Proportional Fair Resource Allocation on an Energy Harvesting Downlink

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***Abstract*—This paper considers the allocation of time slots in a frame, as well as power and rate to multiple receivers on an energy harvesting downlink. Energy arrival times that will occur within the frame are known at the beginning of the frame. The goal is to optimize throughput in a proportionally fair way, taking into account the inherent differences of channel quality among users. Analysis of structural characteristics of the problem reveals that it can be formulated as a biconvex optimization problem, and that it has multiple optima. Due to the biconvex nature of the problem, a Block Coordinate Descent (BCD) based optimization algorithm that converges to an optimal solution is presented. However, finding the optimal allocation with BCD entails a computational complexity that increases sharply in terms of the number of users or slots. Therefore, certain structural characteristics of the optimal power-time allocation policy are derived. Building on those, two simple and computationally scalable heuristics, PTF and ProNTO are proposed. Simulation results suggest that PTF and ProNTO can closely track the performance of BCD which achieves a good balance between total throughput and fairness.**

***Index Terms*—Broadcast channel, energy harvesting, *offline* algorithms, optimization, block coordinate descent, biconvex, proportional fairness, time sharing.**

I. Introduction

MANAGEMENT of energy consumption is vital for the sustainability of many wireless communication systems. Therefore, especially in the past decade, energy efficient scheduling policies have been investigated [28], [6], [27]. Due to recent advances in energy harvesting technologies, emerging communication devices have been powered by rechargeable batteries which are capable of harvesting energy through solar cells, vibration absorption devices, thermoelectric generators, wind power, etc. Although energy harvesting allows sustainable and environmentally friendly deployment of wireless networks, it requires efficient utilization of time-varying energy. Hence, the focus should be shifted from minimizing energy expenditure to optimizing it over time.

It is well known (*e.g.*, [13], [24], [15]) that optimization of a broadcast channel (*e.g.*, the downlink) shared by many users calls for different choices of rate and power allocation to different users depending on the gains, channel conditions, demands of these users, and most importantly, the objective of the optimization. We pose the following problem whose objective is proportional fairness among users: How to allocate among users the transmission power and the proportion of the time between energy harvests, to achieve a good balance between throughput and fairness in an energy harvesting broadcast system. Specifically, we investigate the proportional fairness based utility maximization problem in a time-sharing multi-user additive white Gaussian noise (AWGN) broadcast channel, where the transmitter's battery gets recharged periodically (at known intervals). Energy is assumed to be harvested at the transmitter during the course of transmission. The data, on the other hand, is assumed to be ready at the transmitter before the transmission starts. We focus on finding the optimum *offline* schedule, by assuming that the energy arrival profile at the transmitter is deterministic and known ahead of time in an *offline* manner for a time window, i.e. a *frame*. The times at which harvested energy becomes available and the amounts that become are known in an *offline* fashion, at the beginning of each frame. The challenge of the optimization problem is the set of *causality* constraints introduced by the energy arrival times, *i.e.*, energy may not be used before it is harvested.

There has been considerable recent research effort on optimizing data transmission with an energy harvesting transmitter. In [32], the authors develop a packet scheduling scheme that minimizes the time by which the energy harvesting transmitter delivers all packets to the receiver of a single-user communication system. In [31], the authors extend this work to the multi-user case and, propose an iterative approach that reduces the two-user broadcast problem into a single-user problem as much as possible, and then, utilizes the single-user solution in [32]. [1] treats the time minimization problem for the two-user broadcast channel differently, as it proposes an iterative solution technique by considering two energy arrival slots at a time. These approaches are extended by [20] and [25] to the case of a transmitter with a finite capacity battery. [19] extends [32] one step further to propose the directional water-filling algorithm, which is able to find the optimal energy management schemes for energy harvesting systems operating in fading channels, with finite capacity rechargeable batteries. Both [25] and [19] investigate the following dual *offline* problems; maximizing the number of bits transmitted with a given deadline constraint, and minimizing the transmission completion time with a given number of bits to transmit.

Unlike the broadcast related studies mentioned above, [33]

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investigates the dual problems in a multiple access communication system. By using the generalized iterative backward waterfilling algorithm [33], the transmission completion time minimization problem can be simplified into convex optimization problems, and solved efficiently. [26] solves the short-term throughput maximization problem for a battery-limited energy harvesting transmitter in a single link topology.

In [11], the authors consider the problem of energy allocation over a finite horizon for point-to-point wireless communications, taking into account a time varying channel and energy source, so as to maximize the throughput. In [7], Gatzianas et. al. consider an infinite-horizon *online* throughput maximization problem for a rechargeable sensor network. The authors propose a queue stabilizing transmission policy with decoupled admission control and energy allocation to maximize a function of the long term rate achieved per link. Chen et. al. [5] claim that infinite-horizon based solutions can be highly inefficient, especially in the context of networks with energy replenishment. Hence, unlike [7], [5] investigates the finite-horizon throughput maximization problem for a rechargeable sensor network.

This work differs from the previously mentioned studies particularly in its aim to maximize the throughput in a proportionally fair way, taking into account the inherent differences of channel quality among users. Due to characteristics of the utility function, the problem presented is a *biconvex* problem[1] which is nonconvex, and has multiple optima. This allowes us to decompose the problem into two parts (power allocation, time allocation) and present a Block Coordinate Descent based optimization algorithm, BCD, that converges to a partial optimal solution. Although BCD is guaranteed to converge to a partial optimal solution and thus the partial optimal utility, it is computationally expensive and when there are tens of users and energy arrivals, forming invertible hessian matrices (needed for the optimization of the power variables) may be computationally excessive. Hence, we next restrict our general case assumption to the case where energy interarrival times are equal, so that we can analytically derive the characteristics of the optimal solution, and then, build on those to develop simple heuristics, PTF and ProNTO that closely track the performance of the BCD solution. Note that, not all generality is lost, since harvest amounts are arbitrary and the absence of a harvest in a certain slot can be expressed with a harvest of amount zero for the respective slot. Periodic sampling is consistent with practice as in many energy harvesting systems, transmitters have supercapacitors that can store the harvested energy and supply in every predetermined time window.

We start by describing the system model in the next section. In Section III, we make the problem precise, and study its structure. The BCD algorithm is described in Section IV, followed by a detailed analysis and discussion of the nature of the solution found by BCD. Section V discusses the structure and properties of the optimal solution of the problem. Depending on these properties, PTF and ProNTO heuristics are proposed in Sections VI and VII repectively. In Section VIII, we test the insight gained from analysis about convergence and the nature of the solution, by running the algorithm on numerical examples. We conclude in Section IX with an outline of further directions.

[1]The problem of optimizing a biconvex function over a given (bi)convex or compact set, where a function $f : X \times Y \rightarrow \Re$ is called biconvex if $f(x, y)$ is convex in y for fixed $x \in X$ and is convex in x for fixed $y \in Y$ [21].

II. System Model

There is a single transmitter that transmits to N users by time sharing on a bandwith W. The power spectral density of the background noise is N_o. Channel conditions will be supposed to remain constant during a duration F that will be referred to as a "frame": i.e., g_n, the gain of user n, is chosen to be constant throughout the frame. The transmitter is equipped with a rechargeable battery such that harvested energy becomes available at distinct instances. With some abuse of terminology, the durations between two harvest instants will be called as "slot". The amount of energy harvested from the environment at the beginning of time slot t is E_t, and the length of the t^{th} slot is T_t as illustrated in Fig. 1.

The figure shows the details of a specific frame within a timeline. Note that, the slot lengths do not necessarily need to be equal as energy arrivals may occur in different moments in time. In general, we do not restrict our problem formulation to the case of periodic energy arrivals ($T_t = T$ for all $t \in \{1, \ldots, K\}$). In Section V however, we focus on the equal slots case, to reveal the characteristics of the optimal solution of the proposed problem. For a given frame, the transmitter chooses a power level p_t and a time allocation vector $\tau_t = (\tau_{1t}, ..., \tau_{Nt})$, for each time slot t of the frame, where $p_{nt} = p_t$ is the transmission power for user n during slot t and, τ_{nt} is the time allocated for transmission to user n during slot t.

III. Problem Statement and Structure

We define the total achievable rate for user n (the number of bits transmitted to user n within a frame), $R_n = \sum_{t=1}^{K} \tau_{nt} W \log_2\left(1 + \frac{p_t g_n}{N_o W}\right)$. Our goal is to maximize a total utility, i.e., the log-sum of the user rates $\sum_{n=1}^{N} log_2(R_n)$, which is known to result in proportional fairness [16]. Thus, we define the constrained optimization problem, Problem 1, where Eq. (1) represents the nonnegativity constraints for $t = 1, ..., K$, $n = 1, ..., N$. The equations in (2), called time constraints, ensure that the total time allocated to users does not exceed the slot length, and, every user gets a non-zero time allocation during the frame. Finally, the equations in (3), called energy causality constraints, ensure no energy is consumed before becoming available.

