II. Synchronous Generators

- ***Synchronous machines*** are principally used as ***alternating current (AC) generators***. They supply the electric power used by all sectors of modern societies: industrial, commercial, agricultural, and domestic.

- ***Synchronous generators*** usually operate together (or in parallel), forming a large power system supplying electrical energy to the loads or consumers.

- ***Synchronous generators*** are built in large units, their rating ranging from tens to hundreds of megawatts.

- ***Synchronous generator*** converts mechanical power to ac electric power. The source of mechanical power, ***the prime mover***, may be a diesel engine, a steam turbine, a water turbine, or any similar device.

- For high-speed machines, the prime movers are usually ***steam turbines*** employing fossil or nuclear energy resources.

- Low-speed machines are often driven by ***hydro-turbines*** that employ water power for generation.

- Smaller synchronous machines are sometimes used for private generation and as standby units, with diesel engines or gas turbines as prime movers.

Types of Synchronous Machine

According to the ***arrangement of the field and armature windings***, synchronous machines may be classified as ***rotating-armature type*** or ***rotating-field type***.

Rotating-Armature Type: The armature winding is on the rotor and the field system is on the stator.

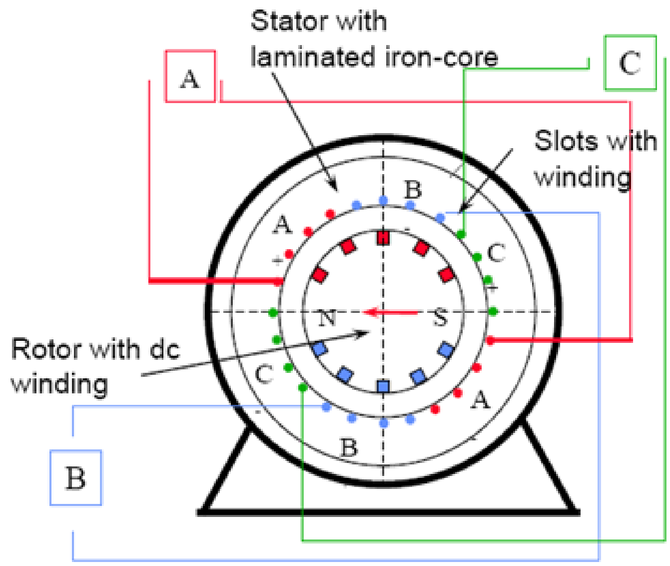
Rotating-Field Type: The armature winding is on the stator and the field system is on the rotor.

According to the shape of the field, synchronous machines may be classified as ***cylindrical-rotor* (*non-salient pole*) *machines*** and ***salient-pole machines***

Round Rotor Machine

- The stator is a ring shaped laminated iron-core with slots.
- Three phase windings are placed in the slots.
- Round solid iron rotor with slots.
- A single winding is placed in the slots. Dc current is supplied through slip rings.

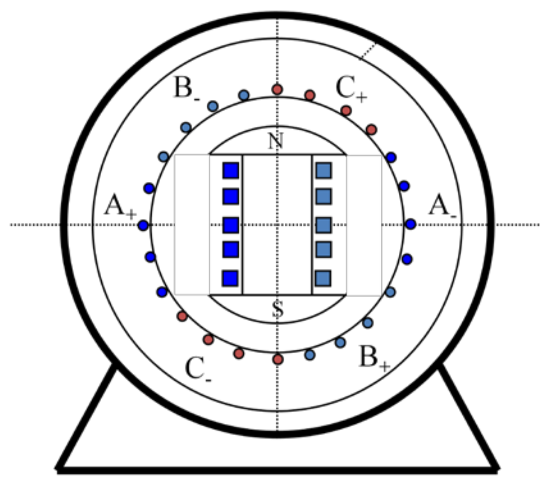
Concept (two poles)



Salient Rotor Machine

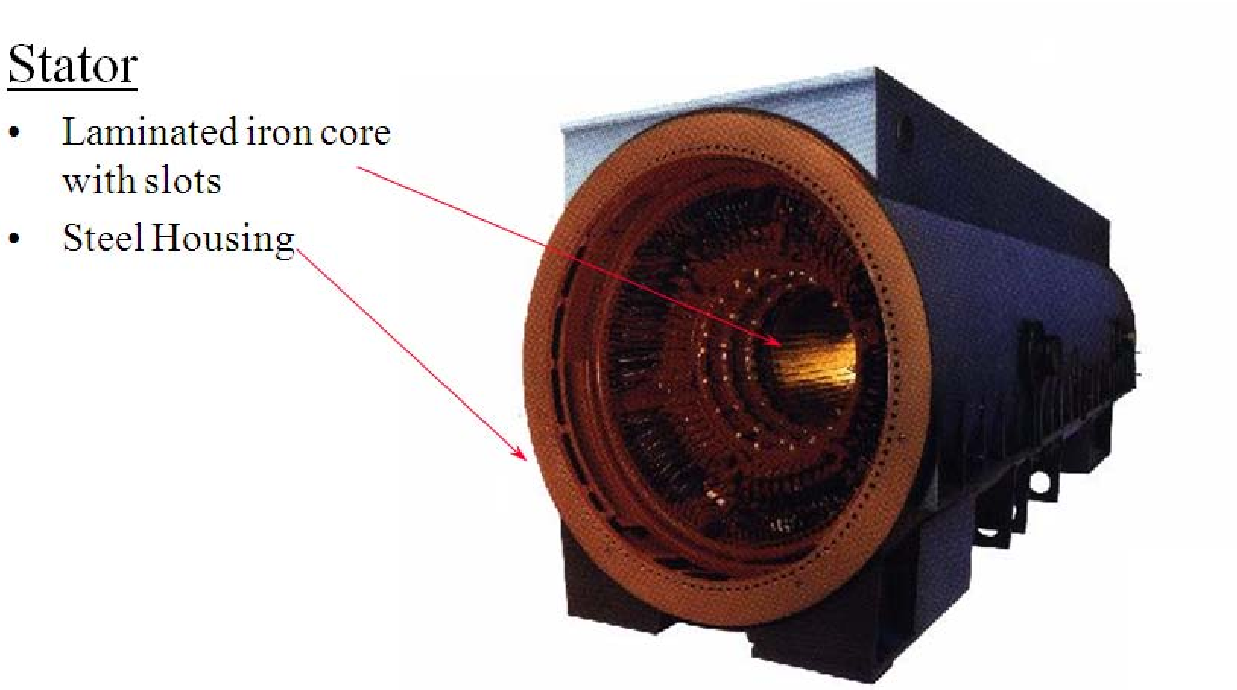
- The stator has a laminated iron-core with slots and three phase windings placed in the slots.
- The rotor has salient poles excited by dc current.
- DC current is supplied to the rotor through slip-rings and brushes.
- The number of poles varies between 2 - 128.

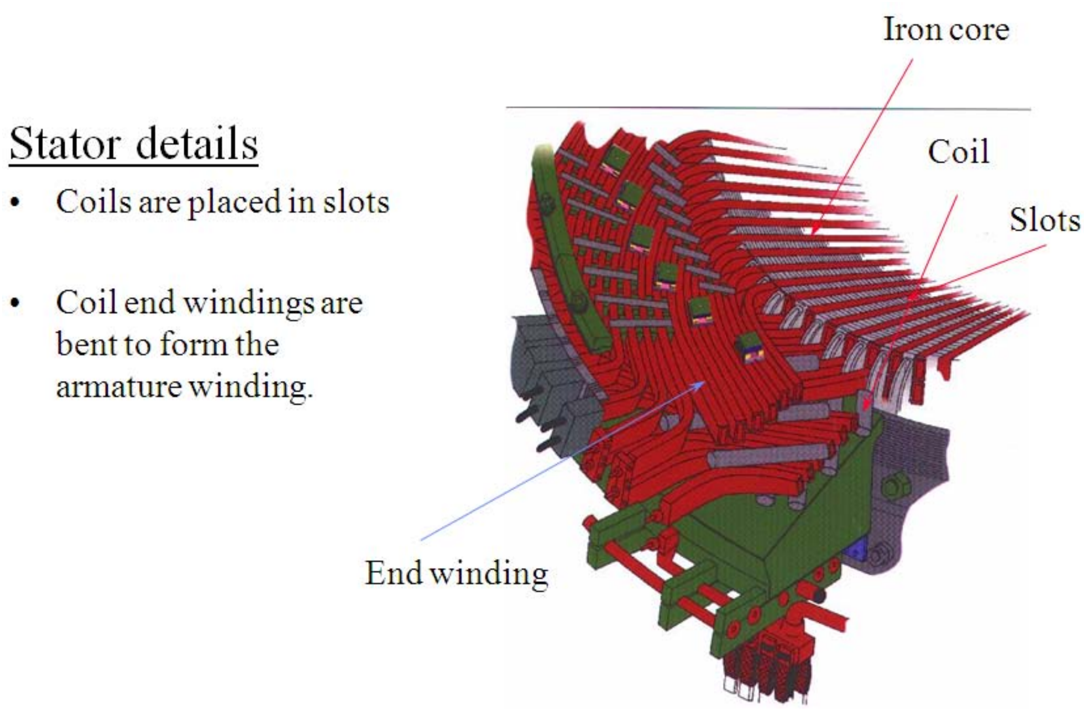
- **Concept (two poles)**



Construction

- The winding consists of copper bars insulated with mica and epoxy resin.
- The conductors are secured by steel wedges.
- The iron core is supported by a steel housing.

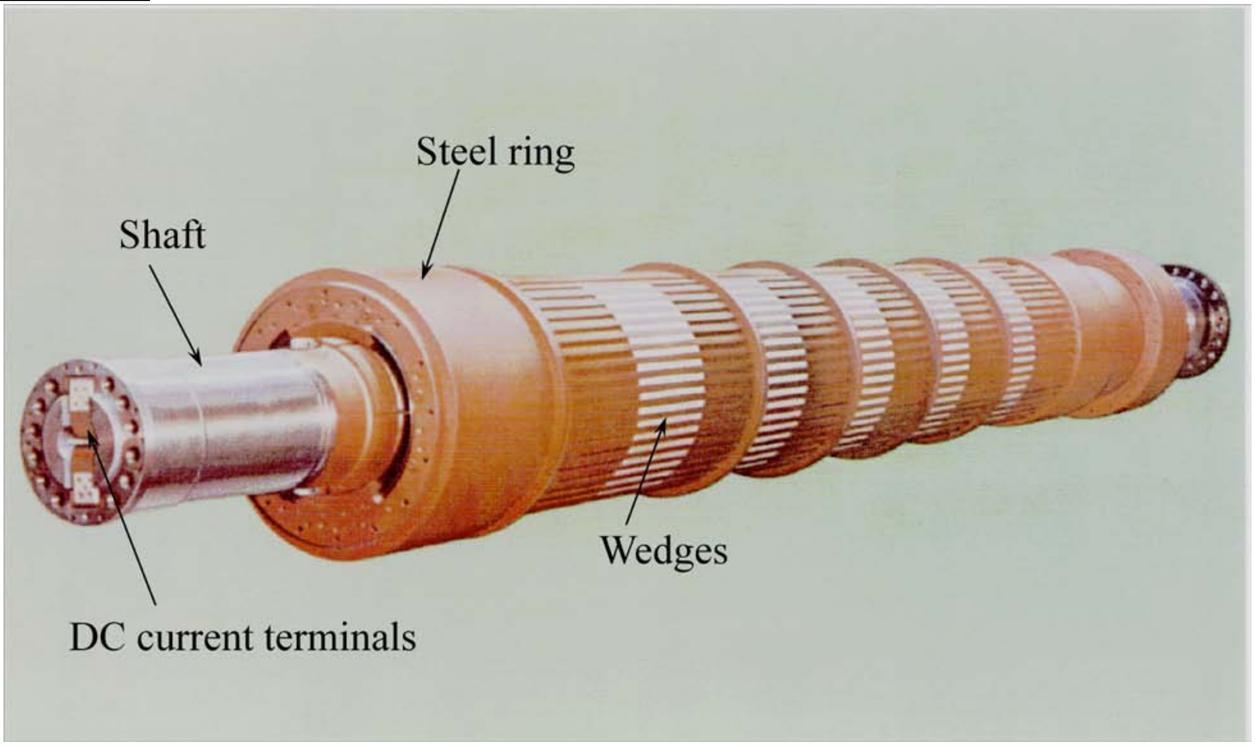




Round rotor

- **The round rotor is used for large high speed (3600rpm) machines.**
- **A forged iron core (not laminated,DC) is installed on the shaft.**
- **Slots are milled in the iron and insulated copper bars are placed in the slots.**
- The slots are closed by wedges and re-enforced with steel rings.

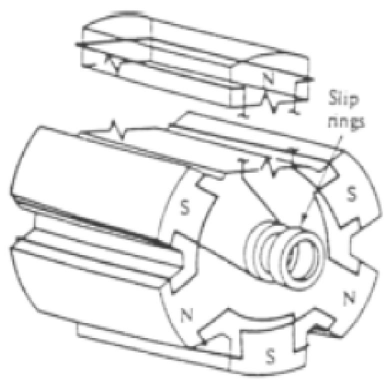
Round rotor



Salient pole rotor construction

- **The poles are bolted to the shaft.**
- **Each pole has a DC winding.**
- **The DC winding is connected to the slip-rings (not shown).**
- **A DC source supplies the winding with DC through brushes pressed into the slip ring.**
- **A fan is installed on the shaft to assure air circulation and effective cooling.**
- **Low speed, large hydro-generators may have more than one hundred poles.**
- **These generators are frequently mounted vertically**

Salient Rotor



Field Excitation and Exciters

- DC field excitation is an important part of the overall design of a synchronous generator
- The field excitation must ensure not only a stable AC terminal voltage, but must also respond to sudden load changes
- Rapid field excitation response is important

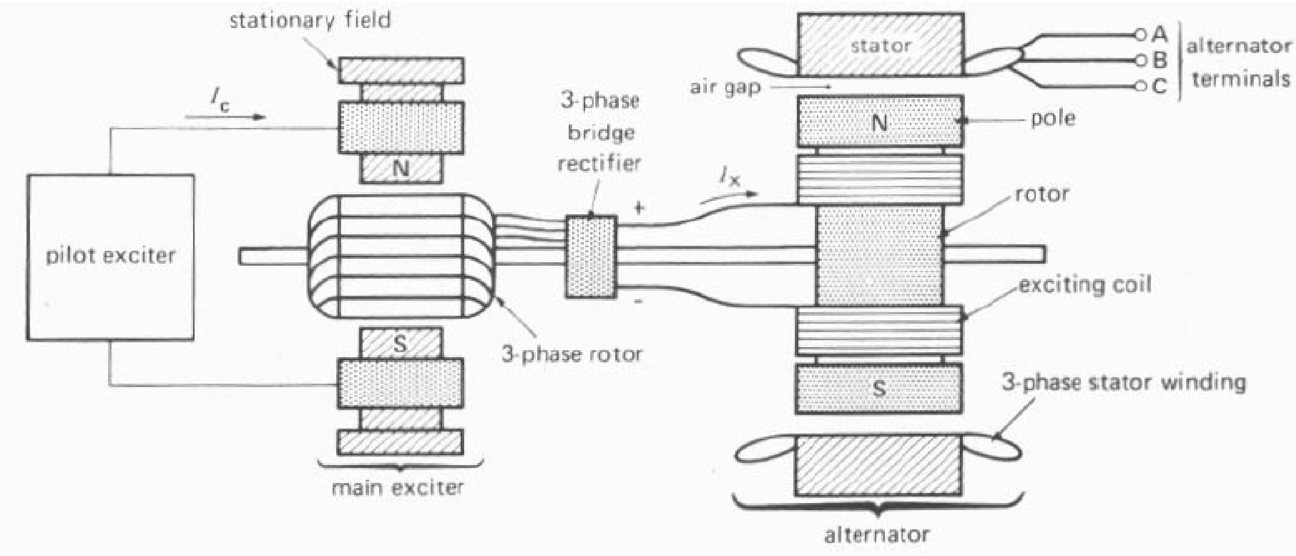
Three methods of excitation

1. ***slip rings*** link the rotor's field winding to an external dc source
2. ***dc generator exciter***

- a dc generator is built on the same shaft as the ac generator's rotor
- a commutator rectifies the current that is sent to the field winding

3. ***brushless exciter***

- an ac generator with fixed field winding and a rotor with a three phase circuit
- diode/SCR rectification supplies dc current to the field windings



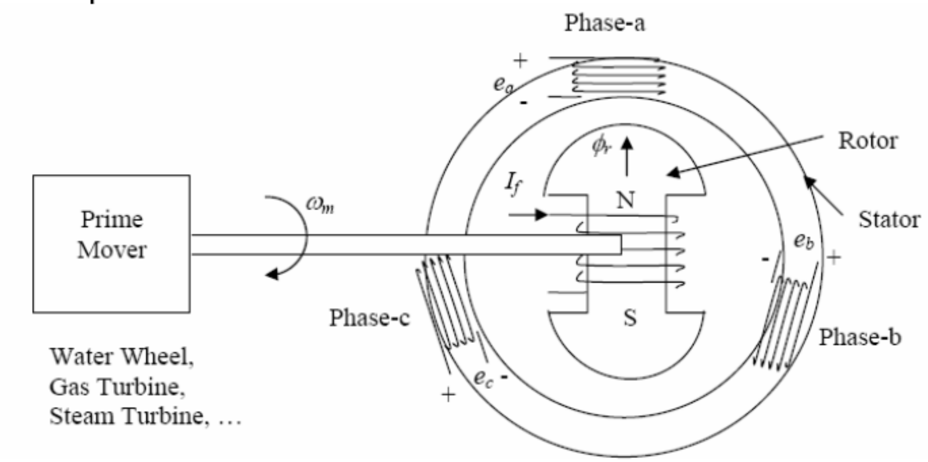
Typical brushless exciter system

Ventilation or Cooling of an Alternator

- The slow speed salient pole alternators are ventilated by the fan action of the salient poles which provide circulating air.
- Cylindrical rotor alternators are usually long, and the problem of air flow requires very special attention.
- The cooling medium, air or hydrogen is cooled by passing over pipes through which cooling water is circulated and ventilation of the alternator.
- Hydrogen is normally used as cooling medium in all the turbine-driven alternators because hydrogen provides better cooling than air and increases the efficiency and decreases the windage losses.
- Liquid cooling is used for the stators of cylindrical rotor generators.

