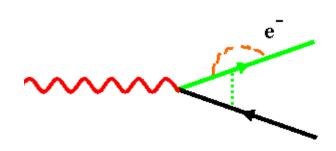
III. Inelastic losses and many-body effects in x-ray spectra

J. J. Rehr







Inelastic losses and many-body effects in x-ray spectra

TALK:

I. Introduction

Many-body effects in XAS

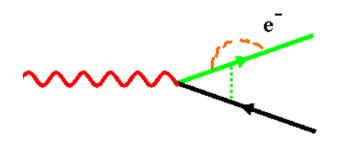
II. Inelastic losses& satellites

Cumulant expansion beyond GW

III. Particle-hole theory: BSE Particle-hole cumulant

Intrinsic, extrinsic losses and interference

I. Introduction:



Key many-body effects

Core-hole effects -----

Excitonic effects, Screening

• Self-energy $\Sigma(E)$ ----

Mean-free path, energy shifts

Phonons, disorder

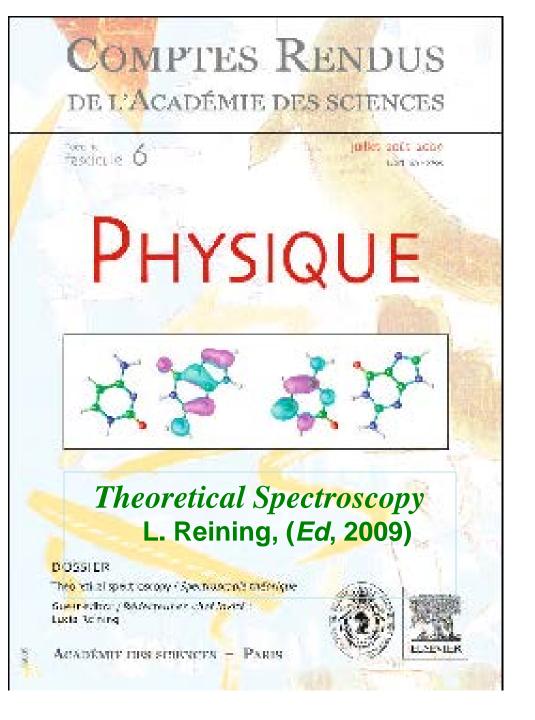
Debye-Waller factors

Excitations ---

Inelastic losses & satellites

"You can judge a many-body theory by how it treats the satellites."

Lars Hedin (1995)

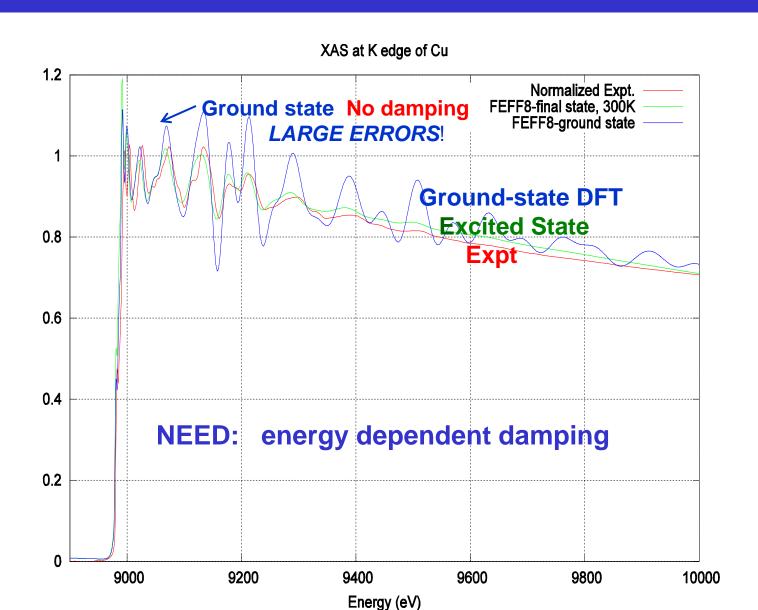


Mini-review

Quasi-particletheory of XAS

JJR et al., Comptes Rendus Physique **10**, 548 (2009)

Motivation: Failure of ground-state DFT in XAS; need for inelastic losses



Starting point for core-XAS calculations: Quasi-particle final state Green's function

Golden rule for XAS via Wave functions

$$\mu(E) \sim \Sigma_f |\langle i|\hat{\epsilon} \cdot \mathbf{r}|f\rangle|^2 \delta(E - E_f)$$



Paradigm shift:

Golden rule via Green's Functions $G = 1/(E - h' - \Sigma)$

$$\mu(E) \sim -\frac{1}{\pi} \text{Im} \langle i | \hat{\epsilon} \cdot \mathbf{r}' \, G(\mathbf{r}', \mathbf{r}, E) \, \hat{\epsilon} \cdot \mathbf{r} | i \rangle$$

Final state h' includes core-hole **AND** energy dependent self energy $\Sigma(E)$