Problem 1:

$$\text{Maximize: } U(\overline{\tau}, \overline{p}) = \sum_{n=1}^{N} \log_2\left(\sum_{t=1}^{K} \tau_{nt} W \log_2\left(1 + \frac{g_n p_t}{N_o W}\right)\right)$$

$$\text{subject to: } \tau_{nt} \geq 0 \ , \ p_t \geq 0 \quad (1)$$

$$\sum_{n=1}^{N} \tau_{nt} = T_t \ , \ \sum_{t=1}^{K} \tau_{nt} \geq \epsilon \quad (2)$$

$$\sum_{i=1}^{t} p_i T_i \leq \sum_{i=1}^{t} E_i \quad (3)$$

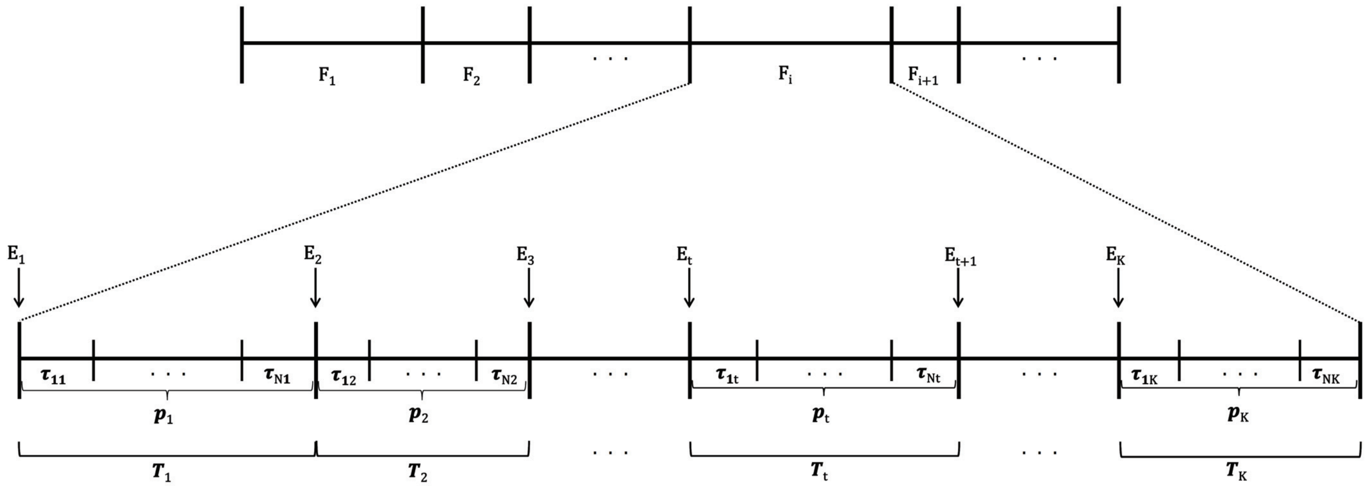

Fig. 1. Multiple frames in a timeline. The highlighted frame, frame i, includes K energy arivals. The time between consecutive arrivals is allocated to N users.

Please note that, Problem 1 can be written as a minimization problem in which the function to be minimized is $-U(\overline{\tau},\overline{p})$. Unfortunately, (1) is a nonlinear non-convex problem with potentially multiple local minima, some of which are also globally optimum. Thus, we can only expect that by proper choice of the initial value, our algorithm converges to a stationary point that is nearby the true optimum. In order to develop such an algorithm, we first decompose the problem into two parts (power allocation, time allocation) and determine some characteristics that will be useful in understanding the problem structure better. Fortunately, these characteristics lead us to Corollary 1, which we exploit to determine the most appropriate algorithm for Problem 1.

A. Structure of the Optimal Power Allocation Problem

In this section, we assume that the time allocation, $\overline{\tau}$, is determined, and try to characterize the structure of the optimal power allocation problem for this $\overline{\tau}$. When the only variables are power variables, Problem 1 reduces to the following constrained optimization problem, whose characteristics are summarized in Theorem 1.

Problem 2:

$$\text{Maximize: } U(\overline{p}) = \sum_{n=1}^{N} f_n(\overline{p})$$

$$\text{subject to: } p_t \geq 0 \ , \quad \sum_{i=1}^{t} p_i T_i \leq \sum_{i=1}^{t} E_i \tag{4}$$

where $t = 1, ..., K$, f_n is a function of the total number of bits sent to user n, and, R_{nt} represents the rate of link n in the t^{th} slot.

$$f_n(\overline{p}) = \log_2 \left(\sum_{t=1}^{K} \tau_{nt} R_{nt} \right) \tag{5}$$

$$R_{nt} = W log_2 \left(1 + L_n p_t\right) \quad \text{where} \quad L_n = \frac{g_n}{N_o W} \tag{6}$$

Theorem 1: Problem 2 can be formulated as a strictly convex optimization problem. Thus, there exists only one global optimum for a given time allocation.

Proof: This result can be obtained by any textbook on convex optimization [4]. Hence, due to space constraints, we refer the interested reader to [23] for the details of our proof. ■

B. Structure of the Optimal Time Allocation Problem

In this section, we assume that the power allocation across all slots has been determined. Then, we determine the characteristics of the time allocation. Thus, Problem 1 reduces to Problem 3, where the only variables are the time variables:

Problem 3:

$$\text{Maximize: } U(\overline{\tau}) = \sum_{n=1}^{N} s_n(\overline{\tau})$$

$$\text{subject to: } \tau_{nt} \geq 0 \ , \quad \sum_{n=1}^{N} \tau_{nt} = T_t \ , \quad \sum_{t=1}^{K} \tau_{nt} \geq \epsilon \tag{7}$$

where $t = 1, ..., K$, $n = 1, ..., N$ and, s_n is a function of the time variables:

$$s_n(\overline{\tau}) = \log_2 \left(\sum_{t=1}^{K} \tau_{nt} R_{nt} \right) \tag{8}$$

and R_{nt}'s (defined in Eq. 6) are known constants. Theorem 2 below records the convexity of Problem 3, and leads to one of the main results of this paper, Corollary 1.

Theorem 2: Problem 3 can be formulated as a convex optimization problem. Thus, all local optima are global optima.

Proof: As in Theorem 1, due to space constraints, we refer the reader to [23] for the proof. ■

Note that, Problem 3 is convex, but not necessarily *strictly* convex. Therefore, rather than a unique global optimum, there may be multiple local optima which are all also globally optimum.

Corollary 1: Problem 1 can be formulated as a biconvex optimization problem.

Proof: The proof is provided in Appendix A. ■

IV. Solution Method

In the previous section, we have shown that $-U(\overline{\tau}, \overline{p})$ is a biconvex function. While not convex, such functions [9], [21] admit efficient coordinate descent algorithms that solve a convex program at each step. In this section, we present a block coordinate descent based algorithm, shortly BCD, for solving Problem 1. Our BCD algorithm operates explicitly as follows:

1) Start from any valid time allocation, for example assign each time slot to different user in the form of TDMA. Assuming that all of the energy E_t is used up until the end of period t, the power is determined. This power setting satisfies Eq. (3).
2) Keep τ_{nt} fixed for all n and t. Optimize $U(\overline{\tau}, \overline{p})$ with respect to $p_t, t = 1, \ldots, K$ and the constraints given by (3).
3) Repeat the following for all $t = 1, \ldots, K$.: Keep τ_{ni} fixed for all $n = 1, \ldots, N$ and $i \neq t$. Also keep p_t fixed for all t. Maximize $U(\overline{\tau}, \overline{p})$ with respect to $\tau_{nt}, n = 1, \ldots, N$ and constraint in Eq. (2).
4) If the variables have converged, stop. Otherwise, go to Step 2.

For optimization of the time variables, the Lagrange multiplier method is used. The optimization of the power variables is accomplished by using the Sequential Unconstrained Minimization Technique (SUMT) [2], which converts a constrained optimization problem into an unconstrained one, by adding the constraints to the objective function as a "penalty". It then uses an unconstrained optimization algorithm (i.e., Newton's method) [4], [17] to solve the problem iteratively.