Principle of Operation

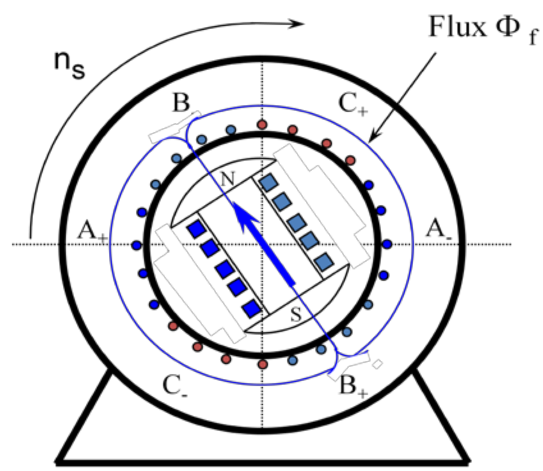
1) From an external source, the field winding is supplied with a DC current -> excitation.
2) Rotor (field) winding is mechanically turned (rotated) at synchronous speed.
3) The rotating magnetic field produced by the field current induces voltages in the outer stator (armature) winding. The frequency of these voltages is in synchronism with the rotor speed.



Operation concept

- The rotor is supplied by DC current I_f that generates a DC flux Φ_f.
- The rotor is driven by a turbine with a constant speed of n_s.
- The rotating field flux induces a voltage in the stator winding.
- The frequency of the induced voltage depends upon the speed.

Operation (two poles)



- The frequency - speed relation is **$f = (p / 120)\ n = p\ n / 120$**
  p is the number of poles.
- Typical rotor speeds are 3600 rpm for 2-pole, 1800 rpm for 4 pole and 450 rpm for 16 poles.

- **The rms. value of the induced voltages are:**

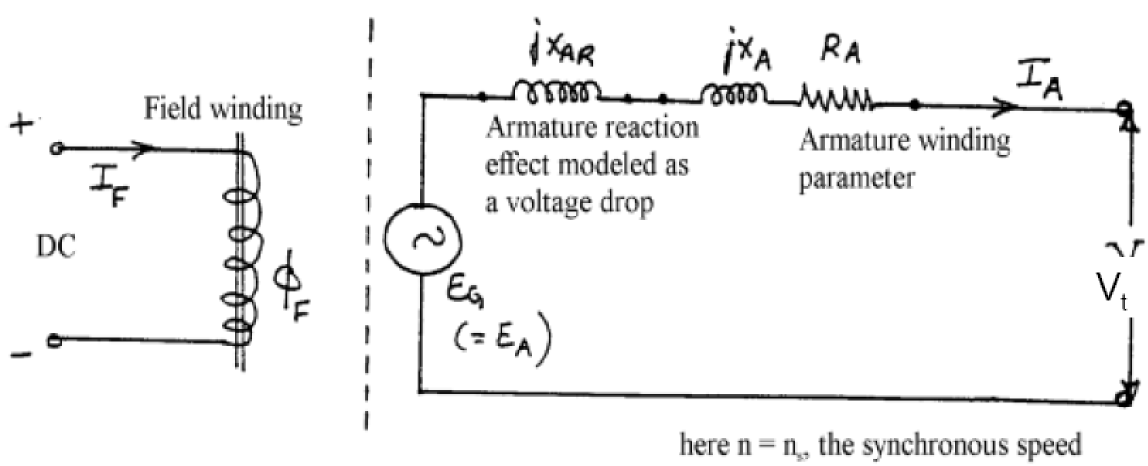
$$E_{an} = E_{rms}\, e^{iO\deg} \qquad E_{bn} = E_{rms}\, e^{-i120\deg} \qquad E_{cn} = E_{rms}\, e^{-i240\deg}$$

- **where:** $E_{rms} = \dfrac{k_w\, \omega N_a \Phi_f}{\sqrt{2}} = 4.44\ f\ N_a\, \Phi_f\, k_w$

k_w = 0.85-0.95 is the winding factor.

Synchronous Generators Equivalent Circuit (round rotor)

1) DC current in the field winding produces the main flux, ϕ_f.
2) ϕ_f induces an emf, E_G, in the armature winding.
3) Depending on the load condition, the armature current I_A is established. In the following discussions, it is assumed to be a lagging power factor.
4) I_A produces its own flux due to armature reaction, E_{AR} is the induced emf by ϕ_{AR}.
5) The resulting phasor, $E_{resultant} = E_G + E_{AR}$ is the "true" induced emf that is available.



If the machine is 3 phase, the same equivalent circuit is used. After solving the single-phase circuit, then proper 3 phase values (Line-line voltage or 3-pahse power) should be calculated.

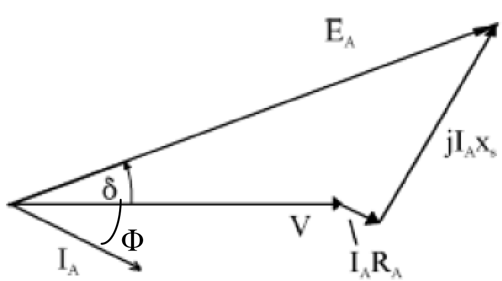
Phasor Diagrams (single phase):

$$V_t = E_A - I_A jX_A - I_A jX_{AR} - I_A R_A = E_A - jX_s I_A - I_A R_A$$

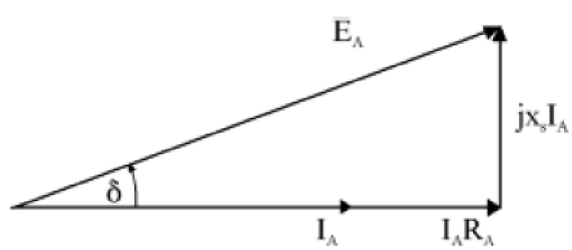
$$V_t = E_A - I_A\left(R_A + jX_s\right)$$

where, $(X_{AR} + X_A)$ = synchronous reactance, X_s.

Inductive Load



Resistive Load



δ= power angle

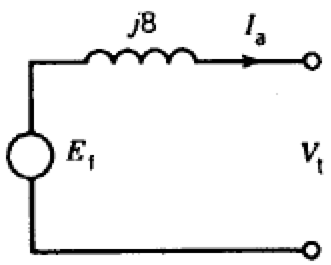
EXAMPLE

A 3ϕ, 5 kVA, 208 V, four-pole, 60 Hz, star-connected synchronous machine has negligible stator winding resistance and a synchronous reactance of 8 ohms per phase at rated terminal voltage.

Determine the excitation voltage and the power angle when the machine is delivering rated kVA at 0.8 PF lagging. Draw the phasor diagram for this condition.

Solution

The per-phase equivalent circuit for the synchronous generator



$$V_t = \frac{208}{\sqrt{3}} = 120 \text{ V/phase}$$

Stator current at rated kVA;

$$I_a = \frac{5000}{\sqrt{3} \times 208} = 13.9 \text{ A}$$

$$\phi = -36.9^\circ \quad \text{for lagging pf of 0.8}$$

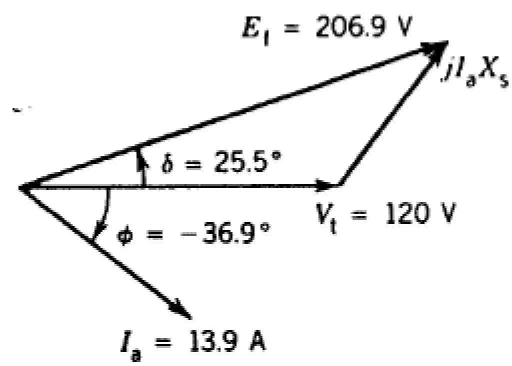
$$E_f = V_t\angle 0^\circ + I_a jX_s$$

$$= 120\angle 0^\circ + 13.9\angle -36.9^\circ \cdot 8\angle 90^\circ$$

$$= 206.9\angle 25.5^\circ$$

Excitation voltage $\quad E_f = 206.9$ V/phase

Power angle $\quad \delta = +25.5^\circ$



H.W

A four pole, three-phase synchronous generator is rated 250 MVA, its terminal voltage is 24 kV, the synchronous reactance is: 125%.

- Calculate the synchronous reactance in ohm.
- Calculate the rated current and the line to ground terminal voltage.
- Draw the equivalent circuit.
- Calculate the induced voltage, E_f, at rated load and pf = 0.8 lag.

(Ans: X_{syn}=2.88Ω, I_g=6.01$\angle$-36.87°KA, E_{gn}=27.93$\angle$29.74KV)

Armature Reaction in Synchronous Machines

Armature reaction refers to

- the influence on the magnetic field in the air gap when the phase windings a, b, and c on the stator are connected across a load.
- The flux produced by the armature winding reacts with the flux set up by the poles on the rotor, causing the total flux to change.

➢ ***The generator delivers a load at a unity power factor.***

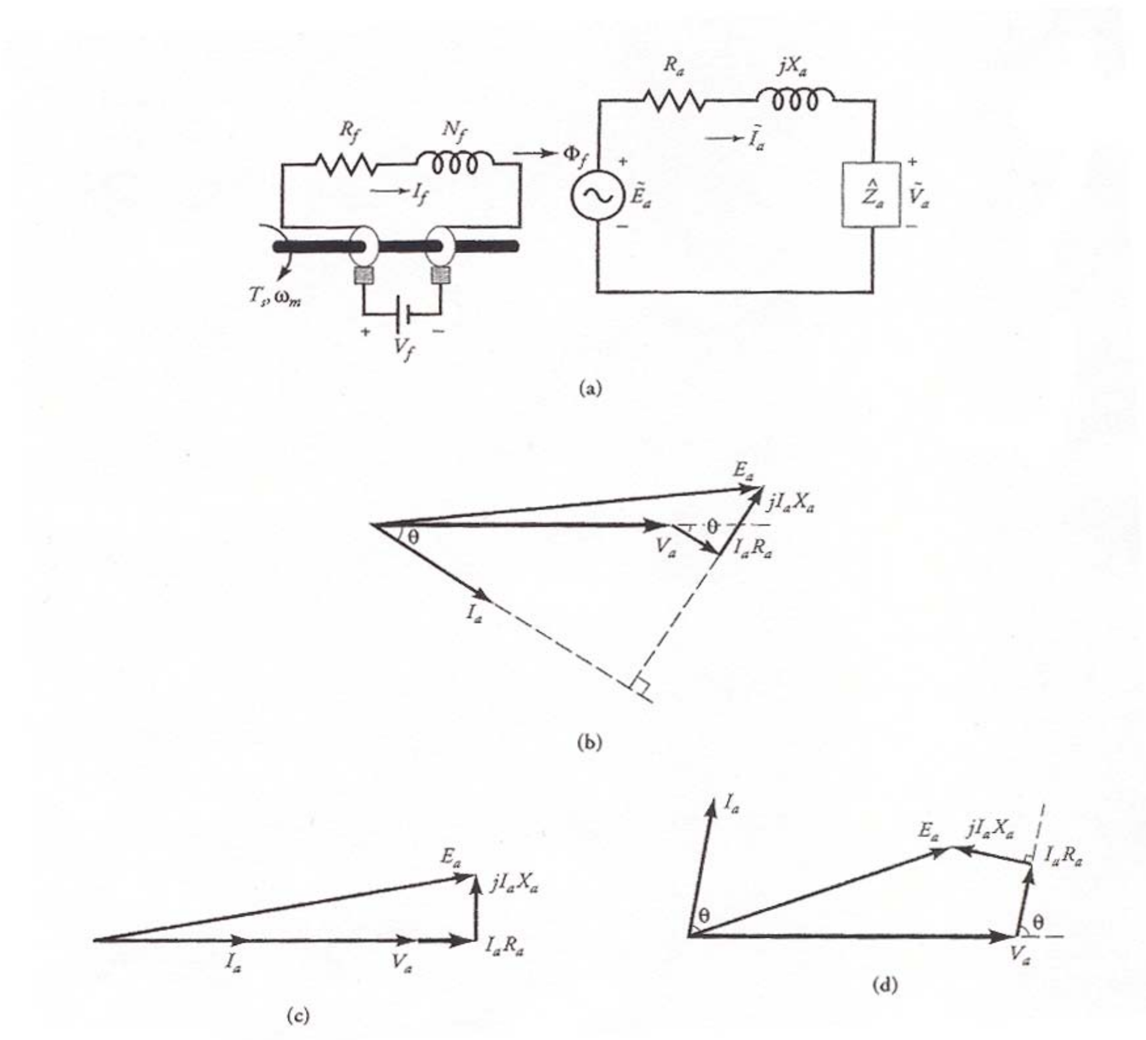


Figure.1 *(a)* The per-phase equivalent circuit of a synchronous generator without armature reaction while depicting the revolving field produced by the rotor. The phasor diagrams for a *(b)* lagging pf, *(c)* unity pf, and *(d)* leading pf.

(a) If Φ_p is the flux per pole in the generator under no load, then the generated voltage E_a must lag Φ_p by 90^o, as shown in Figure 2.

(b) Since the power factor is unity, the phase current $\tilde{I}_a$ is in phase with the terminal phase voltage $\tilde{V}_a$.

(c) As the phase current $\tilde{I}_a$ passes through the armature winding, its magnetomotive force (mmf) produces a flux Φ_{ar} which is in phase with $\tilde{I}_a$. The effective flux Φ_e per pole in the generator is the algebraic sum of the two fluxes; that is, $\Phi_e = \Phi_p + \Phi_{ar}$, as shown in the figure.

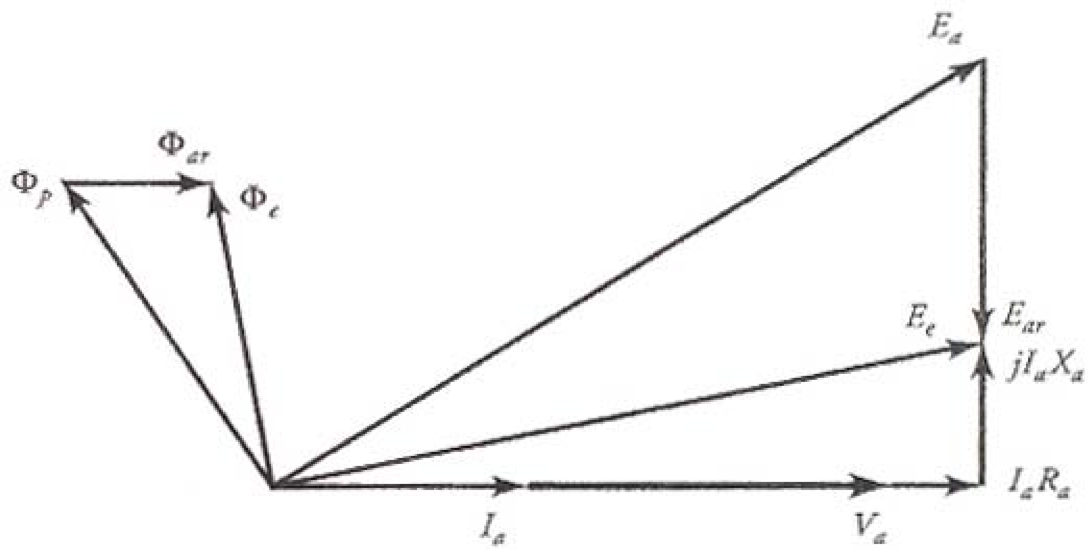


Figure 2: Phasor diagram depicting the effect of armature reaction when the power factor is unity.

(d) The flux Φ_{ar}, in turn, induces an emf $\tilde{E}_{ar}$ in the armature winding. $\tilde{E}_{ar}$ is called the **armature reaction** emf. The armature reaction emf $\tilde{E}_{ar}$ lags the flux Φ_{ar} by 90^o. Hence the effective generated voltage per-phase $\tilde{E}_e$ is the algebraic sum of the no-load voltage $\tilde{E}_a$ and the armature reaction emf $\tilde{E}_{ar}$. That is, $\tilde{E}_e = \tilde{E}_a + \tilde{E}_{ar}$. An equivalent circuit showing the armature reaction emf is given in Figure 3.

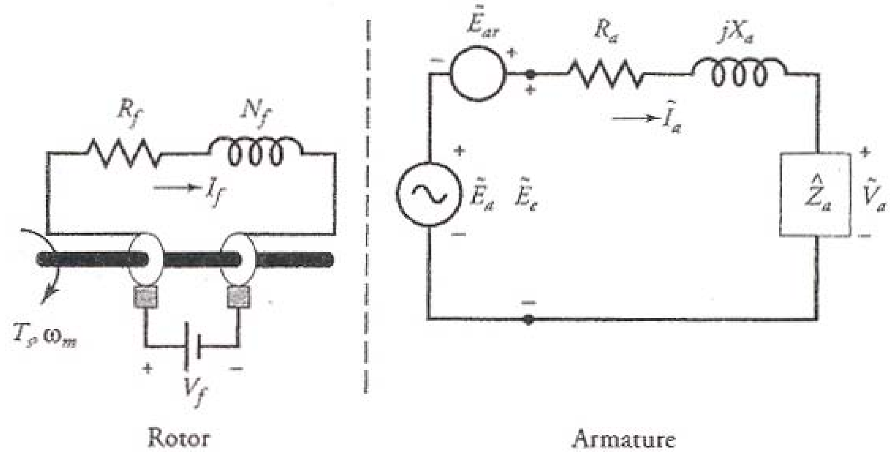


Figure 3: A per-phase equivalent circuit showing the induced emf in the armature winding due to the armature reaction.

(e) The per-phase terminal voltage $\tilde{V}_a$ is obtained by subtracting the voltage drops $\tilde{I}_a R_a$ and $j\tilde{I}_a X_a$ from $\tilde{E}_e$. In other words,

$$\tilde{E}_e = \tilde{V}_a + \tilde{I}_a (R_a + j X_a)$$

From the phasor diagram, it should be obvious that the armature reaction has reduced the effective flux per pole when the power factor of the load is unity.

Also, the terminal voltage is smaller than the generated voltage.

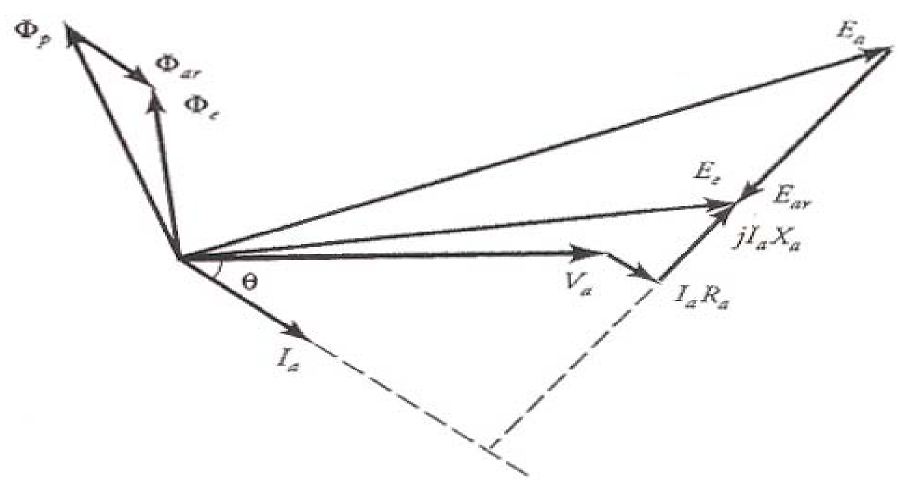


Figure 4: The phasor diagram showing the effect of armature reaction when the power factor is lagging.

