by the autocoherence conditions and that the dispersion in predictions across experts is driven by their ideological differences. The SPF is a panel of macroeconomic predictions by a large number of forecasters. It can be used in a cross section to analyse the dispersion in forecasts, and its longitudinal dimension can be used to understand how models evolve over time. In what follows I will use those data to answer three questions:

1. What kind of autocoherence conditions are imposed on those forecasts?
2. Can we point to a correlation between the forecasts and some measure of the forecaster's ideological position or self-interest?

[17] This can be seen by looking at the following reduced form optimization problem: $\min_{\hat{\theta}} c(\hat{\theta}-\theta)^2 + (s\hat{\theta}-\varphi)^2$, where θ is the true parameter value, $\hat{\theta}$ the perceived one, $s\hat{\theta}$ the outcome (up to a constant), φ the target outcome, and c the cost of deviating from the truth. The optimal value of $\hat{\theta}$ is $\frac{c\theta+s\varphi}{c+s^2}$, with a radically different behavior around $s=0$ (criticality) depending on whether c is positive vs. zero.

3. How do the models evolve over time, under the influence of new empirical observations and changes in the policy regime?

5.1 The basic methodology

Each observation in the SPF is a year x quarter x individual forecasters. The available variables include forecasts for GDP, inflation, unemployment, GDP components, up to 6 quarters (short-run) and 4 years. The data set is broken down into four files corresponding to four different time periods: 1968:4-1979:4, 1980:1-1989:1, 1990:1-1999:4, 2000:1-2009:4....There is a lot of commonality in the individual identifiers between the first two, as well as the last two, files, but very little otherwise. Therefore, it is natural to aggregate these four files into two samples, one corresponding to 1968:4-1989:1b (Sample 1), the other to 1990:1-2009:4 (Sample 2).

The data set only contains forecasts, not the actual models used by the forecasters. One first step in the analysis is to recover some characteristics of the models from the forecasts. To do so, I assume that all forecasts are based on publicly observable signals that are common to all forecasters. Given a proxy for the signals, this allows to estimate how each forecaster reacts to the signal, which is a function of the parameters of the model used by the forecaster. For example, in the above model, p^e is proportional to the signal z, and the coefficient depends on the perceived model's parameters. These estimations yield a "pseudo-model" for each forecaster, which consists of the composite coefficients which determine the forecaster's response to the signal. If the autocoherence conditions impose restrictions on how the actual perceived parameters vary across forecasters, then we can hope to observe some restrictions for the composite coefficients.

In practice, the common signal is proxied by the average 1-year ahead GDP growth forecast across forecasters for the current quarter. This delivers a time series for the signal, which is then centered and normalized. Then, for each forecaster, I run a simple regression of the 1-year ahead forecasts for inflation, real GDP growth, and unemployment growth on that signal. In each sample, and for each forecaster with enough observations, this delivers a pseudo model with six coefficients, namely the slopes and intercepts of those regressions. After eliminating aberrant estimates, I get 122 pseudo-models in Sample 1 and 111 in Sample 2. Finally, I search for pair-wise relationships among those coefficients that could indicate autocoherence restrictions. The following Table summarizes the variables of the pseudo-model and their meanings[18].

Variable	Meaning
a_1	Response of inflation forecast to (standardized) average real GDP growth forecast
a_2	Response of real GDP forecast to average real GDP growth forecast
a_3	Response of unemployment growth forecast to average real GDP growth forecast
c_1	Average real inflation forecast
c_2	Average real GDP growth forecast
c_3	Average unemployment growth forecast

Table 6 – The Pseudo-model parameters

Variable	Mean	Std Dev	Min	Max
a_1	-0.064	0.048	-0.17	0.12
a_2	0.16	0.04	0.02	0.26
a_3	-0.89	0.27	-1.56	0.06
c_1	0.049	0.0095	0.02	0.07
c_2	0.03	0.0084	-0.018	0.058
c_3	0.032	0.037	-0.054	0.155

Table 7 – Descriptive statistics for the pseudo-models, Sample 1

Variable	Mean	Std Dev	Min	Max
a_1	-0.005	0.032	-0.11	0.09
a_2	0.076	0.027	-0.016	0.12
a_3	-0.52	0.26	-1.20	0.21
c_1	0.024	0.005	0.012	0.039
c_2	0.025	0.0034	0.013	0.035
c_3	0.017	0.043	-0.11	0.109

Table 8 – Descriptive statistics for the pseudo-models, Sample 2

The summary statistics in Tables 7 and 8 tell us something about how reliable macroeconomic theory is. For example, we note that a_2 is positive and has a

[18] Given that the signal variable is standardized, the unconditional expectation of the forecast for variable i by a forecaster with pseudo-model $\{a_i, c_i\}$ is c_i. That does not mean, however, that c_i is the sample mean of those forecasts.

low dispersion, which tells us that there is a lot of commonality between the GDP growth forecasts; when the average forecast goes up, the forecasts of all forecasters go up. Similarly, the dispersion in the intercept c_2 is very small, suggesting a lot of agreement across forecasters over the long run growth rate of the economy (which may be interpreted as an autocoherence condition pinning down the value of c_2). On the other hand, the response of inflation forecasts is very heterogeneous: it has a wide dispersion and is as likely to be positive as negative, suggesting that there is little consensus about the Phillips curve. Its mean is negative, suggesting that on average output growth is considered deflationary in this period, which is probably due to the importance of supplmy shocks. These features are roughly shared by the two samples, but we note that the standard deviations go down in Sample 2, suggesting there is less disagreement across forecasters. Note also the large increase in the algebraic value of a_1 between the two periods, which makes the pseudo models more aligned with a "standard" Phillips curve, although the average remains slightly negative.

5.2 Searching for autocoherence restrictions

Are there relationships among the pseudo-models parameters that would indicate the existence of autocoherence conditions? A simple way to detect those is to run pairwise simple regressions of one coefficient on another. The results of such regressions are reported in Tables 9 and 10. Only statistically significant results are reported.

	a_1	a_2	a_3	c_1	c_2	c_3
a_1						
a_2						
a_3		-2.49 (-4.3)				
c_1	-0.09 (-5.51)					
c_2	-0.04 (-2.83)	-0.04 (-2.27)				
c_3		0.2 (2.4)	-0.03 (-2.42)		-0.9 (-2.31)	

Table 9 - Sample 1

	a_1	a_2	a_3	c_1	c_2	c_3
a_1						
a_2						
a_3		-3.73 (-4.3)				
c_1	-0.05 (-4.33)					
c_2						
c_3	0.39 (3.2)		-0.033 (-2.2)	-4.95 (-7.3)	-5.40 (-4.9)	

Table 10 - Sample 2

In the models discussed in the previous section, the averages of all variables were assumed to be common knowledge and normalized to zero; the only coefficients for which the perceived value could differ from the actual one were the ones involving the response of one variable to another variable. Here, though, the averages, captured by the c parameters, differ across forecasters, which potentially introduces a new type of AC restrictions. In what follows I discuss the trade-offs that emerge from Tables 9 and 10 by distinguishing between three kinds of relationships: relationships between response coefficients, relationships between averages, and mixed relationships between a constant and a response.

5.2.1 Relationships between responses

The only robust such relationship that emerges, in the two samples, is a negative one between a_3 and a_2. Since a_3 is typically negative and a_2 typically positive, this means that their absolute values are positively correlated across forecasters. Forecasters who have a more volatile real GDP forecast, as captured by a larger $|a_2|$, must also have a more volatile unemployment growth forecast. Presumably this means that there exists some reasonably stable relationship between output growth and unemployment growth, such as Okun's law or a production function, and that any joint forecast for unemployment growth and GDP growth must be consistent with that relationship.

On the other hand, no AC condition seems to link a_2 or a_3 with a_1, suggesting there is much less of a convincing evidence for a stable inflation-output relationship.

5.2.2 Relationships between averages

Such relationships involve only c_3, the average *growth rate* of unemployment. Most theories would imply that c_3 is zero unless there is a secular trend in labor market institutions such as unemployment benefits or minimum wages. Yet c_3 is on average positive in both samples, has a substantial variance, and is negatively correlated with c_2 (in both samples) and c_1 in sample 2. If we accept the idea of secular growth in the natural rate of unemployment, then it is just natural that experts who believe in a larger growth rate of unemployment also believe in lower secular growth rates of output, hence the negative relationship between c_3 and c_2. This again can be seen as an AC condition matching a production function-like positive link between employment and output. More surprising is the negative link between c_1 and c_3 in sample 2. At face value this might be interpreted as a long-run trade-off between inflation and unemployment. But this is not accurate since c_1 is the average level of inflation, not its rate of change. A more proper interpretation, thus, is a negative link between the inflation rate and the change in unemployment. This is exactly the sort of Phillips curve predicted by models of hysteresis (Blanchard and Summers, 1986). Thus if it were the case that such a relationship were accurately identified and common knowledge, that negative link could be interpreted as an autocoherence condition. However, this is rather far fectched since there is little evidence for hysteresis in the strict sense and it is hard to find a robust link between inflation and unemployment, let alone inflation and the change in unemployment.

REFERENCES

Buera, Francisco, Alexander Monge-Naranjo, and Giorgio Primiceri (2011), "Learning the wealth of nations", *Econometrica*, Vol. 79, No. 1, 1–45

Crawford, Vincent and Joel Sobel (1982), "Strategic information transmission", *Econometrica*

Hansen Lars Peter, Thomas J. Sargent, Gauhar Turmuhambetova, Noah Williams (2006), "Robust control and model misspecification" *Journal of Economic Theory* 128, 45 – 90

Fudenberg. D. and D.K. Levine (1993) "Self-Confirming Equilibrium" *Econometrica*, 61 , 523-546.

——————————— (2007), "Self-confirming equilibrium and the Lucas critique"

Piketty, Thomas (1995) : "Social mobility and redistributive politics", *Quarterly journal of economics*, 110, 3, 551-584.