Regarding the issue of convergence, Problem 1 is a biconvex optimization problem and as such potentially, there exist many local optima. Therefore, convergence to the global optimum is not guaranteed. However, provided that some conditions are satisfied, convergence to a partial optimum (see Definition 1) is guaranteed. As also discussed by Lin in [14], convergence to a stationary (or critical) point, for block coordinate descent methods requires sub-problems to have unique solutions ([22], [3]), but this property does not hold here: Although sub-problem 2 is strictly convex, 3 is not strictly convex (only convex). Fortunately, for the case of two blocks, Grippo and Sciandrone [10] have shown that this uniqueness condition is not needed. Hence, BCD converges to a stationary point of Problem 1. As a stationary point can be minimum, maximum, or a saddle point, this convergence result may not be sufficient. However, we can still use the following definition and theorem (Definition 4.1 and Theorem 4.2 of [9], respectively) to build a stronger result. For this, let $X \subseteq \Re^n$ and $Y \subseteq \Re^m$ be two nonempty sets, let $B \subseteq X \times Y$, and, let B_x and B_y denote the x-sections and y-sections of B, respectively.

Definition 1: Let $f : B \rightarrow \Re$ be a given function and let $(x^*, y^*) \in B$. Then, (x^*, y^*) is called a partial optimum of f on B, if

$$f(x^*, y^*) \leq f(x, y^*) \quad \forall x \in B_{y^*} \quad and, \\ f(x^*, y^*) \leq f(x^*, y) \quad \forall y \in B_{x^*} \tag{9}$$

Theorem 3: Let B be a biconvex set and let $f : B \rightarrow \Re$, be a differentiable, biconvex function. Then, each stationary point of f is a partial optimum.

Hence, we conclude that the BCD algorithm surely converges to a partial optimum of Problem 1. Furthermore, Theorem 4.9 of [9] shows that, when subproblems are solvable, for BCD-like algorithms[2] (There are only two block of variables, and, sequentially one block of variables is minimized under corresponding constraints and the other block is fixed), if the sequence generated by the algorithm is contained in a compact set, then the sequence has at least one accumulation point. The theorem further states that; when one of the subproblems is strictly convex, all accumulation points are partial optima, and have the same function value (Note that while a global optimum is a partial optimum by definition, it may not be an accumulation point. In that case, all the partial optima that are in the set of accumulation points have strictly lower values than optimum function value.) Hence, we conclude that the BCD algorithm surely converges to an accumulation point, which is also partial optimum, of Problem 1, and all accumulation points (a set of partial optima) yield the same utility value. Note that although the final allocation, $(\overline{\tau^*}, \overline{p^*})$ generated by the BCD algorithm, might be a partial optimum, it neither has to be a global nor a local optimum to the given biconvex optimization problem. Because, although the set of accumulation points BCD converges to are partial optima and have the same function value, there may be other partial optima that may have different function values. Depending on the starting point of the algorithm, BCD may converge to a set that includes the global optimum, or a different set that includes local optima, or just partial optima. According to [9], there exists a theorem, originally developed by Wendell and Hurter [29], that describes the connection between partial and local optima for the following biconvex minimization problem,

$$min \{f(x, y) : \; x \in X \subseteq \Re^n, \; y \in Y \subseteq \Re^m\} \tag{10}$$

However, as also noted in [9], the given local optimality condition is in general not sufficient. Indeed, Wiesemann claims in [30] (p. 92) that, even the verification whether a particular solution to a biconvex problem is locally optimal is NP-complete. Gorski et.al. [9], on the other hand, claims that to find the global optimum of a biconvex minimization problem by a BCD-like algorithm (ACS [9]), a multistart version of BCD can be used. But, still, there is no guarantee to find the global optimum within a reasonable amount of time or to be sure that the actual best minimum is the global one. Hence, it seems justified to settle for the modest goal to find a partial optimum in our case.

V. Structure and Properties of the Optimal Solution

In this section, we analyze the structure and properties of the hybrid power-time allocation policy. For this, we let $\overline{R_n} = [R_{n1} \; R_{n2} \; \ldots \; R_{nK}]^T$, where R_{nt} is as defined in Eq. (6), and

[2]ACS (Alternate Convex Search) algorithm proposed in [9].

$\overline{\tau_n} = [\tau_{n1}\ \tau_{n2}\ \ldots\ \tau_{nK}]^T$. Then, the utility in Problem 1 can be rewritten as below, where $U_n = log_2(\overline{\tau_n}^T \overline{R_n})$ is the utility of user n.

$$U = \sum_{n=1}^{N} log_2(\overline{\tau_n}^T \overline{R_n}) = U_1 + U_2 + \ldots + U_N \tag{11}$$

In order to reveal characteristics related to the optimal solution that will help us develop computationally efficient and close-to-optimal heuristics, we decompose the problem into two parts (similarly as in Section III): power allocation and time allocation.

A. Structure of an Optimal Power Allocation Policy

In this section, we analyze the structure and properties of the optimal power allocation policy. In order to do this, we assume that the time allocation is determined, and try to characterize the structure of the optimal solution of the power allocation problem for this time allocation. Clearly, Problem 1 reduces to Problem 2, with $f_n(\overline{p}) = U_n(\overline{p})$. Note that, as Problem 2 has a unique optimum, the optimal power allocation changes for every given time allocation. In Theorem 4, we claim that there exists an optimum solution of Problem 1 with a nondecreasing power schedule. Lemma 1 not only helps us to prove our claim but also reveals that Problem 1 has multiple optima. From the proof of Lemma 1, the attentive reader can observe that any feasible permutation[3] of the optimal schedule $(\overline{\tau}^*, \overline{p}^*)$, described in Theorem 4, is also optimal.

Theorem 4: When all slots have equal length ($T_j = T,\ for\ \forall j \in \{1, ..., K\}$), there exists an optimal schedule $(\overline{\tau}^*, \overline{p}^*)$ such that $\overline{p}^*$ is nondecreasing, (e.g., $\overline{p}^* = (p_1, ..., p_K)$ where $p_1 \leq p_2 \leq ... \leq p_K$).

Proof: The proof is provided in Appendix B, and rests on Lemma 1 below. ■

We shall need the following definition of a permutation of a vector sorted in nondecreasing order of elements, for stating Lemma 1.

Definition 2: Given a vector $\overline{R_n} = [R_{n1}\ R_{n2}\ \ldots\ R_{nK}]^T$, we define $\overline{R_n}^{\uparrow} = [R_{n\pi(1)}\ R_{n\pi(2)}\ \ldots\ R_{n\pi(K)}]^T$ where $\overline{R_n}^{\uparrow}$ is a permutation of $\overline{R_n}$, such that $R_{n\pi(1)} \leq R_{n\pi(2)} \leq \ldots \leq R_{n\pi(K)}$.

Lemma 1: When all slots have equal length ($T_j = T,\ for\ \forall j \in \{1, ..., K\}$), for any given schedule $(\overline{\tau}, \mathcal{P}_{\mathcal{C}})$, we can find such $\overline{\tau_n'}, \overline{R_n'}$ (where $\overline{R_n'} = \overline{R_n}^{\uparrow}$) that $(\overline{\tau_n'})^T \overline{R_n'} = \overline{\tau_n}^T \overline{R_n}$ for all $n = 1, \ldots, N$; i.e., the utility, U, does not change. Hence, if $(\overline{\tau_n}^*, \overline{R_n}^*)$ is optimal, then $(\overline{\tau_n'}^*, \overline{R_n'}^*)$ is also optimal.

Proof: The proof is provided in Appendix C. ■

B. Structure of an Optimal Time Allocation Policy

In this section, we assume that the power allocation through the slots is determined. Then, given that the power variables are known constants, we determine the structure and properties of the optimal time allocation policy. Thus, Problem 1 reduces to Problem 3 with $s_n(\overline{\tau}) = U_n(\overline{\tau})$. As Problem 3 is convex, the analysis can rely on KKT (Karush-Kuhn-Tucker) optimality conditions, which must be satisfied by the global optimum. We start by forming the Lagrangian as in Eq. (12), where μ's are the Lagrange multipliers, and, the total number of constraints[4] is $N(K+1) + K$.

After defining the Lagrangian as in Eq. (12), one can construct the KKT conditions for the optimal solution. Due to space limitations, we do not list the conditions here but refer the interested reader to the associated technical report [23] for the details. Please note that the optimal time allocation should jointly satisfy the set of equations that arise from KKT conditions. Clearly, as the number of users, N, and, the number of slots, K, increase, the number of equations increases dramatically making it cumbersome to write analytical solutions. Therefore, for the sake of conciseness, we continue the analysis with the special case of two users and two slots which allows us to construct the characteristics of the optimal time allocation policy.

Consider two consequtive slots with different power levels. Let us call the one with the least power *the weak slot*, and the one with the highest power *the strong slot*. When the slots have equal length ($T_1 = T_2 = T$), the optimal policy has the properties described in Lemma 2.

