EL5123 --- Image Processing

Image Restoration

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Partly based on
A. K. Jain, Fundamentals of Digital Image Processing, and
Gonzalez/Woods, Digital Image Processing
Figures from Gonzalez/Woods, Digital Image Processing

Lecture Outline

- Introduction
- Image degradation model
  - Blurring caused by finite camera exposure
  - Blurring caused by motion
- Inverse filtering
- Wiener filter

Examples

FIGURE 5.29 (a) 8-bit image corrupted by motion blur and additive noise. (b) Result of inverse filtering. (c) Result of Wiener filtering. (d)–(f) Same sequence, but with noise variance one order of magnitude less. (g)–(i) Same sequence, but noise variance reduced by five orders of magnitude from (a). Note in (h) how the deblurred image is quite visible through a "curtain" of noise.

Degradation Model

- General model
  - $g(x,y) = T(f(x,y)) + n(x,y)$
  - T( ) may not be linear

- Modeling T ( ) by a filtering operation
  - $g(x,y) = f(x,y) * h(x,y) + n(x,y)$
  - This means that the degradation operator is linear and shift invariant
  - h(x,y) is unknown and needs to be estimated (system identification problem)

- Given h(x,y) and some statistics of n(x,y), how to recover f(x,y) from g(x,y)?
- How to estimate h(x,y) and statistics of n(x,y)?

Degradation due to Finite Size Sensor

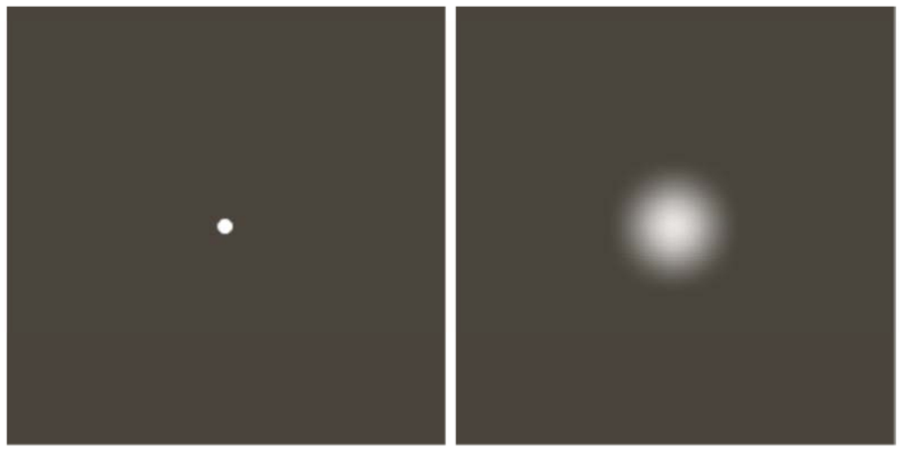
- Ideally the value of a pixel should be the light intensity at a infinitesimal point in the imaged scene.
- Each sensor in a CCD array integrates the light intensity in a small area surrounding a point with possibly non-equal weighting
- The point spread function (PSF) is the image captured when there is only one single point with high intensity in the scene.
- This PSF is the degradation filter.
- Typically h(x,y) due to sensor PSF is low-pass and is often approximated by a Gaussian filter
  - h(x,y)= e^-k (x^2+y^2)

Typical PSF

- Figure 5.24

a b

FIGURE 5.24 Degradation estimation by impulse characterization. (a) An impulse of light (shown magnified). (b) Imaged (degraded) impulse.



Degradation Due to Motion Blur

- The sensor integrates the intensity value of a point over a certain exposure time T
- When there are moving objects in the imaged scene at high speed (relative to exposure time), we see motion blur.
- To reduce the motion blur, one can reduce the exposure time

a b

FIGURE 5.26 (a) Original image. (b) Result of blurring using the function in Eq. (5.6-11) with $a = b = 0.1$ and $T = 1$.

What is the h(x,y) corresponding to motion blur?

- Suppose the sensor has infinitely small aperture, but an exposure time of T
- The object is moving with speed of v_x and v_y
- Derive in class
- Also see textbook