Many-pole GW Self-energy $\Sigma(E)^*$

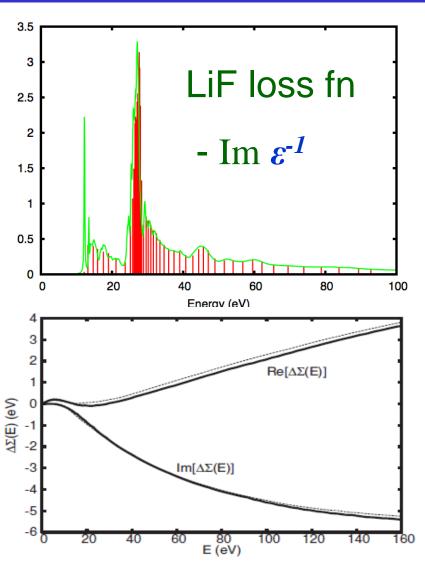
Efficient GW approximation for "Extrinsic Losses"

Sum of plasmon-pole models matched to loss function

$$\Sigma(E) = iGW = \Sigma' - i\Gamma$$

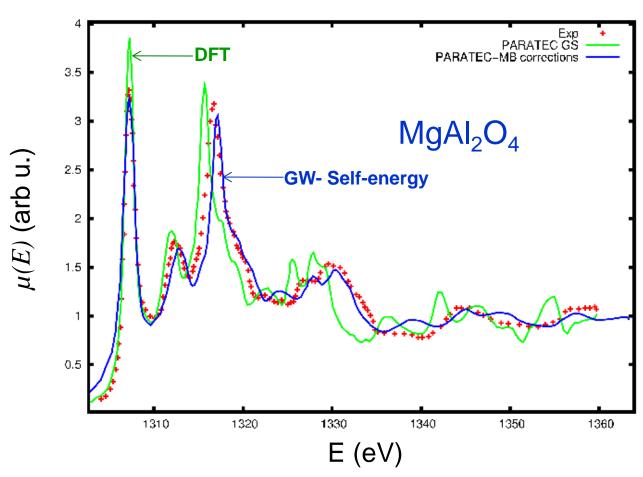
$$W = \varepsilon^{-1} v$$

Extension of Hedin-Lundqvist GW plasmon-pole model



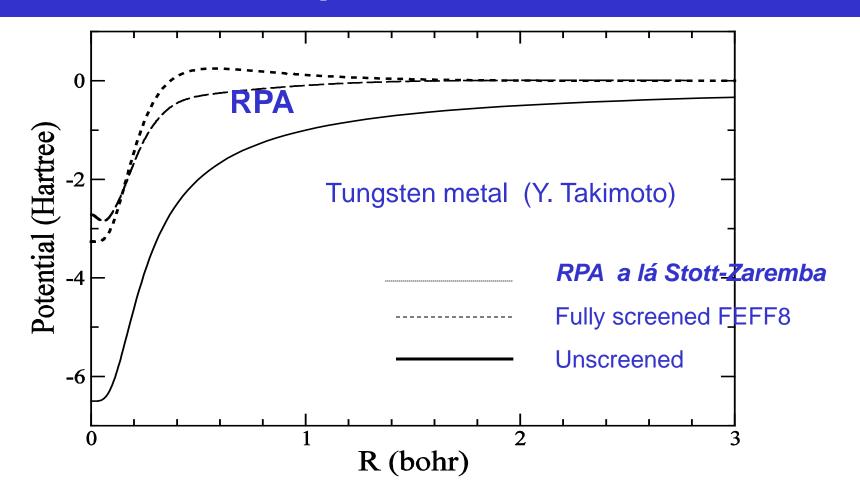
*J.J. Kas et. al, Phys Rev B **76**, 195116 (2007)

Self-energy fixes systematic shifts & broadening due to self-energy in XAS



*J. J. Kas, J. Vinson, N. Trcera, D. Cabaret, E. L. Shirley, and J. J. Rehr, Journal of Physics: Conference Series **190**, 012009 (2009)

Core-hole potential - RPA W



cf. Screened core hole W in Bethe-Salpeter Eq Improves on final state rule, Z+1, half-core hole

Phonon effects: Debye Waller factors in XAS

An Initio Determination of Extended X-Ray Absorption Fine Structure Debye-Waller Factors*



Fernando D. Vila, G. Shu, and John J. Rehr Department of Physics, University of Washington, Seattle, WA 98195



H. H. Rossner and H. J. Krappe Hahn-Meitner-Institut Berlin, Glienicker Strasse 100, D-14109 Berlin, Germany (Dated: August 23, 2005)

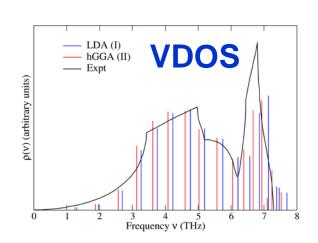
$$\sigma^2 = \frac{\hbar}{\mu_i} \int_0^\infty \rho(\omega^2) \coth \frac{\beta \hbar \omega}{2} d\omega$$

$$\rho(\omega^2) = \langle Q_i | \delta(\omega^2 - D) | Q_i \rangle$$

$$= \{6 - \text{step Lanczos recursion}\}$$

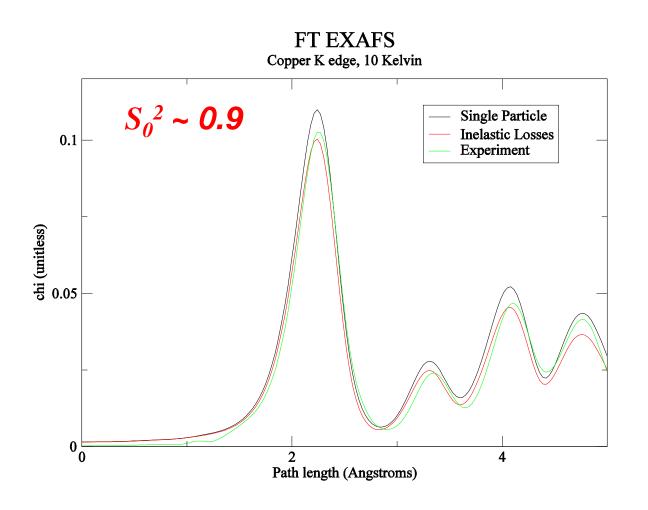
D dynamical matrix < ABINIT</p>

Many pole model for phonons



*Phys. Rev. B **76**, 014301 (2007)