Lemma 2: In an optimal schedule, time allocation over the two slots (of equal length) has the following properties:

1) The weak slot is assigned to only one of the users. The strong slot, however, is shared between users. When both power levels are equal; if one slot is assigned to user 1 (user 2), the other slot is assigned to user 2 (user 1).
2) To whom the the weak slot will be assigned depends on two criteria: first, $\Gamma_n = \frac{R_{n2}}{R_{n1}}$, which is the ratio of user n's rate in the second slot to that in the first, and second, whether the strong slot is before or after the weak slot. When the weak slot preceeds the strong slot, it is assigned to the user with the smaller Γ. Otherwise (implying the decrease in power level), it is assigned to the user with the higher Γ.
3) In a strong slot, the user that did not (or will not) receive any data in the weak slot is favored, i.e., more than half of the slot is assigned to that user. In order to preserve fairness, this favoring operation is done by considering Γ_1 and Γ_2.

Proof: The proof is provided in Appendix D. ■

VI. PTF Heuristic

In this section, we develop the Power-Time-Fair (PTF) heuristic, based on the characteristics (discovered in Section V) of an optimal power/time allocation schedule. PTF operates as follows:

1) **For Power Allocation:** Assign nondecreasing powers through the slots, as follows:
   a) From a slot, say i, to the next one $i+1$: If harvested energy decreases, defer a Δ amount of energy from slot i to slot $i+1$ to equalize the power levels. Do this until all powers are nondecreasing, and, form a virtual nondecreasing harvest order.
   b) By using the virtual harvest order, assign nondecreasing powers through the slots, i.e., in each slot,

[3] A feasible permutation is any permutation of a given schedule that does not violate the constraints described in Eqns. (1)-(3).

[4] There are K equality constraints and $NK + N$ inequality constraints.

$$L(\overline{\tau}, \overline{\lambda}, \overline{\mu}) = -U(\overline{\tau}) + \sum_{j=1}^{K} \sum_{i=1}^{N} \mu_{(N(j-1)+i)} \tau_{ij} + \sum_{j=NK+1}^{NK+N} \mu_j (\epsilon - \sum_{t=1}^{K} \tau_{(j-NK)t}) + \sum_{i=1}^{K} \lambda_i (\sum_{n=1}^{N} \tau_{ni} - T_i) \quad (12)$$

spend what you virtually harvested at the beginning of that slot.

2) **For Time Allocation:** For the allocation found in 1), let, $B_{nt} = R_{nt}T$ be the number of bits that would be sent to user n if the whole slot (of length T) was allocated to that user. Assign the first slot to the user who has the maximum rate, R_{nt}, in that slot. For the other slots, apply the following: At the beginning of each slot, $t \in \{2, \ldots, K\}$, determine the user with the maximum β where, $\beta_n = \frac{B_{nt}}{\sum_{i=1}^{t} B_{ni}}$. Then, assign the whole slot to that user. If multiple users share the same β, then, allocate the slot to the user with the best channel.

VII. ProNTO Heuristic

In this section, we develop a fast and simple heuristic, ProNTO (Powers Nondecreasing - Time Ordered), based on the characteristics discovered in Section V-A and the simulation results obtained by running BCD algorithm for periodic energy arrivals. ProNTO operates as follows:

1) **For Power Allocation:** Assign nondecreasing powers through the slots by using the energy harvest data, as done in part (1) of PTF algorithm.
2) **For Time Allocation:** Order the users, $u_1, \ldots, u_N$, according to their channel quality and form a user priority vector, $\overline{u^{\downarrow}} = [u_1^{\downarrow}, \ldots, u_N^{\downarrow}]$ where $u_1^{\downarrow}$ represents the user with the best channel. As $K > N$, Allocate every user $\theta = floor(K/N)$ slots as follows: The first θ slots are allocated to $u_1^{\downarrow}$, the next θ slots are allocated to $u_2^{\downarrow}$, etc. Add the remaining $mod(K,N)$ slots to the most powerful $mod(K,N)$ users' slots. For example; Let $K = 12$ and $N = 5$, and the path losses of the users to be 13 dB, 17 dB, 10 dB, 12 dB, 20 dB respectively. Then, the first 3 slots are allocated to user 3, the next 3 slots are allocated to user 4, the following 2 slots are allocated to user 1, 9^{th} and 10^{th} slots are allocated to user 2, and the last 2 slots are allocated to user 5.

Both PTF and ProNTO adopt a nondecreasing power allocation which is reminiscent of the optimal policies in [25], [32], i.e., the transmission power remains constant between energy harvests, and, only potentially changes when new energy arrives. Thus, PTF and ProNTO differ only in time allocation part. The time allocation method used in ProNTO is proposed according to the following observation: when a partial optimum obtained by BCD algorithm is modified as described in Lemma 1 and its proof, to form the nondecreasing optimal schedule, the time allocation becomes ordered, e.g., as shown in Table III.

VIII. Numerical and Simulation Results

In this section, we present the numerical and simulation results related to BCD algorithm, and, the proposed heuristics, PTF and ProNTO. Throughout our simulations we use the following setup: $W = 1kHz$, $N_o = 10^{-6}W/Hz$. Unless otherwise stated, all powers are in Watts and all energies are in Joules.

A. Performance of the BCD Algorithm

First, we suppose that there are five users in the system and 10 energy arrivals in 100 secs (frame length). The arrivals are $\overline{E} = [20, 100, 1, 1, 1, 70, 100, 1, 10, 40]$ joules in the $[1^{st}, 2^{nd}, \ldots, 10^{th}]$ slots respectively. The first user is the strongest one, and, other users are ordered in a such way that the preceding user is twice as strong as the previous one, i.e., path losses of the users are; 25, 28, 31, 34, 37 dB respectively. The starting point of the algorithm is the "Spend What You Get" policy (proposed by Gorlatova et. al. [8]) combined with TDMA time allocation. This policy corresponds to using all energy in the epoch it was harvested in, and will be referred to in the rest as SG+TDMA. We performed simulations both for unequal and equal slot lengths. In our simulations, we use the following sequence of slot sizes for 10 slots; $S_1 = [10, 12, 5, 7, 4, 15, 20, 2, 10, 15]$ and $S_2 = [25, 44, 14, 7, 3, 32, 47, 19, 26, 38]$, for the case of unequal slot lengths, and, $\widetilde{S_1} = [10, 10, \ldots, 10]$ and $\widetilde{S_2} = [25.5, 25.5, \ldots, 25.5]$ for the case of equal slot lengths. Note that, S_1 and $\widetilde{S_1}$ have the frame length of 100 secs, whereas S_2 and $\widetilde{S_2}$ have 255 secs.

In order to observe the effect of frame length and different slot lengths, in Figure 2, we show how utility improves through the iterations for all of the aforementioned slot length sequences, i.e., S_1, $\widetilde{S_1}$, S_2, $\widetilde{S_2}$. The fast convergence of the BCD algorithm is evident in this figure, i.e., the utilities seem to rarely change after just a few iterations. The optimal schedules (power and time), optimal utility and thus, the utility improvement (when compared to SG+TDMA) obtained by BCD, for all four sequences are presented in Table I.

However, the periodic energy arrivals assumption allows us to develop two heuristics that closely track the performance of BCD algorithm. Hence, from now on, we present results for the case of periodic energy arrivals. We investigate the throughput improvement of the users for increasing path losses. The results, for $\widetilde{S_1}$, are illustrated in Figure 3.

In the figure, the Mean Path Loss, is computed as $\widetilde{L} = \frac{1}{N}\sum_{i=1}^{N} L_i$ where L_i represents the path loss of user i. As seen from the figure, with minor decrease in the throughput improvement of the stronger users, the weak users receive much more bits than that they used to receive with SG policy and TDMA. As $\widetilde{L}$ increases, the overall throughput improvement also increases. For instance, when $\widetilde{L} = 31$, User 1, User 3, User 4, and, User 5 enjoy approximately 3 %, 1621 %, 361 %, 80 % throughput improvement respectively, while User 1 suffers only 32 % of loss. Clearly, BCD is a proportionally fair algorithm which tries to maximize the utility by meeting certain demands of every user.

We next assume that some amount of energy ($\epsilon < E < \infty$ where ϵ is an infinitely small value) is harvested ev-

TABLE I

THE RESULTS OF BCD ALGORITHM FOR FOUR DIFFERENT SLOT LENGTH SEQUENCES.

