ACHARYA NAGARJUNA UNIVERSITY

CURRICULUM B.A / B.Sc. MATHEMATICS - PAPER - III

LINEAR ALGEBRA AND VECTOR CALCULUS

(Syllabus for the academic years 2010-2011 and onwards) 90 Hours

PART A : LINEAR ALGEBRA

UNIT - I (20 Hours)

Vector spaces, General properties of vector spaces, Vector subspaces, Algebra of subspaces, linear combination of vectors. Linear span, linear sum of two subspaces, Linear independence and dependence of vectors, Basis of vector space, Finite dimensional vector spaces, Dimension of a vector space, Dimension of a subspace.

UNIT - II (25 Hours)

Linear transformations, linear operators, Range and nullspace of transformation, Rank and nullity of linear transformations, Linear transformations as vectors, Product of linear transformations, Invertable linear transformation. Inner product spaces, Euclidean and unitary spaces, Norm or length of a vector, Schwartz inequality, Orthogonality, Orthonormal set, complete orthonormal set, Gram - Schmidt orthogonalisation process.

Pescribed text book : Linear Algebra by J.N. Sharma and A.R. Vasishtha, Krishna Prakashan Mandir, Meerut - 250002.

Reference Books : 1. Linear Algebra by Kenneth Hoffman and Ray Kunze, Pearson Education (low priced edition), New Delhi.
2. Linear Algebra by Stephen H. Friedberg, Arnold J. Insal and Lawrance E. Spence, Prentice Hall of India Pvt. Ltd. 4th edition -2004.

PART B : MULTIPLE INTEGRALS AND VECTOR CALCULUS

UNIT - III (20 Hours)

Vector differentiation. Ordinary derivatives of vectors, Space curves, Continuity, Differentiability, Gradient, Divergence, Curl operators, Formulae involving these operators.

Pescribed test Book : Vector Analysis by Murray. R. Spiegel, Schaum series Publishing company, Related topics in Chapters 3 and 4.

Reference Books : 1. Text book of vector Analysis by Shanti Narayana and P.K. Mittal, S.Chand & Company Ltd, New Delhi.
2. Mathematical Analysis by S.C. Mallik and Savitha Arora, Wiley Eastern Ltd.

UNIT - IV (25 Hours)

Multiple integrals : Introduction, the concept of a plane curve, line integral - Sufficient condition for the existence of the integral. The area of a subset of R^2, Calculation of double integrals, Jordan curve, Area, Change of the order of integration, Double integral as a limit, Change of variable in a double integration. Vector integration, Theorems of Gauss and Stokes, Green's theorem in plane and applications of these theorems.

Pescribed test Book : 1. A Course of Mathematical Analysis by Santhi Narayana and P.K. Mittal, S. Chand Publications. Related topics in Chapter - 16.
2. Vector Analysis by Murray. R. Spiegel, Schaum series Publishing company. Related topics in Chapters - 5 and 6.

Reference Books : 1. Text book of vector Analysis by Shanti Narayana and P.K. Mittal, S.Chand & Company Ltd, New Delhi.

2. Mathematical Analysis by S.C. Mallik and Savitha Arora, Wiley Eastern Ltd.

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ACHARYA NAGARJUNA UNIVERSITY

B.A / B.Sc. MATHEMATICS - PAPER - III

LINEAR ALGEBRA AND VECTOR CALCULUS

QUESTION BANK FOR PRACTICALS

UNIT - I (VECTOR SPACES)

1. Show that a field K can be regarded as a vector space over any subfield F of K.
2. Show that the set M of all $m \times n$ matrices with their elements as real numbers is a vector space over the field F of real numbers with respect to addition of matrices as addition of vectors and multiplication of a matrix by a scalar as scalar multiplication.
3. Show that the set V_n of all ordered n-tuples over a field F is a vector space w.r.t. addition of n-tuples as addition of vectors and multiplication of an n-tuple by a scalar as scalar multiplication.
4. Let S be a nonempty set and F be a field. Let V be the set of all functions from S into F. If addition and scalar multiplication are defined as $(f+g)(x) = f(x) + g(x) \forall x \in S$ and $(cf)(x) = cf(x) \forall x \in S$ where $f, g \in V$, $c \in F$, then show that V is a vector space.
5. If F is a field then show that $W = \{(a_1, a_2, 0) : a_1, a_2 \in F\}$ is a subspace of $V_3(F)$.
6. Let F be a field and a_1, a_2, a_3 be three fixed elements of F. Show that $W = \{(x_1, x_2, x_3) : x_1, x_2, x_3 \in F$ and $a_1x_1 + a_2x_2 + a_3x_3 = 0\}$ is a subspace of $V_3(F)$.
7. i) If S and T are two subsets of a vector space $V(F)$ then show that $S \subseteq T \Rightarrow L(S) \subseteq L(T)$.