PROBLEM: Amplitude discrepancy in EXAFS Observed fine structure smaller than QP theory



II. Inelastic losses and satellites

Q How to treat losses *beyond* the GW-quasi-particle approximation ?

Approach: Improved Green's function G(E) including satellites in spectral function

$$A(\omega) = (1/\pi) \operatorname{Im} G(E)$$

Two methods: GW + Dyson Eq.

Cumulant expansion

Which Green's function?

GW + Dyson vs **Cumulant***

GW

$$G(\omega) = G_0 + G_0 \Sigma G$$

$$\Sigma^{GW} = iGW$$

$$W = \epsilon^{-1}v$$

No vertex $\Gamma = 1$

Cumulant

$$G(t) = G_0(t) e^{C(t)}$$

$$C \sim /\text{Im } \Sigma^{GW} /$$

Implicit vertex

^{*}Recent review and new derivation, see J. Zhou et al. J. Chem. Phys. 143, 184109 (2015).

Answer: XPS expt: Cumulant wins

Phys Rev Lett **77**, 2268 (1996)

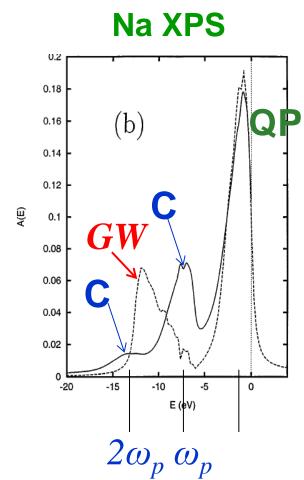
Multiple Plasmon Satellites in Na and Al Spectral Functions from *Ab Initio* Cumulant Expansion

F. Aryasetiawan, 1,2 L. Hedin, 1 and K. Karlsson³

Quasi-particle peaks of **both GW** and **C** agree with XPS expt

GW fails for satellites: only **one** satellite at wrong energy

Cumulant model agrees with experiment: multiple satellites ω_p apart



Reviews & references for cumulant Green's fn

J. Phys.: Condens. Matter 11 (1999) R489–R528.

On correlation effects in electron spectroscopies and the GW approximation

Lars Hedin

Department of Theoretical Physics, Lund University, Sölvegatan 14A, 223 62 Lund, Sweden

THE JOURNAL OF CHEMICAL PHYSICS 143, 184109 (2015)

Dynamical effects in electron spectroscopy

Jianqiang Sky Zhou,^{1,2,a)} J. J. Kas,^{2,3} Lorenzo Sponza,⁴ Igor Reshetnyak,^{1,2} Matteo Guzzo,⁵ Christine Giorgetti,^{1,2} Matteo Gatti,^{1,2,6} Francesco Sottile,^{1,2} J. J. Rehr,^{2,3} and Lucia Reining^{1,2}

¹Laboratoire des Solides Irradiés, École Polytechnique, CNRS, CEA-DSM-IRAMIS, Université Paris-Saclay, F-91128 Palaiseau, France

²European Theoretical Spectroscopy Facility (ETSF)

³Department of Physics, University of Washington, Seattle, Washington 98195-1560, USA

⁴Department of Physics, King's College London, London WC2R 2LS, United Kingdom

⁵Institut für Physik und IRIS Adlershof, Humboldt-Universität zu Berlin, D-12489 Berlin, Germany

⁶Synchrotron SOLEIL, L'Orme des Merisiers, Saint-Aubin, BP 48, F-91192 Gif-sur-Yvette, France

Why does it work: Quasi-boson approximation

IDEA: Neutral excitations - plasmons, phonons, etc. can be represented as **bosons**

Theorem:* Cumulant representation of core-hole Green's function is EXACT for electrons coupled to bosons *D. C. Langreth, *Phys. Rev.* B **1,** 471 (1970)

Corollary: also valid for valence with recoil approximation.

Physics:** GW approximation describes an electronic-polaron: electrons coupled to density fluctuations modeled as bosons

B. I. Lundqvist, *Phys. Kondens. Mater.* **6 193 (1967)

Cumulant expansion properties

$$G_k(t) = e^{i\epsilon_k^0 t} e^{C(t)}$$

$$C(t) = \int d\omega' \beta(\omega') \frac{e^{i\omega't} - i\omega't - 1}{\int_{\omega'}^{2}}$$
Landau formula for $C(t)$

Excitation spectra (GW
$$\Sigma$$
) $\beta_k(\omega) = \frac{1}{\pi} |\text{Im } \Sigma_k(\omega + \epsilon_k)|$

Spectral Function

$$A_k(\omega) = \int \frac{dt}{2\pi} e^{i(\omega - \epsilon_k)t} \exp\left\{ \int d\omega' \beta(\omega') \frac{e^{i\omega't} - i\omega't - 1}{{\omega'}^2} \right\}$$