By following the above sequence of events, we can obtain the phasor diagrams for the lagging (Figure 4) and the leading (Figure 5) power factors. From these figures it is evident that the resultant flux is (smaller/larger) with armature reaction for the (lagging/leading) power factor than without it. In addition, the terminal voltage $\tilde{V}_a$ is (higher/lower) than the generated voltage $\tilde{E}_a$ when the power factor is (leading/ lagging). Since the flux per pole Φ_p is different for each of the three load conditions, the field current I_f must be adjusted each time the load is changed.

Since the armature reaction emf $\tilde{E}_{ar}$ lags the current $\tilde{I}_a$ by 90°, we can also express it as

$$\tilde{E}_{ar} = -\,j\tilde{I}_a X_m$$

where X_m, a constant of proportionality, is known as the **magnetization reactance**.

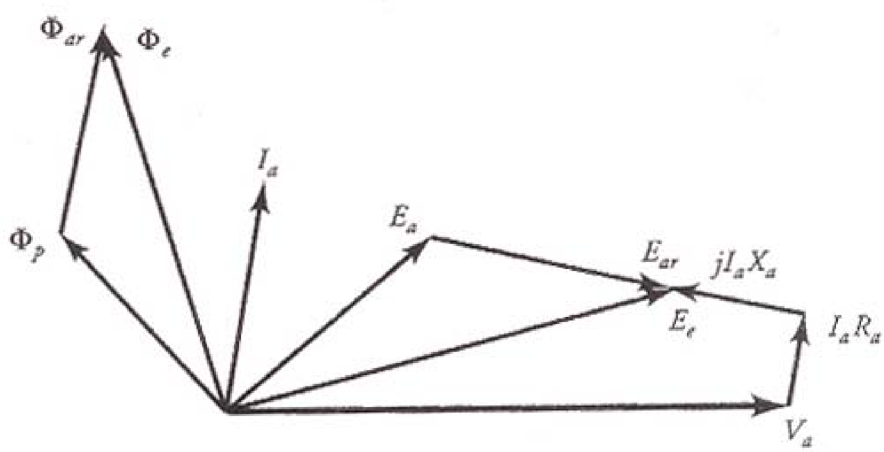


Figure 5: The phasor diagram showing the effect of armature reaction when the power factor of the load is leading.

Both the magnetization reactance and the leakage reactance are present at the same time. It is rather difficult to separate one reactance from the other. For this reason, the two reactances are combined together and the sum

$$X_s = X_m + X_a$$

is called the **synchronous reactance**. The synchronous reactance is usually very large compared with the resistance of the armature winding. We can now define the **synchronous impedance** on a per-phase basis as $\hat{Z}_s = R_a + jX_s$

Synchronous Generator Tests

To obtain the ***parameters of a synchronous generator***, we perform three simple tests as described below.

❖ The Resistance Test

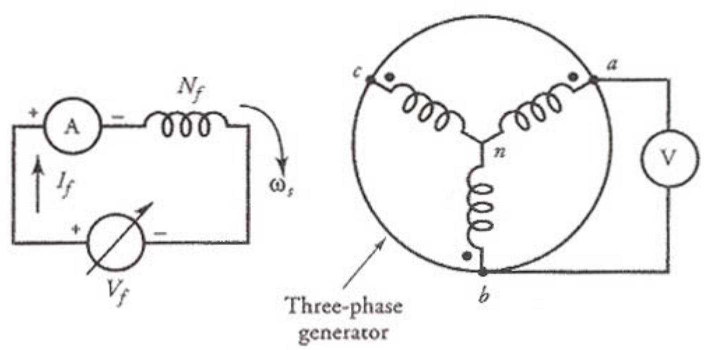
This test is conducted to measure-winding resistance of a synchronous generator when it is at rest and the field winding is open. The resistance is measured between two lines at a time and the average of the three resistance readings is taken to be the measured value of the resistance, R_L, from line to line. If the generator is Y-connected, the per-phase resistance is

$$R_a = 0.5\,R_L$$

❖ The Open-Circuit Test

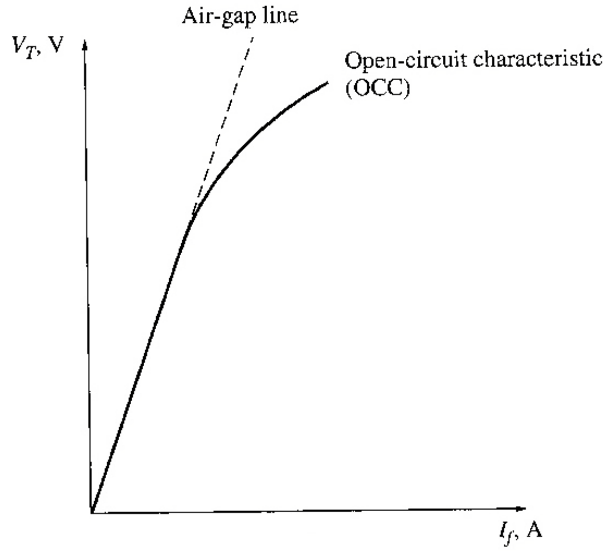
The open-circuit test, or the **no-load test**, is performed by

1) Generator is rotated at the rated speed.
2) No load is connected at the terminals.
3) Field current is increased from 0 to maximum.
4) Record values of the terminal voltage and field current value.



Circuit diagram to perform open-circuit test.

With the terminals open, $I_A=0$, so $E_A = V_\phi$. It is thus possible to construct a plot of E_A or V_T vs I_F graph. This plot is called open-circuit characteristic (OCC) of a generator. With this characteristic, it is possible to find the internal generated voltage of the generator for any given field current.



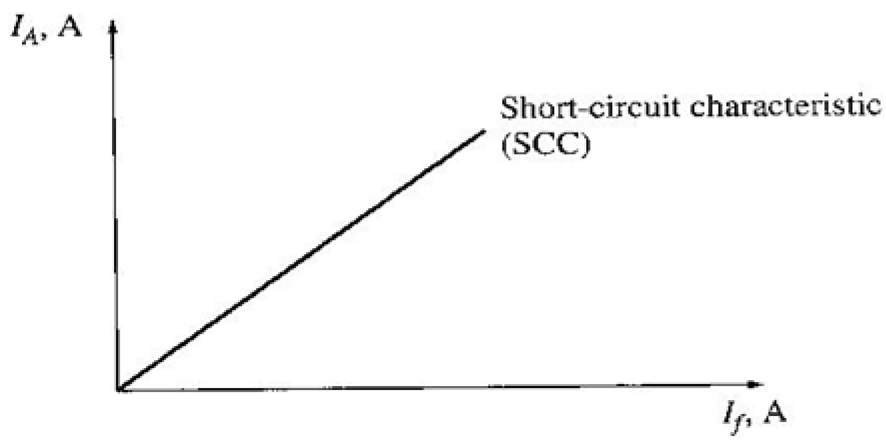
Open-circuit characteristic (OCC) of a generator

The OCC follows a straight-line relation as long as the magnetic circuit of the synchronous generator does not saturate. Since, in the linear region, most of the applied mmf is consumed by the air-gap, the straight line is appropriately called the ***air-gap line***.

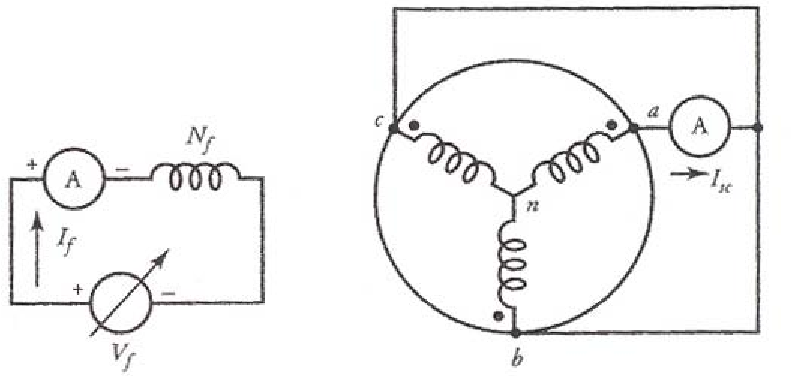
❖ The Short-Circuit Test

The short-circuit test provides information about the current capabilities of a synchronous generator. It is performed by

1) Generator is rotated at rated speed.
2) Adjust field current to 0.
3) Short circuit the terminals.
4) Measure armature current or line current as the field current is increased.



SCC is essentially a straight line. To understand why this characteristic is a straight line, look at the equivalent circuit below when the terminals are short circuited.



Circuit diagram to perform short-circuit test.

When the terminals are short circuited, the armature current I_A is:

$$I_A = \frac{E_A}{R_A + jX_S}$$

And its magnitude is:

$$I_A = \frac{E_A}{\sqrt{R_A^2 + X_S^2}}$$

➔ ***From both tests,*** here we can find the internal machine impedance (E_A from OCC, I_A from SCC):

$$Z_S = \sqrt{R_A^2 + X_S^2} = \frac{E_A}{I_A}$$

Since $X_s >> R_A$, the equation reduces to:

$$X_s \approx \frac{E_A}{I_A} = \frac{V_{\phi oc}}{I_A}$$

Short Circuit Ratio

Ratio of the field current required for the rated voltage at open circuit to the field current required for rated armature current at short circuit.

$$SCR = \frac{I_{f,OC}}{I_{f,SC}}$$

$$So, SCR = \frac{1}{X_s}$$

Example *A 3-phase synchronous generator produces an open-circuit line voltage of 6928 V when the do exciting current is 50 A. The ac terminals are then short-circuited, and the three line currents are found to be 800 A.*

a. Calculate the synchronous reactance per phase.

b. Calculate the terminal voltage if three 12 Ω resistors are connected in wye across the terminals.

Solution:

a. The line-to-neutral induced voltage is

$$E_o = E_L / \sqrt{3} = 6928 / \sqrt{3} = 4000V$$

When the terminals are short-circuited, the only impedance limiting the current flow is that due to the synchronous reactance. Consequently,

$$X_S = E_o / I = 4000 / 80 = 5\Omega$$

b. The equivalent circuit per phase is shown in Fig. 1

The impedance of the circuit is:

$$Z = \sqrt{R^2 + X_S^2} = \sqrt{12^2 + 5^2} = 13\Omega$$

The current is :

$$I = E_o / Z = 4000/13 = 308A$$

The voltage across the load resistor is

$$E = IR = 308 * 12 = 3696\ V$$

The line voltage under load is:

$$E_L = \sqrt{3}E = \sqrt{3} * 3696 = 6402\ V$$

The schematic diagram of Fig. 1 helps us visualize what is happening in the actual circuit.

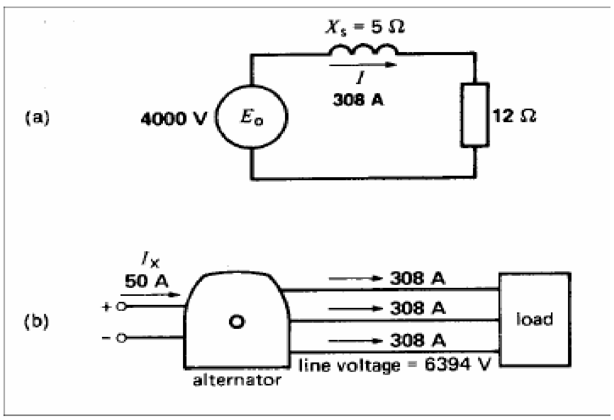


Fig. 1 See Example b. Actual line voltages and currents.

Example: The following data are taken from the open- and short-circuit characteristics of a 45-kVA, three-phase, Y-connected, 220-V (line-to-line), six-pole, 60-Hz synchronous machine. From the open-circuit characteristic:

Line-to-line voltage = 220 V Field current = 2.84 A

From the short-circuit characteristic:

Armature current, A	118	152
Field current, A	2.20	2.84

From the air-gap line:

Field current = 2.20 A Line-to-line voltage = 202 V

Compute the unsaturated value of the synchronous reactance, its saturated value at rated voltage, and the short-circuit ratio. Express the synchronous reactance in ohms per phase and in per unit on the machine rating as a base.

Solution

At a field current of 2.20 A the line-to-neutral voltage on the air-gap line is

$$V_{a,ag} = \frac{202}{\sqrt{3}} = 116.7 \text{ V}$$

and for the same field current the armature current on short circuit is

$$I_{a,sc} = 118 \text{ A}$$

$$X_{s,u} = \frac{116.7}{118} = 0.987 \ \Omega/\text{phase}$$

Note that rated armature current is

$$I_{a,rated} = \frac{45{,}000}{\sqrt{3} \times 220} = 118 \text{ A}$$

Therefore, la, sc = 1.00 per unit. The corresponding air-gap-line voltage is

$$V_{a,ag} = \frac{202}{220} = 0.92 \text{ per unit}$$

in per unit $X_{s,u} = \frac{0.92}{1.00} = 0.92$ per unit

The saturated synchronous reactance can be found from the open- and short-circuit characteristics

$$X_s = \frac{V_{a,rated}}{I'_a} = \frac{(220/\sqrt{3})}{152} = 0.836 \ \Omega/\text{phase}$$

In per unit $I'_a = \frac{152}{118} = 1.29$, and

$$X_s = \frac{1.00}{1.29} = 0.775 \text{ per unit}$$

Finally, from the open- and short-circuit characteristics, the short-circuit ratio is given by

$$SCR = \frac{2.84}{2.20} = 1.29$$

The inverse of the short-circuit ratio is equal to the per-unit saturated synchronous reactance

$$X_s = \frac{1}{SCR} = \frac{1}{1.29} = 0.775 \text{ per unit}$$

H.W

Calculate the saturated synchronous reactance (in Ω/phase and per unit) of a 85 kVA synchronous machine which achieves its rated open-circuit voltage of 460 V at a field current 8.7 A and which achieves rated short-circuit current at a field current of 11.2 A. **[Answer: Xs = 3.21 Ω/phase = 1.29 per unit]**

Voltage regulation of Alternator

The voltage regulation of an Alternator is defined as the change in terminal voltage from no-load to load condition expressed as per-unit or percentage of terminal voltage at load condition; the speed and excitation conditions remaining same.

$$\text{Voltage Regulation, V. R.} = \frac{V_{NL} - V_{FL}}{V_{FL}} \times 100\%$$

$$= \frac{V_{NL} - V_{FL}}{V_{FL}} \quad \textbf{Per unit}$$

Determination of Voltage Regulation

The following are the three methods which are used to determine the voltage regulation of smooth cylindrical type Alternators

1. **Synchronous impedance / EMF method**
2. **Ampere-turn / MMF method**
3. **Potier / ZPF method**

1. Synchronous impedance / EMF method

Synchronous impedance is calculated from OCC and SCC as

$Z_s = E_0/I_{sc}$(for same I_f)

A compromised value of Z_s is normally estimated by taking the ratio of (E_0/I_{sc}) at normal field current I_f. A normal field current I_f is one which gives rated voltage V_r on open circuit.

$Z_s = V_r/I_{sc}$

Advantages:

- Simple no load tests (for obtaining OCC and SCC) are to be conducted
- Calculation procedure is much simpler

Disadvantages:

- The value of voltage regulation obtained by this method is always higher than the actual value.

EXAMPLE . *A 3-phase, 1500 kVA, star-connected, 50-Hz, 2300 V alternator has a resistance between each pair of terminals as measured by direct current is* **0.16 Ω**. *Assume that the effective resistance is 1.5 times the ohmic resistance. A field current of 70 A produces a short-circuit current equal to full-load current of 376 A in each line. The same field current produces an e.m.f. of 700 V on open circuit. Determine the synchronous reactance of the machine and its full load regulation at 0.8 power factor lagging.*

SOLUTION. $$Z_s = \frac{\text{open-circuit e.m.f. per phase}}{\text{short circuit armature current}}$$

$$= \frac{(700/\sqrt{3})}{376} = 1.075\ \Omega$$

Ohmic resistance per phase $= \dfrac{0.16}{2} = 0.08\ \Omega$

Effective resistance per phase

$$R_a = 1.5 \times 0.08 = 0.12\ \Omega$$

Synchronous reactance

$$X_s = \sqrt{Z_s^2 - R_a^2} = \sqrt{1.075^2 - 0.12^2} = 1.068\ \Omega$$

$$S_{3\Phi} = \sqrt{3}\ V_L I_L$$

$$1500 \times 10^3 = \sqrt{3} \times 2300 I_L,\ \ I_L = 376\ \text{A}$$

Rated voltage per phase

$$V_p = 2300\sqrt{3} = 1328\ \text{V}$$

Phase current $\quad I_{ap} = I_L = 376$ A

$$\mathbf{E_p} = \mathbf{V_p} + \mathbf{I_{ap}} \mathbf{Z_s}$$

Let $\mathbf{V_p}$ be taken as reference phasor :

$$\mathbf{V}_p = V_p \angle 0^\circ = 1328\angle 0^\circ\ \text{V} = 1328 + j0\ \text{V}$$

$$\mathbf{I}_{ap} = I_{ap} \angle -\cos^{-1}0.8 = 376\ \angle -36.87^\circ\ \text{A}$$

$$Z_s = R_a + jX_s = 0.12 + j1.068 = 1.075\ \angle 83.59^\circ\ \Omega$$

$$E_p = 1328 + j0 + (376\ \angle -36.87^\circ)\ (1.075\ \angle 83.59^\circ\ ) = 1328 + 404.2\angle 46.72^\circ$$

$$= 1328 + 277.1 + j\ 294.26 = 1605.1 + j294.26 = 1631\ \angle 10.39^\circ\ \text{V}$$

Percentage regulation

$$= \frac{E_p - V_p}{V_p} \times 100$$

$$= \frac{1631 - 1328}{1328} \times 100 = 22.8\%$$

Alternative method of calculating E_p

$$E_p = \sqrt{(V_p \cos\phi + I_a R_a)^2 + (V_p \sin\phi + I_a X_s)^2}$$

$$= \sqrt{(1328 \times 0.8 + 376 \times 0.12)^2 + (1328 \times 0.6 + 376 \times 1.068)^2} = 1631\ \text{V}$$