		Slot 1	Slot 2	Slot 3	Slot 4	Slot 5	Slot 6	Slot 7	Slot 8	Slot 9	Slot 10	Utility	Utility Imp. (%)
Time Allocation	Users vs. Slot Lengths	10	12	5	7	4	15	20	2	10	15		
	1	10	12	0	0	0	0	3.1288	0	0	0		
	2	0	0	0	0	0	5.7638	16.8712	0	0	0		
	3	0	0	0	7	4	9.2362	0	0	0.1987	0		
	4	0	0	0	0	0	0	0	0	9.8013	6.9208	75.7273	8.5449
	5	0	0	5	0	0	0	0	2	0	8.0792		
Power Allocation		2	2.0910	5.9724	3.4337	3.4337	3.1027	2.5535	5.9723	4.4268	5.1636		
Time Allocation	Users vs. Slot Lengths	10	10	10	10	10	10	10	10	10	10		
	1	10	10	6.2337	0	0	0	0	0	0	0		
	2	0	0	3.7663	10	10	0	0	0	0	0		
	3	0	0	0	0	0	10	10	0	0	0		
	4	0	0	0	0	0	0	0	10	6.3094	0	75.7325	9.6133
	5	0	0	0	0	0	0	0	0	3.9606	10		
Power Allocation		2	2.0182	2.2189	2.5923	2.5923	3.4327	3.4327	4.6482	5.1876	6.2772		
Time Allocation	Users vs. Slot Lengths	25	44	14	7	3	32	47	19	26	38		
	1	25	44	0	0	0	0	0.9619	0	0	0		
	2	0	0	0	0	0	13.4168	46.0381	0	0	0		
	3	0	0	14	0	0	18.5832	0	19	0	0		
	4	0	0	0	0	3	0	0	0	0	38	78.2339	9.0566
	5	0	0	0	7	0	0	0	0	26	0		
Power Allocation		0.7956	0.7956	1.3866	2.4499	1.8390	1.2129	1.0275	1.3866	2.4499	1.8390		
Time Allocation	Users vs. Slot Lengths	25.5	25.5	25.5	25.5	25.5	25.5	25.5	25.5	25.5	25.5		
	1	25.5	20.8998	0	0	25.5	0	0	0	0	0		
	2	0	4.6002	25.5	25.5	0	4.2309	0	0	0	0		
	3	0	0	0	0	0	21.2692	25.5	3.4418	0	0		
	4	0	0	0	0	0	0	0	22.0582	18.5955	0	78.2314	9.9830
	5	0	0	0	0	0	0	0	0	6.9045	25.5		
Power Allocation		0.7758	0.8157	1.0472	1.0472	0.7758	1.3140	1.3841	1.7590	1.9852	2.5862		

ery 10 seconds ($T = 10$) within a frame, as in $\widetilde{S_1}$. But this time, we use four different frame lengths; 20, 80, 100, 120. For the frame of 20 secs, we use three different energy harvest models; $[0.5, 50]$, $[50, 0.5]$, $[60, 20]$. We define different cases for the remaining three frame lengths; *Regular*, *Bursty*, and, *Very Bursty*. In *Regular*, the harvest amounts are close to each other and form a regular pattern; $E_R = [73, 65, 9, 19, 40, 37, 22, 84, 39, 67, 81, 100]$. In *Bursty*, there are short term sudden decreases and increases in harvest amounts, causing a bursty pattern; $E_B = [20, 100, 1, 1, 1, 70, 100, 1, 10, 40]$. Finally, *Very Bursty* represents an extreme case where the transmitter stays energy-hungry for a long time; $E_V = [90, 2, 0.5, 0.1, 0.3, 0.7, 40, 60]$.

We use the simplest case of two users and two slots ($N = 2$, $K = 2$, frame of 20 secs) to compare the results obtained by BCD algorithm, with the optimal ones presented in Table V. Our objective in doing such a comparison is to prove the accuracy of both theoretical and simulation results. We refer the interested reader to Appendix D for the details of the optimality table, and provide the comparison in Table II.

The first column of Table II shows the amount of the harvests (E_1, E_2). The second column represents the mean path loss (in dB) of the two users. As observed from the table, for a given power allocation, the results found by BCD algorithm and the optimal ones (obtained by KKT optimality conditions) are almost the same, verifying the consistency and

TABLE II
BCD VS. OPTIMAL RESULTS FOR THE SPECIAL CASE OF TWO USERS AND TWO SLOTS.

Harvests	Mean Path loss	Power Allocation by BCD	Time Allocation by BCD	Optimal Time Allocation	Utility by BCD	Optimal Utility
[0.5 50]	20.5	[0.0500 5.0000]	$\begin{bmatrix}10 & 4.4129\\ 0 & 5.5871\end{bmatrix}$	$\begin{bmatrix}10 & 4.4129\\ 0 & 5.5871\end{bmatrix}$	29.8094	29.8094
	26.5	[0.0500 5.0000]	$\begin{bmatrix}10 & 4.7399\\ 0 & 5.2601\end{bmatrix}$	$\begin{bmatrix}10 & 4.7399\\ 0 & 5.2601\end{bmatrix}$	28.4062	28.4062
[0.5 50]	20.5	[2.2993 2.7507]	$\begin{bmatrix}10 & 0.2428\\ 0 & 9.7572\end{bmatrix}$	$\begin{bmatrix}10 & 0.2431\\ 0 & 9.7569\end{bmatrix}$	30.9401	30.9401
	26.5	[2.2466 2.8034]	$\begin{bmatrix}10 & 0.4291\\ 0 & 9.5709\end{bmatrix}$	$\begin{bmatrix}10 & 0.4295\\ 0 & 9.5705\end{bmatrix}$	29.4618	29.4618
[60 20]	2.5	[3.8238 4.1762]	$\begin{bmatrix}10 & 0.0538\\ 0 & 9.9462\end{bmatrix}$	$\begin{bmatrix}10 & 0.0544\\ 0 & 9.9456\end{bmatrix}$	33.5272	33.5272
	8.5	[3.7879 4.2121]	$\begin{bmatrix}10 & 0.0786\\ 0 & 9.9214\end{bmatrix}$	$\begin{bmatrix}10 & 0.0787\\ 0 & 9.9213\end{bmatrix}$	32.9577	32.9577

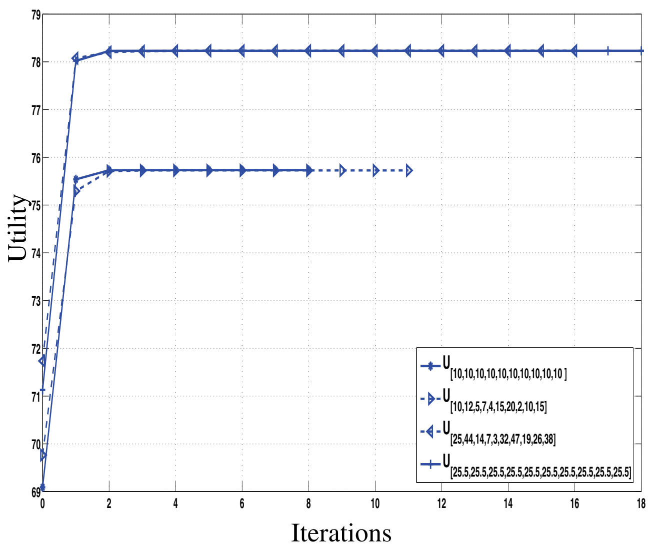

Fig. 2. Utility vs. iterations (N=5, K=10): starting from SG+TDMA, BCD converges to the optimal utility in 11,8,16,18 iterations for the following slot length sequences respectively: S_1, $\widetilde{S_1}$, S_2, $\widetilde{S_2}$.

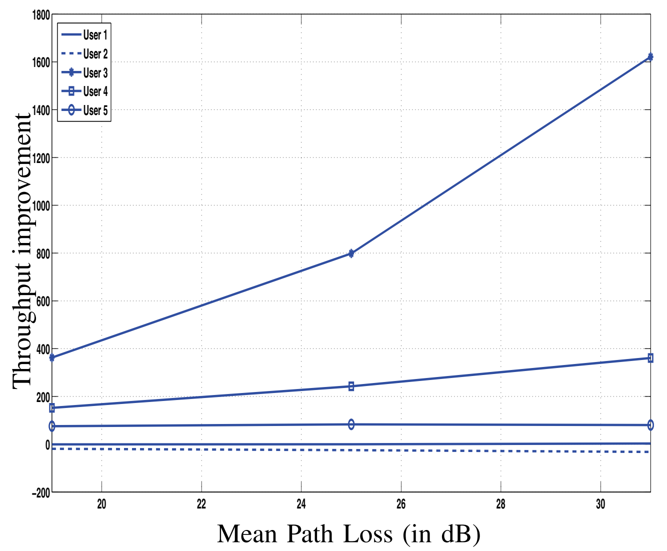

Fig. 3. Throughput improvement vs. mean path loss (N=5, K=10): mean path loss is computed as the mean of the path losses of all users in the system. Results represent the throughput improvement of five users for three different path loss patterns. With minor decrease in the throughput of the stronger users, the weak users receive much more bits than that they used to receive with SG+TDMA.