O. Gunnarsson et al., Phys. Rev. B 50, 10462 (1994)

^{*}For diagrammatic expansion of higher order terms, see e.g.

cf. Retarded Cumulant Approximation*

PHYSICAL REVIEW B 90, 085112 (2014)

Cumulant expansion of the retarded one-electron Green function

J. J. Kas,^{1,*} J. J. Rehr,^{1,2,†} and L. Reining^{3,2,‡}

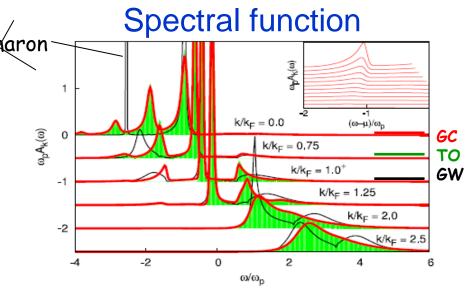
¹Department of Physics, University of Washington, Seattle, Washington 98195, USA

²European Theoretical Spectroscopy Facility (ETSF)

³Laboratoire des Solides Irradiés, École Polytechnique, CNRS, CEA-DSM, F-91128 Palaiseau, France

Retarded GF formalism plasmaron

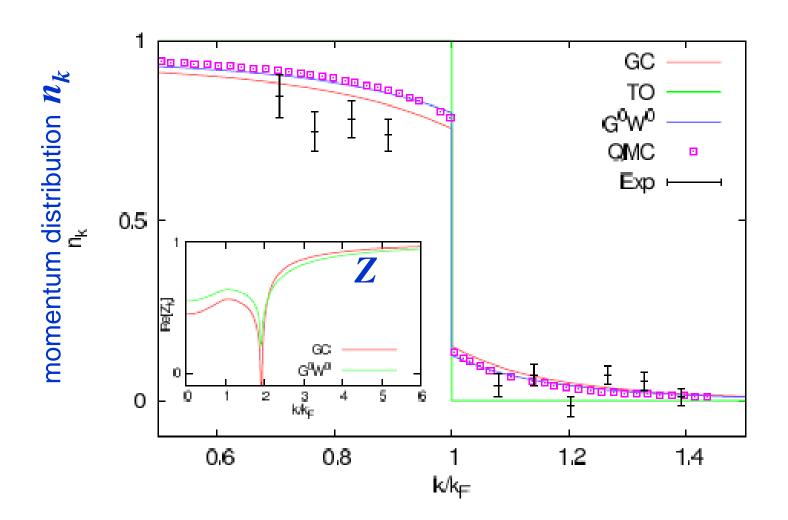
 $G_k^R(t) = -i\theta(t)e^{-i\epsilon_k^{HF}t}e^{\tilde{C}_k^R(t)},$ $\tilde{C}_k^R(t) = \int d\omega \frac{\beta_k(\omega)}{\omega^2} (e^{-i\omega t} + i\omega t - 1),$ $\beta_k(\omega) = \frac{1}{\pi} \left| \operatorname{Im} \Sigma_k^R(\omega + \epsilon_k) \right|,$



Retarded cumulant builds in particle-hole symmetry

Electron-gas quasi-particle properties

Retarded cumulant has good n_k and Z, & pretty good correlation energies



Example: Multiple satellites in XPS of Si

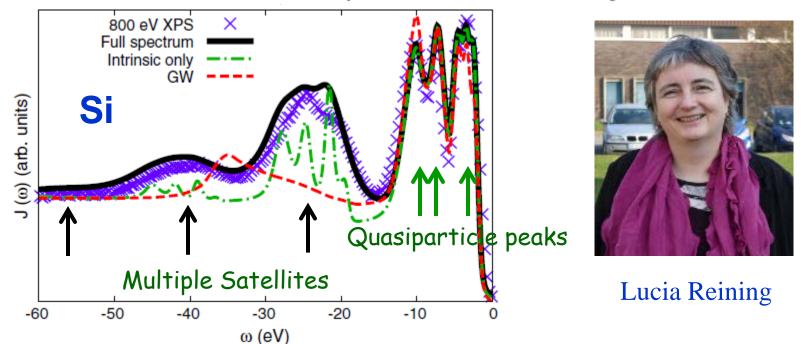
PRL 107, 166401 (2011)

PHYSICAL REVIEW LETTERS

week ending 14 OCTOBER 2011

Valence Electron Photoemission Spectrum of Semiconductors: *Ab Initio*Description of Multiple Satellites

Matteo Guzzo,^{1,2,*} Giovanna Lani,^{1,2} Francesco Sottile,^{1,2} Pina Romaniello,^{3,2} Matteo Gatti,^{4,2} Joshua J. Kas,⁵ John J. Rehr,^{5,2} Mathieu G. Silly,⁶ Fausto Sirotti,⁶ and Lucia Reining^{1,2,†}

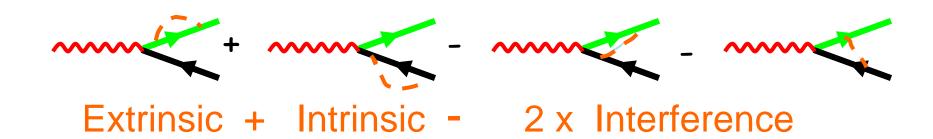


Problems GW: only one broad satellite at wrong position C: position ok but intensity too small

III. Particle-hole cumulant theory

Q: How to calculate *all* inelastic losses and satellites in x-ray spectra?