EXAMPLE . *A 3-phase, star-connected alternator is rated at 1600 kVA, 13500 V. The armature effective resistance and synchronous reactance are 1.5 Ω and 30 Ω respectively per phase. Calculate the percentage regulation for a load of 1280 kW at power factors of (a) 0.8 leading; (b) unity; (c) 0.8 lagging.*

SOLUTION. (*a*) $P_{3\Phi} = \sqrt{3}\, V_L I_L \cos\phi$

$$1280 \times 10^3 = \sqrt{3} \times 13500 I_L \times 0.8$$

$$I_L = \frac{1280 \times 10^3}{\sqrt{3} \times 13500 \times 0.8} = 68.43 \text{ A} = I_a$$

$\cos\phi = 0.8, \quad \sin\phi = 0.6$

$$R_a = 1.5\ \Omega,\ X_s = 30\ \Omega, \quad V_p = \frac{13500}{\sqrt{3}} = 7794.5 \text{ V}$$

For leading power factor

$$E_p^2 = (V_p \cos\phi + I_a R_a)^2 + (-V_p \sin\phi + I_a X_s)^2$$
$$= (7794.5 \times 0.8 + 68.43 \times 1.5)^2 + (-7794.5 \times 0.6 + 68.43 \times 30)^2$$
$$= (6338)^2 + (-2623.8)^2$$
$$E_p = 6859.6 \text{ V}$$

$$\text{Voltage regulation} = \frac{E_p - V_p}{V_p} \times 100 = \frac{6859.6 - 7794.5}{7794.5} \times 100 = -11.99\%$$

(*b*) *Unity power factor*

$$\cos\phi = 1, \sin\phi = 0$$
$$P_{3\Phi} = \sqrt{3}\, V_L I_L \cos\phi$$
$$1280 \times 10^3 = \sqrt{3} \times 13500 I_L \times 1$$
$$I_L = \frac{1280 \times 10^3}{\sqrt{3} \times 13500} = 54.74 A = I_a$$
$$E_p^2 = (V_p + I_a R_a)^2 + (I_a X_s)^2$$
$$= (7794.5 + 54.74 \times 1.5)^2 + (54.74 \times 30)^2 = (7876.6)^2 + (1642.2)^2$$
$$E_p = 8046 \text{ V}$$

Voltage regulation

$$= \frac{E_p - V_p}{V_p} \times 100 = \frac{8046 - 7794.5}{7794.5} \times 100 = 3.227\%$$

(*c*) *Power factor 0.8 lagging*

$$E_p^2 = (V_p \cos\phi + I_a R_a)^2 + (V_p \sin\phi + I_a X_s)^2$$
$$= (7794.5 \times 0.8 + 68.43 \times 1.5)^2 + (7794.5 \times 0.6 + 68.43 \times 30)^2$$
$$= (6338)^2 + (6729.6)^2$$
$$E_p = 9244.4 \text{ V}$$

$$\text{Voltage regulation} = \frac{E_p - V_p}{V_p} \times 100 = \frac{9244.4 - 7794.5}{7794.5} \times 100 = 18.6\%$$

2. *Ampere-turn / MMF method*

The ampere-turn /MMF method is the converse of the EMF method in the sense that instead of having the phasor addition of various voltage drops/EMFs, here the phasor addition of MMF required for the voltage drops are carried out. Further the effect of saturation is also taken care of.

Data required for MMF method are:

- Effective resistance per phase of the 3-phase winding R
- Open circuit characteristic (OCC) at rated speed/frequency
- Short circuit characteristic (SCC) at rated speed/frequency

Compared to the ***EMF*** method, ***MMF*** method, involves more number of complex calculation steps. Further the OCC is referred twice and SCC is referred once while predetermining the voltage regulation for each load condition. Reference of OCC takes care of saturation effect. As this method requires more effort, the final result is very close to the actual value. Hence this method is called optimistic method.

EXAMPLE . *A 3-phase, star-connected, 1000 kVA, 2000 V, 50 Hz alternator gave the following open-circuit and short-circuit test readings :*

Field current	A	10	20	25	30	40	50
O.C. voltage	V	800	1500	1760	2000	2350	2600
S.C. armature current	A		200	250	300		

The armature effective resistance per phase is 0.2 Ω.

Draw the characteristic curves and determine the full-load percentage regulation at

(a) 0.8 power factor lagging, (b) 0.8 power factor leading.

SOLUTION. The O.C.C. and S.C.C. are shown in Fig.

The phase voltage in volts are

$$\frac{800}{\sqrt{3}}, \quad \frac{1500}{\sqrt{3}}, \quad \frac{1760}{\sqrt{3}}, \quad \frac{2000}{\sqrt{3}}, \quad \frac{2350}{\sqrt{3}}, \quad \frac{2600}{\sqrt{3}};$$

or 462, 866, 1016, 1155, 1357, 1501.

Full-load phase voltage $V_p = \frac{2000}{\sqrt{3}} = 1155$ V

$$\text{kVA} = \frac{\sqrt{3}\, V_L\, I_{fl}}{1000}$$

$$1000 = \frac{\sqrt{3} \times 2000 \times I_{fl}}{1000},\; I_{fl} = I_a = 288.7 \text{ A}$$

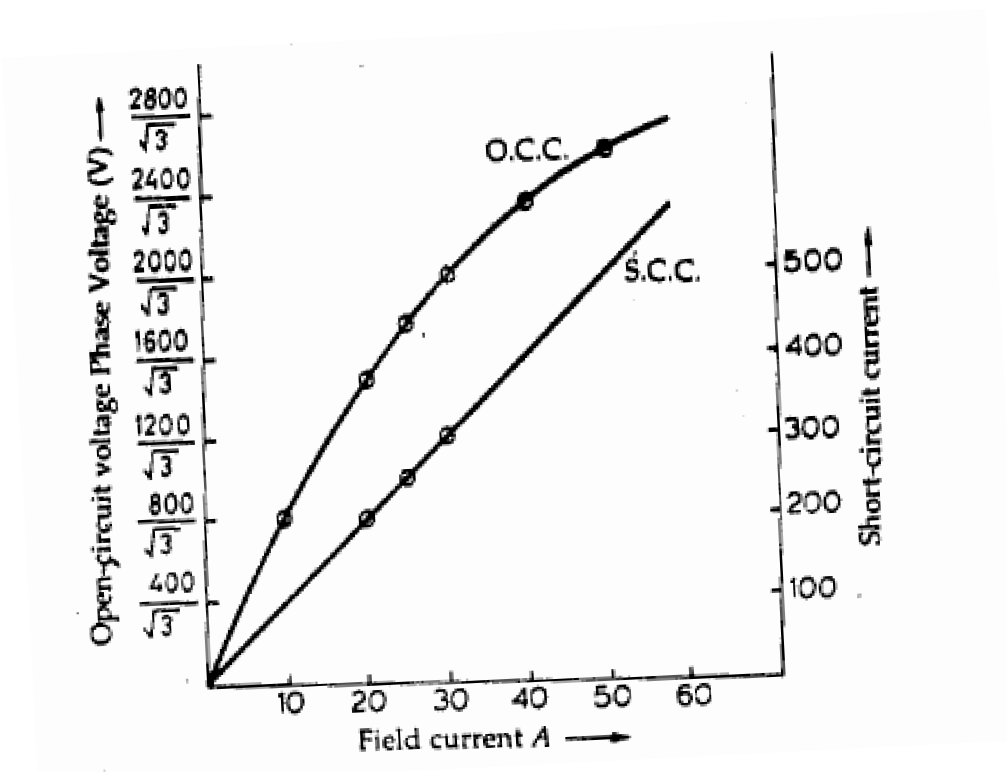
(a) Lagging power factor of 0.8

$$\begin{aligned}\mathbf{E}' = \mathbf{V}_p + \mathbf{I}_a R_a &= 1155 + (288.7 \angle -\cos^{-1} 0.8) \times 0.2 \\ &= 1155 + (57.74 \times 0.8 - j\,57.74 \times 0.6) \\ &= 1155 + 46.2 - j\,34.44 \\ &= 1201.2 - j\,34.44 = 1201.7 \angle -1.6^\circ \text{ V}\end{aligned}$$

Here $\delta = -1.6^\circ$

From the O.C.C., the field current required to produce the voltage of 1201.7 V is 32 A. Therefore $I'_f = 32$ A.

From the S.C.C., the field current required to produce full-load current of 288.7 is 29 A. Therefore $I_{f_2} = 29$ A. For $\cos\phi = 0.8$, $\phi = 36.87^\circ$



From the phasor diagram

$$\mathbf{I}_{f_2} = I_{f_2} \angle 180^\circ - \phi$$
$$= 29 \angle 180^\circ - 36.87^\circ = 29 \angle 143.13^\circ \text{ A}$$
$$= -23.2 + j\,17.4$$
$$\mathbf{I'}_f = I'_f \angle 90^\circ - \alpha$$
$$= 32 \angle 90^\circ - 1.6^\circ = 32 \angle 88.4^\circ \text{ A}$$
$$= 0.89 + j\,31.98$$
$$\mathbf{I}_f = \mathbf{I}_{f_2} + \mathbf{I'}_f$$
$$= -23.2 + j\,17.4 + 0.89 + j\,31.98$$
$$= -22.31 + j\,44.38 = 54.18 \angle 114.3^\circ \text{ A}$$

From the O.C.C., the open circuit phase voltage corresponding to the field current of 54.18 A is 1559 V.

∴ percentage voltage regulation

$$= \frac{E_{op} - V_p}{V_p} \times 100 = \frac{1559 - 1155}{1155} \times 100 = 34.97\%$$

(b) Leading power factor of 0.8

$$\mathbf{E'} = \mathbf{V}_p + \mathbf{I}_a R_a$$
$$= 1155 + (288.7 \angle + \cos^{-1} 0.8) \times 0.2$$
$$= 1155 + 46.2 + j\,34.44$$
$$= 1201.2 + j\,34.44 = 1201.7 \angle +1.6^\circ \text{ V.}$$

From, the phasor diagram

$$\mathbf{I'}_f = I'_f \angle 90^\circ + \alpha = 32 \angle 90^\circ + 1.6 = 32 \angle 91.6^\circ \text{ A}$$
$$= -0.89 + j\,31.98 \text{ A}$$
$$\mathbf{I}_{f_2} = I_{f_2} \angle 180^\circ + \phi$$
$$= 29 \angle 180^\circ + 36.87^{\circ\circ} = 29 \angle 216.87^\circ \text{ A}$$
$$= -23.2 - j\,17.4 \text{ A}$$
$$\mathbf{I}_f = \mathbf{I}_{f_2} + \mathbf{I'}_f = -0.89 + j\,31.98 - 23.2 - j\,17.4$$
$$= -24.09 + j\,14.58 \text{ A}$$
$$= 28.15 \angle 31.18^\circ \text{ A}$$

From the O.C.C., the open-circuit phase voltage corresponding to a field current of 28.15 A is 1098 V.

Percentage voltage regulation

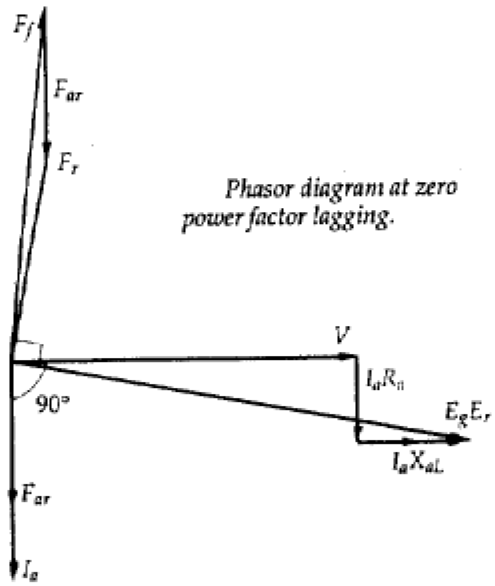
$$= \frac{E_{op} - V_E}{V_p} \times 100 = \frac{1098 - 1155}{1155} \times 100 = -5.02\%$$

ZERO-POWER FACTOR CHARACTERISTIC (ZPFC)

The **zero-power factor characteristic (ZPFC)** of an alternator is a curve of the armature terminal voltage per phase plotted against the field current obtained by operating the machine with constant rated armature current at synchronous speed and zero lagging power factor. The ZPFC is sometimes called **Potier Characteristic** after its originator.

For maintaining very low power factor, alternator is loaded by means of reactors or alternatively by an underexcited synchronous motor. The shape of ZPFC is very much like that of the O.C.C. displaced downwards and to the right.

The phasor diagram corresponding to zero-power factor lagging load



Phasor diagram at zero power factor lagging.

the terminal phase voltage **V** is taken as the reference **phasor.** At zero power factor lagging, the armature current $\mathbf{I}_a$ lags behind **V by 90°. Draw** $\mathbf{I}_a R_a$ parallel to $\mathbf{I}_a$ and $\mathbf{I}_a X_{aL}$ perpendicular to $\mathbf{I}_a$.

$$\mathbf{V} + \mathbf{I}_a R_a + \mathbf{I}_a X_{aL} = \mathbf{E}_g$$

$\mathbf{E}_g$ is the generated voltage per phase.

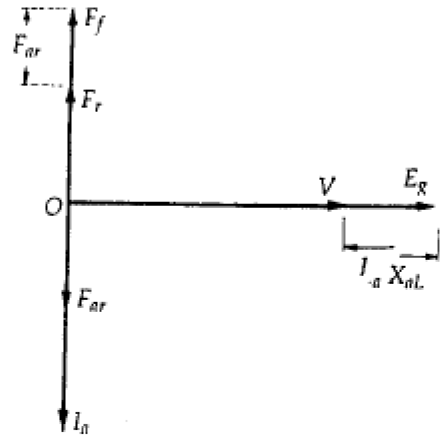
$\mathbf{F}_{ar}$ = armature reaction mmf. It is in phase with $\mathbf{I}_a$.

$\mathbf{F}_f$ = mmf of the mainfield winding (field mmf)

$\mathbf{F}_r$ = resultant mmf

The field mmf $\mathbf{F}_f$ is obtained by subtracting $\mathbf{F}_{ar}$ form $\mathbf{F}_r$, so that $\mathbf{F}_r = \mathbf{F}_f + \mathbf{F}_{ar}$

If the armature resistance R_a is neglected, the resulting phasor **diagram**



Phasor diagram at ZPF lagging with R_a neglected.

it is seen that the terminal phase voltage **V**, the **reactance** voltage drop $\mathbf{I_a X_{aL}}$ and the generated voltage $\mathbf{E_g}$ are all in phase. **Therefore V is** practically equal to the numerical (arithmetical) difference between $\mathbf{E_g}$ **and** $\mathbf{I_a X_{aL}}$.

$$V = E_g - I_a X_{aL} \qquad (1)$$

Also the three mmf phasors $\mathbf{F_f}$, $\mathbf{F_r}$ and $\mathbf{F_{ar}}$ are in phase. Their **magnitudes are** related by the equation

$$F_f = F_r + F_{ar} \qquad (2)$$

The arithmetical relations given in Eqs. (1) and (2) **form the basis** for the Potier triangle.

Equation (2) can be converted into its equivalent field-current **form by** dividing its both sides by T_f, the effective number of turns per pole **on the rotor** field.

$$\therefore \qquad \frac{F_f}{T_f} = \frac{F_r}{T_f} + \frac{F_{ar}}{T_f}$$

or

$$I_f = I_r + I_{ar} \qquad (3)$$

POTIER TRIANGLE

In Fig.1 , consider a point b on the ZPFC conesponding to rated terminal **voltage** V and a field current of $OM = I_f = \frac{F_f}{T_f}$. If, for this condition of operation, the **armature** reaction mmf has a value expressed in equivalent field current of $LM \left(= I_{ar} = \frac{F_{ar}}{T_f} \right)$, then the equivalent field current of the resultant mmf would be $OL \left(= I_r = \frac{F_r}{T_f} \right)$

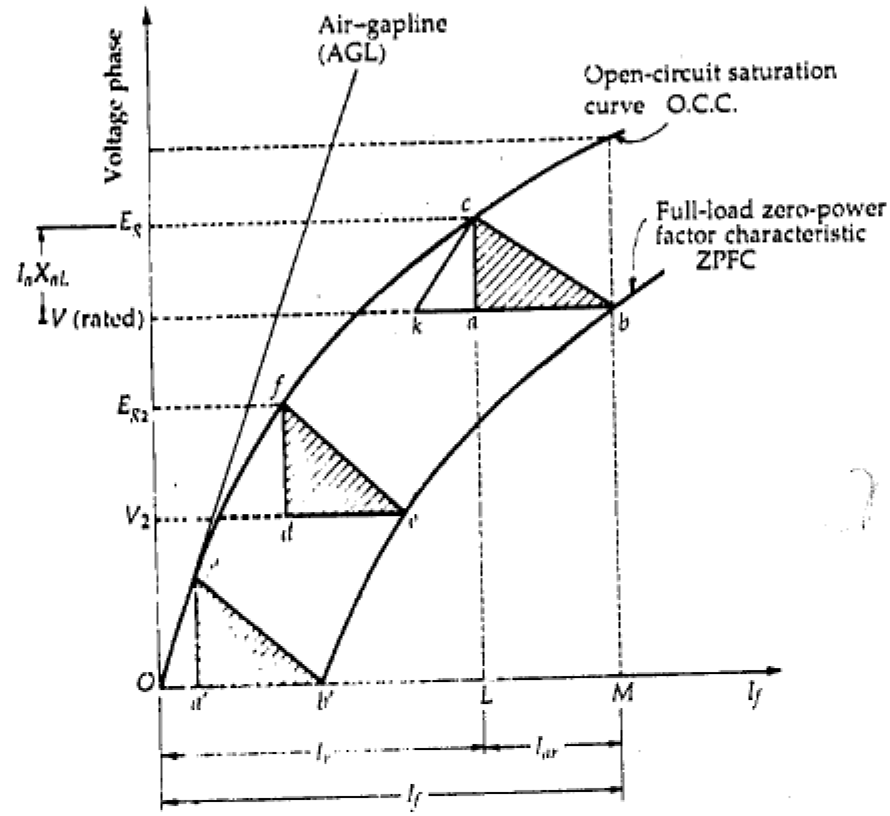


Fig. 1Potier triangle

This field current *OL* would result in a generated voltage E_g (= *Lc*) from the no load saturation curve. Since, for lagging zero-power-factor operation

$$\mathbf{E}_g = \mathbf{V} + \mathbf{I}_a \mathbf{X}_{aL}$$

the vertical distance *ac* must be equal to the leakage-reactance voltage drop $I_a X_{aL}$ where I_a is the rated armature current.

$$\therefore \quad X_{aL} = \frac{\text{voltage } ac \text{ per phase}}{\text{rated armature current}}$$

The triangle formed by the vertices *a*, *b*, *c* is called the *Potier triangle*.