6 APPENDIX

6.1 Proof of Proposition 1

Let $f(\hat{a}) = \bar{\varphi}\hat{a}^3 - (a\varphi)\hat{a}^2 + \varphi\bar{\varphi}\hat{a} - \varphi^2 a$. Since $f(0) < 0$ and $\lim_{\hat{a}\to+\infty} f(\hat{a}) = +\infty$, there always exists a positive solution. Furthermore, $f'(\hat{a}) = 3\bar{\varphi}\hat{a}^2 - 2(a\varphi)\hat{a} + \varphi\bar{\varphi}$. If $(a\varphi)^2 - 3\varphi\bar{\varphi}^2 < 0$, (i.e. $\bar{\varphi} > \frac{a\sqrt{\varphi}}{\sqrt{3}}$), then $f'() > 0$ throughout. If that holds, the solution is unique. One can straightforwardly check that $f(a) = 0$ if $\varphi = \bar{\varphi}$. Finally, differentiating, we get

$$f'(\hat{a})\frac{d\hat{a}}{d\bar{\varphi}} + \hat{a}^3 + \varphi\hat{a} = 0.$$

Since $f'() > 0$, it follows that $\frac{d\hat{a}}{d\bar{\varphi}} < 0$.

QED

6.2 Derivation of the autocoherence conditions

1. Variance of z

$$\begin{aligned} Ez^2 &= 1 \\ &= \omega^2\sigma_u^2 + \sigma_\varepsilon^2 \\ &= \hat{E}z^2 \\ &= \hat{\omega}^2\hat{\sigma}_u^2 + \hat{\sigma}_\varepsilon^2. \end{aligned} \tag{31}$$

2. Covariance between z and y

$$\begin{aligned} Eyz &= a_{yu}\omega\sigma_u^2 + a_{y\varepsilon}\sigma_\varepsilon^2 \\ &= \hat{E}yz \\ &= \hat{a}_{yu}\hat{\omega}\hat{\sigma}_u^2 + \hat{a}_{y\varepsilon}\hat{\sigma}_\varepsilon^2 \end{aligned}$$

Using (31), (32) can be rewritten

$$\begin{aligned} (a_{y\varepsilon}\omega + 1)\omega\sigma_u^2 + a_{y\varepsilon}(1-\omega^2\sigma_u^2) &= (\hat{a}_{y\varepsilon}\hat{\omega} + 1)\hat{\omega}\hat{\sigma}_u^2 + \hat{a}_{y\varepsilon}(1-\hat{\omega}^2\hat{\sigma}_u^2), \\ &\Longleftrightarrow \\ \omega\sigma_u^2 + a_{y\varepsilon} &= \hat{\omega}\hat{\sigma}_u^2 + \hat{a}_{y\varepsilon}. \end{aligned} \tag{32}$$

3. Covariance between z and p

$$\begin{aligned} Epz &= a_{pu}\omega\sigma_u^2 + a_{p\varepsilon}\sigma_\varepsilon^2 \\ &= \hat{E}pz \\ &= \hat{a}_{pu}\hat{\omega}\hat{\sigma}_u^2 + \hat{a}_{p\varepsilon}\hat{\sigma}_\varepsilon^2. \end{aligned}$$

Using similar steps as above, we can see that this is equivalent to

$$\frac{\omega\sigma_u^2}{\delta} + a_{p\varepsilon} = \frac{\hat{\omega}\hat{\sigma}_u^2}{\hat{\delta}} + \hat{a}_{p\varepsilon}. \tag{33}$$

4. Covariance between y and p

$$\begin{aligned}
Epy &= a_{yu}a_{pu}\sigma_u^2 + a_{y\varepsilon}a_{p\varepsilon}\sigma_\varepsilon^2 + \frac{\rho(\rho-1)}{\delta}\sigma_v^2 \\
&= (a_{y\varepsilon}\omega + 1)(\frac{1}{\delta} + \omega a_{p\varepsilon})\sigma_u^2 + a_{y\varepsilon}a_{p\varepsilon}\sigma_\varepsilon^2 + \frac{\rho(\rho-1)}{\delta}\sigma_v^2 \\
&= \left(\frac{1}{\delta} + \omega a_{p\varepsilon} + a_{y\varepsilon}\omega\right)\sigma_u^2 + a_{y\varepsilon}a_{p\varepsilon} + \frac{\rho(\rho-1)}{\delta}\sigma_v^2 \\
&= \hat{E}py \\
&= \left(\frac{1}{\hat{\delta}} + \hat{\omega}\hat{a}_{p\varepsilon} + \hat{a}_{y\varepsilon}\hat{\omega}\right)\hat{\sigma}_u^2 + \hat{a}_{y\varepsilon}\hat{a}_{p\varepsilon} + \frac{\hat{\rho}(\hat{\rho}-1)}{\hat{\delta}}\hat{\sigma}_v^2.
\end{aligned} \tag{34}$$

5. Variance of y

$$\begin{aligned}
Ey^2 &= a_{yu}^2\sigma_u^2 + a_{y\varepsilon}^2\sigma_\varepsilon^2 + \rho^2\sigma_v^2 \\
&= (a_{y\varepsilon}\omega + 1)^2\sigma_u^2 + a_{y\varepsilon}^2(1 - \omega^2\sigma_\varepsilon^2) + \rho^2\sigma_v^2 \\
&= (1 + 2a_{y\varepsilon}\omega)\sigma_u^2 + a_{y\varepsilon}^2 + \rho^2\sigma_v^2 \\
&= \hat{E}y^2 \\
&= (1 + 2\hat{a}_{y\varepsilon}\hat{\omega})\hat{\sigma}_u^2 + \hat{a}_{y\varepsilon}^2 + \hat{\rho}^2\hat{\sigma}_v^2.
\end{aligned} \tag{35}$$

6. Variance of p.

Note that this autocoherence condition can always be matched by picking the right value of $\hat{\sigma}_\eta^2$, regardless of the other parameters of the perceived model. I write it for the record.

$$\begin{aligned}
Ep^2 &= a_{pu}^2\sigma_u^2 + a_{p\varepsilon}^2\sigma_\varepsilon^2 + \frac{(\rho-1)^2}{\delta^2}\sigma_v^2 + \sigma_\eta^2 \\
&= (\frac{1}{\delta^2} + \frac{2a_{p\varepsilon}\omega}{\delta})\sigma_u^2 + a_{p\varepsilon}^2 + \frac{(\rho-1)^2}{\delta^2}\sigma_v^2 \\
&= \hat{E}p^2 \\
&= (\frac{1}{\hat{\delta}^2} + \frac{2\hat{a}_{p\varepsilon}\hat{\omega}}{\hat{\delta}})\hat{\sigma}_u^2 + \hat{a}_{p\varepsilon}^2 + \frac{(\hat{\rho}-1)^2}{\hat{\delta}^2}\hat{\sigma}_v^2.
\end{aligned} \tag{36}$$

6.3 Proof of Proposition 2

As proved in footnote 8, we condition (33) is equivalent to

$$\frac{\hat{\omega}\hat{\sigma}_u^2 + \gamma\hat{a}}{\hat{\delta} + \hat{b} - \hat{\mu}} = \frac{\omega\sigma_u^2 + \gamma a}{\delta + b - \mu}. \tag{37}$$

Using the definition of $a_{y\varepsilon}$ and $\hat{a}_{y\varepsilon}$, we can rewrite (32) as

$$\omega\sigma_u^2 + \gamma a - b\left(\frac{\hat{\omega}\hat{\sigma}_u^2 + \gamma\hat{a}}{\hat{\delta} + \hat{b} - \hat{\mu}}\right) = \hat{\omega}\hat{\sigma}_u^2 + \gamma\hat{a} - \hat{b}\left(\frac{\hat{\omega}\hat{\sigma}_u^2 + \gamma\hat{a}}{\hat{\delta} + \hat{b} - \hat{\mu}}\right)$$

Replacing $\hat{\omega}\hat{\sigma}_u^2 + \gamma\hat{a}$ with $\frac{\omega\sigma_u^2+\gamma a}{\delta+b-\mu}(\hat{\delta} + \hat{b} - \hat{\mu})$ and rearranging, we indeed get $\hat{\delta} - \hat{\mu} = \delta - \mu$.

QED

6.4 The price block revealed case

Assume $\hat{\omega} = \omega, \hat{\sigma}_u = \sigma_u$, and $\hat{\delta} = \delta$. We know from Proposition 2 that $\hat{\mu} = \mu$. From (31) we get

$$\hat{\sigma}_\varepsilon^2 = \sigma_\varepsilon^2.$$

From (32) we get

$$a_{y\varepsilon} = \hat{a}_{y\varepsilon}, \tag{38}$$

Similarly, for (33) to hold we need

$$a_{p\varepsilon} = \hat{a}_{p\varepsilon}. \tag{39}$$

This in turn implies $a_{yu} = \hat{a}_{yu}$ and $a_{pu} = \hat{a}_{pu}$.

Finally, (34) and (35) yield

$$\begin{aligned} \frac{\rho(\rho-1)}{\delta}\sigma_v^2 &= \frac{\hat{\rho}(\hat{\rho}-1)}{\hat{\delta}}\hat{\sigma}_v^2; \\ \hat{\rho}^2\hat{\sigma}_v^2 &= \rho^2\sigma_v^2 \end{aligned}$$

The solution to this system is

$$\begin{aligned}\rho &= \hat{\rho}; \\ \hat{\sigma}_v^2 &= \sigma_v^2.\end{aligned}$$

From (37) we get

$$\omega\sigma_u^2(\hat{b}-b) = \gamma\left[\hat{a}(\delta+b-\mu) - a(\hat{\delta}+\hat{b}-\hat{\mu})\right]. \tag{40}$$

Recall, from (20), that

$$\gamma = -\hat{a}\frac{(\delta-\mu)^2}{\varphi\left(\delta+\hat{b}-\mu\right)^2 + \hat{a}^2\left(\delta-\mu\right)^2}\omega\sigma_u^2. \tag{41}$$

Substituting, we get the cubic equation that has been solved numerically:

$$(\hat{b}-b)\left(\varphi\left(\delta+\hat{b}-\mu\right)^2 + \hat{a}^2\left(\delta-\mu\right)^2\right) + \hat{a}(\delta-\mu)^2\left[(\hat{a}-a)(\delta-\mu) + \hat{a}b - a\hat{b}\right] = 0.$$

Finally, the above conditions trivially imply that the remaining condition (36) holds.