Problem: Single-particle cumulant in XPS (or XAS) only has intrinsic (or extrinsic) losses and ignores interference: Need to include all losses.



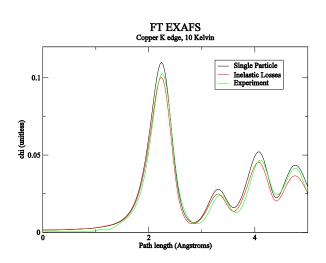
Hedin suggestion: quasi-boson method with intrinsic, extrinsic and interference

Explanation of XAFS many-body amplitude factor: $\chi_{exp} = \chi_{th} * S_0^2$

EXTRINSIC AND INTRINSIC PROCESSES IN EXAFS
Lars Hedin, Dept of Theoretical Physics, University of Lund, Sweden

Physica B 158 (1989) 344-346 North-Holland, Amsterdam

The importance of correlation effects in spectroscopies like EXAFS and photoemission is well recognized. The two main mechanisms are shake-off (in which we include skake-up) when the photoelectron is created, and energy loss of the propagating electron. Shake-off is clearly impossible at threshold, due to lack of energy. For photoemission often the "Spicer three-step model" is used, (1) creation of the photoelectron (including shake-off), (2) propagation to the surface (including losses), and (3) passage through the surface (including losses). Langreth [1] has pointed out that one should add the amplitudes for (2) and (3), and not, as in the Spicer model, convolute their squares, the probabilities. This effect is important primarely at threshold.



*J.J. Rehr, E.A. Stern, R.L. Martin, and E.R. Davidson, Phys. Rev. B 17,560 (1978)

Starting point: GW/BSE Particle-hole Green's function w/o satellites

$$-\operatorname{Im} \epsilon^{-1}(\mathbf{q}, \omega) = \frac{4\pi}{q^2} \operatorname{Im} \langle \Psi_0 | \hat{D}^{\dagger} \frac{1}{E_0 + \omega - \hat{H} + i\gamma} \hat{D} | \Psi_0 \rangle$$

Ingredients: Particle-Hole Hamiltonian

$$H=h_e$$
 - h_h + V_{eh} $h_{e/h}=\varepsilon_{nk}+\Sigma_{nk}$ Σ GW self-energy $V_{eh}=V_x+W$ Particle-hole interaction

OCEAN: core-level GW/BSE code

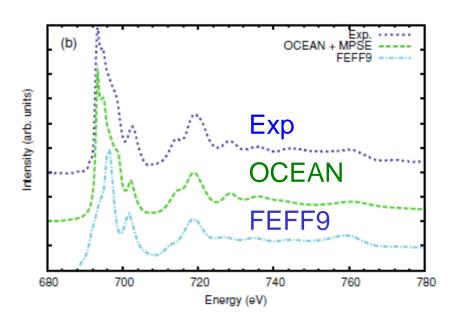
PHYSICAL REVIEW B 83, 115106 (2011)

Bethe-Salpeter equation calculations of core excitation spectra

J. Vinson, J. J. Rehr, and J. J. Kas

Department of Physics, University of Washington, Seattle, Washington 98195, USA

E. L. Shirley
National Institute of Standards and Technology (NIST), Gaithersburg, Maryland 20899, USA



*Obtaining Core Excitations from ABINIT and NBSE

PW-PP + PAW + MPSE + NBSE

*J. Vinson et al. Phys. Rev. B83, 115106 (2011)

Particle-hole cumulant in XPS*

Europhys J. B **85**, 324 (2012)

Plasmon Satellites in Valence-band Photoemission Spectroscopy

Ab Initio study of the photon-energy dependence in semiconductors

Matteo Guzzo^{1,2}, Joshua J. Kas³, Francesco Sottile^{1,2}, Mathieu G. Silly⁴, Fausto Sirotti⁴, John J. Rehr³, and Lucia Reining^{1,2}

$$\langle J_k(\omega) \rangle = \sum_i |M_{ik_0}|^2 \int_0^\infty e^{-a} \int_{-\infty}^\infty e^{i(\omega_0 - \epsilon_k + \epsilon_i)t}$$
$$\times \exp\left[\int \gamma_{ik}(\omega)(e^{-i\omega t} - 1) \ d\omega\right] \ dt \ dz_c$$

Kernel $\gamma(\omega)$ with extrinsic, intrinsic and interference terms

$$\gamma_{ik}(\omega) = \sum_{\mathbf{q}} |g_{\mathbf{q}}|^2 \delta(\omega - \omega_q) = \gamma_i^{int} + \gamma_k^{ext} + \gamma_{ik}^{inf}$$

*L. Hedin, J. Michiels, and J. Inglesfield, Phys. Rev. B 58, 15 565 (1998).