The ZPFC may be used in conjunction with the O.C.C. to find the armature reaction mmf and the approximate leakage reactance voltage of the machine (Fig. 1). The construction is as follows :

1. Take a point *b* on the ZPFC preferably well upon the knee of the curve.
2. Draw *bk* equal to *b′ O* (*b′* is the point for zero voltage, full-load current). That is, *Ob′* is the short-circuit excitation, F_{SC}.
3. Through *k* draw *kc* parallel to *Oc′* to meet O.C.C. in *c*.
4. Drop the perpendicular *ca* on to *bk*.

5. Then, to scale, ca is the leakage reactance drop $I_a X_{aL}$ and ab is the armature reaction mmf F_{aR} or field current I_{faR} equivalent to armature reaction mmf at rated current.

The effect of field leakage flux in combination with the armature leakage flux gives rise to an equivalent leakage reactance X_p, known as the **Potier reactance**. It is greater than the armature leakage reactance.

Also, Potier reactance $X_p = \dfrac{\text{voltage drop per phase } (= ac)}{(\text{ZPF rated armature current per phase } I_a)}$

For cylindrical-rotor machines, Potier reactance X_p is approximately equal to leakage reactance X_{aL}. In salient-pole machines, X_p may be as large as 3 times X_{aL}.

PROCEDURE TO OBTAIN THE REGULATION BY ZERO-POWER FACTOR METHOD

The following procedure is used to obtain regulation by the zero-power factor method :

The phasor diagram for lagging power cos ϕ is drawn as shown in Fig. 2

In the phasor diagram :

OA = **V** = terminal phase voltage at full load. It is taken as reference phasor and drawn horizontally.

OB = $\mathbf{I}_a$ = full-load current lagging behind **V** by an angle ϕ, cos ϕ is the power factor of the load

AC = voltage drop $\mathbf{I}_a R_a$ in the armature resistance (if R_a is given). It is drawn parallel to $\mathbf{I}_a$ (OB)

CD = $\mathbf{I}_a X_{aL}$ = leakage reactance voltage drop. It is perpendicular to AC.

Join OD. It represents the generated e.m.f. E_g

Find the field excitation current I_r corresponding to this generated emf E_g from the O.C.C.

Draw OG (equal to I_r perpendicular to OD).

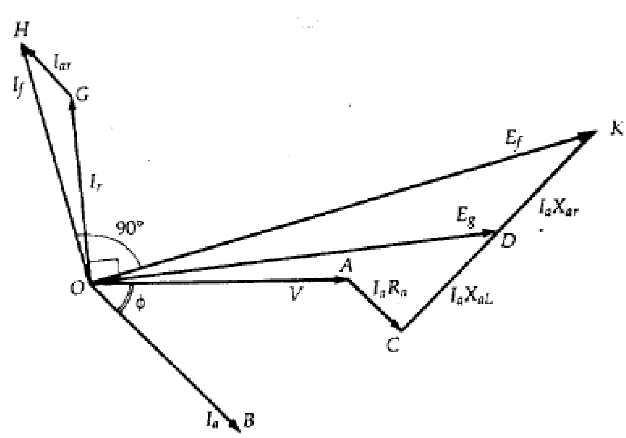


Fig.2

Draw GH parallel to load current OB $(= I_a)$ to represent excitation (field current) equivalent to full-load armature reaction I_{ar}. OH gives the total field current I_f.

If the load is thrown off, then terminal voltage will be equal to generated emf, corresponding to field excitation OH.

Determine the emf E_f $(= OK)$ corresponding to field excitation OH from the O.C.C. Phasor OK will lag behind phasor OH by 90°. DK represents the *voltage drop due to armature reaction.*

Now voltage regulation is obtained from the relation :

Percentage voltage regulation $= \dfrac{E_f - V}{V} \times 100\%$

EXAMPLE . *A 5000 kVA, 6600 V, 3-phase, star-connected alternator has a resistance of* 0.75 Ω *per phase. Estimate by zero power factor method the regulation for a load of 500 A at power factor (a) unity, (b) 0.9 leading, (c) 0.71 lagging, from the following open-circuit and full load, zero power factor curves :*

Field current, A	*Open-circuit terminal voltage, V*	*Saturation curve, zero p.f., V*
32	3100	0
50	4900	1850
75	6600	4250
100	7500	5800
140	8300	7000

SOLUTION. The O.C.C. and the ZPFC are plotted as drown in Fig. 3

Draw a horizontal line at rated line voltage of 6600 V to meet the ZPFC at b. On this line take $bk = Ob' = 32$ A.

Ob' is the field current required to circulate full-load current on S.C.

Draw a line kc parallel to Oc' (the initial slope of the O.C.C.) to meet the O.C.C. at c. Draw the perpendicular ca on the line kb. Hence abc is the Potier's triangle. In this triangle,

ab = field current required to overcome armature reaction on load

$= I_{ar} = 25$ A

and $ac = 900$ V (line-to-line) $= \dfrac{900°}{\sqrt{3}}$ V per phase

∴ Leakage impedance voltage drop

$I_a X_L = \dfrac{900}{\sqrt{3}}$ V per phase

$= 579.6$ V, $I_a = 500$ A

$X_L = \dfrac{900}{\sqrt{3} \times 500} = 1.039\ \Omega$

Taking I_a as reference phasor $\mathbf{I}_a = I_a \angle 0° = 500 \angle 0°$ A $= 500 + j\,0$

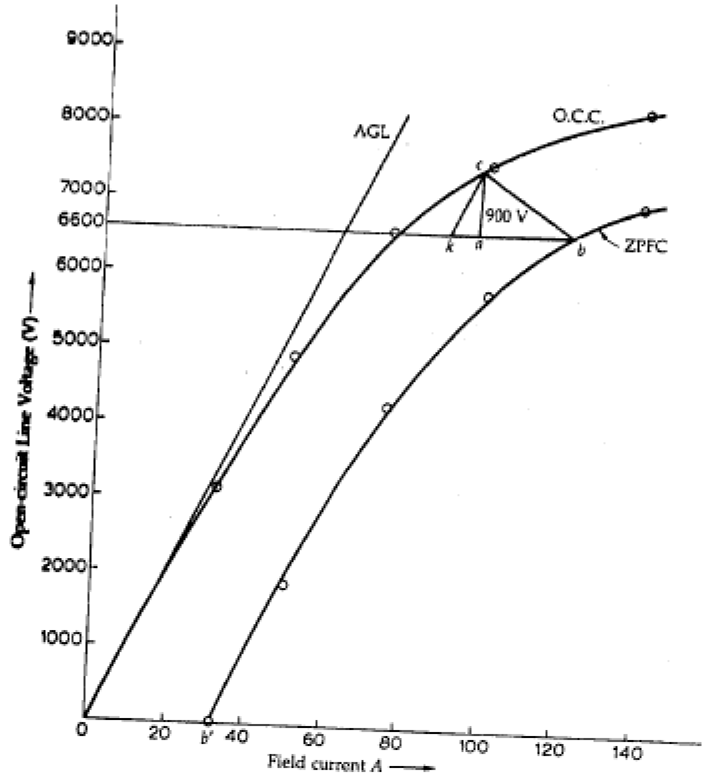


Fig. 3.

(a) Unity power factor

$$\mathbf{V}_p = V_p \angle 0^\circ = \frac{6600}{\sqrt{3}} \angle 0^\circ = 3810.6$$

$$\mathbf{E}_{gp} = \mathbf{V}_p + \mathbf{I}_a\, \mathbf{Z}_L = V_p + I_a\,(R_a + j\,X_L) = V_p + I_a\,R_a + j\,I_a\,X_L$$

$$= 3810.6 + 500 \times 0.075 + j\,519.6$$

$$= 3848.1 + j\,519.6 = 3883 \angle 7.69^\circ \text{ V}$$

$$E_{gl} = \sqrt{3} \times 3883 = 6725 \text{ V}$$

From the O.C.C., the field current corresponding to the line voltage of 6725 V is 78 A.

This current leads $\mathbf{E}_g$ by 90°

$\therefore$ $\mathbf{I}_r = I_r \angle 90^\circ + 7.69^\circ = 78 \angle 97.69^\circ$ A

The current $\mathbf{I}_{ar}$ is in phase with $\mathbf{I}_a$.

$\therefore$ $\mathbf{I}_{ar} = I_{ar} \angle 0^\circ = 25 \angle 0^\circ$ A

We have $\mathbf{I}_f + \mathbf{I}_{ar} = \mathbf{I}_r$

$$\mathbf{I}_f = \mathbf{I}_r - \mathbf{I}_{ar}$$

$$= 78 \angle 97.69^\circ - 25 \angle 0^\circ$$

$$= -10.44 + j\,77.3 - 25$$

$$= -35.44 + j\,77.3 = 85 \angle 114.6^\circ \text{ A}$$

From the O.C.C., corresponding to a field current of 85 A, the voltage $E_{fl} = 7000$ V (line-line)

$$E_{fp} = \frac{7000}{\sqrt{3}} \text{ V} = 4041.6 \text{ V}$$

$$\therefore \quad \text{voltage regulation } = \frac{E_{fp} - V_p}{V_p} \times 100$$

$$= \frac{4041.6 - 3810.6}{3810.6} \times 100 = 6.06\%$$

(*b*) 0.9 p.f. leading

$$\cos\phi = 0.9, \qquad \phi = 25.84°, \qquad \mathbf{I}_a = I_a \angle 0°$$

$$\mathbf{V}_p = 3810.6 \angle -25.84° = 3429.6 - j\,1660.9$$

$$\mathbf{E}_{gp} = \mathbf{V}_p + \mathbf{I}_a \mathbf{Z}_L = V_p + I_a (R_a + j X_L)$$

$$= (3429.6 - j\,1660.9) + (500 \times 0.075 + j\,519.6)$$

$$= 3467.1 - j\,1141.3 = 3650 \angle -18.2° \text{ V}$$

The corresponding line voltage

$$E_{gl} = \sqrt{3}\, E_{gp} = \sqrt{3} \times 3650 = 6321.8 \text{ V}$$

From the O.C.C., the field current corresponding to the line voltage of 6321.8 V is 71 A. This current leads E_g by 90°

$$\therefore \quad \mathbf{I}_r = I_r \angle 90° - 18.2° = 71 \angle 71.8° \text{ A}$$

The current $\mathbf{I}_{ar}$ is in phase with $\mathbf{I}_a$.

$$\mathbf{I}_{ar} = I_{ar} \angle 0° = 25 \angle 0° \text{ A}$$

We have $\mathbf{I}_f + \mathbf{I}_{ar} = \mathbf{I}_r$

$$\mathbf{I}_f = \mathbf{I}_r - \mathbf{I}_{ar}$$

$$= 71 \angle 71.8° - 25 \angle 0°$$

$$= 22.2 + j\,67.5 - 25$$

$$= -2.8 + j\,67.5$$

$$= 67.6 \angle 92.38° \text{ A}$$

From the O.C.C., corresponding to a field current of 67.6 A, the voltage $E_{fl} = 6000$ V (line-to-line)

Corresponding phase voltage $E_{fp} = \frac{6000}{\sqrt{3}} = 3464$ V

$$\therefore \quad \text{voltage regulation } = \frac{E_{fp} - V_p}{V_p} \times 100$$

$$= \frac{3464 - 3810.6}{3810.6} \times 100 = -9.1\%$$

(c) 0.71 p.f. lagging

$$\cos\phi = 0.71, \quad \phi = 44.77^\circ$$

$$\mathbf{I}_a = I_a \angle 0^\circ = 500 \angle 0^\circ \text{ A}$$

$$\mathbf{V}_p = V_p \angle +\phi^\circ = 3810.6 \angle 44.77^\circ = 2705.3 + j\,2683.7$$

$$\mathbf{E}_{gp} = \mathbf{V}_p + \mathbf{I}_a \mathbf{Z}_L = \mathbf{V}_p + I_a (R_a + j\,X_L)$$

$$= \mathbf{V}_p + I_a R_a + j\,I_a X_L$$

$$= (2705.3 + j\,2683.7) + 500 \times 0.075 + j\,519.6$$

$$= 2742.8 + j\,3203.3 = 4217 \angle 49.4^\circ \text{ V}$$

$$E_{gl} = \sqrt{3}\,E_{gp} = \sqrt{3} \times 4217 = 7303.8 \text{ V}$$

From the O.C.C., the field current corresponding to the line voltage of 07.8 V is 95 A.

$$\therefore \quad \mathbf{I}_r = 95 \angle 90 + 49.4^\circ = 95 \angle 139.4^\circ \text{ A}$$

$$\mathbf{I}_{ar} = 25 \angle 0^\circ \text{ A}$$

$$\mathbf{I}_f = \mathbf{I}_r - \mathbf{I}_{ar} = 95 \angle 139.4^\circ - 25 \angle 0^\circ$$

$$= -72.1 + j\,61.8 - 25 = -97.1 + j\,61.8 = 115 \angle 147.5^\circ \text{ A}$$

From the O.C.C., corresponding to a field current of 115 A, $E_{fl} = 7900$ V

$$\therefore \quad \text{voltage regulation} = \frac{E_{fl} - V_t}{V_t} \times 100\%$$

$$= \frac{7900 - 6600}{6600} \times 100 = 19.7\%$$

H.W . *The table gives data for open-circuit and load zero power factor tests on a 6-pole, 440 V, 50 Hz, 3-phase star-connected alternator. The effective ohmic resistance between any two terminals of the armature is 0.3 Ω.*

Field current (A)	2	4	6	7	8	10	12	14	16	18
O.C.terminal voltage (V)	156	288	396	440	474	530	568	592	—	—
S.C.line current (A)	11	22	34	40	46	57	69	80		
Zero p.f. terminal voltage (V)	—	—	—	0	80	206	314	398	460	504

Find the regulation at full-load current of 40 A ai 0.8 power factor lagging using

(a) synchronous impedance method,

(b) mmf method,

(c) Potier-triangle method

Power flow transfer equations for a synchronous generator

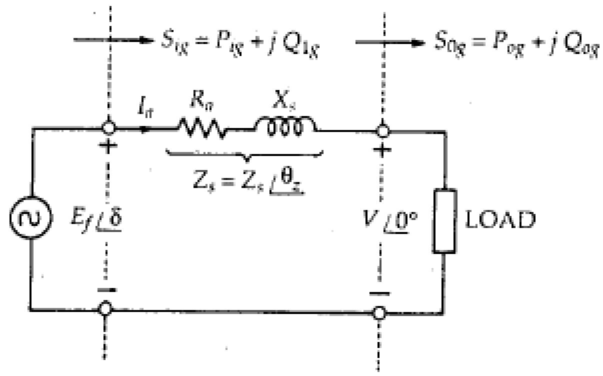


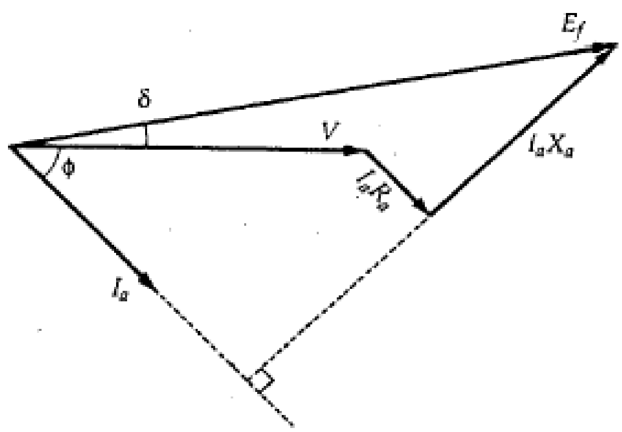
Fig.1 *Circuit model of cylindrical rotor synchronous generator.*

Let V = terminal voltage per phase

E_f = excitation voltage per phase

I_a = armature current

δ = phase angle between $\mathbf{E_f}$ and $\mathbf{V}$



The phasor diagram at lagging power factor

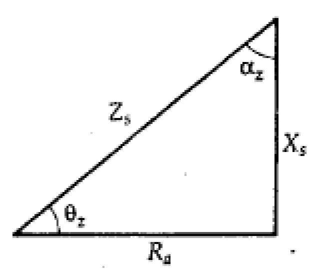
For a synchronous generator $\mathbf{E}_f$ leads $\mathbf{V}$ by angle δ.

$$\mathbf{V} = V \angle 0,\ \mathbf{E}_f = E_f \angle \delta$$

The synchronous impedance is given by

$$\mathbf{Z}_s = R_a + j\,X_s = Z_s \angle \theta_z$$

The impedance triangle is shown in Fig. 1.



$$\theta_z = \tan^{-1}\frac{X_s}{R_a}$$

$$\alpha_z = 90° - \theta_z = \tan^{-1}\frac{R_a}{X_s}$$

Let the subscripts *i*, *o*, *g* denote input, output, generator.
By *KVL* in the network of Fig.1.

$$\mathbf{E}_f = \mathbf{V} + \mathbf{Z}_s \mathbf{I}_a$$

$$\mathbf{I}_a = \frac{\mathbf{E}_f - \mathbf{V}}{\mathbf{Z}_s}$$

Complex Power Output of the Generator Per Phase (S_{og})

$$S_{og} = P_{og} + j\,Q_{og} = \mathbf{V}\,I_a$$

$$= \mathbf{V}\left(\frac{\mathbf{E}_f - \mathbf{V}}{\mathbf{Z}_s}\right)^*$$

$$= V\angle 0^\circ \left(\frac{E_f\angle\delta - V\angle 0^\circ}{Z_s\angle\theta_z}\right)$$

$$= V\angle 0^\circ \left(\frac{E_f}{Z_s}\angle\delta - \theta_z - \frac{V}{Z_s}\angle -\theta_z\right)$$

$$= \frac{V E_f}{Z_s}\angle\theta_z - \delta - \frac{V^2}{Z_f}\angle\theta_z$$

$$\therefore \quad P_{og} + j\,Q_{og} = \frac{V E_f}{Z_s}\cos(\theta_z - \delta) + j\,\frac{V E_f}{Z_s}\sin(\theta_z - \delta) - \frac{V^2}{Z_s}(\cos\theta_z + j\sin\theta_z)$$

Real Output Power Per Phase of the Generator (P_{og})

$$P_{og} = \frac{V E_f}{Z_s}\cos(\theta_z - \delta) - \frac{V^2}{Z_s}\cos\theta_z$$

Since $\cos\theta_z = \dfrac{R_a}{Z_s}$

$$P_{og} = \frac{V E_f}{Z_s}\cos(\theta_z - \delta) - \frac{V^2}{Z_s^2}R_a$$

But $\theta_z = 90^\circ - \alpha_z$

$$\therefore \quad P_{og} = \frac{V E_f}{Z_s}\cos(90^\circ - \overline{\delta + \alpha_z}) - \frac{V^2}{Z_s^2}R_a$$

or

$$P_{og} = \frac{V E_f}{Z_s}\sin(\delta + \alpha_z) - \frac{V^2}{Z_s^2}R_a$$

P_{og} is also called the electrical power developed by the generator.