6.5 Linearization of the AC conditions in variant B

The six AC conditions are

$$\begin{aligned}
1 &= \hat{\omega}^2\hat{\sigma}_u^2 + \hat{\lambda}^2\hat{\sigma}_v^2; && (42)\\
a_{yu}\omega\sigma_u^2 - a_{yv}\lambda\sigma_v^2 &= \hat{a}_{yu}\hat{\omega}\hat{\sigma}_u^2 - \hat{a}_{yv}\hat{\lambda}\hat{\sigma}_v^2; && (43)\\
a_{pu}\omega\sigma_u^2 - a_{pv}\lambda\sigma_v^2 &= \hat{a}_{pu}\hat{\omega}\hat{\sigma}_u^2 - \hat{a}_{pv}\hat{\lambda}\hat{\sigma}_v^2; && (44)\\
a_{yu}a_{pu}\sigma_u^2 + a_{yv}a_{pv}\sigma_v^2 &= \hat{a}_{yu}\hat{a}_{pu}\hat{\sigma}_u^2 + \hat{a}_{yv}\hat{a}_{pv}\hat{\sigma}_v^2; && (45)\\
a_{yu}^2\sigma_u^2 + a_{yv}^2\sigma_v^2 &= \hat{a}_{yu}^2\hat{\sigma}_u^2 + \hat{a}_{yv}^2\hat{\sigma}_v^2; && (46)\\
a_{pu}^2\sigma_u^2 + a_{pv}^2\sigma_v^2 &= \hat{a}_{pu}^2\hat{\sigma}_u^2 + \hat{a}_{pv}^2\hat{\sigma}_v^2. && (47)
\end{aligned}$$

Using the definitions in Table 4 and 5 to rearrange (44), and defining $c = \frac{\omega\sigma_u^2 - \lambda(\rho-1)\sigma_v^2}{\delta+b-\mu}$, we see that (44) is equivalent to

$$\hat{c} + \frac{\gamma\hat{a}}{\hat{\delta} + \hat{b} - \hat{\mu}} = c + \frac{\gamma a}{\delta + b - \mu}. \tag{48}$$

This expression can be conveniently substituted into the expressions in Tables 4 and 5 to reduce the number of hatted parameters that appear. We get the following:

Coefficients	Expression
a_{yu}	$1 + \omega\gamma a - b\omega(c + \frac{\gamma a}{\delta+b-\mu})$
a_{yv}	$\rho - \gamma\lambda a + b\lambda\left(c + \frac{\gamma a}{\delta+b-\mu}\right)$
a_{pu}	$\frac{1}{\delta} + (\mu - b)\frac{\omega}{\delta}\left(c + \frac{\gamma a}{\delta+b-\mu}\right) + \frac{\gamma a\omega}{\delta}$
a_{pv}	$\frac{\rho-1}{\delta} - (\mu - b)\frac{\lambda}{\delta}\left(c + \frac{\gamma a}{\delta+b-\mu}\right) - \frac{\gamma a\lambda}{\delta}$
$\hat{a}_{yu}$	$1 + \hat{\omega}\gamma\hat{a} - \hat{b}\hat{\omega}(c + \frac{\gamma a}{\delta+b-\mu})$
$\hat{a}_{yv}$	$\hat{\rho} - \gamma\hat{\lambda}\hat{a} + \hat{b}\hat{\lambda}\left(c + \frac{\gamma a}{\delta+b-\mu}\right)$
$\hat{a}_{pu}$	$\frac{1}{\hat{\delta}} + (\hat{\mu} - \hat{b})\frac{\hat{\omega}}{\hat{\delta}}\left(c + \frac{\gamma a}{\delta+b-\mu}\right) + \frac{\gamma\hat{a}\hat{\omega}}{\hat{\delta}}$
$\hat{a}_{pv}$	$\frac{\hat{\rho}-1}{\hat{\delta}} - (\hat{\mu} - \hat{b})\frac{\hat{\lambda}}{\hat{\delta}}\left(c + \frac{\gamma a}{\delta+b-\mu}\right) - \frac{\gamma\hat{a}\hat{\lambda}}{\hat{\delta}}$

From now on we will take into account that $\hat{a} = a$ and $\hat{b} = b$. Using this Table we can then compute $\Delta\hat{a}_{yu} = \hat{a}_{yu} - a_{yu}$, etc.[19] We get

$$\begin{aligned}
\Delta\hat{a}_{yu} &\approx \left(\frac{\gamma a(\delta-\mu)}{\delta+b-\mu} - bc\right)\Delta\hat{\omega}; \\
\Delta\hat{a}_{pu} &\approx -\frac{\Delta\hat{\delta}}{\delta^2}\left(1 + (\mu-b)\omega c + \frac{\gamma a\omega\delta}{\delta+b-\mu}\right) + \frac{\Delta\hat{\mu}}{\delta}\omega(c + \frac{\gamma a}{\delta+b-\mu}) \\
&\quad + \frac{\Delta\hat{\omega}}{\delta}\left((\mu-b)c + \frac{\gamma a\delta}{\delta+b-\mu}\right); \\
\Delta\hat{a}_{yv} &\approx \Delta\hat{\rho} + \Delta\hat{\lambda}\left(bc - \frac{\gamma a(\delta-\mu)}{\delta+b-\mu}\right); \\
\Delta\hat{a}_{pv} &\approx \frac{\Delta\hat{\rho}}{\delta} - \frac{\Delta\hat{\delta}}{\delta^2}\left(\rho - 1 - (\mu-b)\lambda c - \frac{\gamma a\lambda\delta}{\delta+b-\mu}\right) \\
&\quad - \frac{\Delta\hat{\mu}}{\delta}\lambda(c + \frac{\gamma a}{\delta+b-\mu}) - \frac{\Delta\hat{\lambda}}{\delta}\left((\mu-b)c + \frac{\gamma a\delta}{\delta+b-\mu}\right).
\end{aligned}$$

[19] Note that a small deviation between the perceived and correct model changes γ marginally, hence a_{yu} is different from its value under the correct model, and thus $\Delta\hat{a}_{yu}$ is not equal to the difference between $\hat{a}_{yu}$ and the value of a_{yu} under the correct model.

We can also compute

$$\begin{aligned}\Delta\hat{c} \quad &\approx \quad -\frac{\omega\sigma_u^2-\lambda(\rho-1)\sigma_v^2}{(\delta+b-\mu)^2}\left(\Delta\hat{\delta}-\Delta\hat{\mu}\right)\\ &\quad +\frac{1}{\delta+b-\mu}\left[\sigma_u^2\Delta\hat{\omega}+2\omega\sigma_u\Delta\hat{\sigma}_u-\lambda\sigma_v^2\Delta\hat{\rho}-(\rho-1)\sigma_v^2\Delta\hat{\lambda}-2\lambda(\rho-1)\sigma_v\Delta\hat{\sigma}_v\right].\end{aligned}$$

Finally, substituting (48) into (43) and rearranging using the definitions in Tables 4 and 5 we get the following:

$$\left(c+\frac{\gamma a}{\delta+b-\mu}\right)\left[\hat{\delta}-\hat{\mu}-\delta+\mu\right]=\hat{\lambda}\hat{\sigma}_v^2-\lambda\sigma_v^2. \tag{49}$$

Next, we differentiate (42)-(47), substituting (48) and (49) for (44) and (43) respectively, and replacing $\Delta\hat{a}_{yu}$,etc,as well as $\Delta\hat{c}$ by their expressions above. We get six linear equations that are expressed as

$$A.(\Delta(\hat{\delta}-\hat{\mu}),\Delta\hat{\lambda},\Delta\hat{\sigma}_v,\Delta\hat{\omega},\Delta\hat{\sigma}_u,\Delta\hat{\rho})'=\Delta\hat{\delta}.w,$$

where the nonzero coefficients of $A:6\times 6$, and $w:6\times 1$ are the following:

$A_{12}=\lambda\sigma_v^2$; $A_{13}=\lambda^2\sigma_v$; $A_{14}=\omega\sigma_u^2$; $A_{15}=\omega^2\sigma_u$.

$A_{21}=c+\frac{\gamma a}{\delta+b-\mu}$; $A_{22}=-\sigma_v^2$; $A_{23}=-2\lambda\sigma_v$.

$A_{31}=-\frac{\omega\sigma_u^2-\lambda(\rho-1)\sigma_v^2}{(\delta+b-\mu)^2}-\frac{\gamma a}{(\delta+b-\mu)^2}$; $A_{32}=-\frac{(\rho-1)\sigma_v^2}{\delta+b-\mu}$; $A_{33}=-\frac{2\lambda(\rho-1)\sigma_v}{\delta+b-\mu}$; $A_{34}=\frac{\sigma_u^2}{\delta+b-\mu}$; $A_{35}=\frac{2\omega\sigma_u}{\delta+b-\mu}$; $A_{36}=-\frac{\lambda\sigma_v^2}{\delta+b-\mu}$.

$A_{41}=\left(c+\frac{\gamma a}{\delta+b-\mu}\right)\left(\frac{\lambda a_{yv}\sigma_v^2-\omega a_{yu}\sigma_u^2}{\delta}\right)$; $A_{42}=a_{pv}\sigma_v^2(bc-\frac{a\gamma(\delta-\mu)}{\delta+b-\mu})-\frac{a_{yv}\sigma_v^2}{\delta}\left((\mu-b)c+\frac{a\gamma\delta}{\delta+b-\mu}\right)$; $A_{43}=2a_{yv}a_{pv}\sigma_v$; $A_{44}=a_{pu}\sigma_u^2(\frac{a\gamma(\delta-\mu)}{\delta+b-\mu}-bc)+\frac{a_{yu}\sigma_u^2}{\delta}\left((\mu-b)c+\frac{a\gamma\delta}{\delta+b-\mu}\right)$; $A_{45}=2a_{yu}a_{pu}\sigma_u$; $A_{46}=a_{pv}\sigma_v^2+\frac{a_{yv}}{\delta}\sigma_v^2$.