Quasi-boson method for particle-hole GF*

• Excitations: $H_v = \Sigma_n \omega_n a_n^{\dagger} a_n$

 $V^n \to -\mathrm{Im} \ \varepsilon^{-1}(\omega_n, q_n)$

• Electrons: $h' = \sum_{k} \epsilon_k c_k^{\dagger} c_k$

- fluctuation potentials*
- \bullet e-boson coupling $V_{pv}=\Sigma_{nkk'}\left[V_{kk'}^{n}a_{n}^{\dagger}+(V_{kk'}^{n})^{*}a_{n}\right]c_{k}^{\dagger}c_{k'}$
- Core-hole-boson coupling: $V_{vc} = -\sum_n V_{bb}^n \left(a_n^\dagger + a_n \right)$

Partition contributions into Intrinsic + Extrinsic + Interference

$$\gamma_K(\omega) = \sum_{q} \left| V^q \tilde{g}(\omega - \omega_q) - \frac{V_{cc}^q}{\omega_q} \right|^2 \delta(\omega - \omega_q) = \gamma_c(\omega) + \gamma_k(\omega) + \gamma_{ck}(\omega)$$

* L. Hedin, J. Michiels, and J. Inglesfield, Phys. Rev. B 58, 15 565 (1998)

Example: Satellites in XPS of Si again

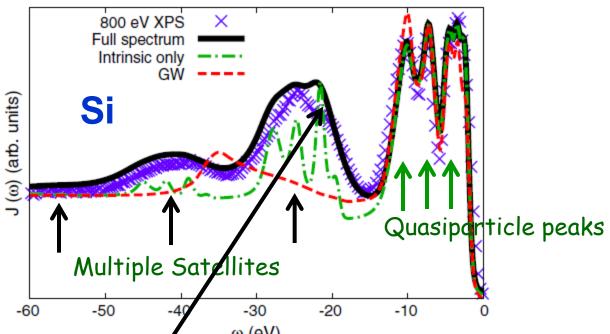
PRL 107, 166401 (2011)

PHYSICAL REVIEW LETTERS

week ending 14 OCTOBER 2011

Valence Electron Photoemission Spectrum of Semiconductors: *Ab Initio*Description of Multiple Satellites

Matteo Guzzo,^{1,2,*} Giovanna Lani,^{1,2} Francesco Sottile,^{1,2} Pina Romaniello,^{3,2} Matteo Gatti,^{4,2} Joshua J. Kas,⁵ John J. Rehr,^{5,2} Mathieu G. Silly,⁶ Fausto Sirotti,⁶ and Lucia Reining^{1,2,†}



Success for particle-hole cumulant: good agreement only if extrinsic and interference terms are included

Particle-hole cumulant in XAS*

PHYSICAL REVIEW B 94, 035156 (2016)

Particle-hole cumulant approach for inelastic losses in x-ray spectra

J. J. Kas, J. J. Rehr, and J. B. Curtis²

¹Department of Physics, University of Washington, Seattle, Washington 98195-1560, USA ²Department of Physics, University of Rochester, Rochester, New York 14927, USA

$$\tilde{G}_K(t) = \tilde{G}_K^0(t)e^{\tilde{C}_K(t)}$$

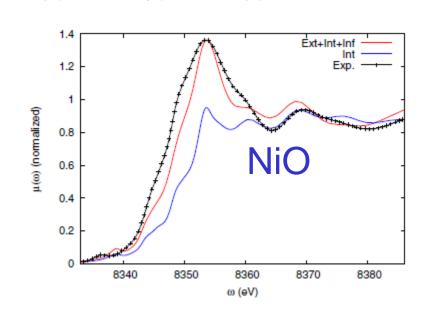
$$\tilde{C}_K(t) = \int d\omega \, \gamma_K(\omega) (e^{i\omega t} - i\omega t - 1)$$

$$\tilde{C}_K(t) = C_c(t) + C_k(t) + C_{ck}$$

All losses in particle-hole spectral function A_K

$$\mu(\omega) = \int d\omega' \, \tilde{A}_K(\omega') \mu_K(\omega - \omega')$$

* cf. L. Campbell, L. Hedin, J. J. Rehr, and W. Bardyszewski, Phys. Rev. B **65**, 064107 (2002)



Theory of many-body amplitude factor

Many-body XAS ≈ Convolution

$$\mu(\omega) = \int_0^\infty d\omega' \tilde{A}(\omega, \omega') \mu_{qp}(\omega - \omega')$$
$$\equiv \langle \mu_{qp}(\omega) \rangle \approx \mu_{qp}(\omega) S_0^2(\omega)$$

• Explains crossover: **adiabatic** $S_{\theta}^{2}(\omega) = 1$

to sudden transition $S_0^2(\omega) \approx 0.9$

$$|g_q|^2 = |g_q^{ext}|^2 + |g_q^{intrin}|^2 \cdot 2 g_q^{ext} g_q^{intrin}$$

Interference reduces loss!

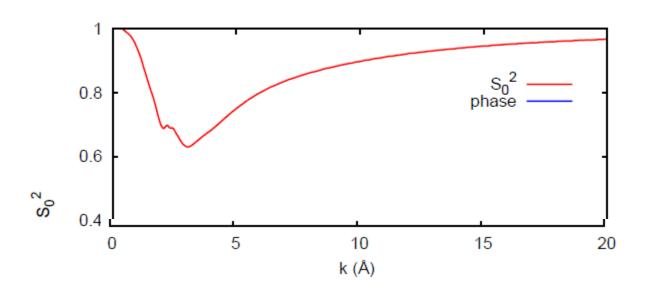
Many-body amplitude factor S_{θ}^{2}

MS Nano Proceedings, Springer (in press 2017)

Cumulant approach for inelastic losses in x-ray spectra

John J. Rehr and Joshua J. Kas

$$S_0^2(R) = \int_0^\omega d\omega' \tilde{A}(\omega, \omega') e^{i2[k(\omega - \omega') - k(\omega)]R}$$