Reactive Output Power Per Phase of the Generator (Q_{og})

$$Q_{og} = \frac{V E_f}{Z_s} \sin(\theta_z - \delta) - \frac{V_2}{Z_s} \sin\theta_z$$

Since $\sin\theta_z = \frac{X_s}{Z_s}$

$$Q_{og} = \frac{V E_f}{Z_s} \sin(\theta_z - \delta) - \frac{V_2}{Z_s^2} X_s$$

But $\theta_z = 90° - \alpha_z$

$$\therefore \quad Q_{og} = \frac{V E_f}{Z_s} \sin(90° - \overline{\delta + \alpha_z}) - \frac{V^2}{Z_s^2} X_s$$

or

$$Q_{og} = \frac{V E_f}{Z_s} \cos(\delta + \alpha_z) - \frac{V_2}{Z_s^2}$$

Complex power input to generator per phase (S_{ig})

$$S_{ig} = P_{ig} + j\,Q_{ig} = \mathbf{E}_f \mathbf{I}_a^*$$

$$= \mathbf{E}_f \angle\delta \left(\frac{E_f}{Z_s} \angle\theta_z - \delta - \frac{V}{Z_s} \angle\theta_z \right)$$

or

$$P_i + j\,Q_{ig} = \frac{E_f^2}{Z_s} \angle\theta_z - \frac{V E_f}{Z_s} \angle\theta_z + \delta$$

$$\therefore\ P_{ig} + j\,Q_{ig} = \frac{E_f^2}{Z_s} \cos\theta_z + j\frac{E_f^2}{Z_s} \sin\theta_z - \left[\frac{V E_f}{Z_s} \cos(\theta_z + \delta) + j\frac{V E_f}{Z_s} \sin(\theta_z + \delta) \right]$$

Real power input to generator per phase (P_{ia})

$$P_{ig} = \frac{E_f^2}{Z_s} \cos\theta_z - \frac{V E_f}{Z_s} \cos(\theta_z + \delta)$$

Since $\cos\theta_z = \frac{R_a}{Z_s}$

$$P_{ig} = \frac{E_f^2}{Z_s} R_a - \frac{V E_f}{Z_s} \cos(\theta_z + \delta)$$

But $\theta_z = 90° - \alpha_z$

$$\therefore \quad P_{ig} = \frac{E_f^2}{Z_s} R_a - \frac{V E_f}{Z_s} \cos(90° + \overline{\delta - \alpha_z})$$

or

$$P_{ig} = \frac{E_f^2}{Z_s} R_a + \frac{V E_f}{Z_s} \sin(\delta - \alpha_z)$$

Reactive power input to generator per phase (Q_{ig})

$$Q_{ig} = \frac{E_f^2}{Z_s} \sin \theta_z - \frac{V E_f}{Z_s} \sin (\theta_z + \delta)$$

But $\sin \theta_z = \dfrac{X_s}{Z_s}$

$$\therefore \qquad Q_{ig} = \frac{E_f^2}{Z_s^2} X_s - \frac{V E_f}{Z_s} \sin (\theta_z + \delta)$$

Since $\theta_z = 90° - \alpha_z$

$\sin (\theta_z + \delta) = \sin (90° + \delta - \alpha_z) = \cos (\delta - \alpha_z)$

$$\therefore \qquad Q_{ig} = \frac{E_f^2}{Z_s^2} X_s - \frac{V E_f}{Z_s} \cos (\delta - \alpha_z)$$

Mechanical power input to generator = P_{ig} + rotational losses

The rotational losses include friction, windage and core losses.

Maximum power output of the generator per phase $P_{og\,(max)}$

For maximum power output of the generator

$$\frac{d P_{og}}{d \delta} = 0 \quad \text{and} \quad \frac{d^2 P_{og}}{d \delta^2} < 0$$

Differentiating Eq. $P_{og} = \dfrac{V E_f}{Z_s} \sin (\delta + \alpha_z) - \dfrac{V^2}{Z_s^2} R_a$ w.r.t. δ and equating it to zero we get

$$\frac{d}{d \delta}\left[\frac{V E_f}{Z_s} \sin (\delta + \alpha_z) - \frac{V^2}{Z_s^2} R_a \right] = 0$$

Since V, E_f, Z_s and R_a are constants

$$\frac{V E_f}{Z_s} \cos (\delta + \alpha_z) = 0$$

or

$$\cos (\delta + \alpha_z) = 0$$

$$\delta + \alpha_z = 90°$$

$$\delta = 90° - \alpha_z = \theta_z$$

Thus for maximum power output of generator

load angle δ = impedance angle θ_z

$$P_{og\,(max)} = \frac{V E_f}{Z_s} - \frac{V^2}{Z_s^2} R_a$$

This occus at $\delta = \theta_z$.

Maximum power input to generator per phase $P_{ig(max)}$

For maximum power input to the generator $\frac{d P_{ig}}{d\delta} = 0$ and $\frac{d^2 P_{ig}}{d\delta^2} < 0$

Differentiating Eq $P_{ig} = \frac{E_f^2}{Z_s} R_a + \frac{V E_f}{Z_s} \sin(\delta - \alpha_z)$ w.r.t. δ and equating it to zero we get

$$\frac{d}{d\delta}\left[\frac{E_f^2}{Z_s^2} R_a + \frac{V E_f}{Z_s} \sin(\delta - \alpha_z)\right] = 0$$

$$\frac{V E_f}{Z_s} \cos(\delta - \alpha_z) = 0$$

$$\delta - \alpha_z = 90^\circ$$

$$\delta = 90^\circ + \alpha_z = 90^\circ + 90^\circ - \theta_z = 180^\circ - \theta_z$$

Thus for maximum power input to the generator

load angle $\delta = 180^\circ -$ impedance angle θ_z

$$P_{i_{g(max)}} = \frac{E_f^2}{Z_s^2} R_a + \frac{V E_f}{Z_s}$$

Power flow equations for a generator with armature resistance neglected

In practical polyphase synchronous machines $R_a < X_s$ and R_a can be neglected in the power flow equations.

When armature resistance R_a is neglected $Z_s = X_s$, $\alpha_z = 0$

$$P_{og} = \frac{V E_f}{X_s} \sin\delta$$

$$Q_{og} = \frac{V E_f}{X_s} \cos\delta - \frac{V^2}{X_s}$$

$$P_{ig} = \frac{V E_f}{X_s} \sin\delta = P_{og}$$

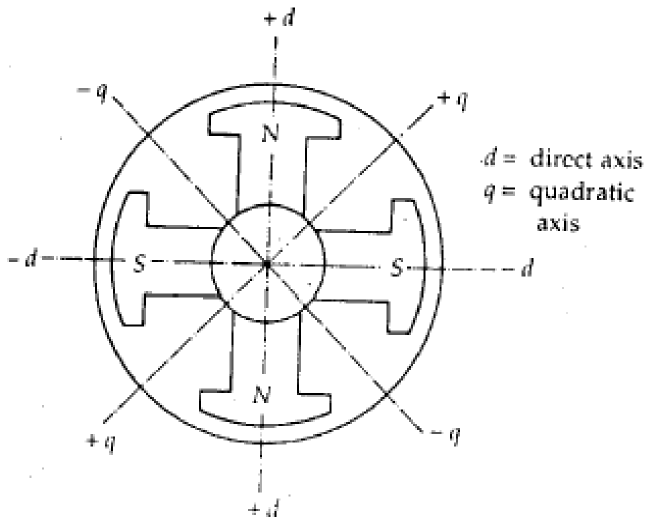
$$Q_{ig} = \frac{E_f^2}{X_s} - \frac{V E_f}{X_s} \cos\delta$$

Also, $$P_{og\,(max)} = \frac{V E_f}{X_s} = P_{ig\,(max)}$$

Salient-Pole Synchronous Generator — Two-Reaction Theory

MAGNETIC AXES OF THE ROTOR

The axis of symmetry of the north magnetic poles of the rotor is called the ***direct axis*** or ***d*-axis.** *The axis of symmetry of the south magnetic poles is the* ***negative d-axis.*** *The axis of symmetry halfway between adjacent north and south poles is called the* ***'quadrature axis'*** or ***q-axis.***



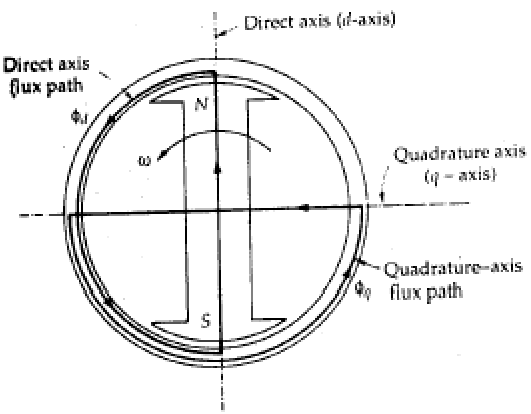
TWO-REACTION THEORY

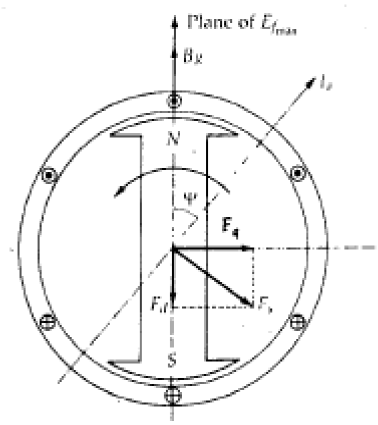
Two-reaction theory was proposed by Andre Blondel. The theory proposes to resolve the given armature mmfs into two mutually perpendicular components, with one located *along the axis* of the rotor salient pole. It is known as the **direct-axis (or *d*-axis) component**. The other component is located *perpendicular* to the axis of the rotor salient pole. It is known as the **quadrature-axis (or *q*-axis) component.** The *d*-axis component of the armature mmf F_a is denoted by F_d and the *q*-axis component by F_q. The component F_d is either magnetizing or demagnetizing. The component F_q results in a cross-megnetizing effect.

$$F_d = F_a \sin \psi$$

$$F_q = F_a \cos \psi$$

Salient-pole generators, such as hydroelectric generators, have armature inductances that are a function of rotor position, making analysis one step more complicated. The key to analysis of such machines is to separate mmf and flux into two orthogonal components. The two components are aligned with the direct axis and the quadrature axis of the machine. The direct axis is aligned with the field winding, while the quadrature axis leads the direct by 90°.





The direct and quadrature axis stator fluxes produce voltages in the stator winding by armature reaction.

Let E_{ad} = direct-axis component of armature reaction voltage

E_{aq} = quadrature-axis component of armature reaction voltage

Since each armature reaction voltage is directly proportional to its stator current and lags behind the stator current by 90°, therefore armature reaction voltages can be written as

$$\mathbf{E}_{ad} = -j\, X_{ad}\, \mathbf{I}_d \qquad (1)$$

$$\mathbf{E}_{aq} = -j\, X_{aq}\, \mathbf{I}_q \qquad (2)$$

where X_{ad} = armature reaction reactance in the direct axis per phase

X_{aq} = armature reaction reactance in the quadrature axis per phase

The total voltage induced in the stator is the sum of emf induced by the field excitation and these two emfs. That is

$$\mathbf{E}' = \mathbf{E}_f + \mathbf{E}_{ad} + \mathbf{E}_{aq} \qquad (3)$$

or

$$\mathbf{E}' = \mathbf{E}_f - j\, X_{ad}\, \mathbf{I}_d - j\, X_{aq}\, \mathbf{I}_q \qquad (4)$$

The voltage E' is equal to the terminal voltage **V** plus the voltage drops in the resistance and leakage reactance of the armature, so that

$$E' = V + R_a I_a + j X_l I_a \tag{5}$$

The armature current I_a is split into two components, one in phase with the excitation voltage E_f and the other in phase quadrature to it.

If I_q = the q-axis component of I_a in phase with E_f

I_d = the d-axis component of I_a lagging E_f by 90°

$$\therefore \quad I_a = I_d + I_q \tag{6}$$

Combination of Eqs. (4) and (5) gives

$$E_f = V + R_a I_a + j X_l I_a + j X_{ad} I_d + j X_{aq} I_q \tag{7}$$

Combination of Eqs. (3.39.6) and (3.39.7) gives

$$E_f = V + R_a (I_d + I_q) + j X_l (I_d + I_q) + j X_{ad} I_d + j X_{aq} I_q$$

$$= V + R_a (I_d + I_q) + j (X_l + X_{ad}) I_d + j (X_l + X_{aq}) I_q \tag{8}$$

Let

$$\boxed{X_d = X_l + X_{ad}} \tag{9}$$

$$\boxed{X_q = X_l + X_{aq}} \tag{10}$$

*The reactance X_d is called the **direct-axis synchronous reactance** and the reactance X_q is called the **quadrature-axis synchronous reactance**.*

Combination of Eqs. (8), (9) and (10) gives

$$E_f = V + R_a I_d + R_a I_q + j X_d I_d + j X_q I_q \tag{11}$$

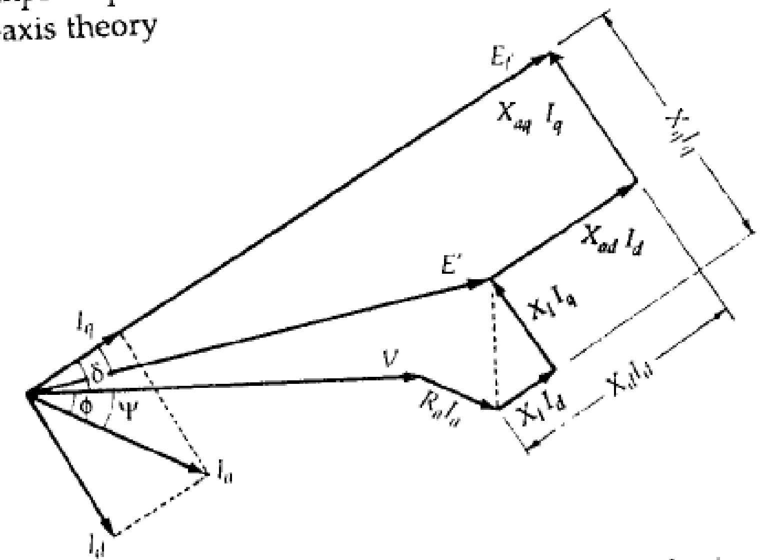
or

$$E_f = V + R_a I_a + j X_d I_d + j X_q I_q \tag{12}$$

Equation (11) is the final form of the *voltage equation for a salient-pole synchronous generator.*

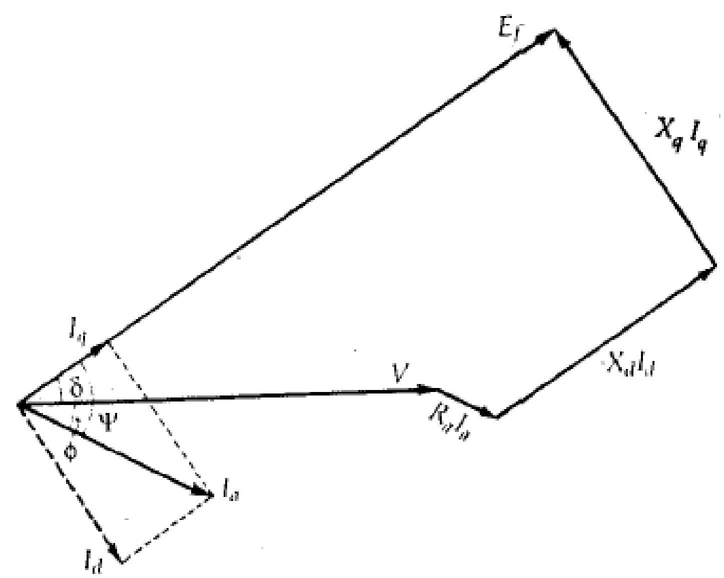
Phasor diagram

The complete phasor diagram of a salient-pole synchronous generator based on two-axis theory



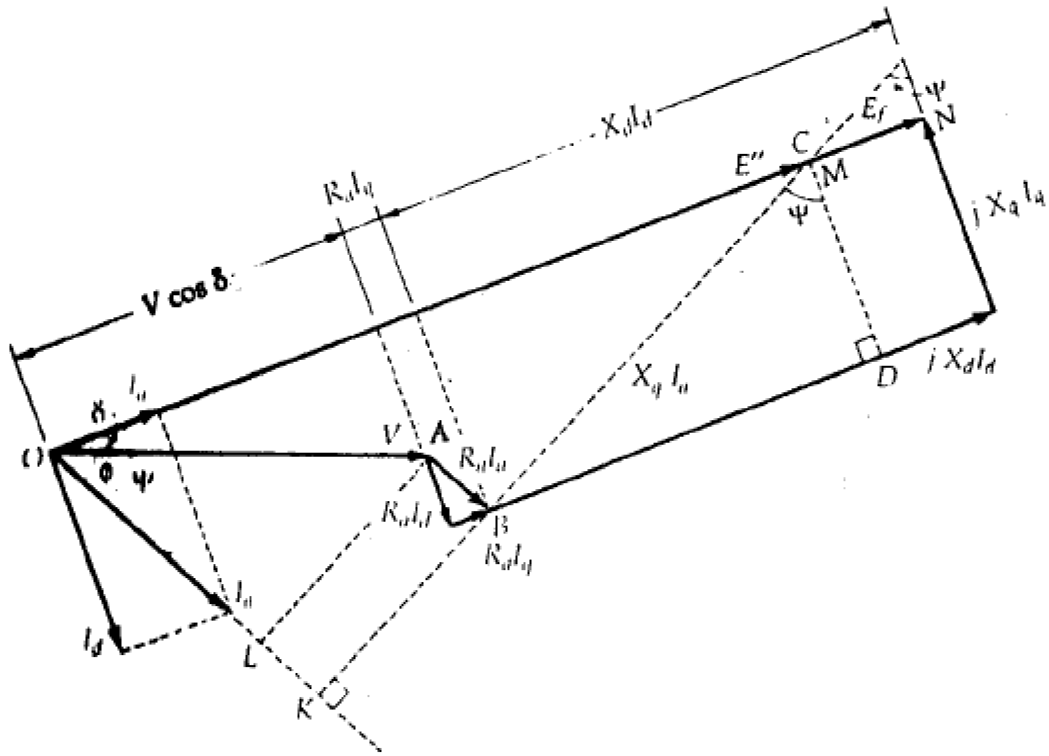
Phasor diagram of a salient-pole synchronous generator at lagging power factor.