   ii) If S is a subset of a vector space $V(F)$ then show that $L(L(S)) = L(S)$.
8. If S and T are two subsets of a vector space $V(F)$ then show that $L(S \cup T) = L(S) + L(T)$.
9. Express the vector $\alpha = (1, -2, 5)$ as a linear combination of the vectors

   $\alpha_1 = (1,1,1), \alpha_2 = (1,2,3), \alpha_3 = (2,-1,1)$.
10. If $\alpha = (1,2,1), \beta = (3,1,5), \gamma = (3,-4,7)$ then show that the subspaces of $V_3(R)$ spanned by $S = \{\alpha, \beta\}$ and $T = \{\alpha, \beta, \gamma\}$ are the same.
11. Show that the system of three vectors $(1,3,2), (1,-7,-8), (2,1,-1)$ of $V_3(R)$ is linearly dependent.
12. Show that $(1,1,2,4), (2,-1,-5,2), (1,-1,4,0), (2,1,1,6)$ are linearly dependent in $V_4(R)$.
13. Show that $\{(1,2,0), (0,3,1), (-1,0,1)\}$ are linearly independent in $V_3(R)$.
14. Show that the vectors $(1,2,1), (2,1,0), (1,-1,2)$ form a basis for R^3.
15. Show that $\{(2,1,0), (2,1,1), (2,2,1)\}$ form a basis of V_3 over reals and express the vector $(1,2,1)$ as a linear combination of these basis vectors.
16. Show that $(1,1,1,1), (0,1,1,1), (0,0,1,1), (0,0,0,1)$ is a basis of V_4 over reals. Express $\alpha = (2,3,4,1)$ as a linear combination of these basis vectors.
17. Extend the set $\{(1,2,3,0), (2,-1,0,1)\}$ to form a basis for $V_4(R)$.
18. Find a basis for the subspace spanned by the vectors $(1,2,0), (-1,0,1), (0,2,1)$ in $V_3(R)$.
19. If W_1 and W_2 are subspaces of $V_4(R)$ generated by the sets $\{(1,1,-1,2), (2,1,3,0), (3,2,2,2)\}$ and $\{(1,-1,0,1), (-1,1,0,-1)\}$ respectively then find $dim(W_1 + W_2)$ and obtain a basis for $W_1 + W_2$.
20. $W = \{(x, y, 0, 0) : x, y \in R\}$. Write a basis for the quotient space R^4/W.

UNIT - II (LINEAR TRANSFORMATIONS AND INNER PRODUCT SPACES)

21. Show that the mapping $T: V_3(R) \to V_2(R)$ defined as $T(a_1, a_2, a_3) = (3a_1 - 2a_2 + a_3, a_1 - 3a_2 - 2a_3)$ is a linear transformation from $V_3(R)$ into $V_2(R)$.

22. Find a linear transformation $T: R^2 \to R^2$ such that $T(1,0) = (1,1)$ and $T(0,1) = (-1,2)$. Prove that T maps the square with vertices $(0, 0), (1, 0), (1, 1), (0, 1)$ into a parallelogram.

23. Let T be a linear transformation from a vector space $U(F)$ into a vector space $V(F)$

    i) If W is a subspace of U then show that $T(W)$ is a subspace of V.

    ii) If X is a subspace of V then show that $\{\alpha \in U : T(\alpha) \in X\}$ is a subspace of U.

24. If W is a subspace of a vector space V and if a linear transformation $T: V \to V/W$ is defined by $T(x) = W + \alpha$ for all $\alpha \in V$, find the kernel of T.

25. Show that the mapping $T: R^3 \to R^3$ defined by $T(x, y, z) = (x + 2y - z, y + z, x + y - 2z)$ is a linear transformation. Find its rank, nullity and verify $rank\ T + nullity\ T = dim\ \mathrm{R}^3$.