optimality of the algorithm. The attentive reader can observe from Table II that, when harvests decrease from one slot to another, the optimal powers tend to be nondecreasing. Hence in that case, the algorithm seems to be converged to the nondecreasing optimal discussed in Theorem 4. Note that, this nondecreasing optimal could also be obtained by using the modification method explained in Lemma 1. By using that method, we modify the results obtained by BCD to reveal the optimal (nondecreasing) power and time allocation policies for increasing number of users. For our analysis, we use three different path loss patterns, called, *Low*, *Moderate*, *High* respectively. In *Low*, the strongest user in the system has 13 dB path loss, and, every new user that joins the system deviates by 3 dB from the previous one (has 3 dB more path loss than the preceding user). In *Moderate*, the strongest user has 19 dB path loss, and, every new user deviates by 3 dB. Finally, in *High*, the strongest user has 25 dB path loss, and, every new user deviates 3 dB. Due to space limitations, we present only the *Bursty-Moderate* case's results in Table III. As illustrated, when the number of users increase, BCD tends to assign increasing powers rather than nondecreasing. One can also see from the table that, no matter how many users exist in the system, ordering powers in nondecreasing order, causes the time allocation to be ordered too. By ordered, we mean that the first slot(s) are allocated to the user with the best channel, the next slot(s) are allocated to the user with the second best channel, etc. , and the last slot(s) are allocated to the user with the worst channel. This observation constitutes the main motivation for the ProNTO heuristic.

B. Performances of the PTF and ProNTO Heuristics

We next use the above-mentioned energy harvesting cases to compare the PTF and ProNTO heuristics' performances to that of BCD's. We start by testing the utility and throughput improvement (over SG+TDMA) performances of the heuristics for increasing path losses. For this, we set the number of users to two, i.e., $N = 2$. The results are presented in Figure 4 and Figure 5, respectively. In both figures, the

TABLE III
OPTIMAL TIME AND POWER ALLOCATION POLICIES VS. NUMBER OF USERS: FOUND BY BCD ALGORITHM AND MODIFIED ACCORDING TO LEMMA 1.

No. of Users	T.A. / P.A.	Users/ Slots	Slot 1	Slot 2	Slot 3	Slot 4	Slot 5	Slot 6	Slot 7	Slot 8	Slot 9	Slot 10
2	T.A.	1	10	10	10	10	10	3.6666	0	0	0	0
		2	0	0	0	0	0	6.3334	10	10	10	10
	P.A.		2	2.5750	2.5750	2.5750	2.5750	4.2117	4.4720	4.4720	4.4720	4.4720
4	T.A.	1	10	10	8.5777	0	0	0	0	0	0	0
		2	0	0	1.4223	10	10	4.7598	0	0	0	0
		3	0	0	0	0	0	5.2401	10	8.3789	0	0
		4	0	0	0	0	0	0	0	1.6211	10	10
	P.A.		2	2.3132	2.3810	2.8028	2.8028	3.6343	4.0501	4.2070	5.0742	5.0742
6	T.A.	1	10	10	0.5333	0	0	0	0	0	0	0
		2	0	0	9.4666	10	0	0	0	0	0	0
		3	0	0	0	0	10	7.3867	0	0	0	0
		4	0	0	0	0	0	2.6132	10	3.3768	0	0
		5	0	0	0	0	0	0	0	6.6231	7.7727	0
		6	0	0	0	0	0	0	0	0	2.2272	10
	P.A.		1.8639	1.8639	2.2338	2.2553	3.0596	3.2585	3.8704	4.0829	5.3135	6.5977

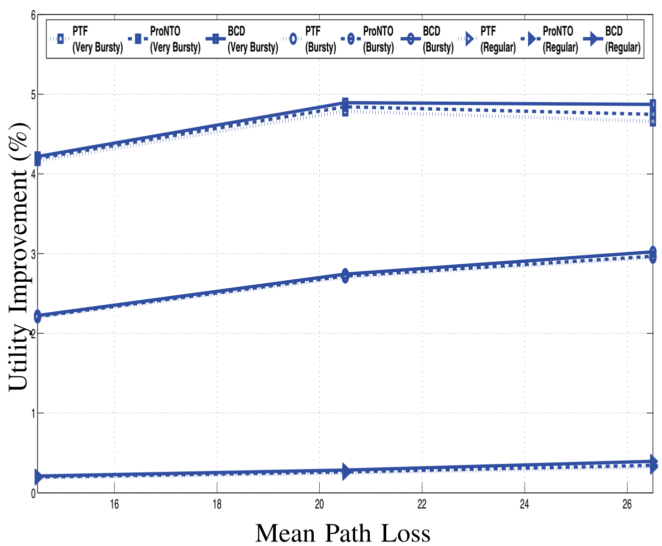

Fig. 4. Utility improvement (BCD, PTF, ProNTO) vs. mean path loss for $N = 2$: the effect of mean path loss on utility improvement for the three energy harvesting cases; *Regular*, *Bursty*, *Very Bursty*.

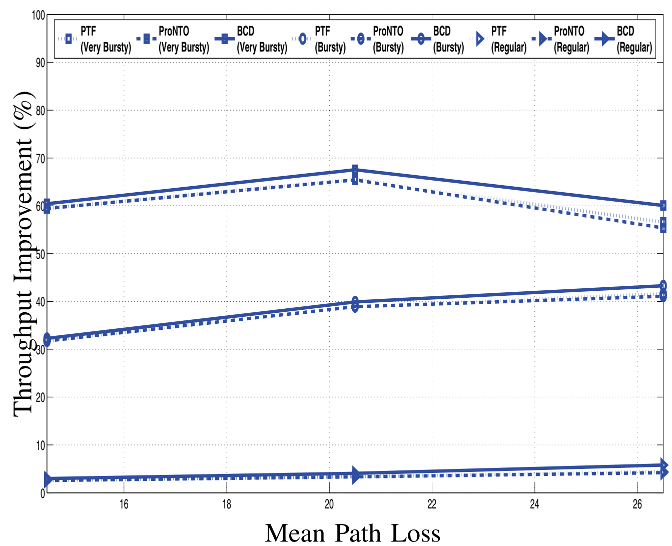

Fig. 5. Throughput improvement (BCD, PTF, ProNTO) vs. mean path loss for $N = 2$: the effect of mean path loss on throughput improvement for the three energy harvesting cases; *Regular*, *Bursty*, *Very Bursty*.

Mean Path Loss, is computed as $\widetilde{L} = \frac{1}{N}\sum_{i=1}^{N} L_i$ where L_i represents the path loss of user i. Hence, the three mean path losses seen in the figures represent the *Low*, *Moderate* and *High* cases. One can observe from Figure 4 that, the utility improvements of all algorithms tend to increase (or at least stay constant) when path loss increases, and the utility improvement performances of the proposed heuristics are very close to that of BCD's. For the chosen cases, ProNTO outperforms PTF. This is more obvious for the *Very Bursty* case. The corresponding throughput improvements are shown in Figure 5. As illustrated, for the case of $N = 2$, even with $\approx 5\%$ of utility improvement, a $\approx 65\%$ of improvement in total throughput is possible. Note that, in all cases, the performances are very close to each other.

In order to determine the effect of number of users to the performances of our proposed heuristics, we next perform a series of simulations by considering all energy harvesting cases (*Regular*, *Bursty*, *Very Bursty*) and different number of users. By taking average over all energy harvesting cases, we present the average utility improvement results in Figure 6, for the *Moderate* case. As illustrated in the figure, when the number of users increase, the average utility improvements of all schemes also increase. Note that, both heuristics closely track the BCD algorithm. When there are few users in the system, PTF and ProNTO are competitive. However, when there are more users, ProNTO seems to outperform PTF in terms of average utility improvement. At all instances, ProNTO is within the 1% neighbourhood of the BCD algorithm.