The simplified phasor diagram based on Eq. (11)



Simplified phasor diagram of a salient-pole synchronous generator at lagging power factor.

Determination of δ from the phasor diagram



$$\mathbf{E''} = \mathbf{OC} = \mathbf{OA} + \mathbf{AB} + \mathbf{BC}$$
$$= V + R_a \mathbf{I}_a + j X_q \mathbf{I}_a$$
$$= V + (R_a + j X_q) \mathbf{I}_a$$

For lagging power factor cos ϕ .

$$\mathbf{I}_a = I_a \angle -\phi = I_a \cos\phi - j I_a \sin\phi$$
$$\mathbf{E''} = \mathbf{OC} = V + (R_a + j X_q)(I_a \cos\phi - j I_a \sin\phi)$$
$$= (V + I_a R_a \cos\phi + I_a X_q \sin\phi) + j(X_q I_a \cos\phi - I_a R_a \sin\phi)$$
$$\tan\delta = \frac{Im\,\mathbf{OC}}{Re\,\mathbf{OC}} = \frac{Im\,\mathbf{E''}}{Re\,\mathbf{E''}}$$

or
$$\tan\delta = \frac{X_q I_a \cos\phi - I_a R_a \sin\phi}{V + X_q I_a \sin\phi + I_a R_a \cos\phi}$$

Hence $\mathbf{E}_f = [E'' + (X_d - X_q) I_d] \angle\delta$

If the armature resistance is neglected

$$\tan\delta = \frac{X_q I_a \cos\phi}{V + X_q I_a \sin\phi}$$

EXAMPLE 1 . *A 1500 kVA, star-connected, 2300 V, 3-phase, salient-pole synchronous generator has reactances $X_d = 1.95\ \Omega$ and $X_q = 1.40\ \Omega$ per phase. All losses may be neglected. Find the excitation voltage for operation at rated kVA and power factor of 0.85 lagging.*

SOLUTION. $V_p = \dfrac{2300}{\sqrt{3}} = 1328$ V

$$(kVA)_{3\phi} = \frac{3\, V_p\, I_a}{1000}$$

$$1500 = \frac{3 \times 1328\, I_a}{1000},\quad I_a = 376.5 \text{ A}$$

Let $\mathbf{V}_p$ be the reference phasor.

$$\mathbf{V}_p = V_p \angle 0^\circ = 1328 \angle 0^\circ$$

$$\cos\phi = 0.85, \qquad \phi = 31.8^\circ$$

$$\therefore \qquad \mathbf{I}_a = I_a \angle -\phi = 376.5 \angle -31.8^\circ \text{ A}$$

$$= 320 - j\,198.4$$

From the phasor diagram

$$\mathbf{E}'' = \mathbf{OC} = \mathbf{OA} + \mathbf{AB} + \mathbf{BC}$$

$$= \mathbf{V}_p + 0 + j\,X_q I_a$$

$$= (1328 + j\,0) + j\,(1.40)\,(320 - j\,198.4)$$

$$= 1328 + 277.8 + j\,448 = 1605.8 + j\,448 = 1667 \angle 15.6^\circ \text{ V.}$$

The phase difference between $\mathbf{E}''$ and $\mathbf{I}_a$ is angle ψ.

$$\psi = \delta + \phi = 15.6^\circ + 31.8^\circ = 47.4^\circ$$

$$I_d = I_a \sin\psi = 376.5 \sin 47.4^\circ = 277.14 \text{ A}$$

$$(X_d - X_q)\, I_q = (1.95 - 1.40) \times 277.14 = 152.4 \text{ V}$$

Since $\mathbf{E}_f$, $\mathbf{E}''$ and $j\,(X_d - X_q)\, I_d$ are in phase we add the magnitudes.

$$\therefore \qquad E_f = E + (X_d - X_q)\, I_d = 1667 + 152.4 = 1819.4 \text{ V.}$$

EXAMPLE 2 . *An alternator has a direct-axis synchronous reactance of 0.8 per unit and a quadrature-axis synchronous reactance of 0.5 per unit. Determine the per unit open-circuit voltage for full load at a lagging power factor of 0.8. Neglect saturation.*

SOLUTION. Let $\mathbf{V}_p$ be the reference phasor

$\therefore$ $$\mathbf{V}_p = 1 \angle 0^\circ = 1 + j\,0 \text{ pu}$$

$$I_a = 1 \text{ pu at } 0.8 \text{ lagging pf}$$

$\therefore$ $$\mathbf{I}_a = 1 \angle -\cos^{-1} 0.8 = 1 \angle -36.9^\circ \text{ pu}$$

$$I_a X_d = 0.8 \text{ pu}$$

$$I_a X_q = 0.5 \text{ pu}$$

$$\begin{aligned} \mathrm{E}'' &= \mathbf{V}_p + j\,\mathbf{I}_a X_q \\ &= 1 + j\,0 + j\,1 \angle -36.9^\circ \times 0.5 \\ &= 1 + 0.5 \angle 90^\circ - 36.9^\circ = 1 + 0.5 \angle 53.1^\circ \\ &= 1 + 0.3 + j\,0.4 = 1.3 + j\,0.4 = 1.36 \angle 17.1^\circ \text{ V} \end{aligned}$$

$\therefore$ $$\delta = 17.1^\circ$$

From the phasor diagram

$$\psi = \phi + \delta = 36.9^\circ + 17.1 = 54^\circ$$

$$I_d = I_a \sin \psi = 1 \times \sin 54^\circ = 0.809$$

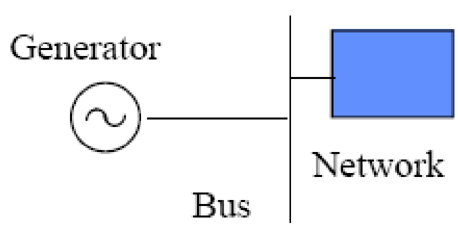
$\therefore$ $$\begin{aligned} E_f &= E'' + (X_d - X_q)\, I_d \\ &= 1.36 + (0.8 - 0.5) \times 0.805 = 1.60 \text{ pu.} \end{aligned}$$

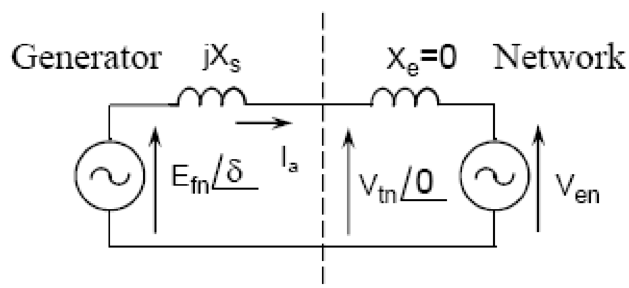
Power-angle Characteristics of Synchronous Generators

Power angle Characteristics Round Rotor Machine

- A synchronous machine supplies an electric network with constant voltage under steady state conditions.
- The terminal voltage in the machine is kept constant by the regulation of the field current.
- The generator speed is constant, at the synchronous speed determined by the network frequency and the number of poles in the machine.
- An increase of input mechanical power increases the torque. Calculate the output power variation with the input power.

- The external network is represented by a voltage source and an equivalent reactance. A large network's impedance is very small, we assume $X_e = 0$. and $V_{en} = V_{tn.}$ The equivalent circuit is:





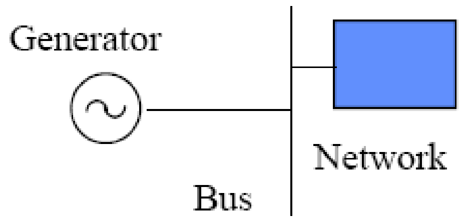
- Using the equivalent circuit the current is: $\mathbf{I} = \dfrac{\mathbf{E_{fn}}\, e^{i\delta} - \mathbf{V_{tn}}}{i\,\mathbf{X_{syn}} + i\,\mathbf{X_{nt}}}$

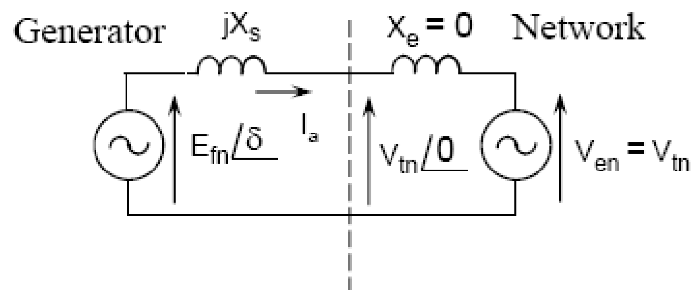
- The complex power delivered by the generator is:

$$S = 3\mathbf{V_{tn}}\,\bar{\mathbf{I}} = 3\,\mathbf{V_{tn}}\left[\frac{\mathbf{E_{fn}}\,\mathbf{e}^{-i\delta} - \mathbf{V_{tn}}}{-\mathbf{i}\,\mathbf{X_s}}\right] \qquad \mathbf{X_s} = \mathbf{X_{syn}} + \mathbf{X_{nt}}$$

- After simplification we get:

$$S = 3\,\frac{\mathbf{E_{fn}}\cdot\mathbf{V_{tn}}}{\mathbf{X_s}}\cdot\sin\delta + \mathbf{j}\cdot 3\cdot\left[\frac{\mathbf{E_{fn}}\mathbf{V_{tn}}}{\mathbf{X_s}}\cdot\cos\delta - \frac{\mathbf{V_{tn}}^{2}}{\mathbf{X_s}}\right]$$





- The real and reactive power are

$$\mathbf{P} = 3\,\frac{\mathbf{E_{fn}}\cdot\mathbf{V_{tn}}}{\mathbf{X_s}}\cdot\sin\delta$$

$$\mathbf{Q} = \mathbf{j}\cdot 3\cdot\left[\frac{\mathbf{E_{fn}}\mathbf{V_{tn}}}{\mathbf{X_s}}\cdot\cos\delta - \frac{\mathbf{V_{tn}}^{2}}{\mathbf{X_s}}\right]$$

- The real power is maximum if $\delta = 90^0$.

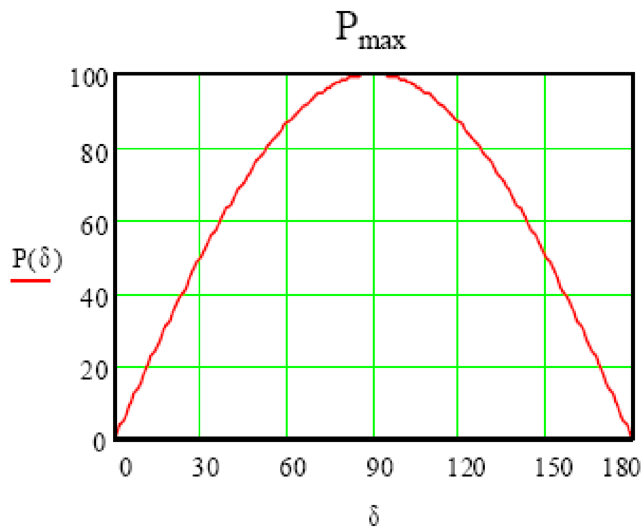
- The maximum torque is:

$T_{max} = P_{max} / \omega$

Power angle Characteristics

Round Rotor Machine

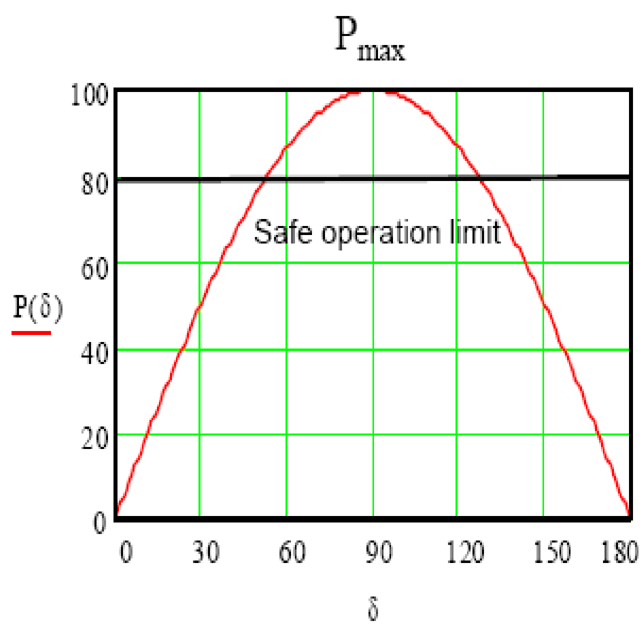
- The P(δ) curve shows that the increase of power increases the angle between the induced voltage and the terminal voltage.
- The power is maximum when δ =90º
- The further increase of input power forces the generator out of synchronism. This generates large current and mechanical forces.
- This angle corresponds to the angle between the field flux and the stator generated rotating flux.



Power angle Characteristics

Round Rotor Machine

- The angle δ, called power angle and it corresponds to the angle between the field flux and the stator generated rotating flux.
- The maximum power is the static stability limit of the system.
- Safe operation requires a 15-20% power reserve.



Salient-Pole Synchronous Generator

To simplify the derivation of expressions for the power and torque developed by a salient pole synchronous machine, neglect R_a and the core losses. The phasor diagram with E_f as reference. The complex power per phase is

$$\begin{aligned} S &= V_t I_a^* \\ &= |V_t|\underline{/-\delta}(|I_q| - j|I_d|)^* \\ &= |V_t|\underline{/-\delta}(|I_q| + j|I_d|) \end{aligned} \quad (1)$$

$$|I_d| = \frac{|E_f| - |V_t|\cos\delta}{X_d}$$

$$|I_q| = \frac{|V_t|\sin\delta}{X_q}$$

If these values of I_d and I_q are substituted in Eq. (1)

$$\begin{aligned} S &= \frac{|V_t|^2}{X_q}\sin\delta\underline{/-\delta} + \frac{|V_t||E_f|}{X_d}\underline{/90^\circ - \delta} - \frac{|V_t|^2}{X_d}\cos\delta\underline{/90^\circ - \delta} \quad (2) \\ &= P + jQ \end{aligned}$$

where P is the real power per phase and from Eq. (2)

$$\begin{aligned} P &= \frac{|V_t||E_f|}{X_d}\sin\delta + \frac{|V_t|^2(X_d - X_q)}{2X_dX_q}\sin 2\delta \\ &= P_f + P_r \end{aligned}$$

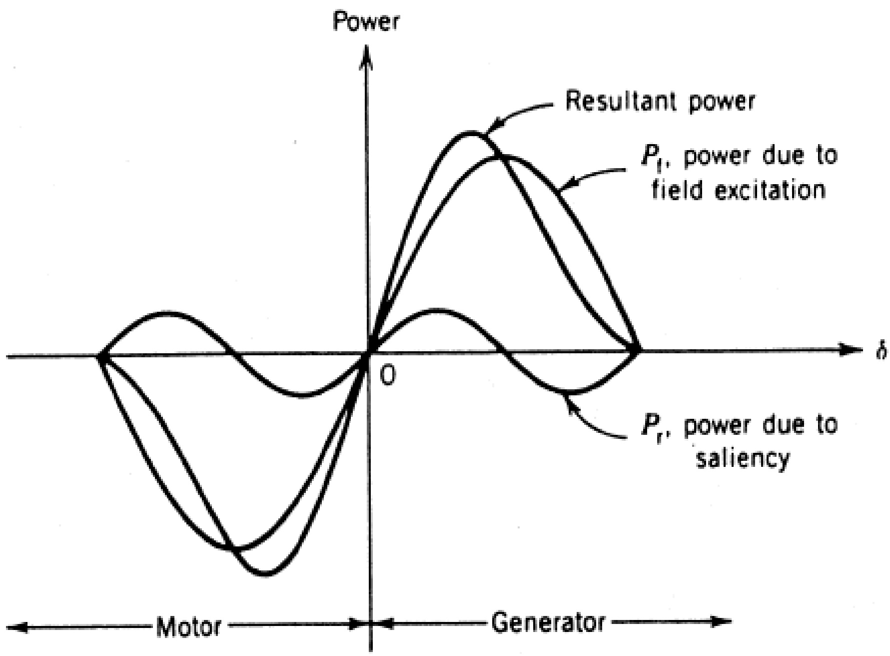
and Q is the reactive power per phase and from Eq. (2)

$$Q = \frac{|V_t||E_f|}{X_d}\cos\delta - |V_t|^2\left|\frac{\sin^2\delta}{X_q} + \frac{\cos^2\delta}{X_d}\right|$$

If $X_d = X_q$ (i.e., no saliency), then

$$P = \frac{|V_t||E_f|}{X_d} \sin \delta$$

$$Q = \frac{|V_t||E_f|}{X_d} \cos \delta - \frac{|V_t|^2}{X_d}$$



Power–angle characteristic of a salient pole synchronous machine.

EXAMPLE

A 3ϕ, 5 kVA, 208 V, four-pole, 60 Hz, star-connected synchronous machine has negligible stator winding resistance and a synchronous reactance of 8 ohms per phase at rated terminal voltage.

What is the steady-state (or static) stability limit? What are the corresponding values of the stator (or armature) current, power factor, and reactive power at this maximum power transfer condition?

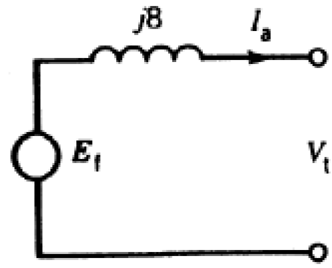
Solution

$$V_t = \frac{208}{\sqrt{3}} = 120 \text{ V/phase}$$

Stator current at rated kVA;

$$I_a = \frac{5000}{\sqrt{3} \times 208} = 13.9 \text{ A}$$

$$\phi = -36.9^\circ \quad \text{for lagging pf of 0.8}$$



$$E_f = V_t\underline{/0^\circ} + I_a jX_s$$

$$= 120\underline{/0^\circ} + 13.9\underline{/-36.9^\circ} \cdot 8\underline{/90^\circ}$$

$$= 206.9\underline{/25.5^\circ}$$

Excitation voltage $\quad E_f = 206.9$ V/phase

Power angle $\quad \delta = +25.5^\circ$

the maximum power transfer occurs at $\delta = 90^\circ$.

$$P_{max} = \frac{3E_f V_t}{X_s} = \frac{3 \times 206.9 \times 120}{8} = 9.32 \text{ kW}$$

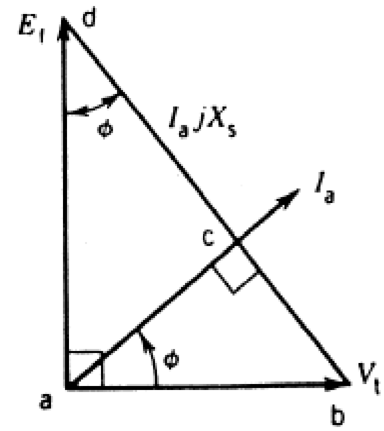
$$I_a = \frac{E_f - V_t}{jX_s} = \frac{206.9\underline{/+90^\circ} - 120\underline{/0^\circ}}{8\underline{/90^\circ}}$$

$$= 29.9\underline{/30.1^\circ} \text{ A}$$

Stator current $\quad I_a = 29.9$ A

Power factor $= \cos 30.1^\circ = 0.865$ leading

The stator current and power factor can also be obtained by drawing the phasor diagram for the maximum power transfer condition. The phasor diagram is shown in Fig.