$A_{52}=a_{yv}\sigma_v^2(bc-\frac{a\gamma(\delta-\mu)}{\delta+b-\mu})$; $A_{53}=a_{yv}^2\sigma_v$; $A_{54}=a_{yu}\sigma_u^2(\frac{a\gamma(\delta-\mu)}{\delta+b-\mu}-bc)$; $A_{55}=a_{yu}^2\sigma_u$; $A_{56}=a_{yv}\sigma_v^2$.

$A_{61}=\left(c+\frac{\gamma a}{\delta+b-\mu}\right)\left(\frac{\lambda a_{pv}\sigma_v^2-\omega a_{pu}\sigma_u^2}{\delta}\right)$; $A_{62}=-\frac{a_{pv}\sigma_v^2}{\delta}\left((\mu-b)c+\frac{a\gamma\delta}{\delta+b-\mu}\right)$; $A_{63}=a_{pv}^2\sigma_v$; $A_{64}=\frac{a_{pu}\sigma_u^2}{\delta}\left((\mu-b)c+\frac{a\gamma\delta}{\delta+b-\mu}\right)$; $A_{65}=a_{pu}^2\sigma_u$; $A_{66}=\frac{a_{pv}\sigma_v^2}{\delta}$.

$w_4=\frac{\sigma_u^2a_{yu}}{\delta^2}\left(1+(\mu-b)\omega c+\frac{\gamma a\omega\delta}{\delta+b-\mu}\right)+\left(c+\frac{\gamma a}{\delta+b-\mu}\right)\left(\frac{\lambda\sigma_v^2a_{yv}-\omega\sigma_u^2a_{yu}}{\delta}\right)$

$+\frac{\sigma_v^2 a_{yv}}{\delta^2}\left(\rho - 1 - (\mu - b)\lambda c - \frac{\gamma a \lambda \delta}{\delta + b - \mu}\right);$

$w_6 = \frac{\sigma_u^2 a_{pu}}{\delta^2}\left(1 + (\mu - b)\omega c + \frac{\gamma a \omega \delta}{\delta + b - \mu}\right) + \left(c + \frac{\gamma a}{\delta + b - \mu}\right)\left(\frac{\lambda \sigma_v^2 a_{pv} - \omega \sigma_u^2 a_{pu}}{\delta}\right)$

$+\frac{\sigma_v^2 a_{pv}}{\delta^2}\left(\rho - 1 - (\mu - b)\lambda c - \frac{\gamma a \lambda \delta}{\delta + b - \mu}\right).$

In the above, γ is computed at the correct model: $\gamma = \gamma_0$.

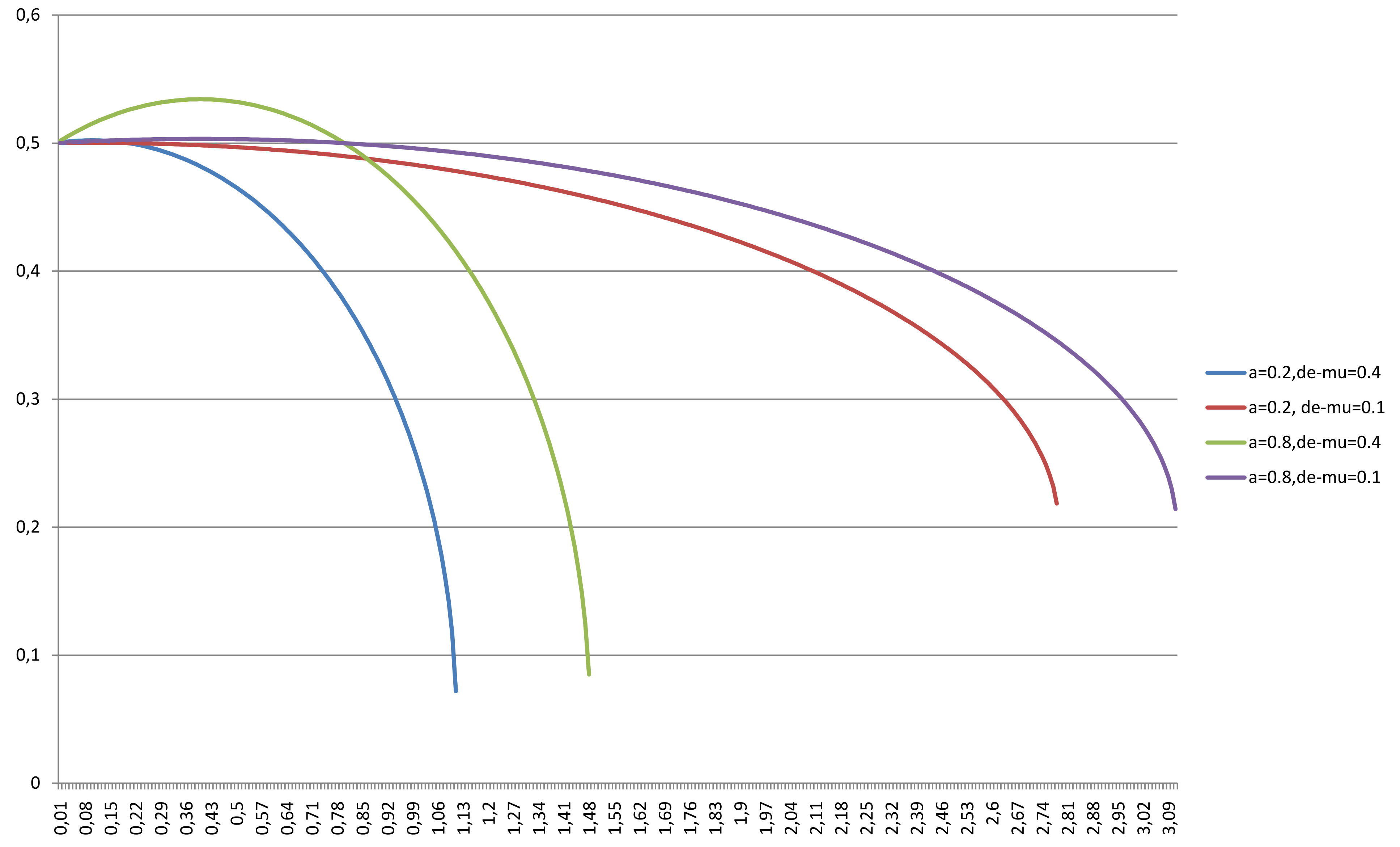
To compute the coefficient m in (30) we use (29) and note that

$$\frac{\partial \gamma}{\partial \varphi} = -\gamma \frac{(\delta + b - \mu)^2}{\varphi(\delta + b - \mu)^2 + a^2(\delta - \mu)^2}$$

and that

$$\begin{aligned}
\nabla_v \gamma &= \left(\frac{\partial \gamma}{\partial(\hat{\delta} - \hat{\mu})}, \frac{\partial \gamma}{\partial \hat{\lambda}}, \frac{\partial \gamma}{\partial \hat{\sigma}_v}, \frac{\partial \gamma}{\partial \hat{\omega}}, \frac{\partial \gamma}{\partial \hat{\sigma}_u}, \frac{\partial \gamma}{\partial \hat{\rho}}\right)' \\
&= \frac{a}{\varphi(\delta + b - \mu)^2 + a^2(\delta - \mu)^2}\begin{pmatrix} 2\lambda\sigma_v^2\rho(\delta - \mu) + b\lambda\sigma_v^2 - 2(\delta - \mu)\omega\sigma_u^2 \\ \sigma_v^2\left(\rho(\delta - \mu)^2 + b(\delta - \mu)\right) \\ 2\sigma_v\lambda\left(\rho(\delta - \mu)^2 + b(\delta - \mu)\right) \\ -(\delta - \mu)^2\sigma_u^2 \\ -2(\delta - \mu)^2\omega\sigma_u \\ (\delta - \mu)^2\lambda\sigma_v^2 \end{pmatrix} \\
&\quad - \frac{\gamma}{\varphi(\delta + b - \mu)^2 + a^2(\delta - \mu)^2}\begin{pmatrix} \left(2\varphi(\delta + b - \mu) + 2a^2(\delta - \mu)\right) \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \end{pmatrix}.
\end{aligned}$$