26. Show that the mapping $T: V_2(R) \to V_3(R)$ defined as $T(a,b) = (a+b, a-b, b)$ is a linear transformation from $V_2(R)$ into $V_3(R)$. Find the range, rank, null space and nullity of T.

27. If $T: V_4(R) \to V_3(R)$ is a linear transformation defined by $T(a,b,c,d) = (a - b + c + d,\ a + 2c - d,\ a + b + 3c - 3d)$ for $a,b,c,d \in R$ then verify $rank\ T + nullity\ T = dim\ V_4(R)$.

28. If $T: V_3(R) \to V_3(R)$ is a linear transformation defined by $T(a,b,c) = (3a, a - b, 2a + b + c)$ then show that $(T^2 - I)(T - 3I) = \hat{0}$.

29. Let $T: R^3 \to R^3$ be a linear transformation defined by $T(x_1, x_2, x_3) = (3x_1, x_1 - x_2, 2x_1 + x_2 + x_3)$. Is T invertible? If so find a rule for T^{-1} like the one which defined T.

30. Let $T: V_3(R) \to V_3(R)$ be defined by $T(x_1, x_2, x_3) = (3x_1 + x_3, -2x_1 + x_2, -x_1 + 2x_2 + 4x_3)$. Prove that T is invertible and find T^{-1}.

31. If $\alpha = (a_1, a_2, \ldots, a_n), \beta = (b_1, b_2, \ldots, b_n) \in V_n(C)$ then show that $(\alpha, \beta) = a_1\bar{b}_1 + a_2\bar{b}_2 + \ldots + a_n\bar{b}_n$ defines an inner product on $V_n(C)$.

32. If $\alpha = (a_1, a_2), \beta = (b_1, b_2) \in V_2(R)$ then show that $(\alpha, \beta) = a_1b_1 - a_2b_1 - a_1b_2 + 4a_2b_2$ defines an inner product on $V_2(R)$.

33. Let $V(C)$ be the vector space of all continuous complex valued functions on [0, 1]. If $f(t), g(t) \in V$ then show that $(f(t), g(t)) = \int_0^1 f(t)\overline{g(t)}\,dt$ defines an inner product on $V(C)$.

34. Find the unit vector corresponding to $(2 - i, 3 + 2i, 2 + \sqrt{3}i)$ of $V_3(C)$ with respect to the standard inner product.

35. Show that the absolute value of the cosine of an angle cannot be greater than 1.

36. Let $V(R)$ be the vector space of polynomials with inner product defined by $(f, g) = \int_0^1 f(t)\, g(t)\, dt$ for $f, g \in V$. If $f(x) = x^2 + x - 4, g(x) = x - 1 \forall x \in [0,1]$ then find $(f, g), \|f\|$ and $\|g\|$.

37. Show that $\{(1,0,0),(0,1,0),(0,0,1)\}$ is an orthonormal set in the inner product space $V_3(R)$.

38. Apply the Gram-Schmidt process to the vectors $\beta_1=(1,0,1), \beta_2=(1,0,-1), \beta_3=(0,3,4)$, to obtain an orthonormal basis for $V_3(R)$ with the standard inner product.

39. Apply the Gram-Schmidt process to the vectors $\{(2,1,3),(1,2,3),(1,1,1)\}$ to obtain an orthonormal basis for $V_3(R)$ with the standard inner product.

40. If W_1 and W_2 are two subspaces of a finite dimensional inner product space, prove that i) $(W_1+W_2)^{\perp}=W_1^{\perp}\cap W_2^{\perp}$ ii) $(W_1\cap W_2)^{\perp}=W_1^{\perp}+W_2^{\perp}$.

UNIT - III (VECTOR DIFFERENTIATION)

41. If $\mathbf{r}=e^{-1}\mathbf{i}+\log(t^2+1)\mathbf{j}-\tan t\,\mathbf{k}$ then find $\frac{d\mathbf{r}}{dt}, \frac{d^2\mathbf{r}}{dt^2}, \left|\frac{d\mathbf{r}}{dt}\right|, \left|\frac{d^2\mathbf{r}}{dt^2}\right|$ at $t=0$.