Intrinsic losses: real-time TDDFT cumulant

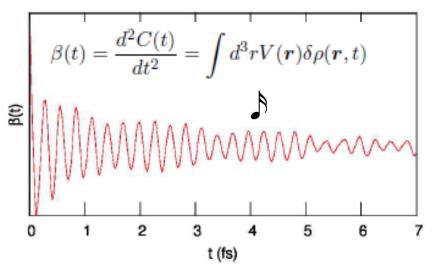
PHYSICAL REVIEW B 91, 121112(R) (2015)

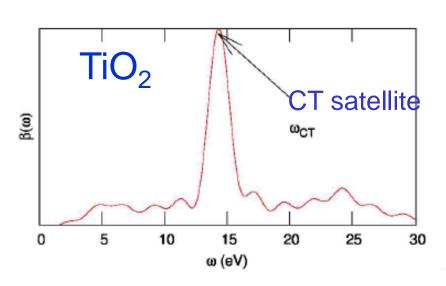
Real-time cumulant approach for charge-transfer satellites in x-ray photoemission spectra

J. J. Kas, F. D. Vila, J. J. Rehr, and S. A. Chambers Department of Physics, University of Washington, Seattle, Washington 98195-1560, USA Physical Sciences Division, Pacific Northwest National Laboratory, Richland, Washington 99352, USA

Langreth cumulant in time-domain*

$$C(t) = \sum_{\boldsymbol{q},\boldsymbol{q'}} V_{\boldsymbol{q}}^* V_{\boldsymbol{q'}} \int d\omega S(\boldsymbol{q},\boldsymbol{q'},\omega) \frac{e^{i\omega t} - i\omega t - 1}{\omega^2} = \int d\omega \beta(\omega) \frac{e^{i\omega t} - i\omega t - 1}{\omega^2}$$

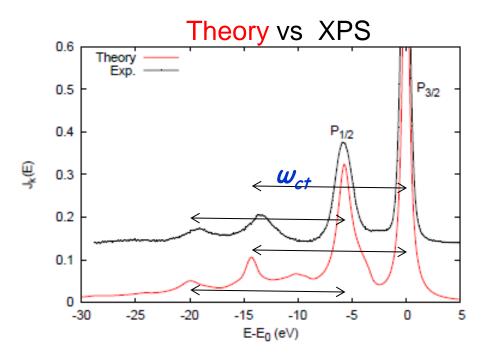




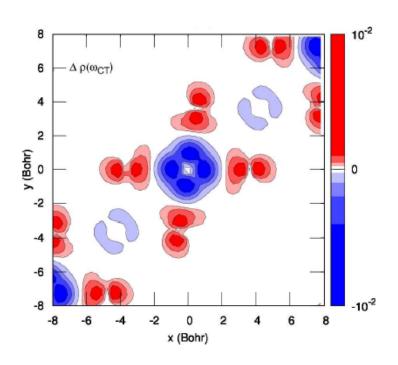
*D. C. Langreth, Phys. Rev. B 1, 471 (1970)

Satellites and real-space interpretation

RT TDDFT Cumulant

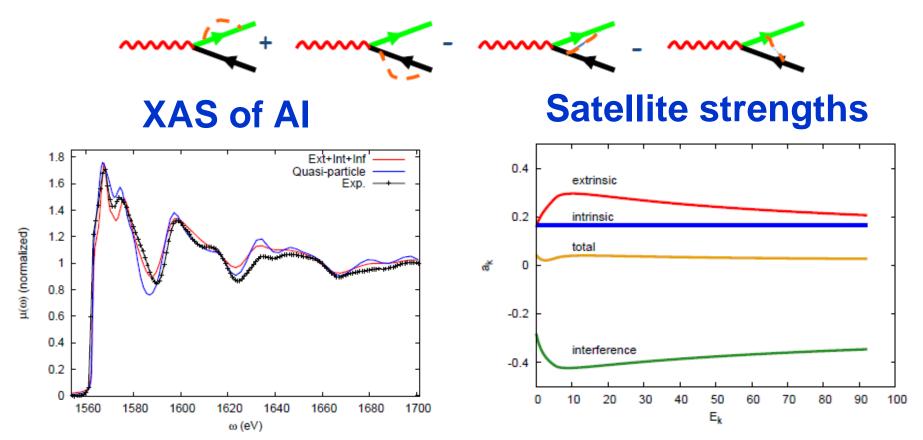


Charge transfer fluctuations



Interpretation: satellites arise from charge density fluctuations between ligand and metal at frequency $\sim \omega_{c\tau}$ due to suddenly turned-on core-hole

Extrinsic, intrinsic and interference terms



Particle-hole cumulant explains cancellation of extrinsic and intrinsic losses at threshold and

crossover: adiabatic

to sudden approximation

Examples: high accuracy XPS and XAS

Phys Rev B (in press, 2017)

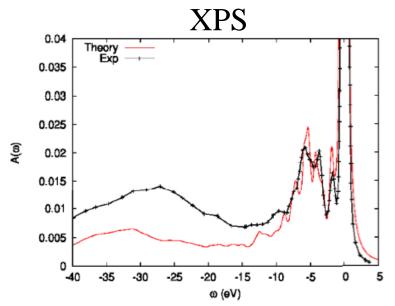
High-resolution valence and core excitation spectra of solid C60