Figure 1 -- The a/b trade-off



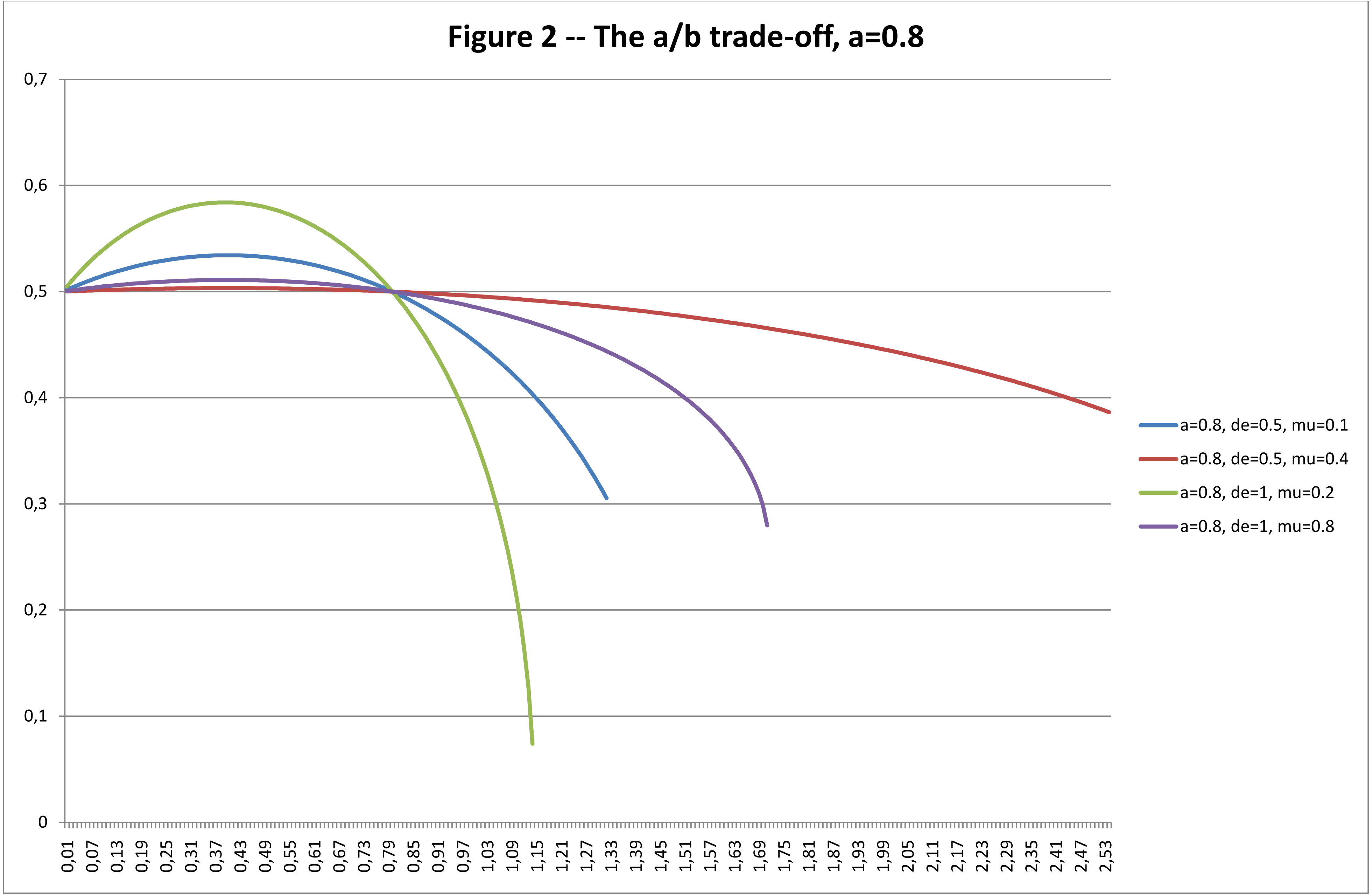


Figure 2 -- The a/b trade-off, a=0.8
0,7
0,6
0,5
0,4
0,3
0,2
0,1
0
0,01
0,07
0,13
0,19
0,25
0,31
0,37
0,43
0,49
0,55
0,61
0,67
0,73
0,79
0,85
0,91
0,97
1,03
1,09
1,15
1,21
1,27
1,33
1,39
1,45
1,51
1,57
1,63
1,69
1,75
1,81
1,87
1,93
1,99
2,05
2,11
2,17
2,23
2,29
2,35
2,41
2,47
2,53
a=0.8, de=0.5, mu=0.1
a=0.8, de=0.5, mu=0.4
a=0.8, de=1, mu=0.2
a=0.8, de=1, mu=0.8

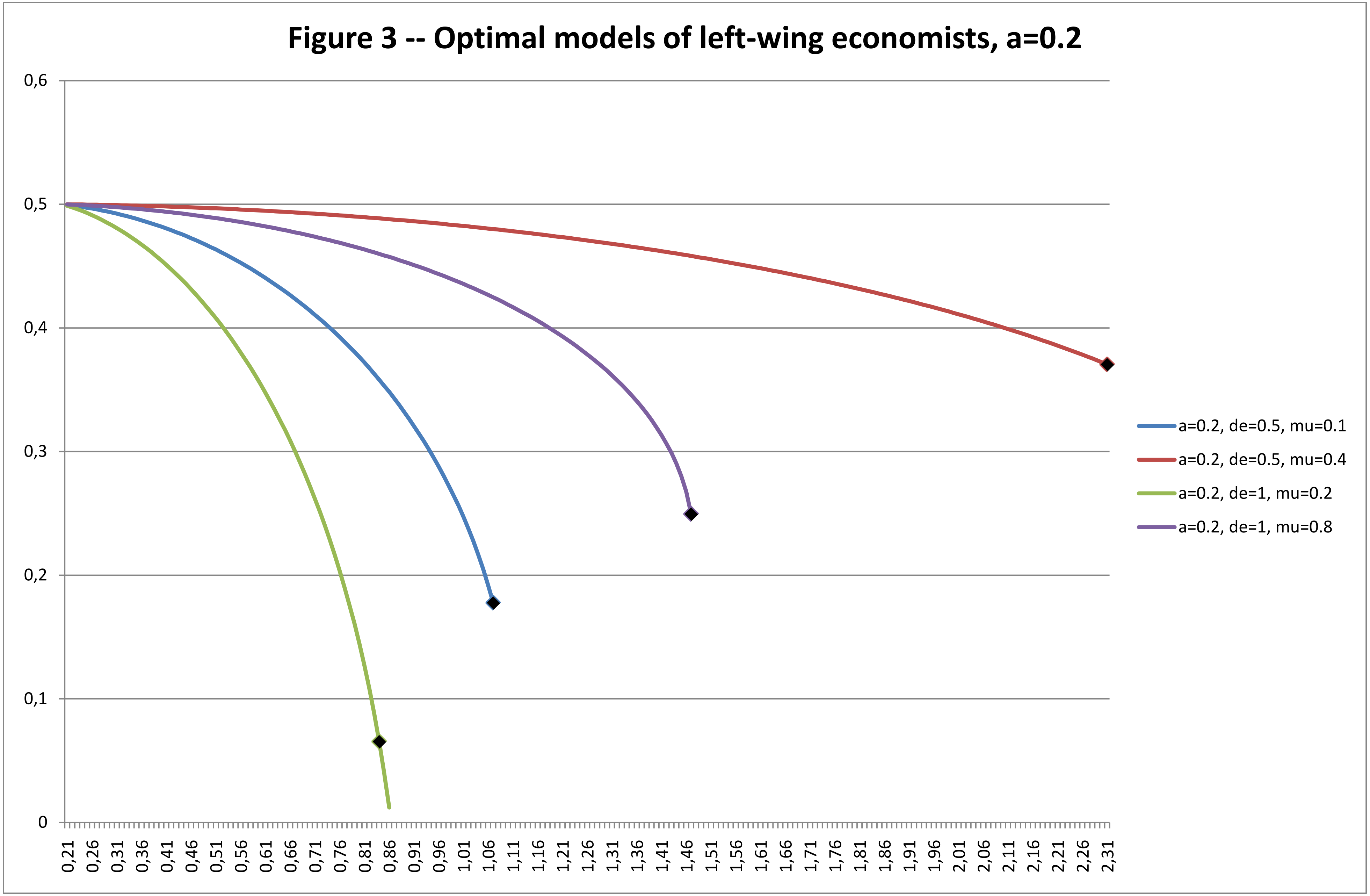
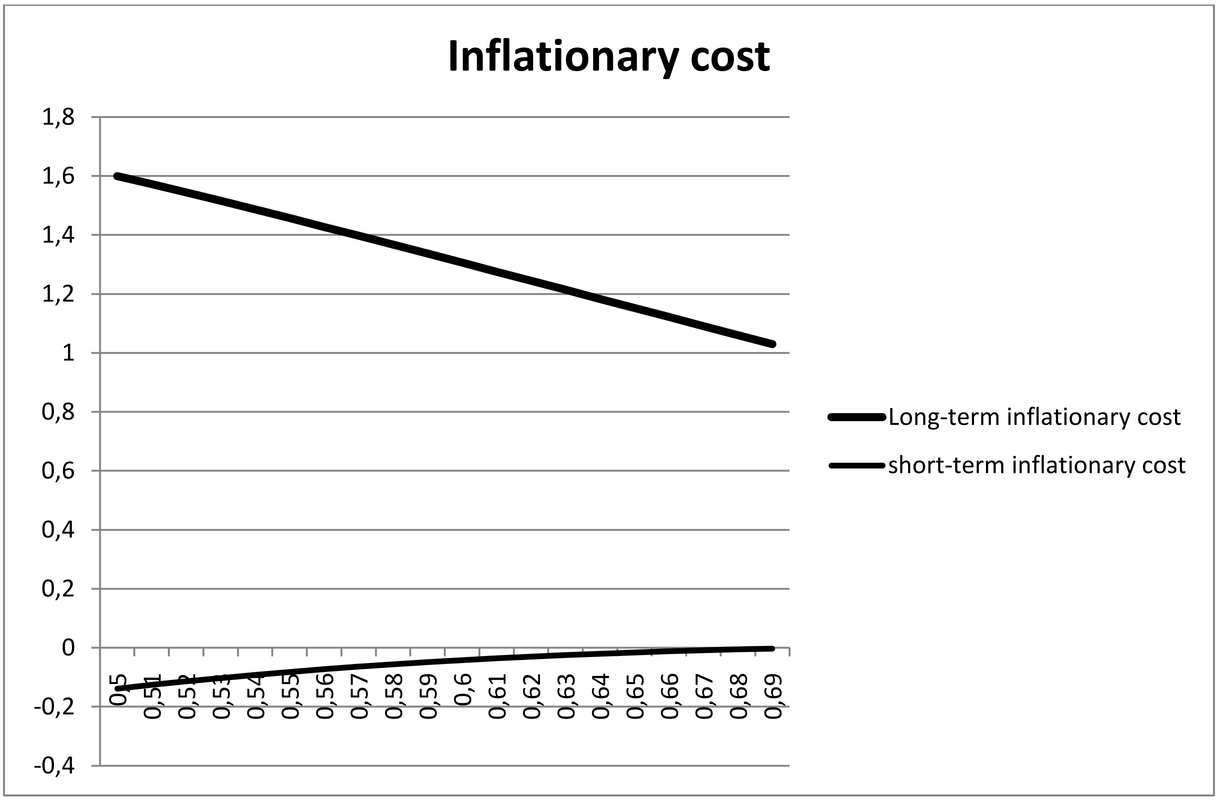


Figure 3 -- Optimal models of left-wing economists, a=0.2
0,6
0,5
0,4
0,3
0,2
0,1
0
0,21
0,26
0,31
0,36
0,41
0,46
0,51
0,56
0,61
0,66
0,71
0,76
0,81
0,86
0,91
0,96
1,01
1,06
1,11
1,16
1,21
1,26
1,31
1,36
1,41
1,46
1,51
1,56
1,61
1,66
1,71
1,76
1,81
1,86
1,91
1,96
2,01
2,06
2,11
2,16
2,21
2,26
2,31
a=0.2, de=0.5, mu=0.1
a=0.2, de=0.5, mu=0.4
a=0.2, de=1, mu=0.2
a=0.2, de=1, mu=0.8

Figure 4 – Ideological sensitivities, a = 0.7 ; b = 0.5 ; ω = 1; λ=1; σ_u^2 = 0.5; δ = 0.7 ; ρ = 1.