42. If $\mathbf{A}=5t^2\mathbf{i}+t\mathbf{j}-t^3\mathbf{k}$ and $\mathbf{B}=\sin t\,\mathbf{i}-\cos t\,\mathbf{j}$ then find i) $\frac{d}{dt}(\mathbf{A}\cdot\mathbf{B})$ ii) $\frac{d}{dt}(\mathbf{A}\times\mathbf{B})$

43. If $\mathbf{r}=a\cos t\,\mathbf{i}+a\sin t\,\mathbf{j}+at\tan\theta\,\mathbf{k}$ then find $\left|\frac{d\mathbf{r}}{dt}\times\frac{d^2\mathbf{r}}{dt^2}\right|$ and $\left[\frac{d\mathbf{r}}{dt}\,\frac{d^2\mathbf{r}}{dt^2}\,\frac{d^3\mathbf{r}}{dt^3}\right]$.

44. If $\mathbf{A}=\sin t\,\mathbf{i}-\cos t\,\mathbf{j}+t\,\mathbf{k}$, $\mathbf{B}=\cos t\,\mathbf{i}-\sin t\,\mathbf{j}-3\mathbf{k}$ and $\mathbf{C}=2\mathbf{i}+3\mathbf{j}-\mathbf{k}$ then find $\frac{d}{dt}[\mathbf{A}\times(\mathbf{B}\times\mathbf{C})]$ at $t=0$.

45. If $\mathbf{f}=\cos xy\,\mathbf{i}+(3xy-2x^2)\,\mathbf{j}-(3x+2y)\,\mathbf{k}$ then find $\frac{\partial^2\mathbf{f}}{\partial x^2}, \frac{\partial^2\mathbf{f}}{\partial x\partial y}, \frac{\partial^2\mathbf{f}}{\partial y^2}$.

46. If $\mathbf{f}=2x^2\,\mathbf{i}-3yz+xz^2\,\mathbf{k}$ and $\varphi=2z-x^3y$ then find $\frac{\partial\mathbf{f}}{\partial x}\cdot\left(\mathbf{i}\frac{\partial\varphi}{\partial x}+\mathbf{j}\frac{\partial\varphi}{\partial y}+\mathbf{k}\frac{\partial\varphi}{\partial z}\right)$ and $\frac{\partial\mathbf{f}}{\partial z}\times\left(\mathbf{i}\frac{\partial\varphi}{\partial x}+\mathbf{j}\frac{\partial\varphi}{\partial y}+\mathbf{k}\frac{\partial\varphi}{\partial z}\right)$ at $(1,-1,1)$.

47. If $a=x+y+z, b=x^2+y^2+z^2, c=xy+yz+zx$ then show that $[\nabla a\ \nabla b\ \nabla c]=0$.

48. Find the directional derivative of $\varphi=x^2yz+4xz^2$ at the point $(1,-2,-1)$ in the direction of $2\mathbf{i}-\mathbf{j}-2\mathbf{k}$.

49. Find the maximum value of the directional derivative of $\varphi=2x^2-y-z^4$ at $(2,-1,1)$.

50. Find the directional derivative of $f=x^2-y^2+2z^2$ at $P(1,2,3)$ in the direction of $\vec{PQ}$ where $Q=(5,0,4)$.

51. Find the directional derivative of $\varphi=xy^2+yz^2$ along the tangent to the curve $x=t, y=t^2, z=t^3$ at $(1,1,1)$.

52. Find the angle between the surfaces of the spheres $x^2+y^2+z^2=29$, $x^2+y^2+z^2+4x-6y-8z-47=0$ at the point $(4,-3,2)$.

53. Find the angle between the surfaces $x^2+y^2+z^2=9$ and $x^2+y^2-z=3$ at $(2,-1,2)$.

54. Show that $r^n\mathbf{r}$ is irrotational. Find when it is solenoidal.

55. Show that i) $div\ \mathbf{r}=3$ ii) $curl\ \mathbf{r}=\mathbf{0}$ iii) $grad\ (\mathbf{r}\cdot\mathbf{a})=\mathbf{a}$ iv) $div\ (\mathbf{r}\times\mathbf{a})=0$

56. i) Find p if $(x+3y)\mathbf{i}+(y-2z)\mathbf{j}+(x+pz)\mathbf{k}$ is solenoidal.

    ii) Find the constants a,b,c so that $(x+2y+az)\mathbf{i}+(bx-3y-z)\mathbf{j}+(4x+cy+2z)\mathbf{k}$ is irrotational.