Because $\delta = +90°$, E_f leads V_t by 90°. The distance bd between phasors V_t and E_f is the voltage drop I_aX_s and the current phasor I_a is in quadrature with I_aX_s.

From the phasor diagram,

$$|I_aX_s|^2 = |E_f|^2 + |V_t|^2$$

$$I_a = \left(\frac{206.9^2 + 120^2}{8^2}\right)^{1/2} = 29.9 \text{ A}$$

From the two triangles abc and abd,

$$\underline{/bac} = \underline{/adb} = \phi$$

$$\tan\phi = \frac{ab}{ad} = \frac{120}{206.9} = 0.58$$

$$\phi = 30.1°$$

$$\text{PF} = \cos 30.1° = 0.865 \quad \text{lead}$$

Example . *A 3-phase synchronous generator is delivering a power of 0.9 infinite bus at rated voltage and at pf 0.8 lagging. The generator has $X_d = 1.0$ $X_q = 0.6$ p.u. Determine the load angle and the excitation voltage.*

In case loss of excitation takes place, will the generator remain in synchronism ?

Solution. In per unit system,

$$V_t I_a \cos\theta = \text{Power}$$

or $$1 \times I_a \times 0.8 = 0.9$$

$\therefore$ $$I_a = 1.125 \text{ p.u.}$$

It is seen from the phasor diagram of Fig.

with $r_a = 0$ that $$\tan(\delta+\theta) = \frac{I_a X_q + V_t \sin\theta}{V_t \cos\theta} = \frac{1.125 \times 0.6 + 1 \times 0.6}{1 \times 0.8}$$

or $$(\delta + \theta) = 57.894°$$

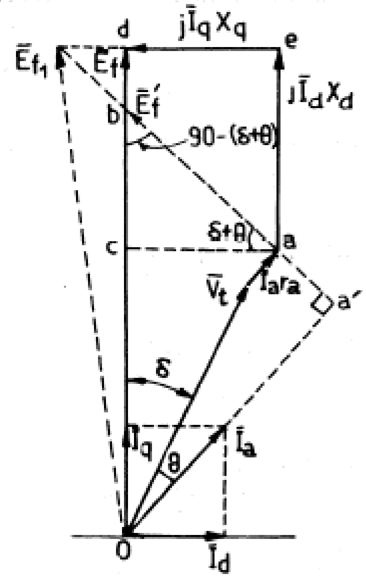
$\therefore$ $$\delta = 57.894° - \cos^{-1}(0.8) = 57.894° - 36.87° = 21.024°.$$

Also $$I_d = I_a \sin(\delta+\theta) = 1.125 \sin(57.894°) = 0.953 \text{ p.u.}$$

$\therefore$ $$E_f = V_t \cos\delta + I_d X_d = 1 \times \cos 21.024° + 0.953 \times 1.0 = 1.8864 \text{ p.u.}$$

When loss of excitation takes place, $E_f = 0$ and the maximum power is then given by

$$\frac{1}{2} V_t^2 \left(\frac{1}{X_q} - \frac{1}{X_d}\right) \sin 2\delta$$



$$= \frac{1}{2}\left(\frac{1}{0.6} - \frac{1}{1}\right) \sin 90° = 0.333 \text{ p.u.}$$

loss of excitation, the maximum power that the reluctance generator can deliver to ɪus is 0.333 p.u. As this is less than 0.9 p.u., the generator will lose synchronism.

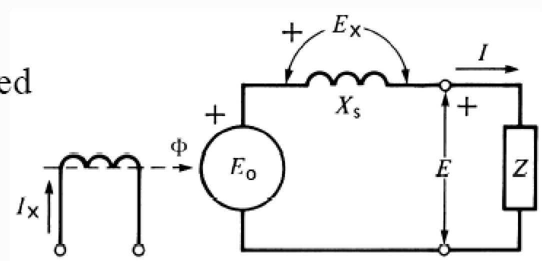
Parallel Operation of Synchronous Generators

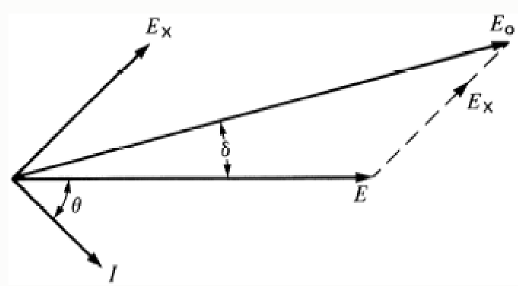
- Generators are rarely used in isolated situations. More commonly, generators are used in parallel, often massively in parallel, such as in the power grid. The following steps must be adhered to:
- when adding a generator to an existing power grid:

1) RMS line voltages of the two generators must be the same.
2) Phase sequence must be the same.
3) Phase angles of the corresponding phases must be the same.
4) Frequency must be the same.

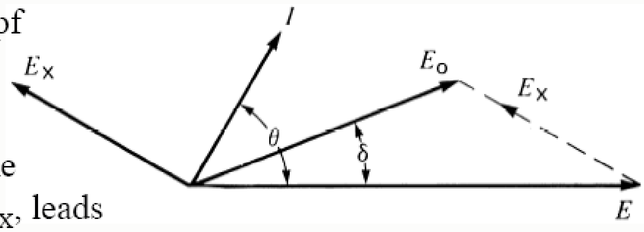
Generator under Load

- The behavior of a synchronous generator depends upon the connected load
  - two basic load categories
    - isolated loads
    - infinite bus
  - isolated loads with a lagging pf
    - current lags the terminal voltage, E
    - the voltage drop across the synchronous reactance, E_X, leads the current by 90°
    - the induced voltage, E_0, generated by the flux, Φ, is equal to the phasor sum of E and E_X





- isolated loads with a leading pf
  - current leads the terminal voltage, E
  - the voltage drop across the synchronous reactance, E_X, leads the current by 90°
  - the induced voltage, E_0, generated by the flux, Φ, is equal to the phasor sum of E and E_X
- note that E_0 always leads E by the angle δ
  - for lagging loads E_0 is greater than E
  - for leading loads E is greater than E_0



Synchronization of a Generator

- Often two or more generators are connected in parallel to supply a common load in large utility systems
  - connecting a generator to other generators is called paralleling
  - many paralleled generators behaves like an infinite bus
    - voltage and frequency are constant and can not be easily altered
  - before connecting a generator to an electrical grid, it must be *synchronized*
- the generator frequency is equal to the system frequency
- the generator voltage is equal to the system voltage
- the generator voltage is in phase with the system voltage
- the phase sequence of the generator is the same as that of the system

Synchronizing may be achieved with the help of ***synchronizing lamps***, the ***rotary lamp method*** being the most popular. Alternatively, a device known as the ***synchroscope*** may conveniently be used to facilitate synchronizing.

- To synchronize a generator
  - adjust the speed regulator of the prime mover so that frequencies are close
  - adjust the excitation so that generator voltage and system voltage are equal
  - observe the phase angle by means of a synchroscope, which indicates the phase angle between two voltages
    - the pointer rotates proportional to the frequency difference
    - a zero mark indicates a zero degree phase angle
    - the speed regulator is adjusted so that the pointer barely creeps across the dial
  - on the zero mark, the line circuit breaker is closed

Connecting to an Infinite Bus

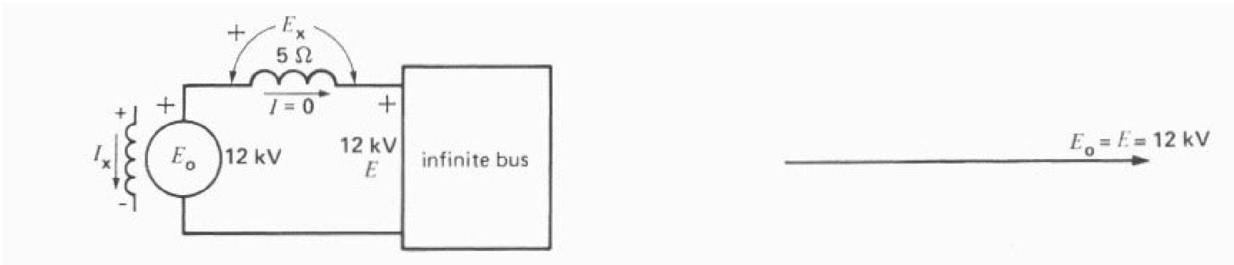
- An infinite bus system is so powerful that it imposes its own
  - voltage magnitude and frequency
  - once an apparatus is connected to an infinite bus, it becomes part of it
  - for a synchronized generator, the operator can only vary two machine parameters
    - the field excitation current, I_X
    - the prime-mover's mechanical torque, T

- Varying the exciting current
  - impacts the induced voltage E_0
  - causes a current to flow that is 90 degrees out-of-phase due to the synchronous reactance

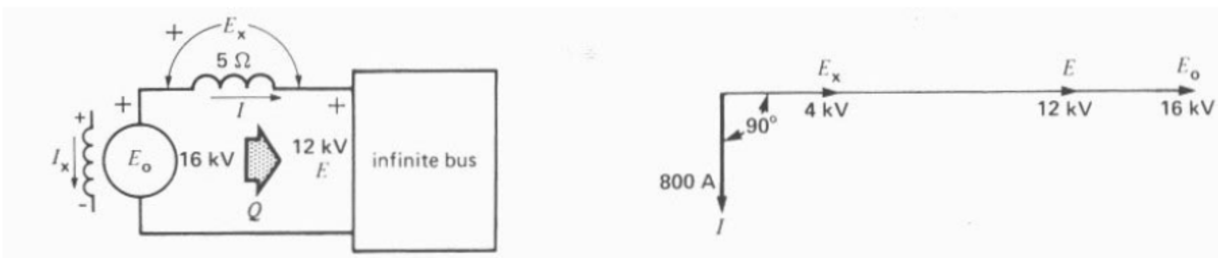
$$I = \frac{E_0 - E}{jX_S}$$

  - does not affect the flow of active (real) power
  - does cause reactive power to flow

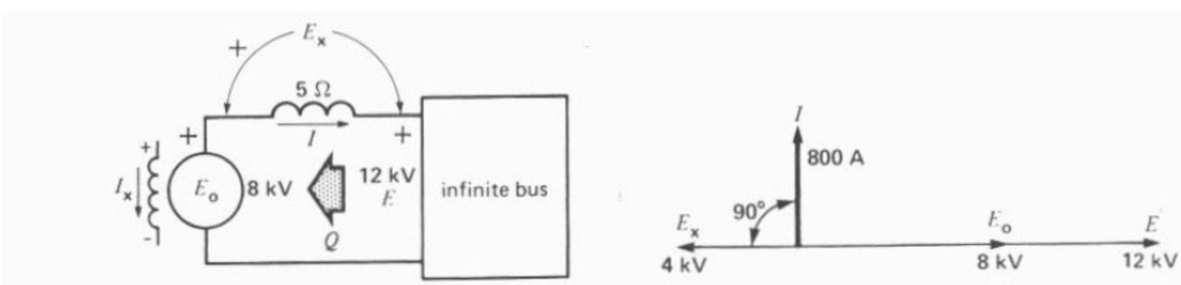
- Generator floating on an infinite bus:



Over-excited generator floating on an infinite bus:

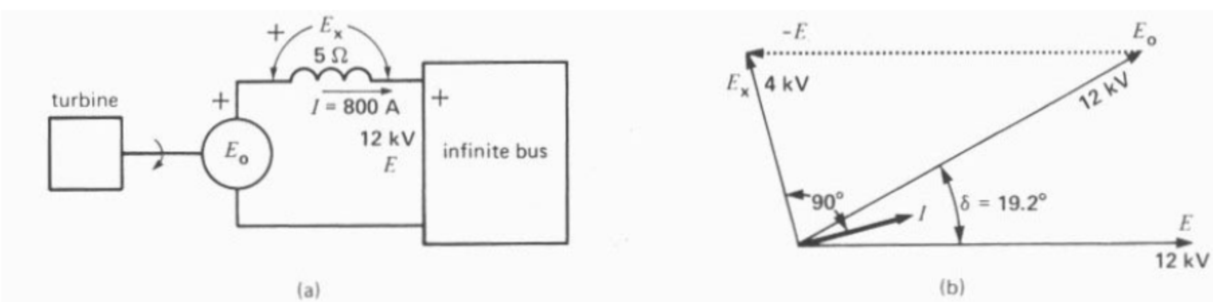


Under-excited generator floating on an infinite bus:



- Varying the mechanical torque
  - by opening up the control valve of the prime-mover, an increase torque is developed
  - the rotor will accelerate, E_0 will increase in value and begin to slip ahead of phasor E, leading by a phase angle δ
  - Although both voltages have similar values, the phase angle produces a difference of potential across the synchronous reactance
    - a current will flow, but this time almost in phase with E
    - real (active) power will flow

Mechanical torque exerted on the generator:



a. Turbine driving the generator.
b. Phasor diagram showing the torque angle δ.

Example:

A 13.8-kV 10-MVA 0.8-PF-lagging 60-Hz two-pole Y-connected steam-turbine generator has a synchronous reactance of 12 Ω per phase and an armature resistance of 1.5 Ω per phase. This generator is operating in parallel with a large power system (infinite bus).

(a) What is the magnitude of E_A at rated conditions?

(b) What is the torque angle of the generator at rated conditions?

(c) If the field current is constant, what is the maximum power possible out of this generator? How much reserve power or torque does this generator have at full load?

(d) At the absolute maximum power possible, how much reactive power will this generator be supplying or consuming? Sketch the corresponding phasor diagram. (Assume I_F is still unchanged.)

SOLUTION

(a) The phase voltage of this generator at rated conditions is

$$V_\phi = \frac{13{,}800\ \text{V}}{\sqrt{3}} = 7967\ \text{V}$$

The armature current per phase at rated conditions is

$$I_A = \frac{S}{\sqrt{3}\ V_T} = \frac{10{,}000{,}000\ \text{VA}}{\sqrt{3}\,(13{,}800\ \text{V})} = 418\ \text{A}$$

Therefore, the internal generated voltage at rated conditions is

$$\mathbf{E}_A = \mathbf{V}_\phi + R_A\mathbf{I}_A + jX_S\mathbf{I}_A$$
$$\mathbf{E}_A = 7967\angle 0° + (1.5\ \Omega)(418\angle -36.87°\ \text{A}) + j(12.0\ \Omega)(418\angle -36.87°\ \text{A})$$
$$\mathbf{E}_A = 12{,}040\angle 17.6°\ \text{V}$$

The magnitude of E_A is 12,040 V.

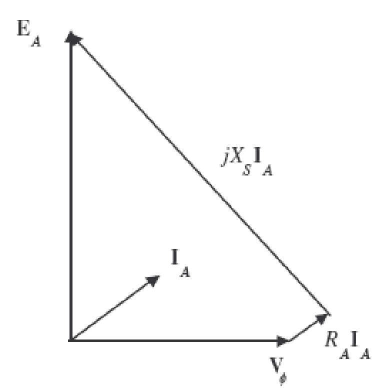
(b) The torque angle of the generator at rated conditions is $\delta = 17.6°$.

(c) Ignoring R_A, the *maximum output power* of the generator is given by

$$P_{\text{MAX}} = \frac{3\ V_\phi\ E_A}{X_S} = \frac{3(7967\ \text{V})(12{,}040\ \text{V})}{12\ \Omega} = 24.0\ \text{MW}$$

The power at maximum load is 8 MW, so the maximum output power is three times the full load output power.

(d) The phasor diagram at these conditions is shown below:



Under these conditions, the armature current is

$$\mathbf{I}_A = \frac{\mathbf{E}_A - \mathbf{V}_\phi}{R_A + jX_S} = \frac{12{,}040\angle 90°\ \text{V - }7967\angle 0°\ \text{V}}{1.5 + j12.0\ \Omega} = 1194\angle 40.6°\ \text{A}$$

The reactive power produced by the generator at this point is

$$Q = 3\ V_\phi\ I_A\ \sin\theta = 3(7967\ \text{V})(1194\ \text{A})\ \sin(0° - 40.6°) = -18.6\ \text{MVAR}$$

The generator is actually consuming reactive power at this time.

Example: Two alternators running in parallel supply lighting load of 2500 KW and a motor load of 5000 KW at 0.707 P.F. one machine is loaded to 4000 KW at a P.F. of 0.8 lagging. What is the KW output and P.F. of the other machine?
Solution: ***For first machine.***

Load power (or KW) of lighting load, p = 2500 KW

Load reactive power (or KVAR) of lighting load, $Q = \frac{P}{\cos\emptyset}\sin\emptyset$

$$\because \cos\emptyset = 1 \; for\ lighting\ load$$

$$\therefore \emptyset = 0 \; and \;\; \sin\emptyset = \sin 0 = 0$$

$$\therefore Q\ of\ lighting\ load = \frac{2500}{1}\times 0 = 0$$

For second machine

P of motor load=5000 KW

$P.F. \cos\emptyset = 0.707 \qquad\qquad \therefore \sin\emptyset = 0.707$

$$\therefore Q\ of\ motor\ load = \frac{P}{\cos\emptyset}\times\sin\emptyset = \frac{5000}{0.707}\times 0.707 = 5000\ KVAR$$

$$Total\ load = 2500 + 5000 = 7500\ KW$$

$$Total\ KVAR = 0 + 5000 = 5000\ KVAR$$

Load sharing

Load taking by 1st machine= 4000 KW

$$\therefore Q\ of\ 1st\ machine = \frac{4000}{0.8}\times 0.6 = 3000\ KVAR$$

$\therefore$ 2nd machine will supply

$$= 7500 - 4000 = 3500\ KW$$

$$and \qquad Q = 5000 - 3000 = 2000\ KVAR$$

$$Since \qquad Q = \frac{P\times\sin\emptyset}{\cos\emptyset} = P\times\tan\emptyset$$

$$or \qquad \tan\emptyset = \frac{Q}{P} = \frac{2000}{3500} = 0.57142$$

$$or \qquad \emptyset = \tan^{-1} 0.57142 = 29.7448^o$$

$$\therefore p.F., \cos\emptyset = \cos 29.7448^o = 0.8682\ lagging$$