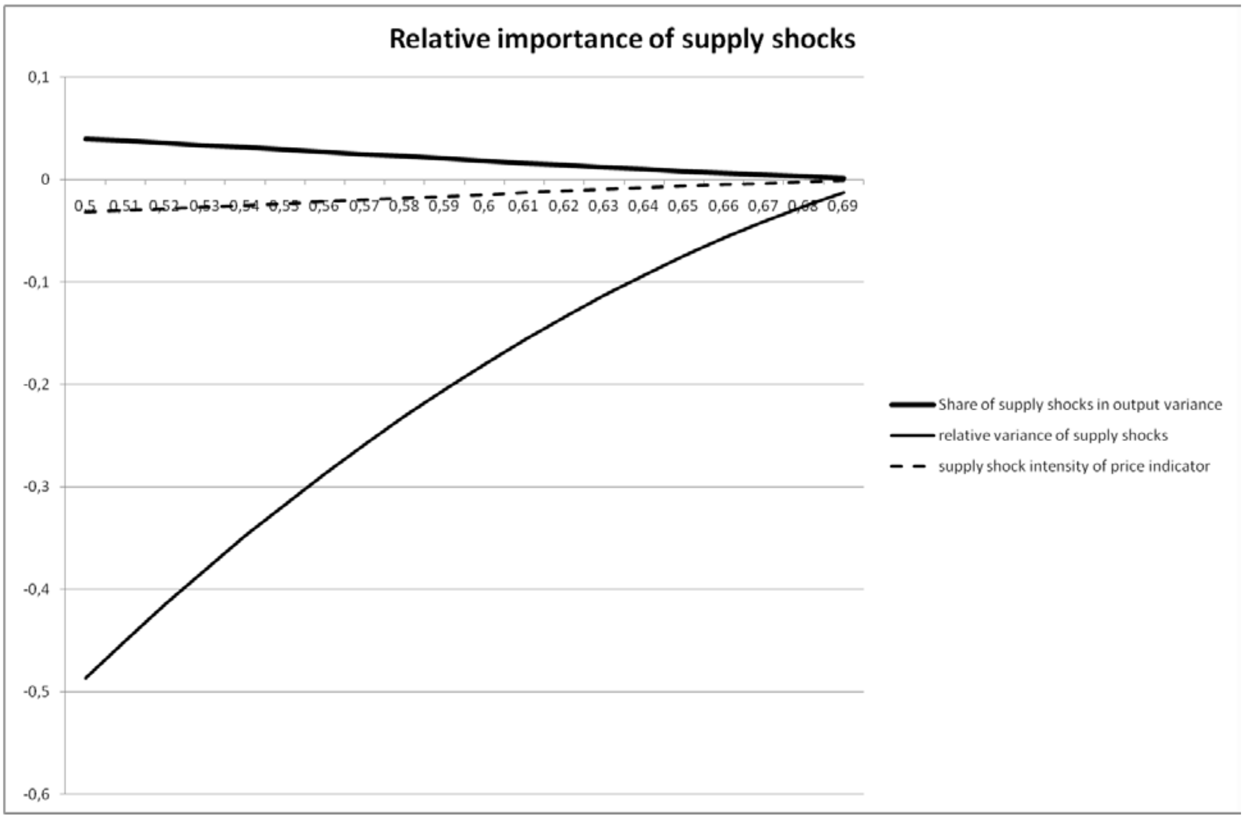
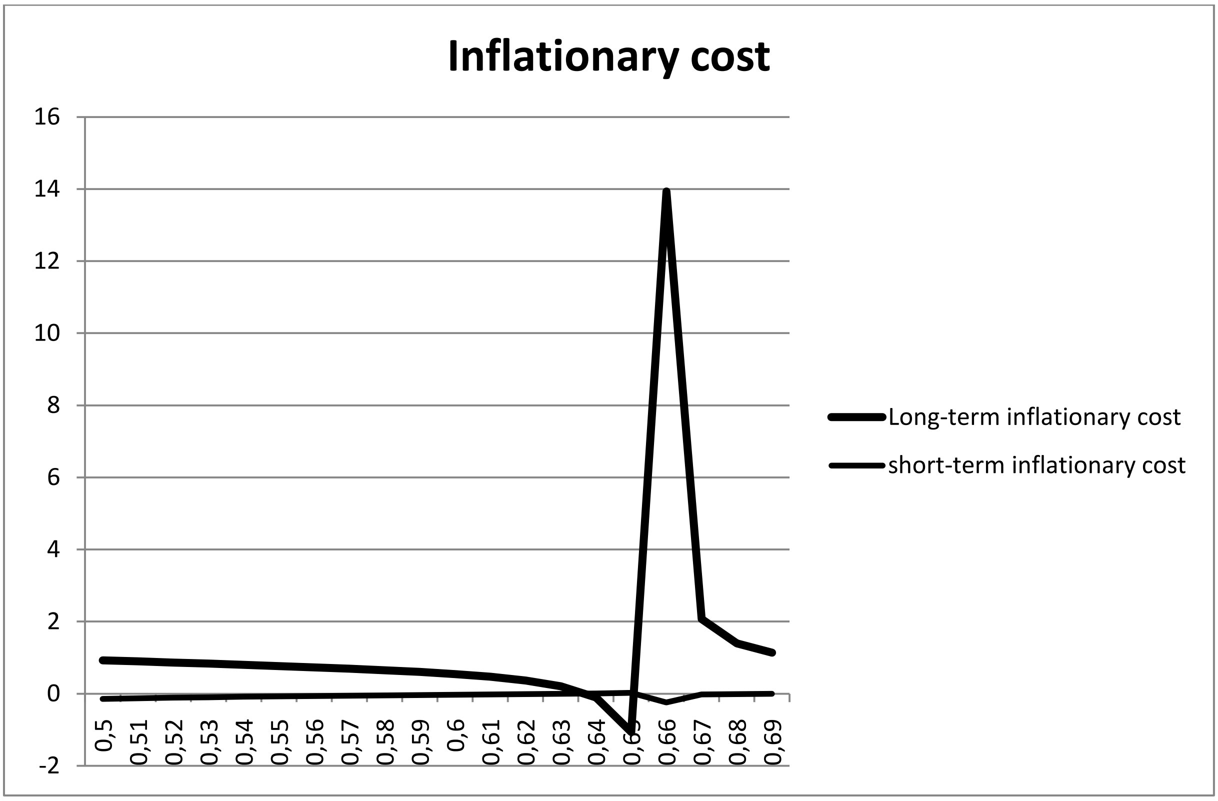


Figure 5 – Ideological sensitivities, a = 0.7 ; b = 0.5 ; ω = 1; λ=1; σ_u^2 = 0.9; δ = 0.7 ; ρ = 1.