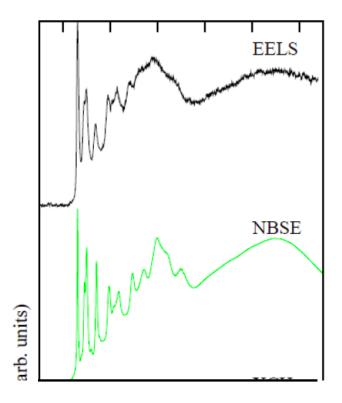
via first-principles calculations and experiment

F. Fossard, K. Gilmore, G. Hug, J J. Kas, J J Rehr, E L Shirley and F D Vila

Particle-hole cumulant

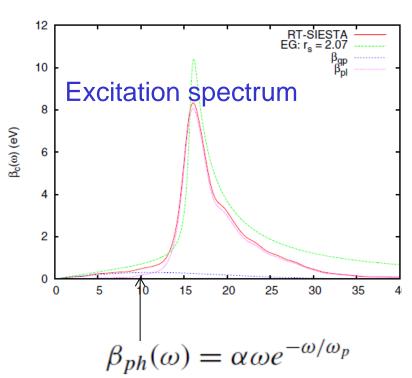
RT-TDDFT intrinsic cumulant





X-ray Edge Singularities in metals

Low energy particle-hole excitations in cumulant **explain** edge singularities in XPS and XAS of metals



$$C_{ph}(t) = -i\alpha\omega_p t - \alpha \ln(1 - i\omega_p t) \qquad A_{ph}(\omega) = e^{-a_{pl}} \frac{e^{-\tilde{\omega}/\omega_p}}{\Gamma(\alpha)} \frac{\omega_p^{-\alpha}}{\tilde{\omega}^{1-\alpha}}$$

cf Doniach-Sunjic line-shape in XPS

Correlated systems

Standard approximation: Hubbard-model

PHYSICAL REVIEW B 85, 165123 (2012)

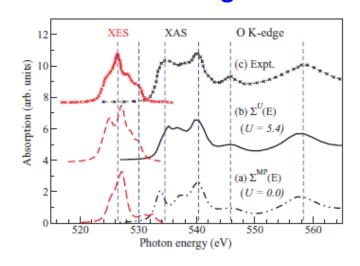
Hubbard model corrections in real-space x-ray spectroscopy theory

Towfiq Ahmed, J. J. Kas, and J. J. Rehr

O K-edge MnO

Hubbard *U* as self-energy correction

$$V^{U}(\mathbf{r}, E) = V^{SCF}(\mathbf{r}) + \Sigma^{GW}(E) + \Sigma^{U}_{lm\sigma}(E)$$



cf. H. Jiang, Rinke et al. Phys. Rev. B 82, 045108 (2010).

Alternative approach: cumulant

Question: Does the particle-hole cumulant method work for correlated *d*- and *f*- systems?

Hedin's answer * MAYBE

"Calculation similar to core case ... but with more complicated fluctuation potentials ...

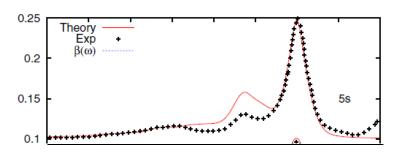
$$V^n \to -\mathrm{Im} \ \varepsilon^{-1}(\omega_n, q_n)$$

... not question of principle, but of computational work..."

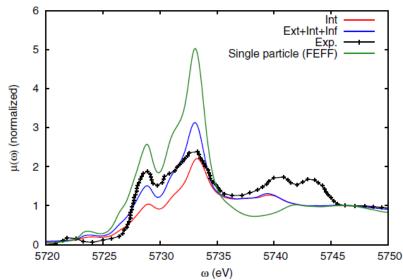
* L. Hedin, J. Phys.: Condens. Matter **11**, R489 (1999)

Particle-hole cumulant for CeO₂*

Ce 5s XPS of CeO₂

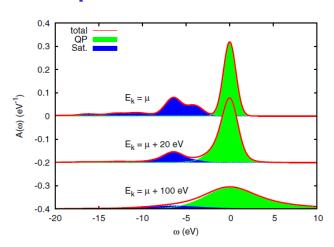


Ce L₃ XAS of CeO₂

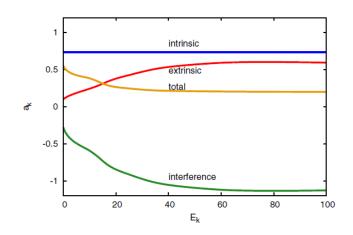


*J. Kas et al. Phys Rev B 94, 035156 (2016)

Spectral function



Spectral weights



Conclusions

Many-body corrections including self-energy shifts, and inelastic losses, and Debye-Waller factors are essential for quantitative agreement with experimental x-ray spectra

Particle-hole cumulant theory approximation can explain all losses (extrinsic, intrinsic and interference) in x-ray spectra.

All losses can be lumped into a spectral function $A_{\it K}(\omega)$ AND can be added ex post facto

$$\mu(\omega) = \int d\omega' \, \tilde{A}_K(\omega') \mu_K(\omega - \omega')$$

Acknowledgments

Supported by DOE BSE DE-FG02-97ER45623

Thanks to

J.J. Kas

J. Vinson

T. Fujikawa

S. Story

M. Verstraete et al.

L. Reining G. Bertsch

K. Gilmore L. Campbell

F. Vila E. Shirley

S. Biermann M Guzzo

J. Sky Zhou C. Draxl

& especially the ETSF