57. If $\mathbf{F}=grad(x^3+y^3+z^3-3xyz)$ then find $div\,\mathbf{F}, curl\,\mathbf{F}$.

58. If $\mathbf{F}=3xyz^3\mathbf{i}+4x^3y\mathbf{j}-xy^2z\mathbf{k}$ the find $\nabla(\nabla\cdot\mathbf{F})$ and $\nabla\times(\nabla\times\mathbf{F})$ at $(-1,2,1)$. Also verify that $\nabla(\nabla\cdot\mathbf{F})=\nabla\times(\nabla\times\mathbf{F})+\nabla^2\mathbf{F}$.

59. If $\mathbf{a}$ is a constant vector then show that $Curl\dfrac{\mathbf{a}\times\mathbf{r}}{r^3}=\dfrac{-\mathbf{a}}{r^3}+3\dfrac{\mathbf{r}}{r^5}(\mathbf{a}\cdot\mathbf{r})$.

60. Show that $\nabla^2(r^n)=n(n+1)r^{n-2}$.

UNIT - IV (VECTOR INTEGRATION AND MULTIPLE INTEGRALS)

61. Evaluate $\int_C (3xy\,dx-y^2\,dy)$, where C is the curve in the xy - plane, $y=2x^2$ from (0, 0) to (1, 2).

62. Evaluate $\int_C (xy\,dx+yz\,dy+zx\,dz)$, where C is $x=t, y=t^2, z=t^3, t$ varying from -1 to 1.

63. Evaluate $\int_C (2x^2+y^2)\,dx+(3y-4x)\,dy$ around the triangle ABC whose vertices are $A=(0,0), B=(2,0), C=(2,1)$.

64. Evaluate $\iint xy(x^2+y^2)\,dx\,dy$ over $[0,a;0,b]$.

65. Evaluate $\iint xy(x+y)\,dx\,dy$ over the area between $y=x^2$ and $y=x$.

66. Evaluate $\iint (x+y)^2\,dx\,dy$ over the area bounded by the ellipse $\dfrac{x^2}{a^2}+\dfrac{y^2}{b^2}=1$.

67. Change the order of integration in the double integral $\int_0^\infty\int_x^\infty \dfrac{e^{-y}}{y}\,dx\,dy$ and hence find the value.

68. Evaluate $\iint \dfrac{\sqrt{(a^2b^2-b^2x^2-a^2y^2)}}{\sqrt{(a^2b^2+b^2x^2+a^2y^2)}}\,dx\,dy$ the field of integration being the positive quadrant of the ellipse $x^2/a^2+y^2/b^2=1$.

69. Using the transformation $x+y=u, x-y=v$, evaluate $\iint e^{\frac{x-y}{x+y}}\,dx\,dy$ over the region bounded by $x=0, y=0, x+y=1$.

70. Evaluate the integral $\int_0^{4a}\int_{y^2/4a}^{y} \dfrac{x^2-y^2}{x^2+y^2}\,dx\,dy$ by changing to polar coordinates.

71. If $\mathbf{F} = 3xy\mathbf{i} - y^2\mathbf{j}$, evaluate $\int_C \mathbf{F} \cdot d\mathbf{r}$ where C is the curve $y = 2x^2$ in the xy-plane from $(0, 0)$ to $(1, 2)$.

72. If $\mathbf{F} = (x^2 + y^2)\mathbf{i} - 2xy\,\mathbf{j}$, evaluate $\oint_C \mathbf{F} \cdot d\mathbf{r}$ where the curve C is the rectangle in the xy-plane bounded by $y = 0, y = b, x = 0, x = a$.

73. Evaluate $\int_S \mathbf{F} \cdot \mathbf{N}\,dS$ where $\mathbf{F} = z\mathbf{i} + x\mathbf{j} - 3y^2z\mathbf{k}$ and S is the surface $x^2 + y^2 = 16$ included in the first octant between $z = 0$ and $z = 5$.

74. If $\mathbf{F} = 2xz\mathbf{i} - x\mathbf{j} + y^2\mathbf{k}$, evaluate $\int \mathbf{F}\,dV$ where V is the region bounded by the surfaces $x = 0, y = 0, y = 6, z = x^2, z = 4$.

75. Compute $\oint_S (ax^2 + by^2 + cz^2)\,dS$ over the sphere $x^2 + y^2 + z^2 = 1$ by using Gauss's divergence theorem.