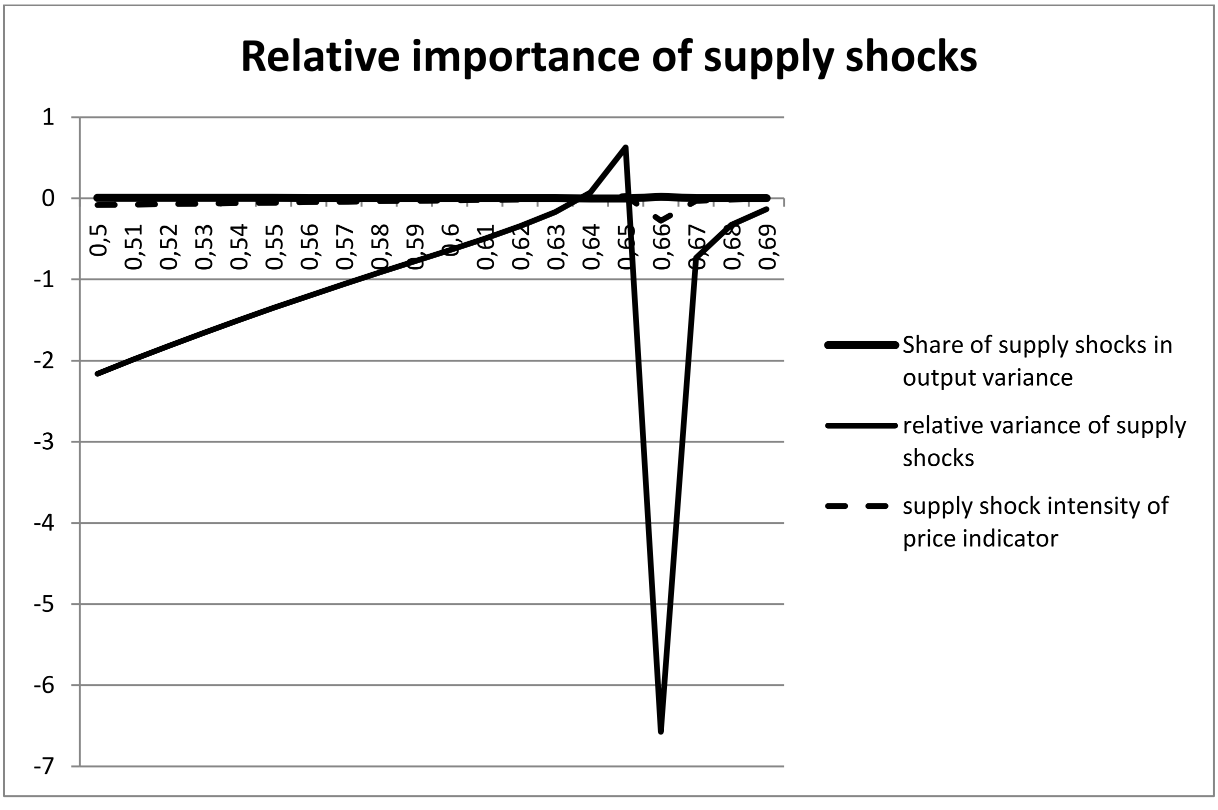
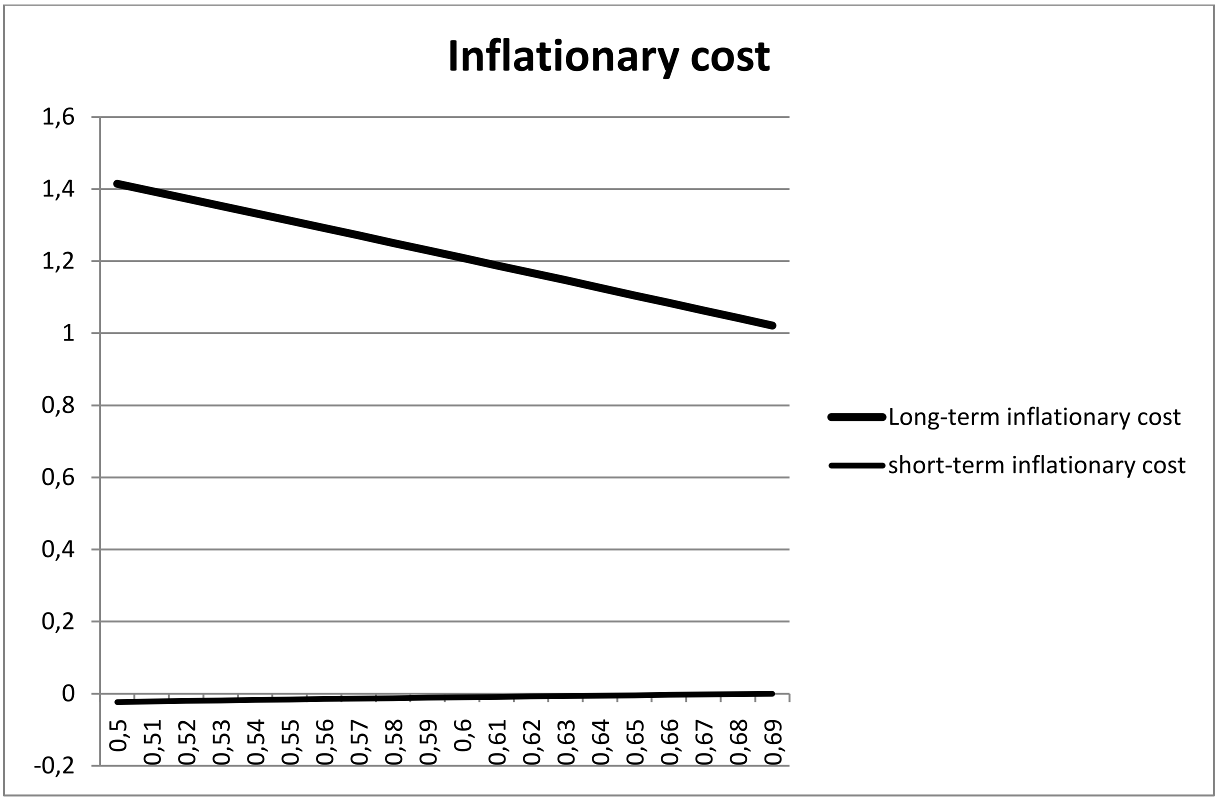


Figure 6 – Ideological sensitivities, a = 0.7 ; b = 0.5 ; ω = 1; λ=1; σ_u^2 = 0.1; δ = 0.7 ; ρ = 1.



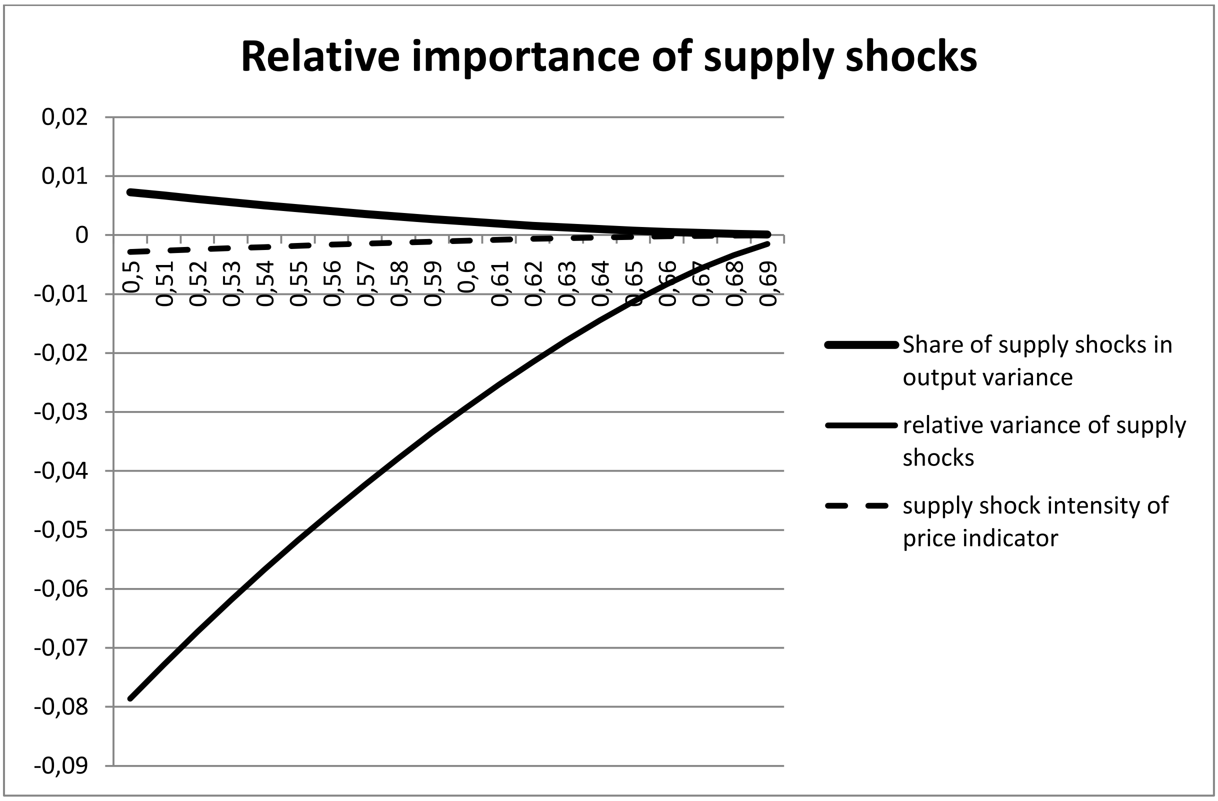
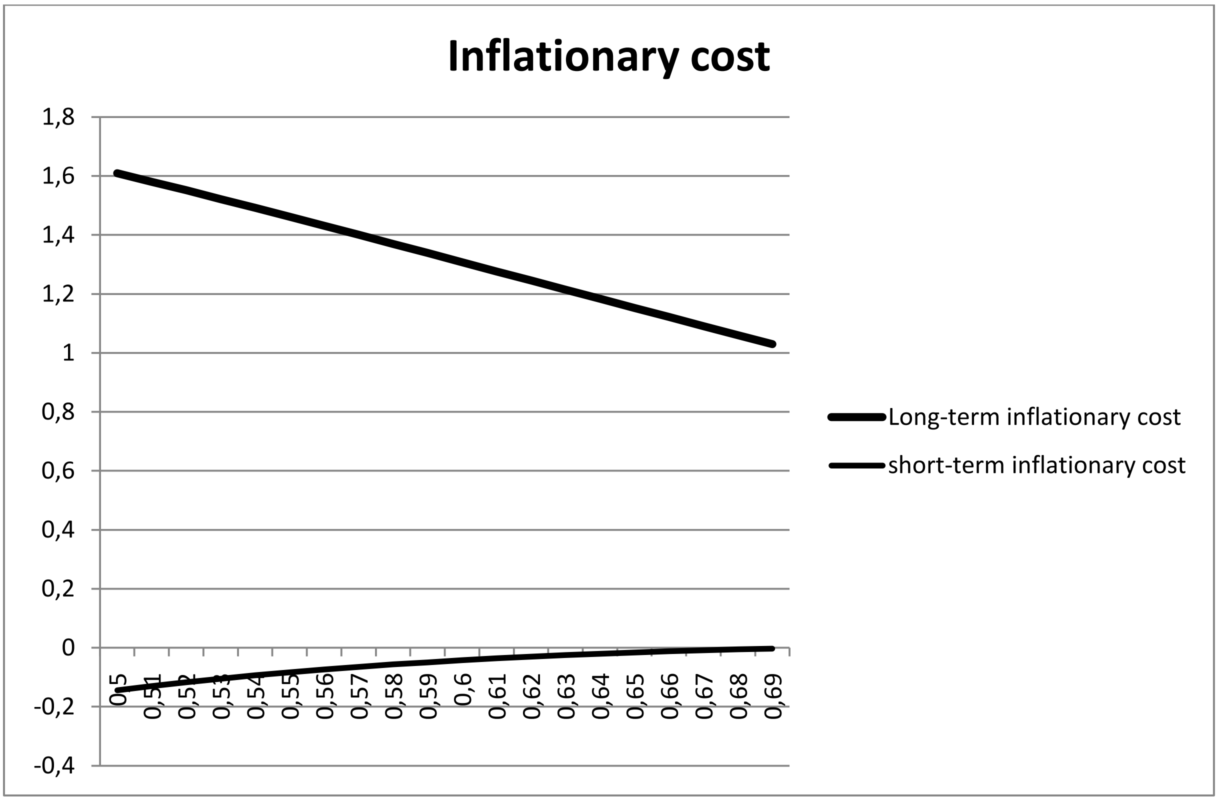


Figure 7 – Ideological sensitivities, a = 1 ; b = 0.5 ; ω = 1; λ=1; σ_u^2 = 0.5; δ = 0.7 ; ρ = 1.