What if the sensor also has a non-zero aperture

- Derive in class

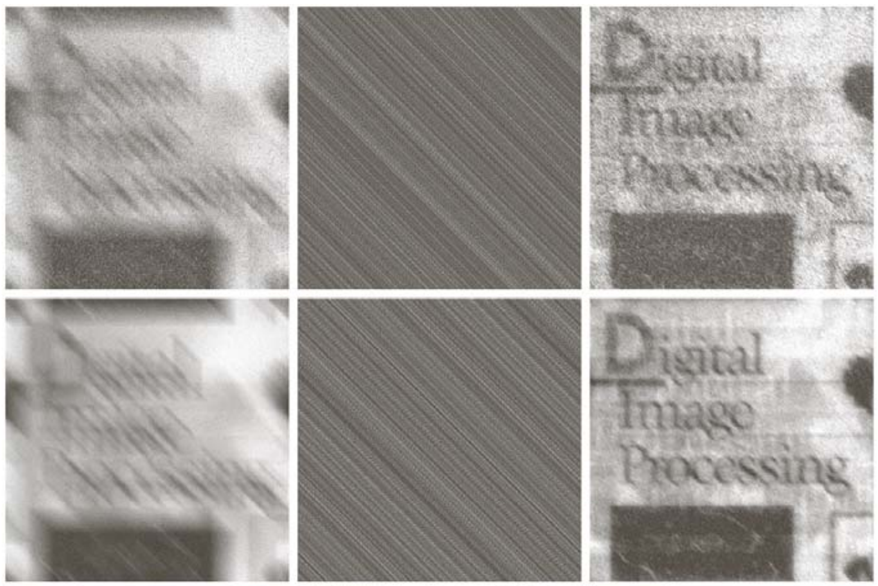
Restoration Methods: Inverse Filter

- Recall the degradation model
  - g(x,y)=h(x,y)*f(x,y)+n(x,y)
  - G(u,v) = H(u,v) F(u,v) + N(u,v)
- Ignoring the noise term
  - G(u,v) ~ H(u,v) F(u,v)
  - \hat F(u,v) = G(u,v) / H(u,v) = Q(u,v) G(u,v)
    - With Q(u,v) = 1/H(u,v) (Inverse Filter)
  - \hat f(x,y) = q(x,y) * g(x,y)
    - q(x,y)* h(x,y)=delta(x,y)

What is the problem of inverse filter

\hat F(u,v) = G(u,v) / H(u,v) = F(u,v)+N(u,v)/H(u,v)

If H(u,v)~ =0 for some u,v, then the noise all be amplified.



a b c
d e f
g h i

FIGURE 5.29 (a) 8-bit image corrupted by motion blur and additive noise. (b) Result of inverse filtering. (c) Result of Wiener filtering. (d)–(f) Same sequence, but with noise variance one order of magnitude less. (g)–(i) Same sequence, but noise variance reduced by five orders of magnitude from (a). Note in (h) how the deblurred image is quite visible through a "curtain" of noise.

Simple Fix of the Inverse Filter Problem

- Assuming the original image is bandlimited up to a certain maximum bandwidth, B
- If $H(u,v) \neq 0$ within the bandwidth
- Then
  - $\hat F(u,v) = \{ G(u,v)/H(u,v),\ u^2+v^2 \le B^2,$
  - $0,$ otherwise

a b c

FIGURE 5.28 Comparison of inverse and Wiener filtering. (a) Result of full inverse filtering of Fig. 5.25(b). (b) Radially limited inverse filter result. (c) Wiener filter result.

Pseudo Inverse Filter

- Instead of cutting off the bandwidth of the inverse filter, identify frequency region where H(u,v) is close to zero, and set the filter (in frequency domain) to zero at those frequencies
  - Q(u,v) = { 1/H(u,v), if |H(u,v)|>=T,
  - 0 , otherwise
  - \hat F(u,v) = { G(u,v)/H(u,v), if |H(u,v)|>=T,
  - 0 , otherwise

MMSE (Wiener) Filter

- Determine q(x,y) or Q(u,v), such that the mean square error between original f(x,y) and estimated \hat f(x,y) is minimized
  - E= \sum_{x,y} (f(x,y)-\hat f(x,y))^2
- Solution (Wiener Filter)
  - Q(u,v) = H*(u,v) / (|H(u,v)|^2+S_n(u,v) / S_f(u,v))
  - See Jain pp.276-278 for derivation
  - Requires the knowledge of the power spectrum of the original signal and the noise.
    - S_f(u,v)= FT{R_f (s,t)}, R_f(s,t)=E{f(x,y) f*(x+s, y+t)}
    - S_n(u,v)= FT{R_n (s,t)}, R_n(s,t)=E{n(x,y) n*(x+s, y+t)}
  - Common Approximation:
    - Assuming S_n(u,v) / S_f(u,v)= K (constant), choose K to yield the best results (visually)

Sample Results

a b c
d e f
g h i

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Other Restoration Methods

- Constrained least square
- See textbook

What if motion is not homogeneous

- Different parts experience different motions
- Different degradation filter in different regions
- Estimate motion for each region separately
- Use corresponding deblurring filter in each region
- Use overlapping regions to avoid block artifacts

How to implement restoration filters?

- Typically, we can at best derive the desired filter in the frequency domain
- There is no closed form solution for the filter in the space domain
- Usually we implement the filter in the DFT domain
  - Must pad zeros to the input image and perform DFT on the zero-padded images.

Homework

1. Consider a digital camera, which samples the image plane with 1mmx1mm resolution, and produces the value of each pixel by averaging the light intensity over a square region of size 1mmx1mm to produce a pixel value. What is the degradation filter h(x,y)? What is its Fourier transform H(u,v)?
2. For the above degradation system, derive the inverse filter, and the bandlimitting inverse filter, and the pseudo inverse filter, and the Wiener filter. You only need to specify your filter in the frequency domain, and you can represent your solution in terms of parameters B, T, L.
3. Suppose a camera is taking pictures of an object moving with a constant speed of vx=10m/s, vy=20 m/s, and the camera exposure time is 1 ms, What is the equivalent degradation filter, both in frequency domain and in spatial domain?
4. Repeat the last problem, but assume the camera has a aperture size of 1mm x 1mm.

Reading

- R. Gonzalez, “Digital Image Processing,” Section 5.5-5.10
- Jain, Fundamentals of digital image processing, Sec. 8.1-8.8.