76. Verify Gauss's divergence theorem to evaluate $\int_S \{(x^3 - yz)\mathbf{i} - 2x^2y\mathbf{j} + z\mathbf{k}\} \cdot \mathbf{N}\,dS$ over the surface of a cube bounded by the coordinate planes $x = y = z = a$.

77. Evaluate $\oint_C (\cos x \sin y - xy)dx + \sin x \cos y\,dy$, by Green's theorem, where C is the circle $x^2 + y^2 = 1$.

78. Verify Green's theorem in the plane for $\oint_C (3x^2 - 8y^2)\,dx + (4y - 6xy)\,dy$ where C is the region bounded by $y = \sqrt{x}$ and $y = x^2$.

79. Verify Stoke's theorem for $\mathbf{F} = -y^3\mathbf{i} + x^3\mathbf{j}$, where S is the circular disc $x^2 + y^2 \le 1, z = 0$.

80. Apply Stoke's theorem to evaluate $\int_C (y\,dx + z\,dy + x\,dz)$ where C is the curve of intersection of $x^2 + y^2 + z^2 = a^2$ and $x + z = a$.

ACHARYA NAGARJUNA UNIVERSITY

B.A / B.Sc. DEGREE EXAMINATION, THEORY MODEL PAPER

(Examination at the end of third year, for 2010 - 2011 onwards)

MATHEMATICS - III

PAPER III - LINEAR ALGEBRA AND VECTOR CALCULUS

Time : 3 Hours **Max. Marks : 100**

SECTION - A (6 x 6 = 36 Marks)

Answer any ***SIX*** questions. Each question carries 6 marks

1. Let W_1 and W_2 be two subspaces of a vector space $V(F)$. Then prove that $W_1 \cup W_2$ is a subspace of $V(F)$ iff $W_1 \subseteq W_2$ or $W_2 \subseteq W_1$.
2. Prove that every finite dimensional vector space has a basis.
3. If $T: V_3(R) \to V_2(R)$ is defined as $T(x_1, x_2, x_3) = (x_1 - x_2, x_1 + x_3)$, prove that T is a linear transformation.
4. State and prove Schwarz's inequality.
5. If $\mathbf{A} = 5t^2\mathbf{i} + t\mathbf{j} - t^3\mathbf{k}$ and $\mathbf{B} = \sin t\,\mathbf{i} - \cos t\,\mathbf{j} + 5t\,\mathbf{k}$ then find i) $\frac{d}{dt}(\mathbf{A}\cdot\mathbf{B})$ ii) $\frac{d}{dt}(\mathbf{A}\times\mathbf{B})$.
6. Find the angle between the surfaces of the spheres $x^2 + y^2 + z^2 = 29$, $x^2 + y^2 + z^2 + 4x - 6y - 8z - 47 = 0$ at the point $(4, -3, 2)$.
7. Change the order of integration in the double integral $\int_0^\infty \int_x^\infty \frac{e^{-y}}{y}\,dx\,dy$ and hence find the value.
8. Evaluate $\int_C \mathbf{A}\cdot d\mathbf{r}$ where C is the line joining (0, 0, 0) and (2, 1, 1), given

   $\mathbf{A} = (2y+3)\mathbf{i} + xz\mathbf{j} + (yz - x)\mathbf{k}$.

SECTION - B (4 x 16 = 64 Marks)

Answer ***ALL*** questions. Each question carries 16 marks

9.(a) Prove that the necessary and sufficient condition for a nonempty subset W of a vector space $V(F)$ to be a subspace of V is that $a, b \in F, \alpha, \beta \in W \Rightarrow a\alpha + b\beta \in W$.

(b) Show that $(1,0,-1), (2,1,3), (-1,0,0), (1,0,1)$ are linearly dependent.

OR

10.(a) If W_1 and W_2 are two subspaces of a finite dimensional vector space $V(F)$ then prove that $dim\ (W_1 + W_2) = dim\ W_1 + dim\ W_2 - dim\ (W_1 \cap W_2)$.

(b) Show that the vectors $\alpha_1 = (1,1,1), \alpha_2 = (-1,1,0), \alpha_3 = (1,0,-1)$ form a basis of R^3 and express $(4, 5, 6)$ in terms of $\alpha_1, \alpha_2, \alpha_3$.

11.(a) If T is a linear transformation from a vector space $U(F)$ into a vector space $V(F)$ and U is finite dimensional then prove that $rank\ (T) + nullity\ (T) = dim\ U$.