Although we aim at proportional fairness in this work, it may be interesting to analyse max-min fairnesses of the proposed algorithms, PTF and ProNTO. Jain's index (FI) is a well-known measure of fairness [12], [18]. FI takes the value of 1 when there is a complete fair allocation, and, it is defined as $FI = \frac{(\sum_{i=1}^{N} x_i)^2}{N \cdot \sum_{i=1}^{N} x_i^2}$. For computing FI, we use the no. of bits transmitted to the users, $x_i = 2^{U_i}$ for $i = 1, \ldots, N$, where U_i is as defined in Eq. (11). From Table

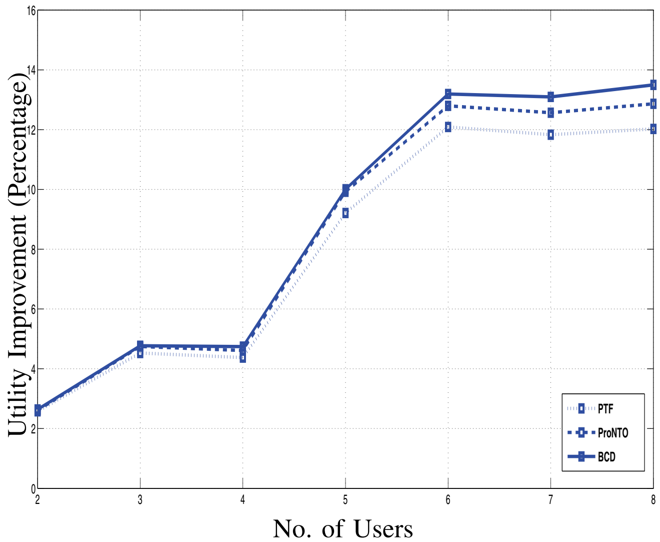

Fig. 6. Average utility improvement (PTF, ProNTO, BCD) vs. no. of users: the average is taken over *Regular*, *Bursty*, *Very Bursty* cases. The average utility improvements of the proposed algorithms over SG+TDMA, for increasing number of users, are compared. Utility improvment increases with increasing number of users.

IV, it is clear that SG+TDMA is the worst choice in terms of fairness. Although low path losses embrace lower utility improvement, they mainly allow both PTF and ProNTO to be very efficient in terms of fairness. However, as observed from the table, when all three cases are considered, PTF seems to be more fair than ProNTO is. Hence, ProNTO seems to trade of fairness for utility improvement. It can also be inferred from Figure 6 and Table IV that, when ProNTO outperforms PTF in terms of utility improvement, the difference between two heuristics is not high. However, this is not the case for fairness, i.e., when PTF outperforms ProNTO, the difference can be considered as high. Hence, although ProNTO seems more promising in terms of utility improvement, depending on system requirements, one can still choose PTF over ProNTO for more fairness.

IX. Conclusion

This paper investigated the proportional fair power and time allocation problem in an energy harvesting broadcast system. The paper focuses on finding the optimum *offline* schedule for this problem, by assuming that the energy harvesting times and the corresponding harvested energy amounts are known at the beginning of each frame. Detailed analysis of structural characteristics of the problem has been performed, which revealed that it can be formulated as a biconvex optimization problem, and that it has multiple optima. Furthermore, an algorithm based on block coordinate descent (BCD), that surely converges to a partial optima of the problem, has been showed. Building on the problem formulation and BCD, the optimal resource allocation policy was further studied and, the existence of an optimal nondecreasing power schedule and, an ordered time allocation schedule were proved. This allowed us to propose two alternative efficient and scalable heuristics, PTF and ProNTO. The computational ease of these algorithms were observed in numerical examples, while the policies they result in coincide with the structural properties we have shown the optimal to have. Simulation results indicate that, despite their simplistic design, PTF and ProNTO heuristics can closely track the performance of the optimal BCD algorithm. In our examples, which were computed for small or moderate problem sizes, both PTF and ProNTO took one or two orders of magnitude smaller time to converge than BCD, which has to compute a Hessian. Typically, ProNTO outperforms PTF in terms of utility improvement, whereas the latter is fairer. The utility improvement difference between BCD and ProNTO is shown to be less than 1% at all instances.

An interesting future direction could be the development of an *online* algorithm that will bypass the need for *offline* knowledge about the energy harvesting statistics. This algorithm may use energy harvesting prediction algorithms to predict the energy that will arrive in the future.

Appendix A
Proof of Corollary 1

Showing that $-U(\overline{\tau},\overline{p})$ is biconvex will be enough to show that Problem 1 can be formulated as a biconvex optimization problem. $-U(\overline{\tau},\overline{p})$ is a function of two set of variables; $\overline{\tau}$ and $\overline{p}$. From Theorem 1, given $\overline{\tau}$, $-U(\overline{p})$ is convex. Similarly, from Theorem 2, given $\overline{p}$, $-U(\overline{\tau})$ is convex. Hence, $-U(\overline{\tau},\overline{p})$ is biconvex, which completes the proof.

Appendix B
Proof of Theorem 4

The proof is done by contradiction. For any given time allocation $\overline{\tau}$, consider a given power sequence, $\mathcal{P}_{\mathcal{C}} = (p_1,...,p_{d-1},p_d,...,p_K)$, in which the power level decreases at some time, say $d > 1$. In such a case, we can defer some energy, $0 < \Delta \leq p_{d-1}T_{d-1}$, from the $(d-1)^{th}$ slot to the d^{th} slot forming a modified schedule, $\mathcal{P}'_{\mathcal{C}} = (p_1,...,p'_{d-1},p'_d,...,p_K)$, that will not violate the energy causality conditions (as shown in Fig. 7). Clearly, we can continue this deferral operation until $p'_{d-1} < p'_d$ and still not violate the energy causality conditions. Applying the same method for every possible decrease leads us to a nondecreasing schedule, $\mathcal{P}^{\uparrow}_{\mathcal{C}} = (p'_1,...,p'_{d-1},p'_d,...,p'_K)$, where $p'_1 \leq p'_2 \leq ... \leq p'_K$. From Lemma 1, $U(\overline{\tau},\mathcal{P}_{\mathcal{C}}) = U(\overline{\tau}^{\mathcal{P}^{\uparrow}_{\mathcal{C}}},\mathcal{P}^{\uparrow}_{\mathcal{C}})$. Thus, for time allocation $\overline{\tau}^* = \overline{\tau}^{\mathcal{P}^{\uparrow}_{\mathcal{C}}}$, $\mathcal{P}^{\uparrow}_{\mathcal{C}}$ is optimal. This completes the proof.

Appendix C
Proof of Lemma 1

Let, $\overline{R'_n} = \overline{R_n}^{\uparrow}$ where $\overline{R_n}^{\uparrow}$ is as defined in Definition 2. Note that, Definition 2 forces

$$log_2(1+L_np'_1) \leq \ldots \leq log_2(1+L_np'_l) \leq \ldots \leq log_2(1+L_np'_K) \quad (13)$$

$$p'_1 \leq \ldots \leq p'_l \leq \ldots \leq p'_K \quad (14)$$

Hence, sorting $\overline{R_n}$ in increasing order, forces nondecreasing powers (ordered schedule $\mathcal{P}^{\uparrow}_{\mathcal{C}}$ mentioned previously), which indeed forces all other $\overline{R_i}$ (where $i \in \{1,\ldots,i-1,i+1,\ldots,N\}$) to be sorted in increasing order,

TABLE IV

FAIRNESS INDEX (SG+TDMA, PTF, PRONTO, BCD) VS. NO. OF USERS: THE FAIRNESS OF PTF AND PRONTO HEURISTICS ARE COMPARED TO THAT OF SG+TDMA'S AND BCD'S, THROUGH FI.

Number of Users	Fairness Index (FI)											
	Regular				Bursty				Very Bursty			
	SG+TDMA	PTF	ProNTO	BCD	SG+TDMA	PTF	ProNTO	BCD	SG+TDMA	PTF	ProNTO	BCD
2	0.9989	0.9949	0.9997	0.9911	1.0000	0.9944	0.9998	0.9915	0.9079	0.9844	0.9997	0.9855
3	0.9667	0.9813	0.9931	0.9744	0.8079	0.9972	0.9501	0.9672	0.6398	0.9755	0.9633	0.9635
4	0.9333	0.9484	0.9781	0.9439	0.6520	0.9018	0.8917	0.9288	0.8035	0.8822	0.9642	0.9032
5	0.7487	0.9650	0.8568	0.9034	0.5554	0.8969	0.9360	0.8921	0.5764	0.8913	0.8308	0.8455
6	0.8425	0.8291	0.9059	0.8426	0.5594	0.7804	0.7842	0.8147	0.3123	0.9141	0.6706	0.7941
7	0.6796	0.8567	0.7783	0.7842	0.5399	0.8627	0.6613	0.7115	0.1958	0.8098	0.5695	0.7186
8	0.5800	0.8172	0.6582	0.7100	0.3554	0.7736	0.5627	0.6355	0.2456	0.6257	0.6915	0.6466