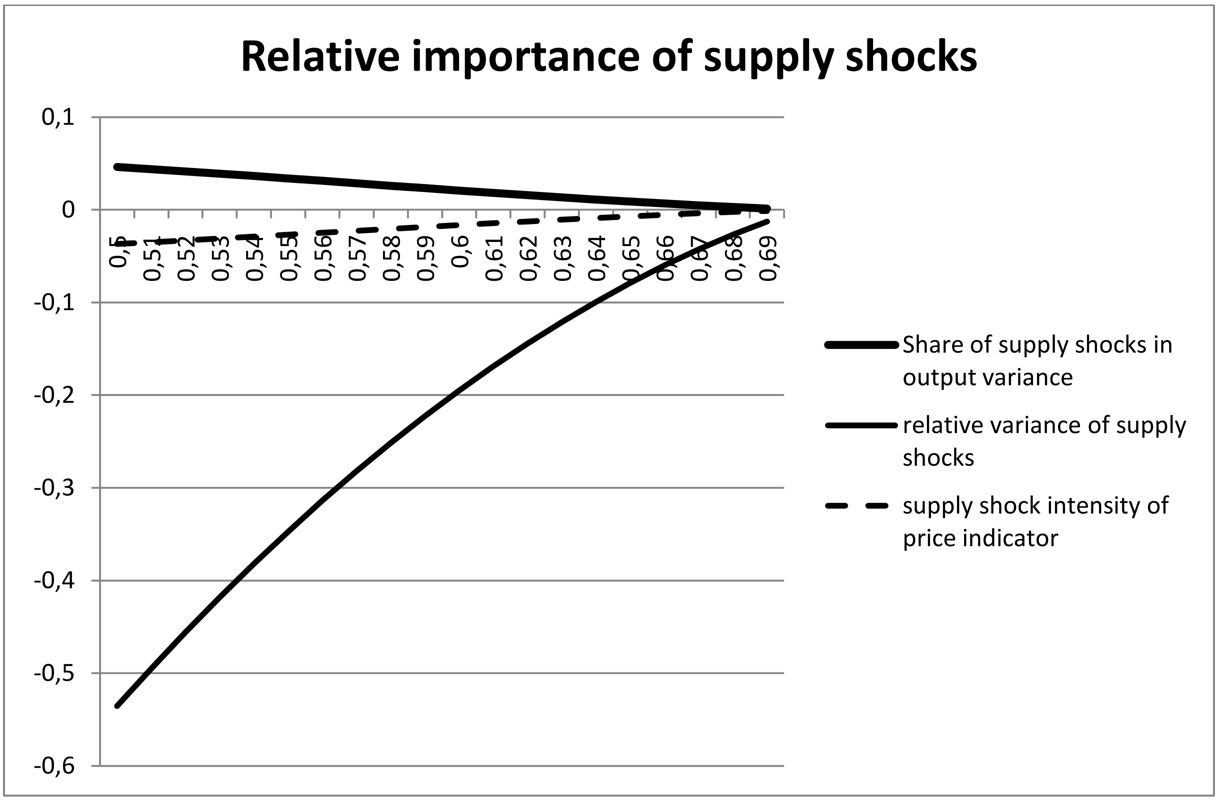
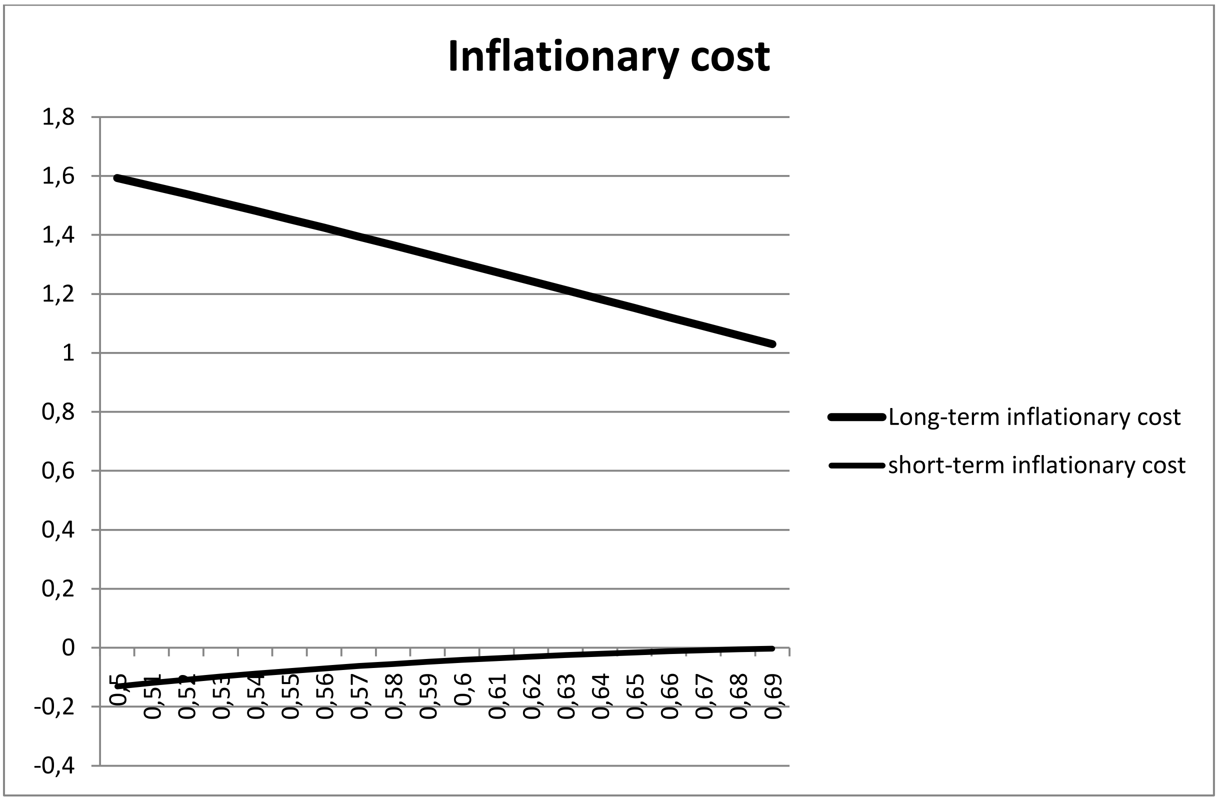


Figure 8 – Ideological sensitivities, a = 0.3 ; b = 0.5 ; ω = 1; λ=1; σ_u^2 = 0.5; δ = 0.7 ; ρ = 1.



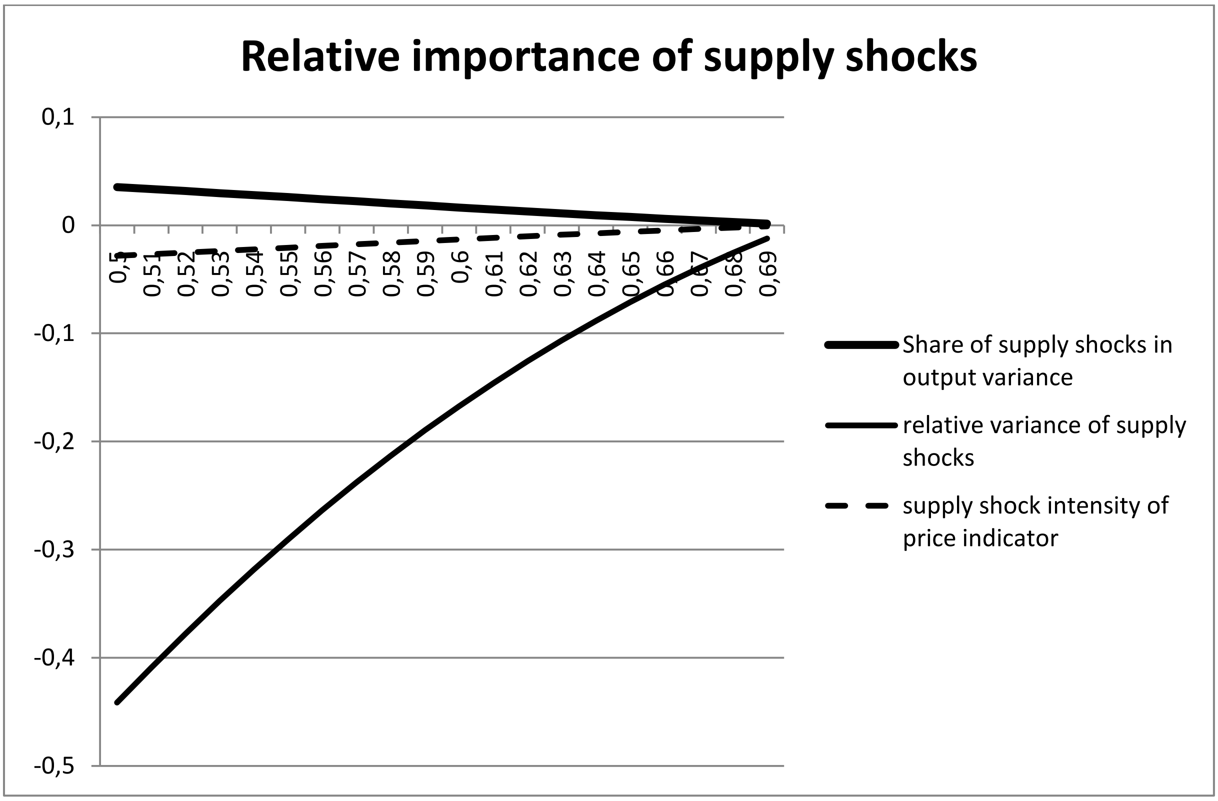


Figure 9 – Ideological sensitivities, a = 0.7 ; b = 0.1 ; ω = 1; λ=1; σ_u^2 = 0.5; δ = 0.7 ; ρ = 1.