(b) Let $T: V_3(R) \to V_3(R)$ be defined by $T(x_1, x_2, x_3) = (3x_1 + x_3, -2x_1, + x_2, -x_1 + 2x_2 + 4x_3)$. Prove that T is invertible and find T^{-1}.

OR

12.(a) State and prove Bessal's inequality.

(b) Apply the Gram-Schmidt process to the vectors {(2, 1, 3), (1, 2, 3), (1, 1, 1)} to obtain an orthonormal basis for $V_3(R)$ with the standard inner product.

13.(a) If **A, B** are two differentiable vector point functions then prove that

$div(\mathbf{A} \times \mathbf{B}) = \mathbf{B} \cdot (curl\, \mathbf{A}) - \mathbf{A} \cdot (curl\, \mathbf{B})$

(b) Find the directional derivative of $\varphi = xy + yz + zx$ at the point $(1, 2, 0)$ in the direction of $\mathbf{i} + 2\mathbf{j} + 2\mathbf{k}$

OR

14.(a) If **F** is a differentiable vector point function then prove that $curl(curl\, \mathbf{F}) = grad(div\, \mathbf{F}) - \nabla^2 \mathbf{F}$.

(b) Find div **F** and $curl$ **F** where $\mathbf{F} = xy^2\mathbf{i} + 2x^2yz\mathbf{j} - 3yz^2\mathbf{k}$ at $(1, -1, 1)$.

15.(a) State and prove Gauss's divergence theorem.

(b) Verify Green's theorem in the plane for $\oint_C (3x^2 - 8y^2)\, dx + (4y - 6xy)\, dy$ where C is the region bounded by $y = \sqrt{x}$ and $y = x^2$.

OR

16.(a) State and prove Stoke's theorem.

(b) Evaluate $\iint xy(x + y)\, dxdy$ over the area between $y = x^2$ and $y = x$.

ACHARYA NAGARJUNA UNIVERSITY

B.A / B.Sc. DEGREE EXAMINATION, PRACTICAL MODEL PAPER

(Practical Examination at the end of third year, for 2010- 2011 onwards)

MATHEMATICS - III

PAPER III - LINEAR ALGEBRA AND VECTOR CALCULUS

Time : 3 Hours **Max. Marks : 30**

Answer ***ALL*** questions. Each question carries $7\frac{1}{2}$ marks. $4\times 7\frac{1}{2}=30$ M

1 (a) Let F be a field and a_1, a_2, a_3 be three fixed elements of F. Show that $W = \{(x_1, x_2, x_3) : x_1, x_2, x_3 \in F$ and $a_1x_1 + a_2x_2 + a_3x_3 = 0\}$ is a subspace of $V_3(F)$.

OR

(b) Show that the vectors $(1,2,1),(2,1,0),(1,-1,2)$ form a basis for R^3.

2 (a) If $\alpha = (a_1, a_2), \beta = (b_1, b_2) \in V_2(R)$ then show that $(\alpha, \beta) = a_1b_1 - a_2b_1 - a_1b_2 + 4a_2b_2$ defines an inner product on $V_2(R)$.

OR

(b) Let $T : R^3 \to R^3$ be a linear transformation defined by $T(x_1, x_2, x_3) = (3x_1, x_1 - x_2, 2x_1 + x_2 + x_3)$. Is T invertible? If so find a rule for T^{-1} like the one which defined T.

3 (a) Find the angle between the surfaces $x^2 + y^2 + z^2 = 9$ and $x^2 + y^2 - z = 3$ at $(2,-1,2)$.

OR

(b) If $\mathbf{F} = grad(x^3 + y^3 + z^3 - 3xyz)$ then find $div\,\mathbf{F}, curl\,\mathbf{F}$.

4 (a) Evaluate $\oint_C (\cos x \sin y - xy)dx + \sin x \cos y\, dy$, by Green's theorem, where C is the circle $x^2 + y^2 = 1$.

OR

(b) Compute $\oint_S (ax^2 + by^2 + cz^2)\, dS$ over the sphere $x^2 + y^2 + z^2 = 1$ by using Gauss's divergence theorem.

Written exam	**: 30 Marks**
For record	**: 10 Marks**
For viva-voce	**: 10 Marks**
Total marks	**: 50 Marks**