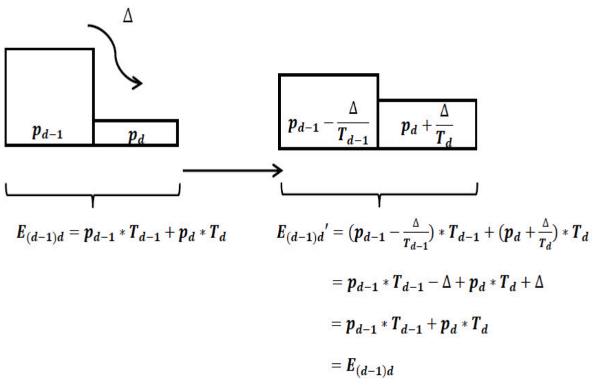

Fig. 7. Maintaining energy causality after energy deferral.

to form $\overline{R_i'}$. Now, we have new rates, $\overline{R_i'}$ for all users $i = 1, \ldots, N$. Remember that the utility of a user is defined as in Eq. (11). Thus, changing the order of $\overline{R_i}$ vector does not change the value of U_i if the order of $\overline{\tau_i}$ is also changed so that the previous element pairs are matched again. Let us explain this, with an example. Let $R_{i2} < R_{i1}$, $R_{NK} < R_{i2}$, and, $R_{i1} \leq R_{i3} \leq \ldots \leq R_{i(K-1)}$. Then, $\overline{\tau_i'}$, and, $\overline{R_i'}$ vectors are defined as $\overline{R_i'} = [R_{iK}\ R_{i2}\ R_{i1}\ R_{i3}\ \ldots\ R_{i(K-1)}]^T$ and $\overline{\tau_i'} = [\tau_{iK}\ \tau_{i2}\ \tau_{i1}\ \tau_{i3}\ \ldots\ \tau_{i(K-1)}]^T$. Thus, we have

$$\begin{aligned}\overline{\tau_i'}^T \overline{R_i'} &= \tau_{iK} R_{iK} + \tau_{i2} R_{i2} + \tau_{i1} R_{i1} + \ldots + \tau_{i(K-1)} R_{i(K-1)} \\ &= \tau_{i1} R_{i1} + \tau_{i2} R_{i2} + \ldots + \tau_{i(K-1)} R_{i(K-1)} + \tau_{iK} R_{iK} \\ &= \overline{\tau_i}^T \overline{R_i} \end{aligned} \tag{15}$$

where $\overline{\tau_i}$ and $\overline{R_i}$ are as defined in Eq. (11). As it can be observed, $U_i = U_i'$ as long as $\overline{R_i'} = \overline{R_i}^{\uparrow}$ and $\overline{\tau_i'} = (\overline{\tau_i})^{\overline{R_i}^{\uparrow}}$. Here, $\overline{\tau_i}^{\overline{R_i}^{\uparrow}}$ indicates the $\overline{\tau_i}$ vector ordered according to $\overline{R_i}^{\uparrow}$. Under these circumstances, $U_i = U_i'$ for all $i = 1, \ldots, N$, and, the overall utility does not change, $U = U'$. This completes the proof.

APPENDIX D
PROOF OF LEMMA 2

For this proof, we use the KKT optimality conditions. Let, $A_n = \tau_{n1}^* R_{n1} + \tau_{n2}^* R_{n2}$. Then, for ($N = 2$, $K = 2$), the set of KKT conditions described in [23] reduces to Eqns. (16a)-(16d).

$$\frac{\partial L}{\partial \tau_{nt}} = \frac{1}{ln2} \frac{R_{nt}}{A_n} + \mu_{2(t-1)+n}^* + \mu_{n+4}^* - \lambda_t^* = 0 \tag{16a}$$

$$\mu_i^* \geq 0\ ,\ \tau_{nt}^* \geq 0 \tag{16b}$$

$$\tau_{n1}^* + \tau_{n2}^* \geq \epsilon\ ,\ \tau_{1t}^* + \tau_{2t}^* = T \tag{16c}$$

$$\mu_{2(t-1)+n}^* \tau_{nt}^* = 0\ ,\ \mu_{4+n}^* (\tau_{n1}^* + \tau_{n2}^* - \epsilon) = 0 \tag{16d}$$

for $i = 1, \ldots, 6$, $n = 1, 2$ and $t = 1, 2$. Combining the set of equations described above leads us to the following optimality conditions for the time allocation:

$$\mu_{2t-1}^* \tau_{1t}^* = 0 \tag{17a}$$

$$\left(\frac{R_{1t}}{A_1 ln2} - \frac{R_{2t}}{A_2 ln2} + \mu_{2t-1}^* \right) (T - \tau_{1t}^*) = 0 \tag{17b}$$

Solving the set of equations in Eq. (17), one can obtain the desired relation between power allocation and time allocation, as illustrated in Table V. Due to the convex nature of the problem, the solutions presented in Table V represent the global optima, when the rate improvements of the users, Γ_n, are equal. By inspecting Table V, one can observe the properties mentioned in Lemma 2. All cases are summarized in Table V, which completes the proof.

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TABLE V
OVERALL OPTIMALITY CONDITIONS FOR THE SPECIAL CASE OF TWO USERS AND TWO SLOTS ($T_1 = T_2$): CATEGORIZED ACCORDING TO THE RELATION BETWEEN THE POWERS ALLOCATED IN THE FIRST AND SECOND SLOTS. FOR A GIVEN POWER ALLOCATION, THE OPTIMAL TIME ALLOCATION DIFFERS ACCORDING TO THE RELATION BETWEEN THE RATE IMPROVEMENTS OF THE USERS.

Power Relation (Slot 1 vs. Slot 2)	Users' Rate Improvement Relation (User 1 vs. User 2)	Slot 1		Slot 2		Utility
		User 1 τ_{11}	User 2 τ_{21}	User 1 τ_{12}	User 2 τ_{22}	
$p_1 < p_2$	$\Gamma_1 < \Gamma_2$	T	0	$\frac{T}{2}\left(1-\frac{1}{\Gamma_1}\right)$	$\frac{T}{2}\left(1+\frac{1}{\Gamma_1}\right)$	$log_2\left(\frac{R_{22}}{R_{12}}(R_{11}+R_{12})^2\right)+2log_2\left(\frac{T}{2}\right)$
	$\Gamma_1 = \Gamma_2$	T	0	$\frac{T}{2}\left(1-\frac{1}{\Gamma_1}\right)$	$\frac{T}{2}\left(1+\frac{1}{\Gamma_1}\right)$	$log_2\left((R_{11}+R_{12})(R_{21}+R_{22})\right)+2log_2\left(\frac{T}{2}\right)$
		0	T	$\frac{T}{2}\left(1+\frac{1}{\Gamma_2}\right)$	$\frac{T}{2}\left(1-\frac{1}{\Gamma_2}\right)$	$log_2\left((R_{11}+R_{12})(R_{21}+R_{22})\right)+2log_2\left(\frac{T}{2}\right)$
	$\Gamma_1 > \Gamma_2$	0	T	$\frac{T}{2}\left(1+\frac{1}{\Gamma_2}\right)$	$\frac{T}{2}\left(1-\frac{1}{\Gamma_2}\right)$	$log_2\left(\frac{R_{12}}{R_{22}}(R_{21}+R_{22})^2\right)+2log_2\left(\frac{T}{2}\right)$
$p_1 = p_2$	$\Gamma_1 < \Gamma_2$	T	0	0	T	$log_2(R_{11}R_{22})+2log_2(T)$
	$\Gamma_1 = \Gamma_2$	T	0	0	T	$log_2(R_{11}R_{22})+2log_2(T)$
		0	T	T	0	$log_2(R_{11}R_{22})+2log_2(T)$
	$\Gamma_1 > \Gamma_2$	0	T	T	0	$log_2(R_{12}R_{21})+2log_2(T)$
$p_1 > p_2$	$\Gamma_1 < \Gamma_2$	$\frac{T}{2}(1+\Gamma_2)$	$\frac{T}{2}(1-\Gamma_2)$	0	T	$log_2\left(\frac{R_{11}}{R_{21}}(R_{21}+R_{22})^2\right)+2log_2\left(\frac{T}{2}\right)$
	$\Gamma_1 = \Gamma_2$	$\frac{T}{2}(1-\Gamma_2)$	$\frac{T}{2}(1+\Gamma_2)$	T	0	$log_2\left((R_{11}+R_{12})(R_{21}+R_{22})\right)+2log_2\left(\frac{T}{2}\right)$
		$\frac{T}{2}(1+\Gamma_1)$	$\frac{T}{2}(1-\Gamma_1)$	0	T	$log_2\left((R_{11}+R_{12})(R_{21}+R_{22})\right)+2log_2\left(\frac{T}{2}\right)$
	$\Gamma_1 > \Gamma_2$	$\frac{T}{2}(1-\Gamma_1)$	$\frac{T}{2}(1+\Gamma_1)$	T	0	$log_2\left(\frac{R_{21}}{R_{11}}(R_{11}+R_{12})^2\right)+2log_2\left(\frac{T}{2}\right)$

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